Computer algebra independent integration tests

1-Algebraic-functions/1.1-Binomial-products/1.1.3-General/1.1.3.3-a+b-x^n-^p-c+d-x^n-^q

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Contents

Chapter 1

Introduction

Local contents

This report gives the result of running the computer algebra independent integration test. Elementary Algebraic integrals version.

The download section below contains links to download the problems in plain text format used for all CAS systems.

The number of integrals in this report is [239]. This is test number [13].

1.1 Listing of CAS systems tested

The following systems were tested at this time.

1. Mathematica 12.3.1 (64 bit) on windows 10.
2. Rubi 4.16.1 in Mathematica 12.3.1 on windows 10.
3. Maple 2021.1 (64 bit) on windows 10.
4. Maxima 5.45 on Linux. (via sagemath 9.3)
5. Fricas 1.3.7 on Linux (via sagemath 9.3)
6. Giac/Xcas 1.7 on Linux. (via sagemath 9.3)
7. Sympy 1.8 under Python 3.8.8 using Anaconda distribution on Ubuntu.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 under windows 10 (64 bit)
9. `IntegrateAlgebraic` under Mathematica 12.3.1 on windows 10. `https://github.com/stblake/algebraic_integration`. September 15, 2021 version.

Maxima, Fricas and Giac/Xcas were called from inside SageMath. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (239)	0.00 (0)
Mathematica	100.00 (239)	0.00 (0)
Fricas	89.12 (213)	10.88 (26)
Maple	73.64 (176)	26.36 (63)
Mupad	70.29 (168)	29.71 (71)
Maxima	69.87 (167)	30.13 (72)
IntegrateAlgebraic	69.04 (165)	30.96 (74)
Giac	56.07 (134)	43.93 (105)
Sympy	47.28 (113)	% 52.72 (126)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

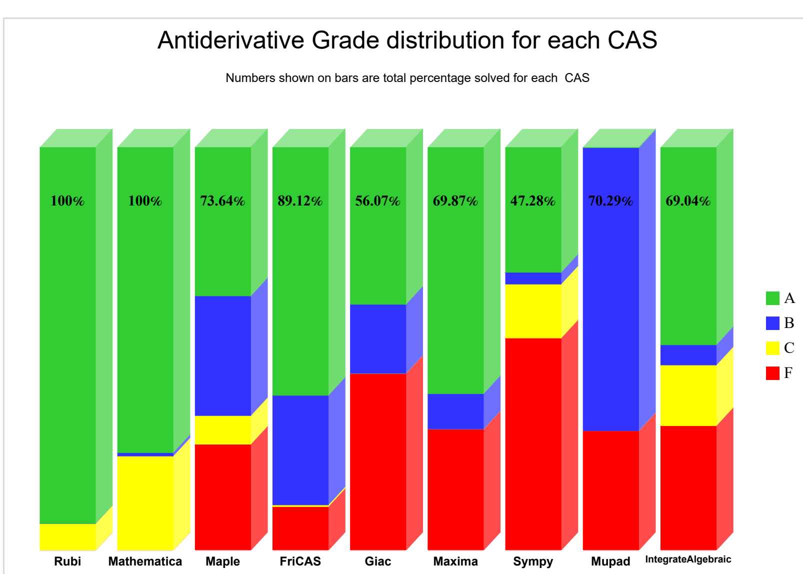
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

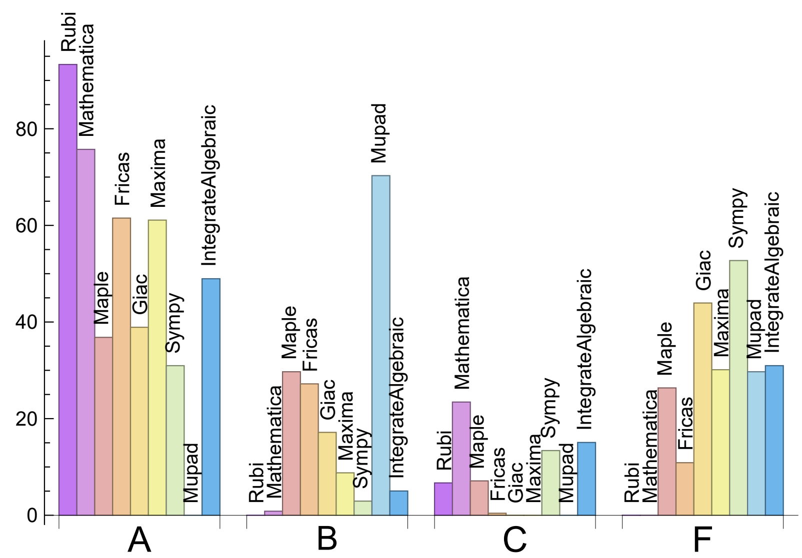
System	% A grade	% B grade	% C grade	% F grade
Rubi	93.31	0.00	6.69	0.00
Mathematica	75.73	0.84	23.43	0.00
Fricas	61.51	27.20	0.42	10.88
Maxima	61.09	8.79	0.00	30.13
IntegrateAlgebraic	48.95	5.02	15.06	30.96
Giac	38.91	17.15	0.00	43.93
Maple	36.82	29.71	7.11	26.36
Sympy	30.96	2.93	13.39	52.72
Mupad	N/A	70.29	0.00	29.71

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



The following table shows the distribution of the different types of failure for each CAS. There are 3 types of reasons why it can fail. The first is when CAS returns back the input within the time limit, which means it could not solve it. This the typical normal failure **F** .

The second is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned **F(-1)**.

The third is due to an exception generated. Assigned **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in CAS. This type of error requires more investigations to determine the cause.

System	Number failed	Percentage normal failure	Percentage time-out failure	Percentage exception failure
Rubi	0	0.00 %	0.00 %	0.00 %
Mathematica	0	0.00 %	0.00 %	0.00 %
Maple	63	98.41 %	1.59 %	0.00 %
Fricas	26	0.00 %	96.15 %	3.85 %
IntegrateAlgebraic	74	98.65 %	1.35 %	0.00 %
Giac	105	78.10 %	0.00 %	21.90 %
Maxima	72	100.00 %	0.00 %	0.00 %
Sympy	126	43.65 %	56.35 %	0.00 %
Mupad	71	100.00 %	0.00 %	0.00 %

Table 1.4: Failure statistics for each CAS

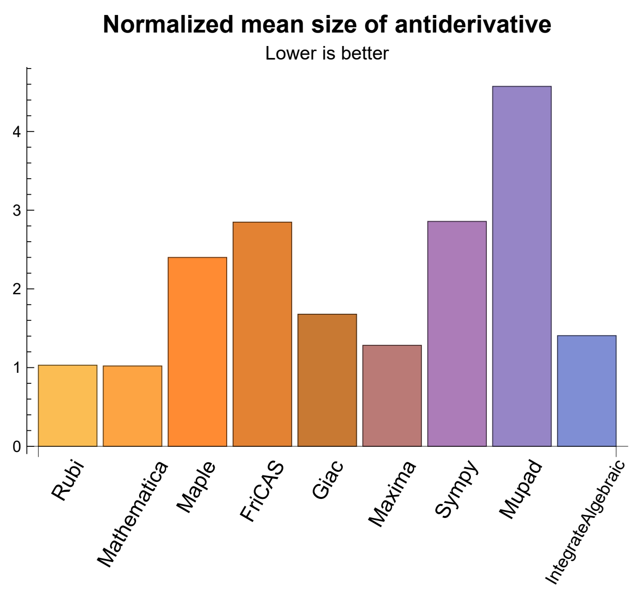
1.3 Performance

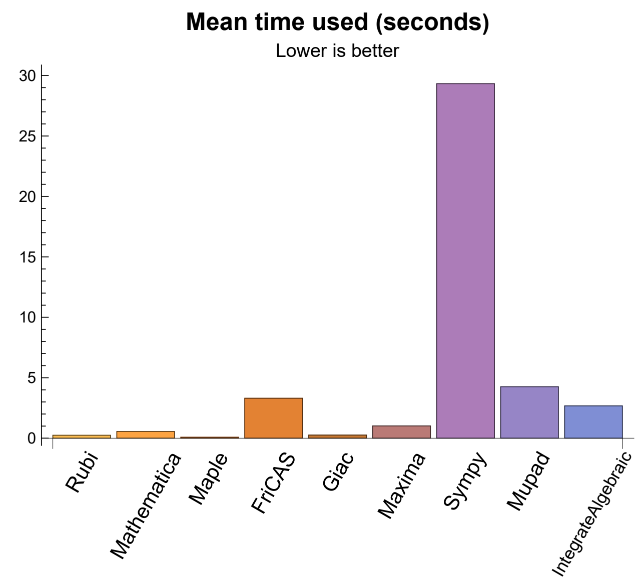
The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.24	170.02	1.03	122.00	1.00
Mathematica	0.55	168.35	1.02	117.00	0.94
Maple	0.08	420.53	2.40	189.50	1.47
Maxima	1.02	185.69	1.28	144.00	1.14
Fricas	3.30	553.31	2.85	231.00	2.10
Sympy	29.33	321.99	2.86	162.00	1.20
Giac	0.26	261.01	1.68	200.00	1.36
Mupad	4.26	1179.55	4.57	150.50	1.24
IntegrateAlgebraic	2.68	217.68	1.41	166.00	1.37

Table 1.5: Time and leaf size performance for each CAS

The following are bar charts for the normalized leafsize and time used columns from the above table.




Mean time used (seconds)
Lower is better
30
25
20
15
10
5
0
Rubi
Mathematica
Maple
FriCAS
Giac
Maxima
Sympy
Mupad
IntegrateAlgebraic

1.4 list of integrals that has no closed form antiderivative

{}

1.5 list of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

IntegrateAlgebraic {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {34, 61, 62, 63, 66, 67, 68, 69, 72, 73, 74, 75, 76, 80, 81}

Mathematica {34, 56, 61, 62, 63, 65, 66, 67, 68, 69, 70, 72, 73, 74, 75, 76, 77, 80, 81, 110, 111, 112, 113, 115, 116, 117, 118, 119, 120, 122, 123, 209}

IntegrateAlgebraic {}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

Mupad Verification phase not implemented yet.

1.7 Timing

The command `AboluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since these integrals are run in a batch mode, using an automated script, and by using sagemath (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 14 such integrals out of total 705, or about 2 percent. This pecrentage can be higher or lower depending on the specific input test file.

Such integrals can be indentified by looking at the output of the integration in each section for Maxima. The exception message will indicate of the error is due to the interactive question being asked or not.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/ for reference.

1.9.2 Important note about FriCAS and Giac/XCAS results

There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error `Exception raised: NotImplementedError`

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the buildin system function `LeafSize` is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special buildin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy and Giac antiderivatives is determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which is called directly from Python, the following code is used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
         leafCount =1
```

1.9.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative, Maple was used to determine the leaf size of Mupad output by post processing.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

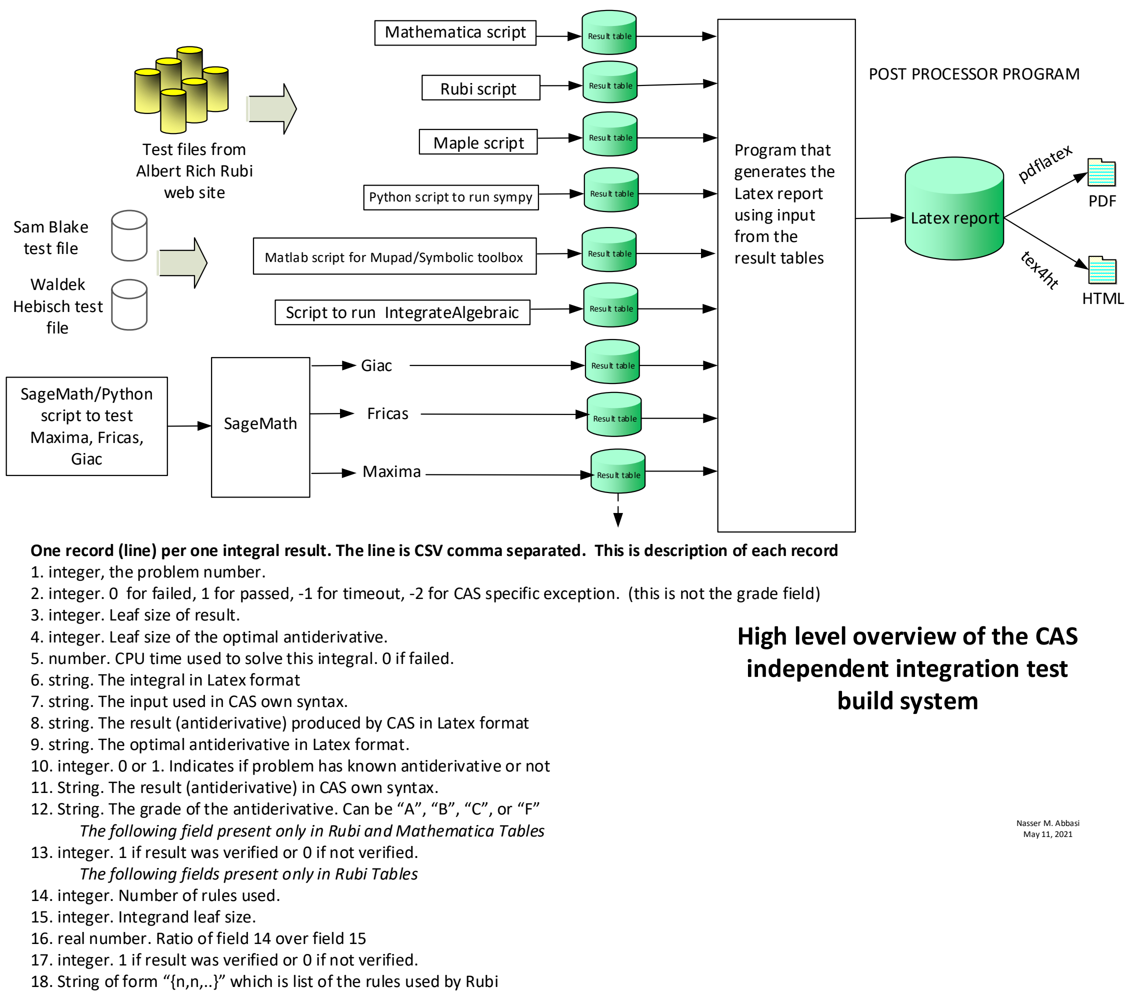
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



Chapter 2

detailed summary tables of results

Local contents

2.1 List of integrals sorted by grade for each CAS

Local contents

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 64, 65, 70, 71, 77, 78, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239 }

B grade: { }

C grade: { 34, 61, 62, 63, 66, 67, 68, 69, 72, 73, 74, 75, 76, 79, 80, 81 }

F grade: { }

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 28, 30, 31, 32, 33, 35, 36, 37, 38, 39, 40, 41, 42, 45, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 64, 67, 73, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 114, 117, 124, 125, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 156, 162, 169, 170, 171, 172, 173, 174, 175, 176, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 189, 190, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 210, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 233, 235, 237, 238, 239 }

B grade: { 209, 211 }

C grade: { 27, 29, 34, 43, 44, 46, 61, 62, 63, 65, 66, 68, 69, 70, 71, 72, 74, 75, 76, 77, 78, 79, 80, 110, 111, 112, 113, 115, 116, 118, 119, 120, 121, 122, 123, 126, 155, 157, 158, 159, 160, 161, 163, 164, 165, 166, 167, 168, 177, 188, 191, 192, 193, 232, 234, 236 }

F grade: { }

2.1.3 Maple

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 17, 18, 19, 24, 25, 26, 30, 31, 32, 33, 39, 40, 41, 42, 47, 48, 49, 50, 56, 57, 58, 59, 60, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 96, 100, 101, 102, 107, 108, 109, 110, 111, 127, 151, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 194, 195, 196, 206, 208, 210, 212, 213, 214, 216, 218, 223, 226, 228, 231, 233, 235, 236 }

B grade: { 11, 12, 14, 15, 16, 20, 21, 22, 23, 94, 95, 97, 98, 99, 103, 104, 105, 106, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 189, 197, 198, 199, 220, 222, 224, 230, 232, 234 }

C grade: { 124, 200, 201, 202, 203, 204, 205, 207, 209, 211, 215, 217, 219, 221, 225, 227, 229 }

F grade: { 13, 27, 28, 29, 34, 35, 36, 37, 38, 43, 44, 45, 46, 51, 52, 53, 54, 55, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 125, 126, 186, 187, 188, 190, 191, 192, 193, 237, 238, 239 }

2.1.4 Maxima

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 29, 30, 31, 39, 40, 46, 47, 48, 49, 50, 55, 56, 57, 58, 59, 60, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 127, 128, 129, 130, 134, 135, 136, 137, 141, 142, 143, 144, 148, 151, 155, 156, 158, 162, 163, 164, 165, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236 }

B grade: { 26, 27, 28, 32, 33, 35, 36, 37, 38, 41, 42, 43, 44, 45, 51, 52, 53, 54, 149, 150, 157 }

C grade: { }

F grade: { 34, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 131, 132, 133, 138, 139, 140, 145, 146, 147, 152, 153, 154, 159, 160, 161, 166, 167, 168, 169, 170, 171, 186, 187, 188, 189, 190, 191, 192, 193, 237, 238, 239 }

2.1.5 FriCAS

A grade: { 1, 2, 3, 4, 5, 6, 8, 9, 10, 11, 14, 15, 16, 17, 18, 19, 24, 25, 30, 31, 32, 33, 35, 36, 39, 40, 41, 42, 43, 44, 45, 47, 48, 49, 50, 51, 52, 53, 56, 57, 58, 59, 60, 82, 83, 84, 85, 86, 90, 91, 92, 93, 127, 128, 129, 130, 131, 132, 134, 135, 136, 137, 138, 139, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 155, 156, 157, 158, 162, 163, 164, 165, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 182, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 230, 231, 232, 233, 234, 235, 236, 238, 239 }

B grade: { 7, 12, 13, 20, 21, 22, 23, 27, 28, 29, 34, 37, 38, 46, 54, 55, 61, 62, 63, 68, 69, 75, 76, 87, 88, 89, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 110, 111, 112, 113, 118, 119, 124, 133, 140, 154, 159, 160, 161, 166, 167, 168, 179, 180, 181, 183, 184, 185, 186, 229 }

C grade: { 237 }

F grade: { 26, 64, 65, 66, 67, 70, 71, 72, 73, 74, 77, 78, 79, 80, 81, 109, 114, 115, 116, 117, 120, 121, 122, 123, 125, 126 }

2.1.6 Sympy

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 20, 21, 22, 23, 24, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 104, 105, 106, 107, 127, 128, 129, 130, 134, 135, 136, 141, 142, 143, 144, 148, 149, 150, 151, 158, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 184, 185, 189 }

B grade: { 30, 47, 137, 157, 164, 165, 172 }

C grade: { 27, 28, 29, 35, 36, 43, 44, 45, 46, 51, 52, 53, 206, 208, 209, 210, 211, 212, 213, 216, 218, 219, 220, 221, 222, 223, 226, 228, 229, 230, 231, 235 }

F grade: { 19, 25, 26, 31, 32, 33, 34, 37, 38, 39, 40, 41, 42, 48, 49, 50, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 101, 102, 103, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 131, 132, 133, 138, 139, 140, 145, 146, 147, 152, 153, 154, 155, 156, 159, 160, 161, 162, 163, 166, 167, 168, 169, 170, 171, 183, 186, 187, 188, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 207, 214, 215, 217, 224, 225, 227, 232, 233, 234, 236, 237, 238, 239 }

2.1.7 Giac

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 99, 100, 101, 102, 105, 106, 107, 108, 148, 149, 150, 152, 153, 154, 155, 156, 157, 158, 159, 161, 162, 163, 164, 165, 166, 168, 172, 173, 174, 175, 176, 177, 178, 197, 198, 203, 205, 206, 207, 208, 209, 210, 211, 215, 216, 217, 218, 219, 220, 221, 225, 227, 235 }

B grade: { 97, 98, 103, 104, 109, 130, 133, 140, 147, 151, 160, 167, 179, 180, 181, 182, 183, 184, 185, 194, 195, 196, 199, 200, 201, 202, 204, 212, 213, 214, 222, 223, 224, 226, 228, 229, 230, 231, 232, 233, 234 }

C grade: { }

F grade: { 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 131, 132, 134, 135, 136, 137, 138, 139, 141, 142, 143, 144, 145, 146, 169, 170, 171, 186, 187, 188, 189, 190, 191, 192, 193, 236, 237, 238, 239 }

2.1.8 Mupad

A grade: { }

B grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 30, 31, 32, 33, 39, 40, 41, 42, 47, 48, 49, 50, 56, 57, 58, 59, 60, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 189, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 226, 228, 231, 233, 235, 236 }

C grade: { }

F grade: { 27, 28, 29, 34, 35, 36, 37, 38, 43, 44, 45, 46, 51, 52, 53, 54, 55, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 171, 186, 187, 188, 190, 191, 192, 225, 227, 229, 230, 232, 234, 237, 238, 239 }

2.1.9 IntegrateAlgebraic

A grade: { 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 110, 124, 125, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 174, 175, 176, 177, 178, 199, 200, 201, 202, 203, 204, 206, 208, 209, 210, 211, 212, 213, 214, 215, 217, 219, 220, 221, 222, 223, 224, 225, 227, 229, 230, 232, 233, 234, 235 }

B grade: { 194, 195, 196, 197, 198, 205, 207, 216, 218, 226, 228, 231 }

C grade: { 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 237, 238 }

F grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 126, 172, 173, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 236, 239 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column `N.S.` in the table below, which stands for **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To help make the table fit, `Mathematica` was abbreviated to `MMA` and `IntegrateAlgebraic` to `I.A.`

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	94	94	94	97	96	97	104	97	87	0
N.S.	1	1.00	1.00	1.03	1.02	1.03	1.11	1.03	0.93	0.00
time (sec)	N/A	0.072	0.049	0.038	0.475	0.933	0.125	0.173	0.051	0.000

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	70	70	70	73	70	74	80	74	66	0
N.S.	1	1.00	1.00	1.04	1.00	1.06	1.14	1.06	0.94	0.00
time (sec)	N/A	0.043	0.015	0.044	0.562	0.849	0.079	0.148	0.034	0.000

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	50	50	50	49	48	50	51	50	48	0
N.S.	1	1.00	1.00	0.98	0.96	1.00	1.02	1.00	0.96	0.00
time (sec)	N/A	0.028	0.008	0.044	0.481	0.635	0.074	0.189	0.048	0.000

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	28	28	28	25	24	26	26	26	25	0
N.S.	1	1.00	1.00	0.89	0.86	0.93	0.93	0.93	0.89	0.00
time (sec)	N/A	0.013	0.005	0.040	0.536	0.846	0.066	0.149	0.037	0.000

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	144	144	128	195	128	369	71	133	123	0
N.S.	1	1.00	0.89	1.35	0.89	2.56	0.49	0.92	0.85	0.00
time (sec)	N/A	0.094	0.123	0.048	1.277	1.137	0.417	0.202	1.383	0.001

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	169	169	145	221	158	537	97	160	143	0
N.S.	1	1.00	0.86	1.31	0.93	3.18	0.57	0.95	0.85	0.00
time (sec)	N/A	0.082	0.096	0.052	1.157	1.021	0.578	0.191	1.398	0.000

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	197	197	175	249	192	743	133	180	173	0
N.S.	1	1.00	0.89	1.26	0.97	3.77	0.68	0.91	0.88	0.00
time (sec)	N/A	0.106	0.131	0.059	1.434	1.175	0.777	0.191	1.396	0.000

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	122	122	122	125	124	132	139	132	116	0
N.S.	1	1.00	1.00	1.02	1.02	1.08	1.14	1.08	0.95	0.00
time (sec)	N/A	0.071	0.019	0.050	0.547	0.734	0.091	0.159	1.199	0.000

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	82	82	82	87	82	91	90	91	75	0
N.S.	1	1.00	1.00	1.06	1.00	1.11	1.10	1.11	0.91	0.00
time (sec)	N/A	0.046	0.012	0.037	0.707	0.818	0.084	0.208	0.042	0.000

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	50	50	50	49	48	50	51	50	48	0
N.S.	1	1.00	1.00	0.98	0.96	1.00	1.02	1.00	0.96	0.00
time (sec)	N/A	0.029	0.007	0.039	0.574	0.510	0.072	0.159	0.045	0.000

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	173	173	167	334	189	505	156	211	152	0
N.S.	1	1.00	0.97	1.93	1.09	2.92	0.90	1.22	0.88	0.00
time (sec)	N/A	0.128	0.142	0.046	1.230	0.680	0.679	0.194	1.386	0.000

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	B	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	203	203	210	367	226	771	189	233	191	0
N.S.	1	1.00	1.03	1.81	1.11	3.80	0.93	1.15	0.94	0.00
time (sec)	N/A	0.244	0.221	0.062	1.282	1.006	1.135	0.190	1.412	0.000

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F(-1)	A	B	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	258	258	234	0	267	1067	233	264	249	0
N.S.	1	1.00	0.91	0.00	1.03	4.14	0.90	1.02	0.97	0.00
time (sec)	N/A	0.233	0.276	180.000	1.321	0.854	1.623	0.197	1.428	0.001

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	252	252	253	661	364	873	371	391	250	0
N.S.	1	1.00	1.00	2.62	1.44	3.46	1.47	1.55	0.99	0.00
time (sec)	N/A	0.191	0.127	0.047	1.189	1.226	1.310	0.201	1.430	0.000

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	208	208	203	486	273	700	257	296	192	0
N.S.	1	1.00	0.98	2.34	1.31	3.37	1.24	1.42	0.92	0.00
time (sec)	N/A	0.148	0.106	0.046	1.319	1.024	1.003	0.185	1.402	0.001

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	173	173	167	334	190	507	156	211	152	0
N.S.	1	1.00	0.97	1.93	1.10	2.93	0.90	1.22	0.88	0.00
time (sec)	N/A	0.124	0.149	0.043	1.321	0.795	0.690	0.324	1.375	0.000

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	145	145	129	195	128	390	71	133	123	0
N.S.	1	1.00	0.89	1.34	0.88	2.69	0.49	0.92	0.85	0.00
time (sec)	N/A	0.078	0.075	0.046	1.075	0.955	0.436	0.191	1.379	0.000

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	288	288	224	222	293	254	447	278	1364	0
N.S.	1	1.00	0.78	0.77	1.02	0.88	1.55	0.97	4.74	0.00
time (sec)	N/A	0.147	0.107	0.050	1.235	1.170	79.722	0.266	7.705	0.001

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	346	346	336	406	489	432	0	443	2589	0
N.S.	1	1.00	0.97	1.17	1.41	1.25	0.00	1.28	7.48	0.00
time (sec)	N/A	0.270	0.235	0.054	1.298	18.476	0.000	0.203	16.807	0.001

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	B	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	320	320	313	905	509	1619	546	529	416	0
N.S.	1	1.00	0.98	2.83	1.59	5.06	1.71	1.65	1.30	0.00
time (sec)	N/A	0.298	0.303	0.060	1.346	1.113	12.429	0.190	0.390	0.001

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	B	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	267	267	260	708	397	1316	405	412	302	0
N.S.	1	1.00	0.97	2.65	1.49	4.93	1.52	1.54	1.13	0.00
time (sec)	N/A	0.226	0.238	0.057	1.156	1.063	8.537	0.190	1.493	0.001

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	B	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	234	234	227	529	306	1027	291	319	240	0
N.S.	1	1.00	0.97	2.26	1.31	4.39	1.24	1.36	1.03	0.00
time (sec)	N/A	0.220	0.156	0.053	1.219	0.925	4.330	0.200	0.296	0.001

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	B	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	203	203	205	367	220	768	189	227	191	0
N.S.	1	1.00	1.01	1.81	1.08	3.78	0.93	1.12	0.94	0.00
time (sec)	N/A	0.231	0.198	0.053	1.395	0.983	2.556	0.213	1.467	0.001

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	169	169	145	221	158	537	97	160	143	0
N.S.	1	1.00	0.86	1.31	0.93	3.18	0.57	0.95	0.85	0.00
time (sec)	N/A	0.084	0.091	0.054	1.439	0.949	1.418	0.171	1.428	0.001

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	346	346	337	406	489	440	0	443	2492	0
N.S.	1	1.00	0.97	1.17	1.41	1.27	0.00	1.28	7.20	0.00
time (sec)	N/A	0.255	0.195	0.056	1.290	18.528	0.000	0.201	15.930	0.001

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	F(-1)	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	419	419	381	606	784	0	0	664	3637	0
N.S.	1	1.00	0.91	1.45	1.87	0.00	0.00	1.58	8.68	0.00
time (sec)	N/A	0.493	0.630	0.062	1.257	0.000	0.000	0.222	24.310	0.001

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	B	B	C	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	112	112	62	0	322	399	80	0	-1	165
N.S.	1	1.00	0.55	0.00	2.88	3.56	0.71	0.00	-0.01	1.47
time (sec)	N/A	0.032	0.106	0.429	1.219	1.064	4.984	0.000	0.000	0.389

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	B	B	C	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	91	91	134	0	244	363	76	0	-1	149
N.S.	1	1.00	1.47	0.00	2.68	3.99	0.84	0.00	-0.01	1.64
time (sec)	N/A	0.020	0.084	0.366	1.216	1.017	5.525	0.000	0.000	0.366

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	A	B	C	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	85	85	62	0	130	372	70	0	-1	142
N.S.	1	1.00	0.73	0.00	1.53	4.38	0.82	0.00	-0.01	1.67
time (sec)	N/A	0.013	0.037	0.387	1.254	0.792	16.137	0.000	0.000	0.291

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	47	47	28	25	50	44	190	0	27	28
N.S.	1	1.00	0.60	0.53	1.06	0.94	4.04	0.00	0.57	0.60
time (sec)	N/A	0.009	0.015	0.046	0.523	1.056	91.681	0.000	1.348	0.223

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-1)	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	55	55	40	37	85	69	0	0	44	40
N.S.	1	1.00	0.73	0.67	1.55	1.25	0.00	0.00	0.80	0.73
time (sec)	N/A	0.013	0.021	0.043	0.624	0.669	0.000	0.000	1.425	0.293

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	A	F(-1)	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	74	74	51	48	119	91	0	0	58	51
N.S.	1	1.00	0.69	0.65	1.61	1.23	0.00	0.00	0.78	0.69
time (sec)	N/A	0.019	0.022	0.048	0.584	1.015	0.000	0.000	1.391	0.428

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	A	F(-1)	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	93	93	62	59	153	113	0	0	73	62
N.S.	1	1.00	0.67	0.63	1.65	1.22	0.00	0.00	0.78	0.67
time (sec)	N/A	0.026	0.026	0.046	0.538	0.970	0.000	0.000	1.371	0.630

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	C	F	F	B	F	F	F	A
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	398	58	151	0	0	644	0	0	-1	515
N.S.	1	0.15	0.38	0.00	0.00	1.62	0.00	0.00	-0.00	1.29
time (sec)	N/A	0.026	0.134	0.698	0.000	54.975	0.000	0.000	0.000	3.122

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	B	A	C	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	139	139	151	0	552	421	126	0	-1	176
N.S.	1	1.00	1.09	0.00	3.97	3.03	0.91	0.00	-0.01	1.27
time (sec)	N/A	0.057	0.137	0.375	1.527	1.253	9.174	0.000	0.000	0.565

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	B	A	C	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	120	120	141	0	436	399	121	0	-1	165
N.S.	1	1.00	1.18	0.00	3.63	3.32	1.01	0.00	-0.01	1.38
time (sec)	N/A	0.042	0.078	0.384	1.396	1.655	7.599	0.000	0.000	0.481

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	B	B	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	113	113	137	0	296	412	0	0	-1	158
N.S.	1	1.00	1.21	0.00	2.62	3.65	0.00	0.00	-0.01	1.40
time (sec)	N/A	0.042	0.082	0.532	1.373	1.729	0.000	0.000	0.000	0.499

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	B	B	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	110	110	131	0	180	521	0	0	-1	144
N.S.	1	1.00	1.19	0.00	1.64	4.74	0.00	0.00	-0.01	1.31
time (sec)	N/A	0.040	0.063	0.533	1.354	1.165	0.000	0.000	0.000	0.393

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-1)	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	76	76	40	37	105	67	0	0	44	40
N.S.	1	1.00	0.53	0.49	1.38	0.88	0.00	0.00	0.58	0.53
time (sec)	N/A	0.021	0.015	0.042	0.498	0.890	0.000	0.000	1.432	0.348

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-1)	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	105	105	51	48	155	91	0	0	56	51
N.S.	1	1.00	0.49	0.46	1.48	0.87	0.00	0.00	0.53	0.49
time (sec)	N/A	0.035	0.032	0.050	0.585	1.078	0.000	0.000	1.389	0.529

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	A	F(-1)	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	98	98	62	59	206	113	0	0	71	62
N.S.	1	1.00	0.63	0.60	2.10	1.15	0.00	0.00	0.72	0.63
time (sec)	N/A	0.035	0.035	0.046	0.574	1.249	0.000	0.000	1.436	0.760

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	A	F(-1)	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	117	117	73	70	257	135	0	0	86	73
N.S.	1	1.00	0.62	0.60	2.20	1.15	0.00	0.00	0.74	0.62
time (sec)	N/A	0.044	0.040	0.049	0.510	0.804	0.000	0.000	1.427	1.189

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	B	A	C	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	174	174	75	0	406	482	170	0	-1	227
N.S.	1	1.00	0.43	0.00	2.33	2.77	0.98	0.00	-0.01	1.30
time (sec)	N/A	0.059	0.068	0.374	1.678	1.210	10.424	0.000	0.000	0.707

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	B	A	C	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	141	141	72	0	322	424	82	0	-1	200
N.S.	1	1.00	0.51	0.00	2.28	3.01	0.58	0.00	-0.01	1.42
time (sec)	N/A	0.045	0.095	0.375	1.437	1.095	5.347	0.000	0.000	0.668

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	B	A	C	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	111	111	141	0	244	362	78	0	-1	176
N.S.	1	1.00	1.27	0.00	2.20	3.26	0.70	0.00	-0.01	1.59
time (sec)	N/A	0.030	0.154	0.395	1.198	1.308	4.439	0.000	0.000	0.514

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	A	B	C	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	99	99	61	0	134	488	71	0	-1	158
N.S.	1	1.00	0.62	0.00	1.35	4.93	0.72	0.00	-0.01	1.60
time (sec)	N/A	0.024	0.052	0.380	1.224	1.059	12.828	0.000	0.000	0.370

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	47	47	37	34	51	54	190	0	33	37
N.S.	1	1.00	0.79	0.72	1.09	1.15	4.04	0.00	0.70	0.79
time (sec)	N/A	0.010	0.036	0.049	0.470	1.231	82.052	0.000	1.370	0.296

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-1)	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	91	91	59	57	86	87	0	0	87	60
N.S.	1	1.00	0.65	0.63	0.95	0.96	0.00	0.00	0.96	0.66
time (sec)	N/A	0.028	0.039	0.045	0.608	0.719	0.000	0.000	1.424	0.394

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-1)	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	121	121	80	81	120	121	0	0	105	84
N.S.	1	1.00	0.66	0.67	0.99	1.00	0.00	0.00	0.87	0.69
time (sec)	N/A	0.035	0.034	0.042	0.499	0.915	0.000	0.000	1.460	0.572

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-1)	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	151	151	100	105	154	155	0	0	132	108
N.S.	1	1.00	0.66	0.70	1.02	1.03	0.00	0.00	0.87	0.72
time (sec)	N/A	0.047	0.062	0.046	0.657	1.298	0.000	0.000	1.453	0.874

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	B	A	C	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	262	262	238	0	672	717	270	0	-1	325
N.S.	1	1.00	0.91	0.00	2.56	2.74	1.03	0.00	-0.00	1.24
time (sec)	N/A	0.165	5.190	0.385	1.294	1.224	13.131	0.000	0.000	1.159

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	B	A	C	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	219	219	203	0	552	634	131	0	-1	283
N.S.	1	1.00	0.93	0.00	2.52	2.89	0.60	0.00	-0.00	1.29
time (sec)	N/A	0.166	5.171	0.385	1.479	1.245	7.275	0.000	0.000	0.895

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	B	A	C	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	175	175	172	0	436	554	126	0	-1	240
N.S.	1	1.00	0.98	0.00	2.49	3.17	0.72	0.00	-0.01	1.37
time (sec)	N/A	0.096	5.150	0.381	1.275	1.258	6.437	0.000	0.000	0.772

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	B	B	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	159	159	168	0	301	652	0	0	-1	222
N.S.	1	1.00	1.06	0.00	1.89	4.10	0.00	0.00	-0.01	1.40
time (sec)	N/A	0.102	5.163	0.559	1.137	1.170	0.000	0.000	0.000	0.782

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	A	B	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	152	152	180	0	190	719	0	0	-1	211
N.S.	1	1.00	1.18	0.00	1.25	4.73	0.00	0.00	-0.01	1.39
time (sec)	N/A	0.068	5.247	0.550	1.193	1.299	0.000	0.000	0.000	0.688

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-1)	F	B	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	78	78	126	76	109	103	0	0	148	79
N.S.	1	1.00	1.62	0.97	1.40	1.32	0.00	0.00	1.90	1.01
time (sec)	N/A	0.021	0.097	0.049	0.508	1.191	0.000	0.000	1.427	0.570

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-1)	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	174	174	106	115	159	152	0	0	176	118
N.S.	1	1.00	0.61	0.66	0.91	0.87	0.00	0.00	1.01	0.68
time (sec)	N/A	0.073	5.106	0.046	0.712	0.995	0.000	0.000	1.450	0.834

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-1)	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	211	211	138	156	210	200	0	0	217	159
N.S.	1	1.00	0.65	0.74	1.00	0.95	0.00	0.00	1.03	0.75
time (sec)	N/A	0.127	5.176	0.047	0.503	1.065	0.000	0.000	1.430	1.258

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-1)	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	253	253	169	197	261	246	0	0	257	200
N.S.	1	1.00	0.67	0.78	1.03	0.97	0.00	0.00	1.02	0.79
time (sec)	N/A	0.207	5.153	0.049	0.643	1.407	0.000	0.000	1.481	1.865

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-1)	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	109	109	120	134	182	166	0	0	271	137
N.S.	1	1.00	1.10	1.23	1.67	1.52	0.00	0.00	2.49	1.26
time (sec)	N/A	0.036	0.043	0.053	0.626	0.895	0.000	0.000	1.556	1.141

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	C	F	F	B	F(-1)	F	F	C
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	331	62	655	0	0	643	0	0	-1	618
N.S.	1	0.19	1.98	0.00	0.00	1.94	0.00	0.00	-0.00	1.87
time (sec)	N/A	0.028	1.232	0.701	0.000	36.577	0.000	0.000	0.000	14.445

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	C	F	F	B	F	F	F	C
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	273	60	443	0	0	535	0	0	-1	550
N.S.	1	0.22	1.62	0.00	0.00	1.96	0.00	0.00	-0.00	2.01
time (sec)	N/A	0.027	0.711	0.559	0.000	3.507	0.000	0.000	0.000	7.775

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	C	F	F	B	F	F	F	C
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	233	59	161	0	0	469	0	0	-1	487
N.S.	1	0.25	0.69	0.00	0.00	2.01	0.00	0.00	-0.00	2.09
time (sec)	N/A	0.027	0.224	0.570	0.000	1.579	0.000	0.000	0.000	4.355

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F	F(-1)	F	F	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	148	207	168	0	0	0	0	0	-1	320
N.S.	1	1.40	1.14	0.00	0.00	0.00	0.00	0.00	-0.01	2.16
time (sec)	N/A	0.192	0.151	0.526	0.000	0.000	0.000	0.000	0.000	1.912

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F	F(-1)	F	F	F	C
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	179	238	256	0	0	0	0	0	-1	352
N.S.	1	1.33	1.43	0.00	0.00	0.00	0.00	0.00	-0.01	1.97
time (sec)	N/A	0.193	0.801	0.574	0.000	0.000	0.000	0.000	0.000	2.701

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	C	F	F	F(-1)	F	F	F	C
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	226	621	621	0	0	0	0	0	-1	399
N.S.	1	2.75	2.75	0.00	0.00	0.00	0.00	0.00	-0.00	1.77
time (sec)	N/A	2.580	2.673	0.558	0.000	0.000	0.000	0.000	0.000	3.789

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	A	F	F	F(-1)	F	F	F	C
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	280	1172	277	0	0	0	0	0	-1	471
N.S.	1	4.19	0.99	0.00	0.00	0.00	0.00	0.00	-0.00	1.68
time (sec)	N/A	6.642	5.746	0.605	0.000	0.000	0.000	0.000	0.000	8.096

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	C	F	F	B	F(-1)	F	F	C
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	351	62	698	0	0	819	0	0	-1	680
N.S.	1	0.18	1.99	0.00	0.00	2.33	0.00	0.00	-0.00	1.94
time (sec)	N/A	0.029	1.083	0.398	0.000	19.124	0.000	0.000	0.000	14.087

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	C	F	F	B	F(-1)	F	F	C
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	301	60	450	0	0	631	0	0	-1	645
N.S.	1	0.20	1.50	0.00	0.00	2.10	0.00	0.00	-0.00	2.14
time (sec)	N/A	0.028	0.643	0.582	0.000	2.323	0.000	0.000	0.000	7.331

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F	F(-1)	F	F	F	C
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	182	241	78	0	0	0	0	0	-1	356
N.S.	1	1.32	0.43	0.00	0.00	0.00	0.00	0.00	-0.01	1.96
time (sec)	N/A	0.210	0.041	0.538	0.000	0.000	0.000	0.000	0.000	2.398

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F	F(-1)	F	F	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	217	276	99	0	0	0	0	0	-1	397
N.S.	1	1.27	0.46	0.00	0.00	0.00	0.00	0.00	-0.00	1.83
time (sec)	N/A	0.204	0.145	0.555	0.000	0.000	0.000	0.000	0.000	2.675

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	C	F	F	F(-1)	F	F	F	C
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	261	625	625	0	0	0	0	0	-1	452
N.S.	1	2.39	2.39	0.00	0.00	0.00	0.00	0.00	-0.00	1.73
time (sec)	N/A	1.934	2.032	0.600	0.000	0.000	0.000	0.000	0.000	4.081

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	A	F	F	F(-1)	F	F	F	C
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	324	1214	288	0	0	0	0	0	-1	551
N.S.	1	3.75	0.89	0.00	0.00	0.00	0.00	0.00	-0.00	1.70
time (sec)	N/A	5.692	5.944	0.594	0.000	0.000	0.000	0.000	0.000	9.704

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	C	F	F	F(-1)	F(-1)	F	F	C
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	541	62	1171	0	0	0	0	0	-1	1081
N.S.	1	0.11	2.16	0.00	0.00	0.00	0.00	0.00	-0.00	2.00
time (sec)	N/A	0.027	2.341	0.395	0.000	0.000	0.000	0.000	0.000	102.412

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	C	F	F	B	F(-1)	F	F	C
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	458	62	908	0	0	1246	0	0	-1	970
N.S.	1	0.14	1.98	0.00	0.00	2.72	0.00	0.00	-0.00	2.12
time (sec)	N/A	0.028	1.838	0.412	0.000	114.550	0.000	0.000	0.000	76.857

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	C	F	F	B	F(-1)	F	F	C
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	391	62	651	0	0	954	0	0	-1	804
N.S.	1	0.16	1.66	0.00	0.00	2.44	0.00	0.00	-0.00	2.06
time (sec)	N/A	0.028	1.164	0.607	0.000	12.614	0.000	0.000	0.000	28.127

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F	F(-1)	F(-1)	F	F	C
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	217	276	79	0	0	0	0	0	-1	385
N.S.	1	1.27	0.36	0.00	0.00	0.00	0.00	0.00	-0.00	1.77
time (sec)	N/A	0.239	0.033	0.572	0.000	0.000	0.000	0.000	0.000	3.310

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F	F(-1)	F(-1)	F	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	267	326	153	0	0	0	0	0	-1	444
N.S.	1	1.22	0.57	0.00	0.00	0.00	0.00	0.00	-0.00	1.66
time (sec)	N/A	0.243	0.219	0.546	0.000	0.000	0.000	0.000	0.000	3.661

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	C	F	F	F(-1)	F	F	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	307	167	168	0	0	0	0	0	-1	536
N.S.	1	0.54	0.55	0.00	0.00	0.00	0.00	0.00	-0.00	1.75
time (sec)	N/A	0.310	0.380	0.579	0.000	0.000	0.000	0.000	0.000	5.651

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	C	F	F	F(-1)	F(-1)	F	F	C
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	377	428	428	0	0	0	0	0	-1	644
N.S.	1	1.14	1.14	0.00	0.00	0.00	0.00	0.00	-0.00	1.71
time (sec)	N/A	2.742	3.084	0.592	0.000	0.000	0.000	0.000	0.000	8.919

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	A	F	F	F(-1)	F(-1)	F	F	C
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	463	1990	337	0	0	0	0	0	-1	791
N.S.	1	4.30	0.73	0.00	0.00	0.00	0.00	0.00	-0.00	1.71
time (sec)	N/A	8.662	5.878	0.559	0.000	0.000	0.000	0.000	0.000	27.991

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F(-1)	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	53	53	52	71	0	91	0	0	131	0
N.S.	1	1.00	0.98	1.34	0.00	1.72	0.00	0.00	2.47	0.00
time (sec)	N/A	0.019	0.038	0.041	0.000	1.348	0.000	0.000	1.905	0.527

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	94	94	94	97	96	98	107	98	88	0
N.S.	1	1.00	1.00	1.03	1.02	1.04	1.14	1.04	0.94	0.00
time (sec)	N/A	0.068	0.024	0.040	0.651	0.977	0.090	0.156	1.303	0.000

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	70	70	70	73	70	74	76	74	66	0
N.S.	1	1.00	1.00	1.04	1.00	1.06	1.09	1.06	0.94	0.00
time (sec)	N/A	0.049	0.017	0.039	0.647	0.897	0.084	0.150	1.242	0.000

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	50	50	50	49	48	50	53	50	48	0
N.S.	1	1.00	1.00	0.98	0.96	1.00	1.06	1.00	0.96	0.00
time (sec)	N/A	0.029	0.012	0.036	0.605	0.865	0.078	0.161	0.048	0.000

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	28	28	28	25	24	26	26	26	25	0
N.S.	1	1.00	1.00	0.89	0.86	0.93	0.93	0.93	0.89	0.00
time (sec)	N/A	0.014	0.009	0.045	0.493	0.759	0.065	0.150	0.036	0.000

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	223	223	196	266	212	639	87	245	720	0
N.S.	1	1.00	0.88	1.19	0.95	2.87	0.39	1.10	3.23	0.00
time (sec)	N/A	0.151	0.179	0.048	1.278	1.288	0.659	0.165	1.480	0.000

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	245	245	212	295	236	711	112	266	740	0
N.S.	1	1.00	0.87	1.20	0.96	2.90	0.46	1.09	3.02	0.00
time (sec)	N/A	0.147	0.202	0.053	1.158	1.417	0.849	0.169	1.520	0.001

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	273	273	243	314	271	787	151	286	762	0
N.S.	1	1.00	0.89	1.15	0.99	2.88	0.55	1.05	2.79	0.00
time (sec)	N/A	0.174	0.223	0.054	1.211	1.321	1.031	0.194	1.581	0.001

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	154	154	154	163	158	173	185	173	146	0
N.S.	1	1.00	1.00	1.06	1.03	1.12	1.20	1.12	0.95	0.00
time (sec)	N/A	0.114	0.033	0.041	0.546	1.050	0.117	0.150	0.067	0.000

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	122	122	122	125	124	132	139	132	116	0
N.S.	1	1.00	1.00	1.02	1.02	1.08	1.14	1.08	0.95	0.00
time (sec)	N/A	0.077	0.023	0.035	0.544	1.077	0.098	0.151	1.297	0.000

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	82	82	82	87	82	91	97	91	75	0
N.S.	1	1.00	1.00	1.06	1.00	1.11	1.18	1.11	0.91	0.00
time (sec)	N/A	0.049	0.017	0.041	0.695	0.559	0.090	0.166	0.046	0.000

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	50	50	50	49	48	50	53	50	48	0
N.S.	1	1.00	1.00	0.98	0.96	1.00	1.06	1.00	0.96	0.00
time (sec)	N/A	0.030	0.008	0.038	0.478	0.534	0.080	0.149	0.045	0.000

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	B	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	253	253	231	436	286	1239	187	353	1081	0
N.S.	1	1.00	0.91	1.72	1.13	4.90	0.74	1.40	4.27	0.00
time (sec)	N/A	0.194	0.131	0.048	1.285	1.182	1.119	0.198	1.485	0.001

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	B	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	291	291	298	475	319	1335	219	376	1254	0
N.S.	1	1.00	1.02	1.63	1.10	4.59	0.75	1.29	4.31	0.00
time (sec)	N/A	0.366	0.216	0.056	1.104	0.932	1.979	0.174	1.536	0.001

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	349	349	319	499	361	1411	264	407	1401	0
N.S.	1	1.00	0.91	1.43	1.03	4.04	0.76	1.17	4.01	0.00
time (sec)	N/A	0.266	0.227	0.058	1.220	1.202	5.850	0.212	1.660	0.001

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	B	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	332	332	322	837	489	2477	435	617	1822	0
N.S.	1	1.00	0.97	2.52	1.47	7.46	1.31	1.86	5.49	0.00
time (sec)	N/A	0.267	0.224	0.047	1.437	1.220	3.585	0.484	1.515	0.001

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	B	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	288	288	271	627	385	1855	303	481	1433	0
N.S.	1	1.00	0.94	2.18	1.34	6.44	1.05	1.67	4.98	0.00
time (sec)	N/A	0.223	0.166	0.046	1.210	1.479	1.696	0.173	1.488	0.001

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	B	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	253	253	231	436	287	1240	187	353	1081	0
N.S.	1	1.00	0.91	1.72	1.13	4.90	0.74	1.40	4.27	0.00
time (sec)	N/A	0.190	0.132	0.046	1.093	1.431	1.084	0.177	1.466	0.001

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	223	223	196	266	212	639	87	245	720	0
N.S.	1	1.00	0.88	1.19	0.95	2.87	0.39	1.10	3.23	0.00
time (sec)	N/A	0.138	0.151	0.048	1.126	0.744	0.610	0.165	0.221	0.001

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	449	449	340	320	365	1356	0	437	6153	0
N.S.	1	1.00	0.76	0.71	0.81	3.02	0.00	0.97	13.70	0.00
time (sec)	N/A	0.268	0.143	0.056	1.434	1.742	0.000	0.209	2.755	0.001

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	513	513	498	550	481	3299	0	667	21975	0
N.S.	1	1.00	0.97	1.07	0.94	6.43	0.00	1.30	42.84	0.00
time (sec)	N/A	0.420	0.335	0.056	1.566	58.700	0.000	0.195	4.004	0.001

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	B	F(-1)	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	407	407	391	1118	644	3222	0	798	2490	0
N.S.	1	1.00	0.96	2.75	1.58	7.92	0.00	1.96	6.12	0.00
time (sec)	N/A	0.396	0.453	0.060	1.446	1.489	0.000	0.179	1.707	0.001

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	B	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	357	357	341	885	521	2580	471	642	2043	0
N.S.	1	1.00	0.96	2.48	1.46	7.23	1.32	1.80	5.72	0.00
time (sec)	N/A	0.367	0.349	0.062	1.454	1.370	47.538	0.174	0.302	0.001

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	B	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	317	317	301	669	405	1938	337	496	1616	0
N.S.	1	1.00	0.95	2.11	1.28	6.11	1.06	1.56	5.10	0.00
time (sec)	N/A	0.317	0.254	0.055	1.309	1.108	6.989	0.179	1.530	0.001

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	B	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	291	291	297	475	319	1335	219	376	1254	0
N.S.	1	1.00	1.02	1.63	1.10	4.59	0.75	1.29	4.31	0.00
time (sec)	N/A	0.377	0.227	0.058	1.261	1.380	2.174	0.168	0.298	0.001

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	245	245	212	295	236	711	112	266	740	0
N.S.	1	1.00	0.87	1.20	0.96	2.90	0.46	1.09	3.02	0.00
time (sec)	N/A	0.153	0.172	0.053	1.348	0.830	0.965	0.166	1.533	0.001

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	513	513	499	550	470	3299	0	667	21975	0
N.S.	1	1.00	0.97	1.07	0.92	6.43	0.00	1.30	42.84	0.00
time (sec)	N/A	0.428	0.347	0.058	1.457	59.892	0.000	0.217	3.817	0.001

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	F(-1)	F(-1)	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	596	596	629	784	670	0	0	967	37266	0
N.S.	1	1.00	1.06	1.32	1.12	0.00	0.00	1.62	62.53	0.00
time (sec)	N/A	0.739	6.192	0.066	1.292	0.000	0.000	0.218	5.617	0.001

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	F	B	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	103	103	155	103	0	315	0	0	-1	103
N.S.	1	1.00	1.50	1.00	0.00	3.06	0.00	0.00	-0.01	1.00
time (sec)	N/A	0.056	0.171	0.208	0.000	3.924	0.000	0.000	0.000	0.431

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	F	B	F	F	F	C
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	116	116	155	158	0	339	0	0	-1	105
N.S.	1	1.00	1.34	1.36	0.00	2.92	0.00	0.00	-0.01	0.91
time (sec)	N/A	0.023	0.167	0.221	0.000	3.544	0.000	0.000	0.000	0.419

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F	B	F	F	F	C
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	211	211	364	0	0	2381	0	0	-1	327
N.S.	1	1.00	1.73	0.00	0.00	11.28	0.00	0.00	-0.00	1.55
time (sec)	N/A	0.225	0.629	0.587	0.000	10.959	0.000	0.000	0.000	1.193

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F	B	F	F	F	C
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	173	173	161	0	0	844	0	0	-1	281
N.S.	1	1.00	0.93	0.00	0.00	4.88	0.00	0.00	-0.01	1.62
time (sec)	N/A	0.103	0.180	0.575	0.000	1.831	0.000	0.000	0.000	0.902

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F	F(-1)	F	F	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	105	105	84	0	0	0	0	0	-1	213
N.S.	1	1.00	0.80	0.00	0.00	0.00	0.00	0.00	-0.01	2.03
time (sec)	N/A	0.057	0.045	0.566	0.000	0.000	0.000	0.000	0.000	0.825

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F	F(-1)	F	F	F	C
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	134	134	256	0	0	0	0	0	-1	243
N.S.	1	1.00	1.91	0.00	0.00	0.00	0.00	0.00	-0.01	1.81
time (sec)	N/A	0.096	0.577	0.598	0.000	0.000	0.000	0.000	0.000	1.575

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F	F(-1)	F	F	F	C
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	180	180	621	0	0	0	0	0	-1	283
N.S.	1	1.00	3.45	0.00	0.00	0.00	0.00	0.00	-0.01	1.57
time (sec)	N/A	0.203	2.368	0.603	0.000	0.000	0.000	0.000	0.000	3.494

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F	F(-1)	F	F	F	C
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	233	233	231	0	0	0	0	0	-1	356
N.S.	1	1.00	0.99	0.00	0.00	0.00	0.00	0.00	-0.00	1.53
time (sec)	N/A	0.295	5.439	0.615	0.000	0.000	0.000	0.000	0.000	5.179

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F	B	F(-1)	F	F	C
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	280	280	560	0	0	3308	0	0	-1	379
N.S.	1	1.00	2.00	0.00	0.00	11.81	0.00	0.00	-0.00	1.35
time (sec)	N/A	0.359	0.956	0.430	0.000	47.324	0.000	0.000	0.000	2.764

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F	B	F(-1)	F	F	C
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	230	230	358	0	0	1667	0	0	-1	356
N.S.	1	1.00	1.56	0.00	0.00	7.25	0.00	0.00	-0.00	1.55
time (sec)	N/A	0.175	0.638	0.646	0.000	3.507	0.000	0.000	0.000	1.685

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F	F(-1)	F	F	F	C
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	135	135	78	0	0	0	0	0	-1	243
N.S.	1	1.00	0.58	0.00	0.00	0.00	0.00	0.00	-0.01	1.80
time (sec)	N/A	0.076	0.060	0.608	0.000	0.000	0.000	0.000	0.000	1.129

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F	F(-1)	F	F	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	162	162	99	0	0	0	0	0	-1	260
N.S.	1	1.00	0.61	0.00	0.00	0.00	0.00	0.00	-0.01	1.60
time (sec)	N/A	0.109	0.140	0.595	0.000	0.000	0.000	0.000	0.000	2.118

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F	F(-1)	F	F	F	C
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	205	205	625	0	0	0	0	0	-1	304
N.S.	1	1.00	3.05	0.00	0.00	0.00	0.00	0.00	-0.00	1.48
time (sec)	N/A	0.184	2.265	0.596	0.000	0.000	0.000	0.000	0.000	3.916

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F	F(-1)	F	F	F	C
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	266	266	1216	0	0	0	0	0	-1	394
N.S.	1	1.00	4.57	0.00	0.00	0.00	0.00	0.00	-0.00	1.48
time (sec)	N/A	0.293	5.005	0.609	0.000	0.000	0.000	0.000	0.000	6.846

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F	B	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	53	53	44	211	0	208	0	0	-1	53
N.S.	1	1.00	0.83	3.98	0.00	3.92	0.00	0.00	-0.02	1.00
time (sec)	N/A	0.018	0.019	3.245	0.000	16.480	0.000	0.000	0.000	0.233

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F	F(-1)	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	57	57	48	0	0	0	0	0	-1	57
N.S.	1	1.00	0.84	0.00	0.00	0.00	0.00	0.00	-0.02	1.00
time (sec)	N/A	0.022	0.025	0.619	0.000	0.000	0.000	0.000	0.000	0.439

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	545	545	48	0	0	0	0	0	-1	0
N.S.	1	1.00	0.09	0.00	0.00	0.00	0.00	0.00	-0.00	0.00
time (sec)	N/A	1.085	0.020	0.566	0.000	0.000	0.000	0.000	0.000	180.013

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	F(-2)	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	143	143	118	248	164	306	454	0	173	135
N.S.	1	1.00	0.83	1.73	1.15	2.14	3.17	0.00	1.21	0.94
time (sec)	N/A	0.123	0.190	0.063	1.453	0.863	57.222	0.000	2.593	0.187

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	A	A	F(-2)	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	99	99	84	191	126	208	121	0	99	90
N.S.	1	1.00	0.85	1.93	1.27	2.10	1.22	0.00	1.00	0.91
time (sec)	N/A	0.067	0.158	0.059	1.191	0.755	36.578	0.000	1.901	0.172

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	A	A	F(-2)	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	74	74	52	163	106	128	87	0	92	56
N.S.	1	1.00	0.70	2.20	1.43	1.73	1.18	0.00	1.24	0.76
time (sec)	N/A	0.048	0.042	0.057	1.371	0.854	41.308	0.000	1.962	0.114

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	A	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	39	39	39	74	50	99	42	64	58	43
N.S.	1	1.00	1.00	1.90	1.28	2.54	1.08	1.64	1.49	1.10
time (sec)	N/A	0.019	0.073	0.049	1.201	0.748	2.189	0.200	0.076	0.001

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	A	F	F(-2)	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	104	104	100	287	0	482	0	0	149	110
N.S.	1	1.00	0.96	2.76	0.00	4.63	0.00	0.00	1.43	1.06
time (sec)	N/A	0.111	0.244	0.108	0.000	0.937	0.000	0.000	1.631	0.229

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	A	F	F(-2)	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	147	147	122	943	0	801	0	0	1195	138
N.S.	1	1.00	0.83	6.41	0.00	5.45	0.00	0.00	8.13	0.94
time (sec)	N/A	0.209	0.386	0.066	0.000	0.907	0.000	0.000	2.263	0.386

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	B	F(-1)	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	213	213	330	1972	0	1749	0	820	1895	215
N.S.	1	1.00	1.55	9.26	0.00	8.21	0.00	3.85	8.90	1.01
time (sec)	N/A	0.340	0.823	0.068	0.000	1.076	0.000	0.412	3.729	1.350

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	A	A	F(-2)	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	164	164	159	353	190	380	1817	0	327	194
N.S.	1	1.00	0.97	2.15	1.16	2.32	11.08	0.00	1.99	1.18
time (sec)	N/A	0.142	0.213	0.060	1.488	0.763	123.483	0.000	3.878	0.243

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	A	A	F(-2)	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	126	126	106	260	152	268	534	0	197	132
N.S.	1	1.00	0.84	2.06	1.21	2.13	4.24	0.00	1.56	1.05
time (sec)	N/A	0.080	0.260	0.060	1.160	0.606	94.065	0.000	2.579	0.205

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	A	A	F(-2)	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	100	100	73	205	132	164	163	0	81	82
N.S.	1	1.00	0.73	2.05	1.32	1.64	1.63	0.00	0.81	0.82
time (sec)	N/A	0.063	0.091	0.057	1.326	0.868	56.146	0.000	2.512	0.161

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	A	B	F(-2)	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	54	54	46	100	63	100	92	0	34	50
N.S.	1	1.00	0.85	1.85	1.17	1.85	1.70	0.00	0.63	0.93
time (sec)	N/A	0.025	0.025	0.052	1.202	0.802	2.692	0.000	1.501	0.001

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	A	F	F(-2)	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	106	106	102	528	0	519	0	0	556	116
N.S.	1	1.00	0.96	4.98	0.00	4.90	0.00	0.00	5.25	1.09
time (sec)	N/A	0.128	0.236	0.061	0.000	0.904	0.000	0.000	1.684	0.275

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	A	F(-1)	F(-2)	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	156	156	143	834	0	769	0	0	448	160
N.S.	1	1.00	0.92	5.35	0.00	4.93	0.00	0.00	2.87	1.03
time (sec)	N/A	0.222	0.387	0.062	0.000	0.929	0.000	0.000	2.165	0.432

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	B	F(-1)	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	209	209	168	1817	0	1765	0	727	1664	189
N.S.	1	1.00	0.80	8.69	0.00	8.44	0.00	3.48	7.96	0.90
time (sec)	N/A	0.345	0.604	0.067	0.000	1.060	0.000	0.611	3.472	0.565

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	A	A	F(-2)	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	198	198	201	457	219	494	5513	0	487	252
N.S.	1	1.00	1.02	2.31	1.11	2.49	27.84	0.00	2.46	1.27
time (sec)	N/A	0.158	0.255	0.067	1.207	0.858	158.704	0.000	6.049	0.283

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	A	A	F(-2)	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	152	152	121	336	181	350	1841	0	271	173
N.S.	1	1.00	0.80	2.21	1.19	2.30	12.11	0.00	1.78	1.14
time (sec)	N/A	0.102	0.158	0.060	1.251	0.816	112.031	0.000	3.791	0.263

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	A	A	F(-2)	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	125	125	94	253	161	222	520	0	99	106
N.S.	1	1.00	0.75	2.02	1.29	1.78	4.16	0.00	0.79	0.85
time (sec)	N/A	0.077	0.112	0.060	1.505	0.663	82.773	0.000	3.480	0.185

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	A	A	F(-2)	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	71	71	64	120	78	139	99	0	34	68
N.S.	1	1.00	0.90	1.69	1.10	1.96	1.39	0.00	0.48	0.96
time (sec)	N/A	0.034	0.062	0.058	1.249	0.885	4.364	0.000	1.630	0.001

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	A	F(-1)	F(-2)	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	134	134	116	859	0	659	0	0	1427	130
N.S.	1	1.00	0.87	6.41	0.00	4.92	0.00	0.00	10.65	0.97
time (sec)	N/A	0.221	0.270	0.056	0.000	1.149	0.000	0.000	2.156	0.298

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	A	F(-1)	F(-2)	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	166	166	145	1323	0	1001	0	0	1153	166
N.S.	1	1.00	0.87	7.97	0.00	6.03	0.00	0.00	6.95	1.00
time (sec)	N/A	0.234	0.430	0.063	0.000	1.153	0.000	0.000	2.311	0.390

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	A	F(-1)	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	237	237	191	1638	0	1445	0	945	1476	240
N.S.	1	1.00	0.81	6.91	0.00	6.10	0.00	3.99	6.23	1.01
time (sec)	N/A	0.373	0.849	0.071	0.000	0.964	0.000	0.489	3.439	0.516

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	126	126	95	535	166	233	386	158	107	104
N.S.	1	1.00	0.75	4.25	1.32	1.85	3.06	1.25	0.85	0.83
time (sec)	N/A	0.090	0.147	0.063	1.327	0.556	89.999	0.204	1.727	0.206

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	B	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	73	73	66	348	129	158	114	99	63	72
N.S.	1	1.00	0.90	4.77	1.77	2.16	1.56	1.36	0.86	0.99
time (sec)	N/A	0.054	0.087	0.063	1.244	0.613	83.772	0.183	1.621	0.163

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	B	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	51	51	53	173	109	115	82	78	88	55
N.S.	1	1.00	1.04	3.39	2.14	2.25	1.61	1.53	1.73	1.08
time (sec)	N/A	0.033	0.039	0.059	1.298	0.847	60.252	0.197	1.977	0.107

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	43	43	43	71	67	98	44	71	66	47
N.S.	1	1.00	1.00	1.65	1.56	2.28	1.02	1.65	1.53	1.09
time (sec)	N/A	0.020	0.018	0.046	1.208	0.782	3.088	0.239	1.441	0.001

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	108	108	104	228	0	542	0	134	1183	114
N.S.	1	1.00	0.96	2.11	0.00	5.02	0.00	1.24	10.95	1.06
time (sec)	N/A	0.096	0.246	0.064	0.000	0.771	0.000	0.181	1.982	0.218

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	A	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	172	172	150	1135	0	1163	0	300	3813	172
N.S.	1	1.00	0.87	6.60	0.00	6.76	0.00	1.74	22.17	1.00
time (sec)	N/A	0.218	0.775	0.069	0.000	1.057	0.000	0.220	3.536	0.634

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	B	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	250	250	216	2269	0	2307	0	352	2890	268
N.S.	1	1.00	0.86	9.08	0.00	9.23	0.00	1.41	11.56	1.07
time (sec)	N/A	0.400	1.746	0.069	0.000	2.223	0.000	0.288	5.479	1.654

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	B	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	132	132	92	969	200	336	0	222	172	130
N.S.	1	1.00	0.70	7.34	1.52	2.55	0.00	1.68	1.30	0.98
time (sec)	N/A	0.101	0.075	0.067	1.259	1.542	0.000	0.209	1.908	0.277

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	94	90	81	789	164	272	0	160	120	101
N.S.	1	0.96	0.86	8.39	1.74	2.89	0.00	1.70	1.28	1.07
time (sec)	N/A	0.079	0.103	0.064	1.157	0.616	0.000	0.226	1.834	0.277

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	B	B	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	76	76	48	387	144	210	224	127	71	77
N.S.	1	1.00	0.63	5.09	1.89	2.76	2.95	1.67	0.93	1.01
time (sec)	N/A	0.050	0.024	0.056	1.374	0.892	81.344	0.196	2.438	0.176

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	B	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	60	61	36	198	85	156	71	86	34	63
N.S.	1	1.02	0.60	3.30	1.42	2.60	1.18	1.43	0.57	1.05
time (sec)	N/A	0.031	0.013	0.058	1.183	0.834	4.794	0.165	1.868	0.001

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	B	F	B	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	147	147	106	962	0	1075	0	200	3000	159
N.S.	1	1.00	0.72	6.54	0.00	7.31	0.00	1.36	20.41	1.08
time (sec)	N/A	0.194	0.072	0.074	0.000	1.322	0.000	0.206	2.684	0.344

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	B	F	B	F(-1)	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	224	224	164	3119	0	2321	0	424	4274	264
N.S.	1	1.00	0.73	13.92	0.00	10.36	0.00	1.89	19.08	1.18
time (sec)	N/A	0.323	0.145	0.073	0.000	2.360	0.000	0.256	6.202	0.855

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	B	F	B	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	320	320	239	5158	0	4093	0	516	8936	412
N.S.	1	1.00	0.75	16.12	0.00	12.79	0.00	1.61	27.92	1.29
time (sec)	N/A	0.525	0.321	0.079	0.000	4.782	0.000	0.309	9.488	1.344

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	143	143	145	1150	228	483	0	203	194	167
N.S.	1	1.00	1.01	8.04	1.59	3.38	0.00	1.42	1.36	1.17
time (sec)	N/A	0.151	0.430	0.061	1.341	0.819	0.000	0.256	2.050	0.283

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	B	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	122	118	97	588	190	407	0	163	144	134
N.S.	1	0.97	0.80	4.82	1.56	3.34	0.00	1.34	1.18	1.10
time (sec)	N/A	0.095	0.082	0.065	1.308	0.985	0.000	0.205	2.219	0.246

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	B	A	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	103	103	60	541	170	331	1479	145	87	103
N.S.	1	1.00	0.58	5.25	1.65	3.21	14.36	1.41	0.84	1.00
time (sec)	N/A	0.065	0.035	0.064	1.251	0.628	155.819	0.220	2.910	0.203

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	B	A	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	79	82	38	271	101	225	774	98	34	78
N.S.	1	1.04	0.48	3.43	1.28	2.85	9.80	1.24	0.43	0.99
time (sec)	N/A	0.038	0.025	0.068	1.291	0.829	7.925	0.239	1.722	0.001

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	B	F	B	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	201	201	118	1767	0	1990	0	247	5387	237
N.S.	1	1.00	0.59	8.79	0.00	9.90	0.00	1.23	26.80	1.18
time (sec)	N/A	0.315	0.084	0.071	0.000	4.965	0.000	0.219	4.622	0.458

Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	B	F	B	F(-1)	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	287	287	178	4644	0	3887	0	576	5789	393
N.S.	1	1.00	0.62	16.18	0.00	13.54	0.00	2.01	20.17	1.37
time (sec)	N/A	0.448	0.189	0.080	0.000	5.151	0.000	0.326	8.729	1.040

Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	B	F	B	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	409	409	239	7300	0	6171	0	523	4284	587
N.S.	1	1.00	0.58	17.85	0.00	15.09	0.00	1.28	10.47	1.44
time (sec)	N/A	0.702	0.387	0.079	0.000	16.664	0.000	0.285	8.227	1.599

Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	A	F	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	123	123	167	253	0	890	0	0	4674	179
N.S.	1	1.00	1.36	2.06	0.00	7.24	0.00	0.00	38.00	1.46
time (sec)	N/A	0.094	1.401	0.096	0.000	1.776	0.000	0.000	22.224	0.399

Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	A	F	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	81	81	81	155	0	247	0	0	478	147
N.S.	1	1.00	1.00	1.91	0.00	3.05	0.00	0.00	5.90	1.81
time (sec)	N/A	0.052	0.068	0.096	0.000	1.186	0.000	0.000	6.583	0.428

Problem 171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	A	F	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	122	122	87	280	0	319	0	0	-1	158
N.S.	1	1.00	0.71	2.30	0.00	2.61	0.00	0.00	-0.01	1.30
time (sec)	N/A	0.080	0.106	0.080	0.000	1.199	0.000	0.000	0.000	0.426

Problem 172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	39	39	40	45	33	98	82	33	32	0
N.S.	1	1.00	1.03	1.15	0.85	2.51	2.10	0.85	0.82	0.00
time (sec)	N/A	0.020	0.032	0.046	1.305	0.739	0.330	0.153	0.069	0.001

Problem 173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	145	145	129	195	128	390	71	133	123	0
N.S.	1	1.00	0.89	1.34	0.88	2.69	0.49	0.92	0.85	0.00
time (sec)	N/A	0.106	0.102	0.050	1.294	0.925	0.454	0.222	0.274	0.001

Problem 174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	49	49	41	59	47	48	82	49	49	53
N.S.	1	1.00	0.84	1.20	0.96	0.98	1.67	1.00	1.00	1.08
time (sec)	N/A	0.049	0.072	0.052	0.638	0.816	0.299	0.195	0.074	0.032

Problem 175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	26	26	26	21	20	20	24	20	20	31
N.S.	1	1.00	1.00	0.81	0.77	0.77	0.92	0.77	0.77	1.19
time (sec)	N/A	0.016	0.013	0.040	0.550	0.805	0.196	0.205	0.034	0.013

Problem 176	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	17	17	17	24	23	23	27	24	13	17
N.S.	1	1.00	1.00	1.41	1.35	1.35	1.59	1.41	0.76	1.00
time (sec)	N/A	0.015	0.006	0.048	0.564	0.620	0.266	0.182	1.458	0.015

Problem 177	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	104	104	22	66	71	71	102	71	90	104
N.S.	1	1.00	0.21	0.63	0.68	0.68	0.98	0.68	0.87	1.00
time (sec)	N/A	0.086	0.007	0.044	1.135	0.824	5.721	0.166	1.497	0.134

Problem 178	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	30	30	30	23	22	22	26	23	22	32
N.S.	1	1.00	1.00	0.77	0.73	0.73	0.87	0.77	0.73	1.07
time (sec)	N/A	0.019	0.031	0.042	0.558	0.559	0.184	0.152	0.041	0.023

Problem 179	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	132	132	110	138	186	527	2744	740	131	0
N.S.	1	1.00	0.83	1.05	1.41	3.99	20.79	5.61	0.99	0.00
time (sec)	N/A	0.111	0.284	0.053	0.614	0.860	3.673	0.216	1.638	0.504

Problem 180	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	99	99	90	104	140	319	1540	450	99	0
N.S.	1	1.00	0.91	1.05	1.41	3.22	15.56	4.55	1.00	0.00
time (sec)	N/A	0.073	0.128	0.055	0.503	0.740	3.346	0.193	1.557	0.111

Problem 181	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	70	70	70	74	94	175	726	232	71	0
N.S.	1	1.00	1.00	1.06	1.34	2.50	10.37	3.31	1.01	0.00
time (sec)	N/A	0.045	0.102	0.054	0.440	0.913	1.956	0.188	1.535	0.065

Problem 182	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	40	40	37	43	48	69	236	83	38	0
N.S.	1	1.00	0.92	1.08	1.20	1.72	5.90	2.08	0.95	0.00
time (sec)	N/A	0.021	0.075	0.045	0.510	0.942	0.650	0.169	1.483	0.042

Problem 183	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	F(-1)	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	158	158	149	164	242	667	0	947	157	0
N.S.	1	1.00	0.94	1.04	1.53	4.22	0.00	5.99	0.99	0.00
time (sec)	N/A	0.130	0.211	0.057	0.622	0.876	0.000	0.243	1.705	0.601

Problem 184	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	112	112	105	117	168	370	1765	539	108	0
N.S.	1	1.00	0.94	1.04	1.50	3.30	15.76	4.81	0.96	0.00
time (sec)	N/A	0.082	0.197	0.055	0.604	0.837	77.474	0.225	1.566	0.619

Problem 185	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	70	70	70	74	94	175	726	232	71	0
N.S.	1	1.00	1.00	1.06	1.34	2.50	10.37	3.31	1.01	0.00
time (sec)	N/A	0.047	0.108	0.049	0.633	0.797	2.737	0.272	1.531	0.077

Problem 186	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F	B	F(-1)	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	178	178	218	0	0	478	0	0	-1	0
N.S.	1	1.00	1.22	0.00	0.00	2.69	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.086	0.142	0.828	0.000	0.910	0.000	0.000	0.000	0.070

Problem 187	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F	A	F(-1)	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	116	116	113	0	0	231	0	0	-1	0
N.S.	1	1.00	0.97	0.00	0.00	1.99	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.036	0.083	0.797	0.000	0.635	0.000	0.000	0.000	0.064

Problem 188	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F	A	F(-1)	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	58	58	82	0	0	85	0	0	-1	0
N.S.	1	1.00	1.41	0.00	0.00	1.47	0.00	0.00	-0.02	0.00
time (sec)	N/A	0.014	0.137	0.598	0.000	0.913	0.000	0.000	0.000	0.049

Problem 189	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	A	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	18	18	18	53	0	31	211	0	75	0
N.S.	1	1.00	1.00	2.94	0.00	1.72	11.72	0.00	4.17	0.00
time (sec)	N/A	0.003	0.036	0.073	0.000	0.725	33.049	0.000	1.756	0.019

Problem 190	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F	A	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	57	57	55	0	0	108	0	0	-1	0
N.S.	1	1.00	0.96	0.00	0.00	1.89	0.00	0.00	-0.02	0.00
time (sec)	N/A	0.027	0.069	0.904	0.000	0.852	0.000	0.000	0.000	0.600

Problem 191	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F	A	F(-1)	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	327	327	136	0	0	400	0	0	-1	0
N.S.	1	1.00	0.42	0.00	0.00	1.22	0.00	0.00	-0.00	0.00
time (sec)	N/A	0.184	0.463	0.766	0.000	0.874	0.000	0.000	0.000	0.067

Problem 192	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F	A	F(-1)	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	127	127	94	0	0	173	0	0	-1	0
N.S.	1	1.00	0.74	0.00	0.00	1.36	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.064	0.170	0.632	0.000	0.851	0.000	0.000	0.000	0.049

Problem 193	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F	A	F(-1)	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	50	50	55	0	0	68	0	0	64	0
N.S.	1	1.00	1.10	0.00	0.00	1.36	0.00	0.00	1.28	0.00
time (sec)	N/A	0.012	0.029	0.605	0.000	0.903	0.000	0.000	1.763	0.017

Problem 194	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	152	164	110	92	178	114	0	621	152	340
N.S.	1	1.08	0.72	0.61	1.17	0.75	0.00	4.09	1.00	2.24
time (sec)	N/A	0.118	0.104	0.043	0.500	0.957	0.000	0.760	1.761	0.221

Problem 195	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	109	118	88	68	124	90	0	495	118	246
N.S.	1	1.08	0.81	0.62	1.14	0.83	0.00	4.54	1.08	2.26
time (sec)	N/A	0.087	0.060	0.046	0.498	0.823	0.000	0.748	1.736	0.183

Problem 196	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	67	72	62	44	70	66	0	361	83	152
N.S.	1	1.07	0.93	0.66	1.04	0.99	0.00	5.39	1.24	2.27
time (sec)	N/A	0.044	0.046	0.046	0.619	0.634	0.000	0.371	1.641	0.142

Problem 197	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	A	F	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	80	80	85	174	52	80	0	78	248	161
N.S.	1	1.00	1.06	2.18	0.65	1.00	0.00	0.98	3.10	2.01
time (sec)	N/A	0.078	0.243	0.101	1.300	0.704	0.000	0.320	3.598	0.129

Problem 198	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	A	F	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	96	114	114	182	98	85	0	157	584	267
N.S.	1	1.19	1.19	1.90	1.02	0.89	0.00	1.64	6.08	2.78
time (sec)	N/A	0.084	0.056	0.067	1.462	0.910	0.000	0.421	6.890	0.193

Problem 199	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	A	F(-1)	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	121	164	137	226	162	100	0	324	1004	206
N.S.	1	1.36	1.13	1.87	1.34	0.83	0.00	2.68	8.30	1.70
time (sec)	N/A	0.104	0.092	0.066	1.353	0.803	0.000	0.455	15.557	0.181

Problem 200	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	A	A	F	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	208	208	161	298	246	138	0	558	2314	385
N.S.	1	1.00	0.77	1.43	1.18	0.66	0.00	2.68	11.12	1.85
time (sec)	N/A	0.149	0.309	0.095	0.548	1.416	0.000	0.627	39.151	0.300

Problem 201	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	A	A	F	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	159	159	135	240	192	112	0	432	1681	297
N.S.	1	1.00	0.85	1.51	1.21	0.70	0.00	2.72	10.57	1.87
time (sec)	N/A	0.123	0.191	0.067	0.666	0.818	0.000	0.458	42.568	0.239

Problem 202	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	A	A	F	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	114	114	129	182	137	88	0	288	734	207
N.S.	1	1.00	1.13	1.60	1.20	0.77	0.00	2.53	6.44	1.82
time (sec)	N/A	0.047	0.279	0.059	0.543	0.871	0.000	0.350	17.425	0.185

Problem 203	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	104	104	101	153	105	83	0	110	243	200
N.S.	1	1.00	0.97	1.47	1.01	0.80	0.00	1.06	2.34	1.92
time (sec)	N/A	0.087	0.089	0.064	1.500	0.740	0.000	0.395	3.486	0.204

Problem 204	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	A	A	F(-1)	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	84	84	105	153	75	100	0	171	236	145
N.S.	1	1.00	1.25	1.82	0.89	1.19	0.00	2.04	2.81	1.73
time (sec)	N/A	0.080	0.086	0.066	1.472	0.795	0.000	0.383	3.438	0.171

Problem 205	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	A	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	125	125	117	191	153	96	0	172	1154	255
N.S.	1	1.00	0.94	1.53	1.22	0.77	0.00	1.38	9.23	2.04
time (sec)	N/A	0.085	0.143	0.106	0.554	0.884	0.000	0.231	32.625	0.206

Problem 206	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	C	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	103	103	70	57	95	55	216	108	108	196
N.S.	1	1.00	0.68	0.55	0.92	0.53	2.10	1.05	1.05	1.90
time (sec)	N/A	0.075	0.043	0.046	0.633	1.046	62.839	0.223	2.440	0.132

Problem 207	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	A	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	87	87	98	147	113	77	0	121	720	175
N.S.	1	1.00	1.13	1.69	1.30	0.89	0.00	1.39	8.28	2.01
time (sec)	N/A	0.069	0.105	0.078	0.537	0.826	0.000	0.373	22.496	0.158

Problem 208	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	C	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	65	65	52	38	54	37	202	59	66	122
N.S.	1	1.00	0.80	0.58	0.83	0.57	3.11	0.91	1.02	1.88
time (sec)	N/A	0.040	0.054	0.044	0.469	0.776	41.910	0.222	2.359	0.098

Problem 209	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	C	A	A	C	A	B	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	47	47	101	103	74	55	182	69	293	88
N.S.	1	1.00	2.15	2.19	1.57	1.17	3.87	1.47	6.23	1.87
time (sec)	N/A	0.020	0.212	0.066	0.497	0.875	45.512	0.176	12.685	0.134

Problem 210	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	C	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	46	46	66	62	29	48	162	45	77	65
N.S.	1	1.00	1.43	1.35	0.63	1.04	3.52	0.98	1.67	1.41
time (sec)	N/A	0.061	0.033	0.072	1.152	0.830	40.135	0.224	3.865	0.081

Problem 211	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	C	A	A	C	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	33	33	73	77	44	56	148	58	61	66
N.S.	1	1.00	2.21	2.33	1.33	1.70	4.48	1.76	1.85	2.00
time (sec)	N/A	0.057	0.030	0.072	1.177	0.869	35.169	0.216	2.593	0.093

Problem 212	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	C	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	60	60	77	84	45	57	141	114	297	87
N.S.	1	1.00	1.28	1.40	0.75	0.95	2.35	1.90	4.95	1.45
time (sec)	N/A	0.063	0.071	0.082	1.226	0.839	63.618	0.213	8.668	0.100

Problem 213	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	C	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	62	62	51	37	54	52	146	116	53	122
N.S.	1	1.00	0.82	0.60	0.87	0.84	2.35	1.87	0.85	1.97
time (sec)	N/A	0.062	0.018	0.050	1.233	0.849	61.444	0.206	2.440	0.113

Problem 214	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-1)	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	99	99	102	125	85	78	0	268	650	175
N.S.	1	1.00	1.03	1.26	0.86	0.79	0.00	2.71	6.57	1.77
time (sec)	N/A	0.078	0.102	0.074	1.107	0.591	0.000	0.221	21.455	0.165

Problem 215	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	A	A	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	164	164	148	240	196	115	0	203	1682	297
N.S.	1	1.00	0.90	1.46	1.20	0.70	0.00	1.24	10.26	1.81
time (sec)	N/A	0.120	0.124	0.099	0.568	0.778	0.000	0.280	42.656	0.243

Problem 216	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	C	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	118	118	87	68	124	66	240	124	130	246
N.S.	1	1.00	0.74	0.58	1.05	0.56	2.03	1.05	1.10	2.08
time (sec)	N/A	0.086	0.062	0.046	0.551	0.782	70.843	0.251	2.700	0.151

Problem 217	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	A	A	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	118	118	121	182	142	90	0	140	1048	209
N.S.	1	1.00	1.03	1.54	1.20	0.76	0.00	1.19	8.88	1.77
time (sec)	N/A	0.097	0.098	0.075	0.639	1.205	0.000	0.233	25.513	0.189

Problem 218	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	C	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	72	72	61	43	69	42	223	65	76	146
N.S.	1	1.00	0.85	0.60	0.96	0.58	3.10	0.90	1.06	2.03
time (sec)	N/A	0.046	0.038	0.040	0.645	0.764	44.702	0.198	2.663	0.126

Problem 219	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	A	A	C	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	68	68	119	124	89	63	199	79	417	103
N.S.	1	1.00	1.75	1.82	1.31	0.93	2.93	1.16	6.13	1.51
time (sec)	N/A	0.032	0.349	0.071	0.459	0.863	41.776	0.265	10.800	0.139

Problem 220	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	A	C	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	56	56	87	108	37	61	178	55	108	75
N.S.	1	1.00	1.55	1.93	0.66	1.09	3.18	0.98	1.93	1.34
time (sec)	N/A	0.069	0.037	0.077	1.342	0.847	39.158	0.209	3.967	0.112

Problem 221	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	A	A	C	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	57	57	90	97	55	68	165	66	77	75
N.S.	1	1.00	1.58	1.70	0.96	1.19	2.89	1.16	1.35	1.32
time (sec)	N/A	0.075	0.041	0.069	1.257	1.085	36.825	0.217	2.945	0.119

Problem 222	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	A	C	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	76	76	102	158	60	73	162	141	457	104
N.S.	1	1.00	1.34	2.08	0.79	0.96	2.13	1.86	6.01	1.37
time (sec)	N/A	0.073	0.102	0.072	1.292	0.813	69.669	0.494	7.500	0.125

Problem 223	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	C	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	75	75	66	49	75	67	170	137	79	146
N.S.	1	1.00	0.88	0.65	1.00	0.89	2.27	1.83	1.05	1.95
time (sec)	N/A	0.069	0.031	0.041	1.319	0.744	70.802	0.244	2.765	0.136

Problem 224	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	A	F(-1)	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	123	123	144	227	114	100	0	325	1005	209
N.S.	1	1.00	1.17	1.85	0.93	0.81	0.00	2.64	8.17	1.70
time (sec)	N/A	0.095	0.108	0.070	1.208	0.857	0.000	0.271	19.135	0.191

Problem 225	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	A	A	F(-1)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	161	161	119	316	196	190	0	214	-1	297
N.S.	1	1.00	0.74	1.96	1.22	1.18	0.00	1.33	-0.01	1.84
time (sec)	N/A	0.123	0.162	0.087	0.492	0.863	0.000	0.398	0.000	0.264

Problem 226	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	C	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	115	115	72	68	123	80	226	200	90	236
N.S.	1	1.00	0.63	0.59	1.07	0.70	1.97	1.74	0.78	2.05
time (sec)	N/A	0.094	0.054	0.049	0.464	0.663	177.265	0.435	2.801	0.162

Problem 227	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	A	A	F(-1)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	152	152	90	254	138	159	0	147	-1	196
N.S.	1	1.00	0.59	1.67	0.91	1.05	0.00	0.97	-0.01	1.29
time (sec)	N/A	0.115	0.121	0.077	0.500	0.666	0.000	0.321	0.000	0.190

Problem 228	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	C	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	76	76	45	43	69	56	201	152	67	173
N.S.	1	1.00	0.59	0.57	0.91	0.74	2.64	2.00	0.88	2.28
time (sec)	N/A	0.054	0.067	0.043	0.576	0.770	136.305	0.280	2.747	0.141

Problem 229	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	A	B	C	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	63	63	86	160	76	129	182	113	-1	87
N.S.	1	1.00	1.37	2.54	1.21	2.05	2.89	1.79	-0.02	1.38
time (sec)	N/A	0.032	0.244	0.073	0.548	0.837	112.361	0.280	0.000	0.146

Problem 230	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	A	C	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	65	65	84	188	58	101	172	115	-1	87
N.S.	1	1.00	1.29	2.89	0.89	1.55	2.65	1.77	-0.02	1.34
time (sec)	N/A	0.081	0.044	0.076	1.463	0.813	136.445	0.362	0.000	0.111

Problem 231	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	C	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	67	67	51	48	71	103	165	219	73	146
N.S.	1	1.00	0.76	0.72	1.06	1.54	2.46	3.27	1.09	2.18
time (sec)	N/A	0.077	0.027	0.050	1.365	0.781	136.133	0.459	2.866	0.154

Problem 232	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	B	A	A	F(-1)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	117	117	75	315	104	138	0	211	-1	196
N.S.	1	1.00	0.64	2.69	0.89	1.18	0.00	1.80	-0.01	1.68
time (sec)	N/A	0.098	0.032	0.091	1.265	0.716	0.000	0.542	0.000	0.183

Problem 233	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-1)	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	119	119	77	73	125	132	0	242	104	236
N.S.	1	1.00	0.65	0.61	1.05	1.11	0.00	2.03	0.87	1.98
time (sec)	N/A	0.096	0.031	0.046	1.342	0.804	0.000	0.728	2.897	0.181

Problem 234	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	B	A	A	F(-1)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	166	166	78	387	162	165	0	402	-1	297
N.S.	1	1.00	0.47	2.33	0.98	0.99	0.00	2.42	-0.01	1.79
time (sec)	N/A	0.120	0.033	0.084	1.480	0.884	0.000	0.794	0.000	0.248

Problem 235	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	C	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	40	40	56	53	23	39	148	40	72	60
N.S.	1	1.00	1.40	1.32	0.58	0.98	3.70	1.00	1.80	1.50
time (sec)	N/A	0.054	0.026	0.069	1.351	0.885	30.114	0.171	3.652	0.069

Problem 236	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	F(-1)	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	53	53	244	66	79	65	0	0	96	0
N.S.	1	1.00	4.60	1.25	1.49	1.23	0.00	0.00	1.81	0.00
time (sec)	N/A	0.091	0.314	0.053	1.244	0.858	0.000	0.000	3.274	24.091

Problem 237	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F	C	F	F	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	36	36	36	0	0	69	0	0	-1	44
N.S.	1	1.00	1.00	0.00	0.00	1.92	0.00	0.00	-0.03	1.22
time (sec)	N/A	0.020	0.020	0.857	0.000	0.871	0.000	0.000	0.000	7.622

Problem 238	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F	A	F	F	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	75	75	75	0	0	50	0	0	-1	58
N.S.	1	1.00	1.00	0.00	0.00	0.67	0.00	0.00	-0.01	0.77
time (sec)	N/A	0.050	0.058	0.877	0.000	0.725	0.000	0.000	0.000	10.946

Problem 239	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F	A	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	96	104	103	0	0	180	0	0	-1	0
N.S.	1	1.08	1.07	0.00	0.00	1.88	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.122	0.394	2.167	0.000	0.846	0.000	0.000	0.000	1.414

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [177] had the largest ratio of [.5294]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	2	1	1.00	17	0.059
2	A	2	1	1.00	17	0.059
3	A	2	1	1.00	17	0.059
4	A	2	1	1.00	15	0.067
5	A	7	7	1.00	17	0.412
6	A	7	7	1.00	17	0.412
7	A	8	8	1.00	17	0.471
8	A	2	1	1.00	19	0.053
9	A	2	1	1.00	19	0.053
10	A	2	1	1.00	17	0.059
11	A	8	7	1.00	19	0.368
12	A	9	8	1.00	19	0.421
13	A	8	8	1.00	19	0.421
14	A	8	7	1.00	19	0.368
15	A	8	7	1.00	19	0.368
16	A	8	7	1.00	19	0.368
17	A	7	7	1.00	17	0.412
18	A	13	7	1.00	19	0.368
19	A	14	8	1.00	19	0.421
20	A	9	8	1.00	19	0.421
21	A	9	8	1.00	19	0.421
22	A	9	8	1.00	19	0.421
23	A	9	8	1.00	19	0.421
24	A	7	7	1.00	17	0.412
25	A	14	8	1.00	19	0.421
26	A	15	9	1.00	19	0.474
27	A	3	3	1.00	20	0.150
28	A	2	2	1.00	20	0.100
29	A	2	2	1.00	20	0.100
30	A	2	2	1.00	20	0.100
31	A	3	3	1.00	20	0.150
32	A	4	3	1.00	20	0.150
33	A	5	3	1.00	20	0.150
34	C	2	2	0.15	22	0.091
35	A	4	4	1.00	22	0.182
36	A	3	3	1.00	22	0.136
37	A	3	3	1.00	22	0.136
38	A	3	3	1.00	22	0.136
39	A	3	2	1.00	22	0.091

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
40	A	4	3	1.00	22	0.136
41	A	5	4	1.00	22	0.182
42	A	6	4	1.00	22	0.182
43	A	4	3	1.00	19	0.158
44	A	3	3	1.00	19	0.158
45	A	2	2	1.00	19	0.105
46	A	2	2	1.00	19	0.105
47	A	2	2	1.00	19	0.105
48	A	3	3	1.00	19	0.158
49	A	4	3	1.00	19	0.158
50	A	5	3	1.00	19	0.158
51	A	5	4	1.00	21	0.190
52	A	4	4	1.00	21	0.190
53	A	3	3	1.00	21	0.143
54	A	3	3	1.00	21	0.143
55	A	3	3	1.00	21	0.143
56	A	3	2	1.00	21	0.095
57	A	4	3	1.00	21	0.143
58	A	5	4	1.00	21	0.190
59	A	6	4	1.00	21	0.190
60	A	4	2	1.00	21	0.095
61	C	2	2	0.19	21	0.095
62	C	2	2	0.22	21	0.095
63	C	2	2	0.25	21	0.095
64	A	7	7	1.40	21	0.333
65	A	8	8	1.33	21	0.381
66	C	2	2	2.75	21	0.095
67	C	2	2	4.19	21	0.095
68	C	2	2	0.18	21	0.095
69	C	2	2	0.20	21	0.095
70	A	8	8	1.32	21	0.381
71	A	8	8	1.27	21	0.381
72	C	2	2	2.39	21	0.095
73	C	2	2	3.75	21	0.095
74	C	2	2	0.11	21	0.095
75	C	2	2	0.14	21	0.095
76	C	2	2	0.16	21	0.095
77	A	9	8	1.27	21	0.381
78	A	9	9	1.22	21	0.429
79	C	2	2	0.54	21	0.095
80	C	2	2	1.14	21	0.095
81	C	2	2	4.30	21	0.095
82	A	1	1	1.00	50	0.020
83	A	2	1	1.00	17	0.059
84	A	2	1	1.00	17	0.059
85	A	2	1	1.00	17	0.059
86	A	2	1	1.00	15	0.067
87	A	10	7	1.00	17	0.412

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
88	A	10	7	1.00	17	0.412
89	A	11	8	1.00	17	0.471
90	A	2	1	1.00	19	0.053
91	A	2	1	1.00	19	0.053
92	A	2	1	1.00	19	0.053
93	A	2	1	1.00	17	0.059
94	A	11	7	1.00	19	0.368
95	A	12	8	1.00	19	0.421
96	A	11	8	1.00	19	0.421
97	A	11	7	1.00	19	0.368
98	A	11	7	1.00	19	0.368
99	A	11	7	1.00	19	0.368
100	A	10	7	1.00	17	0.412
101	A	19	7	1.00	19	0.368
102	A	20	8	1.00	19	0.421
103	A	12	8	1.00	19	0.421
104	A	12	8	1.00	19	0.421
105	A	12	8	1.00	19	0.421
106	A	12	8	1.00	19	0.421
107	A	10	7	1.00	17	0.412
108	A	20	8	1.00	19	0.421
109	A	21	9	1.00	19	0.474
110	A	4	4	1.00	25	0.160
111	A	1	1	1.00	25	0.040
112	A	10	9	1.00	21	0.429
113	A	9	8	1.00	21	0.381
114	A	4	4	1.00	21	0.190
115	A	5	5	1.00	21	0.238
116	A	7	7	1.00	21	0.333
117	A	8	7	1.00	21	0.333
118	A	11	10	1.00	21	0.476
119	A	10	9	1.00	21	0.429
120	A	5	5	1.00	21	0.238
121	A	5	5	1.00	21	0.238
122	A	7	7	1.00	21	0.333
123	A	8	7	1.00	21	0.333
124	A	4	4	1.00	17	0.235
125	A	4	4	1.00	26	0.154
126	A	7	7	1.00	21	0.333
127	A	6	6	1.00	21	0.286
128	A	6	6	1.00	21	0.286
129	A	5	5	1.00	19	0.263
130	A	4	4	1.00	11	0.364
131	A	7	6	1.00	21	0.286
132	A	8	7	1.00	21	0.333
133	A	9	7	1.00	21	0.333
134	A	7	7	1.00	21	0.333
135	A	7	6	1.00	21	0.286

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
136	A	6	5	1.00	19	0.263
137	A	5	5	1.00	11	0.454
138	A	7	6	1.00	21	0.286
139	A	8	7	1.00	21	0.333
140	A	9	7	1.00	21	0.333
141	A	8	7	1.00	21	0.333
142	A	8	6	1.00	21	0.286
143	A	7	5	1.00	19	0.263
144	A	6	5	1.00	11	0.454
145	A	8	7	1.00	21	0.333
146	A	8	7	1.00	21	0.333
147	A	9	8	1.00	21	0.381
148	A	5	5	1.00	21	0.238
149	A	5	5	1.00	21	0.238
150	A	4	4	1.00	19	0.210
151	A	4	4	1.00	11	0.364
152	A	7	6	1.00	21	0.286
153	A	8	7	1.00	21	0.333
154	A	9	7	1.00	21	0.333
155	A	5	5	1.00	21	0.238
156	A	5	5	0.96	21	0.238
157	A	5	5	1.00	19	0.263
158	A	5	4	1.02	11	0.364
159	A	8	7	1.00	21	0.333
160	A	9	8	1.00	21	0.381
161	A	10	8	1.00	21	0.381
162	A	5	5	1.00	21	0.238
163	A	6	6	0.97	21	0.286
164	A	6	5	1.00	19	0.263
165	A	6	4	1.04	11	0.364
166	A	9	7	1.00	21	0.333
167	A	10	8	1.00	21	0.381
168	A	11	8	1.00	21	0.381
169	A	8	8	1.00	23	0.348
170	A	4	4	1.00	23	0.174
171	A	5	5	1.00	23	0.217
172	A	3	3	1.00	17	0.176
173	A	8	8	1.00	17	0.471
174	A	3	2	1.00	21	0.095
175	A	3	2	1.00	17	0.118
176	A	4	4	1.00	17	0.235
177	A	9	9	1.00	17	0.529
178	A	4	3	1.00	17	0.176
179	A	2	1	1.00	17	0.059
180	A	2	1	1.00	17	0.059
181	A	2	1	1.00	17	0.059
182	A	2	1	1.00	15	0.067
183	A	2	1	1.00	19	0.053

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
184	A	2	1	1.00	19	0.053
185	A	2	1	1.00	17	0.059
186	A	4	2	1.00	25	0.080
187	A	3	2	1.00	25	0.080
188	A	2	2	1.00	23	0.087
189	A	1	1	1.00	15	0.067
190	A	1	1	1.00	69	0.014
191	A	5	3	1.00	25	0.120
192	A	3	3	1.00	23	0.130
193	A	2	2	1.00	15	0.133
194	A	6	4	1.08	31	0.129
195	A	4	4	1.08	31	0.129
196	A	2	2	1.07	29	0.069
197	A	5	5	1.00	31	0.161
198	A	5	5	1.19	31	0.161
199	A	5	4	1.36	31	0.129
200	A	9	8	1.00	31	0.258
201	A	7	7	1.00	31	0.226
202	A	5	5	1.00	28	0.179
203	A	5	5	1.00	31	0.161
204	A	6	6	1.00	31	0.194
205	A	5	5	1.00	29	0.172
206	A	4	4	1.00	29	0.138
207	A	3	3	1.00	29	0.103
208	A	2	2	1.00	27	0.074
209	A	2	2	1.00	26	0.077
210	A	3	3	1.00	29	0.103
211	A	2	2	1.00	29	0.069
212	A	3	3	1.00	29	0.103
213	A	2	2	1.00	29	0.069
214	A	5	5	1.00	29	0.172
215	A	8	7	1.00	31	0.226
216	A	4	4	1.00	31	0.129
217	A	6	6	1.00	31	0.194
218	A	2	2	1.00	29	0.069
219	A	4	4	1.00	28	0.143
220	A	3	3	1.00	31	0.097
221	A	4	4	1.00	31	0.129
222	A	3	3	1.00	31	0.097
223	A	2	2	1.00	31	0.065
224	A	5	5	1.00	31	0.161
225	A	8	8	1.00	31	0.258
226	A	4	4	1.00	31	0.129
227	A	7	7	1.00	31	0.226
228	A	2	2	1.00	29	0.069
229	A	4	4	1.00	28	0.143
230	A	3	3	1.00	31	0.097
231	A	2	2	1.00	31	0.065

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
232	A	5	5	1.00	31	0.161
233	A	4	4	1.00	31	0.129
234	A	7	7	1.00	31	0.226
235	A	3	3	1.00	31	0.097
236	A	1	1	1.00	57	0.018
237	A	3	3	1.00	32	0.094
238	A	4	4	1.00	41	0.098
239	A	2	2	1.08	76	0.026

Chapter 3

Listing of integrals

Local contents

3.1 $\int \left(a+bx^3\right)\left(c+dx^3\right)^4\,dx$

Optimal. Leaf size=94

$$\frac{1}{4}c^3x^4(4ad+bc)+\frac{2}{7}c^2dx^7(3ad+2bc)+\frac{1}{13}d^3x^{13}(ad+4bc)+\frac{1}{5}cd^2x^{10}(2ad+3bc)+ac^4x+\frac{1}{16}bd^4x^{16}$$

Rubi [A] time = 0.07, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {373}

$$\frac{2}{7}c^2dx^7(3ad+2bc)+\frac{1}{4}c^3x^4(4ad+bc)+\frac{1}{13}d^3x^{13}(ad+4bc)+\frac{1}{5}cd^2x^{10}(2ad+3bc)+ac^4x+\frac{1}{16}bd^4x^{16}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)*(c + d*x^3)^4,x]
```

```
[Out] a*c^4*x + (c^3*(b*c + 4*a*d)*x^4)/4 + (2*c^2*d*(2*b*c + 3*a*d)*x^7)/7 + (c*
d^2*(3*b*c + 2*a*d)*x^10)/5 + (d^3*(4*b*c + a*d)*x^13)/13 + (b*d^4*x^16)/16
```

Rule 373

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\int \left(a+bx^3\right)\left(c+dx^3\right)^4\,dx = \int \left(ac^4+c^3(bc+4ad)x^3+2c^2d(2bc+3ad)x^6+2cd^2(3bc+2ad)x^9+d^3(4bc+ad)x\right.$$
$$= ac^4x+\frac{1}{4}c^3(bc+4ad)x^4+\frac{2}{7}c^2d(2bc+3ad)x^7+\frac{1}{5}cd^2(3bc+2ad)x^{10}+\frac{1}{13}d^3(4bc+a$$

Mathematica [A] time = 0.05, size = 94, normalized size = 1.00

$$\frac{1}{4}c^3x^4(4ad+bc)+\frac{2}{7}c^2dx^7(3ad+2bc)+\frac{1}{13}d^3x^{13}(ad+4bc)+\frac{1}{5}cd^2x^{10}(2ad+3bc)+ac^4x+\frac{1}{16}bd^4x^{16}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)*(c + d*x^3)^4,x]
```

```
[Out] a*c^4*x + (c^3*(b*c + 4*a*d)*x^4)/4 + (2*c^2*d*(2*b*c + 3*a*d)*x^7)/7 + (c*
d^2*(3*b*c + 2*a*d)*x^10)/5 + (d^3*(4*b*c + a*d)*x^13)/13 + (b*d^4*x^16)/16
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+bx^3\right)\left(c+dx^3\right)^4\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x^3)*(c + d*x^3)^4,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x^3)*(c + d*x^3)^4, x]
```

fricas [A] time = 0.93, size = 97, normalized size = 1.03

$$\frac{1}{16}x^{16}d^4b+\frac{4}{13}x^{13}d^3cb+\frac{1}{13}x^{13}d^4a+\frac{3}{5}x^{10}d^2c^2b+\frac{2}{5}x^{10}d^3ca+\frac{4}{7}x^7dc^3b+\frac{6}{7}x^7d^2c^2a+\frac{1}{4}x^4c^4b+x^4dc^3a+xc^4a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*(d*x^3+c)^4,x, algorithm="fricas")
```

```
[Out] 1/16*x^16*d^4*b + 4/13*x^13*d^3*c*b + 1/13*x^13*d^4*a + 3/5*x^10*d^2*c^2*b
+ 2/5*x^10*d^3*c*a + 4/7*x^7*d*c^3*b + 6/7*x^7*d^2*c^2*a + 1/4*x^4*c^4*b +
x^4*d*c^3*a + x*c^4*a
```

giac [A] time = 0.17, size = 97, normalized size = 1.03

$$\frac{1}{16}\,bd^4x^{16}+\frac{4}{13}\,bcd^3x^{13}+\frac{1}{13}\,ad^4x^{13}+\frac{3}{5}\,bc^2d^2x^{10}+\frac{2}{5}\,acd^3x^{10}+\frac{4}{7}\,bc^3dx^7+\frac{6}{7}\,ac^2d^2x^7+\frac{1}{4}\,bc^4x^4+ac^3dx^4+ac^4x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*(d*x^3+c)^4,x, algorithm="giac")
```

```
[Out] 1/16*b*d^4*x^16 + 4/13*b*c*d^3*x^13 + 1/13*a*d^4*x^13 + 3/5*b*c^2*d^2*x^10
+ 2/5*a*c*d^3*x^10 + 4/7*b*c^3*d*x^7 + 6/7*a*c^2*d^2*x^7 + 1/4*b*c^4*x^4 +
a*c^3*d*x^4 + a*c^4*x
```

maple [A] time = 0.04, size = 97, normalized size = 1.03

$$\frac{b\,d^4x^{16}}{16}+\frac{\left(a\,d^4+4bc\,d^3\right)x^{13}}{13}+\frac{\left(4ac\,d^3+6c^2d^2b\right)x^{10}}{10}+\frac{\left(6a\,c^2d^2+4c^3db\right)x^7}{7}+a\,c^4x+\frac{\left(4a\,c^3d+b\,c^4\right)x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)*(d*x^3+c)^4,x)
```

```
[Out] 1/16*b*d^4*x^16+1/13*(a*d^4+4*b*c*d^3)*x^13+1/10*(4*a*c*d^3+6*b*c^2*d^2)*x^
10+1/7*(6*a*c^2*d^2+4*b*c^3*d)*x^7+1/4*(4*a*c^3*d+b*c^4)*x^4+a*c^4*x
```

maxima [A] time = 0.47, size = 96, normalized size = 1.02

$$\frac{1}{16}\,bd^4x^{16}+\frac{1}{13}\left(4\,bcd^3+ad^4\right)x^{13}+\frac{1}{5}\left(3\,bc^2d^2+2\,acd^3\right)x^{10}+\frac{2}{7}\left(2\,bc^3d+3\,ac^2d^2\right)x^7+ac^4x+\frac{1}{4}\left(bc^4+4\,ac^3d\right)x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*(d*x^3+c)^4,x, algorithm="maxima")
```

```
[Out] 1/16*b*d^4*x^16 + 1/13*(4*b*c*d^3 + a*d^4)*x^13 + 1/5*(3*b*c^2*d^2 + 2*a*c*
d^3)*x^10 + 2/7*(2*b*c^3*d + 3*a*c^2*d^2)*x^7 + a*c^4*x + 1/4*(b*c^4 + 4*a*
c^3*d)*x^4
```

mupad [B] time = 0.05, size = 87, normalized size = 0.93

$$x^4\left(\frac{b\,c^4}{4}+a\,d\,c^3\right)+x^{13}\left(\frac{a\,d^4}{13}+\frac{4\,b\,c\,d^3}{13}\right)+\frac{b\,d^4\,x^{16}}{16}+a\,c^4\,x+\frac{2\,c^2\,d\,x^7\,\left(3\,a\,d+2\,b\,c\right)}{7}+\frac{c\,d^2\,x^{10}\,\left(2\,a\,d+3\,b\,c\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)*(c + d*x^3)^4,x)
```

```
[Out] x^4*((b*c^4)/4 + a*c^3*d) + x^13*((a*d^4)/13 + (4*b*c*d^3)/13) + (b*d^4*x^1
6)/16 + a*c^4*x + (2*c^2*d*x^7*(3*a*d + 2*b*c))/7 + (c*d^2*x^10*(2*a*d + 3*
b*c))/5
```

sympy [A] time = 0.13, size = 104, normalized size = 1.11

$$ac^4x+\frac{bd^4x^{16}}{16}+x^{13}\left(\frac{ad^4}{13}+\frac{4bcd^3}{13}\right)+x^{10}\left(\frac{2acd^3}{5}+\frac{3bc^2d^2}{5}\right)+x^7\left(\frac{6ac^2d^2}{7}+\frac{4bc^3d}{7}\right)+x^4\left(ac^3d+\frac{bc^4}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)*(d*x**3+c)**4,x)
```

```
[Out] a*c**4*x + b*d**4*x**16/16 + x**13*(a*d**4/13 + 4*b*c*d**3/13) + x**10*(2*a
*c*d**3/5 + 3*b*c**2*d**2/5) + x**7*(6*a*c**2*d**2/7 + 4*b*c**3*d/7) + x**4
*(a*c**3*d + b*c**4/4)
```

3.2 $\int \left(a+b x^3\right) \left(c+d x^3\right)^3 \, dx$

Optimal. Leaf size=70

$$\frac{1}{4} c^2 x^4 (3 a d+b c)+\frac{1}{10} d^2 x^{10} (a d+3 b c)+\frac{3}{7} c d x^7 (a d+b c)+a c^3 x+\frac{1}{13} b d^3 x^{13}$$

Rubi [A] time = 0.04, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {373}

$$\frac{1}{4} c^2 x^4 (3 a d+b c)+\frac{1}{10} d^2 x^{10} (a d+3 b c)+\frac{3}{7} c d x^7 (a d+b c)+a c^3 x+\frac{1}{13} b d^3 x^{13}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)*(c + d*x^3)^3,x]
```

```
[Out] a*c^3*x + (c^2*(b*c + 3*a*d)*x^4)/4 + (3*c*d*(b*c + a*d)*x^7)/7 + (d^2*(3*b
*c + a*d)*x^10)/10 + (b*d^3*x^13)/13
```

Rule 373

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned} \int \left(a+b x^3\right) \left(c+d x^3\right)^3 \, dx &= \int \left(a c^3+c^2 (b c+3 a d) x^3+3 c d (b c+a d) x^6+d^2 (3 b c+a d) x^9+b d^3 x^{12}\right) \, dx\\ &= a c^3 x+\frac{1}{4} c^2 (b c+3 a d) x^4+\frac{3}{7} c d (b c+a d) x^7+\frac{1}{10} d^2 (3 b c+a d) x^{10}+\frac{1}{13} b d^3 x^{13} \end{aligned}$$

Mathematica [A] time = 0.01, size = 70, normalized size = 1.00

$$\frac{1}{4} c^2 x^4 (3 a d+b c)+\frac{1}{10} d^2 x^{10} (a d+3 b c)+\frac{3}{7} c d x^7 (a d+b c)+a c^3 x+\frac{1}{13} b d^3 x^{13}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)*(c + d*x^3)^3,x]
```

```
[Out] a*c^3*x + (c^2*(b*c + 3*a*d)*x^4)/4 + (3*c*d*(b*c + a*d)*x^7)/7 + (d^2*(3*b
*c + a*d)*x^10)/10 + (b*d^3*x^13)/13
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+b x^3\right) \left(c+d x^3\right)^3 \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x^3)*(c + d*x^3)^3,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x^3)*(c + d*x^3)^3, x]
```

fricas [A] time = 0.85, size = 74, normalized size = 1.06

$$\frac{1}{13} x^{13} d^3 b+\frac{3}{10} x^{10} d^2 c b+\frac{1}{10} x^{10} d^3 a+\frac{3}{7} x^7 d c^2 b+\frac{3}{7} x^7 d^2 c a+\frac{1}{4} x^4 c^3 b+\frac{3}{4} x^4 d c^2 a+x c^3 a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*(d*x^3+c)^3,x, algorithm="fricas")
```

```
[Out] 1/13*x^13*d^3*b + 3/10*x^10*d^2*c*b + 1/10*x^10*d^3*a + 3/7*x^7*d*c^2*b + 3
/7*x^7*d^2*c*a + 1/4*x^4*c^3*b + 3/4*x^4*d*c^2*a + x*c^3*a
```

giac [A] time = 0.15, size = 74, normalized size = 1.06

$$\frac{1}{13}\,bd^3x^{13}+\frac{3}{10}\,bcd^2x^{10}+\frac{1}{10}\,ad^3x^{10}+\frac{3}{7}\,bc^2dx^7+\frac{3}{7}\,acd^2x^7+\frac{1}{4}\,bc^3x^4+\frac{3}{4}\,ac^2dx^4+ac^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*(d*x^3+c)^3,x, algorithm="giac")
```

```
[Out] 1/13*b*d^3*x^13 + 3/10*b*c*d^2*x^10 + 1/10*a*d^3*x^10 + 3/7*b*c^2*d*x^7 + 3
/7*a*c*d^2*x^7 + 1/4*b*c^3*x^4 + 3/4*a*c^2*d*x^4 + a*c^3*x
```

maple [A] time = 0.04, size = 73, normalized size = 1.04

$$\frac{b\,d^3x^{13}}{13}+\frac{\left(a\,d^3+3bc\,d^2\right)x^{10}}{10}+\frac{\left(3ac\,d^2+3b\,c^2d\right)x^7}{7}+a\,c^3x+\frac{\left(3a\,c^2d+b\,c^3\right)x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)*(d*x^3+c)^3,x)
```

```
[Out] 1/13*b*d^3*x^13+1/10*(a*d^3+3*b*c*d^2)*x^10+1/7*(3*a*c*d^2+3*b*c^2*d)*x^7+1
/4*(3*a*c^2*d+b*c^3)*x^4+a*c^3*x
```

maxima [A] time = 0.56, size = 70, normalized size = 1.00

$$\frac{1}{13}\,bd^3x^{13}+\frac{1}{10}\left(3\,bcd^2+ad^3\right)x^{10}+\frac{3}{7}\left(bc^2d+acd^2\right)x^7+ac^3x+\frac{1}{4}\left(bc^3+3\,ac^2d\right)x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*(d*x^3+c)^3,x, algorithm="maxima")
```

```
[Out] 1/13*b*d^3*x^13 + 1/10*(3*b*c*d^2 + a*d^3)*x^10 + 3/7*(b*c^2*d + a*c*d^2)*x
^7 + a*c^3*x + 1/4*(b*c^3 + 3*a*c^2*d)*x^4
```

mupad [B] time = 0.03, size = 66, normalized size = 0.94

$$x^4\left(\frac{b\,c^3}{4}+\frac{3\,a\,d\,c^2}{4}\right)+x^{10}\left(\frac{a\,d^3}{10}+\frac{3\,b\,c\,d^2}{10}\right)+\frac{b\,d^3\,x^{13}}{13}+a\,c^3\,x+\frac{3\,c\,d\,x^7\,\left(a\,d+b\,c\right)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)*(c + d*x^3)^3,x)
```

```
[Out] x^4*((b*c^3)/4 + (3*a*c^2*d)/4) + x^10*((a*d^3)/10 + (3*b*c*d^2)/10) + (b*d
^3*x^13)/13 + a*c^3*x + (3*c*d*x^7*(a*d + b*c))/7
```

sympy [A] time = 0.08, size = 80, normalized size = 1.14

$$ac^3x+\frac{bd^3x^{13}}{13}+x^{10}\left(\frac{ad^3}{10}+\frac{3bcd^2}{10}\right)+x^7\left(\frac{3acd^2}{7}+\frac{3bc^2d}{7}\right)+x^4\left(\frac{3ac^2d}{4}+\frac{bc^3}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)*(d*x**3+c)**3,x)
```

```
[Out] a*c**3*x + b*d**3*x**13/13 + x**10*(a*d**3/10 + 3*b*c*d**2/10) + x**7*(3*a*
c*d**2/7 + 3*b*c**2*d/7) + x**4*(3*a*c**2*d/4 + b*c**3/4)
```

3.3 $\int \left(a+bx^3\right)\left(c+dx^3\right)^2\,dx$

Optimal. Leaf size=50

$$\frac{1}{7}dx^7(ad+2bc)+\frac{1}{4}cx^4(2ad+bc)+ac^2x+\frac{1}{10}bd^2x^{10}$$

Rubi [A] time = 0.03, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {373}

$$\frac{1}{7}dx^7(ad+2bc)+\frac{1}{4}cx^4(2ad+bc)+ac^2x+\frac{1}{10}bd^2x^{10}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)*(c + d*x^3)^2,x]
```

```
[Out] a*c^2*x + (c*(b*c + 2*a*d)*x^4)/4 + (d*(2*b*c + a*d)*x^7)/7 + (b*d^2*x^10)/
10
```

Rule 373

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \left(a+bx^3\right)\left(c+dx^3\right)^2\,dx &= \int \left(ac^2+c(bc+2ad)x^3+d(2bc+ad)x^6+bd^2x^9\right)dx\\ &= ac^2x+\frac{1}{4}c(bc+2ad)x^4+\frac{1}{7}d(2bc+ad)x^7+\frac{1}{10}bd^2x^{10}\end{aligned}$$

Mathematica [A] time = 0.01, size = 50, normalized size = 1.00

$$\frac{1}{7}dx^7(ad+2bc)+\frac{1}{4}cx^4(2ad+bc)+ac^2x+\frac{1}{10}bd^2x^{10}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)*(c + d*x^3)^2,x]
```

```
[Out] a*c^2*x + (c*(b*c + 2*a*d)*x^4)/4 + (d*(2*b*c + a*d)*x^7)/7 + (b*d^2*x^10)/
10
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+bx^3\right)\left(c+dx^3\right)^2\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x^3)*(c + d*x^3)^2,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x^3)*(c + d*x^3)^2, x]
```

fricas [A] time = 0.63, size = 50, normalized size = 1.00

$$\frac{1}{10}x^{10}d^2b+\frac{2}{7}x^7dcb+\frac{1}{7}x^7d^2a+\frac{1}{4}x^4c^2b+\frac{1}{2}x^4dca+xc^2a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*(d*x^3+c)^2,x, algorithm="fricas")
```

```
[Out] 1/10*x^10*d^2*b + 2/7*x^7*d*c*b + 1/7*x^7*d^2*a + 1/4*x^4*c^2*b + 1/2*x^4*d
*c*a + x*c^2*a
```

giac [A] time = 0.19, size = 50, normalized size = 1.00

$$\frac{1}{10}\,bd^2x^{10}+\frac{2}{7}\,bcdx^7+\frac{1}{7}\,ad^2x^7+\frac{1}{4}\,bc^2x^4+\frac{1}{2}\,acdx^4+ac^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*(d*x^3+c)^2,x, algorithm="giac")
```

```
[Out] 1/10*b*d^2*x^10 + 2/7*b*c*d*x^7 + 1/7*a*d^2*x^7 + 1/4*b*c^2*x^4 + 1/2*a*c*d
*x^4 + a*c^2*x
```

maple [A] time = 0.04, size = 49, normalized size = 0.98

$$\frac{b\,d^2x^{10}}{10}+\frac{\left(a\,d^2+2bcd\right)x^7}{7}+a\,c^2x+\frac{\left(2acd+b\,c^2\right)x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)*(d*x^3+c)^2,x)
```

```
[Out] 1/10*b*d^2*x^10+1/7*(a*d^2+2*b*c*d)*x^7+1/4*(2*a*c*d+b*c^2)*x^4+a*c^2*x
```

maxima [A] time = 0.48, size = 48, normalized size = 0.96

$$\frac{1}{10}\,bd^2x^{10}+\frac{1}{7}\left(2\,bcd+ad^2\right)x^7+\frac{1}{4}\left(bc^2+2\,acd\right)x^4+ac^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*(d*x^3+c)^2,x, algorithm="maxima")
```

```
[Out] 1/10*b*d^2*x^10 + 1/7*(2*b*c*d + a*d^2)*x^7 + 1/4*(b*c^2 + 2*a*c*d)*x^4 + a
*c^2*x
```

mupad [B] time = 0.05, size = 48, normalized size = 0.96

$$x^4\left(\frac{b\,c^2}{4}+\frac{a\,d\,c}{2}\right)+x^7\left(\frac{a\,d^2}{7}+\frac{2\,b\,c\,d}{7}\right)+\frac{b\,d^2\,x^{10}}{10}+a\,c^2\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)*(c + d*x^3)^2,x)
```

```
[Out] x^4*((b*c^2)/4 + (a*c*d)/2) + x^7*((a*d^2)/7 + (2*b*c*d)/7) + (b*d^2*x^10)/
10 + a*c^2*x
```

sympy [A] time = 0.07, size = 51, normalized size = 1.02

$$ac^2x+\frac{bd^2x^{10}}{10}+x^7\left(\frac{ad^2}{7}+\frac{2bcd}{7}\right)+x^4\left(\frac{acd}{2}+\frac{bc^2}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)*(d*x**3+c)**2,x)
```

```
[Out] a*c**2*x + b*d**2*x**10/10 + x**7*(a*d**2/7 + 2*b*c*d/7) + x**4*(a*c*d/2 +
b*c**2/4)
```

3.4 $\int \left(a+bx^3\right)\left(c+dx^3\right)\,dx$

Optimal. Leaf size=28

$$\frac{1}{4}x^4(ad+bc)+acx+\frac{1}{7}bdx^7$$

Rubi [A] time = 0.01, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {373}

$$\frac{1}{4}x^4(ad+bc)+acx+\frac{1}{7}bdx^7$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)*(c + d*x^3),x]
```

```
[Out] a*c*x + ((b*c + a*d)*x^4)/4 + (b*d*x^7)/7
```

Rule 373

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \left(a+bx^3\right)\left(c+dx^3\right)\,dx &= \int \left(ac+(bc+ad)x^3+bdx^6\right)\,dx\\ &= acx+\frac{1}{4}(bc+ad)x^4+\frac{1}{7}bdx^7\end{aligned}$$

Mathematica [A] time = 0.01, size = 28, normalized size = 1.00

$$\frac{1}{4}x^4(ad+bc)+acx+\frac{1}{7}bdx^7$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)*(c + d*x^3),x]
```

```
[Out] a*c*x + ((b*c + a*d)*x^4)/4 + (b*d*x^7)/7
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+bx^3\right)\left(c+dx^3\right)\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x^3)*(c + d*x^3),x]
```

```
[Out] IntegrateAlgebraic[(a + b*x^3)*(c + d*x^3), x]
```

fricas [A] time = 0.85, size = 26, normalized size = 0.93

$$\frac{1}{7}x^7db+\frac{1}{4}x^4cb+\frac{1}{4}x^4da+xca$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*(d*x^3+c),x, algorithm="fricas")
[Out] 1/7*x^7*d*b + 1/4*x^4*c*b + 1/4*x^4*d*a + x*c*a
```

giac [A] time = 0.15, size = 26, normalized size = 0.93

$$\frac{1}{7}\,bdx^7 + \frac{1}{4}\,bcx^4 + \frac{1}{4}\,adx^4 + acx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*(d*x^3+c),x, algorithm="giac")
[Out] 1/7*b*d*x^7 + 1/4*b*c*x^4 + 1/4*a*d*x^4 + a*c*x
```

maple [A] time = 0.04, size = 25, normalized size = 0.89

$$\frac{bd\,x^7}{7} + \frac{(ad+bc)\,x^4}{4} + acx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)*(d*x^3+c),x)
[Out] a*c*x+1/4*(a*d+b*c)*x^4+1/7*b*d*x^7
```

maxima [A] time = 0.54, size = 24, normalized size = 0.86

$$\frac{1}{7}\,bdx^7 + \frac{1}{4}\,(bc+ad)x^4 + acx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*(d*x^3+c),x, algorithm="maxima")
[Out] 1/7*b*d*x^7 + 1/4*(b*c + a*d)*x^4 + a*c*x
```

mupad [B] time = 0.04, size = 25, normalized size = 0.89

$$\frac{b\,d\,x^7}{7} + \left(\frac{a\,d}{4} + \frac{b\,c}{4}\right)\,x^4 + a\,c\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)*(c + d*x^3),x)
[Out] x^4*((a*d)/4 + (b*c)/4) + a*c*x + (b*d*x^7)/7
```

sympy [A] time = 0.07, size = 26, normalized size = 0.93

$$acx + \frac{bdx^7}{7} + x^4\left(\frac{ad}{4} + \frac{bc}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)*(d*x**3+c),x)
[Out] a*c*x + b*d*x**7/7 + x**4*(a*d/4 + b*c/4)
```

3.5 $\int \frac{a+bx^3}{c+dx^3} \, dx$

Optimal. Leaf size=144

$$\frac{(bc-ad)\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}\,x+d^{2/3}x^2\right)}{6c^{2/3}d^{4/3}}-\frac{(bc-ad)\log\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{3c^{2/3}d^{4/3}}+\frac{(bc-ad)\tan^{-1}\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}\,x}{\sqrt{3}\sqrt[3]{c}}\right)}{\sqrt{3}\,c^{2/3}d^{4/3}}+\frac{bx}{d}$$

Rubi [A] time = 0.09, antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.412, Rules used = {388, 200, 31, 634, 617, 204, 628}

$$\frac{(bc-ad)\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}\,x+d^{2/3}x^2\right)}{6c^{2/3}d^{4/3}}-\frac{(bc-ad)\log\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{3c^{2/3}d^{4/3}}+\frac{(bc-ad)\tan^{-1}\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}\,x}{\sqrt{3}\sqrt[3]{c}}\right)}{\sqrt{3}\,c^{2/3}d^{4/3}}+\frac{bx}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)/(c + d*x^3),x]
```

```
[Out] (b*x)/d + ((b*c - a*d)*ArcTan[(c^(1/3) - 2*d^(1/3)*x)/(Sqrt[3]*c^(1/3))])/(
Sqrt[3]*c^(2/3)*d^(4/3)) - ((b*c - a*d)*Log[c^(1/3) + d^(1/3)*x])/(3*c^(2/3
)*d^(4/3)) + ((b*c - a*d)*Log[c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2])/(
6*c^(2/3)*d^(4/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$
\begin{aligned}
\int \frac{a+bx^3}{c+dx^3}\,dx &= \frac{bx}{d}-\frac{(bc-ad)\int \frac{1}{c+dx^3}\,dx}{d}\\
&= \frac{bx}{d}-\frac{(bc-ad)\int \frac{1}{\sqrt[3]{c}+\sqrt[3]{d}x}\,dx}{3c^{2/3}d}-\frac{(bc-ad)\int \frac{2\sqrt[3]{c}-\sqrt[3]{d}x}{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}\,dx}{3c^{2/3}d}\\
&= \frac{bx}{d}-\frac{(bc-ad)\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{2/3}d^{4/3}}+\frac{(bc-ad)\int \frac{-\sqrt[3]{c}\sqrt[3]{d}+2d^{2/3}x}{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}\,dx}{6c^{2/3}d^{4/3}}-\frac{(bc-ad)\int \frac{1}{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x\ldots}}{2\sqrt[3]{c}d}\\
&= \frac{bx}{d}-\frac{(bc-ad)\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{2/3}d^{4/3}}+\frac{(bc-ad)\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{6c^{2/3}d^{4/3}}-\frac{(bc-ad)\,\mathrm{Subst}\left(\int \ldots\right.}{c^{2/\ldots}}\\
&= \frac{bx}{d}+\frac{(bc-ad)\tan^{-1}\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{\sqrt{3}\sqrt[3]{c}}\right)}{\sqrt{3}c^{2/3}d^{4/3}}-\frac{(bc-ad)\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{2/3}d^{4/3}}+\frac{(bc-ad)\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+\ldots\right.}{6c^{2/3}d^{4/3}}
\end{aligned}
$$

Mathematica [A] time = 0.12, size = 128, normalized size = 0.89

$$
\frac{(bc-ad)\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)-2(bc-ad)\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)+2\sqrt{3}(bc-ad)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}}{\sqrt{3}}\right)+6bc^{2/3}\sqrt[3]{d}x}{6c^{2/3}d^{4/3}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)/(c + d*x^3),x]
```

```
[Out] (6*b*c^(2/3)*d^(1/3)*x + 2*Sqrt[3]*(b*c - a*d)*ArcTan[(1 - (2*d^(1/3)*x)/c^
(1/3))/Sqrt[3]] - 2*(b*c - a*d)*Log[c^(1/3) + d^(1/3)*x] + (b*c - a*d)*Log[
c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2])/(6*c^(2/3)*d^(4/3))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$
\int \frac{a+bx^3}{c+dx^3}\,dx
$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x^3)/(c + d*x^3),x]
```

```
[Out] IntegrateAlgebraic[(a + b*x^3)/(c + d*x^3), x]
```

fricas [A] time = 1.14, size = 369, normalized size = 2.56

$$
\left[\frac{6bc^2dx-3\sqrt{\frac{1}{3}}\left(bc^2d-acd^2\right)\sqrt{-\frac{\left(c^2d\right)^{\frac{1}{3}}}{d}}\log\left(\frac{2cdx^3-3\left(c^2d\right)^{\frac{1}{3}}cx-c^2+3\sqrt{\frac{1}{3}}\left(2cdx^2+\left(c^2d\right)^{\frac{2}{3}}x-\left(c^2d\right)^{\frac{1}{3}}c\right)\sqrt{-\frac{\left(c^2d\right)^{\frac{1}{3}}}{d}}}{dx^3+c}\right)+\left(c^2d\right)^{\frac{2}{3}}(bc-ad)\log\left(cdx^2-\left(c^2d\right)^{\frac{2}{3}}x+\left(c^2d\right)^{\frac{1}{3}}c\right)-2\left(c^2d\right)^{\frac{2}{3}}(bc-ad)\log\left(cdx+\left(c^2d\right)^{\frac{2}{3}}\right)}{6c^2d^2}, \frac{6bc^2dx-6\sqrt{\frac{1}{3}}\left(bc^2d-acd^2\right)\sqrt{\frac{\left(c^2d\right)^{\frac{1}{3}}}{d}}\arctan\left(\frac{\sqrt{\frac{1}{3}}\left(2\left(c^2d\right)^{\frac{2}{3}}x-\left(c^2d\right)^{\frac{1}{3}}c\right)\sqrt{\frac{\left(c^2d\right)^{\frac{1}{3}}}{d}}}{c^2}\right)+\left(c^2d\right)^{\frac{2}{3}}(bc-ad)\log\left(cdx^2-\left(c^2d\right)^{\frac{2}{3}}x+\left(c^2d\right)^{\frac{1}{3}}c\right)-2\left(c^2d\right)^{\frac{2}{3}}(bc-ad)\log\left(cdx+\left(c^2d\right)^{\frac{2}{3}}\right)}{6c^2d^2}\right]
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] [1/6*(6*b*c^2*d*x - 3*sqrt(1/3)*(b*c^2*d - a*c*d^2)*sqrt(-(c^2*d)^(1/3)/d)*
log((2*c*d*x^3 - 3*(c^2*d)^(1/3)*c*x - c^2 + 3*sqrt(1/3)*(2*c*d*x^2 + (c^2*
d)^(2/3)*x - (c^2*d)^(1/3)*c)*sqrt(-(c^2*d)^(1/3)/d))/(d*x^3 + c)) + (c^2*d
)^(2/3)*(b*c - a*d)*log(c*d*x^2 - (c^2*d)^(2/3)*x + (c^2*d)^(1/3)*c) - 2*(c
^2*d)^(2/3)*(b*c - a*d)*log(c*d*x + (c^2*d)^(2/3)))/(c^2*d^2), 1/6*(6*b*c^2
*d*x - 6*sqrt(1/3)*(b*c^2*d - a*c*d^2)*sqrt((c^2*d)^(1/3)/d)*arctan(sqrt(1/
3)*(2*(c^2*d)^(2/3)*x - (c^2*d)^(1/3)*c)*sqrt((c^2*d)^(1/3)/d)/c^2) + (c^2*
d)^(2/3)*(b*c - a*d)*log(c*d*x^2 - (c^2*d)^(2/3)*x + (c^2*d)^(1/3)*c) - 2*(
c^2*d)^(2/3)*(b*c - a*d)*log(c*d*x + (c^2*d)^(2/3)))/(c^2*d^2)]
```

giac [A] time = 0.20, size = 133, normalized size = 0.92

$$\frac{\sqrt{3}\,(bc-ad)\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{3\left(-cd^2\right)^{\frac{2}{3}}}+\frac{(bc-ad)\log\left(x^2+x\left(-\frac{c}{d}\right)^{\frac{1}{3}}+\left(-\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6\left(-cd^2\right)^{\frac{2}{3}}}+\frac{bx}{d}+\frac{(bc-ad)\left(-\frac{c}{d}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right|\right)}{3cd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] 1/3*sqrt(3)*(b*c - a*d)*arctan(1/3*sqrt(3)*(2*x + (-c/d)^(1/3))/(-c/d)^(1/3
))/(-c*d^2)^(2/3) + 1/6*(b*c - a*d)*log(x^2 + x*(-c/d)^(1/3) + (-c/d)^(2/3)
)/(-c*d^2)^(2/3) + b*x/d + 1/3*(b*c - a*d)*(-c/d)^(1/3)*log(abs(x - (-c/d)^
(1/3)))/(c*d)
```

maple [A] time = 0.05, size = 195, normalized size = 1.35

$$\frac{\sqrt{3}\,a\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{c}{d}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3\left(\frac{c}{d}\right)^{\frac{2}{3}}d}+\frac{a\ln\left(x+\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(\frac{c}{d}\right)^{\frac{2}{3}}d}-\frac{a\ln\left(x^2-\left(\frac{c}{d}\right)^{\frac{1}{3}}x+\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6\left(\frac{c}{d}\right)^{\frac{2}{3}}d}-\frac{\sqrt{3}\,bc\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{c}{d}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3\left(\frac{c}{d}\right)^{\frac{2}{3}}d^2}-\frac{bc\ln\left(x+\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(\frac{c}{d}\right)^{\frac{2}{3}}d^2}+\frac{bc\ln\left(x^2-\left(\frac{c}{d}\right)^{\frac{1}{3}}x+\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6\left(\frac{c}{d}\right)^{\frac{2}{3}}d^2}+\frac{bx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)/(d*x^3+c),x)
```

```
[Out] b*x/d+1/3/d/(c/d)^(2/3)*ln(x+(c/d)^(1/3))*a-1/3/d^2/(c/d)^(2/3)*ln(x+(c/d)^
(1/3))*b*c-1/6/d/(c/d)^(2/3)*ln(x^2-(c/d)^(1/3)*x+(c/d)^(2/3))*a+1/6/d^2/(c
/d)^(2/3)*ln(x^2-(c/d)^(1/3)*x+(c/d)^(2/3))*b*c+1/3/d/(c/d)^(2/3)*3^(1/2)*a
rctan(1/3*3^(1/2)*(2/(c/d)^(1/3)*x-1))*a-1/3/d^2/(c/d)^(2/3)*3^(1/2)*arctan
(1/3*3^(1/2)*(2/(c/d)^(1/3)*x-1))*b*c
```

maxima [A] time = 1.28, size = 128, normalized size = 0.89

$$\frac{bx}{d}-\frac{\sqrt{3}\,(bc-ad)\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{3d^2\left(\frac{c}{d}\right)^{\frac{2}{3}}}+\frac{(bc-ad)\log\left(x^2-x\left(\frac{c}{d}\right)^{\frac{1}{3}}+\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6d^2\left(\frac{c}{d}\right)^{\frac{2}{3}}}-\frac{(bc-ad)\log\left(x+\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3d^2\left(\frac{c}{d}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] b*x/d - 1/3*sqrt(3)*(b*c - a*d)*arctan(1/3*sqrt(3)*(2*x - (c/d)^(1/3))/(c/d
)^(1/3))/(d^2*(c/d)^(2/3)) + 1/6*(b*c - a*d)*log(x^2 - x*(c/d)^(1/3) + (c/d
```

```
)^(2/3))/(d^2*(c/d)^(2/3)) - 1/3*(b*c - a*d)*log(x + (c/d)^(1/3))/(d^2*(c/d
)^(2/3))
```

mupad [B] time = 1.38, size = 123, normalized size = 0.85

$$\frac{b\,x}{d}+\frac{\ln\left(d^{1/3}\,x+c^{1/3}\right)\,\left(a\,d-b\,c\right)}{3\,c^{2/3}\,d^{4/3}}-\frac{\ln\left(c^{1/3}-2\,d^{1/3}\,x+\sqrt{3}\,c^{1/3}\,1\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(a\,d-b\,c\right)}{3\,c^{2/3}\,d^{4/3}}+\frac{\ln\left(2\,d^{1/3}\,x-c^{1/3}+\sqrt{3}\,c^{1/3}\,1\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(a\,d-b\,c\right)}{3\,c^{2/3}\,d^{4/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)/(c + d*x^3),x)
```

```
[Out] (b*x)/d + (log(d^(1/3)*x + c^(1/3))*(a*d - b*c))/(3*c^(2/3)*d^(4/3)) - (log
(3^(1/2)*c^(1/3)*1i - 2*d^(1/3)*x + c^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(a*d -
b*c))/(3*c^(2/3)*d^(4/3)) + (log(3^(1/2)*c^(1/3)*1i + 2*d^(1/3)*x - c^(1/3)
)*((3^(1/2)*1i)/2 - 1/2)*(a*d - b*c))/(3*c^(2/3)*d^(4/3))
```

sympy [A] time = 0.42, size = 71, normalized size = 0.49

$$\frac{bx}{d}+\operatorname{RootSum}\left(27t^3c^2d^4-a^3d^3+3a^2bcd^2-3ab^2c^2d+b^3c^3,\left(t\mapsto t\log\left(\frac{3tcd}{ad-bc}+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)/(d*x**3+c),x)
```

```
[Out] b*x/d + RootSum(27*_t**3*c**2*d**4 - a**3*d**3 + 3*a**2*b*c*d**2 - 3*a*b**2
*c**2*d + b**3*c**3, Lambda(_t, _t*log(3*_t*c*d/(a*d - b*c) + x)))
```

3.6 $\int \frac{a+bx^3}{\left(c+dx^3\right)^2}\,dx$

Optimal. Leaf size=169

$$-\frac{(2ad+bc)\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}\,x+d^{2/3}x^2\right)}{18c^{5/3}d^{4/3}}+\frac{(2ad+bc)\log\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{9c^{5/3}d^{4/3}}-\frac{(2ad+bc)\tan^{-1}\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}\,x}{\sqrt{3}\sqrt[3]{c}}\right)}{3\sqrt{3}\,c^{5/3}d^{4/3}}-\frac{x(bc}{3cd\,(c}$$

Rubi [A] time = 0.08, antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.412, Rules used = {385, 200, 31, 634, 617, 204, 628}

$$-\frac{(2ad+bc)\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}\,x+d^{2/3}x^2\right)}{18c^{5/3}d^{4/3}}+\frac{(2ad+bc)\log\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{9c^{5/3}d^{4/3}}-\frac{(2ad+bc)\tan^{-1}\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}\,x}{\sqrt{3}\sqrt[3]{c}}\right)}{3\sqrt{3}\,c^{5/3}d^{4/3}}-\frac{x(bc-ad)}{3cd\left(c+dx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)/(c + d*x^3)^2,x]
```

```
[Out] -((b*c - a*d)*x)/(3*c*d*(c + d*x^3)) - ((b*c + 2*a*d)*ArcTan[(c^(1/3) - 2*d
^(1/3)*x)/(Sqrt[3]*c^(1/3))])/(3*Sqrt[3]*c^(5/3)*d^(4/3)) + ((b*c + 2*a*d)*
Log[c^(1/3) + d^(1/3)*x])/(9*c^(5/3)*d^(4/3)) - ((b*c + 2*a*d)*Log[c^(2/3)
- c^(1/3)*d^(1/3)*x + d^(2/3)*x^2])/(18*c^(5/3)*d^(4/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{a+bx^3}{\left(c+dx^3\right)^2}\,dx = -\frac{(bc-ad)x}{3cd\left(c+dx^3\right)}+\frac{(bc+2ad)\int \frac{1}{c+dx^3}\,dx}{3cd}$$

$$= -\frac{(bc-ad)x}{3cd\left(c+dx^3\right)}+\frac{(bc+2ad)\int \frac{1}{\sqrt[3]{c}+\sqrt[3]{d}\,x}\,dx}{9c^{5/3}d}+\frac{(bc+2ad)\int \frac{2\sqrt[3]{c}-\sqrt[3]{d}\,x}{c^{2/3}-\sqrt[3]{c}\,\sqrt[3]{d}\,x+d^{2/3}x^2}\,dx}{9c^{5/3}d}$$

$$= -\frac{(bc-ad)x}{3cd\left(c+dx^3\right)}+\frac{(bc+2ad)\log\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{9c^{5/3}d^{4/3}}-\frac{(bc+2ad)\int \frac{-\sqrt[3]{c}\,\sqrt[3]{d}+2d^{2/3}x}{c^{2/3}-\sqrt[3]{c}\,\sqrt[3]{d}\,x+d^{2/3}x^2}\,dx}{18c^{5/3}d^{4/3}}+\frac{(bc+2a}{}$$

$$= -\frac{(bc-ad)x}{3cd\left(c+dx^3\right)}+\frac{(bc+2ad)\log\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{9c^{5/3}d^{4/3}}-\frac{(bc+2ad)\log\left(c^{2/3}-\sqrt[3]{c}\,\sqrt[3]{d}\,x+d^{2/3}x^2\right)}{18c^{5/3}d^{4/3}}+-$$

$$= -\frac{(bc-ad)x}{3cd\left(c+dx^3\right)}-\frac{(bc+2ad)\tan^{-1}\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}\,x}{\sqrt{3}\,\sqrt[3]{c}}\right)}{3\sqrt{3}\,c^{5/3}d^{4/3}}+\frac{(bc+2ad)\log\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{9c^{5/3}d^{4/3}}-\frac{(bc+2ad)\log}{}$$

Mathematica [A] time = 0.10, size = 145, normalized size = 0.86

$$\frac{-(2ad+bc)\log\left(c^{2/3}-\sqrt[3]{c}\,\sqrt[3]{d}\,x+d^{2/3}x^2\right)-\frac{6c^{2/3}\sqrt[3]{d}\,x(bc-ad)}{c+dx^3}+2(2ad+bc)\log\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)-2\sqrt{3}\,(2ad+bc)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\,x}{\sqrt[3]{c}}}{\sqrt{3}}\right)}{18c^{5/3}d^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)/(c + d*x^3)^2,x]
```

```
[Out] ((-6*c^(2/3)*d^(1/3)*(b*c - a*d)*x)/(c + d*x^3) - 2*Sqrt[3]*(b*c + 2*a*d)*A
rcTan[(1 - (2*d^(1/3)*x)/c^(1/3))/Sqrt[3]] + 2*(b*c + 2*a*d)*Log[c^(1/3) +
d^(1/3)*x] - (b*c + 2*a*d)*Log[c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2])/
(18*c^(5/3)*d^(4/3))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a+bx^3}{\left(c+dx^3\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x^3)/(c + d*x^3)^2,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x^3)/(c + d*x^3)^2, x]
```

fricas [A] time = 1.02, size = 537, normalized size = 3.18

$$\left[\frac{3\sqrt{\frac{1}{3}}\left(bc^3d+2ac^2d^2+\left(bc^2d^2+2acd^3\right)x^3\right)\sqrt{-\frac{\left(c^2d\right)^{\frac{1}{3}}}{d}}\log\left(\frac{2cdx^3-3\left(c^2d\right)^{\frac{1}{3}}cx-c^2+3\sqrt{\frac{1}{3}}\left(2cdx^2+\left(c^2d\right)^{\frac{2}{3}}x-\left(c^2d\right)^{\frac{1}{3}}c\right)\sqrt{-\frac{\left(c^2d\right)^{\frac{1}{3}}}{d}}}{dx^3+c}\right)-\left(\left(bcd+2ad^2\right)x^3+bc^2+2acd\right)\left(c^2d\right)^{\frac{2}{3}}\log\left(cdx^2-\left(c^2d\right)^{\frac{2}{3}}x+\left(c^2d\right)^{\frac{1}{3}}c\right)+2\left(\left(bcd+2ad^2\right)x^3+bc^2+2acd\right)\left(c^2d\right)^{\frac{2}{3}}\log\left(cdx+\left(c^2d\right)^{\frac{2}{3}}\right)-6\left(bc^3d-ac^2d^2\right)x}{18\left(c^3d^3x^3+c^4d^2\right)},\ \frac{6\sqrt{\frac{1}{3}}\left(bc^3d+2ac^2d^2+\left(bc^2d^2+2acd^3\right)x^3\right)\sqrt{\frac{\left(c^2d\right)^{\frac{1}{3}}}{d}}\arctan\left(\frac{\sqrt{\frac{1}{3}}\left(2\left(c^2d\right)^{\frac{2}{3}}x-\left(c^2d\right)^{\frac{1}{3}}c\right)\sqrt{\frac{\left(c^2d\right)^{\frac{1}{3}}}{d}}}{c^2}\right)-\left(\left(bcd+2ad^2\right)x^3+bc^2+2acd\right)\left(c^2d\right)^{\frac{2}{3}}\log\left(cdx^2-\left(c^2d\right)^{\frac{2}{3}}x+\left(c^2d\right)^{\frac{1}{3}}c\right)+2\left(\left(bcd+2ad^2\right)x^3+bc^2+2acd\right)\left(c^2d\right)^{\frac{2}{3}}\log\left(cdx+\left(c^2d\right)^{\frac{2}{3}}\right)-6\left(bc^3d-ac^2d^2\right)x}{18\left(c^3d^3x^3+c^4d^2\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)/(d*x^3+c)^2,x, algorithm="fricas")
```

```
[Out] [1/18*(3*sqrt(1/3)*(b*c^3*d + 2*a*c^2*d^2 + (b*c^2*d^2 + 2*a*c*d^3)*x^3)*sq
rt(-(c^2*d)^(1/3)/d)*log((2*c*d*x^3 - 3*(c^2*d)^(1/3)*c*x - c^2 + 3*sqrt(1/
3)*(2*c*d*x^2 + (c^2*d)^(2/3)*x - (c^2*d)^(1/3)*c)*sqrt(-(c^2*d)^(1/3)/d))/
(d*x^3 + c)) - ((b*c*d + 2*a*d^2)*x^3 + b*c^2 + 2*a*c*d)*(c^2*d)^(2/3)*log(
c*d*x^2 - (c^2*d)^(2/3)*x + (c^2*d)^(1/3)*c) + 2*((b*c*d + 2*a*d^2)*x^3 + b
*c^2 + 2*a*c*d)*(c^2*d)^(2/3)*log(c*d*x + (c^2*d)^(2/3)) - 6*(b*c^3*d - a*c
^2*d^2)*x)/(c^3*d^3*x^3 + c^4*d^2), 1/18*(6*sqrt(1/3)*(b*c^3*d + 2*a*c^2*d^
2 + (b*c^2*d^2 + 2*a*c*d^3)*x^3)*sqrt((c^2*d)^(1/3)/d)*arctan(sqrt(1/3)*(2*
(c^2*d)^(2/3)*x - (c^2*d)^(1/3)*c)*sqrt((c^2*d)^(1/3)/d)/c^2) - ((b*c*d + 2
*a*d^2)*x^3 + b*c^2 + 2*a*c*d)*(c^2*d)^(2/3)*log(c*d*x^2 - (c^2*d)^(2/3)*x
+ (c^2*d)^(1/3)*c) + 2*((b*c*d + 2*a*d^2)*x^3 + b*c^2 + 2*a*c*d)*(c^2*d)^(2
/3)*log(c*d*x + (c^2*d)^(2/3)) - 6*(b*c^3*d - a*c^2*d^2)*x)/(c^3*d^3*x^3 +
c^4*d^2)]
```

giac [A] time = 0.19, size = 160, normalized size = 0.95

$$-\frac{\sqrt{3}\left(bc+2\,ad\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{9\left(-cd^2\right)^{\frac{2}{3}}c}-\frac{\left(bc+2\,ad\right)\log\left(x^2+x\left(-\frac{c}{d}\right)^{\frac{1}{3}}+\left(-\frac{c}{d}\right)^{\frac{2}{3}}\right)}{18\left(-cd^2\right)^{\frac{2}{3}}c}-\frac{\left(bc+2\,ad\right)\left(-\frac{c}{d}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right|\right)}{9\,c^2d}-\frac{bcx-adx}{3\left(dx^3+c\right)cd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)/(d*x^3+c)^2,x, algorithm="giac")
```

```
[Out] -1/9*sqrt(3)*(b*c + 2*a*d)*arctan(1/3*sqrt(3)*(2*x + (-c/d)^(1/3))/(-c/d)^(
1/3))/((-c*d^2)^(2/3)*c) - 1/18*(b*c + 2*a*d)*log(x^2 + x*(-c/d)^(1/3) + (-
c/d)^(2/3))/((-c*d^2)^(2/3)*c) - 1/9*(b*c + 2*a*d)*(-c/d)^(1/3)*log(abs(x -
 (-c/d)^(1/3)))/(c^2*d) - 1/3*(b*c*x - a*d*x)/((d*x^3 + c)*c*d)
```

maple [A] time = 0.05, size = 221, normalized size = 1.31

$$\frac{2\sqrt{3}\,a\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{c}{d}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{9\left(\frac{c}{d}\right)^{\frac{2}{3}}cd}+\frac{2a\ln\left(x+\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{9\left(\frac{c}{d}\right)^{\frac{2}{3}}cd}-\frac{a\ln\left(x^2-\left(\frac{c}{d}\right)^{\frac{1}{3}}x+\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}{9\left(\frac{c}{d}\right)^{\frac{2}{3}}cd}+\frac{\sqrt{3}\,b\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{c}{d}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{9\left(\frac{c}{d}\right)^{\frac{2}{3}}d^2}+\frac{b\ln\left(x+\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{9\left(\frac{c}{d}\right)^{\frac{2}{3}}d^2}-\frac{b\ln\left(x^2-\left(\frac{c}{d}\right)^{\frac{1}{3}}x+\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}{18\left(\frac{c}{d}\right)^{\frac{2}{3}}d^2}+\frac{\left(ad-bc\right)x}{3\left(d\,x^3+c\right)cd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)/(d*x^3+c)^2,x)
```

```
[Out] 1/3*(a*d-b*c)/d/c*x/(d*x^3+c)+2/9/c/d/(c/d)^(2/3)*ln(x+(c/d)^(1/3))*a+1/9/d
^2/(c/d)^(2/3)*ln(x+(c/d)^(1/3))*b-1/9/c/d/(c/d)^(2/3)*ln(x^2-(c/d)^(1/3)*x
+(c/d)^(2/3))*a-1/18/d^2/(c/d)^(2/3)*ln(x^2-(c/d)^(1/3)*x+(c/d)^(2/3))*b+2/
9/c/d/(c/d)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(c/d)^(1/3)*x-1))*a+1/9/d^2
/(c/d)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(c/d)^(1/3)*x-1))*b
```

maxima [A] time = 1.16, size = 158, normalized size = 0.93

$$-\frac{\left(bc-ad\right)x}{3\left(cd^2x^3+c^2d\right)}+\frac{\sqrt{3}\left(bc+2\,ad\right)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{9\,cd^2\left(\frac{c}{d}\right)^{\frac{2}{3}}}-\frac{\left(bc+2\,ad\right)\log\left(x^2-x\left(\frac{c}{d}\right)^{\frac{1}{3}}+\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}{18\,cd^2\left(\frac{c}{d}\right)^{\frac{2}{3}}}+\frac{\left(bc+2\,ad\right)\log\left(x+\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{9\,cd^2\left(\frac{c}{d}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)/(d*x^3+c)^2,x, algorithm="maxima")
```

```
[Out] -1/3*(b*c - a*d)*x/(c*d^2*x^3 + c^2*d) + 1/9*sqrt(3)*(b*c + 2*a*d)*arctan(1
/3*sqrt(3)*(2*x - (c/d)^(1/3))/(c/d)^(1/3))/(c*d^2*(c/d)^(2/3)) - 1/18*(b*c
 + 2*a*d)*log(x^2 - x*(c/d)^(1/3) + (c/d)^(2/3))/(c*d^2*(c/d)^(2/3)) + 1/9*
(b*c + 2*a*d)*log(x + (c/d)^(1/3))/(c*d^2*(c/d)^(2/3))
```

mupad [B]   time = 1.40, size = 143, normalized size = 0.85

$$\frac{\ln\left(d^{1/3}\,x+c^{1/3}\right)\,\left(2\,a\,d+b\,c\right)}{9\,c^{5/3}\,d^{4/3}}-\frac{\ln\left(c^{1/3}-2\,d^{1/3}\,x+\sqrt{3}\,c^{1/3}\,1{}\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,\left(2\,a\,d+b\,c\right)}{9\,c^{5/3}\,d^{4/3}}+\frac{\ln\left(2\,d^{1/3}\,x-c^{1/3}+\sqrt{3}\,c^{1/3}\,1{}\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,\left(2\,a\,d+b\,c\right)}{9\,c^{5/3}\,d^{4/3}}+\frac{x\,\left(a\,d-b\,c\right)}{3\,c\,d\,\left(d\,x^3+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)/(c + d*x^3)^2,x)
```

```
[Out] (log(d^(1/3)*x + c^(1/3))*(2*a*d + b*c))/(9*c^(5/3)*d^(4/3)) - (log(3^(1/2)
*c^(1/3)*1i - 2*d^(1/3)*x + c^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(2*a*d + b*c))/
(9*c^(5/3)*d^(4/3)) + (log(3^(1/2)*c^(1/3)*1i + 2*d^(1/3)*x - c^(1/3))*((3^
(1/2)*1i)/2 - 1/2)*(2*a*d + b*c))/(9*c^(5/3)*d^(4/3)) + (x*(a*d - b*c))/(3*
c*d*(c + d*x^3))
```

sympy [A]   time = 0.58, size = 97, normalized size = 0.57

$$\frac{x\left(ad-bc\right)}{3c^{2}d+3cd^{2}x^{3}}+\operatorname{RootSum}{\left(729t^{3}c^{5}d^{4}-8a^{3}d^{3}-12a^{2}bcd^{2}-6ab^{2}c^{2}d-b^{3}c^{3}, \left(t\mapsto t\log{\left(\frac{9tc^{2}d}{2ad+bc}+x\right)}\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)/(d*x**3+c)**2,x)
```

```
[Out] x*(a*d - b*c)/(3*c**2*d + 3*c*d**2*x**3) + RootSum(729*_t**3*c**5*d**4 - 8*
a**3*d**3 - 12*a**2*b*c*d**2 - 6*a*b**2*c**2*d - b**3*c**3, Lambda(_t, _t*l
og(9*_t*c**2*d/(2*a*d + b*c) + x)))
```

3.7 $\int \frac{a+bx^3}{\left(c+dx^3\right)^3}\,dx$

Optimal. Leaf size=197

$$-\frac{(5ad+bc)\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}\,x+d^{2/3}x^2\right)}{54c^{8/3}d^{4/3}}+\frac{(5ad+bc)\log\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{27c^{8/3}d^{4/3}}-\frac{(5ad+bc)\tan^{-1}\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}\,x}{\sqrt{3}\sqrt[3]{c}}\right)}{9\sqrt{3}\,c^{8/3}d^{4/3}}+\frac{x(5a}{18c^2d\,(}$$

Rubi [A] time = 0.11, antiderivative size = 197, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.471, Rules used = {385, 199, 200, 31, 634, 617, 204, 628}

$$-\frac{(5ad+bc)\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}\,x+d^{2/3}x^2\right)}{54c^{8/3}d^{4/3}}+\frac{(5ad+bc)\log\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{27c^{8/3}d^{4/3}}-\frac{(5ad+bc)\tan^{-1}\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}\,x}{\sqrt{3}\sqrt[3]{c}}\right)}{9\sqrt{3}\,c^{8/3}d^{4/3}}+\frac{x(5ad+bc)}{18c^2d\left(c+dx^3\right)}-\frac{x(bc-ad)}{6cd\left(c+dx^3\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)/(c + d*x^3)^3,x]
```

```
[Out] -((b*c - a*d)*x)/(6*c*d*(c + d*x^3)^2) + ((b*c + 5*a*d)*x)/(18*c^2*d*(c + d
*x^3)) - ((b*c + 5*a*d)*ArcTan[(c^(1/3) - 2*d^(1/3)*x)/(Sqrt[3]*c^(1/3))])/
(9*Sqrt[3]*c^(8/3)*d^(4/3)) + ((b*c + 5*a*d)*Log[c^(1/3) + d^(1/3)*x])/(27*
c^(8/3)*d^(4/3)) - ((b*c + 5*a*d)*Log[c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)
*x^2])/(54*c^(8/3)*d^(4/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
```

```
p, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}
\int \frac{a+bx^3}{\left(c+dx^3\right)^3}\,dx &= -\frac{(bc-ad)x}{6cd\left(c+dx^3\right)^2}+\frac{(bc+5ad)\int \frac{1}{\left(c+dx^3\right)^2}\,dx}{6cd} \\
&= -\frac{(bc-ad)x}{6cd\left(c+dx^3\right)^2}+\frac{(bc+5ad)x}{18c^2d\left(c+dx^3\right)}+\frac{(bc+5ad)\int \frac{1}{c+dx^3}\,dx}{9c^2d} \\
&= -\frac{(bc-ad)x}{6cd\left(c+dx^3\right)^2}+\frac{(bc+5ad)x}{18c^2d\left(c+dx^3\right)}+\frac{(bc+5ad)\int \frac{1}{\sqrt[3]{c}+\sqrt[3]{d}\,x}\,dx}{27c^{8/3}d}+\frac{(bc+5ad)\int \frac{2\sqrt[3]{c}-\sqrt[3]{d}\,x}{c^{2/3}-\sqrt[3]{c}\,\sqrt[3]{d}\,x+d^{2/3}x \cdots}}{27c^{8/3}d} \\
&= -\frac{(bc-ad)x}{6cd\left(c+dx^3\right)^2}+\frac{(bc+5ad)x}{18c^2d\left(c+dx^3\right)}+\frac{(bc+5ad)\log\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{27c^{8/3}d^{4/3}}-\frac{(bc+5ad)\int \frac{-\sqrt[3]{c}\,\sqrt[3]{d}+2d \cdots}{c^{2/3}-\sqrt[3]{c}\,\sqrt[3]{d}\,x+ \cdots}}{54c^{8/3}d^{4/3}} \\
&= -\frac{(bc-ad)x}{6cd\left(c+dx^3\right)^2}+\frac{(bc+5ad)x}{18c^2d\left(c+dx^3\right)}+\frac{(bc+5ad)\log\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{27c^{8/3}d^{4/3}}-\frac{(bc+5ad)\log\left(c^{2/3}-\sqrt[3]{c} \cdots\right.}{54c^{8/3}d^{4/3}} \\
&= -\frac{(bc-ad)x}{6cd\left(c+dx^3\right)^2}+\frac{(bc+5ad)x}{18c^2d\left(c+dx^3\right)}-\frac{(bc+5ad)\tan^{-1}\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}\,x}{\sqrt{3}\,\sqrt[3]{c}}\right)}{9\sqrt{3}\,c^{8/3}d^{4/3}}+\frac{(bc+5ad)\log\left(\sqrt[3]{c}+\sqrt[3]{\cdots}\right.}{27c^{8/3}d^{4/3}}
\end{aligned}$$

Mathematica [A] time = 0.13, size = 175, normalized size = 0.89

$$\frac{-(5ad+bc)\log\left(c^{2/3}-\sqrt[3]{c}\,\sqrt[3]{d}\,x+d^{2/3}x^2\right)-\frac{9c^{5/3}\sqrt[3]{d}\,x(bc-ad)}{\left(c+dx^3\right)^2}+\frac{3c^{2/3}\sqrt[3]{d}\,x(5ad+bc)}{c+dx^3}+2(5ad+bc)\log\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)-2\sqrt{3}\,(5ad+bc)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\,x}{\sqrt[3]{c}}}{\sqrt{3}}\right)}{54c^{8/3}d^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)/(c + d*x^3)^3,x]
```

```
[Out] ((-9*c^(5/3)*d^(1/3)*(b*c - a*d)*x)/(c + d*x^3)^2 + (3*c^(2/3)*d^(1/3)*(b*c
 + 5*a*d)*x)/(c + d*x^3) - 2*Sqrt[3]*(b*c + 5*a*d)*ArcTan[(1 - (2*d^(1/3)*x
)/c^(1/3))/Sqrt[3]] + 2*(b*c + 5*a*d)*Log[c^(1/3) + d^(1/3)*x] - (b*c + 5*a
*d)*Log[c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2])/(54*c^(8/3)*d^(4/3))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a+bx^3}{\left(c+dx^3\right)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x^3)/(c + d*x^3)^3,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x^3)/(c + d*x^3)^3, x]
```

fricas [B] time = 1.18, size = 743, normalized size = 3.77

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)/(d*x^3+c)^3,x, algorithm="fricas")
```

```
[Out] [1/54*(3*(b*c^3*d^2 + 5*a*c^2*d^3)*x^4 + 3*sqrt(1/3)*((b*c^2*d^3 + 5*a*c*d^
4)*x^6 + b*c^4*d + 5*a*c^3*d^2 + 2*(b*c^3*d^2 + 5*a*c^2*d^3)*x^3)*sqrt(-(c^
2*d)^(1/3)/d)*log((2*c*d*x^3 - 3*(c^2*d)^(1/3)*c*x - c^2 + 3*sqrt(1/3)*(2*c
*d*x^2 + (c^2*d)^(2/3)*x - (c^2*d)^(1/3)*c)*sqrt(-(c^2*d)^(1/3)/d))/(d*x^3
+ c)) - ((b*c*d^2 + 5*a*d^3)*x^6 + b*c^3 + 5*a*c^2*d + 2*(b*c^2*d + 5*a*c*d
^2)*x^3)*(c^2*d)^(2/3)*log(c*d*x^2 - (c^2*d)^(2/3)*x + (c^2*d)^(1/3)*c) + 2
*((b*c*d^2 + 5*a*d^3)*x^6 + b*c^3 + 5*a*c^2*d + 2*(b*c^2*d + 5*a*c*d^2)*x^3
)*(c^2*d)^(2/3)*log(c*d*x + (c^2*d)^(2/3)) - 6*(b*c^4*d - 4*a*c^3*d^2)*x)/(
c^4*d^4*x^6 + 2*c^5*d^3*x^3 + c^6*d^2), 1/54*(3*(b*c^3*d^2 + 5*a*c^2*d^3)*x
^4 + 6*sqrt(1/3)*((b*c^2*d^3 + 5*a*c*d^4)*x^6 + b*c^4*d + 5*a*c^3*d^2 + 2*(
b*c^3*d^2 + 5*a*c^2*d^3)*x^3)*sqrt((c^2*d)^(1/3)/d)*arctan(sqrt(1/3)*(2*(c^
2*d)^(2/3)*x - (c^2*d)^(1/3)*c)*sqrt((c^2*d)^(1/3)/d)/c^2) - ((b*c*d^2 + 5*
a*d^3)*x^6 + b*c^3 + 5*a*c^2*d + 2*(b*c^2*d + 5*a*c*d^2)*x^3)*(c^2*d)^(2/3)
*log(c*d*x^2 - (c^2*d)^(2/3)*x + (c^2*d)^(1/3)*c) + 2*((b*c*d^2 + 5*a*d^3)*
x^6 + b*c^3 + 5*a*c^2*d + 2*(b*c^2*d + 5*a*c*d^2)*x^3)*(c^2*d)^(2/3)*log(c*
d*x + (c^2*d)^(2/3)) - 6*(b*c^4*d - 4*a*c^3*d^2)*x)/(c^4*d^4*x^6 + 2*c^5*d^
3*x^3 + c^6*d^2)]
```

giac [A] time = 0.19, size = 180, normalized size = 0.91

$$-\frac{\sqrt{3}\,(bc+5\,ad)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{27\left(-cd^2\right)^{\frac{2}{3}}c^2}-\frac{(bc+5\,ad)\log\left(x^2+x\left(-\frac{c}{d}\right)^{\frac{1}{3}}+\left(-\frac{c}{d}\right)^{\frac{2}{3}}\right)}{54\left(-cd^2\right)^{\frac{2}{3}}c^2}-\frac{(bc+5\,ad)\left(-\frac{c}{d}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right|\right)}{27\,c^3d}+\frac{bcdx^4+5\,ad^2x^4-2\,bc^2x+8\,acdx}{18\left(dx^3+c\right)^2c^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)/(d*x^3+c)^3,x, algorithm="giac")
```

```
[Out] -1/27*sqrt(3)*(b*c + 5*a*d)*arctan(1/3*sqrt(3)*(2*x + (-c/d)^(1/3))/(-c/d)^
(1/3))/((-c*d^2)^(2/3)*c^2) - 1/54*(b*c + 5*a*d)*log(x^2 + x*(-c/d)^(1/3) +
 (-c/d)^(2/3))/((-c*d^2)^(2/3)*c^2) - 1/27*(b*c + 5*a*d)*(-c/d)^(1/3)*log(a
bs(x - (-c/d)^(1/3)))/(c^3*d) + 1/18*(b*c*d*x^4 + 5*a*d^2*x^4 - 2*b*c^2*x +
 8*a*c*d*x)/((d*x^3 + c)^2*c^2*d)
```

maple [A] time = 0.06, size = 249, normalized size = 1.26

$$\frac{5\sqrt{3}\,a\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{c}{d}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{27\left(\frac{c}{d}\right)^{\frac{2}{3}}c^{2}d}+\frac{5a\ln\left(x+\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{27\left(\frac{c}{d}\right)^{\frac{2}{3}}c^{2}d}-\frac{5a\ln\left(x^{2}-\left(\frac{c}{d}\right)^{\frac{1}{3}}x+\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}{54\left(\frac{c}{d}\right)^{\frac{2}{3}}c^{2}d}+\frac{\sqrt{3}\,b\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{c}{d}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{27\left(\frac{c}{d}\right)^{\frac{2}{3}}c\,d^{2}}+\frac{b\ln\left(x+\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{27\left(\frac{c}{d}\right)^{\frac{2}{3}}c\,d^{2}}-\frac{b\ln\left(x^{2}-\left(\frac{c}{d}\right)^{\frac{1}{3}}x+\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}{54\left(\frac{c}{d}\right)^{\frac{2}{3}}c\,d^{2}}+\frac{\frac{(5ad+bc)x^{4}}{18c^{2}}+\frac{(4ad-bc)x}{9cd}}{\left(d\,x^{3}+c\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)/(d*x^3+c)^3,x)
```

```
[Out] (1/18*(5*a*d+b*c)/c^2*x^4+1/9*(4*a*d-b*c)/c/d*x)/(d*x^3+c)^2+5/27/c^2/d/(c/
d)^(2/3)*ln(x+(c/d)^(1/3))*a+1/27/c/d^2/(c/d)^(2/3)*ln(x+(c/d)^(1/3))*b-5/5
4/c^2/d/(c/d)^(2/3)*ln(x^2-(c/d)^(1/3)*x+(c/d)^(2/3))*a-1/54/c/d^2/(c/d)^(2
/3)*ln(x^2-(c/d)^(1/3)*x+(c/d)^(2/3))*b+5/27/c^2/d/(c/d)^(2/3)*3^(1/2)*arct
an(1/3*3^(1/2)*(2/(c/d)^(1/3)*x-1))*a+1/27/c/d^2/(c/d)^(2/3)*3^(1/2)*arctan
(1/3*3^(1/2)*(2/(c/d)^(1/3)*x-1))*b
```

maxima [A] time = 1.43, size = 192, normalized size = 0.97

$$\frac{\left(bcd+5\,ad^{2}\right)x^{4}-2\left(bc^{2}-4\,acd\right)x}{18\left(c^{2}d^{3}x^{6}+2\,c^{3}d^{2}x^{3}+c^{4}d\right)}+\frac{\sqrt{3}\,(bc+5\,ad)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{27\,c^{2}d^{2}\left(\frac{c}{d}\right)^{\frac{2}{3}}}-\frac{(bc+5\,ad)\log\left(x^{2}-x\left(\frac{c}{d}\right)^{\frac{1}{3}}+\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}{54\,c^{2}d^{2}\left(\frac{c}{d}\right)^{\frac{2}{3}}}+\frac{(bc+5\,ad)\log\left(x+\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{27\,c^{2}d^{2}\left(\frac{c}{d}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)/(d*x^3+c)^3,x, algorithm="maxima")
```

```
[Out] 1/18*((b*c*d + 5*a*d^2)*x^4 - 2*(b*c^2 - 4*a*c*d)*x)/(c^2*d^3*x^6 + 2*c^3*d
^2*x^3 + c^4*d) + 1/27*sqrt(3)*(b*c + 5*a*d)*arctan(1/3*sqrt(3)*(2*x - (c/d
)^(1/3))/(c/d)^(1/3))/(c^2*d^2*(c/d)^(2/3)) - 1/54*(b*c + 5*a*d)*log(x^2 -
x*(c/d)^(1/3) + (c/d)^(2/3))/(c^2*d^2*(c/d)^(2/3)) + 1/27*(b*c + 5*a*d)*log
(x + (c/d)^(1/3))/(c^2*d^2*(c/d)^(2/3))
```

mupad [B] time = 1.40, size = 173, normalized size = 0.88

$$\frac{\frac{x^{4}(5\,a\,d+b\,c)}{18\,c^{2}}+\frac{x\,(4\,a\,d-b\,c)}{9\,c\,d}}{c^{2}+2\,c\,d\,x^{3}+d^{2}\,x^{6}}+\frac{\ln\left(d^{1/3}\,x+c^{1/3}\right)(5\,a\,d+b\,c)}{27\,c^{8/3}\,d^{4/3}}-\frac{\ln\left(c^{1/3}-2\,d^{1/3}\,x+\sqrt{3}\,c^{1/3}\,1\mathrm{i}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)(5\,a\,d+b\,c)}{27\,c^{8/3}\,d^{4/3}}+\frac{\ln\left(2\,d^{1/3}\,x-c^{1/3}+\sqrt{3}\,c^{1/3}\,1\mathrm{i}\right)\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)(5\,a\,d+b\,c)}{27\,c^{8/3}\,d^{4/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)/(c + d*x^3)^3,x)
```

```
[Out] ((x^4*(5*a*d + b*c))/(18*c^2) + (x*(4*a*d - b*c))/(9*c*d))/(c^2 + d^2*x^6 +
 2*c*d*x^3) + (log(d^(1/3)*x + c^(1/3))*(5*a*d + b*c))/(27*c^(8/3)*d^(4/3))
 - (log(3^(1/2)*c^(1/3)*1i - 2*d^(1/3)*x + c^(1/3))*((3^(1/2)*1i)/2 + 1/2)*
(5*a*d + b*c))/(27*c^(8/3)*d^(4/3)) + (log(3^(1/2)*c^(1/3)*1i + 2*d^(1/3)*x
 - c^(1/3))*((3^(1/2)*1i)/2 - 1/2)*(5*a*d + b*c))/(27*c^(8/3)*d^(4/3))
```

sympy [A] time = 0.78, size = 133, normalized size = 0.68

$$\frac{x^{4}\left(5ad^{2}+bcd\right)+x\left(8acd-2bc^{2}\right)}{18c^{4}d+36c^{3}d^{2}x^{3}+18c^{2}d^{3}x^{6}}+\operatorname{RootSum}\left(19683t^{3}c^{8}d^{4}-125a^{3}d^{3}-75a^{2}bcd^{2}-15ab^{2}c^{2}d-b^{3}c^{3},\left(t\mapsto t\log\left(\frac{27tc^{3}d}{5ad+bc}+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)/(d*x**3+c)**3,x)
```

```
[Out] (x**4*(5*a*d**2 + b*c*d) + x*(8*a*c*d - 2*b*c**2))/(18*c**4*d + 36*c**3*d**
2*x**3 + 18*c**2*d**3*x**6) + RootSum(19683*_t**3*c**8*d**4 - 125*a**3*d**3
 - 75*a**2*b*c*d**2 - 15*a*b**2*c**2*d - b**3*c**3, Lambda(_t, _t*log(27*_t
*c**3*d/(5*a*d + b*c) + x)))
```

3.8 $\int \left(a+bx^3\right)^2 \left(c+dx^3\right)^3 \, dx$

Optimal. Leaf size=122

$$\frac{1}{10}dx^{10}\left(a^2d^2+6abcd+3b^2c^2\right)+\frac{1}{7}cx^7\left(3a^2d^2+6abcd+b^2c^2\right)+a^2c^3x+\frac{1}{4}ac^2x^4(3ad+2bc)+\frac{1}{13}bd^2x^{13}(2ad+3bc)$$

Rubi [A] time = 0.07, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {373}

$$\frac{1}{10}dx^{10}\left(a^2d^2+6abcd+3b^2c^2\right)+\frac{1}{7}cx^7\left(3a^2d^2+6abcd+b^2c^2\right)+a^2c^3x+\frac{1}{4}ac^2x^4(3ad+2bc)+\frac{1}{13}bd^2x^{13}(2ad+3bc)+\frac{1}{16}b^2d^3x^{16}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^2*(c + d*x^3)^3,x]
```

```
[Out] a^2*c^3*x + (a*c^2*(2*b*c + 3*a*d)*x^4)/4 + (c*(b^2*c^2 + 6*a*b*c*d + 3*a^2
*d^2)*x^7)/7 + (d*(3*b^2*c^2 + 6*a*b*c*d + a^2*d^2)*x^10)/10 + (b*d^2*(3*b*
c + 2*a*d)*x^13)/13 + (b^2*d^3*x^16)/16
```

Rule 373

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \left(a+bx^3\right)^2\left(c+dx^3\right)^3\,dx &= \int \left(a^2c^3+ac^2(2bc+3ad)x^3+c\left(b^2c^2+6abcd+3a^2d^2\right)x^6+d\left(3b^2c^2+6abcd+\right.\right.\\ &= a^2c^3x+\frac{1}{4}ac^2(2bc+3ad)x^4+\frac{1}{7}c\left(b^2c^2+6abcd+3a^2d^2\right)x^7+\frac{1}{10}d\left(3b^2c^2+6ab\right.\end{aligned}$$

Mathematica [A] time = 0.02, size = 122, normalized size = 1.00

$$\frac{1}{10}dx^{10}\left(a^2d^2+6abcd+3b^2c^2\right)+\frac{1}{7}cx^7\left(3a^2d^2+6abcd+b^2c^2\right)+a^2c^3x+\frac{1}{4}ac^2x^4(3ad+2bc)+\frac{1}{13}bd^2x^{13}(2ad+3bc)+\frac{1}{16}b^2d^3x^{16}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^2*(c + d*x^3)^3,x]
```

```
[Out] a^2*c^3*x + (a*c^2*(2*b*c + 3*a*d)*x^4)/4 + (c*(b^2*c^2 + 6*a*b*c*d + 3*a^2
*d^2)*x^7)/7 + (d*(3*b^2*c^2 + 6*a*b*c*d + a^2*d^2)*x^10)/10 + (b*d^2*(3*b*
c + 2*a*d)*x^13)/13 + (b^2*d^3*x^16)/16
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+bx^3\right)^2\left(c+dx^3\right)^3\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x^3)^2*(c + d*x^3)^3,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x^3)^2*(c + d*x^3)^3, x]
```

fricas [A] time = 0.73, size = 132, normalized size = 1.08

$$\frac{1}{16}x^{16}d^3b^2+\frac{3}{13}x^{13}d^2cb^2+\frac{2}{13}x^{13}d^3ba+\frac{3}{10}x^{10}dc^2b^2+\frac{3}{5}x^{10}d^2cba+\frac{1}{10}x^{10}d^3a^2+\frac{1}{7}x^7c^3b^2+\frac{6}{7}x^7dc^2ba+\frac{3}{7}x^7d^2ca^2+\frac{1}{2}x^4c^3ba+\frac{3}{4}x^4dc^2a^2+xc^3a^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*(d*x^3+c)^3,x, algorithm="fricas")
```

```
[Out] 1/16*x^16*d^3*b^2 + 3/13*x^13*d^2*c*b^2 + 2/13*x^13*d^3*b*a + 3/10*x^10*d*c
^2*b^2 + 3/5*x^10*d^2*c*b*a + 1/10*x^10*d^3*a^2 + 1/7*x^7*c^3*b^2 + 6/7*x^7
*d*c^2*b*a + 3/7*x^7*d^2*c*a^2 + 1/2*x^4*c^3*b*a + 3/4*x^4*d*c^2*a^2 + x*c^
3*a^2
```

giac [A] time = 0.16, size = 132, normalized size = 1.08

$$\frac{1}{16}b^2d^3x^{16}+\frac{3}{13}b^2cd^2x^{13}+\frac{2}{13}abd^3x^{13}+\frac{3}{10}b^2c^2dx^{10}+\frac{3}{5}abcd^2x^{10}+\frac{1}{10}a^2d^3x^{10}+\frac{1}{7}b^2c^3x^7+\frac{6}{7}abc^2dx^7+\frac{3}{7}a^2cd^2x^7+\frac{1}{2}abc^3x^4+\frac{3}{4}a^2c^2dx^4+a^2c^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*(d*x^3+c)^3,x, algorithm="giac")
```

```
[Out] 1/16*b^2*d^3*x^16 + 3/13*b^2*c*d^2*x^13 + 2/13*a*b*d^3*x^13 + 3/10*b^2*c^2*
d*x^10 + 3/5*a*b*c*d^2*x^10 + 1/10*a^2*d^3*x^10 + 1/7*b^2*c^3*x^7 + 6/7*a*b
*c^2*d*x^7 + 3/7*a^2*c*d^2*x^7 + 1/2*a*b*c^3*x^4 + 3/4*a^2*c^2*d*x^4 + a^2*
c^3*x
```

maple [A] time = 0.05, size = 125, normalized size = 1.02

$$\frac{b^2d^3x^{16}}{16}+\frac{\left(2ab\,d^3+3b^2c\,d^2\right)x^{13}}{13}+\frac{\left(a^2d^3+6abc\,d^2+3b^2c^2d\right)x^{10}}{10}+\frac{\left(3a^2c\,d^2+6ab\,c^2d+b^2c^3\right)x^7}{7}+a^2c^3x+\frac{\left(3a^2c^2d+2ab\,c^3\right)x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^2*(d*x^3+c)^3,x)
```

```
[Out] 1/16*b^2*d^3*x^16+1/13*(2*a*b*d^3+3*b^2*c*d^2)*x^13+1/10*(a^2*d^3+6*a*b*c*d
^2+3*b^2*c^2*d)*x^10+1/7*(3*a^2*c*d^2+6*a*b*c^2*d+b^2*c^3)*x^7+1/4*(3*a^2*c
^2*d+2*a*b*c^3)*x^4+a^2*c^3*x
```

maxima [A] time = 0.55, size = 124, normalized size = 1.02

$$\frac{1}{16}b^2d^3x^{16}+\frac{1}{13}\left(3\,b^2cd^2+2\,abd^3\right)x^{13}+\frac{1}{10}\left(3\,b^2c^2d+6\,abcd^2+a^2d^3\right)x^{10}+\frac{1}{7}\left(b^2c^3+6\,abc^2d+3\,a^2cd^2\right)x^7+a^2c^3x+\frac{1}{4}\left(2\,abc^3+3\,a^2c^2d\right)x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*(d*x^3+c)^3,x, algorithm="maxima")
```

```
[Out] 1/16*b^2*d^3*x^16 + 1/13*(3*b^2*c*d^2 + 2*a*b*d^3)*x^13 + 1/10*(3*b^2*c^2*d
 + 6*a*b*c*d^2 + a^2*d^3)*x^10 + 1/7*(b^2*c^3 + 6*a*b*c^2*d + 3*a^2*c*d^2)*
x^7 + a^2*c^3*x + 1/4*(2*a*b*c^3 + 3*a^2*c^2*d)*x^4
```

mupad [B] time = 1.20, size = 116, normalized size = 0.95

$$x^7\left(\frac{3\,a^2\,c\,d^2}{7}+\frac{6\,a\,b\,c^2\,d}{7}+\frac{b^2\,c^3}{7}\right)+x^{10}\left(\frac{a^2\,d^3}{10}+\frac{3\,a\,b\,c\,d^2}{5}+\frac{3\,b^2\,c^2\,d}{10}\right)+a^2\,c^3\,x+\frac{b^2\,d^3\,x^{16}}{16}+\frac{a\,c^2\,x^4\,\left(3\,a\,d+2\,b\,c\right)}{4}+\frac{b\,d^2\,x^{13}\,\left(2\,a\,d+3\,b\,c\right)}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^2*(c + d*x^3)^3,x)
```

```
[Out] x^7*((b^2*c^3)/7 + (3*a^2*c*d^2)/7 + (6*a*b*c^2*d)/7) + x^10*((a^2*d^3)/10
+ (3*b^2*c^2*d)/10 + (3*a*b*c*d^2)/5) + a^2*c^3*x + (b^2*d^3*x^16)/16 + (a*
c^2*x^4*(3*a*d + 2*b*c))/4 + (b*d^2*x^13*(2*a*d + 3*b*c))/13
```

sympy [A] time = 0.09, size = 139, normalized size = 1.14

$$a^2c^3x+\frac{b^2d^3x^{16}}{16}+x^{13}\left(\frac{2abd^3}{13}+\frac{3b^2cd^2}{13}\right)+x^{10}\left(\frac{a^2d^3}{10}+\frac{3abcd^2}{5}+\frac{3b^2c^2d}{10}\right)+x^7\left(\frac{3a^2cd^2}{7}+\frac{6abc^2d}{7}+\frac{b^2c^3}{7}\right)+x^4\left(\frac{3a^2c^2d}{4}+\frac{abc^3}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**2*(d*x**3+c)**3,x)
```

```
[Out] a**2*c**3*x + b**2*d**3*x**16/16 + x**13*(2*a*b*d**3/13 + 3*b**2*c*d**2/13)
 + x**10*(a**2*d**3/10 + 3*a*b*c*d**2/5 + 3*b**2*c**2*d/10) + x**7*(3*a**2*
c*d**2/7 + 6*a*b*c**2*d/7 + b**2*c**3/7) + x**4*(3*a**2*c**2*d/4 + a*b*c**3
/2)
```

3.9 $\int \left(a+bx^3\right)^2 \left(c+dx^3\right)^2 \, dx$

Optimal. Leaf size=82

$$\frac{1}{7}x^7\left(a^2d^2+4abcd+b^2c^2\right)+a^2c^2x+\frac{1}{5}bdx^{10}(ad+bc)+\frac{1}{2}acx^4(ad+bc)+\frac{1}{13}b^2d^2x^{13}$$

Rubi [A] time = 0.05, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {373}

$$\frac{1}{7}x^7\left(a^2d^2+4abcd+b^2c^2\right)+a^2c^2x+\frac{1}{5}bdx^{10}(ad+bc)+\frac{1}{2}acx^4(ad+bc)+\frac{1}{13}b^2d^2x^{13}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^2*(c + d*x^3)^2,x]
```

```
[Out] a^2*c^2*x + (a*c*(b*c + a*d)*x^4)/2 + ((b^2*c^2 + 4*a*b*c*d + a^2*d^2)*x^7)
/7 + (b*d*(b*c + a*d)*x^10)/5 + (b^2*d^2*x^13)/13
```

Rule 373

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \left(a+bx^3\right)^2\left(c+dx^3\right)^2\,dx &= \int \left(a^2c^2+2ac(bc+ad)x^3+\left(b^2c^2+4abcd+a^2d^2\right)x^6+2bd(bc+ad)x^9+b^2d^2x^{12}\right) \\ &= a^2c^2x+\frac{1}{2}ac(bc+ad)x^4+\frac{1}{7}\left(b^2c^2+4abcd+a^2d^2\right)x^7+\frac{1}{5}bd(bc+ad)x^{10}+\frac{1}{13}b^2d^2\end{aligned}$$

Mathematica [A] time = 0.01, size = 82, normalized size = 1.00

$$\frac{1}{7}x^7\left(a^2d^2+4abcd+b^2c^2\right)+a^2c^2x+\frac{1}{5}bdx^{10}(ad+bc)+\frac{1}{2}acx^4(ad+bc)+\frac{1}{13}b^2d^2x^{13}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^2*(c + d*x^3)^2,x]
```

```
[Out] a^2*c^2*x + (a*c*(b*c + a*d)*x^4)/2 + ((b^2*c^2 + 4*a*b*c*d + a^2*d^2)*x^7)
/7 + (b*d*(b*c + a*d)*x^10)/5 + (b^2*d^2*x^13)/13
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+bx^3\right)^2\left(c+dx^3\right)^2\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x^3)^2*(c + d*x^3)^2,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x^3)^2*(c + d*x^3)^2, x]
```

fricas [A] time = 0.82, size = 91, normalized size = 1.11

$$\frac{1}{13}x^{13}d^2b^2+\frac{1}{5}x^{10}dcb^2+\frac{1}{5}x^{10}d^2ba+\frac{1}{7}x^7c^2b^2+\frac{4}{7}x^7dcba+\frac{1}{7}x^7d^2a^2+\frac{1}{2}x^4c^2ba+\frac{1}{2}x^4dca^2+xc^2a^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*(d*x^3+c)^2,x, algorithm="fricas")
```

```
[Out] 1/13*x^13*d^2*b^2 + 1/5*x^10*d*c*b^2 + 1/5*x^10*d^2*b*a + 1/7*x^7*c^2*b^2 +
 4/7*x^7*d*c*b*a + 1/7*x^7*d^2*a^2 + 1/2*x^4*c^2*b*a + 1/2*x^4*d*c*a^2 + x*
c^2*a^2
```

giac [A] time = 0.21, size = 91, normalized size = 1.11

$$\frac{1}{13}b^2d^2x^{13} + \frac{1}{5}b^2cdx^{10} + \frac{1}{5}abd^2x^{10} + \frac{1}{7}b^2c^2x^7 + \frac{4}{7}abcdx^7 + \frac{1}{7}a^2d^2x^7 + \frac{1}{2}abc^2x^4 + \frac{1}{2}a^2cdx^4 + a^2c^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*(d*x^3+c)^2,x, algorithm="giac")
```

```
[Out] 1/13*b^2*d^2*x^13 + 1/5*b^2*c*d*x^10 + 1/5*a*b*d^2*x^10 + 1/7*b^2*c^2*x^7 +
 4/7*a*b*c*d*x^7 + 1/7*a^2*d^2*x^7 + 1/2*a*b*c^2*x^4 + 1/2*a^2*c*d*x^4 + a^
2*c^2*x
```

maple [A] time = 0.04, size = 87, normalized size = 1.06

$$\frac{b^2d^2x^{13}}{13} + \frac{\left(2ab\,d^2 + 2b^2cd\right)x^{10}}{10} + \frac{\left(a^2d^2 + 4abcd + b^2c^2\right)x^7}{7} + a^2c^2x + \frac{\left(2a^2cd + 2ab\,c^2\right)x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^2*(d*x^3+c)^2,x)
```

```
[Out] 1/13*b^2*d^2*x^13+1/10*(2*a*b*d^2+2*b^2*c*d)*x^10+1/7*(a^2*d^2+4*a*b*c*d+b^
2*c^2)*x^7+1/4*(2*a^2*c*d+2*a*b*c^2)*x^4+a^2*c^2*x
```

maxima [A] time = 0.71, size = 82, normalized size = 1.00

$$\frac{1}{13}b^2d^2x^{13} + \frac{1}{5}\left(b^2cd + abd^2\right)x^{10} + \frac{1}{7}\left(b^2c^2 + 4\,abcd + a^2d^2\right)x^7 + a^2c^2x + \frac{1}{2}\left(abc^2 + a^2cd\right)x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*(d*x^3+c)^2,x, algorithm="maxima")
```

```
[Out] 1/13*b^2*d^2*x^13 + 1/5*(b^2*c*d + a*b*d^2)*x^10 + 1/7*(b^2*c^2 + 4*a*b*c*d
 + a^2*d^2)*x^7 + a^2*c^2*x + 1/2*(a*b*c^2 + a^2*c*d)*x^4
```

mupad [B] time = 0.04, size = 75, normalized size = 0.91

$$x^7\left(\frac{a^2\,d^2}{7} + \frac{4\,a\,b\,c\,d}{7} + \frac{b^2\,c^2}{7}\right) + a^2\,c^2\,x + \frac{b^2\,d^2\,x^{13}}{13} + \frac{a\,c\,x^4\,\left(a\,d + b\,c\right)}{2} + \frac{b\,d\,x^{10}\,\left(a\,d + b\,c\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^2*(c + d*x^3)^2,x)
```

```
[Out] x^7*((a^2*d^2)/7 + (b^2*c^2)/7 + (4*a*b*c*d)/7) + a^2*c^2*x + (b^2*d^2*x^13
)/13 + (a*c*x^4*(a*d + b*c))/2 + (b*d*x^10*(a*d + b*c))/5
```

sympy [A] time = 0.08, size = 90, normalized size = 1.10

$$a^2c^2x + \frac{b^2d^2x^{13}}{13} + x^{10}\left(\frac{abd^2}{5} + \frac{b^2cd}{5}\right) + x^7\left(\frac{a^2d^2}{7} + \frac{4abcd}{7} + \frac{b^2c^2}{7}\right) + x^4\left(\frac{a^2cd}{2} + \frac{abc^2}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**2*(d*x**3+c)**2,x)
```

```
[Out] a**2*c**2*x + b**2*d**2*x**13/13 + x**10*(a*b*d**2/5 + b**2*c*d/5) + x**7*(
a**2*d**2/7 + 4*a*b*c*d/7 + b**2*c**2/7) + x**4*(a**2*c*d/2 + a*b*c**2/2)
```

3.10 $\int \left(a+bx^3\right)^2 \left(c+dx^3\right)\,dx$

Optimal. Leaf size=50

$$a^2cx+\frac{1}{7}bx^7(2ad+bc)+\frac{1}{4}ax^4(ad+2bc)+\frac{1}{10}b^2dx^{10}$$

Rubi [A] time = 0.03, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {373}

$$a^2cx+\frac{1}{7}bx^7(2ad+bc)+\frac{1}{4}ax^4(ad+2bc)+\frac{1}{10}b^2dx^{10}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^2*(c + d*x^3),x]
```

```
[Out] a^2*c*x + (a*(2*b*c + a*d)*x^4)/4 + (b*(b*c + 2*a*d)*x^7)/7 + (b^2*d*x^10)/
10
```

Rule 373

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \left(a+bx^3\right)^2\left(c+dx^3\right)\,dx &= \int \left(a^2c+a(2bc+ad)x^3+b(bc+2ad)x^6+b^2dx^9\right)\,dx\\ &= a^2cx+\frac{1}{4}a(2bc+ad)x^4+\frac{1}{7}b(bc+2ad)x^7+\frac{1}{10}b^2dx^{10}\end{aligned}$$

Mathematica [A] time = 0.01, size = 50, normalized size = 1.00

$$a^2cx+\frac{1}{7}bx^7(2ad+bc)+\frac{1}{4}ax^4(ad+2bc)+\frac{1}{10}b^2dx^{10}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^2*(c + d*x^3),x]
```

```
[Out] a^2*c*x + (a*(2*b*c + a*d)*x^4)/4 + (b*(b*c + 2*a*d)*x^7)/7 + (b^2*d*x^10)/
10
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+bx^3\right)^2\left(c+dx^3\right)\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x^3)^2*(c + d*x^3),x]
```

```
[Out] IntegrateAlgebraic[(a + b*x^3)^2*(c + d*x^3), x]
```

fricas [A] time = 0.51, size = 50, normalized size = 1.00

$$\frac{1}{10}x^{10}db^2+\frac{1}{7}x^7cb^2+\frac{2}{7}x^7dba+\frac{1}{2}x^4cba+\frac{1}{4}x^4da^2+xca^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*(d*x^3+c),x, algorithm="fricas")
```

```
[Out] 1/10*x^10*d*b^2 + 1/7*x^7*c*b^2 + 2/7*x^7*d*b*a + 1/2*x^4*c*b*a + 1/4*x^4*d
*a^2 + x*c*a^2
```

giac [A] time = 0.16, size = 50, normalized size = 1.00

$$\frac{1}{10}\,b^2dx^{10}+\frac{1}{7}\,b^2cx^7+\frac{2}{7}\,abdx^7+\frac{1}{2}\,abcx^4+\frac{1}{4}\,a^2dx^4+a^2cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*(d*x^3+c),x, algorithm="giac")
```

```
[Out] 1/10*b^2*d*x^10 + 1/7*b^2*c*x^7 + 2/7*a*b*d*x^7 + 1/2*a*b*c*x^4 + 1/4*a^2*d
*x^4 + a^2*c*x
```

maple [A] time = 0.04, size = 49, normalized size = 0.98

$$\frac{b^2d\,x^{10}}{10}+\frac{\left(2abd+b^2c\right)x^7}{7}+a^2cx+\frac{\left(a^2d+2abc\right)x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^2*(d*x^3+c),x)
```

```
[Out] 1/10*b^2*d*x^10+1/7*(2*a*b*d+b^2*c)*x^7+1/4*(a^2*d+2*a*b*c)*x^4+a^2*c*x
```

maxima [A] time = 0.57, size = 48, normalized size = 0.96

$$\frac{1}{10}\,b^2dx^{10}+\frac{1}{7}\left(b^2c+2\,abd\right)x^7+\frac{1}{4}\left(2\,abc+a^2d\right)x^4+a^2cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*(d*x^3+c),x, algorithm="maxima")
```

```
[Out] 1/10*b^2*d*x^10 + 1/7*(b^2*c + 2*a*b*d)*x^7 + 1/4*(2*a*b*c + a^2*d)*x^4 + a
^2*c*x
```

mupad [B] time = 0.04, size = 48, normalized size = 0.96

$$x^4\left(\frac{d\,a^2}{4}+\frac{b\,c\,a}{2}\right)+x^7\left(\frac{c\,b^2}{7}+\frac{2\,a\,d\,b}{7}\right)+\frac{b^2\,d\,x^{10}}{10}+a^2\,c\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^2*(c + d*x^3),x)
```

```
[Out] x^4*((a^2*d)/4 + (a*b*c)/2) + x^7*((b^2*c)/7 + (2*a*b*d)/7) + (b^2*d*x^10)/
10 + a^2*c*x
```

sympy [A] time = 0.07, size = 51, normalized size = 1.02

$$a^2cx+\frac{b^2dx^{10}}{10}+x^7\left(\frac{2abd}{7}+\frac{b^2c}{7}\right)+x^4\left(\frac{a^2d}{4}+\frac{abc}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**2*(d*x**3+c),x)
```

```
[Out] a**2*c*x + b**2*d*x**10/10 + x**7*(2*a*b*d/7 + b**2*c/7) + x**4*(a**2*d/4 +
 a*b*c/2)
```

3.11 $\int \frac{(a+bx^3)^2}{c+dx^3} \, dx$

Optimal. Leaf size=173

$$-\frac{(bc-ad)^2\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{6c^{2/3}d^{7/3}}+\frac{(bc-ad)^2\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{2/3}d^{7/3}}-\frac{(bc-ad)^2\tan^{-1}\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{\sqrt{3}\sqrt[3]{c}}\right)}{\sqrt{3}c^{2/3}d^{7/3}}-\frac{bx(bc-2ad)}{d^2}$$

Rubi [A] time = 0.13, antiderivative size = 173, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {390, 200, 31, 634, 617, 204, 628}

$$-\frac{(bc-ad)^2\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{6c^{2/3}d^{7/3}}+\frac{(bc-ad)^2\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{2/3}d^{7/3}}-\frac{(bc-ad)^2\tan^{-1}\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{\sqrt{3}\sqrt[3]{c}}\right)}{\sqrt{3}c^{2/3}d^{7/3}}-\frac{bx(bc-2ad)}{d^2}+\frac{b^2x^4}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^2/(c + d*x^3),x]
```

```
[Out] -((b*(b*c - 2*a*d)*x)/d^2) + (b^2*x^4)/(4*d) - ((b*c - a*d)^2*ArcTan[(c^(1/
3) - 2*d^(1/3)*x)/(Sqrt[3]*c^(1/3))])/(Sqrt[3]*c^(2/3)*d^(7/3)) + ((b*c - a
*d)^2*Log[c^(1/3) + d^(1/3)*x])/(3*c^(2/3)*d^(7/3)) - ((b*c - a*d)^2*Log[c^
(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2])/(6*c^(2/3)*d^(7/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{(a+bx^3)^2}{c+dx^3}\,dx = \int \left(-\frac{b(bc-2ad)}{d^2} + \frac{b^2x^3}{d} + \frac{b^2c^2-2abcd+a^2d^2}{d^2(c+dx^3)}\right)dx$$

$$= -\frac{b(bc-2ad)x}{d^2} + \frac{b^2x^4}{4d} + \frac{(bc-ad)^2\int \frac{1}{c+dx^3}\,dx}{d^2}$$

$$= -\frac{b(bc-2ad)x}{d^2} + \frac{b^2x^4}{4d} + \frac{(bc-ad)^2\int \frac{1}{\sqrt[3]{c}+\sqrt[3]{d}x}\,dx}{3c^{2/3}d^2} + \frac{(bc-ad)^2\int \frac{2\sqrt[3]{c}-\sqrt[3]{d}x}{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}\,dx}{3c^{2/3}d^2}$$

$$= -\frac{b(bc-2ad)x}{d^2} + \frac{b^2x^4}{4d} + \frac{(bc-ad)^2\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{2/3}d^{7/3}} - \frac{(bc-ad)^2\int \frac{-\sqrt[3]{c}\sqrt[3]{d}+2d^{2/3}x}{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}\,dx}{6c^{2/3}d^{7/3}} +$$

$$= -\frac{b(bc-2ad)x}{d^2} + \frac{b^2x^4}{4d} + \frac{(bc-ad)^2\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{2/3}d^{7/3}} - \frac{(bc-ad)^2\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2}\right.}{6c^{2/3}d^{7/3}}$$

$$= -\frac{b(bc-2ad)x}{d^2} + \frac{b^2x^4}{4d} - \frac{(bc-ad)^2\tan^{-1}\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{\sqrt{3}\sqrt[3]{c}}\right)}{\sqrt{3}c^{2/3}d^{7/3}} + \frac{(bc-ad)^2\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{2/3}d^{7/3}} - \frac{(bc}{}$$

Mathematica [A] time = 0.14, size = 167, normalized size = 0.97

$$\frac{-2(bc-ad)^2\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)-12bc^{2/3}\sqrt[3]{d}x(bc-2ad)+4(bc-ad)^2\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)+4\sqrt{3}(bc-ad)^2\tan^{-1}\left(\frac{2\sqrt[3]{d}x-\sqrt[3]{c}}{\sqrt{3}\sqrt[3]{c}}\right)+3b^2c^{2/3}d^{4/3}x^4}{12c^{2/3}d^{7/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^2/(c + d*x^3),x]
```

```
[Out] (-12*b*c^(2/3)*d^(1/3)*(b*c - 2*a*d)*x + 3*b^2*c^(2/3)*d^(4/3)*x^4 + 4*Sqrt
[3]*(b*c - a*d)^2*ArcTan[(-c^(1/3) + 2*d^(1/3)*x)/(Sqrt[3]*c^(1/3))] + 4*(b
*c - a*d)^2*Log[c^(1/3) + d^(1/3)*x] - 2*(b*c - a*d)^2*Log[c^(2/3) - c^(1/3
)*d^(1/3)*x + d^(2/3)*x^2])/(12*c^(2/3)*d^(7/3))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx^3)^2}{c+dx^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x^3)^2/(c + d*x^3),x]
```

```
[Out] IntegrateAlgebraic[(a + b*x^3)^2/(c + d*x^3), x]
```

fricas [A] time = 0.68, size = 505, normalized size = 2.92

$$\left[\frac{3\,b^{2}c^{2}d^{2}x^{4}+6\,\sqrt{\frac{1}{3}}\left(b^{2}c^{3}d-2\,abc^{2}d^{2}+a^{2}cd^{3}\right)\sqrt{-\frac{(c^{2}d)^{\frac{1}{3}}}{d}}\log\left(\frac{2\,cdx^{3}-3\,(c^{2}d)^{\frac{1}{3}}cx-c^{2}+3\,\sqrt{\frac{1}{3}}\left(2\,cdx^{2}+(c^{2}d)^{\frac{2}{3}}x-(c^{2}d)^{\frac{1}{3}}c\right)\sqrt{-\frac{(c^{2}d)^{\frac{1}{3}}}{d}}}{dx^{3}+c}\right)-2\left(b^{2}c^{2}-2\,abcd+a^{2}d^{2}\right)(c^{2}d)^{\frac{2}{3}}\log\left(cdx^{2}-(c^{2}d)^{\frac{2}{3}}x+(c^{2}d)^{\frac{1}{3}}c\right)+4\left(b^{2}c^{2}-2\,abcd+a^{2}d^{2}\right)(c^{2}d)^{\frac{2}{3}}\log\left(cdx+(c^{2}d)^{\frac{2}{3}}\right)-12\left(b^{2}c^{3}d-2\,abc^{2}d^{2}\right)x}{12\,c^{2}d^{3}},\ \frac{3\,b^{2}c^{2}d^{2}x^{4}+12\,\sqrt{\frac{1}{3}}\left(b^{2}c^{3}d-2\,abc^{2}d^{2}+a^{2}cd^{3}\right)\sqrt{\frac{(c^{2}d)^{\frac{1}{3}}}{d}}\arctan\left(\frac{\sqrt{\frac{1}{3}}\left(2\,(c^{2}d)^{\frac{2}{3}}x-(c^{2}d)^{\frac{1}{3}}c\right)\sqrt{\frac{(c^{2}d)^{\frac{1}{3}}}{d}}}{c^{2}}\right)-2\left(b^{2}c^{2}-2\,abcd+a^{2}d^{2}\right)(c^{2}d)^{\frac{2}{3}}\log\left(cdx^{2}-(c^{2}d)^{\frac{2}{3}}x+(c^{2}d)^{\frac{1}{3}}c\right)+4\left(b^{2}c^{2}-2\,abcd+a^{2}d^{2}\right)(c^{2}d)^{\frac{2}{3}}\log\left(cdx+(c^{2}d)^{\frac{2}{3}}\right)-12\left(b^{2}c^{3}d-2\,abc^{2}d^{2}\right)x}{12\,c^{2}d^{3}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] [1/12*(3*b^2*c^2*d^2*x^4 + 6*sqrt(1/3)*(b^2*c^3*d - 2*a*b*c^2*d^2 + a^2*c*d
^3)*sqrt(-(c^2*d)^(1/3)/d)*log((2*c*d*x^3 - 3*(c^2*d)^(1/3)*c*x - c^2 + 3*s
qrt(1/3)*(2*c*d*x^2 + (c^2*d)^(2/3)*x - (c^2*d)^(1/3)*c)*sqrt(-(c^2*d)^(1/3
)/d))/(d*x^3 + c)) - 2*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*(c^2*d)^(2/3)*log(c*
d*x^2 - (c^2*d)^(2/3)*x + (c^2*d)^(1/3)*c) + 4*(b^2*c^2 - 2*a*b*c*d + a^2*d
^2)*(c^2*d)^(2/3)*log(c*d*x + (c^2*d)^(2/3)) - 12*(b^2*c^3*d - 2*a*b*c^2*d^
2)*x)/(c^2*d^3), 1/12*(3*b^2*c^2*d^2*x^4 + 12*sqrt(1/3)*(b^2*c^3*d - 2*a*b*
c^2*d^2 + a^2*c*d^3)*sqrt((c^2*d)^(1/3)/d)*arctan(sqrt(1/3)*(2*(c^2*d)^(2/3
)*x - (c^2*d)^(1/3)*c)*sqrt((c^2*d)^(1/3)/d)/c^2) - 2*(b^2*c^2 - 2*a*b*c*d
+ a^2*d^2)*(c^2*d)^(2/3)*log(c*d*x^2 - (c^2*d)^(2/3)*x + (c^2*d)^(1/3)*c) +
 4*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*(c^2*d)^(2/3)*log(c*d*x + (c^2*d)^(2/3))
 - 12*(b^2*c^3*d - 2*a*b*c^2*d^2)*x)/(c^2*d^3)]
```

giac [A] time = 0.19, size = 211, normalized size = 1.22

$$-\frac{\sqrt{3}\left(b^{2}c^{2}-2\,abcd+a^{2}d^{2}\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{3\left(-cd^{2}\right)^{\frac{2}{3}}d}-\frac{\left(b^{2}c^{2}-2\,abcd+a^{2}d^{2}\right)\log\left(x^{2}+x\left(-\frac{c}{d}\right)^{\frac{1}{3}}+\left(-\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6\left(-cd^{2}\right)^{\frac{2}{3}}d}-\frac{\left(b^{2}c^{2}d^{2}-2\,abcd^{3}+a^{2}d^{4}\right)\left(-\frac{c}{d}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right|\right)}{3\,cd^{4}}+\frac{b^{2}d^{3}x^{4}-4\,b^{2}cd^{2}x+8\,abd^{3}x}{4\,d^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2/(d*x^3+c),x, algorithm="giac")
```

```
[Out] -1/3*sqrt(3)*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*arctan(1/3*sqrt(3)*(2*x + (-c/
d)^(1/3))/(-c/d)^(1/3))/((-c*d^2)^(2/3)*d) - 1/6*(b^2*c^2 - 2*a*b*c*d + a^2
*d^2)*log(x^2 + x*(-c/d)^(1/3) + (-c/d)^(2/3))/((-c*d^2)^(2/3)*d) - 1/3*(b^
2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*(-c/d)^(1/3)*log(abs(x - (-c/d)^(1/3)))/
(c*d^4) + 1/4*(b^2*d^3*x^4 - 4*b^2*c*d^2*x + 8*a*b*d^3*x)/d^4
```

maple [B] time = 0.05, size = 334, normalized size = 1.93

$$\frac{b^{2}x^{4}}{4d}+\frac{\sqrt{3}\,a^{2}\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{c}{d}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3\left(\frac{c}{d}\right)^{\frac{2}{3}}d}+\frac{a^{2}\ln\left(x+\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(\frac{c}{d}\right)^{\frac{2}{3}}d}-\frac{a^{2}\ln\left(x^{2}-\left(\frac{c}{d}\right)^{\frac{1}{3}}x+\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6\left(\frac{c}{d}\right)^{\frac{2}{3}}d}-\frac{2\sqrt{3}\,abc\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{c}{d}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3\left(\frac{c}{d}\right)^{\frac{2}{3}}d^{2}}-\frac{2abc\ln\left(x+\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(\frac{c}{d}\right)^{\frac{2}{3}}d^{2}}+\frac{abc\ln\left(x^{2}-\left(\frac{c}{d}\right)^{\frac{1}{3}}x+\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}{3\left(\frac{c}{d}\right)^{\frac{2}{3}}d^{2}}+\frac{2abx}{d}+\frac{\sqrt{3}\,b^{2}c^{2}\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{c}{d}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3\left(\frac{c}{d}\right)^{\frac{2}{3}}d^{3}}+\frac{b^{2}c^{2}\ln\left(x+\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(\frac{c}{d}\right)^{\frac{2}{3}}d^{3}}-\frac{b^{2}c^{2}\ln\left(x^{2}-\left(\frac{c}{d}\right)^{\frac{1}{3}}x+\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6\left(\frac{c}{d}\right)^{\frac{2}{3}}d^{3}}-\frac{b^{2}cx}{d^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^2/(d*x^3+c),x)
```

```
[Out] 1/4*b^2*x^4/d+2*b/d*a*x-b^2/d^2*c*x+1/3/d/(c/d)^(2/3)*ln(x+(c/d)^(1/3))*a^2
-2/3/d^2/(c/d)^(2/3)*ln(x+(c/d)^(1/3))*a*b*c+1/3/d^3/(c/d)^(2/3)*ln(x+(c/d)
^(1/3))*b^2*c^2-1/6/d/(c/d)^(2/3)*ln(x^2-(c/d)^(1/3)*x+(c/d)^(2/3))*a^2+1/3
/d^2/(c/d)^(2/3)*ln(x^2-(c/d)^(1/3)*x+(c/d)^(2/3))*a*b*c-1/6/d^3/(c/d)^(2/3
)*ln(x^2-(c/d)^(1/3)*x+(c/d)^(2/3))*b^2*c^2+1/3/d/(c/d)^(2/3)*3^(1/2)*arcta
n(1/3*3^(1/2)*(2/(c/d)^(1/3)*x-1))*a^2-2/3/d^2/(c/d)^(2/3)*3^(1/2)*arctan(1
/3*3^(1/2)*(2/(c/d)^(1/3)*x-1))*a*b*c+1/3/d^3/(c/d)^(2/3)*3^(1/2)*arctan(1/
3*3^(1/2)*(2/(c/d)^(1/3)*x-1))*b^2*c^2
```

maxima [A] time = 1.23, size = 189, normalized size = 1.09

$$\frac{b^{2}dx^{4}-4\left(b^{2}c-2\,abd\right)x}{4\,d^{2}}+\frac{\sqrt{3}\left(b^{2}c^{2}-2\,abcd+a^{2}d^{2}\right)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{3\,d^{3}\left(\frac{c}{d}\right)^{\frac{2}{3}}}-\frac{\left(b^{2}c^{2}-2\,abcd+a^{2}d^{2}\right)\log\left(x^{2}-x\left(\frac{c}{d}\right)^{\frac{1}{3}}+\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6\,d^{3}\left(\frac{c}{d}\right)^{\frac{2}{3}}}+\frac{\left(b^{2}c^{2}-2\,abcd+a^{2}d^{2}\right)\log\left(x+\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\,d^{3}\left(\frac{c}{d}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] 1/4*(b^2*d*x^4 - 4*(b^2*c - 2*a*b*d)*x)/d^2 + 1/3*sqrt(3)*(b^2*c^2 - 2*a*b*
c*d + a^2*d^2)*arctan(1/3*sqrt(3)*(2*x - (c/d)^(1/3))/(c/d)^(1/3))/(d^3*(c/
d)^(2/3)) - 1/6*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*log(x^2 - x*(c/d)^(1/3) + (
c/d)^(2/3))/(d^3*(c/d)^(2/3)) + 1/3*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*log(x +
 (c/d)^(1/3))/(d^3*(c/d)^(2/3))
```

mupad [B] time = 1.39, size = 152, normalized size = 0.88

$$\frac{b^2\,x^4}{4\,d}-x\,\left(\frac{b^2\,c}{d^2}-\frac{2\,a\,b}{d}\right)+\frac{\ln\left(d^{1/3}\,x+c^{1/3}\right)\,{\left(a\,d-b\,c\right)}^2}{3\,c^{2/3}\,d^{7/3}}+\frac{\ln\left(2\,d^{1/3}\,x-c^{1/3}+\sqrt{3}\,c^{1/3}\,1{}\mathrm{i}\right)\,\left(-\frac{1}{6}+\frac{\sqrt{3}\,1{}\mathrm{i}}{6}\right)\,{\left(a\,d-b\,c\right)}^2}{c^{2/3}\,d^{7/3}}-\frac{\ln\left(c^{1/3}-2\,d^{1/3}\,x+\sqrt{3}\,c^{1/3}\,1{}\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,{\left(a\,d-b\,c\right)}^2}{3\,c^{2/3}\,d^{7/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^2/(c + d*x^3),x)
```

```
[Out] (b^2*x^4)/(4*d) - x*((b^2*c)/d^2 - (2*a*b)/d) + (log(d^(1/3)*x + c^(1/3))*(
a*d - b*c)^2)/(3*c^(2/3)*d^(7/3)) + (log(3^(1/2)*c^(1/3)*1i + 2*d^(1/3)*x -
 c^(1/3))*((3^(1/2)*1i)/6 - 1/6)*(a*d - b*c)^2)/(c^(2/3)*d^(7/3)) - (log(3^
(1/2)*c^(1/3)*1i - 2*d^(1/3)*x + c^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(a*d - b*c
)^2)/(3*c^(2/3)*d^(7/3))
```

sympy [A] time = 0.68, size = 156, normalized size = 0.90

$$\frac{b^{2} x^{4}}{4 d} + x \left(\frac{2 a b}{d} - \frac{b^{2} c}{d^{2}}\right) + \operatorname{RootSum} {\left(27 t^{3} c^{2} d^{7} - a^{6} d^{6} + 6 a^{5} b c d^{5} - 15 a^{4} b^{2} c^{2} d^{4} + 20 a^{3} b^{3} c^{3} d^{3} - 15 a^{2} b^{4} c^{4} d^{2} + 6 a b^{5} c^{5} d - b^{6} c^{6}, \left(t \mapsto t \log {\left(\frac{3 t c d^{2}}{a^{2} d^{2} - 2 a b c d + b^{2} c^{2}} + x \right)} \right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**2/(d*x**3+c),x)
```

```
[Out] b**2*x**4/(4*d) + x*(2*a*b/d - b**2*c/d**2) + RootSum(27*_t**3*c**2*d**7 -
a**6*d**6 + 6*a**5*b*c*d**5 - 15*a**4*b**2*c**2*d**4 + 20*a**3*b**3*c**3*d*
*3 - 15*a**2*b**4*c**4*d**2 + 6*a*b**5*c**5*d - b**6*c**6, Lambda(_t, _t*lo
g(3*_t*c*d**2/(a**2*d**2 - 2*a*b*c*d + b**2*c**2) + x)))
```

3.12 $\int \frac{(a+bx^3)^2}{(c+dx^3)^2}\,dx$

Optimal. Leaf size=203

$$\frac{(bc-ad)(ad+2bc)\log\left(c^{2/3}-\sqrt[3]{c}\,\sqrt[3]{d}\,x+d^{2/3}x^2\right)}{9c^{5/3}d^{7/3}}-\frac{2(bc-ad)(ad+2bc)\log\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{9c^{5/3}d^{7/3}}+\frac{2(bc-ad)(ad+2bc)\,\mathrm{ta}}{3\sqrt{3}\,c^{5/3}d}$$

Rubi [A] time = 0.24, antiderivative size = 203, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.421, Rules used = {390, 385, 200, 31, 634, 617, 204, 628}

$$\frac{(bc-ad)(ad+2bc)\log\left(c^{2/3}-\sqrt[3]{c}\,\sqrt[3]{d}\,x+d^{2/3}x^2\right)}{9c^{5/3}d^{7/3}}-\frac{2(bc-ad)(ad+2bc)\log\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{9c^{5/3}d^{7/3}}+\frac{2(bc-ad)(ad+2bc)\tan^{-1}\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}\,x}{\sqrt{3}\,\sqrt[3]{c}}\right)}{3\sqrt{3}\,c^{5/3}d^{7/3}}+\frac{x(bc-ad)^2}{3cd^2\left(c+dx^3\right)}+\frac{b^2x}{d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^2/(c + d*x^3)^2,x]
```

```
[Out] (b^2*x)/d^2 + ((b*c - a*d)^2*x)/(3*c*d^2*(c + d*x^3)) + (2*(b*c - a*d)*(2*b
*c + a*d)*ArcTan[(c^(1/3) - 2*d^(1/3)*x)/(Sqrt[3]*c^(1/3))])/(3*Sqrt[3]*c^(
5/3)*d^(7/3)) - (2*(b*c - a*d)*(2*b*c + a*d)*Log[c^(1/3) + d^(1/3)*x])/(9*c
^(5/3)*d^(7/3)) + ((b*c - a*d)*(2*b*c + a*d)*Log[c^(2/3) - c^(1/3)*d^(1/3)*
x + d^(2/3)*x^2])/(9*c^(5/3)*d^(7/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a+bx^3\right)^2}{\left(c+dx^3\right)^2}\,dx &= \int \left(\frac{b^2}{d^2}-\frac{b^2c^2-a^2d^2+2bd(bc-ad)x^3}{d^2\left(c+dx^3\right)^2}\right)dx\\
&= \frac{b^2x}{d^2}-\frac{\int \frac{b^2c^2-a^2d^2+2bd(bc-ad)x^3}{\left(c+dx^3\right)^2}\,dx}{d^2}\\
&= \frac{b^2x}{d^2}+\frac{(bc-ad)^2x}{3cd^2\left(c+dx^3\right)}-\frac{(2(bc-ad)(2bc+ad))\int \frac{1}{c+dx^3}\,dx}{3cd^2}\\
&= \frac{b^2x}{d^2}+\frac{(bc-ad)^2x}{3cd^2\left(c+dx^3\right)}-\frac{(2(bc-ad)(2bc+ad))\int \frac{1}{\sqrt[3]{c}+\sqrt[3]{d}\,x}\,dx}{9c^{5/3}d^2}-\frac{(2(bc-ad)(2bc+ad))\int \cdots}{9c^{5/3}d^2}\\
&= \frac{b^2x}{d^2}+\frac{(bc-ad)^2x}{3cd^2\left(c+dx^3\right)}-\frac{2(bc-ad)(2bc+ad)\log\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{9c^{5/3}d^{7/3}}+\frac{((bc-ad)(2bc+ad))\int \cdots}{9c^{5/3}d^{7/3}}\\
&= \frac{b^2x}{d^2}+\frac{(bc-ad)^2x}{3cd^2\left(c+dx^3\right)}-\frac{2(bc-ad)(2bc+ad)\log\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{9c^{5/3}d^{7/3}}+\frac{(bc-ad)(2bc+ad)\log(\cdots}{9c^{5/3}\cdots}\\
&= \frac{b^2x}{d^2}+\frac{(bc-ad)^2x}{3cd^2\left(c+dx^3\right)}+\frac{2(bc-ad)(2bc+ad)\tan^{-1}\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}\,x}{\sqrt{3}\sqrt[3]{c}}\right)}{3\sqrt{3}\,c^{5/3}d^{7/3}}-\frac{2(bc-ad)(2bc+ad)\log\cdots}{9c^{5/3}d^{7/3}}
\end{aligned}$$

Mathematica [A] time = 0.22, size = 210, normalized size = 1.03

$$\frac{-\frac{2\left(-a^2d^2-abcd+2b^2c^2\right)\log\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{c^{5/3}}+\frac{2\sqrt{3}\left(-a^2d^2-abcd+2b^2c^2\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\,x}{\sqrt[3]{c}}}{\sqrt{3}}\right)}{c^{5/3}}+\frac{\left(-a^2d^2-abcd+2b^2c^2\right)\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}\,x+d^{2/3}x^2\right)}{c^{5/3}}+\frac{3\sqrt[3]{d}\,x(bc-ad)^2}{c\left(c+dx^3\right)}+9b^2\sqrt[3]{d}\,x}{9d^{7/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^2/(c + d*x^3)^2,x]
```

```
[Out] (9*b^2*d^(1/3)*x + (3*d^(1/3)*(b*c - a*d)^2*x)/(c*(c + d*x^3)) + (2*Sqrt[3]
*(2*b^2*c^2 - a*b*c*d - a^2*d^2)*ArcTan[(1 - (2*d^(1/3)*x)/c^(1/3))/Sqrt[3]
])/c^(5/3) - (2*(2*b^2*c^2 - a*b*c*d - a^2*d^2)*Log[c^(1/3) + d^(1/3)*x])/c
^(5/3) + ((2*b^2*c^2 - a*b*c*d - a^2*d^2)*Log[c^(2/3) - c^(1/3)*d^(1/3)*x +
 d^(2/3)*x^2])/c^(5/3))/(9*d^(7/3))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+bx^3\right)^2}{\left(c+dx^3\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x^3)^2/(c + d*x^3)^2,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x^3)^2/(c + d*x^3)^2, x]
```

fricas [B] time = 1.01, size = 771, normalized size = 3.80

$$\left[\frac{9b^2c^3d^2x^4 - 3\sqrt{\frac{1}{3}}\left(2b^2c^4d - abc^3d^2 - a^2c^2d^3 + \left(2b^2c^3d^2 - abc^2d^3 - a^2cd^4\right)x^3\right)\sqrt{-\frac{\left(c^2d\right)^{\frac{1}{3}}}{d}}\log\left(\frac{2cdx^3 - 3\left(c^2d\right)^{\frac{1}{3}}cx - c^2 + 3\sqrt{\frac{1}{3}}\left(2cdx^2 + \left(c^2d\right)^{\frac{2}{3}}x - \left(c^2d\right)^{\frac{1}{3}}c\right)\sqrt{-\frac{\left(c^2d\right)^{\frac{1}{3}}}{d}}}{dx^3+c}\right) + \left(2b^2c^3 - abc^2d - a^2cd^2 + \left(2b^2c^2d - abcd^2 - a^2d^3\right)x^3\right)\left(c^2d\right)^{\frac{2}{3}}\log\left(cdx^2 - \left(c^2d\right)^{\frac{2}{3}}x + \left(c^2d\right)^{\frac{1}{3}}c\right) - 2\left(2b^2c^3 - abc^2d - a^2cd^2 + \left(2b^2c^2d - abcd^2 - a^2d^3\right)x^3\right)\left(c^2d\right)^{\frac{2}{3}}\log\left(cdx + \left(c^2d\right)^{\frac{2}{3}}\right) + 3\left(4b^2c^4d - 2abc^3d^2 + a^2c^2d^3\right)x}{9\left(c^3d^4x^3 + c^4d^3\right)}, \frac{9b^2c^3d^2x^4 - 6\sqrt{\frac{1}{3}}\left(2b^2c^4d - abc^3d^2 - a^2c^2d^3 + \left(2b^2c^3d^2 - abc^2d^3 - a^2cd^4\right)x^3\right)\sqrt{\frac{\left(c^2d\right)^{\frac{1}{3}}}{d}}\arctan\left(\frac{\sqrt{\frac{1}{3}}\left(2\left(c^2d\right)^{\frac{2}{3}}x - \left(c^2d\right)^{\frac{1}{3}}c\right)\sqrt{\frac{\left(c^2d\right)^{\frac{1}{3}}}{d}}}{c^2}\right) + \left(2b^2c^3 - abc^2d - a^2cd^2 + \left(2b^2c^2d - abcd^2 - a^2d^3\right)x^3\right)\left(c^2d\right)^{\frac{2}{3}}\log\left(cdx^2 - \left(c^2d\right)^{\frac{2}{3}}x + \left(c^2d\right)^{\frac{1}{3}}c\right) - 2\left(2b^2c^3 - abc^2d - a^2cd^2 + \left(2b^2c^2d - abcd^2 - a^2d^3\right)x^3\right)\left(c^2d\right)^{\frac{2}{3}}\log\left(cdx + \left(c^2d\right)^{\frac{2}{3}}\right) + 3\left(4b^2c^4d - 2abc^3d^2 + a^2c^2d^3\right)x}{9\left(c^3d^4x^3 + c^4d^3\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2/(d*x^3+c)^2,x, algorithm="fricas")
```

```
[Out] [1/9*(9*b^2*c^3*d^2*x^4 - 3*sqrt(1/3)*(2*b^2*c^4*d - a*b*c^3*d^2 - a^2*c^2*
d^3 + (2*b^2*c^3*d^2 - a*b*c^2*d^3 - a^2*c*d^4)*x^3)*sqrt(-(c^2*d)^(1/3)/d)
*log((2*c*d*x^3 - 3*(c^2*d)^(1/3)*c*x - c^2 + 3*sqrt(1/3)*(2*c*d*x^2 + (c^2
*d)^(2/3)*x - (c^2*d)^(1/3)*c)*sqrt(-(c^2*d)^(1/3)/d))/(d*x^3 + c)) + (2*b^
2*c^3 - a*b*c^2*d - a^2*c*d^2 + (2*b^2*c^2*d - a*b*c*d^2 - a^2*d^3)*x^3)*(c
^2*d)^(2/3)*log(c*d*x^2 - (c^2*d)^(2/3)*x + (c^2*d)^(1/3)*c) - 2*(2*b^2*c^3
 - a*b*c^2*d - a^2*c*d^2 + (2*b^2*c^2*d - a*b*c*d^2 - a^2*d^3)*x^3)*(c^2*d)
^(2/3)*log(c*d*x + (c^2*d)^(2/3)) + 3*(4*b^2*c^4*d - 2*a*b*c^3*d^2 + a^2*c^
2*d^3)*x)/(c^3*d^4*x^3 + c^4*d^3), 1/9*(9*b^2*c^3*d^2*x^4 - 6*sqrt(1/3)*(2*
b^2*c^4*d - a*b*c^3*d^2 - a^2*c^2*d^3 + (2*b^2*c^3*d^2 - a*b*c^2*d^3 - a^2*
c*d^4)*x^3)*sqrt((c^2*d)^(1/3)/d)*arctan(sqrt(1/3)*(2*(c^2*d)^(2/3)*x - (c^
2*d)^(1/3)*c)*sqrt((c^2*d)^(1/3)/d)/c^2) + (2*b^2*c^3 - a*b*c^2*d - a^2*c*d
^2 + (2*b^2*c^2*d - a*b*c*d^2 - a^2*d^3)*x^3)*(c^2*d)^(2/3)*log(c*d*x^2 - (
c^2*d)^(2/3)*x + (c^2*d)^(1/3)*c) - 2*(2*b^2*c^3 - a*b*c^2*d - a^2*c*d^2 +
(2*b^2*c^2*d - a*b*c*d^2 - a^2*d^3)*x^3)*(c^2*d)^(2/3)*log(c*d*x + (c^2*d)^
(2/3)) + 3*(4*b^2*c^4*d - 2*a*b*c^3*d^2 + a^2*c^2*d^3)*x)/(c^3*d^4*x^3 + c^
4*d^3)]
```

giac [A] time = 0.19, size = 233, normalized size = 1.15

$$\frac{b^2x}{d^2} + \frac{2\sqrt{3}\left(2b^2c^2 - abcd - a^2d^2\right)\arctan\left(\frac{\sqrt{3}\left(2x + \left(-\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{9\left(-cd^2\right)^{\frac{2}{3}}cd} + \frac{\left(2b^2c^2 - abcd - a^2d^2\right)\log\left(x^2 + x\left(-\frac{c}{d}\right)^{\frac{1}{3}} + \left(-\frac{c}{d}\right)^{\frac{2}{3}}\right)}{9\left(-cd^2\right)^{\frac{2}{3}}cd} + \frac{2\left(2b^2c^2 - abcd - a^2d^2\right)\left(-\frac{c}{d}\right)^{\frac{1}{3}}\log\left(\left|x - \left(-\frac{c}{d}\right)^{\frac{1}{3}}\right|\right)}{9c^2d^2} + \frac{b^2c^2x - 2abcdx + a^2d^2x}{3\left(dx^3 + c\right)cd^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2/(d*x^3+c)^2,x, algorithm="giac")
```

```
[Out] b^2*x/d^2 + 2/9*sqrt(3)*(2*b^2*c^2 - a*b*c*d - a^2*d^2)*arctan(1/3*sqrt(3)*
(2*x + (-c/d)^(1/3))/(-c/d)^(1/3))/((-c*d^2)^(2/3)*c*d) + 1/9*(2*b^2*c^2 -
a*b*c*d - a^2*d^2)*log(x^2 + x*(-c/d)^(1/3) + (-c/d)^(2/3))/((-c*d^2)^(2/3)
*c*d) + 2/9*(2*b^2*c^2 - a*b*c*d - a^2*d^2)*(-c/d)^(1/3)*log(abs(x - (-c/d)
^(1/3)))/(c^2*d^2) + 1/3*(b^2*c^2*x - 2*a*b*c*d*x + a^2*d^2*x)/((d*x^3 + c)
*c*d^2)
```

maple [B] time = 0.06, size = 367, normalized size = 1.81

$$\frac{a^{2}x}{3\left(d\,x^{3}+c\right)c}-\frac{2abx}{3\left(d\,x^{3}+c\right)d}+\frac{b^{2}cx}{3\left(d\,x^{3}+c\right)d^{2}}+\frac{2\sqrt{3}\,a^{2}\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{c}{d}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{9\left(\frac{c}{d}\right)^{\frac{2}{3}}cd}+\frac{2a^{2}\ln\left(x+\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{9\left(\frac{c}{d}\right)^{\frac{2}{3}}cd}-\frac{a^{2}\ln\left(x^{2}-\left(\frac{c}{d}\right)^{\frac{1}{3}}x+\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}{9\left(\frac{c}{d}\right)^{\frac{2}{3}}cd}+\frac{2\sqrt{3}\,ab\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{c}{d}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{9\left(\frac{c}{d}\right)^{\frac{2}{3}}d^{2}}+\frac{2ab\ln\left(x+\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{9\left(\frac{c}{d}\right)^{\frac{2}{3}}d^{2}}-\frac{ab\ln\left(x^{2}-\left(\frac{c}{d}\right)^{\frac{1}{3}}x+\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}{9\left(\frac{c}{d}\right)^{\frac{2}{3}}d^{2}}-\frac{4\sqrt{3}\,b^{2}c\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{c}{d}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{9\left(\frac{c}{d}\right)^{\frac{2}{3}}d^{3}}-\frac{4b^{2}c\ln\left(x+\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{9\left(\frac{c}{d}\right)^{\frac{2}{3}}d^{3}}+\frac{2b^{2}c\ln\left(x^{2}-\left(\frac{c}{d}\right)^{\frac{1}{3}}x+\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}{9\left(\frac{c}{d}\right)^{\frac{2}{3}}d^{3}}+\frac{b^{2}x}{d^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^2/(d*x^3+c)^2,x)
```

```
[Out] b^2*x/d^2+1/3/c*x/(d*x^3+c)*a^2-2/3/d*x/(d*x^3+c)*a*b+1/3/d^2*c*x/(d*x^3+c)
*b^2+2/9/d/c/(c/d)^(2/3)*ln(x+(c/d)^(1/3))*a^2+2/9/d^2/(c/d)^(2/3)*ln(x+(c/
d)^(1/3))*a*b-4/9/d^3*c/(c/d)^(2/3)*ln(x+(c/d)^(1/3))*b^2-1/9/d/c/(c/d)^(2/
3)*ln(x^2-(c/d)^(1/3)*x+(c/d)^(2/3))*a^2-1/9/d^2/(c/d)^(2/3)*ln(x^2-(c/d)^(
1/3)*x+(c/d)^(2/3))*a*b+2/9/d^3*c/(c/d)^(2/3)*ln(x^2-(c/d)^(1/3)*x+(c/d)^(2
/3))*b^2+2/9/d/c/(c/d)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(c/d)^(1/3)*x-1)
)*a^2+2/9/d^2/(c/d)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(c/d)^(1/3)*x-1))*a
*b-4/9/d^3*c/(c/d)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(c/d)^(1/3)*x-1))*b^
2
```

maxima [A] time = 1.28, size = 226, normalized size = 1.11

$$\frac{\left(b^{2}c^{2}-2\,abcd+a^{2}d^{2}\right)x}{3\left(cd^{3}x^{3}+c^{2}d^{2}\right)}+\frac{b^{2}x}{d^{2}}-\frac{2\sqrt{3}\left(2\,b^{2}c^{2}-abcd-a^{2}d^{2}\right)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{9\,cd^{3}\left(\frac{c}{d}\right)^{\frac{2}{3}}}+\frac{\left(2\,b^{2}c^{2}-abcd-a^{2}d^{2}\right)\log\left(x^{2}-x\left(\frac{c}{d}\right)^{\frac{1}{3}}+\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}{9\,cd^{3}\left(\frac{c}{d}\right)^{\frac{2}{3}}}-\frac{2\left(2\,b^{2}c^{2}-abcd-a^{2}d^{2}\right)\log\left(x+\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{9\,cd^{3}\left(\frac{c}{d}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2/(d*x^3+c)^2,x, algorithm="maxima")
```

```
[Out] 1/3*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*x/(c*d^3*x^3 + c^2*d^2) + b^2*x/d^2 - 2
/9*sqrt(3)*(2*b^2*c^2 - a*b*c*d - a^2*d^2)*arctan(1/3*sqrt(3)*(2*x - (c/d)^
(1/3))/(c/d)^(1/3))/(c*d^3*(c/d)^(2/3)) + 1/9*(2*b^2*c^2 - a*b*c*d - a^2*d^
2)*log(x^2 - x*(c/d)^(1/3) + (c/d)^(2/3))/(c*d^3*(c/d)^(2/3)) - 2/9*(2*b^2*
c^2 - a*b*c*d - a^2*d^2)*log(x + (c/d)^(1/3))/(c*d^3*(c/d)^(2/3))
```

mupad [B] time = 1.41, size = 191, normalized size = 0.94

$$\frac{b^2\,x}{d^2}+\frac{x\,\left(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2\right)}{3\,c\,\left(d^3\,x^3+c\,d^2\right)}+\frac{2\,\ln\left(d^{1/3}\,x+c^{1/3}\right)\,\left(a\,d-b\,c\right)\,\left(a\,d+2\,b\,c\right)}{9\,c^{5/3}\,d^{7/3}}+\frac{2\,\ln\left(2\,d^{1/3}\,x-c^{1/3}+\sqrt{3}\,c^{1/3}\,1{}\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,\left(a\,d-b\,c\right)\,\left(a\,d+2\,b\,c\right)}{9\,c^{5/3}\,d^{7/3}}-\frac{2\,\ln\left(c^{1/3}-2\,d^{1/3}\,x+\sqrt{3}\,c^{1/3}\,1{}\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,\left(a\,d-b\,c\right)\,\left(a\,d+2\,b\,c\right)}{9\,c^{5/3}\,d^{7/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^2/(c + d*x^3)^2,x)
```

```
[Out] (b^2*x)/d^2 + (x*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d))/(3*c*(c*d^2 + d^3*x^3)) +
 (2*log(d^(1/3)*x + c^(1/3))*(a*d - b*c)*(a*d + 2*b*c))/(9*c^(5/3)*d^(7/3))
 + (2*log(3^(1/2)*c^(1/3)*1i + 2*d^(1/3)*x - c^(1/3))*((3^(1/2)*1i)/2 - 1/2
)*(a*d - b*c)*(a*d + 2*b*c))/(9*c^(5/3)*d^(7/3)) - (2*log(3^(1/2)*c^(1/3)*1
i - 2*d^(1/3)*x + c^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(a*d - b*c)*(a*d + 2*b*c)
)/(9*c^(5/3)*d^(7/3))
```

sympy [A] time = 1.14, size = 189, normalized size = 0.93

$$\frac{b^{2}x}{d^{2}}+\frac{x\left(a^{2}d^{2}-2abcd+b^{2}c^{2}\right)}{3c^{2}d^{2}+3cd^{3}x^{3}}+\operatorname{RootSum}\left(729t^{3}c^{5}d^{7}-8a^{6}d^{6}-24a^{5}bcd^{5}+24a^{4}b^{2}c^{2}d^{4}+88a^{3}b^{3}c^{3}d^{3}-48a^{2}b^{4}c^{4}d^{2}-96ab^{5}c^{5}d+64b^{6}c^{6},\left(t\mapsto t\log\left(\frac{9tc^{2}d^{2}}{2a^{2}d^{2}+2abcd-4b^{2}c^{2}}+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**2/(d*x**3+c)**2,x)
```

```
[Out] b**2*x/d**2 + x*(a**2*d**2 - 2*a*b*c*d + b**2*c**2)/(3*c**2*d**2 + 3*c*d**3
*x**3) + RootSum(729*_t**3*c**5*d**7 - 8*a**6*d**6 - 24*a**5*b*c*d**5 + 24*
a**4*b**2*c**2*d**4 + 88*a**3*b**3*c**3*d**3 - 48*a**2*b**4*c**4*d**2 - 96*
a*b**5*c**5*d + 64*b**6*c**6, Lambda(_t, _t*log(9*_t*c**2*d**2/(2*a**2*d**2
 + 2*a*b*c*d - 4*b**2*c**2) + x)))
```

3.13 $\int \frac{(a+bx^3)^2}{(c+dx^3)^3}\,dx$

Optimal. Leaf size=258

$$-\frac{\left(5a^2d^2+2abcd+2b^2c^2\right)\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{54c^{8/3}d^{7/3}}+\frac{\left(5a^2d^2+2abcd+2b^2c^2\right)\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{27c^{8/3}d^{7/3}}-\frac{\left(5a^2d^2+2a\right.}{}$$

Rubi [A] time = 0.23, antiderivative size = 258, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$, Rules used = {413, 385, 200, 31, 634, 617, 204, 628}

$$-\frac{\left(5a^2d^2+2abcd+2b^2c^2\right)\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{54c^{8/3}d^{7/3}}+\frac{\left(5a^2d^2+2abcd+2b^2c^2\right)\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{27c^{8/3}d^{7/3}}-\frac{\left(5a^2d^2+2abcd+2b^2c^2\right)\tan^{-1}\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{\sqrt{3}\sqrt[3]{c}}\right)}{9\sqrt{3}c^{8/3}d^{7/3}}-\frac{x(bc-ad)(5ad+4bc)}{18c^2d^2\left(c+dx^3\right)}-\frac{x\left(a+bx^3\right)(bc-ad)}{6cd\left(c+dx^3\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^2/(c + d*x^3)^3,x]
```

```
[Out] -((b*c - a*d)*x*(a + b*x^3))/(6*c*d*(c + d*x^3)^2) - ((b*c - a*d)*(4*b*c +
5*a*d)*x)/(18*c^2*d^2*(c + d*x^3)) - ((2*b^2*c^2 + 2*a*b*c*d + 5*a^2*d^2)*A
rcTan[(c^(1/3) - 2*d^(1/3)*x)/(Sqrt[3]*c^(1/3))])/(9*Sqrt[3]*c^(8/3)*d^(7/3
)) + ((2*b^2*c^2 + 2*a*b*c*d + 5*a^2*d^2)*Log[c^(1/3) + d^(1/3)*x])/(27*c^(
8/3)*d^(7/3)) - ((2*b^2*c^2 + 2*a*b*c*d + 5*a^2*d^2)*Log[c^(2/3) - c^(1/3)*
d^(1/3)*x + d^(2/3)*x^2])/(54*c^(8/3)*d^(7/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 413

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[((a*d - c*b)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*n*(p +
 1)), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
```

```
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{\left(a+bx^3\right)^2}{\left(c+dx^3\right)^3}\,dx = -\frac{(bc-ad)x\left(a+bx^3\right)}{6cd\left(c+dx^3\right)^2}+\frac{\int \frac{a(bc+5ad)+2b(2bc+ad)x^3}{\left(c+dx^3\right)^2}\,dx}{6cd}$$

$$= -\frac{(bc-ad)x\left(a+bx^3\right)}{6cd\left(c+dx^3\right)^2}-\frac{(bc-ad)(4bc+5ad)x}{18c^2d^2\left(c+dx^3\right)}+\frac{\left(2b^2c^2+2abcd+5a^2d^2\right)\int \frac{1}{c+dx^3}\,dx}{9c^2d^2}$$

$$= -\frac{(bc-ad)x\left(a+bx^3\right)}{6cd\left(c+dx^3\right)^2}-\frac{(bc-ad)(4bc+5ad)x}{18c^2d^2\left(c+dx^3\right)}+\frac{\left(2b^2c^2+2abcd+5a^2d^2\right)\int \frac{1}{\sqrt[3]{c}+\sqrt[3]{d}\,x}\,dx}{27c^{8/3}d^2}+$$

$$= -\frac{(bc-ad)x\left(a+bx^3\right)}{6cd\left(c+dx^3\right)^2}-\frac{(bc-ad)(4bc+5ad)x}{18c^2d^2\left(c+dx^3\right)}+\frac{\left(2b^2c^2+2abcd+5a^2d^2\right)\log\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right.}{27c^{8/3}d^{7/3}}$$

$$= -\frac{(bc-ad)x\left(a+bx^3\right)}{6cd\left(c+dx^3\right)^2}-\frac{(bc-ad)(4bc+5ad)x}{18c^2d^2\left(c+dx^3\right)}+\frac{\left(2b^2c^2+2abcd+5a^2d^2\right)\log\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right.}{27c^{8/3}d^{7/3}}$$

$$= -\frac{(bc-ad)x\left(a+bx^3\right)}{6cd\left(c+dx^3\right)^2}-\frac{(bc-ad)(4bc+5ad)x}{18c^2d^2\left(c+dx^3\right)}-\frac{\left(2b^2c^2+2abcd+5a^2d^2\right)\tan^{-1}\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}\,x}{\sqrt{3}\sqrt[3]{c}}\right.}{9\sqrt{3}\,c^{8/3}d^{7/3}}$$

Mathematica [A] time = 0.28, size = 234, normalized size = 0.91

$$\frac{2\left(5a^2d^2+2abcd+2b^2c^2\right)\log\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)-2\sqrt{3}\left(5a^2d^2+2abcd+2b^2c^2\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\,x}{\sqrt[3]{c}}}{\sqrt{3}}\right)-\frac{3c^{2/3}\sqrt[3]{d}\,x\left(-a^2d^2\left(8c+5dx^3\right)+2abcd\left(2c-dx^3\right)+b^2c^2\left(4c+7dx^3\right)\right)}{\left(c+dx^3\right)^2}-\left(5a^2d^2+2abcd+2b^2c^2\right)\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}\,x+d^{2/3}x^2\right)}{54c^{8/3}d^{7/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^2/(c + d*x^3)^3,x]
```

```
[Out] ((-3*c^(2/3)*d^(1/3)*x*(2*a*b*c*d*(2*c - d*x^3) - a^2*d^2*(8*c + 5*d*x^3) +
 b^2*c^2*(4*c + 7*d*x^3)))/(c + d*x^3)^2 - 2*Sqrt[3]*(2*b^2*c^2 + 2*a*b*c*d
 + 5*a^2*d^2)*ArcTan[(1 - (2*d^(1/3)*x)/c^(1/3))/Sqrt[3]] + 2*(2*b^2*c^2 +
2*a*b*c*d + 5*a^2*d^2)*Log[c^(1/3) + d^(1/3)*x] - (2*b^2*c^2 + 2*a*b*c*d +
5*a^2*d^2)*Log[c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2])/(54*c^(8/3)*d^(7
/3))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+bx^3\right)^2}{\left(c+dx^3\right)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x^3)^2/(c + d*x^3)^3,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x^3)^2/(c + d*x^3)^3, x]
```

fricas [B] time = 0.85, size = 1067, normalized size = 4.14

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2/(d*x^3+c)^3,x, algorithm="fricas")
```

```
[Out] [-1/54*(3*(7*b^2*c^4*d^2 - 2*a*b*c^3*d^3 - 5*a^2*c^2*d^4)*x^4 - 3*sqrt(1/3)
*(2*b^2*c^5*d + 2*a*b*c^4*d^2 + 5*a^2*c^3*d^3 + (2*b^2*c^3*d^3 + 2*a*b*c^2*
d^4 + 5*a^2*c*d^5)*x^6 + 2*(2*b^2*c^4*d^2 + 2*a*b*c^3*d^3 + 5*a^2*c^2*d^4)*
x^3)*sqrt(-(c^2*d)^(1/3)/d)*log((2*c*d*x^3 - 3*(c^2*d)^(1/3)*c*x - c^2 + 3*
sqrt(1/3)*(2*c*d*x^2 + (c^2*d)^(2/3)*x - (c^2*d)^(1/3)*c)*sqrt(-(c^2*d)^(1/
3)/d))/(d*x^3 + c)) + ((2*b^2*c^2*d^2 + 2*a*b*c*d^3 + 5*a^2*d^4)*x^6 + 2*b^
2*c^4 + 2*a*b*c^3*d + 5*a^2*c^2*d^2 + 2*(2*b^2*c^3*d + 2*a*b*c^2*d^2 + 5*a^
2*c*d^3)*x^3)*(c^2*d)^(2/3)*log(c*d*x^2 - (c^2*d)^(2/3)*x + (c^2*d)^(1/3)*c
) - 2*((2*b^2*c^2*d^2 + 2*a*b*c*d^3 + 5*a^2*d^4)*x^6 + 2*b^2*c^4 + 2*a*b*c^
3*d + 5*a^2*c^2*d^2 + 2*(2*b^2*c^3*d + 2*a*b*c^2*d^2 + 5*a^2*c*d^3)*x^3)*(c
^2*d)^(2/3)*log(c*d*x + (c^2*d)^(2/3)) + 12*(b^2*c^5*d + a*b*c^4*d^2 - 2*a^
2*c^3*d^3)*x)/(c^4*d^5*x^6 + 2*c^5*d^4*x^3 + c^6*d^3), -1/54*(3*(7*b^2*c^4*
d^2 - 2*a*b*c^3*d^3 - 5*a^2*c^2*d^4)*x^4 - 6*sqrt(1/3)*(2*b^2*c^5*d + 2*a*b
*c^4*d^2 + 5*a^2*c^3*d^3 + (2*b^2*c^3*d^3 + 2*a*b*c^2*d^4 + 5*a^2*c*d^5)*x^
6 + 2*(2*b^2*c^4*d^2 + 2*a*b*c^3*d^3 + 5*a^2*c^2*d^4)*x^3)*sqrt((c^2*d)^(1/
3)/d)*arctan(sqrt(1/3)*(2*(c^2*d)^(2/3)*x - (c^2*d)^(1/3)*c)*sqrt((c^2*d)^(
1/3)/d)/c^2) + ((2*b^2*c^2*d^2 + 2*a*b*c*d^3 + 5*a^2*d^4)*x^6 + 2*b^2*c^4 +
 2*a*b*c^3*d + 5*a^2*c^2*d^2 + 2*(2*b^2*c^3*d + 2*a*b*c^2*d^2 + 5*a^2*c*d^3
)*x^3)*(c^2*d)^(2/3)*log(c*d*x^2 - (c^2*d)^(2/3)*x + (c^2*d)^(1/3)*c) - 2*(
(2*b^2*c^2*d^2 + 2*a*b*c*d^3 + 5*a^2*d^4)*x^6 + 2*b^2*c^4 + 2*a*b*c^3*d + 5
*a^2*c^2*d^2 + 2*(2*b^2*c^3*d + 2*a*b*c^2*d^2 + 5*a^2*c*d^3)*x^3)*(c^2*d)^(
2/3)*log(c*d*x + (c^2*d)^(2/3)) + 12*(b^2*c^5*d + a*b*c^4*d^2 - 2*a^2*c^3*d
^3)*x)/(c^4*d^5*x^6 + 2*c^5*d^4*x^3 + c^6*d^3)]
```

giac [A] time = 0.20, size = 264, normalized size = 1.02

$$-\frac{\sqrt{3}\left(2\,b^{2}c^{2}+2\,abcd+5\,a^{2}d^{2}\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{27\left(-cd^{2}\right)^{\frac{2}{3}}c^{2}d}-\frac{\left(2\,b^{2}c^{2}+2\,abcd+5\,a^{2}d^{2}\right)\log\left(x^{2}+x\left(-\frac{c}{d}\right)^{\frac{1}{3}}+\left(-\frac{c}{d}\right)^{\frac{2}{3}}\right)}{54\left(-cd^{2}\right)^{\frac{2}{3}}c^{2}d}-\frac{\left(2\,b^{2}c^{2}+2\,abcd+5\,a^{2}d^{2}\right)\left(-\frac{c}{d}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right|\right)}{27\,c^{3}d^{2}}-\frac{7\,b^{2}c^{2}dx^{4}-2\,abcd^{2}x^{4}-5\,a^{2}d^{3}x^{4}+4\,b^{2}c^{3}x+4\,abc^{2}dx-8\,a^{2}cd^{2}x}{18\left(dx^{3}+c\right)^{2}c^{2}d^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2/(d*x^3+c)^3,x, algorithm="giac")
```

```
[Out] -1/27*sqrt(3)*(2*b^2*c^2 + 2*a*b*c*d + 5*a^2*d^2)*arctan(1/3*sqrt(3)*(2*x +
 (-c/d)^(1/3))/(-c/d)^(1/3))/((-c*d^2)^(2/3)*c^2*d) - 1/54*(2*b^2*c^2 + 2*a
*b*c*d + 5*a^2*d^2)*log(x^2 + x*(-c/d)^(1/3) + (-c/d)^(2/3))/((-c*d^2)^(2/3
```

```
)*c^2*d) - 1/27*(2*b^2*c^2 + 2*a*b*c*d + 5*a^2*d^2)*(-c/d)^(1/3)*log(abs(x
- (-c/d)^(1/3)))/(c^3*d^2) - 1/18*(7*b^2*c^2*d*x^4 - 2*a*b*c*d^2*x^4 - 5*a^
2*d^3*x^4 + 4*b^2*c^3*x + 4*a*b*c^2*d*x - 8*a^2*c*d^2*x)/((d*x^3 + c)^2*c^2
*d^2)
```

maple [F(-1)] time = 180.00, size = 0, normalized size = 0.00

hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^2/(d*x^3+c)^3,x)
```

```
[Out] int((b*x^3+a)^2/(d*x^3+c)^3,x)
```

maxima [A] time = 1.32, size = 267, normalized size = 1.03

$$-\frac{\left(7\,b^2c^2d-2\,abcd^2-5\,a^2d^3\right)x^4+4\left(b^2c^3+abc^2d-2\,a^2cd^2\right)x}{18\left(c^2d^4x^6+2\,c^3d^3x^3+c^4d^2\right)}+\frac{\sqrt{3}\left(2\,b^2c^2+2\,abcd+5\,a^2d^2\right)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{27\,c^2d^3\left(\frac{c}{d}\right)^{\frac{2}{3}}}-\frac{\left(2\,b^2c^2+2\,abcd+5\,a^2d^2\right)\log\left(x^2-x\left(\frac{c}{d}\right)^{\frac{1}{3}}+\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}{54\,c^2d^3\left(\frac{c}{d}\right)^{\frac{2}{3}}}+\frac{\left(2\,b^2c^2+2\,abcd+5\,a^2d^2\right)\log\left(x+\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{27\,c^2d^3\left(\frac{c}{d}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2/(d*x^3+c)^3,x, algorithm="maxima")
```

```
[Out] -1/18*((7*b^2*c^2*d - 2*a*b*c*d^2 - 5*a^2*d^3)*x^4 + 4*(b^2*c^3 + a*b*c^2*d
 - 2*a^2*c*d^2)*x)/(c^2*d^4*x^6 + 2*c^3*d^3*x^3 + c^4*d^2) + 1/27*sqrt(3)*(
2*b^2*c^2 + 2*a*b*c*d + 5*a^2*d^2)*arctan(1/3*sqrt(3)*(2*x - (c/d)^(1/3))/(
c/d)^(1/3))/(c^2*d^3*(c/d)^(2/3)) - 1/54*(2*b^2*c^2 + 2*a*b*c*d + 5*a^2*d^2
)*log(x^2 - x*(c/d)^(1/3) + (c/d)^(2/3))/(c^2*d^3*(c/d)^(2/3)) + 1/27*(2*b^
2*c^2 + 2*a*b*c*d + 5*a^2*d^2)*log(x + (c/d)^(1/3))/(c^2*d^3*(c/d)^(2/3))
```

mupad [B] time = 1.43, size = 249, normalized size = 0.97

$$\frac{\ln\left(d^{1/3}\,x+c^{1/3}\right)\,\left(5\,a^2\,d^2+2\,a\,b\,c\,d+2\,b^2\,c^2\right)}{27\,c^{8/3}\,d^{7/3}}-\frac{\frac{2\,x\,\left(-2\,a^2\,d^2+a\,b\,c\,d+b^2\,c^2\right)}{9\,c\,d^2}-\frac{x^4\,\left(5\,a^2\,d^2+2\,a\,b\,c\,d-7\,b^2\,c^2\right)}{18\,c^2\,d}}{c^2+2\,c\,d\,x^3+d^2\,x^6}+\frac{\ln\left(2\,d^{1/3}\,x-c^{1/3}+\sqrt{3}\,c^{1/3}\,1{}\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,\left(5\,a^2\,d^2+2\,a\,b\,c\,d+2\,b^2\,c^2\right)}{27\,c^{8/3}\,d^{7/3}}-\frac{\ln\left(c^{1/3}-2\,d^{1/3}\,x+\sqrt{3}\,c^{1/3}\,1{}\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,\left(5\,a^2\,d^2+2\,a\,b\,c\,d+2\,b^2\,c^2\right)}{27\,c^{8/3}\,d^{7/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^2/(c + d*x^3)^3,x)
```

```
[Out] (log(d^(1/3)*x + c^(1/3))*(5*a^2*d^2 + 2*b^2*c^2 + 2*a*b*c*d))/(27*c^(8/3)*
d^(7/3)) - ((2*x*(b^2*c^2 - 2*a^2*d^2 + a*b*c*d))/(9*c*d^2) - (x^4*(5*a^2*d
^2 - 7*b^2*c^2 + 2*a*b*c*d))/(18*c^2*d))/(c^2 + d^2*x^6 + 2*c*d*x^3) + (log
(3^(1/2)*c^(1/3)*1i + 2*d^(1/3)*x - c^(1/3))*((3^(1/2)*1i)/2 - 1/2)*(5*a^2*
d^2 + 2*b^2*c^2 + 2*a*b*c*d))/(27*c^(8/3)*d^(7/3)) - (log(3^(1/2)*c^(1/3)*1
i - 2*d^(1/3)*x + c^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(5*a^2*d^2 + 2*b^2*c^2 +
2*a*b*c*d))/(27*c^(8/3)*d^(7/3))
```

sympy [A] time = 1.62, size = 233, normalized size = 0.90

$$\frac{x^4\left(5a^2d^3+2abcd^2-7b^2c^2d\right)+x\left(8a^2cd^2-4abc^2d-4b^2c^3\right)}{18c^4d^2+36c^3d^3x^3+18c^2d^4x^6}+\operatorname{RootSum}\left(19683t^3c^8d^7-125a^6d^6-150a^5bcd^5-210a^4b^2c^2d^4-128a^3b^3c^3d^3-84a^2b^4c^4d^2-24ab^5c^5d-8b^6c^6,\left(t\mapsto t\log\left(\frac{27tc^3d^2}{5a^2d^2+2abcd+2b^2c^2}+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**2/(d*x**3+c)**3,x)
```

```
[Out] (x**4*(5*a**2*d**3 + 2*a*b*c*d**2 - 7*b**2*c**2*d) + x*(8*a**2*c*d**2 - 4*a
*b*c**2*d - 4*b**2*c**3))/(18*c**4*d**2 + 36*c**3*d**3*x**3 + 18*c**2*d**4*
x**6) + RootSum(19683*_t**3*c**8*d**7 - 125*a**6*d**6 - 150*a**5*b*c*d**5 -
 210*a**4*b**2*c**2*d**4 - 128*a**3*b**3*c**3*d**3 - 84*a**2*b**4*c**4*d**2
 - 24*a*b**5*c**5*d - 8*b**6*c**6, Lambda(_t, _t*log(27*_t*c**3*d**2/(5*a**
2*d**2 + 2*a*b*c*d + 2*b**2*c**2) + x)))
```

3.14 $\int \frac{(c+dx^3)^4}{a+bx^3}\,dx$

Optimal. Leaf size=252

$$-\frac{(bc-ad)^4\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{2/3}b^{13/3}}+\frac{(bc-ad)^4\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}b^{13/3}}-\frac{(bc-ad)^4\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}b^{13/3}}+\frac{dx(2bc-a}{}$$

Rubi [A] time = 0.19, antiderivative size = 252, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {390, 200, 31, 634, 617, 204, 628}

$$\frac{d^2x^4\left(a^2d^2-4abcd+6b^2c^2\right)}{4b^3}+\frac{dx(2bc-ad)\left(a^2d^2-2abcd+2b^2c^2\right)}{b^4}-\frac{(bc-ad)^4\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{2/3}b^{13/3}}+\frac{(bc-ad)^4\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}b^{13/3}}-\frac{(bc-ad)^4\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}b^{13/3}}+\frac{d^3x^7(4bc-ad)}{7b^2}+\frac{d^4x^{10}}{10b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3)^4/(a + b*x^3),x]
```

```
[Out] (d*(2*b*c - a*d)*(2*b^2*c^2 - 2*a*b*c*d + a^2*d^2)*x)/b^4 + (d^2*(6*b^2*c^2
 - 4*a*b*c*d + a^2*d^2)*x^4)/(4*b^3) + (d^3*(4*b*c - a*d)*x^7)/(7*b^2) + (d
^4*x^10)/(10*b) - ((b*c - a*d)^4*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^
(1/3))])/(Sqrt[3]*a^(2/3)*b^(13/3)) + ((b*c - a*d)^4*Log[a^(1/3) + b^(1/3)*
x])/(3*a^(2/3)*b^(13/3)) - ((b*c - a*d)^4*Log[a^(2/3) - a^(1/3)*b^(1/3)*x +
 b^(2/3)*x^2])/(6*a^(2/3)*b^(13/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{\left(c+dx^3\right)^4}{a+bx^3}\,dx = \int \left(\frac{d(2bc-ad)\left(2b^2c^2-2abcd+a^2d^2\right)}{b^4}+\frac{d^2\left(6b^2c^2-4abcd+a^2d^2\right)x^3}{b^3}+\frac{d^3(4bc-ad)x^6}{b^2}+\right.$$

$$= \frac{d(2bc-ad)\left(2b^2c^2-2abcd+a^2d^2\right)x}{b^4}+\frac{d^2\left(6b^2c^2-4abcd+a^2d^2\right)x^4}{4b^3}+\frac{d^3(4bc-ad)x^7}{7b^2}+$$

$$= \frac{d(2bc-ad)\left(2b^2c^2-2abcd+a^2d^2\right)x}{b^4}+\frac{d^2\left(6b^2c^2-4abcd+a^2d^2\right)x^4}{4b^3}+\frac{d^3(4bc-ad)x^7}{7b^2}+$$

$$= \frac{d(2bc-ad)\left(2b^2c^2-2abcd+a^2d^2\right)x}{b^4}+\frac{d^2\left(6b^2c^2-4abcd+a^2d^2\right)x^4}{4b^3}+\frac{d^3(4bc-ad)x^7}{7b^2}+$$

$$= \frac{d(2bc-ad)\left(2b^2c^2-2abcd+a^2d^2\right)x}{b^4}+\frac{d^2\left(6b^2c^2-4abcd+a^2d^2\right)x^4}{4b^3}+\frac{d^3(4bc-ad)x^7}{7b^2}+$$

$$= \frac{d(2bc-ad)\left(2b^2c^2-2abcd+a^2d^2\right)x}{b^4}+\frac{d^2\left(6b^2c^2-4abcd+a^2d^2\right)x^4}{4b^3}+\frac{d^3(4bc-ad)x^7}{7b^2}+$$

Mathematica [A] time = 0.13, size = 253, normalized size = 1.00

$$\frac{-\frac{70(bc-ad)^4\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{a^{2/3}}+\frac{140(bc-ad)^4\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{a^{2/3}}+\frac{140\sqrt{3}(bc-ad)^4\tan^{-1}\left(\frac{2\sqrt[3]{b}x-\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{a^{2/3}}+105b^{4/3}d^2x^4\left(a^2d^2-4abcd+6b^2c^2\right)+420\sqrt[3]{b}dx\left(-a^3d^3+4a^2bcd^2-6ab^2c^2d+4b^3c^3\right)+60b^{7/3}d^3x^7(4bc-ad)+42b^{10/3}d^4x^{10}}{420b^{13/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^3)^4/(a + b*x^3),x]
```

```
[Out] (420*b^(1/3)*d*(4*b^3*c^3 - 6*a*b^2*c^2*d + 4*a^2*b*c*d^2 - a^3*d^3)*x + 10
5*b^(4/3)*d^2*(6*b^2*c^2 - 4*a*b*c*d + a^2*d^2)*x^4 + 60*b^(7/3)*d^3*(4*b*c
 - a*d)*x^7 + 42*b^(10/3)*d^4*x^10 + (140*Sqrt[3]*(b*c - a*d)^4*ArcTan[(-a^
(1/3) + 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/a^(2/3) + (140*(b*c - a*d)^4*Log[a
^(1/3) + b^(1/3)*x])/a^(2/3) - (70*(b*c - a*d)^4*Log[a^(2/3) - a^(1/3)*b^(1
/3)*x + b^(2/3)*x^2])/a^(2/3))/(420*b^(13/3))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c+dx^3\right)^4}{a+bx^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x^3)^4/(a + b*x^3),x]
```

```
[Out] IntegrateAlgebraic[(c + d*x^3)^4/(a + b*x^3), x]
```

fricas [A] time = 1.23, size = 873, normalized size = 3.46

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^4/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] [1/420*(42*a^2*b^4*d^4*x^10 + 60*(4*a^2*b^4*c*d^3 - a^3*b^3*d^4)*x^7 + 105*
(6*a^2*b^4*c^2*d^2 - 4*a^3*b^3*c*d^3 + a^4*b^2*d^4)*x^4 + 210*sqrt(1/3)*(a*
b^5*c^4 - 4*a^2*b^4*c^3*d + 6*a^3*b^3*c^2*d^2 - 4*a^4*b^2*c*d^3 + a^5*b*d^4
)*sqrt(-(a^2*b)^(1/3)/b)*log((2*a*b*x^3 - 3*(a^2*b)^(1/3)*a*x - a^2 + 3*sqr
t(1/3)*(2*a*b*x^2 + (a^2*b)^(2/3)*x - (a^2*b)^(1/3)*a)*sqrt(-(a^2*b)^(1/3)/
b))/(b*x^3 + a)) - 70*(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*
b*c*d^3 + a^4*d^4)*(a^2*b)^(2/3)*log(a*b*x^2 - (a^2*b)^(2/3)*x + (a^2*b)^(1
/3)*a) + 140*(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 +
 a^4*d^4)*(a^2*b)^(2/3)*log(a*b*x + (a^2*b)^(2/3)) + 420*(4*a^2*b^4*c^3*d -
 6*a^3*b^3*c^2*d^2 + 4*a^4*b^2*c*d^3 - a^5*b*d^4)*x)/(a^2*b^5), 1/420*(42*a
^2*b^4*d^4*x^10 + 60*(4*a^2*b^4*c*d^3 - a^3*b^3*d^4)*x^7 + 105*(6*a^2*b^4*c
^2*d^2 - 4*a^3*b^3*c*d^3 + a^4*b^2*d^4)*x^4 + 420*sqrt(1/3)*(a*b^5*c^4 - 4*
a^2*b^4*c^3*d + 6*a^3*b^3*c^2*d^2 - 4*a^4*b^2*c*d^3 + a^5*b*d^4)*sqrt((a^2*
b)^(1/3)/b)*arctan(sqrt(1/3)*(2*(a^2*b)^(2/3)*x - (a^2*b)^(1/3)*a)*sqrt((a^
2*b)^(1/3)/b)/a^2) - 70*(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^
3*b*c*d^3 + a^4*d^4)*(a^2*b)^(2/3)*log(a*b*x^2 - (a^2*b)^(2/3)*x + (a^2*b)^
(1/3)*a) + 140*(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3
 + a^4*d^4)*(a^2*b)^(2/3)*log(a*b*x + (a^2*b)^(2/3)) + 420*(4*a^2*b^4*c^3*d
 - 6*a^3*b^3*c^2*d^2 + 4*a^4*b^2*c*d^3 - a^5*b*d^4)*x)/(a^2*b^5)]
```

giac [A] time = 0.20, size = 391, normalized size = 1.55

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^4/(b*x^3+a),x, algorithm="giac")
```

```
[Out] -1/3*sqrt(3)*(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 +
 a^4*d^4)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/((-a*b^2)^(
2/3)*b^3) - 1/6*(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^
3 + a^4*d^4)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/((-a*b^2)^(2/3)*b^3)
- 1/3*(b^10*c^4 - 4*a*b^9*c^3*d + 6*a^2*b^8*c^2*d^2 - 4*a^3*b^7*c*d^3 + a^4
*b^6*d^4)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a*b^10) + 1/140*(14*b^9*
d^4*x^10 + 80*b^9*c*d^3*x^7 - 20*a*b^8*d^4*x^7 + 210*b^9*c^2*d^2*x^4 - 140*
a*b^8*c*d^3*x^4 + 35*a^2*b^7*d^4*x^4 + 560*b^9*c^3*d*x - 840*a*b^8*c^2*d^2*
x + 560*a^2*b^7*c*d^3*x - 140*a^3*b^6*d^4*x)/b^10
```

maple [B] time = 0.05, size = 661, normalized size = 2.62

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c)^4/(b*x^3+a),x)
```

```
[Out] -4/3/b^4/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*a^3*c*
d^3+2/b^3/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*a^2*c
^2*d^2-4/3/b^2/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*
a*c^3*d-1/7*d^4/b^2*x^7*a+4/7*d^3/b*x^7*c-1/6/b/(a/b)^(2/3)*ln(x^2-(a/b)^(1
/3)*x+(a/b)^(2/3))*c^4+1/3/b/(a/b)^(2/3)*ln(x+(a/b)^(1/3))*c^4+4*d/b*c^3*x+
```

```
1/4*d^4/b^3*x^4*a^2+3/2*d^2/b*x^4*c^2-d^4/b^4*a^3*x-4/3/b^4/(a/b)^(2/3)*ln(
x+(a/b)^(1/3))*a^3*c*d^3+2/b^3/(a/b)^(2/3)*ln(x+(a/b)^(1/3))*a^2*c^2*d^2-4/
3/b^2/(a/b)^(2/3)*ln(x+(a/b)^(1/3))*a*c^3*d+2/3/b^4/(a/b)^(2/3)*ln(x^2-(a/b
)^(1/3)*x+(a/b)^(2/3))*a^3*c*d^3-1/b^3/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/
b)^(2/3))*a^2*c^2*d^2+2/3/b^2/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))
*a*c^3*d+1/3/b^5/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1)
)*a^4*d^4-6*d^2/b^2*a*c^2*x-d^3/b^2*x^4*a*c+4*d^3/b^3*a^2*c*x+1/3/b/(a/b)^(
2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*c^4+1/3/b^5/(a/b)^(2/3
)*ln(x+(a/b)^(1/3))*a^4*d^4-1/6/b^5/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^
(2/3))*a^4*d^4+1/10*d^4*x^10/b
```

maxima [A]   time = 1.19, size = 364, normalized size = 1.44

$$\frac{14\,b^3d^4x^{10}+20\left(4\,b^3cd^3-ab^2d^4\right)x^7+35\left(6\,b^3c^2d^2-4\,ab^2cd^3+a^2bd^4\right)x^4+140\left(4\,b^3c^3d-6\,ab^2c^2d^2+4\,a^2bcd^3-a^3d^4\right)x}{140\,b^4}+\frac{\sqrt{3}\left(b^4c^4-4\,ab^3c^3d+6\,a^2b^2c^2d^2-4\,a^3bcd^3+a^4d^4\right)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,b^5\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\left(b^4c^4-4\,ab^3c^3d+6\,a^2b^2c^2d^2-4\,a^3bcd^3+a^4d^4\right)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,b^5\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\left(b^4c^4-4\,ab^3c^3d+6\,a^2b^2c^2d^2-4\,a^3bcd^3+a^4d^4\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,b^5\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^4/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] 1/140*(14*b^3*d^4*x^10 + 20*(4*b^3*c*d^3 - a*b^2*d^4)*x^7 + 35*(6*b^3*c^2*d
^2 - 4*a*b^2*c*d^3 + a^2*b*d^4)*x^4 + 140*(4*b^3*c^3*d - 6*a*b^2*c^2*d^2 +
4*a^2*b*c*d^3 - a^3*d^4)*x)/b^4 + 1/3*sqrt(3)*(b^4*c^4 - 4*a*b^3*c^3*d + 6*
a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*arctan(1/3*sqrt(3)*(2*x - (a/b)^
(1/3))/(a/b)^(1/3))/(b^5*(a/b)^(2/3)) - 1/6*(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^
2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2
/3))/(b^5*(a/b)^(2/3)) + 1/3*(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 -
 4*a^3*b*c*d^3 + a^4*d^4)*log(x + (a/b)^(1/3))/(b^5*(a/b)^(2/3))
```

mupad [B]   time = 1.43, size = 250, normalized size = 0.99

$$x\left(\frac{4\,c^3\,d}{b}-\frac{a\left(\frac{a\left(\frac{a\,d^4}{b^2}-\frac{4\,c\,d^3}{b}\right)}{b}+\frac{6\,c^2\,d^2}{b}\right)}{b}\right)-x^7\left(\frac{a\,d^4}{7\,b^2}-\frac{4\,c\,d^3}{7\,b}\right)+x^4\left(\frac{a\left(\frac{a\,d^4}{b^2}-\frac{4\,c\,d^3}{b}\right)}{4\,b}+\frac{3\,c^2\,d^2}{2\,b}\right)+\frac{d^4\,x^{10}}{10\,b}+\frac{\ln\left(b^{1/3}\,x+a^{1/3}\right)\left(a\,d-b\,c\right)^4}{3\,a^{2/3}\,b^{13/3}}+\frac{\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1{}\mathrm{i}\right)\left(-\frac{1}{6}+\frac{\sqrt{3}\,1{}\mathrm{i}}{6}\right)\left(a\,d-b\,c\right)^4}{a^{2/3}\,b^{13/3}}-\frac{\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1{}\mathrm{i}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\left(a\,d-b\,c\right)^4}{3\,a^{2/3}\,b^{13/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3)^4/(a + b*x^3),x)
```

```
[Out] x*((4*c^3*d)/b - (a*((a*((a*d^4)/b^2 - (4*c*d^3)/b))/b + (6*c^2*d^2)/b))/b)
 - x^7*((a*d^4)/(7*b^2) - (4*c*d^3)/(7*b)) + x^4*((a*((a*d^4)/b^2 - (4*c*d^
3)/b))/(4*b) + (3*c^2*d^2)/(2*b)) + (d^4*x^10)/(10*b) + (log(b^(1/3)*x + a^
(1/3))*(a*d - b*c)^4)/(3*a^(2/3)*b^(13/3)) + (log(3^(1/2)*a^(1/3)*1i + 2*b^
(1/3)*x - a^(1/3))*((3^(1/2)*1i)/6 - 1/6)*(a*d - b*c)^4)/(a^(2/3)*b^(13/3))
 - (log(3^(1/2)*a^(1/3)*1i - 2*b^(1/3)*x + a^(1/3))*((3^(1/2)*1i)/2 + 1/2)*
(a*d - b*c)^4)/(3*a^(2/3)*b^(13/3))
```

sympy [A]   time = 1.31, size = 371, normalized size = 1.47

$$x^7\left(-\frac{ad^4}{7b^2}+\frac{4cd^3}{7b}\right)+x^4\left(\frac{a^2d^4}{4b^3}-\frac{acd^3}{b^2}+\frac{3c^2d^2}{2b}\right)+x\left(-\frac{a^3d^4}{b^4}+\frac{4a^2cd^3}{b^3}-\frac{6ac^2d^2}{b^2}+\frac{4c^3d}{b}\right)+\operatorname{RootSum}\left(27t^3a^2b^{13}-a^{12}d^{12}+12a^{11}bcd^{11}-66a^{10}b^2c^2d^{10}+220a^9b^3c^3d^9-495a^8b^4c^4d^8+792a^7b^5c^5d^7-924a^6b^6c^6d^6+792a^5b^7c^7d^5-495a^4b^8c^8d^4+220a^3b^9c^9d^3-66a^2b^{10}c^{10}d^2+12ab^{11}c^{11}d-b^{12}c^{12},\left(t\mapsto t\log\left(\frac{3tab^4}{a^4d^4-4a^3bcd^3+6a^2b^2c^2d^2-4ab^3c^3d+b^4c^4}+x\right)\right)\right)+\frac{d^4x^{10}}{10b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c)**4/(b*x**3+a),x)
```

```
[Out] x**7*(-a*d**4/(7*b**2) + 4*c*d**3/(7*b)) + x**4*(a**2*d**4/(4*b**3) - a*c*d
**3/b**2 + 3*c**2*d**2/(2*b)) + x*(-a**3*d**4/b**4 + 4*a**2*c*d**3/b**3 - 6
*a*c**2*d**2/b**2 + 4*c**3*d/b) + RootSum(27*_t**3*a**2*b**13 - a**12*d**12
 + 12*a**11*b*c*d**11 - 66*a**10*b**2*c**2*d**10 + 220*a**9*b**3*c**3*d**9
- 495*a**8*b**4*c**4*d**8 + 792*a**7*b**5*c**5*d**7 - 924*a**6*b**6*c**6*d*
*6 + 792*a**5*b**7*c**7*d**5 - 495*a**4*b**8*c**8*d**4 + 220*a**3*b**9*c**9
*d**3 - 66*a**2*b**10*c**10*d**2 + 12*a*b**11*c**11*d - b**12*c**12, Lambda
(_t, _t*log(3*_t*a*b**4/(a**4*d**4 - 4*a**3*b*c*d**3 + 6*a**2*b**2*c**2*d**
2 - 4*a*b**3*c**3*d + b**4*c**4) + x))) + d**4*x**10/(10*b)
```

3.15 $\int \frac{(c+dx^3)^3}{a+bx^3}\, dx$

Optimal. Leaf size=208

$$-\frac{(bc-ad)^3\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{6a^{2/3}b^{10/3}}+\frac{(bc-ad)^3\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3a^{2/3}b^{10/3}}-\frac{(bc-ad)^3\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{2/3}b^{10/3}}+\frac{dx\left(a^2d^2-\right.}{}$$

Rubi [A] time = 0.15, antiderivative size = 208, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {390, 200, 31, 634, 617, 204, 628}

$$\frac{dx\left(a^2d^2-3abcd+3b^2c^2\right)}{b^3}-\frac{(bc-ad)^3\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{6a^{2/3}b^{10/3}}+\frac{(bc-ad)^3\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3a^{2/3}b^{10/3}}-\frac{(bc-ad)^3\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{2/3}b^{10/3}}+\frac{d^2x^4(3bc-ad)}{4b^2}+\frac{d^3x^7}{7b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3)^3/(a + b*x^3),x]
```

```
[Out] (d*(3*b^2*c^2 - 3*a*b*c*d + a^2*d^2)*x)/b^3 + (d^2*(3*b*c - a*d)*x^4)/(4*b^
2) + (d^3*x^7)/(7*b) - ((b*c - a*d)^3*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[
3]*a^(1/3))])/(Sqrt[3]*a^(2/3)*b^(10/3)) + ((b*c - a*d)^3*Log[a^(1/3) + b^(
1/3)*x])/(3*a^(2/3)*b^(10/3)) - ((b*c - a*d)^3*Log[a^(2/3) - a^(1/3)*b^(1/3
)*x + b^(2/3)*x^2])/(6*a^(2/3)*b^(10/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{\left(c+dx^3\right)^3}{a+bx^3}\,dx = \int \left(\frac{d\left(3b^2c^2-3abcd+a^2d^2\right)}{b^3}+\frac{d^2(3bc-ad)x^3}{b^2}+\frac{d^3x^6}{b}+\frac{b^3c^3-3ab^2c^2d+3a^2bcd^2-a^3d^3}{b^3\left(a+bx^3\right)}\right.$$

$$= \frac{d\left(3b^2c^2-3abcd+a^2d^2\right)x}{b^3}+\frac{d^2(3bc-ad)x^4}{4b^2}+\frac{d^3x^7}{7b}+\frac{(bc-ad)^3\int\frac{1}{a+bx^3}\,dx}{b^3}$$

$$= \frac{d\left(3b^2c^2-3abcd+a^2d^2\right)x}{b^3}+\frac{d^2(3bc-ad)x^4}{4b^2}+\frac{d^3x^7}{7b}+\frac{(bc-ad)^3\int\frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{3a^{2/3}b^3}+\frac{(bc-ad\cdots}{\cdots}$$

$$= \frac{d\left(3b^2c^2-3abcd+a^2d^2\right)x}{b^3}+\frac{d^2(3bc-ad)x^4}{4b^2}+\frac{d^3x^7}{7b}+\frac{(bc-ad)^3\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}b^{10/3}}-\frac{(bc\cdots}{\cdots}$$

$$= \frac{d\left(3b^2c^2-3abcd+a^2d^2\right)x}{b^3}+\frac{d^2(3bc-ad)x^4}{4b^2}+\frac{d^3x^7}{7b}+\frac{(bc-ad)^3\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}b^{10/3}}-\frac{(bc\cdots}{\cdots}$$

$$= \frac{d\left(3b^2c^2-3abcd+a^2d^2\right)x}{b^3}+\frac{d^2(3bc-ad)x^4}{4b^2}+\frac{d^3x^7}{7b}-\frac{(bc-ad)^3\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{2/3}b^{10/3}}+\frac{(bc\cdots}{\cdots}$$

Mathematica [A] time = 0.11, size = 203, normalized size = 0.98

$$\frac{\frac{14(ad-bc)^3\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{a^{2/3}}+\frac{28(bc-ad)^3\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{a^{2/3}}+\frac{28\sqrt{3}(bc-ad)^3\tan^{-1}\left(\frac{2\sqrt[3]{b}x-\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{a^{2/3}}+84\sqrt[3]{b}dx\left(a^2d^2-3abcd+3b^2c^2\right)+21b^{4/3}d^2x^4(3bc-ad)+12b^{7/3}d^3x^7}{84b^{10/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^3)^3/(a + b*x^3),x]
```

```
[Out] (84*b^(1/3)*d*(3*b^2*c^2 - 3*a*b*c*d + a^2*d^2)*x + 21*b^(4/3)*d^2*(3*b*c -
 a*d)*x^4 + 12*b^(7/3)*d^3*x^7 + (28*Sqrt[3]*(b*c - a*d)^3*ArcTan[(-a^(1/3)
 + 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/a^(2/3) + (28*(b*c - a*d)^3*Log[a^(1/3)
 + b^(1/3)*x]])/a^(2/3) + (14*(-(b*c) + a*d)^3*Log[a^(2/3) - a^(1/3)*b^(1/3)
*x + b^(2/3)*x^2]])/a^(2/3))/(84*b^(10/3))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c+dx^3\right)^3}{a+bx^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x^3)^3/(a + b*x^3),x]
```

```
[Out] IntegrateAlgebraic[(c + d*x^3)^3/(a + b*x^3), x]
```

fricas [A] time = 1.02, size = 700, normalized size = 3.37

$$\left[\frac{12a^2b^3d^3x^7 + 21\left(3a^2b^3cd^2 - a^3b^2d^3\right)x^4 - 42\sqrt{\frac{1}{3}}\left(ab^4c^3 - 3a^2b^3c^2d + 3a^3b^2cd^2 - a^4bd^3\right)\sqrt{\frac{\left(-a^2b\right)^{\frac{1}{3}}}{b}}\log\left(\frac{2abx^3 + 3\left(-a^2b\right)^{\frac{1}{3}}ax - a^2 - 3\sqrt{\frac{1}{3}}\left(2abx^2 + \left(-a^2b\right)^{\frac{2}{3}}x + \left(-a^2b\right)^{\frac{1}{3}}a\right)\sqrt{\frac{\left(-a^2b\right)^{\frac{1}{3}}}{b}}}{bx^3+a}\right) - 14\left(b^3c^3 - 3ab^2c^2d + 3a^2bcd^2 - a^3d^3\right)\left(-a^2b\right)^{\frac{2}{3}}\log\left(abx^2 - \left(-a^2b\right)^{\frac{2}{3}}x - \left(-a^2b\right)^{\frac{1}{3}}a\right) + 28\left(b^3c^3 - 3ab^2c^2d + 3a^2bcd^2 - a^3d^3\right)\left(-a^2b\right)^{\frac{2}{3}}\log\left(abx + \left(-a^2b\right)^{\frac{2}{3}}\right) + 84\left(3a^2b^3c^2d - 3a^3b^2cd^2 + a^4bd^3\right)x}{84a^2b^4}, \frac{12a^2b^3d^3x^7 + 21\left(3a^2b^3cd^2 - a^3b^2d^3\right)x^4 + 84\sqrt{\frac{1}{3}}\left(ab^4c^3 - 3a^2b^3c^2d + 3a^3b^2cd^2 - a^4bd^3\right)\sqrt{-\frac{\left(-a^2b\right)^{\frac{1}{3}}}{b}}\arctan\left(\frac{\sqrt{\frac{1}{3}}\left(2\left(-a^2b\right)^{\frac{2}{3}}x + \left(-a^2b\right)^{\frac{1}{3}}a\right)\sqrt{-\frac{\left(-a^2b\right)^{\frac{1}{3}}}{b}}}{a^2}\right) - 14\left(b^3c^3 - 3ab^2c^2d + 3a^2bcd^2 - a^3d^3\right)\left(-a^2b\right)^{\frac{2}{3}}\log\left(abx^2 - \left(-a^2b\right)^{\frac{2}{3}}x - \left(-a^2b\right)^{\frac{1}{3}}a\right) + 28\left(b^3c^3 - 3ab^2c^2d + 3a^2bcd^2 - a^3d^3\right)\left(-a^2b\right)^{\frac{2}{3}}\log\left(abx + \left(-a^2b\right)^{\frac{2}{3}}\right) + 84\left(3a^2b^3c^2d - 3a^3b^2cd^2 + a^4bd^3\right)x}{84a^2b^4}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^3/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] [1/84*(12*a^2*b^3*d^3*x^7 + 21*(3*a^2*b^3*c*d^2 - a^3*b^2*d^3)*x^4 - 42*sqr
t(1/3)*(a*b^4*c^3 - 3*a^2*b^3*c^2*d + 3*a^3*b^2*c*d^2 - a^4*b*d^3)*sqrt((-a
^2*b)^(1/3)/b)*log((2*a*b*x^3 + 3*(-a^2*b)^(1/3)*a*x - a^2 - 3*sqrt(1/3)*(2
*a*b*x^2 + (-a^2*b)^(2/3)*x + (-a^2*b)^(1/3)*a)*sqrt((-a^2*b)^(1/3)/b))/(b*
x^3 + a)) - 14*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*(-a^2*b)
^(2/3)*log(a*b*x^2 - (-a^2*b)^(2/3)*x - (-a^2*b)^(1/3)*a) + 28*(b^3*c^3 - 3
*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*(-a^2*b)^(2/3)*log(a*b*x + (-a^2*b)
^(2/3)) + 84*(3*a^2*b^3*c^2*d - 3*a^3*b^2*c*d^2 + a^4*b*d^3)*x)/(a^2*b^4),
1/84*(12*a^2*b^3*d^3*x^7 + 21*(3*a^2*b^3*c*d^2 - a^3*b^2*d^3)*x^4 + 84*sqrt
(1/3)*(a*b^4*c^3 - 3*a^2*b^3*c^2*d + 3*a^3*b^2*c*d^2 - a^4*b*d^3)*sqrt(-(-a
^2*b)^(1/3)/b)*arctan(sqrt(1/3)*(2*(-a^2*b)^(2/3)*x + (-a^2*b)^(1/3)*a)*sqr
t(-(-a^2*b)^(1/3)/b)/a^2) - 14*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a
^3*d^3)*(-a^2*b)^(2/3)*log(a*b*x^2 - (-a^2*b)^(2/3)*x - (-a^2*b)^(1/3)*a) +
 28*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*(-a^2*b)^(2/3)*log(
a*b*x + (-a^2*b)^(2/3)) + 84*(3*a^2*b^3*c^2*d - 3*a^3*b^2*c*d^2 + a^4*b*d^3
)*x)/(a^2*b^4)]
```

giac [A] time = 0.18, size = 296, normalized size = 1.42

$$-\frac{\sqrt{3}\left(b^3c^3 - 3ab^2c^2d + 3a^2bcd^2 - a^3d^3\right)\arctan\left(\frac{\sqrt{3}\left(2x + \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\left(-ab^2\right)^{\frac{2}{3}}b^2} - \frac{\left(b^3c^3 - 3ab^2c^2d + 3a^2bcd^2 - a^3d^3\right)\log\left(x^2 + x\left(-\frac{a}{b}\right)^{\frac{1}{3}} + \left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(-ab^2\right)^{\frac{2}{3}}b^2} - \frac{\left(b^7c^3 - 3ab^6c^2d + 3a^2b^5cd^2 - a^3b^4d^3\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3ab^7} + \frac{4b^6d^3x^7 + 21b^6cd^2x^4 - 7ab^5d^3x^4 + 84b^6c^2dx - 84ab^5cd^2x + 28a^2b^4d^3x}{28b^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^3/(b*x^3+a),x, algorithm="giac")
```

```
[Out] -1/3*sqrt(3)*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*arctan(1/3
*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/((-a*b^2)^(2/3)*b^2) - 1/6*(b^3
*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*log(x^2 + x*(-a/b)^(1/3) +
(-a/b)^(2/3))/((-a*b^2)^(2/3)*b^2) - 1/3*(b^7*c^3 - 3*a*b^6*c^2*d + 3*a^2*b
^5*c*d^2 - a^3*b^4*d^3)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a*b^7) + 1
/28*(4*b^6*d^3*x^7 + 21*b^6*c*d^2*x^4 - 7*a*b^5*d^3*x^4 + 84*b^6*c^2*d*x -
84*a*b^5*c*d^2*x + 28*a^2*b^4*d^3*x)/b^7
```

maple [B] time = 0.05, size = 486, normalized size = 2.34

$$\frac{d^3x^7}{7b} - \frac{ad^3x^4}{4b^2} + \frac{3cd^2x^4}{4b} - \frac{\sqrt{3}\,a^3d^3\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3\left(\frac{a}{b}\right)^{\frac{2}{3}}b^4} - \frac{a^3d^3\ln\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{2}{3}}b^4} + \frac{a^3d^3\ln\left(x^2 - \left(\frac{a}{b}\right)^{\frac{1}{3}}x + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(\frac{a}{b}\right)^{\frac{2}{3}}b^4} + \frac{\sqrt{3}\,a^2cd^2\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{\left(\frac{a}{b}\right)^{\frac{2}{3}}b^3} + \frac{a^2cd^2\ln\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{\left(\frac{a}{b}\right)^{\frac{2}{3}}b^3} - \frac{a^2cd^2\ln\left(x^2 - \left(\frac{a}{b}\right)^{\frac{1}{3}}x + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{2\left(\frac{a}{b}\right)^{\frac{2}{3}}b^3} + \frac{a^2d^3x}{b^3} - \frac{\sqrt{3}\,ac^2d\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{\left(\frac{a}{b}\right)^{\frac{2}{3}}b^2} - \frac{ac^2d\ln\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{\left(\frac{a}{b}\right)^{\frac{2}{3}}b^2} + \frac{ac^2d\ln\left(x^2 - \left(\frac{a}{b}\right)^{\frac{1}{3}}x + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{2\left(\frac{a}{b}\right)^{\frac{2}{3}}b^2} - \frac{3acd^2x}{b^2} + \frac{\sqrt{3}\,c^3\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3\left(\frac{a}{b}\right)^{\frac{2}{3}}b} + \frac{c^3\ln\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{2}{3}}b} - \frac{c^3\ln\left(x^2 - \left(\frac{a}{b}\right)^{\frac{1}{3}}x + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(\frac{a}{b}\right)^{\frac{2}{3}}b} + \frac{3c^2dx}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c)^3/(b*x^3+a),x)
```

```
[Out] 1/7*d^3*x^7/b-1/4*d^3/b^2*x^4*a+3/4*d^2/b*x^4*c+d^3/b^3*a^2*x-3*d^2/b^2*a*c
*x+3*d/b*c^2*x-1/3/b^4/(a/b)^(2/3)*ln(x+(a/b)^(1/3))*a^3*d^3+1/b^3/(a/b)^(2
/3)*ln(x+(a/b)^(1/3))*a^2*c*d^2-1/b^2/(a/b)^(2/3)*ln(x+(a/b)^(1/3))*a*c^2*d
+1/3/b/(a/b)^(2/3)*ln(x+(a/b)^(1/3))*c^3+1/6/b^4/(a/b)^(2/3)*ln(x^2-(a/b)^(
1/3)*x+(a/b)^(2/3))*a^3*d^3-1/2/b^3/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^
(2/3))*a^2*c*d^2+1/2/b^2/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*a*c^
2*d-1/6/b/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*c^3-1/3/b^4/(a/b)^(
2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*a^3*d^3+1/b^3/(a/b)^(2
/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*a^2*c*d^2-1/b^2/(a/b)^(
```

```
2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*a*c^2*d+1/3/b/(a/b)^(2
/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*c^3
```

maxima [A] time = 1.32, size = 273, normalized size = 1.31

$$\frac{4\,b^2d^3x^7+7\,\left(3\,b^2cd^2-abd^3\right)x^4+28\,\left(3\,b^2c^2d-3\,abcd^2+a^2d^3\right)x}{28\,b^3}+\frac{\sqrt{3}\,\left(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3\right)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,b^4\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\left(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3\right)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,b^4\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\left(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,b^4\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^3/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] 1/28*(4*b^2*d^3*x^7 + 7*(3*b^2*c*d^2 - a*b*d^3)*x^4 + 28*(3*b^2*c^2*d - 3*a
*b*c*d^2 + a^2*d^3)*x)/b^3 + 1/3*sqrt(3)*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b
*c*d^2 - a^3*d^3)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(b^4*
(a/b)^(2/3)) - 1/6*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*log(
x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(b^4*(a/b)^(2/3)) + 1/3*(b^3*c^3 - 3*a*b
^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*log(x + (a/b)^(1/3))/(b^4*(a/b)^(2/3))
```

mupad [B] time = 1.40, size = 192, normalized size = 0.92

$$x\left(\frac{3\,c^2\,d}{b}+\frac{a\,\left(\frac{a\,d^3}{b^2}-\frac{3\,c\,d^2}{b}\right)}{b}\right)-x^4\,\left(\frac{a\,d^3}{4\,b^2}-\frac{3\,c\,d^2}{4\,b}\right)+\frac{d^3\,x^7}{7\,b}-\frac{\ln\left(b^{1/3}\,x+a^{1/3}\right)\,{\left(a\,d-b\,c\right)}^3}{3\,a^{2/3}\,b^{10/3}}-\frac{\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1{}\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,{\left(a\,d-b\,c\right)}^3}{3\,a^{2/3}\,b^{10/3}}+\frac{\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1{}\mathrm{i}\right)\,\left(\frac{1}{6}+\frac{\sqrt{3}\,1{}\mathrm{i}}{6}\right)\,{\left(a\,d-b\,c\right)}^3}{a^{2/3}\,b^{10/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3)^3/(a + b*x^3),x)
```

```
[Out] x*((3*c^2*d)/b + (a*((a*d^3)/b^2 - (3*c*d^2)/b))/b) - x^4*((a*d^3)/(4*b^2)
- (3*c*d^2)/(4*b)) + (d^3*x^7)/(7*b) - (log(b^(1/3)*x + a^(1/3))*(a*d - b*c
)^3)/(3*a^(2/3)*b^(10/3)) - (log(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*x - a^(1/3)
)*((3^(1/2)*1i)/2 - 1/2)*(a*d - b*c)^3)/(3*a^(2/3)*b^(10/3)) + (log(3^(1/2)
*a^(1/3)*1i - 2*b^(1/3)*x + a^(1/3))*((3^(1/2)*1i)/6 + 1/6)*(a*d - b*c)^3)/
(a^(2/3)*b^(10/3))
```

sympy [A] time = 1.00, size = 257, normalized size = 1.24

$$x^{4}\left(-\frac{ad^{3}}{4b^{2}}+\frac{3cd^{2}}{4b}\right)+x\left(\frac{a^{2}d^{3}}{b^{3}}-\frac{3acd^{2}}{b^{2}}+\frac{3c^{2}d}{b}\right)+\operatorname{RootSum}{\left(27t^{3}a^{2}b^{10}+a^{9}d^{9}-9a^{8}bcd^{8}+36a^{7}b^{2}c^{2}d^{7}-84a^{6}b^{3}c^{3}d^{6}+126a^{5}b^{4}c^{4}d^{5}-126a^{4}b^{5}c^{5}d^{4}+84a^{3}b^{6}c^{6}d^{3}-36a^{2}b^{7}c^{7}d^{2}+9ab^{8}c^{8}d-b^{9}c^{9}, \left(t\mapsto t\log{\left(-\frac{3tab^{3}}{a^{3}d^{3}-3a^{2}bcd^{2}+3ab^{2}c^{2}d-b^{3}c^{3}}+x\right)}\right)\right)}+\frac{d^{3}x^{7}}{7b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c)**3/(b*x**3+a),x)
```

```
[Out] x**4*(-a*d**3/(4*b**2) + 3*c*d**2/(4*b)) + x*(a**2*d**3/b**3 - 3*a*c*d**2/b
**2 + 3*c**2*d/b) + RootSum(27*_t**3*a**2*b**10 + a**9*d**9 - 9*a**8*b*c*d*
*8 + 36*a**7*b**2*c**2*d**7 - 84*a**6*b**3*c**3*d**6 + 126*a**5*b**4*c**4*d
**5 - 126*a**4*b**5*c**5*d**4 + 84*a**3*b**6*c**6*d**3 - 36*a**2*b**7*c**7*
d**2 + 9*a*b**8*c**8*d - b**9*c**9, Lambda(_t, _t*log(-3*_t*a*b**3/(a**3*d*
*3 - 3*a**2*b*c*d**2 + 3*a*b**2*c**2*d - b**3*c**3) + x))) + d**3*x**7/(7*b
)
```

3.16 $\int \frac{(c+dx^3)^2}{a+bx^3} \, dx$

Optimal. Leaf size=173

$$-\frac{(bc-ad)^2\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{2/3}b^{7/3}}+\frac{(bc-ad)^2\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}b^{7/3}}-\frac{(bc-ad)^2\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}b^{7/3}}+\frac{dx(2bc-a}{b^2}$$

Rubi [A] time = 0.12, antiderivative size = 173, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {390, 200, 31, 634, 617, 204, 628}

$$-\frac{(bc-ad)^2\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{2/3}b^{7/3}}+\frac{(bc-ad)^2\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}b^{7/3}}-\frac{(bc-ad)^2\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}b^{7/3}}+\frac{dx(2bc-ad)}{b^2}+\frac{d^2x^4}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3)^2/(a + b*x^3),x]
```

```
[Out] (d*(2*b*c - a*d)*x)/b^2 + (d^2*x^4)/(4*b) - ((b*c - a*d)^2*ArcTan[(a^(1/3)
- 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(2/3)*b^(7/3)) + ((b*c - a*d)
^2*Log[a^(1/3) + b^(1/3)*x])/(3*a^(2/3)*b^(7/3)) - ((b*c - a*d)^2*Log[a^(2/
3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(2/3)*b^(7/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(c+dx^3\right)^2}{a+bx^3}\,dx &= \int \left(\frac{d(2bc-ad)}{b^2}+\frac{d^2x^3}{b}+\frac{b^2c^2-2abcd+a^2d^2}{b^2\left(a+bx^3\right)}\right)dx \\
&= \frac{d(2bc-ad)x}{b^2}+\frac{d^2x^4}{4b}+\frac{(bc-ad)^2\int \frac{1}{a+bx^3}\,dx}{b^2} \\
&= \frac{d(2bc-ad)x}{b^2}+\frac{d^2x^4}{4b}+\frac{(bc-ad)^2\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{3a^{2/3}b^2}+\frac{(bc-ad)^2\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{3a^{2/3}b^2} \\
&= \frac{d(2bc-ad)x}{b^2}+\frac{d^2x^4}{4b}+\frac{(bc-ad)^2\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}b^{7/3}}-\frac{(bc-ad)^2\int \frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{6a^{2/3}b^{7/3}}+\cdots \\
&= \frac{d(2bc-ad)x}{b^2}+\frac{d^2x^4}{4b}+\frac{(bc-ad)^2\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}b^{7/3}}-\frac{(bc-ad)^2\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}\cdots\right)}{6a^{2/3}b^{7/3}}\cdots \\
&= \frac{d(2bc-ad)x}{b^2}+\frac{d^2x^4}{4b}-\frac{(bc-ad)^2\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{2/3}b^{7/3}}+\frac{(bc-ad)^2\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}b^{7/3}}-\cdots
\end{aligned}$$

Mathematica [A] time = 0.15, size = 167, normalized size = 0.97

$$\frac{-2(bc-ad)^2\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)+3a^{2/3}b^{4/3}d^2x^4-12a^{2/3}\sqrt[3]{b}dx(ad-2bc)+4(bc-ad)^2\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)+4\sqrt{3}(bc-ad)^2\tan^{-1}\left(\frac{2\sqrt[3]{b}x-\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{12a^{2/3}b^{7/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^3)^2/(a + b*x^3),x]
```

```
[Out] (-12*a^(2/3)*b^(1/3)*d*(-2*b*c + a*d)*x + 3*a^(2/3)*b^(4/3)*d^2*x^4 + 4*Sqr
t[3]*(b*c - a*d)^2*ArcTan[(-a^(1/3) + 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))] + 4*(
b*c - a*d)^2*Log[a^(1/3) + b^(1/3)*x] - 2*(b*c - a*d)^2*Log[a^(2/3) - a^(1/
3)*b^(1/3)*x + b^(2/3)*x^2])/(12*a^(2/3)*b^(7/3))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c+dx^3\right)^2}{a+bx^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x^3)^2/(a + b*x^3),x]
```

```
[Out] IntegrateAlgebraic[(c + d*x^3)^2/(a + b*x^3), x]
```

fricas [A] time = 0.80, size = 507, normalized size = 2.93

$$\left[\frac{3\,a^{2}b^{2}d^{2}x^{4}+6\,\sqrt{\frac{1}{3}}\left(ab^{3}c^{2}-2\,a^{2}b^{2}cd+a^{3}bd^{2}\right)\sqrt{-\frac{\left(a^{2}b\right)^{\frac{1}{3}}}{b}}\log\left(\frac{2\,abx^{3}-3\,\left(a^{2}b\right)^{\frac{1}{3}}ax-a^{2}+3\,\sqrt{\frac{1}{3}}\left(2\,abx^{2}+\left(a^{2}b\right)^{\frac{2}{3}}x-\left(a^{2}b\right)^{\frac{1}{3}}a\right)\sqrt{-\frac{\left(a^{2}b\right)^{\frac{1}{3}}}{b}}}{bx^{3}+a}\right)-2\,\left(b^{2}c^{2}-2\,abcd+a^{2}d^{2}\right)\left(a^{2}b\right)^{\frac{2}{3}}\log\left(abx^{2}-\left(a^{2}b\right)^{\frac{2}{3}}x+\left(a^{2}b\right)^{\frac{1}{3}}a\right)+4\,\left(b^{2}c^{2}-2\,abcd+a^{2}d^{2}\right)\left(a^{2}b\right)^{\frac{2}{3}}\log\left(abx+\left(a^{2}b\right)^{\frac{2}{3}}\right)+12\,\left(2\,a^{2}b^{2}cd-a^{3}bd^{2}\right)x}{12\,a^{2}b^{3}},\ \frac{3\,a^{2}b^{2}d^{2}x^{4}+12\,\sqrt{\frac{1}{3}}\left(ab^{3}c^{2}-2\,a^{2}b^{2}cd+a^{3}bd^{2}\right)\sqrt{\frac{\left(a^{2}b\right)^{\frac{1}{3}}}{b}}\arctan\left(\frac{\sqrt{\frac{1}{3}}\left(2\,\left(a^{2}b\right)^{\frac{2}{3}}x-\left(a^{2}b\right)^{\frac{1}{3}}a\right)\sqrt{\frac{\left(a^{2}b\right)^{\frac{1}{3}}}{b}}}{a^{2}}\right)-2\,\left(b^{2}c^{2}-2\,abcd+a^{2}d^{2}\right)\left(a^{2}b\right)^{\frac{2}{3}}\log\left(abx^{2}-\left(a^{2}b\right)^{\frac{2}{3}}x+\left(a^{2}b\right)^{\frac{1}{3}}a\right)+4\,\left(b^{2}c^{2}-2\,abcd+a^{2}d^{2}\right)\left(a^{2}b\right)^{\frac{2}{3}}\log\left(abx+\left(a^{2}b\right)^{\frac{2}{3}}\right)+12\,\left(2\,a^{2}b^{2}cd-a^{3}bd^{2}\right)x}{12\,a^{2}b^{3}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^2/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] [1/12*(3*a^2*b^2*d^2*x^4 + 6*sqrt(1/3)*(a*b^3*c^2 - 2*a^2*b^2*c*d + a^3*b*d
^2)*sqrt(-(a^2*b)^(1/3)/b)*log((2*a*b*x^3 - 3*(a^2*b)^(1/3)*a*x - a^2 + 3*s
qrt(1/3)*(2*a*b*x^2 + (a^2*b)^(2/3)*x - (a^2*b)^(1/3)*a)*sqrt(-(a^2*b)^(1/3
)/b))/(b*x^3 + a)) - 2*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*(a^2*b)^(2/3)*log(a*
b*x^2 - (a^2*b)^(2/3)*x + (a^2*b)^(1/3)*a) + 4*(b^2*c^2 - 2*a*b*c*d + a^2*d
^2)*(a^2*b)^(2/3)*log(a*b*x + (a^2*b)^(2/3)) + 12*(2*a^2*b^2*c*d - a^3*b*d^
2)*x)/(a^2*b^3), 1/12*(3*a^2*b^2*d^2*x^4 + 12*sqrt(1/3)*(a*b^3*c^2 - 2*a^2*
b^2*c*d + a^3*b*d^2)*sqrt((a^2*b)^(1/3)/b)*arctan(sqrt(1/3)*(2*(a^2*b)^(2/3
)*x - (a^2*b)^(1/3)*a)*sqrt((a^2*b)^(1/3)/b)/a^2) - 2*(b^2*c^2 - 2*a*b*c*d
+ a^2*d^2)*(a^2*b)^(2/3)*log(a*b*x^2 - (a^2*b)^(2/3)*x + (a^2*b)^(1/3)*a) +
 4*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*(a^2*b)^(2/3)*log(a*b*x + (a^2*b)^(2/3))
 + 12*(2*a^2*b^2*c*d - a^3*b*d^2)*x)/(a^2*b^3)]
```

giac [A] time = 0.32, size = 211, normalized size = 1.22

$$-\frac{\sqrt{3}\left(b^{2}c^{2}-2\,abcd+a^{2}d^{2}\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,\left(-ab^{2}\right)^{\frac{2}{3}}b}-\frac{\left(b^{2}c^{2}-2\,abcd+a^{2}d^{2}\right)\log\left(x^{2}+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,\left(-ab^{2}\right)^{\frac{2}{3}}b}-\frac{\left(b^{4}c^{2}-2\,ab^{3}cd+a^{2}b^{2}d^{2}\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,ab^{4}}+\frac{b^{3}d^{2}x^{4}+8\,b^{3}cdx-4\,ab^{2}d^{2}x}{4\,b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^2/(b*x^3+a),x, algorithm="giac")
```

```
[Out] -1/3*sqrt(3)*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*arctan(1/3*sqrt(3)*(2*x + (-a/
b)^(1/3))/(-a/b)^(1/3))/((-a*b^2)^(2/3)*b) - 1/6*(b^2*c^2 - 2*a*b*c*d + a^2
*d^2)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/((-a*b^2)^(2/3)*b) - 1/3*(b^
4*c^2 - 2*a*b^3*c*d + a^2*b^2*d^2)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/
(a*b^4) + 1/4*(b^3*d^2*x^4 + 8*b^3*c*d*x - 4*a*b^2*d^2*x)/b^4
```

maple [B] time = 0.04, size = 334, normalized size = 1.93

$$\frac{d^{2}x^{4}}{4b}+\frac{\sqrt{3}\,a^{2}d^{2}\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3\left(\frac{a}{b}\right)^{\frac{2}{3}}b^{3}}+\frac{a^{2}d^{2}\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{2}{3}}b^{3}}-\frac{a^{2}d^{2}\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(\frac{a}{b}\right)^{\frac{2}{3}}b^{3}}-\frac{2\sqrt{3}\,acd\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3\left(\frac{a}{b}\right)^{\frac{2}{3}}b^{2}}-\frac{2acd\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{2}{3}}b^{2}}+\frac{acd\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{2}{3}}b^{2}}-\frac{a\,d^{2}x}{b^{2}}+\frac{\sqrt{3}\,c^{2}\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3\left(\frac{a}{b}\right)^{\frac{2}{3}}b}+\frac{c^{2}\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{2}{3}}b}-\frac{c^{2}\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(\frac{a}{b}\right)^{\frac{2}{3}}b}+\frac{2cdx}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c)^2/(b*x^3+a),x)
```

```
[Out] 1/4*d^2*x^4/b-d^2/b^2*a*x+2*d/b*c*x+1/3/b^3/(a/b)^(2/3)*ln(x+(a/b)^(1/3))*a
^2*d^2-2/3/b^2/(a/b)^(2/3)*ln(x+(a/b)^(1/3))*a*c*d+1/3/b/(a/b)^(2/3)*ln(x+(
a/b)^(1/3))*c^2-1/6/b^3/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*a^2*d
^2+1/3/b^2/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*a*c*d-1/6/b/(a/b)^
(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*c^2+1/3/b^3/(a/b)^(2/3)*3^(1/2)*arc
tan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*a^2*d^2-2/3/b^2/(a/b)^(2/3)*3^(1/2)*ar
ctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*a*c*d+1/3/b/(a/b)^(2/3)*3^(1/2)*arcta
n(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*c^2
```

maxima [A] time = 1.32, size = 190, normalized size = 1.10

$$\frac{bd^{2}x^{4}+4\left(2\,bcd-ad^{2}\right)x}{4\,b^{2}}+\frac{\sqrt{3}\left(b^{2}c^{2}-2\,abcd+a^{2}d^{2}\right)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,b^{3}\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\left(b^{2}c^{2}-2\,abcd+a^{2}d^{2}\right)\log\left(x^{2}-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,b^{3}\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\left(b^{2}c^{2}-2\,abcd+a^{2}d^{2}\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,b^{3}\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^2/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] 1/4*(b*d^2*x^4 + 4*(2*b*c*d - a*d^2)*x)/b^2 + 1/3*sqrt(3)*(b^2*c^2 - 2*a*b*
c*d + a^2*d^2)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(b^3*(a/
b)^(2/3)) - 1/6*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*log(x^2 - x*(a/b)^(1/3) + (
a/b)^(2/3))/(b^3*(a/b)^(2/3)) + 1/3*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*log(x +
 (a/b)^(1/3))/(b^3*(a/b)^(2/3))
```

mupad [B] time = 1.38, size = 152, normalized size = 0.88

$$\frac{d^2\,x^4}{4\,b}-x\,\left(\frac{a\,d^2}{b^2}-\frac{2\,c\,d}{b}\right)+\frac{\ln\left(b^{1/3}\,x+a^{1/3}\right)\,{\left(a\,d-b\,c\right)}^2}{3\,a^{2/3}\,b^{7/3}}+\frac{\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1{}\mathrm{i}\right)\,\left(-\frac{1}{6}+\frac{\sqrt{3}\,1{}\mathrm{i}}{6}\right)\,{\left(a\,d-b\,c\right)}^2}{a^{2/3}\,b^{7/3}}-\frac{\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1{}\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,{\left(a\,d-b\,c\right)}^2}{3\,a^{2/3}\,b^{7/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3)^2/(a + b*x^3),x)
```

```
[Out] (d^2*x^4)/(4*b) - x*((a*d^2)/b^2 - (2*c*d)/b) + (log(b^(1/3)*x + a^(1/3))*(
a*d - b*c)^2)/(3*a^(2/3)*b^(7/3)) + (log(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*x -
 a^(1/3))*((3^(1/2)*1i)/6 - 1/6)*(a*d - b*c)^2)/(a^(2/3)*b^(7/3)) - (log(3^
(1/2)*a^(1/3)*1i - 2*b^(1/3)*x + a^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(a*d - b*c
)^2)/(3*a^(2/3)*b^(7/3))
```

sympy [A] time = 0.69, size = 156, normalized size = 0.90

$$x\left(-\frac{ad^2}{b^2}+\frac{2cd}{b}\right)+\operatorname{RootSum}\left(27t^3a^2b^7-a^6d^6+6a^5bcd^5-15a^4b^2c^2d^4+20a^3b^3c^3d^3-15a^2b^4c^4d^2+6ab^5c^5d-b^6c^6,\left(t\mapsto t\log\left(\frac{3tab^2}{a^2d^2-2abcd+b^2c^2}+x\right)\right)\right)+\frac{d^2x^4}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c)**2/(b*x**3+a),x)
```

```
[Out] x*(-a*d**2/b**2 + 2*c*d/b) + RootSum(27*_t**3*a**2*b**7 - a**6*d**6 + 6*a**
5*b*c*d**5 - 15*a**4*b**2*c**2*d**4 + 20*a**3*b**3*c**3*d**3 - 15*a**2*b**4
*c**4*d**2 + 6*a*b**5*c**5*d - b**6*c**6, Lambda(_t, _t*log(3*_t*a*b**2/(a*
*2*d**2 - 2*a*b*c*d + b**2*c**2) + x))) + d**2*x**4/(4*b)
```

3.17 $\int \frac{c+dx^3}{a+bx^3}\,dx$

Optimal. Leaf size=145

$$-\frac{(bc-ad)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{2/3}b^{4/3}}+\frac{(bc-ad)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}b^{4/3}}-\frac{(bc-ad)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{2/3}b^{4/3}}+\frac{dx}{b}$$

Rubi [A] time = 0.08, antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.412, Rules used = {388, 200, 31, 634, 617, 204, 628}

$$-\frac{(bc-ad)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{2/3}b^{4/3}}+\frac{(bc-ad)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}b^{4/3}}-\frac{(bc-ad)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{2/3}b^{4/3}}+\frac{dx}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3)/(a + b*x^3),x]
```

```
[Out] (d*x)/b - ((b*c - a*d)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(
Sqrt[3]*a^(2/3)*b^(4/3)) + ((b*c - a*d)*Log[a^(1/3) + b^(1/3)*x])/(3*a^(2/3
)*b^(4/3)) - ((b*c - a*d)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(
6*a^(2/3)*b^(4/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{c+dx^3}{a+bx^3}\,dx = \frac{dx}{b} - \frac{(-bc+ad)\int \frac{1}{a+bx^3}\,dx}{b}$$

$$= \frac{dx}{b} + \frac{(bc-ad)\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}\,x}\,dx}{3a^{2/3}b} + \frac{(bc-ad)\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}\,x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2}\,dx}{3a^{2/3}b}$$

$$= \frac{dx}{b} + \frac{(bc-ad)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3a^{2/3}b^{4/3}} - \frac{(bc-ad)\int \frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2}\,dx}{6a^{2/3}b^{4/3}} + \frac{(bc-ad)\int \frac{1}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+ \cdots}}{2\sqrt[3]{a}\,b}$$

$$= \frac{dx}{b} + \frac{(bc-ad)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3a^{2/3}b^{4/3}} - \frac{(bc-ad)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{6a^{2/3}b^{4/3}} + \frac{(bc-ad)\,\mathrm{Subst} \cdots}{\cdots}$$

$$= \frac{dx}{b} - \frac{(bc-ad)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{2/3}b^{4/3}} + \frac{(bc-ad)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3a^{2/3}b^{4/3}} - \frac{(bc-ad)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b} \cdots\right.}{6a^{2/3}b^{4/3}}$$

Mathematica [A] time = 0.08, size = 129, normalized size = 0.89

$$\frac{-(bc-ad)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)+6a^{2/3}\sqrt[3]{b}\,dx+2(bc-ad)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)-2\sqrt{3}\,(bc-ad)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{6a^{2/3}b^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^3)/(a + b*x^3),x]
```

```
[Out] (6*a^(2/3)*b^(1/3)*d*x - 2*Sqrt[3]*(b*c - a*d)*ArcTan[(1 - (2*b^(1/3)*x)/a^
(1/3))/Sqrt[3]] + 2*(b*c - a*d)*Log[a^(1/3) + b^(1/3)*x] - (b*c - a*d)*Log[
a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(2/3)*b^(4/3))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{c+dx^3}{a+bx^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x^3)/(a + b*x^3),x]
```

```
[Out] IntegrateAlgebraic[(c + d*x^3)/(a + b*x^3), x]
```

fricas [A] time = 0.96, size = 390, normalized size = 2.69

$$\left[\frac{6a^2bdx - 3\sqrt{\frac{1}{3}}\left(ab^2c-a^2bd\right)\sqrt{\frac{\left(-a^2b\right)^{\frac{1}{3}}}{b}}\log\left(\frac{2abx^3+3\left(-a^2b\right)^{\frac{1}{3}}ax-a^2-3\sqrt{\frac{1}{3}}\left(2abx^2+\left(-a^2b\right)^{\frac{2}{3}}x+\left(-a^2b\right)^{\frac{1}{3}}a\right)\sqrt{\frac{\left(-a^2b\right)^{\frac{1}{3}}}{b}}}{bx^3+a}\right)-\left(-a^2b\right)^{\frac{2}{3}}(bc-ad)\log\left(abx^2-\left(-a^2b\right)^{\frac{2}{3}}x-\left(-a^2b\right)^{\frac{1}{3}}a\right)+2\left(-a^2b\right)^{\frac{2}{3}}(bc-ad)\log\left(abx+\left(-a^2b\right)^{\frac{2}{3}}\right)}{6a^2b^2}, \frac{6a^2bdx+6\sqrt{\frac{1}{3}}\left(ab^2c-a^2bd\right)\sqrt{-\frac{\left(-a^2b\right)^{\frac{1}{3}}}{b}}\arctan\left(\frac{\sqrt{\frac{1}{3}}\left(2\left(-a^2b\right)^{\frac{2}{3}}x+\left(-a^2b\right)^{\frac{1}{3}}a\right)\sqrt{-\frac{\left(-a^2b\right)^{\frac{1}{3}}}{b}}}{a^2}\right)-\left(-a^2b\right)^{\frac{2}{3}}(bc-ad)\log\left(abx^2-\left(-a^2b\right)^{\frac{2}{3}}x-\left(-a^2b\right)^{\frac{1}{3}}a\right)+2\left(-a^2b\right)^{\frac{2}{3}}(bc-ad)\log\left(abx+\left(-a^2b\right)^{\frac{2}{3}}\right)}{6a^2b^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] [1/6*(6*a^2*b*d*x - 3*sqrt(1/3)*(a*b^2*c - a^2*b*d)*sqrt((-a^2*b)^(1/3)/b)*
log((2*a*b*x^3 + 3*(-a^2*b)^(1/3)*a*x - a^2 - 3*sqrt(1/3)*(2*a*b*x^2 + (-a^
2*b)^(2/3)*x + (-a^2*b)^(1/3)*a)*sqrt((-a^2*b)^(1/3)/b))/(b*x^3 + a)) - (-a
^2*b)^(2/3)*(b*c - a*d)*log(a*b*x^2 - (-a^2*b)^(2/3)*x - (-a^2*b)^(1/3)*a)
+ 2*(-a^2*b)^(2/3)*(b*c - a*d)*log(a*b*x + (-a^2*b)^(2/3)))/(a^2*b^2), 1/6*
(6*a^2*b*d*x + 6*sqrt(1/3)*(a*b^2*c - a^2*b*d)*sqrt(-(-a^2*b)^(1/3)/b)*arct
an(sqrt(1/3)*(2*(-a^2*b)^(2/3)*x + (-a^2*b)^(1/3)*a)*sqrt(-(-a^2*b)^(1/3)/b
)/a^2) - (-a^2*b)^(2/3)*(b*c - a*d)*log(a*b*x^2 - (-a^2*b)^(2/3)*x - (-a^2*
b)^(1/3)*a) + 2*(-a^2*b)^(2/3)*(b*c - a*d)*log(a*b*x + (-a^2*b)^(2/3)))/(a^
2*b^2)]
```

giac [A] time = 0.19, size = 133, normalized size = 0.92

$$-\frac{\sqrt{3}\,(bc-ad)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,\left(-ab^{2}\right)^{\frac{2}{3}}}-\frac{(bc-ad)\log\left(x^{2}+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,\left(-ab^{2}\right)^{\frac{2}{3}}}+\frac{dx}{b}-\frac{(bc-ad)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)/(b*x^3+a),x, algorithm="giac")
```

```
[Out] -1/3*sqrt(3)*(b*c - a*d)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/
3))/(-a*b^2)^(2/3) - 1/6*(b*c - a*d)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3
))/(-a*b^2)^(2/3) + d*x/b - 1/3*(b*c - a*d)*(-a/b)^(1/3)*log(abs(x - (-a/b)
^(1/3)))/(a*b)
```

maple [A] time = 0.05, size = 195, normalized size = 1.34

$$-\frac{\sqrt{3}\,ad\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3\left(\frac{a}{b}\right)^{\frac{2}{3}}b^{2}}-\frac{ad\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{2}{3}}b^{2}}+\frac{ad\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(\frac{a}{b}\right)^{\frac{2}{3}}b^{2}}+\frac{\sqrt{3}\,c\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3\left(\frac{a}{b}\right)^{\frac{2}{3}}b}+\frac{c\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{2}{3}}b}-\frac{c\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(\frac{a}{b}\right)^{\frac{2}{3}}b}+\frac{dx}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c)/(b*x^3+a),x)
```

```
[Out] d*x/b-1/3/b^2/(a/b)^(2/3)*ln(x+(a/b)^(1/3))*a*d+1/3/b/(a/b)^(2/3)*ln(x+(a/b
)^(1/3))*c+1/6/b^2/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*a*d-1/6/b/
(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*c-1/3/b^2/(a/b)^(2/3)*3^(1/2)
*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*a*d+1/3/b/(a/b)^(2/3)*3^(1/2)*arct
an(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*c
```

maxima [A] time = 1.08, size = 128, normalized size = 0.88

$$\frac{dx}{b}+\frac{\sqrt{3}\,(bc-ad)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,b^{2}\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{(bc-ad)\log\left(x^{2}-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,b^{2}\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{(bc-ad)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,b^{2}\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] d*x/b + 1/3*sqrt(3)*(b*c - a*d)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b
)^(1/3))/(b^2*(a/b)^(2/3)) - 1/6*(b*c - a*d)*log(x^2 - x*(a/b)^(1/3) + (a/b
)^(2/3))/(b^2*(a/b)^(2/3)) + 1/3*(b*c - a*d)*log(x + (a/b)^(1/3))/(b^2*(a/b
)^(2/3))
```

mupad [B] time = 1.38, size = 123, normalized size = 0.85

$$\frac{d\,x}{b}-\frac{\ln\left(b^{1/3}\,x+a^{1/3}\right)\,\left(a\,d-b\,c\right)}{3\,a^{2/3}\,b^{4/3}}+\frac{\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(a\,d-b\,c\right)}{3\,a^{2/3}\,b^{4/3}}-\frac{\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(a\,d-b\,c\right)}{3\,a^{2/3}\,b^{4/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3)/(a + b*x^3),x)
```

```
[Out] (d*x)/b - (log(b^(1/3)*x + a^(1/3))*(a*d - b*c))/(3*a^(2/3)*b^(4/3)) + (log
(3^(1/2)*a^(1/3)*1i - 2*b^(1/3)*x + a^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(a*d -
b*c))/(3*a^(2/3)*b^(4/3)) - (log(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*x - a^(1/3)
)*((3^(1/2)*1i)/2 - 1/2)*(a*d - b*c))/(3*a^(2/3)*b^(4/3))
```

sympy [A] time = 0.44, size = 71, normalized size = 0.49

$$\operatorname{RootSum}\left(27t^3a^2b^4+a^3d^3-3a^2bcd^2+3ab^2c^2d-b^3c^3,\left(t\mapsto t\log\left(-\frac{3tab}{ad-bc}+x\right)\right)\right)+\frac{dx}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c)/(b*x**3+a),x)
```

```
[Out] RootSum(27*_t**3*a**2*b**4 + a**3*d**3 - 3*a**2*b*c*d**2 + 3*a*b**2*c**2*d
- b**3*c**3, Lambda(_t, _t*log(-3*_t*a*b/(a*d - b*c) + x))) + d*x/b
```

3.18 $\int \frac{1}{(a+bx^3)(c+dx^3)}\,dx$

Optimal. Leaf size=288

$$-\frac{b^{2/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{2/3}(bc-ad)}+\frac{b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}(bc-ad)}-\frac{b^{2/3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{2/3}(bc-ad)}+\frac{d^{2/3}\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}\right.}{6c^{2/3}(bc-ad)}$$

Rubi [A] time = 0.15, antiderivative size = 288, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {391, 200, 31, 634, 617, 204, 628}

$$-\frac{b^{2/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{2/3}(bc-ad)}+\frac{b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}(bc-ad)}-\frac{b^{2/3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{2/3}(bc-ad)}+\frac{d^{2/3}\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{6c^{2/3}(bc-ad)}-\frac{d^{2/3}\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{2/3}(bc-ad)}+\frac{d^{2/3}\tan^{-1}\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{\sqrt{3}\sqrt[3]{c}}\right)}{\sqrt{3}\,c^{2/3}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^3)*(c + d*x^3)),x]
```

```
[Out] -((b^(2/3)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(2
/3)*(b*c - a*d))) + (d^(2/3)*ArcTan[(c^(1/3) - 2*d^(1/3)*x)/(Sqrt[3]*c^(1/3
))])/(Sqrt[3]*c^(2/3)*(b*c - a*d)) + (b^(2/3)*Log[a^(1/3) + b^(1/3)*x])/(3*
a^(2/3)*(b*c - a*d)) - (d^(2/3)*Log[c^(1/3) + d^(1/3)*x])/(3*c^(2/3)*(b*c -
 a*d)) - (b^(2/3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(2/3
)*(b*c - a*d)) + (d^(2/3)*Log[c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2])/(
6*c^(2/3)*(b*c - a*d))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 391

```
Int[1/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))), x_Symbol] :> Dis
t[b/(b*c - a*d), Int[1/(a + b*x^n), x], x] - Dist[d/(b*c - a*d), Int[1/(c +
 d*x^n), x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{1}{\left(a+bx^3\right)\left(c+dx^3\right)}\,dx = \frac{b\int \frac{1}{a+bx^3}\,dx}{bc-ad} - \frac{d\int \frac{1}{c+dx^3}\,dx}{bc-ad}$$

$$= \frac{b\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}\,x}\,dx}{3a^{2/3}(bc-ad)} + \frac{b\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}\,x}{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}\,dx}{3a^{2/3}(bc-ad)} - \frac{d\int \frac{1}{\sqrt[3]{c}+\sqrt[3]{d}\,x}\,dx}{3c^{2/3}(bc-ad)} - \frac{d\int \frac{2\sqrt[3]{c}-\sqrt[3]{d}\,x}{c^{2/3}-\sqrt[3]{c}\,\sqrt[3]{d}\,x+d^{2/3}}}{3c^{2/3}(bc-ad)}$$

$$= \frac{b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3a^{2/3}(bc-ad)} - \frac{d^{2/3}\log\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{3c^{2/3}(bc-ad)} - \frac{b^{2/3}\int \frac{-\sqrt[3]{a}\,\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}\,dx}{6a^{2/3}(bc-ad)} + \frac{b\int}{}$$

$$= \frac{b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3a^{2/3}(bc-ad)} - \frac{d^{2/3}\log\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{3c^{2/3}(bc-ad)} - \frac{b^{2/3}\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{6a^{2/3}(bc-ad)}$$

$$= -\frac{b^{2/3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{2/3}(bc-ad)} + \frac{d^{2/3}\tan^{-1}\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}\,x}{\sqrt{3}\,\sqrt[3]{c}}\right)}{\sqrt{3}\,c^{2/3}(bc-ad)} + \frac{b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3a^{2/3}(bc-ad)} - \frac{d^{2/3}\log}{3c^{2}}$$

Mathematica [A] time = 0.11, size = 224, normalized size = 0.78

$$\frac{\frac{b^{2/3}\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{a^{2/3}} - \frac{2b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{a^{2/3}} + \frac{2\sqrt{3}\,b^{2/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{a^{2/3}} - \frac{d^{2/3}\log\left(c^{2/3}-\sqrt[3]{c}\,\sqrt[3]{d}\,x+d^{2/3}x^2\right)}{c^{2/3}} + \frac{2d^{2/3}\log\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{c^{2/3}} - \frac{2\sqrt{3}\,d^{2/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\,x}{\sqrt[3]{c}}}{\sqrt{3}}\right)}{c^{2/3}}}{6ad-6bc}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x^3)*(c + d*x^3)),x]
```

```
[Out] ((2*Sqrt[3]*b^(2/3)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/a^(2/3) -
(2*Sqrt[3]*d^(2/3)*ArcTan[(1 - (2*d^(1/3)*x)/c^(1/3))/Sqrt[3]])/c^(2/3) - (
2*b^(2/3)*Log[a^(1/3) + b^(1/3)*x])/a^(2/3) + (2*d^(2/3)*Log[c^(1/3) + d^(1
/3)*x])/c^(2/3) + (b^(2/3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/
a^(2/3) - (d^(2/3)*Log[c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2])/c^(2/3))
/(-6*b*c + 6*a*d)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+bx^3\right)\left(c+dx^3\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/((a + b*x^3)*(c + d*x^3)),x]
```

```
[Out] IntegrateAlgebraic[1/((a + b*x^3)*(c + d*x^3)), x]
```

fricas [A] time = 1.17, size = 254, normalized size = 0.88

$$-\frac{2\sqrt{3}\left(-\frac{b^2}{a^2}\right)^{\frac13}\arctan\left(\frac{2\sqrt{3}ax\left(-\frac{b^2}{a^2}\right)^{\frac23}-\sqrt{3}b}{3b}\right)+2\sqrt{3}\left(\frac{d^2}{c^2}\right)^{\frac13}\arctan\left(\frac{2\sqrt{3}cx\left(\frac{d^2}{c^2}\right)^{\frac23}-\sqrt{3}d}{3d}\right)-\left(-\frac{b^2}{a^2}\right)^{\frac13}\log\left(b^2x^2+abx\left(-\frac{b^2}{a^2}\right)^{\frac13}+a^2\left(-\frac{b^2}{a^2}\right)^{\frac23}\right)-\left(\frac{d^2}{c^2}\right)^{\frac13}\log\left(d^2x^2-cdx\left(\frac{d^2}{c^2}\right)^{\frac13}+c^2\left(\frac{d^2}{c^2}\right)^{\frac23}\right)+2\left(-\frac{b^2}{a^2}\right)^{\frac13}\log\left(bx-a\left(-\frac{b^2}{a^2}\right)^{\frac13}\right)+2\left(\frac{d^2}{c^2}\right)^{\frac13}\log\left(dx+c\left(\frac{d^2}{c^2}\right)^{\frac13}\right)}{6\,(bc-ad)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] -1/6*(2*sqrt(3)*(-b^2/a^2)^(1/3)*arctan(1/3*(2*sqrt(3)*a*x*(-b^2/a^2)^(2/3)
 - sqrt(3)*b)/b) + 2*sqrt(3)*(d^2/c^2)^(1/3)*arctan(1/3*(2*sqrt(3)*c*x*(d^2
/c^2)^(2/3) - sqrt(3)*d)/d) - (-b^2/a^2)^(1/3)*log(b^2*x^2 + a*b*x*(-b^2/a^
2)^(1/3) + a^2*(-b^2/a^2)^(2/3)) - (d^2/c^2)^(1/3)*log(d^2*x^2 - c*d*x*(d^2
/c^2)^(1/3) + c^2*(d^2/c^2)^(2/3)) + 2*(-b^2/a^2)^(1/3)*log(b*x - a*(-b^2/a
^2)^(1/3)) + 2*(d^2/c^2)^(1/3)*log(d*x + c*(d^2/c^2)^(1/3)))/(b*c - a*d)
```

giac [A] time = 0.27, size = 278, normalized size = 0.97

$$-\frac{b\left(-\frac{a}{b}\right)^{\frac13}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac13}\right|\right)}{3\left(abc-a^2d\right)}+\frac{d\left(-\frac{c}{d}\right)^{\frac13}\log\left(\left|x-\left(-\frac{c}{d}\right)^{\frac13}\right|\right)}{3\left(bc^2-acd\right)}+\frac{\left(-ab^2\right)^{\frac13}\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac13}\right)}{3\left(-\frac{a}{b}\right)^{\frac13}}\right)}{\sqrt{3}abc-\sqrt{3}a^2d}-\frac{\left(-cd^2\right)^{\frac13}\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{c}{d}\right)^{\frac13}\right)}{3\left(-\frac{c}{d}\right)^{\frac13}}\right)}{\sqrt{3}bc^2-\sqrt{3}acd}+\frac{\left(-ab^2\right)^{\frac13}\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac13}+\left(-\frac{a}{b}\right)^{\frac23}\right)}{6\left(abc-a^2d\right)}-\frac{\left(-cd^2\right)^{\frac13}\log\left(x^2+x\left(-\frac{c}{d}\right)^{\frac13}+\left(-\frac{c}{d}\right)^{\frac23}\right)}{6\left(bc^2-acd\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] -1/3*b*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a*b*c - a^2*d) + 1/3*d*(-c/
d)^(1/3)*log(abs(x - (-c/d)^(1/3)))/(b*c^2 - a*c*d) + (-a*b^2)^(1/3)*arctan
(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(sqrt(3)*a*b*c - sqrt(3)*a^
2*d) - (-c*d^2)^(1/3)*arctan(1/3*sqrt(3)*(2*x + (-c/d)^(1/3))/(-c/d)^(1/3))
/(sqrt(3)*b*c^2 - sqrt(3)*a*c*d) + 1/6*(-a*b^2)^(1/3)*log(x^2 + x*(-a/b)^(1
/3) + (-a/b)^(2/3))/(a*b*c - a^2*d) - 1/6*(-c*d^2)^(1/3)*log(x^2 + x*(-c/d)
^(1/3) + (-c/d)^(2/3))/(b*c^2 - a*c*d)
```

maple [A] time = 0.05, size = 222, normalized size = 0.77

$$-\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac13}}-1\right)}{3}\right)}{3\,(ad-bc)\left(\frac{a}{b}\right)^{\frac23}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{c}{d}\right)^{\frac13}}-1\right)}{3}\right)}{3\,(ad-bc)\left(\frac{c}{d}\right)^{\frac23}}-\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac13}\right)}{3\,(ad-bc)\left(\frac{a}{b}\right)^{\frac23}}+\frac{\ln\left(x+\left(\frac{c}{d}\right)^{\frac13}\right)}{3\,(ad-bc)\left(\frac{c}{d}\right)^{\frac23}}+\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac13}x+\left(\frac{a}{b}\right)^{\frac23}\right)}{6\,(ad-bc)\left(\frac{a}{b}\right)^{\frac23}}-\frac{\ln\left(x^2-\left(\frac{c}{d}\right)^{\frac13}x+\left(\frac{c}{d}\right)^{\frac23}\right)}{6\,(ad-bc)\left(\frac{c}{d}\right)^{\frac23}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^3+a)/(d*x^3+c),x)
```

```
[Out] -1/3/(a*d-b*c)/(a/b)^(2/3)*ln(x+(a/b)^(1/3))+1/6/(a*d-b*c)/(a/b)^(2/3)*ln(x
^2-(a/b)^(1/3)*x+(a/b)^(2/3))-1/3/(a*d-b*c)/(a/b)^(2/3)*3^(1/2)*arctan(1/3*
3^(1/2)*(2/(a/b)^(1/3)*x-1))+1/3/(a*d-b*c)/(c/d)^(2/3)*ln(x+(c/d)^(1/3))-1/
6/(a*d-b*c)/(c/d)^(2/3)*ln(x^2-(c/d)^(1/3)*x+(c/d)^(2/3))+1/3/(a*d-b*c)/(c/
d)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(c/d)^(1/3)*x-1))
```

maxima [A] time = 1.24, size = 293, normalized size = 1.02

$$\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac13}\right)}{3\left(\frac{a}{b}\right)^{\frac13}}\right)}{3\left(bc\left(\frac{a}{b}\right)^{\frac13}-ad\left(\frac{a}{b}\right)^{\frac13}\right)\left(\frac{a}{b}\right)^{\frac13}}-\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{c}{d}\right)^{\frac13}\right)}{3\left(\frac{c}{d}\right)^{\frac13}}\right)}{3\left(bc\left(\frac{c}{d}\right)^{\frac13}-ad\left(\frac{c}{d}\right)^{\frac13}\right)\left(\frac{c}{d}\right)^{\frac13}}-\frac{\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac13}+\left(\frac{a}{b}\right)^{\frac23}\right)}{6\left(bc\left(\frac{a}{b}\right)^{\frac23}-ad\left(\frac{a}{b}\right)^{\frac23}\right)}+\frac{\log\left(x^2-x\left(\frac{c}{d}\right)^{\frac13}+\left(\frac{c}{d}\right)^{\frac23}\right)}{6\left(bc\left(\frac{c}{d}\right)^{\frac23}-ad\left(\frac{c}{d}\right)^{\frac23}\right)}+\frac{\log\left(x+\left(\frac{a}{b}\right)^{\frac13}\right)}{3\left(bc\left(\frac{a}{b}\right)^{\frac23}-ad\left(\frac{a}{b}\right)^{\frac23}\right)}-\frac{\log\left(x+\left(\frac{c}{d}\right)^{\frac13}\right)}{3\left(bc\left(\frac{c}{d}\right)^{\frac23}-ad\left(\frac{c}{d}\right)^{\frac23}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/((b*c*(a/b)
^(1/3) - a*d*(a/b)^(1/3))*(a/b)^(1/3)) - 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*
x - (c/d)^(1/3))/(c/d)^(1/3))/((b*c*(c/d)^(1/3) - a*d*(c/d)^(1/3))*(c/d)^(1
/3)) - 1/6*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(b*c*(a/b)^(2/3) - a*d*(a
/b)^(2/3)) + 1/6*log(x^2 - x*(c/d)^(1/3) + (c/d)^(2/3))/(b*c*(c/d)^(2/3) -
a*d*(c/d)^(2/3)) + 1/3*log(x + (a/b)^(1/3))/(b*c*(a/b)^(2/3) - a*d*(a/b)^(2
/3)) - 1/3*log(x + (c/d)^(1/3))/(b*c*(c/d)^(2/3) - a*d*(c/d)^(2/3))
```

mupad [B] time = 7.70, size = 1364, normalized size = 4.74

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^3)*(c + d*x^3)),x)
```

```
[Out] log(((-b^2/(a^2*(a*d - b*c)^3))^(1/3)*(9*a^2*b^4*d^6 + 9*b^6*c^2*d^4 - 18*a
*b^5*c*d^5 - 9*b^3*d^3*(x + a*c*(-b^2/(a^2*(a*d - b*c)^3))^(1/3))*(a*d + b*
c)*(a*d - b*c)^4*(-b^2/(a^2*(a*d - b*c)^3))^(2/3)))/3 - 6*b^5*d^5*x)*(-b^2/
(27*a^5*d^3 - 27*a^2*b^3*c^3 + 81*a^3*b^2*c^2*d - 81*a^4*b*c*d^2))^(1/3) +
log(((d^2/(c^2*(a*d - b*c)^3))^(1/3)*(9*a^2*b^4*d^6 + 9*b^6*c^2*d^4 - 18*a*
b^5*c*d^5 - 9*b^3*d^3*(x + a*c*(d^2/(c^2*(a*d - b*c)^3))^(1/3))*(a*d + b*c)
*(a*d - b*c)^4*(d^2/(c^2*(a*d - b*c)^3))^(2/3)))/3 - 6*b^5*d^5*x)*(-d^2/(27
*b^3*c^5 - 27*a^3*c^2*d^3 + 81*a^2*b*c^3*d^2 - 81*a*b^2*c^4*d))^(1/3) + (lo
g(6*b^5*d^5*x + ((3^(1/2)*1i - 1)*(-b^2/(a^2*(a*d - b*c)^3))^(1/3)*(((3^(1/
2)*1i - 1)^2*(81*b^3*d^3*x*(a*d + b*c)*(a*d - b*c)^4 + (81*a*b^3*c*d^3*(3^(
1/2)*1i - 1)*(a*d + b*c)*(a*d - b*c)^4*(-b^2/(a^2*(a*d - b*c)^3))^(1/3))/2)
*(-b^2/(a^2*(a*d - b*c)^3))^(2/3))/36 - 9*a^2*b^4*d^6 - 9*b^6*c^2*d^4 + 18*
a*b^5*c*d^5))/6)*(-b^2/(27*a^5*d^3 - 27*a^2*b^3*c^3 + 81*a^3*b^2*c^2*d - 81
*a^4*b*c*d^2))^(1/3)*(3^(1/2)*1i - 1))/2 - (log(6*b^5*d^5*x - ((3^(1/2)*1i
+ 1)*(-b^2/(a^2*(a*d - b*c)^3))^(1/3)*(((3^(1/2)*1i + 1)^2*(81*b^3*d^3*x*(a
*d + b*c)*(a*d - b*c)^4 - (81*a*b^3*c*d^3*(3^(1/2)*1i + 1)*(a*d + b*c)*(a*d
 - b*c)^4*(-b^2/(a^2*(a*d - b*c)^3))^(1/3))/2)*(-b^2/(a^2*(a*d - b*c)^3))^(
2/3))/36 - 9*a^2*b^4*d^6 - 9*b^6*c^2*d^4 + 18*a*b^5*c*d^5))/6)*(-b^2/(27*a^
5*d^3 - 27*a^2*b^3*c^3 + 81*a^3*b^2*c^2*d - 81*a^4*b*c*d^2))^(1/3)*(3^(1/2)
*1i + 1))/2 + (log(6*b^5*d^5*x + ((3^(1/2)*1i - 1)*(d^2/(c^2*(a*d - b*c)^3)
)^(1/3)*(((3^(1/2)*1i - 1)^2*(81*b^3*d^3*x*(a*d + b*c)*(a*d - b*c)^4 + (81*
a*b^3*c*d^3*(3^(1/2)*1i - 1)*(a*d + b*c)*(a*d - b*c)^4*(d^2/(c^2*(a*d - b*c
)^3))^(1/3))/2)*(d^2/(c^2*(a*d - b*c)^3))^(2/3))/36 - 9*a^2*b^4*d^6 - 9*b^6
*c^2*d^4 + 18*a*b^5*c*d^5))/6)*(-d^2/(27*b^3*c^5 - 27*a^3*c^2*d^3 + 81*a^2*
b*c^3*d^2 - 81*a*b^2*c^4*d))^(1/3)*(3^(1/2)*1i - 1))/2 - (log(6*b^5*d^5*x -
 ((3^(1/2)*1i + 1)*(d^2/(c^2*(a*d - b*c)^3))^(1/3)*(((3^(1/2)*1i + 1)^2*(81
*b^3*d^3*x*(a*d + b*c)*(a*d - b*c)^4 - (81*a*b^3*c*d^3*(3^(1/2)*1i + 1)*(a*
d + b*c)*(a*d - b*c)^4*(d^2/(c^2*(a*d - b*c)^3))^(1/3))/2)*(d^2/(c^2*(a*d -
 b*c)^3))^(2/3))/36 - 9*a^2*b^4*d^6 - 9*b^6*c^2*d^4 + 18*a*b^5*c*d^5))/6)*(
-d^2/(27*b^3*c^5 - 27*a^3*c^2*d^3 + 81*a^2*b*c^3*d^2 - 81*a*b^2*c^4*d))^(1/
3)*(3^(1/2)*1i + 1))/2
```

sympy [A] time = 79.72, size = 447, normalized size = 1.55

$$\operatorname{RootSum}{\left(t^{3} \left(27 a^{5} d^{3} - 81 a^{4} b c d^{2} + 81 a^{3} b^{2} c^{2} d - 27 a^{2} b^{3} c^{3}\right) + b^{2}, \left(t \mapsto t \log{\left(x + \frac{81 t^{4} a^{7} c^{2} d^{5} - 243 t^{4} a^{6} b c^{3} d^{4} + 162 t^{4} a^{5} b^{2} c^{4} d^{3} + 162 t^{4} a^{4} b^{3} c^{5} d^{2} - 243 t^{4} a^{3} b^{4} c^{6} d + 81 t^{4} a^{2} b^{5} c^{7} - 3 t a^{4} d^{4} + 3 t a^{3} b c d^{3} + 3 t a b^{3} c^{3} d - 3 t b^{4} c^{4}}{a^{2} b d^{3} + b^{3} c^{2} d} \right)} \right)\right)} + \operatorname{RootSum}{\left(t^{3} \left(27 a^{3} c^{2} d^{3} - 81 a^{2} b c^{3} d^{2} + 81 a b^{2} c^{4} d - 27 b^{3} c^{5}\right) - d^{2}, \left(t \mapsto t \log{\left(x + \frac{81 t^{4} a^{7} c^{2} d^{5} - 243 t^{4} a^{6} b c^{3} d^{4} + 162 t^{4} a^{5} b^{2} c^{4} d^{3} + 162 t^{4} a^{4} b^{3} c^{5} d^{2} - 243 t^{4} a^{3} b^{4} c^{6} d + 81 t^{4} a^{2} b^{5} c^{7} - 3 t a^{4} d^{4} + 3 t a^{3} b c d^{3} + 3 t a b^{3} c^{3} d - 3 t b^{4} c^{4}}{a^{2} b d^{3} + b^{3} c^{2} d} \right)} \right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**3+a)/(d*x**3+c),x)
```

```
[Out] RootSum(_t**3*(27*a**5*d**3 - 81*a**4*b*c*d**2 + 81*a**3*b**2*c**2*d - 27*a
**2*b**3*c**3) + b**2, Lambda(_t, _t*log(x + (81*_t**4*a**7*c**2*d**5 - 243
*_t**4*a**6*b*c**3*d**4 + 162*_t**4*a**5*b**2*c**4*d**3 + 162*_t**4*a**4*b*
*3*c**5*d**2 - 243*_t**4*a**3*b**4*c**6*d + 81*_t**4*a**2*b**5*c**7 - 3*_t*
a**4*d**4 + 3*_t*a**3*b*c*d**3 + 3*_t*a*b**3*c**3*d - 3*_t*b**4*c**4)/(a**2
```

```
*b*d**3 + b**3*c**2*d)))) + RootSum(_t**3*(27*a**3*c**2*d**3 - 81*a**2*b*c*
*3*d**2 + 81*a*b**2*c**4*d - 27*b**3*c**5) - d**2, Lambda(_t, _t*log(x + (8
1*_t**4*a**7*c**2*d**5 - 243*_t**4*a**6*b*c**3*d**4 + 162*_t**4*a**5*b**2*c
**4*d**3 + 162*_t**4*a**4*b**3*c**5*d**2 - 243*_t**4*a**3*b**4*c**6*d + 81*
_t**4*a**2*b**5*c**7 - 3*_t*a**4*d**4 + 3*_t*a**3*b*c*d**3 + 3*_t*a*b**3*c*
*3*d - 3*_t*b**4*c**4)/(a**2*b*d**3 + b**3*c**2*d))))
```

3.19 $\int \frac{1}{(a+bx^3)(c+dx^3)^2}\,dx$

Optimal. Leaf size=346

$$-\frac{b^{5/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{6a^{2/3}(bc-ad)^2}+\frac{b^{5/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3a^{2/3}(bc-ad)^2}-\frac{b^{5/3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{2/3}(bc-ad)^2}+\frac{d^{2/3}(5bc-2ad)\log\left(c^{2/3}-\right.}{18c^{5/3}(bc-}$$

Rubi [A] time = 0.27, antiderivative size = 346, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.421, Rules used = {414, 522, 200, 31, 634, 617, 204, 628}

$$-\frac{b^{5/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{6a^{2/3}(bc-ad)^2}+\frac{b^{5/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3a^{2/3}(bc-ad)^2}-\frac{b^{5/3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{2/3}(bc-ad)^2}+\frac{d^{2/3}(5bc-2ad)\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}\,x+d^{2/3}x^2\right)}{18c^{5/3}(bc-ad)^2}-\frac{d^{2/3}(5bc-2ad)\log\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{9c^{5/3}(bc-ad)^2}+\frac{d^{2/3}(5bc-2ad)\tan^{-1}\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}\,x}{\sqrt{3}\sqrt[3]{c}}\right)}{3\sqrt{3}\,c^{5/3}(bc-ad)^2}-\frac{dx}{3c\left(c+dx^3\right)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^3)*(c + d*x^3)^2),x]
```

```
[Out] -(d*x)/(3*c*(b*c - a*d)*(c + d*x^3)) - (b^(5/3)*ArcTan[(a^(1/3) - 2*b^(1/3)
*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(2/3)*(b*c - a*d)^2) + (d^(2/3)*(5*b*c -
 2*a*d)*ArcTan[(c^(1/3) - 2*d^(1/3)*x)/(Sqrt[3]*c^(1/3))])/(3*Sqrt[3]*c^(5/
3)*(b*c - a*d)^2) + (b^(5/3)*Log[a^(1/3) + b^(1/3)*x])/(3*a^(2/3)*(b*c - a*
d)^2) - (d^(2/3)*(5*b*c - 2*a*d)*Log[c^(1/3) + d^(1/3)*x])/(9*c^(5/3)*(b*c
- a*d)^2) - (b^(5/3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(
2/3)*(b*c - a*d)^2) + (d^(2/3)*(5*b*c - 2*a*d)*Log[c^(2/3) - c^(1/3)*d^(1/3
)*x + d^(2/3)*x^2])/(18*c^(5/3)*(b*c - a*d)^2)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
```

```
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{\left(a+bx^3\right)\left(c+dx^3\right)^2}\,dx &= -\frac{dx}{3c(bc-ad)\left(c+dx^3\right)}+\frac{\int \frac{3bc-2ad-2bdx^3}{\left(a+bx^3\right)\left(c+dx^3\right)}\,dx}{3c(bc-ad)} \\
&= -\frac{dx}{3c(bc-ad)\left(c+dx^3\right)}+\frac{b^2\int \frac{1}{a+bx^3}\,dx}{(bc-ad)^2}-\frac{(d(5bc-2ad))\int \frac{1}{c+dx^3}\,dx}{3c(bc-ad)^2} \\
&= -\frac{dx}{3c(bc-ad)\left(c+dx^3\right)}+\frac{b^2\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{3a^{2/3}(bc-ad)^2}+\frac{b^2\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{3a^{2/3}(bc-ad)^2}-\frac{(d(5bc-2a}{9c^{5/3}} \\
&= -\frac{dx}{3c(bc-ad)\left(c+dx^3\right)}+\frac{b^{5/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}(bc-ad)^2}-\frac{d^{2/3}(5bc-2ad)\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{9c^{5/3}(bc-ad)^2} \\
&= -\frac{dx}{3c(bc-ad)\left(c+dx^3\right)}+\frac{b^{5/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}(bc-ad)^2}-\frac{d^{2/3}(5bc-2ad)\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{9c^{5/3}(bc-ad)^2} \\
&= -\frac{dx}{3c(bc-ad)\left(c+dx^3\right)}-\frac{b^{5/3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}(bc-ad)^2}+\frac{d^{2/3}(5bc-2ad)\tan^{-1}\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{\sqrt{3}\sqrt[3]{c}}\right)}{3\sqrt{3}c^{5/3}(bc-ad)^2}
\end{aligned}
$$

Mathematica [A] time = 0.24, size = 336, normalized size = 0.97

$$
\frac{-3b^{5/3}c^{5/3}\left(c+dx^3\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)+a^{2/3}d^{2/3}\left(c+dx^3\right)(5bc-2ad)\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)+6a^{2/3}c^{2/3}dx(ad-bc)+2a^{2/3}d^{2/3}\left(c+dx^3\right)(2ad-5bc)\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)-2\sqrt{3}a^{2/3}d^{2/3}\left(c+dx^3\right)(2ad-5bc)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}}{\sqrt{3}}\right)+6b^{5/3}c^{5/3}\left(c+dx^3\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)-6\sqrt{3}b^{5/3}c^{5/3}\left(c+dx^3\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{18a^{2/3}c^{5/3}\left(c+dx^3\right)(bc-ad)^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x^3)*(c + d*x^3)^2),x]
```

```
[Out] (6*a^(2/3)*c^(2/3)*d*(-(b*c) + a*d)*x - 6*Sqrt[3]*b^(5/3)*c^(5/3)*(c + d*x^
3)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] - 2*Sqrt[3]*a^(2/3)*d^(2/3)*
```

```
(-5*b*c + 2*a*d)*(c + d*x^3)*ArcTan[(1 - (2*d^(1/3)*x)/c^(1/3))/Sqrt[3]] +
6*b^(5/3)*c^(5/3)*(c + d*x^3)*Log[a^(1/3) + b^(1/3)*x] + 2*a^(2/3)*d^(2/3)*
(-5*b*c + 2*a*d)*(c + d*x^3)*Log[c^(1/3) + d^(1/3)*x] - 3*b^(5/3)*c^(5/3)*(
c + d*x^3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2] + a^(2/3)*d^(2/3)
*(5*b*c - 2*a*d)*(c + d*x^3)*Log[c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2]
)/(18*a^(2/3)*c^(5/3)*(b*c - a*d)^2*(c + d*x^3))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+bx^3\right)\left(c+dx^3\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/((a + b*x^3)*(c + d*x^3)^2),x]
```

```
[Out] IntegrateAlgebraic[1/((a + b*x^3)*(c + d*x^3)^2), x]
```

fricas [A] time = 18.48, size = 432, normalized size = 1.25

$$\frac{6\sqrt{3}\left(bcdx^3+bc^2\right)\left(\frac{b^2}{a^2}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}ax\left(\frac{b^2}{a^2}\right)^{\frac{2}{3}}-\sqrt{3}b}{3b}\right)-2\sqrt{3}\left(\left(5bcd-2ad^2\right)x^3+5bc^2-2acd\right)\left(\frac{d^2}{c^2}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}cx\left(\frac{d^2}{c^2}\right)^{\frac{2}{3}}-\sqrt{3}d}{3d}\right)-3\left(bcdx^3+bc^2\right)\left(\frac{b^2}{a^2}\right)^{\frac{1}{3}}\log\left(b^2x^2-abx\left(\frac{b^2}{a^2}\right)^{\frac{1}{3}}+a^2\left(\frac{b^2}{a^2}\right)^{\frac{2}{3}}\right)+\left(\left(5bcd-2ad^2\right)x^3+5bc^2-2acd\right)\left(\frac{d^2}{c^2}\right)^{\frac{1}{3}}\log\left(d^2x^2-cdx\left(\frac{d^2}{c^2}\right)^{\frac{1}{3}}+c^2\left(\frac{d^2}{c^2}\right)^{\frac{2}{3}}\right)+6\left(bcdx^3+bc^2\right)\left(\frac{b^2}{a^2}\right)^{\frac{1}{3}}\log\left(bx+a\left(\frac{b^2}{a^2}\right)^{\frac{1}{3}}\right)-2\left(\left(5bcd-2ad^2\right)x^3+5bc^2-2acd\right)\left(\frac{d^2}{c^2}\right)^{\frac{1}{3}}\log\left(dx+c\left(\frac{d^2}{c^2}\right)^{\frac{1}{3}}\right)-6\left(bcd-ad^2\right)x}{18\left(b^2c^4-2abc^3d+a^2c^2d^2+\left(b^2c^3d-2abc^2d^2+a^2cd^3\right)x^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)/(d*x^3+c)^2,x, algorithm="fricas")
```

```
[Out] 1/18*(6*sqrt(3)*(b*c*d*x^3 + b*c^2)*(b^2/a^2)^(1/3)*arctan(1/3*(2*sqrt(3)*a
*x*(b^2/a^2)^(2/3) - sqrt(3)*b)/b) - 2*sqrt(3)*((5*b*c*d - 2*a*d^2)*x^3 + 5
*b*c^2 - 2*a*c*d)*(d^2/c^2)^(1/3)*arctan(1/3*(2*sqrt(3)*c*x*(d^2/c^2)^(2/3)
 - sqrt(3)*d)/d) - 3*(b*c*d*x^3 + b*c^2)*(b^2/a^2)^(1/3)*log(b^2*x^2 - a*b*
x*(b^2/a^2)^(1/3) + a^2*(b^2/a^2)^(2/3)) + ((5*b*c*d - 2*a*d^2)*x^3 + 5*b*c
^2 - 2*a*c*d)*(d^2/c^2)^(1/3)*log(d^2*x^2 - c*d*x*(d^2/c^2)^(1/3) + c^2*(d^
2/c^2)^(2/3)) + 6*(b*c*d*x^3 + b*c^2)*(b^2/a^2)^(1/3)*log(b*x + a*(b^2/a^2)
^(1/3)) - 2*((5*b*c*d - 2*a*d^2)*x^3 + 5*b*c^2 - 2*a*c*d)*(d^2/c^2)^(1/3)*l
og(d*x + c*(d^2/c^2)^(1/3)) - 6*(b*c*d - a*d^2)*x)/(b^2*c^4 - 2*a*b*c^3*d +
 a^2*c^2*d^2 + (b^2*c^3*d - 2*a*b*c^2*d^2 + a^2*c*d^3)*x^3)
```

giac [A] time = 0.20, size = 443, normalized size = 1.28

$$-\frac{b^2\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\left(ab^2c^2-2a^2bcd+a^3d^2\right)}+\frac{\left(-ab^2\right)^{\frac{1}{3}}b\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}ab^2c^2-2\sqrt{3}a^2bcd+\sqrt{3}a^3d^2}+\frac{\left(-ab^2\right)^{\frac{1}{3}}b\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(ab^2c^2-2a^2bcd+a^3d^2\right)}+\frac{\left(5bcd-2ad^2\right)\left(-\frac{c}{d}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right|\right)}{9\left(b^2c^4-2abc^3d+a^2c^2d^2\right)}-\frac{\left(5\left(-cd^2\right)^{\frac{1}{3}}bc-2\left(-cd^2\right)^{\frac{1}{3}}ad\right)\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{3\left(\sqrt{3}b^2c^4-2\sqrt{3}abc^3d+\sqrt{3}a^2c^2d^2\right)}-\frac{\left(5\left(-cd^2\right)^{\frac{1}{3}}bc-2\left(-cd^2\right)^{\frac{1}{3}}ad\right)\log\left(x^2+x\left(-\frac{c}{d}\right)^{\frac{1}{3}}+\left(-\frac{c}{d}\right)^{\frac{2}{3}}\right)}{18\left(b^2c^4-2abc^3d+a^2c^2d^2\right)}-\frac{dx}{3\left(dx^3+c\right)\left(bc^2-acd\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)/(d*x^3+c)^2,x, algorithm="giac")
```

```
[Out] -1/3*b^2*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a*b^2*c^2 - 2*a^2*b*c*d +
 a^3*d^2) + (-a*b^2)^(1/3)*b*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)
^(1/3))/(sqrt(3)*a*b^2*c^2 - 2*sqrt(3)*a^2*b*c*d + sqrt(3)*a^3*d^2) + 1/6*(
-a*b^2)^(1/3)*b*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a*b^2*c^2 - 2*a^2
*b*c*d + a^3*d^2) + 1/9*(5*b*c*d - 2*a*d^2)*(-c/d)^(1/3)*log(abs(x - (-c/d)
^(1/3)))/(b^2*c^4 - 2*a*b*c^3*d + a^2*c^2*d^2) - 1/3*(5*(-c*d^2)^(1/3)*b*c
- 2*(-c*d^2)^(1/3)*a*d)*arctan(1/3*sqrt(3)*(2*x + (-c/d)^(1/3))/(-c/d)^(1/3
))/(sqrt(3)*b^2*c^4 - 2*sqrt(3)*a*b*c^3*d + sqrt(3)*a^2*c^2*d^2) - 1/18*(5*
(-c*d^2)^(1/3)*b*c - 2*(-c*d^2)^(1/3)*a*d)*log(x^2 + x*(-c/d)^(1/3) + (-c/d
)^(2/3))/(b^2*c^4 - 2*a*b*c^3*d + a^2*c^2*d^2) - 1/3*d*x/((d*x^3 + c)*(b*c^
2 - a*c*d))
```

maple [A] time = 0.05, size = 406, normalized size = 1.17

$$\frac{ad^2x}{3\left(ad-bc\right)^2\left(dx^3+c\right)c}-\frac{bdx}{3\left(ad-bc\right)^2\left(dx^3+c\right)}+\frac{2\sqrt{3}\,ad\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{c}{d}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{9\left(ad-bc\right)^2\left(\frac{c}{d}\right)^{\frac{2}{3}}c}+\frac{2ad\ln\left(x+\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{9\left(ad-bc\right)^2\left(\frac{c}{d}\right)^{\frac{2}{3}}c}-\frac{ad\ln\left(x^2-\left(\frac{c}{d}\right)^{\frac{1}{3}}x+\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}{9\left(ad-bc\right)^2\left(\frac{c}{d}\right)^{\frac{2}{3}}c}+\frac{\sqrt{3}\,b\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3\left(ad-bc\right)^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{5\sqrt{3}\,b\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{c}{d}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{9\left(ad-bc\right)^2\left(\frac{c}{d}\right)^{\frac{2}{3}}}+\frac{b\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(ad-bc\right)^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{5b\ln\left(x+\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{9\left(ad-bc\right)^2\left(\frac{c}{d}\right)^{\frac{2}{3}}}-\frac{b\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(ad-bc\right)^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{5b\ln\left(x^2-\left(\frac{c}{d}\right)^{\frac{1}{3}}x+\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}{18\left(ad-bc\right)^2\left(\frac{c}{d}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^3+a)/(d*x^3+c)^2,x)
```

```
[Out] 1/3*b/(a*d-b*c)^2/(a/b)^(2/3)*ln(x+(a/b)^(1/3))-1/6*b/(a*d-b*c)^2/(a/b)^(2/
3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3*b/(a*d-b*c)^2/(a/b)^(2/3)*3^(1/2)*
arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))+1/3*d^2/(a*d-b*c)^2/c*x/(d*x^3+c)*a
-1/3*d/(a*d-b*c)^2*x/(d*x^3+c)*b+2/9*d/(a*d-b*c)^2/c/(c/d)^(2/3)*ln(x+(c/d)
^(1/3))*a-5/9/(a*d-b*c)^2/(c/d)^(2/3)*ln(x+(c/d)^(1/3))*b-1/9*d/(a*d-b*c)^2
/c/(c/d)^(2/3)*ln(x^2-(c/d)^(1/3)*x+(c/d)^(2/3))*a+5/18/(a*d-b*c)^2/(c/d)^(
2/3)*ln(x^2-(c/d)^(1/3)*x+(c/d)^(2/3))*b+2/9*d/(a*d-b*c)^2/c/(c/d)^(2/3)*3^
(1/2)*arctan(1/3*3^(1/2)*(2/(c/d)^(1/3)*x-1))*a-5/9/(a*d-b*c)^2/(c/d)^(2/3)
*3^(1/2)*arctan(1/3*3^(1/2)*(2/(c/d)^(1/3)*x-1))*b
```

maxima [A] time = 1.30, size = 489, normalized size = 1.41

$$\frac{\sqrt{3}\, b \arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\left(b^{2}c^{2}\left(\frac{a}{b}\right)^{\frac{1}{3}}-2\,abcd\left(\frac{a}{b}\right)^{\frac{1}{3}}+a^{2}d^{2}\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)\left(\frac{a}{b}\right)^{\frac{1}{3}}}-\frac{\sqrt{3}\,(5\,bc-2\,ad)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{9\left(b^{2}c^{3}\left(\frac{c}{d}\right)^{\frac{1}{3}}-2\,abc^{2}d\left(\frac{c}{d}\right)^{\frac{1}{3}}+a^{2}cd^{2}\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)\left(\frac{c}{d}\right)^{\frac{1}{3}}}-\frac{dx}{3\left(bc^{3}-ac^{2}d+\left(bc^{2}d-acd^{2}\right)x^{3}\right)}-\frac{b\log\left(x^{2}-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(b^{2}c^{2}\left(\frac{a}{b}\right)^{\frac{2}{3}}-2\,abcd\left(\frac{a}{b}\right)^{\frac{2}{3}}+a^{2}d^{2}\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}+\frac{(5\,bc-2\,ad)\log\left(x^{2}-x\left(\frac{c}{d}\right)^{\frac{1}{3}}+\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}{18\left(b^{2}c^{3}\left(\frac{c}{d}\right)^{\frac{2}{3}}-2\,abc^{2}d\left(\frac{c}{d}\right)^{\frac{2}{3}}+a^{2}cd^{2}\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}+\frac{b\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(b^{2}c^{2}\left(\frac{a}{b}\right)^{\frac{2}{3}}-2\,abcd\left(\frac{a}{b}\right)^{\frac{2}{3}}+a^{2}d^{2}\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}-\frac{(5\,bc-2\,ad)\log\left(x+\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{9\left(b^{2}c^{3}\left(\frac{c}{d}\right)^{\frac{2}{3}}-2\,abc^{2}d\left(\frac{c}{d}\right)^{\frac{2}{3}}+a^{2}cd^{2}\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)/(d*x^3+c)^2,x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(3)*b*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/((b^2*c^2
*(a/b)^(1/3) - 2*a*b*c*d*(a/b)^(1/3) + a^2*d^2*(a/b)^(1/3))*(a/b)^(1/3)) -
1/9*sqrt(3)*(5*b*c - 2*a*d)*arctan(1/3*sqrt(3)*(2*x - (c/d)^(1/3))/(c/d)^(1
/3))/((b^2*c^3*(c/d)^(1/3) - 2*a*b*c^2*d*(c/d)^(1/3) + a^2*c*d^2*(c/d)^(1/3
))*(c/d)^(1/3)) - 1/3*d*x/(b*c^3 - a*c^2*d + (b*c^2*d - a*c*d^2)*x^3) - 1/6
*b*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(b^2*c^2*(a/b)^(2/3) - 2*a*b*c*d*
(a/b)^(2/3) + a^2*d^2*(a/b)^(2/3)) + 1/18*(5*b*c - 2*a*d)*log(x^2 - x*(c/d)
^(1/3) + (c/d)^(2/3))/(b^2*c^3*(c/d)^(2/3) - 2*a*b*c^2*d*(c/d)^(2/3) + a^2*
c*d^2*(c/d)^(2/3)) + 1/3*b*log(x + (a/b)^(1/3))/(b^2*c^2*(a/b)^(2/3) - 2*a*
b*c*d*(a/b)^(2/3) + a^2*d^2*(a/b)^(2/3)) - 1/9*(5*b*c - 2*a*d)*log(x + (c/d
)^(1/3))/(b^2*c^3*(c/d)^(2/3) - 2*a*b*c^2*d*(c/d)^(2/3) + a^2*c*d^2*(c/d)^(
2/3))
```

mupad [B] time = 16.81, size = 2589, normalized size = 7.48

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^3)*(c + d*x^3)^2),x)
```

```
[Out] log((((((27*b^3*d^3*x*(a*d - b*c)^3*(3*b^2*c^2 - 2*a^2*d^2 + 3*a*b*c*d))/c
+ (27*a*b^3*c^4*d^3*(a*d + b*c)*(a*d - b*c)^5*((d^2*(2*a*d - 5*b*c)^3)/(c^5
*(a*d - b*c)^6))^(1/3))/(b*c^4 - a*c^3*d))*((d^2*(2*a*d - 5*b*c)^3)/(c^5*(a
*d - b*c)^6))^(2/3))/81 - (b^4*d^4*(8*a^3*d^3 - 27*b^3*c^3 + 98*a*b^2*c^2*d
 - 52*a^2*b*c*d^2))/(3*b*c^4 - 3*a*c^3*d))*((d^2*(2*a*d - 5*b*c)^3)/(c^5*(a
*d - b*c)^6))^(1/3))/9 + (2*b^6*d^5*x*(4*a^3*d^3 - 85*b^3*c^3 + 84*a*b^2*c^
2*d - 30*a^2*b*c*d^2))/(9*c^3*(a*d - b*c)^4))*((8*a^3*d^5 - 125*b^3*c^3*d^2
 + 150*a*b^2*c^2*d^3 - 60*a^2*b*c*d^4)/(729*b^6*c^11 + 729*a^6*c^5*d^6 - 43
74*a^5*b*c^6*d^5 + 10935*a^2*b^4*c^9*d^2 - 14580*a^3*b^3*c^8*d^3 + 10935*a^
4*b^2*c^7*d^4 - 4374*a*b^5*c^10*d))^(1/3) + log((((((27*b^3*d^3*x*(a*d - b*
c)^3*(3*b^2*c^2 - 2*a^2*d^2 + 3*a*b*c*d))/c + (81*a*b^3*c^4*d^3*(a*d + b*c)
*(a*d - b*c)^5*(b^5/(a^2*(a*d - b*c)^6))^(1/3))/(b*c^4 - a*c^3*d))*(b^5/(a^
2*(a*d - b*c)^6))^(2/3))/9 - (b^4*d^4*(8*a^3*d^3 - 27*b^3*c^3 + 98*a*b^2*c^
2*d - 52*a^2*b*c*d^2))/(3*b*c^4 - 3*a*c^3*d))*(b^5/(a^2*(a*d - b*c)^6))^(1/
3))/3 + (2*b^6*d^5*x*(4*a^3*d^3 - 85*b^3*c^3 + 84*a*b^2*c^2*d - 30*a^2*b*c*
d^2))/(9*c^3*(a*d - b*c)^4))*(b^5/(27*a^8*d^6 + 27*a^2*b^6*c^6 - 162*a^3*b^
5*c^5*d + 405*a^4*b^4*c^4*d^2 - 540*a^5*b^3*c^3*d^3 + 405*a^6*b^2*c^2*d^4 -
 162*a^7*b*c*d^5))^(1/3) + (log(((3^(1/2)*1i - 1)*(((3^(1/2)*1i - 1)^2*((27
*b^3*d^3*x*(a*d - b*c)^3*(3*b^2*c^2 - 2*a^2*d^2 + 3*a*b*c*d))/c + (27*a*b^3
```

```
*c^4*d^3*(3^(1/2)*1i - 1)*(a*d + b*c)*(a*d - b*c)^5*((d^2*(2*a*d - 5*b*c)^3
)/(c^5*(a*d - b*c)^6))^(1/3))/(2*(b*c^4 - a*c^3*d)))*((d^2*(2*a*d - 5*b*c)^
3)/(c^5*(a*d - b*c)^6))^(2/3))/324 - (b^4*d^4*(8*a^3*d^3 - 27*b^3*c^3 + 98*
a*b^2*c^2*d - 52*a^2*b*c*d^2))/(3*b*c^4 - 3*a*c^3*d))*((d^2*(2*a*d - 5*b*c)
^3)/(c^5*(a*d - b*c)^6))^(1/3))/18 + (2*b^6*d^5*x*(4*a^3*d^3 - 85*b^3*c^3 +
 84*a*b^2*c^2*d - 30*a^2*b*c*d^2))/(9*c^3*(a*d - b*c)^4))*(3^(1/2)*1i - 1)*
((8*a^3*d^5 - 125*b^3*c^3*d^2 + 150*a*b^2*c^2*d^3 - 60*a^2*b*c*d^4)/(729*b^
6*c^11 + 729*a^6*c^5*d^6 - 4374*a^5*b*c^6*d^5 + 10935*a^2*b^4*c^9*d^2 - 145
80*a^3*b^3*c^8*d^3 + 10935*a^4*b^2*c^7*d^4 - 4374*a*b^5*c^10*d))^(1/3))/2 -
 (log(((3^(1/2)*1i + 1)*(((3^(1/2)*1i + 1)^2*((27*b^3*d^3*x*(a*d - b*c)^3*(
3*b^2*c^2 - 2*a^2*d^2 + 3*a*b*c*d))/c - (27*a*b^3*c^4*d^3*(3^(1/2)*1i + 1)*
(a*d + b*c)*(a*d - b*c)^5*((d^2*(2*a*d - 5*b*c)^3)/(c^5*(a*d - b*c)^6))^(1/
3))/(2*(b*c^4 - a*c^3*d)))*((d^2*(2*a*d - 5*b*c)^3)/(c^5*(a*d - b*c)^6))^(2
/3))/324 - (b^4*d^4*(8*a^3*d^3 - 27*b^3*c^3 + 98*a*b^2*c^2*d - 52*a^2*b*c*d
^2))/(3*b*c^4 - 3*a*c^3*d))*((d^2*(2*a*d - 5*b*c)^3)/(c^5*(a*d - b*c)^6))^(
1/3))/18 - (2*b^6*d^5*x*(4*a^3*d^3 - 85*b^3*c^3 + 84*a*b^2*c^2*d - 30*a^2*b
*c*d^2))/(9*c^3*(a*d - b*c)^4))*(3^(1/2)*1i + 1)*((8*a^3*d^5 - 125*b^3*c^3*
d^2 + 150*a*b^2*c^2*d^3 - 60*a^2*b*c*d^4)/(729*b^6*c^11 + 729*a^6*c^5*d^6 -
 4374*a^5*b*c^6*d^5 + 10935*a^2*b^4*c^9*d^2 - 14580*a^3*b^3*c^8*d^3 + 10935
*a^4*b^2*c^7*d^4 - 4374*a*b^5*c^10*d))^(1/3))/2 + (log(((3^(1/2)*1i - 1)*((
(3^(1/2)*1i - 1)^2*((27*b^3*d^3*x*(a*d - b*c)^3*(3*b^2*c^2 - 2*a^2*d^2 + 3*
a*b*c*d))/c + (81*a*b^3*c^4*d^3*(3^(1/2)*1i - 1)*(a*d + b*c)*(a*d - b*c)^5*
(b^5/(a^2*(a*d - b*c)^6))^(1/3))/(2*(b*c^4 - a*c^3*d)))*(b^5/(a^2*(a*d - b*
c)^6))^(2/3))/36 - (b^4*d^4*(8*a^3*d^3 - 27*b^3*c^3 + 98*a*b^2*c^2*d - 52*a
^2*b*c*d^2))/(3*b*c^4 - 3*a*c^3*d))*(b^5/(a^2*(a*d - b*c)^6))^(1/3))/6 + (2
*b^6*d^5*x*(4*a^3*d^3 - 85*b^3*c^3 + 84*a*b^2*c^2*d - 30*a^2*b*c*d^2))/(9*c
^3*(a*d - b*c)^4))*(3^(1/2)*1i - 1)*(b^5/(27*a^8*d^6 + 27*a^2*b^6*c^6 - 162
*a^3*b^5*c^5*d + 405*a^4*b^4*c^4*d^2 - 540*a^5*b^3*c^3*d^3 + 405*a^6*b^2*c^
2*d^4 - 162*a^7*b*c*d^5))^(1/3))/2 - (log(((3^(1/2)*1i + 1)*(((3^(1/2)*1i +
 1)^2*((27*b^3*d^3*x*(a*d - b*c)^3*(3*b^2*c^2 - 2*a^2*d^2 + 3*a*b*c*d))/c -
 (81*a*b^3*c^4*d^3*(3^(1/2)*1i + 1)*(a*d + b*c)*(a*d - b*c)^5*(b^5/(a^2*(a*
d - b*c)^6))^(1/3))/(2*(b*c^4 - a*c^3*d)))*(b^5/(a^2*(a*d - b*c)^6))^(2/3))
/36 - (b^4*d^4*(8*a^3*d^3 - 27*b^3*c^3 + 98*a*b^2*c^2*d - 52*a^2*b*c*d^2))/
(3*b*c^4 - 3*a*c^3*d))*(b^5/(a^2*(a*d - b*c)^6))^(1/3))/6 - (2*b^6*d^5*x*(4
*a^3*d^3 - 85*b^3*c^3 + 84*a*b^2*c^2*d - 30*a^2*b*c*d^2))/(9*c^3*(a*d - b*c
)^4))*(3^(1/2)*1i + 1)*(b^5/(27*a^8*d^6 + 27*a^2*b^6*c^6 - 162*a^3*b^5*c^5*
d + 405*a^4*b^4*c^4*d^2 - 540*a^5*b^3*c^3*d^3 + 405*a^6*b^2*c^2*d^4 - 162*a
^7*b*c*d^5))^(1/3))/2 + (d*x)/(3*c*(c + d*x^3)*(a*d - b*c))
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**3+a)/(d*x**3+c)**2,x)
[Out] Timed out
```

3.20 $\int \frac{(c+dx^3)^5}{(a+bx^3)^2}\,dx$

Optimal. Leaf size=320

$$-\frac{(bc-ad)^4(13ad+2bc)\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{18a^{5/3}b^{16/3}}+\frac{(bc-ad)^4(13ad+2bc)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{9a^{5/3}b^{16/3}}-\frac{(bc-ad)^4(13a}{3}$$

Rubi [A] time = 0.30, antiderivative size = 320, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$, Rules used = {390, 385, 200, 31, 634, 617, 204, 628}

$$\frac{d^3x^4\left(3a^2d^2-10abcd+10b^2c^2\right)}{4b^4}+\frac{d^2x\left(15a^2bcd^2-4a^3d^3-20ab^2c^2d+10b^3c^3\right)}{b^5}-\frac{(bc-ad)^4(13ad+2bc)\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{18a^{5/3}b^{16/3}}+\frac{(bc-ad)^4(13ad+2bc)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{9a^{5/3}b^{16/3}}-\frac{(bc-ad)^4(13ad+2bc)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{5/3}b^{16/3}}+\frac{d^4x^7(5bc-2ad)}{7b^3}+\frac{x(bc-ad)^5}{3ab^5\left(a+bx^3\right)}+\frac{d^5x^{10}}{10b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3)^5/(a + b*x^3)^2,x]
```

```
[Out] (d^2*(10*b^3*c^3 - 20*a*b^2*c^2*d + 15*a^2*b*c*d^2 - 4*a^3*d^3)*x)/b^5 + (d
^3*(10*b^2*c^2 - 10*a*b*c*d + 3*a^2*d^2)*x^4)/(4*b^4) + (d^4*(5*b*c - 2*a*d
)*x^7)/(7*b^3) + (d^5*x^10)/(10*b^2) + ((b*c - a*d)^5*x)/(3*a*b^5*(a + b*x^
3)) - ((b*c - a*d)^4*(2*b*c + 13*a*d)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[
3]*a^(1/3))])/(3*Sqrt[3]*a^(5/3)*b^(16/3)) + ((b*c - a*d)^4*(2*b*c + 13*a*d
)*Log[a^(1/3) + b^(1/3)*x])/(9*a^(5/3)*b^(16/3)) - ((b*c - a*d)^4*(2*b*c +
13*a*d)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(18*a^(5/3)*b^(16/3
))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
```

```
0] && GeQ[p, -q]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{\left(c+dx^3\right)^5}{\left(a+bx^3\right)^2}\,dx = \int \left(\frac{d^2\left(10b^3c^3-20ab^2c^2d+15a^2bcd^2-4a^3d^3\right)}{b^5}+\frac{d^3\left(10b^2c^2-10abcd+3a^2d^2\right)x^3}{b^4}+d^4(5$$

$$=\frac{d^2\left(10b^3c^3-20ab^2c^2d+15a^2bcd^2-4a^3d^3\right)x}{b^5}+\frac{d^3\left(10b^2c^2-10abcd+3a^2d^2\right)x^4}{4b^4}+d^4(5bc$$

$$=\frac{d^2\left(10b^3c^3-20ab^2c^2d+15a^2bcd^2-4a^3d^3\right)x}{b^5}+\frac{d^3\left(10b^2c^2-10abcd+3a^2d^2\right)x^4}{4b^4}+d^4(5bc$$

$$=\frac{d^2\left(10b^3c^3-20ab^2c^2d+15a^2bcd^2-4a^3d^3\right)x}{b^5}+\frac{d^3\left(10b^2c^2-10abcd+3a^2d^2\right)x^4}{4b^4}+d^4(5bc$$

$$=\frac{d^2\left(10b^3c^3-20ab^2c^2d+15a^2bcd^2-4a^3d^3\right)x}{b^5}+\frac{d^3\left(10b^2c^2-10abcd+3a^2d^2\right)x^4}{4b^4}+d^4(5bc$$

$$=\frac{d^2\left(10b^3c^3-20ab^2c^2d+15a^2bcd^2-4a^3d^3\right)x}{b^5}+\frac{d^3\left(10b^2c^2-10abcd+3a^2d^2\right)x^4}{4b^4}+d^4(5bc$$

$$=\frac{d^2\left(10b^3c^3-20ab^2c^2d+15a^2bcd^2-4a^3d^3\right)x}{b^5}+\frac{d^3\left(10b^2c^2-10abcd+3a^2d^2\right)x^4}{4b^4}+d^4(5bc$$

Mathematica [A] time = 0.30, size = 313, normalized size = 0.98

$$\frac{-\frac{70(bc-ad)^4(13ad+2bc)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{a^{5/3}}+\frac{140(bc-ad)^4(13ad+2bc)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{a^{5/3}}+\frac{140\sqrt{3}(bc-ad)^4(13ad+2bc)\tan^{-1}\left(\frac{2\sqrt[3]{b}x-\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{a^{5/3}}+315b^{4/3}d^3x^4\left(3a^2d^2-10abcd+10b^2c^2\right)+1260\sqrt[3]{b}d^2x\left(-4a^3d^3+15a^2bcd^2-20ab^2c^2d+10b^3c^3\right)+180b^{7/3}d^4x^7(5bc-2ad)+\frac{420\sqrt[3]{b}x(bc-ad)^5}{a\left(a+bx^3\right)}+126b^{10/3}d^5x^{10}}{1260b^{16/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^3)^5/(a + b*x^3)^2,x]
```

```
[Out] (1260*b^(1/3)*d^2*(10*b^3*c^3 - 20*a*b^2*c^2*d + 15*a^2*b*c*d^2 - 4*a^3*d^3
)*x + 315*b^(4/3)*d^3*(10*b^2*c^2 - 10*a*b*c*d + 3*a^2*d^2)*x^4 + 180*b^(7/
3)*d^4*(5*b*c - 2*a*d)*x^7 + 126*b^(10/3)*d^5*x^10 + (420*b^(1/3)*(b*c - a*
d)^5*x)/(a*(a + b*x^3)) + (140*Sqrt[3]*(b*c - a*d)^4*(2*b*c + 13*a*d)*ArcTa
n[(-a^(1/3) + 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/a^(5/3) + (140*(b*c - a*d)^4
*(2*b*c + 13*a*d)*Log[a^(1/3) + b^(1/3)*x])/a^(5/3) - (70*(b*c - a*d)^4*(2*
b*c + 13*a*d)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/a^(5/3))/(126
0*b^(16/3))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c+dx^3\right)^5}{\left(a+bx^3\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x^3)^5/(a + b*x^3)^2,x]
```

```
[Out] IntegrateAlgebraic[(c + d*x^3)^5/(a + b*x^3)^2, x]
```

fricas [B] time = 1.11, size = 1619, normalized size = 5.06

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^5/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] [1/1260*(126*a^3*b^5*d^5*x^13 + 18*(50*a^3*b^5*c*d^4 - 13*a^4*b^4*d^5)*x^10
 + 45*(70*a^3*b^5*c^2*d^3 - 50*a^4*b^4*c*d^4 + 13*a^5*b^3*d^5)*x^7 + 315*(4
0*a^3*b^5*c^3*d^2 - 70*a^4*b^4*c^2*d^3 + 50*a^5*b^3*c*d^4 - 13*a^6*b^2*d^5)
*x^4 + 210*sqrt(1/3)*(2*a^2*b^6*c^5 + 5*a^3*b^5*c^4*d - 40*a^4*b^4*c^3*d^2
+ 70*a^5*b^3*c^2*d^3 - 50*a^6*b^2*c*d^4 + 13*a^7*b*d^5 + (2*a*b^7*c^5 + 5*a
^2*b^6*c^4*d - 40*a^3*b^5*c^3*d^2 + 70*a^4*b^4*c^2*d^3 - 50*a^5*b^3*c*d^4 +
 13*a^6*b^2*d^5)*x^3)*sqrt(-(a^2*b)^(1/3)/b)*log((2*a*b*x^3 - 3*(a^2*b)^(1/
3)*a*x - a^2 + 3*sqrt(1/3)*(2*a*b*x^2 + (a^2*b)^(2/3)*x - (a^2*b)^(1/3)*a)*
sqrt(-(a^2*b)^(1/3)/b))/(b*x^3 + a)) - 70*(2*a*b^5*c^5 + 5*a^2*b^4*c^4*d -
40*a^3*b^3*c^3*d^2 + 70*a^4*b^2*c^2*d^3 - 50*a^5*b*c*d^4 + 13*a^6*d^5 + (2*
b^6*c^5 + 5*a*b^5*c^4*d - 40*a^2*b^4*c^3*d^2 + 70*a^3*b^3*c^2*d^3 - 50*a^4*
b^2*c*d^4 + 13*a^5*b*d^5)*x^3)*(a^2*b)^(2/3)*log(a*b*x^2 - (a^2*b)^(2/3)*x
+ (a^2*b)^(1/3)*a) + 140*(2*a*b^5*c^5 + 5*a^2*b^4*c^4*d - 40*a^3*b^3*c^3*d^
2 + 70*a^4*b^2*c^2*d^3 - 50*a^5*b*c*d^4 + 13*a^6*d^5 + (2*b^6*c^5 + 5*a*b^5
*c^4*d - 40*a^2*b^4*c^3*d^2 + 70*a^3*b^3*c^2*d^3 - 50*a^4*b^2*c*d^4 + 13*a^
5*b*d^5)*x^3)*(a^2*b)^(2/3)*log(a*b*x + (a^2*b)^(2/3)) + 420*(a^2*b^6*c^5 -
 5*a^3*b^5*c^4*d + 40*a^4*b^4*c^3*d^2 - 70*a^5*b^3*c^2*d^3 + 50*a^6*b^2*c*d
^4 - 13*a^7*b*d^5)*x)/(a^3*b^7*x^3 + a^4*b^6), 1/1260*(126*a^3*b^5*d^5*x^13
 + 18*(50*a^3*b^5*c*d^4 - 13*a^4*b^4*d^5)*x^10 + 45*(70*a^3*b^5*c^2*d^3 - 5
0*a^4*b^4*c*d^4 + 13*a^5*b^3*d^5)*x^7 + 315*(40*a^3*b^5*c^3*d^2 - 70*a^4*b^
4*c^2*d^3 + 50*a^5*b^3*c*d^4 - 13*a^6*b^2*d^5)*x^4 + 420*sqrt(1/3)*(2*a^2*b
^6*c^5 + 5*a^3*b^5*c^4*d - 40*a^4*b^4*c^3*d^2 + 70*a^5*b^3*c^2*d^3 - 50*a^6
*b^2*c*d^4 + 13*a^7*b*d^5 + (2*a*b^7*c^5 + 5*a^2*b^6*c^4*d - 40*a^3*b^5*c^3
*d^2 + 70*a^4*b^4*c^2*d^3 - 50*a^5*b^3*c*d^4 + 13*a^6*b^2*d^5)*x^3)*sqrt((a
^2*b)^(1/3)/b)*arctan(sqrt(1/3)*(2*(a^2*b)^(2/3)*x - (a^2*b)^(1/3)*a)*sqrt(
(a^2*b)^(1/3)/b)/a^2) - 70*(2*a*b^5*c^5 + 5*a^2*b^4*c^4*d - 40*a^3*b^3*c^3*
d^2 + 70*a^4*b^2*c^2*d^3 - 50*a^5*b*c*d^4 + 13*a^6*d^5 + (2*b^6*c^5 + 5*a*b
^5*c^4*d - 40*a^2*b^4*c^3*d^2 + 70*a^3*b^3*c^2*d^3 - 50*a^4*b^2*c*d^4 + 13*
a^5*b*d^5)*x^3)*(a^2*b)^(2/3)*log(a*b*x^2 - (a^2*b)^(2/3)*x + (a^2*b)^(1/3)
*a) + 140*(2*a*b^5*c^5 + 5*a^2*b^4*c^4*d - 40*a^3*b^3*c^3*d^2 + 70*a^4*b^2*
c^2*d^3 - 50*a^5*b*c*d^4 + 13*a^6*d^5 + (2*b^6*c^5 + 5*a*b^5*c^4*d - 40*a^2
*b^4*c^3*d^2 + 70*a^3*b^3*c^2*d^3 - 50*a^4*b^2*c*d^4 + 13*a^5*b*d^5)*x^3)*(
a^2*b)^(2/3)*log(a*b*x + (a^2*b)^(2/3)) + 420*(a^2*b^6*c^5 - 5*a^3*b^5*c^4*
```

```
d + 40*a^4*b^4*c^3*d^2 - 70*a^5*b^3*c^2*d^3 + 50*a^6*b^2*c*d^4 - 13*a^7*b*d
^5)*x)/(a^3*b^7*x^3 + a^4*b^6)]
```

giac [A] time = 0.19, size = 529, normalized size = 1.65

$$-\frac{\sqrt{3}\left(2 b^{5} c^{5}+5 a b^{4} c^{4} d-40 a^{2} b^{3} c^{3} d^{2}+70 a^{3} b^{2} c^{2} d^{3}-50 a^{4} b c d^{4}+13 a^{5} d^{5}\right) \arctan\left(\frac{\sqrt{3}\left(2 x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\left(-a b^{2}\right)^{\frac{2}{3}} a b^{4}}-\frac{\left(2 b^{5} c^{5}+5 a b^{4} c^{4} d-40 a^{2} b^{3} c^{3} d^{2}+70 a^{3} b^{2} c^{2} d^{3}-50 a^{4} b c d^{4}+13 a^{5} d^{5}\right) \log\left(x^{2}+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\left(-a b^{2}\right)^{\frac{2}{3}} a b^{4}}-\frac{\left(2 b^{5} c^{5}+5 a b^{4} c^{4} d-40 a^{2} b^{3} c^{3} d^{2}+70 a^{3} b^{2} c^{2} d^{3}-50 a^{4} b c d^{4}+13 a^{5} d^{5}\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}} \log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9 a^{2} b^{5}}+\frac{b^{5} c^{5} x-5 a b^{4} c^{4} d x+10 a^{2} b^{3} c^{3} d^{2} x-10 a^{3} b^{2} c^{2} d^{3} x+5 a^{4} b c d^{4} x-a^{5} d^{5} x}{3\left(b x^{3}+a\right) a b^{5}}+\frac{14 b^{18} d^{5} x^{10}+100 b^{18} c d^{4} x^{7}-40 a b^{17} d^{5} x^{7}+350 b^{18} c^{2} d^{3} x^{4}-350 a b^{17} c d^{4} x^{4}+105 a^{2} b^{16} d^{5} x^{4}+1400 b^{18} c^{3} d^{2} x-2800 a b^{17} c^{2} d^{3} x+2100 a^{2} b^{16} c d^{4} x-560 a^{3} b^{15} d^{5} x}{140 b^{20}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^5/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] -1/9*sqrt(3)*(2*b^5*c^5 + 5*a*b^4*c^4*d - 40*a^2*b^3*c^3*d^2 + 70*a^3*b^2*c
^2*d^3 - 50*a^4*b*c*d^4 + 13*a^5*d^5)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3
))/(-a/b)^(1/3))/((-a*b^2)^(2/3)*a*b^4) - 1/18*(2*b^5*c^5 + 5*a*b^4*c^4*d -
 40*a^2*b^3*c^3*d^2 + 70*a^3*b^2*c^2*d^3 - 50*a^4*b*c*d^4 + 13*a^5*d^5)*log
(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/((-a*b^2)^(2/3)*a*b^4) - 1/9*(2*b^5*c
^5 + 5*a*b^4*c^4*d - 40*a^2*b^3*c^3*d^2 + 70*a^3*b^2*c^2*d^3 - 50*a^4*b*c*d
^4 + 13*a^5*d^5)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a^2*b^5) + 1/3*(b
^5*c^5*x - 5*a*b^4*c^4*d*x + 10*a^2*b^3*c^3*d^2*x - 10*a^3*b^2*c^2*d^3*x +
5*a^4*b*c*d^4*x - a^5*d^5*x)/((b*x^3 + a)*a*b^5) + 1/140*(14*b^18*d^5*x^10
+ 100*b^18*c*d^4*x^7 - 40*a*b^17*d^5*x^7 + 350*b^18*c^2*d^3*x^4 - 350*a*b^1
7*c*d^4*x^4 + 105*a^2*b^16*d^5*x^4 + 1400*b^18*c^3*d^2*x - 2800*a*b^17*c^2*
d^3*x + 2100*a^2*b^16*c*d^4*x - 560*a^3*b^15*d^5*x)/b^20
```

maple [B] time = 0.06, size = 905, normalized size = 2.83

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c)^5/(b*x^3+a)^2,x)
```

```
[Out] -50/9/b^5*a^3/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*c
*d^4+70/9/b^4*a^2/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1
))*c^2*d^3-40/9/b^3*a/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)
*x-1))*c^3*d^2-2/7*d^5/b^3*x^7*a+5/7*d^4/b^2*x^7*c+3/4*d^5/b^4*x^4*a^2+5/2*
d^3/b^2*x^4*c^2-4*d^5/b^5*a^3*x+10*d^2/b^2*c^3*x+1/3/a*x/(b*x^3+a)*c^5-35/9
/b^4*a^2/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*c^2*d^3+20/9/b^3*a/(
a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*c^3*d^2-50/9/b^5*a^3/(a/b)^(2/
3)*ln(x+(a/b)^(1/3))*c*d^4+70/9/b^4*a^2/(a/b)^(2/3)*ln(x+(a/b)^(1/3))*c^2*d
^3-40/9/b^3*a/(a/b)^(2/3)*ln(x+(a/b)^(1/3))*c^3*d^2+25/9/b^5*a^3/(a/b)^(2/3
)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*c*d^4-10/3/b^3*a^2*x/(b*x^3+a)*c^2*d^3+
10/3/b^2*a*x/(b*x^3+a)*c^3*d^2+13/9/b^6*a^4/(a/b)^(2/3)*3^(1/2)*arctan(1/3*
3^(1/2)*(2/(a/b)^(1/3)*x-1))*d^5+5/9/b^2/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(
1/2)*(2/(a/b)^(1/3)*x-1))*c^4*d+2/9/b/a/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1
/2)*(2/(a/b)^(1/3)*x-1))*c^5+5/3/b^4*a^3*x/(b*x^3+a)*c*d^4-13/18/b^6*a^4/(a
/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*d^5-5/18/b^2/(a/b)^(2/3)*ln(x^2
-(a/b)^(1/3)*x+(a/b)^(2/3))*c^4*d-1/9/b/a/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+
(a/b)^(2/3))*c^5+13/9/b^6*a^4/(a/b)^(2/3)*ln(x+(a/b)^(1/3))*d^5+5/9/b^2/(a/
b)^(2/3)*ln(x+(a/b)^(1/3))*c^4*d+2/9/b/a/(a/b)^(2/3)*ln(x+(a/b)^(1/3))*c^5-
5/2*d^4/b^3*x^4*a*c+15*d^4/b^4*a^2*c*x-1/3/b^5*a^4*x/(b*x^3+a)*d^5-5/3/b*x/
(b*x^3+a)*c^4*d-20*d^3/b^3*a*c^2*x+1/10*d^5*x^10/b^2
```

maxima [A] time = 1.35, size = 509, normalized size = 1.59

$$\frac{\left(b^{5} c^{5}-5 a b^{4} c^{4} d+10 a^{2} b^{3} c^{3} d^{2}-10 a^{3} b^{2} c^{2} d^{3}+5 a^{4} b c d^{4}-a^{5} d^{5}\right) x}{3\left(a b^{6} x^{3}+a^{2} b^{5}\right)}+\frac{14 b^{3} d^{5} x^{10}+20\left(5 b^{3} c d^{4}-2 a b^{2} d^{5}\right) x^{7}+35\left(10 b^{3} c^{2} d^{3}-10 a b^{2} c d^{4}+3 a^{2} b d^{5}\right) x^{4}+140\left(10 b^{3} c^{3} d^{2}-20 a b^{2} c^{2} d^{3}+15 a^{2} b c d^{4}-4 a^{3} d^{5}\right) x}{140 b^{5}}+\frac{\sqrt{3}\left(2 b^{5} c^{5}+5 a b^{4} c^{4} d-40 a^{2} b^{3} c^{3} d^{2}+70 a^{3} b^{2} c^{2} d^{3}-50 a^{4} b c d^{4}+13 a^{5} d^{5}\right) \arctan\left(\frac{\sqrt{3}\left(2 x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9 a b^{6}\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\left(2 b^{5} c^{5}+5 a b^{4} c^{4} d-40 a^{2} b^{3} c^{3} d^{2}+70 a^{3} b^{2} c^{2} d^{3}-50 a^{4} b c d^{4}+13 a^{5} d^{5}\right) \log\left(x^{2}-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18 a b^{6}\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\left(2 b^{5} c^{5}+5 a b^{4} c^{4} d-40 a^{2} b^{3} c^{3} d^{2}+70 a^{3} b^{2} c^{2} d^{3}-50 a^{4} b c d^{4}+13 a^{5} d^{5}\right) \log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9 a b^{6}\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^5/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] 1/3*(b^5*c^5 - 5*a*b^4*c^4*d + 10*a^2*b^3*c^3*d^2 - 10*a^3*b^2*c^2*d^3 + 5*
a^4*b*c*d^4 - a^5*d^5)*x/(a*b^6*x^3 + a^2*b^5) + 1/140*(14*b^3*d^5*x^10 + 2
```

```
0*(5*b^3*c*d^4 - 2*a*b^2*d^5)*x^7 + 35*(10*b^3*c^2*d^3 - 10*a*b^2*c*d^4 + 3
*a^2*b*d^5)*x^4 + 140*(10*b^3*c^3*d^2 - 20*a*b^2*c^2*d^3 + 15*a^2*b*c*d^4 -
 4*a^3*d^5)*x)/b^5 + 1/9*sqrt(3)*(2*b^5*c^5 + 5*a*b^4*c^4*d - 40*a^2*b^3*c^
3*d^2 + 70*a^3*b^2*c^2*d^3 - 50*a^4*b*c*d^4 + 13*a^5*d^5)*arctan(1/3*sqrt(3
)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a*b^6*(a/b)^(2/3)) - 1/18*(2*b^5*c^5 +
5*a*b^4*c^4*d - 40*a^2*b^3*c^3*d^2 + 70*a^3*b^2*c^2*d^3 - 50*a^4*b*c*d^4 +
13*a^5*d^5)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a*b^6*(a/b)^(2/3)) + 1/
9*(2*b^5*c^5 + 5*a*b^4*c^4*d - 40*a^2*b^3*c^3*d^2 + 70*a^3*b^2*c^2*d^3 - 50
*a^4*b*c*d^4 + 13*a^5*d^5)*log(x + (a/b)^(1/3))/(a*b^6*(a/b)^(2/3))
```

mupad [B] time = 0.39, size = 416, normalized size = 1.30

$$x\left(\frac{10\,c^3\,d^2}{b^2}-\frac{2\,a\left(\frac{2\,a\left(\frac{2\,a\,d^5}{b^3}-\frac{5\,c\,d^4}{b^2}\right)}{b}-\frac{a^2\,d^5}{b^4}+\frac{10\,c^2\,d^3}{b^2}\right)}{b}+\frac{a^2\left(\frac{2\,a\,d^5}{b^3}-\frac{5\,c\,d^4}{b^2}\right)}{b^2}\right)-x^7\left(\frac{2\,a\,d^5}{7\,b^3}-\frac{5\,c\,d^4}{7\,b^2}\right)+x^4\left(\frac{a\left(\frac{2\,a\,d^5}{b^3}-\frac{5\,c\,d^4}{b^2}\right)}{2\,b}-\frac{a^2\,d^5}{4\,b^4}+\frac{5\,c^2\,d^3}{2\,b^2}\right)+\frac{d^5\,x^{10}}{10\,b^2}-\frac{x\left(a^5\,d^5-5\,a^4\,b\,c\,d^4+10\,a^3\,b^2\,c^2\,d^3-10\,a^2\,b^3\,c^3\,d^2+5\,a\,b^4\,c^4\,d-b^5\,c^5\right)}{3\,a\left(b^6\,x^3+a\,b^5\right)}+\frac{\ln\left(b^{1/3}\,x+a^{1/3}\right)\left(a\,d-b\,c\right)^4\,\left(13\,a\,d+2\,b\,c\right)}{9\,a^{5/3}\,b^{16/3}}-\frac{\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1{}\mathrm{i}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\left(a\,d-b\,c\right)^4\,\left(13\,a\,d+2\,b\,c\right)}{9\,a^{5/3}\,b^{16/3}}+\frac{\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1{}\mathrm{i}\right)\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\left(a\,d-b\,c\right)^4\,\left(13\,a\,d+2\,b\,c\right)}{9\,a^{5/3}\,b^{16/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3)^5/(a + b*x^3)^2,x)
```

```
[Out] x*((10*c^3*d^2)/b^2 - (2*a*((2*a*((2*a*d^5)/b^3 - (5*c*d^4)/b^2))/b - (a^2*
d^5)/b^4 + (10*c^2*d^3)/b^2))/b + (a^2*((2*a*d^5)/b^3 - (5*c*d^4)/b^2))/b^2
) - x^7*((2*a*d^5)/(7*b^3) - (5*c*d^4)/(7*b^2)) + x^4*((a*((2*a*d^5)/b^3 -
(5*c*d^4)/b^2))/(2*b) - (a^2*d^5)/(4*b^4) + (5*c^2*d^3)/(2*b^2)) + (d^5*x^1
0)/(10*b^2) - (x*(a^5*d^5 - b^5*c^5 - 10*a^2*b^3*c^3*d^2 + 10*a^3*b^2*c^2*d
^3 + 5*a*b^4*c^4*d - 5*a^4*b*c*d^4))/(3*a*(a*b^5 + b^6*x^3)) + (log(b^(1/3)
*x + a^(1/3))*(a*d - b*c)^4*(13*a*d + 2*b*c))/(9*a^(5/3)*b^(16/3)) - (log(3
^(1/2)*a^(1/3)*1i - 2*b^(1/3)*x + a^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(a*d - b*
c)^4*(13*a*d + 2*b*c))/(9*a^(5/3)*b^(16/3)) + (log(3^(1/2)*a^(1/3)*1i + 2*b
^(1/3)*x - a^(1/3))*((3^(1/2)*1i)/2 - 1/2)*(a*d - b*c)^4*(13*a*d + 2*b*c))/
(9*a^(5/3)*b^(16/3))
```

sympy [A] time = 12.43, size = 546, normalized size = 1.71

$$x^7\left(-\frac{2ad^5}{7b^3}+\frac{5cd^4}{7b^2}\right)+x^4\left(\frac{3a^2d^5}{4b^4}-\frac{5acd^4}{2b^3}+\frac{5c^2d^3}{2b^2}\right)+x\left(-\frac{4a^3d^5}{b^5}+\frac{15a^2cd^4}{b^4}-\frac{20ac^2d^3}{b^3}+\frac{10c^3d^2}{b^2}\right)+\frac{x\left(-a^5d^5+5a^4bcd^4-10a^3b^2c^2d^3+10a^2b^3c^3d^2-5ab^4c^4d+b^5c^5\right)}{3a^2b^5+3ab^6x^3}+\operatorname{RootSum}\left(729t^3a^5b^{16}-2197a^{15}d^{15}+25350a^{14}bcd^{14}-132990a^{13}b^2c^2d^{13}+418280a^{12}b^3c^3d^{12}-874635a^{11}b^4c^4d^{11}+1271886a^{10}b^5c^5d^{10}-1302400a^9b^6c^6d^9+922680a^8b^7c^7d^8-422235a^7b^8c^8d^7+97570a^6b^9c^9d^6+7194a^5b^{10}c^{10}d^5-10200a^4b^{11}c^{11}d^4+1435a^3b^{12}c^{12}d^3+330a^2b^{13}c^{13}d^2-60ab^{14}c^{14}d-8b^{15}c^{15},\left(t\mapsto t\log\left(\frac{9ta^2b^5}{13a^5d^5-50a^4bcd^4+70a^3b^2c^2d^3-40a^2b^3c^3d^2+5ab^4c^4d+2b^5c^5}+x\right)\right)\right)+\frac{d^5x^{10}}{10b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c)**5/(b*x**3+a)**2,x)
```

```
[Out] x**7*(-2*a*d**5/(7*b**3) + 5*c*d**4/(7*b**2)) + x**4*(3*a**2*d**5/(4*b**4)
- 5*a*c*d**4/(2*b**3) + 5*c**2*d**3/(2*b**2)) + x*(-4*a**3*d**5/b**5 + 15*a
**2*c*d**4/b**4 - 20*a*c**2*d**3/b**3 + 10*c**3*d**2/b**2) + x*(-a**5*d**5
+ 5*a**4*b*c*d**4 - 10*a**3*b**2*c**2*d**3 + 10*a**2*b**3*c**3*d**2 - 5*a*b
**4*c**4*d + b**5*c**5)/(3*a**2*b**5 + 3*a*b**6*x**3) + RootSum(729*_t**3*a
**5*b**16 - 2197*a**15*d**15 + 25350*a**14*b*c*d**14 - 132990*a**13*b**2*c*
*2*d**13 + 418280*a**12*b**3*c**3*d**12 - 874635*a**11*b**4*c**4*d**11 + 12
71886*a**10*b**5*c**5*d**10 - 1302400*a**9*b**6*c**6*d**9 + 922680*a**8*b**
7*c**7*d**8 - 422235*a**7*b**8*c**8*d**7 + 97570*a**6*b**9*c**9*d**6 + 7194
*a**5*b**10*c**10*d**5 - 10200*a**4*b**11*c**11*d**4 + 1435*a**3*b**12*c**1
2*d**3 + 330*a**2*b**13*c**13*d**2 - 60*a*b**14*c**14*d - 8*b**15*c**15, La
mbda(_t, _t*log(9*_t*a**2*b**5/(13*a**5*d**5 - 50*a**4*b*c*d**4 + 70*a**3*b
**2*c**2*d**3 - 40*a**2*b**3*c**3*d**2 + 5*a*b**4*c**4*d + 2*b**5*c**5) + x
))) + d**5*x**10/(10*b**2)
```

3.21 $\int \frac{(c+dx^3)^4}{(a+bx^3)^2} \, dx$

Optimal. Leaf size=267

$$-\frac{(bc-ad)^3(5ad+bc)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{9a^{5/3}b^{13/3}}+\frac{2(bc-ad)^3(5ad+bc)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{5/3}b^{13/3}}-\frac{2(bc-ad)^3(5ad}{3\ldots}$$

Rubi [A] time = 0.23, antiderivative size = 267, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$, Rules used = {390, 385, 200, 31, 634, 617, 204, 628}

$$\frac{d^2x\left(3a^2d^2-8abcd+6b^2c^2\right)}{b^4}-\frac{(bc-ad)^3(5ad+bc)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{9a^{5/3}b^{13/3}}+\frac{2(bc-ad)^3(5ad+bc)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{5/3}b^{13/3}}-\frac{2(bc-ad)^3(5ad+bc)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{5/3}b^{13/3}}+\frac{d^3x^4(2bc-ad)}{2b^3}+\frac{x(bc-ad)^4}{3ab^4\left(a+bx^3\right)}+\frac{d^4x^7}{7b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3)^4/(a + b*x^3)^2,x]
```

```
[Out] (d^2*(6*b^2*c^2 - 8*a*b*c*d + 3*a^2*d^2)*x)/b^4 + (d^3*(2*b*c - a*d)*x^4)/(
2*b^3) + (d^4*x^7)/(7*b^2) + ((b*c - a*d)^4*x)/(3*a*b^4*(a + b*x^3)) - (2*(
b*c - a*d)^3*(b*c + 5*a*d)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))
])/(3*Sqrt[3]*a^(5/3)*b^(13/3)) + (2*(b*c - a*d)^3*(b*c + 5*a*d)*Log[a^(1/3
) + b^(1/3)*x])/(9*a^(5/3)*b^(13/3)) - ((b*c - a*d)^3*(b*c + 5*a*d)*Log[a^(
2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(9*a^(5/3)*b^(13/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{\left(c+dx^3\right)^4}{\left(a+bx^3\right)^2}\,dx = \int \left(\frac{d^2\left(6b^2c^2-8abcd+3a^2d^2\right)}{b^4}+\frac{2d^3(2bc-ad)x^3}{b^3}+\frac{d^4x^6}{b^2}+\frac{(bc-ad)^3(bc+3ad)+4bd(bc-}{b^4\left(a+bx^3\right)^2}\right.$$

$$= \frac{d^2\left(6b^2c^2-8abcd+3a^2d^2\right)x}{b^4}+\frac{d^3(2bc-ad)x^4}{2b^3}+\frac{d^4x^7}{7b^2}+\frac{\int \frac{(bc-ad)^3(bc+3ad)+4bd(bc-ad)^3x^3}{\left(a+bx^3\right)^2}\,dx}{b^4}$$

$$= \frac{d^2\left(6b^2c^2-8abcd+3a^2d^2\right)x}{b^4}+\frac{d^3(2bc-ad)x^4}{2b^3}+\frac{d^4x^7}{7b^2}+\frac{(bc-ad)^4x}{3ab^4\left(a+bx^3\right)}+\frac{\left(2(bc-ad)^3(bc+\right.}{3a}$$

$$= \frac{d^2\left(6b^2c^2-8abcd+3a^2d^2\right)x}{b^4}+\frac{d^3(2bc-ad)x^4}{2b^3}+\frac{d^4x^7}{7b^2}+\frac{(bc-ad)^4x}{3ab^4\left(a+bx^3\right)}+\frac{\left(2(bc-ad)^3(bc+\right.}{9a}$$

$$= \frac{d^2\left(6b^2c^2-8abcd+3a^2d^2\right)x}{b^4}+\frac{d^3(2bc-ad)x^4}{2b^3}+\frac{d^4x^7}{7b^2}+\frac{(bc-ad)^4x}{3ab^4\left(a+bx^3\right)}+\frac{2(bc-ad)^3(bc+5}{9a}$$

$$= \frac{d^2\left(6b^2c^2-8abcd+3a^2d^2\right)x}{b^4}+\frac{d^3(2bc-ad)x^4}{2b^3}+\frac{d^4x^7}{7b^2}+\frac{(bc-ad)^4x}{3ab^4\left(a+bx^3\right)}+\frac{2(bc-ad)^3(bc+5}{9a}$$

$$= \frac{d^2\left(6b^2c^2-8abcd+3a^2d^2\right)x}{b^4}+\frac{d^3(2bc-ad)x^4}{2b^3}+\frac{d^4x^7}{7b^2}+\frac{(bc-ad)^4x}{3ab^4\left(a+bx^3\right)}-\frac{2(bc-ad)^3(bc+5}{3\sqrt{3}}$$

Mathematica [A] time = 0.24, size = 260, normalized size = 0.97

$$\frac{\frac{14(ad-bc)^3(5ad+bc)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{a^{5/3}}+\frac{28(bc-ad)^3(5ad+bc)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{a^{5/3}}+\frac{28\sqrt{3}(bc-ad)^3(5ad+bc)\tan^{-1}\left(\frac{2\sqrt[3]{b}x-\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{a^{5/3}}+126\sqrt[3]{b}d^2x\left(3a^2d^2-8abcd+6b^2c^2\right)+63b^{4/3}d^3x^4(2bc-ad)+\frac{42\sqrt[3]{b}x(bc-ad)^4}{a\left(a+bx^3\right)}+18b^{7/3}d^4x^7}{126b^{13/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^3)^4/(a + b*x^3)^2,x]
```

```
[Out] (126*b^(1/3)*d^2*(6*b^2*c^2 - 8*a*b*c*d + 3*a^2*d^2)*x + 63*b^(4/3)*d^3*(2*
b*c - a*d)*x^4 + 18*b^(7/3)*d^4*x^7 + (42*b^(1/3)*(b*c - a*d)^4*x)/(a*(a +
```

```
b*x^3)) + (28*Sqrt[3]*(b*c - a*d)^3*(b*c + 5*a*d)*ArcTan[(-a^(1/3) + 2*b^(1
/3)*x)/(Sqrt[3]*a^(1/3))])/a^(5/3) + (28*(b*c - a*d)^3*(b*c + 5*a*d)*Log[a^
(1/3) + b^(1/3)*x])/a^(5/3) + (14*(-(b*c) + a*d)^3*(b*c + 5*a*d)*Log[a^(2/3
) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/a^(5/3))/(126*b^(13/3))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c+dx^3\right)^4}{\left(a+bx^3\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x^3)^4/(a + b*x^3)^2,x]
```

```
[Out] IntegrateAlgebraic[(c + d*x^3)^4/(a + b*x^3)^2, x]
```

fricas [B] time = 1.06, size = 1316, normalized size = 4.93

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^4/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] [1/126*(18*a^3*b^4*d^4*x^10 + 9*(14*a^3*b^4*c*d^3 - 5*a^4*b^3*d^4)*x^7 + 63
*(12*a^3*b^4*c^2*d^2 - 14*a^4*b^3*c*d^3 + 5*a^5*b^2*d^4)*x^4 - 42*sqrt(1/3)
*(a^2*b^5*c^4 + 2*a^3*b^4*c^3*d - 12*a^4*b^3*c^2*d^2 + 14*a^5*b^2*c*d^3 - 5
*a^6*b*d^4 + (a*b^6*c^4 + 2*a^2*b^5*c^3*d - 12*a^3*b^4*c^2*d^2 + 14*a^4*b^3
*c*d^3 - 5*a^5*b^2*d^4)*x^3)*sqrt((-a^2*b)^(1/3)/b)*log((2*a*b*x^3 + 3*(-a^
2*b)^(1/3)*a*x - a^2 - 3*sqrt(1/3)*(2*a*b*x^2 + (-a^2*b)^(2/3)*x + (-a^2*b)
^(1/3)*a)*sqrt((-a^2*b)^(1/3)/b))/(b*x^3 + a)) - 14*(a*b^4*c^4 + 2*a^2*b^3*
c^3*d - 12*a^3*b^2*c^2*d^2 + 14*a^4*b*c*d^3 - 5*a^5*d^4 + (b^5*c^4 + 2*a*b^
4*c^3*d - 12*a^2*b^3*c^2*d^2 + 14*a^3*b^2*c*d^3 - 5*a^4*b*d^4)*x^3)*(-a^2*b
)^(2/3)*log(a*b*x^2 - (-a^2*b)^(2/3)*x - (-a^2*b)^(1/3)*a) + 28*(a*b^4*c^4
+ 2*a^2*b^3*c^3*d - 12*a^3*b^2*c^2*d^2 + 14*a^4*b*c*d^3 - 5*a^5*d^4 + (b^5*
c^4 + 2*a*b^4*c^3*d - 12*a^2*b^3*c^2*d^2 + 14*a^3*b^2*c*d^3 - 5*a^4*b*d^4)*
x^3)*(-a^2*b)^(2/3)*log(a*b*x + (-a^2*b)^(2/3)) + 42*(a^2*b^5*c^4 - 4*a^3*b
^4*c^3*d + 24*a^4*b^3*c^2*d^2 - 28*a^5*b^2*c*d^3 + 10*a^6*b*d^4)*x)/(a^3*b^
6*x^3 + a^4*b^5), 1/126*(18*a^3*b^4*d^4*x^10 + 9*(14*a^3*b^4*c*d^3 - 5*a^4*
b^3*d^4)*x^7 + 63*(12*a^3*b^4*c^2*d^2 - 14*a^4*b^3*c*d^3 + 5*a^5*b^2*d^4)*x
^4 + 84*sqrt(1/3)*(a^2*b^5*c^4 + 2*a^3*b^4*c^3*d - 12*a^4*b^3*c^2*d^2 + 14*
a^5*b^2*c*d^3 - 5*a^6*b*d^4 + (a*b^6*c^4 + 2*a^2*b^5*c^3*d - 12*a^3*b^4*c^2
*d^2 + 14*a^4*b^3*c*d^3 - 5*a^5*b^2*d^4)*x^3)*sqrt(-(-a^2*b)^(1/3)/b)*arcta
n(sqrt(1/3)*(2*(-a^2*b)^(2/3)*x + (-a^2*b)^(1/3)*a)*sqrt(-(-a^2*b)^(1/3)/b)
/a^2) - 14*(a*b^4*c^4 + 2*a^2*b^3*c^3*d - 12*a^3*b^2*c^2*d^2 + 14*a^4*b*c*d
^3 - 5*a^5*d^4 + (b^5*c^4 + 2*a*b^4*c^3*d - 12*a^2*b^3*c^2*d^2 + 14*a^3*b^2
*c*d^3 - 5*a^4*b*d^4)*x^3)*(-a^2*b)^(2/3)*log(a*b*x^2 - (-a^2*b)^(2/3)*x -
(-a^2*b)^(1/3)*a) + 28*(a*b^4*c^4 + 2*a^2*b^3*c^3*d - 12*a^3*b^2*c^2*d^2 +
14*a^4*b*c*d^3 - 5*a^5*d^4 + (b^5*c^4 + 2*a*b^4*c^3*d - 12*a^2*b^3*c^2*d^2
+ 14*a^3*b^2*c*d^3 - 5*a^4*b*d^4)*x^3)*(-a^2*b)^(2/3)*log(a*b*x + (-a^2*b)^
(2/3)) + 42*(a^2*b^5*c^4 - 4*a^3*b^4*c^3*d + 24*a^4*b^3*c^2*d^2 - 28*a^5*b^
2*c*d^3 + 10*a^6*b*d^4)*x)/(a^3*b^6*x^3 + a^4*b^5)]
```

giac [A] time = 0.19, size = 412, normalized size = 1.54

$$-\frac{2\sqrt{3}\left(b^4c^4+2ab^3c^3d-12a^2b^2c^2d^2+14a^3bcd^3-5a^4d^4\right)\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\left(-ab^2\right)^{\frac{2}{3}}ab^3}-\frac{\left(b^4c^4+2ab^3c^3d-12a^2b^2c^2d^2+14a^3bcd^3-5a^4d^4\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{9\left(-ab^2\right)^{\frac{2}{3}}ab^3}-\frac{2\left(b^4c^4+2ab^3c^3d-12a^2b^2c^2d^2+14a^3bcd^3-5a^4d^4\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9a^2b^4}+\frac{b^4c^4x-4ab^3c^3dx+6a^2b^2c^2d^2x-4a^3bcd^3x+a^4d^4x}{3\left(bx^3+a\right)ab^4}+\frac{2b^{12}d^4x^7+14b^{12}cd^3x^4-7ab^{11}d^4x^4+84b^{12}c^2d^2x-112ab^{11}cd^3x+42a^2b^{10}d^4x}{14b^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^4/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] -2/9*sqrt(3)*(b^4*c^4 + 2*a*b^3*c^3*d - 12*a^2*b^2*c^2*d^2 + 14*a^3*b*c*d^3
 - 5*a^4*d^4)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/((-a*b^
2)^(2/3)*a*b^3) - 1/9*(b^4*c^4 + 2*a*b^3*c^3*d - 12*a^2*b^2*c^2*d^2 + 14*a^
3*b*c*d^3 - 5*a^4*d^4)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/((-a*b^2)^(
2/3)*a*b^3) - 2/9*(b^4*c^4 + 2*a*b^3*c^3*d - 12*a^2*b^2*c^2*d^2 + 14*a^3*b*
c*d^3 - 5*a^4*d^4)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a^2*b^4) + 1/3*
(b^4*c^4*x - 4*a*b^3*c^3*d*x + 6*a^2*b^2*c^2*d^2*x - 4*a^3*b*c*d^3*x + a^4*
d^4*x)/((b*x^3 + a)*a*b^4) + 1/14*(2*b^12*d^4*x^7 + 14*b^12*c*d^3*x^4 - 7*a
*b^11*d^4*x^4 + 84*b^12*c^2*d^2*x - 112*a*b^11*c*d^3*x + 42*a^2*b^10*d^4*x)
/b^14
```

maple [B] time = 0.06, size = 708, normalized size = 2.65

$$\frac{d^4x^7}{7b^2}-\frac{ad^4x^4}{2b^3}+\frac{cd^3x^4}{b^2}+\frac{a^3d^4x}{3\left(bx^3+a\right)b^4}-\frac{4a^2cd^3x}{3\left(bx^3+a\right)b^3}+\frac{2ac^2d^2x}{\left(bx^3+a\right)b^2}+\frac{c^4x}{3\left(bx^3+a\right)a}-\frac{4c^3dx}{3\left(bx^3+a\right)b}-\frac{10\sqrt{3}\,a^3d^4\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{9\left(\frac{a}{b}\right)^{\frac{2}{3}}b^5}-\frac{10a^3d^4\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\left(\frac{a}{b}\right)^{\frac{2}{3}}b^5}+\frac{5a^3d^4\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{9\left(\frac{a}{b}\right)^{\frac{2}{3}}b^5}+\frac{28\sqrt{3}\,a^2cd^3\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{9\left(\frac{a}{b}\right)^{\frac{2}{3}}b^4}+\frac{28a^2cd^3\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\left(\frac{a}{b}\right)^{\frac{2}{3}}b^4}-\frac{14a^2cd^3\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{9\left(\frac{a}{b}\right)^{\frac{2}{3}}b^4}+\frac{3a^2d^4x}{b^4}-\frac{8\sqrt{3}\,ac^2d^2\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3\left(\frac{a}{b}\right)^{\frac{2}{3}}b^3}-\frac{8ac^2d^2\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{2}{3}}b^3}+\frac{4ac^2d^2\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{2}{3}}b^3}-\frac{8acd^3x}{b^3}+\frac{2\sqrt{3}\,c^4\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{9\left(\frac{a}{b}\right)^{\frac{2}{3}}ab}+\frac{2c^4\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\left(\frac{a}{b}\right)^{\frac{2}{3}}ab}-\frac{c^4\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{9\left(\frac{a}{b}\right)^{\frac{2}{3}}ab}+\frac{4\sqrt{3}\,c^3d\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{9\left(\frac{a}{b}\right)^{\frac{2}{3}}b^2}+\frac{4c^3d\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\left(\frac{a}{b}\right)^{\frac{2}{3}}b^2}-\frac{2c^3d\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{9\left(\frac{a}{b}\right)^{\frac{2}{3}}b^2}+\frac{6c^2d^2x}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c)^4/(b*x^3+a)^2,x)
```

```
[Out] -8/3/b^3*a/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*c^2*
d^2+28/9/b^4*a^2/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1)
)*c*d^3+6*d^2/b^2*c^2*x-1/2*d^4/b^3*x^4*a+d^3/b^2*x^4*c+1/3/a*x/(b*x^3+a)*c
^4+3*d^4/b^4*a^2*x-10/9/b^5*a^3/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(
a/b)^(1/3)*x-1))*d^4+4/9/b^2/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b
)^(1/3)*x-1))*c^3*d+2/9/b/a/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)
^(1/3)*x-1))*c^4-4/3/b^3*a^2*x/(b*x^3+a)*c*d^3+2/b^2*a*x/(b*x^3+a)*c^2*d^2+
28/9/b^4*a^2/(a/b)^(2/3)*ln(x+(a/b)^(1/3))*c*d^3-8/3/b^3*a/(a/b)^(2/3)*ln(x
+(a/b)^(1/3))*c^2*d^2-14/9/b^4*a^2/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(
2/3))*c*d^3+4/3/b^3*a/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*c^2*d^2
+5/9/b^5*a^3/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*d^4-10/9/b^5*a^3
/(a/b)^(2/3)*ln(x+(a/b)^(1/3))*d^4-2/9/b^2/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x
+(a/b)^(2/3))*c^3*d-1/9/b/a/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*c
^4+1/3/b^4*a^3*x/(b*x^3+a)*d^4-4/3/b*x/(b*x^3+a)*c^3*d-8*d^3/b^3*a*c*x+4/9/
b^2/(a/b)^(2/3)*ln(x+(a/b)^(1/3))*c^3*d+2/9/b/a/(a/b)^(2/3)*ln(x+(a/b)^(1/3
))*c^4+1/7*d^4*x^7/b^2
```

maxima [A] time = 1.16, size = 397, normalized size = 1.49

$$\frac{\left(b^4c^4-4\,ab^3c^3d+6\,a^2b^2c^2d^2-4\,a^3bcd^3+a^4d^4\right)x}{3\left(ab^5x^3+a^2b^4\right)}+\frac{2\,b^2d^4x^7+7\left(2\,b^2cd^3-abd^4\right)x^4+14\left(6\,b^2c^2d^2-8\,abcd^3+3\,a^2d^4\right)x}{14\,b^4}+\frac{2\sqrt{3}\left(b^4c^4+2\,ab^3c^3d-12\,a^2b^2c^2d^2+14\,a^3bcd^3-5\,a^4d^4\right)\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,ab^5\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\left(b^4c^4+2\,ab^3c^3d-12\,a^2b^2c^2d^2+14\,a^3bcd^3-5\,a^4d^4\right)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{9\,ab^5\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{2\left(b^4c^4+2\,ab^3c^3d-12\,a^2b^2c^2d^2+14\,a^3bcd^3-5\,a^4d^4\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\,ab^5\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^4/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] 1/3*(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)
*x/(a*b^5*x^3 + a^2*b^4) + 1/14*(2*b^2*d^4*x^7 + 7*(2*b^2*c*d^3 - a*b*d^4)*
x^4 + 14*(6*b^2*c^2*d^2 - 8*a*b*c*d^3 + 3*a^2*d^4)*x)/b^4 + 2/9*sqrt(3)*(b^
4*c^4 + 2*a*b^3*c^3*d - 12*a^2*b^2*c^2*d^2 + 14*a^3*b*c*d^3 - 5*a^4*d^4)*ar
ctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a*b^5*(a/b)^(2/3)) - 1/9
*(b^4*c^4 + 2*a*b^3*c^3*d - 12*a^2*b^2*c^2*d^2 + 14*a^3*b*c*d^3 - 5*a^4*d^4
)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a*b^5*(a/b)^(2/3)) + 2/9*(b^4*c^4
 + 2*a*b^3*c^3*d - 12*a^2*b^2*c^2*d^2 + 14*a^3*b*c*d^3 - 5*a^4*d^4)*log(x +
 (a/b)^(1/3))/(a*b^5*(a/b)^(2/3))
```

mupad [B] time = 1.49, size = 302, normalized size = 1.13

$$x\left(\frac{2\,a\left(\frac{2\,a\,d^4}{b^3}-\frac{4\,c\,d^3}{b^2}\right)}{b}-\frac{a^2\,d^4}{b^4}+\frac{6\,c^2\,d^2}{b^2}\right)-x^4\left(\frac{a\,d^4}{2\,b^3}-\frac{c\,d^3}{b^2}\right)+\frac{d^4\,x^7}{7\,b^2}+\frac{x\left(a^4\,d^4-4\,a^3\,b\,c\,d^3+6\,a^2\,b^2\,c^2\,d^2-4\,a\,b^3\,c^3\,d+b^4\,c^4\right)}{3\,a\left(b^5\,x^3+a\,b^4\right)}-\frac{2\ln\left(b^{1/3}\,x+a^{1/3}\right)\left(a\,d-b\,c\right)^3\left(5\,a\,d+b\,c\right)}{9\,a^{5/3}\,b^{13/3}}+\frac{2\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(a\,d-b\,c\right)^3\left(5\,a\,d+b\,c\right)}{9\,a^{5/3}\,b^{13/3}}-\frac{2\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(a\,d-b\,c\right)^3\left(5\,a\,d+b\,c\right)}{9\,a^{5/3}\,b^{13/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3)^4/(a + b*x^3)^2,x)
```

```
[Out] x*((2*a*((2*a*d^4)/b^3 - (4*c*d^3)/b^2))/b - (a^2*d^4)/b^4 + (6*c^2*d^2)/b^
2) - x^4*((a*d^4)/(2*b^3) - (c*d^3)/b^2) + (d^4*x^7)/(7*b^2) + (x*(a^4*d^4
+ b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3))/(3*a*(a*b^4
 + b^5*x^3)) - (2*log(b^(1/3)*x + a^(1/3))*(a*d - b*c)^3*(5*a*d + b*c))/(9*
a^(5/3)*b^(13/3)) + (2*log(3^(1/2)*a^(1/3)*1i - 2*b^(1/3)*x + a^(1/3))*((3^
(1/2)*1i)/2 + 1/2)*(a*d - b*c)^3*(5*a*d + b*c))/(9*a^(5/3)*b^(13/3)) - (2*l
og(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*x - a^(1/3))*((3^(1/2)*1i)/2 - 1/2)*(a*d
- b*c)^3*(5*a*d + b*c))/(9*a^(5/3)*b^(13/3))
```

sympy [A] time = 8.54, size = 405, normalized size = 1.52

$$x^4\left(-\frac{ad^4}{2b^3}+\frac{cd^3}{b^2}\right)+x\left(\frac{3a^2d^4}{b^4}-\frac{8acd^3}{b^3}+\frac{6c^2d^2}{b^2}\right)+\frac{x\left(a^4d^4-4a^3bcd^3+6a^2b^2c^2d^2-4ab^3c^3d+b^4c^4\right)}{3a^2b^4+3ab^5x^3}+\operatorname{RootSum}\left(729t^3a^5b^{13}+1000a^{12}d^{12}-8400a^{11}bcd^{11}+30720a^{10}b^2c^2d^{10}-63472a^9b^3c^3d^9+79848a^8b^4c^4d^8-60192a^7b^5c^5d^7+22848a^6b^6c^6d^6+288a^5b^7c^7d^5-3528a^4b^8c^8d^4+752a^3b^9c^9d^3+192a^2b^{10}c^{10}d^2-48ab^{11}c^{11}d-8b^{12}c^{12},\left(t\mapsto t\log\left(-\frac{9ta^2b^4}{10a^4d^4-28a^3bcd^3+24a^2b^2c^2d^2-4ab^3c^3d-2b^4c^4}+x\right)\right)\right)+\frac{d^4x^7}{7b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c)**4/(b*x**3+a)**2,x)
```

```
[Out] x**4*(-a*d**4/(2*b**3) + c*d**3/b**2) + x*(3*a**2*d**4/b**4 - 8*a*c*d**3/b*
*3 + 6*c**2*d**2/b**2) + x*(a**4*d**4 - 4*a**3*b*c*d**3 + 6*a**2*b**2*c**2*
d**2 - 4*a*b**3*c**3*d + b**4*c**4)/(3*a**2*b**4 + 3*a*b**5*x**3) + RootSum
(729*_t**3*a**5*b**13 + 1000*a**12*d**12 - 8400*a**11*b*c*d**11 + 30720*a**
10*b**2*c**2*d**10 - 63472*a**9*b**3*c**3*d**9 + 79848*a**8*b**4*c**4*d**8
- 60192*a**7*b**5*c**5*d**7 + 22848*a**6*b**6*c**6*d**6 + 288*a**5*b**7*c**
7*d**5 - 3528*a**4*b**8*c**8*d**4 + 752*a**3*b**9*c**9*d**3 + 192*a**2*b**1
0*c**10*d**2 - 48*a*b**11*c**11*d - 8*b**12*c**12, Lambda(_t, _t*log(-9*_t*
a**2*b**4/(10*a**4*d**4 - 28*a**3*b*c*d**3 + 24*a**2*b**2*c**2*d**2 - 4*a*b
**3*c**3*d - 2*b**4*c**4) + x))) + d**4*x**7/(7*b**2)
```

3.22 $\int \frac{(c+dx^3)^3}{(a+bx^3)^2}\,dx$

Optimal. Leaf size=234

$$-\frac{(bc-ad)^2(7ad+2bc)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18a^{5/3}b^{10/3}}+\frac{(bc-ad)^2(7ad+2bc)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{5/3}b^{10/3}}-\frac{(bc-ad)^2(7ad+}{3\sqrt{3}}$$

Rubi [A] time = 0.22, antiderivative size = 234, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.421, Rules used = {390, 385, 200, 31, 634, 617, 204, 628}

$$-\frac{(bc-ad)^2(7ad+2bc)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18a^{5/3}b^{10/3}}+\frac{(bc-ad)^2(7ad+2bc)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{5/3}b^{10/3}}-\frac{(bc-ad)^2(7ad+2bc)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{5/3}b^{10/3}}+\frac{d^2x(3bc-2ad)}{b^3}+\frac{x(bc-ad)^3}{3ab^3\left(a+bx^3\right)}+\frac{d^3x^4}{4b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3)^3/(a + b*x^3)^2,x]
```

```
[Out] (d^2*(3*b*c - 2*a*d)*x)/b^3 + (d^3*x^4)/(4*b^2) + ((b*c - a*d)^3*x)/(3*a*b^
3*(a + b*x^3)) - ((b*c - a*d)^2*(2*b*c + 7*a*d)*ArcTan[(a^(1/3) - 2*b^(1/3)
*x)/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(5/3)*b^(10/3)) + ((b*c - a*d)^2*(2*b*
c + 7*a*d)*Log[a^(1/3) + b^(1/3)*x])/(9*a^(5/3)*b^(10/3)) - ((b*c - a*d)^2*
(2*b*c + 7*a*d)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(18*a^(5/3)
*b^(10/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{\left(c+dx^3\right)^3}{\left(a+bx^3\right)^2}\,dx = \int \left(\frac{d^2(3bc-2ad)}{b^3}+\frac{d^3x^3}{b^2}+\frac{(bc-ad)^2(bc+2ad)+3bd(bc-ad)^2x^3}{b^3\left(a+bx^3\right)^2}\right)dx$$

$$=\frac{d^2(3bc-2ad)x}{b^3}+\frac{d^3x^4}{4b^2}+\frac{\int \frac{(bc-ad)^2(bc+2ad)+3bd(bc-ad)^2x^3}{\left(a+bx^3\right)^2}\,dx}{b^3}$$

$$=\frac{d^2(3bc-2ad)x}{b^3}+\frac{d^3x^4}{4b^2}+\frac{(bc-ad)^3x}{3ab^3\left(a+bx^3\right)}+\frac{\left((bc-ad)^2(2bc+7ad)\right)\int \frac{1}{a+bx^3}\,dx}{3ab^3}$$

$$=\frac{d^2(3bc-2ad)x}{b^3}+\frac{d^3x^4}{4b^2}+\frac{(bc-ad)^3x}{3ab^3\left(a+bx^3\right)}+\frac{\left((bc-ad)^2(2bc+7ad)\right)\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}\,x}\,dx}{9a^{5/3}b^3}+$$

$$=\frac{d^2(3bc-2ad)x}{b^3}+\frac{d^3x^4}{4b^2}+\frac{(bc-ad)^3x}{3ab^3\left(a+bx^3\right)}+\frac{(bc-ad)^2(2bc+7ad)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{9a^{5/3}b^{10/3}}-$$

$$=\frac{d^2(3bc-2ad)x}{b^3}+\frac{d^3x^4}{4b^2}+\frac{(bc-ad)^3x}{3ab^3\left(a+bx^3\right)}+\frac{(bc-ad)^2(2bc+7ad)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{9a^{5/3}b^{10/3}}-$$

$$=\frac{d^2(3bc-2ad)x}{b^3}+\frac{d^3x^4}{4b^2}+\frac{(bc-ad)^3x}{3ab^3\left(a+bx^3\right)}-\frac{(bc-ad)^2(2bc+7ad)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{5/3}b^{10/3}}+$$

Mathematica [A] time = 0.16, size = 227, normalized size = 0.97

$$\frac{-\frac{2(bc-ad)^2(7ad+2bc)\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{a^{5/3}}+\frac{4(bc-ad)^2(7ad+2bc)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{a^{5/3}}+\frac{4\sqrt{3}\,(bc-ad)^2(7ad+2bc)\tan^{-1}\left(\frac{2\sqrt[3]{b}\,x-\sqrt[3]{a}}{\sqrt{3}\,\sqrt[3]{a}}\right)}{a^{5/3}}+36\sqrt[3]{b}\,d^2x(3bc-2ad)+\frac{12\sqrt[3]{b}\,x(bc-ad)^3}{a\left(a+bx^3\right)}+9b^{4/3}d^3x^4}{36b^{10/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^3)^3/(a + b*x^3)^2,x]
```

```
[Out] (36*b^(1/3)*d^2*(3*b*c - 2*a*d)*x + 9*b^(4/3)*d^3*x^4 + (12*b^(1/3)*(b*c -
a*d)^3*x)/(a*(a + b*x^3)) + (4*Sqrt[3]*(b*c - a*d)^2*(2*b*c + 7*a*d)*ArcTan
[(-a^(1/3) + 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/a^(5/3) + (4*(b*c - a*d)^2*(2
*b*c + 7*a*d)*Log[a^(1/3) + b^(1/3)*x])/a^(5/3) - (2*(b*c - a*d)^2*(2*b*c +
 7*a*d)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/a^(5/3))/(36*b^(10/
3))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c+dx^3\right)^3}{\left(a+bx^3\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x^3)^3/(a + b*x^3)^2,x]
```

```
[Out] IntegrateAlgebraic[(c + d*x^3)^3/(a + b*x^3)^2, x]
```

fricas [B] time = 0.93, size = 1027, normalized size = 4.39

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^3/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] [1/36*(9*a^3*b^3*d^3*x^7 + 9*(12*a^3*b^3*c*d^2 - 7*a^4*b^2*d^3)*x^4 + 6*sqr
t(1/3)*(2*a^2*b^4*c^3 + 3*a^3*b^3*c^2*d - 12*a^4*b^2*c*d^2 + 7*a^5*b*d^3 +
(2*a*b^5*c^3 + 3*a^2*b^4*c^2*d - 12*a^3*b^3*c*d^2 + 7*a^4*b^2*d^3)*x^3)*sqr
t(-(a^2*b)^(1/3)/b)*log((2*a*b*x^3 - 3*(a^2*b)^(1/3)*a*x - a^2 + 3*sqrt(1/3
)*(2*a*b*x^2 + (a^2*b)^(2/3)*x - (a^2*b)^(1/3)*a)*sqrt(-(a^2*b)^(1/3)/b))/(
b*x^3 + a)) - 2*(2*a*b^3*c^3 + 3*a^2*b^2*c^2*d - 12*a^3*b*c*d^2 + 7*a^4*d^3
 + (2*b^4*c^3 + 3*a*b^3*c^2*d - 12*a^2*b^2*c*d^2 + 7*a^3*b*d^3)*x^3)*(a^2*b
)^(2/3)*log(a*b*x^2 - (a^2*b)^(2/3)*x + (a^2*b)^(1/3)*a) + 4*(2*a*b^3*c^3 +
 3*a^2*b^2*c^2*d - 12*a^3*b*c*d^2 + 7*a^4*d^3 + (2*b^4*c^3 + 3*a*b^3*c^2*d
- 12*a^2*b^2*c*d^2 + 7*a^3*b*d^3)*x^3)*(a^2*b)^(2/3)*log(a*b*x + (a^2*b)^(2
/3)) + 12*(a^2*b^4*c^3 - 3*a^3*b^3*c^2*d + 12*a^4*b^2*c*d^2 - 7*a^5*b*d^3)*
x)/(a^3*b^5*x^3 + a^4*b^4), 1/36*(9*a^3*b^3*d^3*x^7 + 9*(12*a^3*b^3*c*d^2 -
 7*a^4*b^2*d^3)*x^4 + 12*sqrt(1/3)*(2*a^2*b^4*c^3 + 3*a^3*b^3*c^2*d - 12*a^
4*b^2*c*d^2 + 7*a^5*b*d^3 + (2*a*b^5*c^3 + 3*a^2*b^4*c^2*d - 12*a^3*b^3*c*d
^2 + 7*a^4*b^2*d^3)*x^3)*sqrt((a^2*b)^(1/3)/b)*arctan(sqrt(1/3)*(2*(a^2*b)^
(2/3)*x - (a^2*b)^(1/3)*a)*sqrt((a^2*b)^(1/3)/b)/a^2) - 2*(2*a*b^3*c^3 + 3*
a^2*b^2*c^2*d - 12*a^3*b*c*d^2 + 7*a^4*d^3 + (2*b^4*c^3 + 3*a*b^3*c^2*d - 1
2*a^2*b^2*c*d^2 + 7*a^3*b*d^3)*x^3)*(a^2*b)^(2/3)*log(a*b*x^2 - (a^2*b)^(2/
3)*x + (a^2*b)^(1/3)*a) + 4*(2*a*b^3*c^3 + 3*a^2*b^2*c^2*d - 12*a^3*b*c*d^2
 + 7*a^4*d^3 + (2*b^4*c^3 + 3*a*b^3*c^2*d - 12*a^2*b^2*c*d^2 + 7*a^3*b*d^3)
*x^3)*(a^2*b)^(2/3)*log(a*b*x + (a^2*b)^(2/3)) + 12*(a^2*b^4*c^3 - 3*a^3*b^
3*c^2*d + 12*a^4*b^2*c*d^2 - 7*a^5*b*d^3)*x)/(a^3*b^5*x^3 + a^4*b^4)]
```

giac [A] time = 0.20, size = 319, normalized size = 1.36

$$-\frac{\sqrt{3}\left(2\,b^{3}c^{3}+3\,ab^{2}c^{2}d-12\,a^{2}bcd^{2}+7\,a^{3}d^{3}\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,\left(-ab^{2}\right)^{\frac{2}{3}}ab^{2}}-\frac{\left(2\,b^{3}c^{3}+3\,ab^{2}c^{2}d-12\,a^{2}bcd^{2}+7\,a^{3}d^{3}\right)\log\left(x^{2}+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,\left(-ab^{2}\right)^{\frac{2}{3}}ab^{2}}-\frac{\left(2\,b^{3}c^{3}+3\,ab^{2}c^{2}d-12\,a^{2}bcd^{2}+7\,a^{3}d^{3}\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\,a^{2}b^{3}}+\frac{b^{3}c^{3}x-3\,ab^{2}c^{2}dx+3\,a^{2}bcd^{2}x-a^{3}d^{3}x}{3\left(bx^{3}+a\right)ab^{3}}+\frac{b^{6}d^{3}x^{4}+12\,b^{6}cd^{2}x-8\,ab^{5}d^{3}x}{4\,b^{8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^3/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] -1/9*sqrt(3)*(2*b^3*c^3 + 3*a*b^2*c^2*d - 12*a^2*b*c*d^2 + 7*a^3*d^3)*arcta
n(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/((-a*b^2)^(2/3)*a*b^2) - 1
/18*(2*b^3*c^3 + 3*a*b^2*c^2*d - 12*a^2*b*c*d^2 + 7*a^3*d^3)*log(x^2 + x*(-
a/b)^(1/3) + (-a/b)^(2/3))/((-a*b^2)^(2/3)*a*b^2) - 1/9*(2*b^3*c^3 + 3*a*b^
```

```
2*c^2*d - 12*a^2*b*c*d^2 + 7*a^3*d^3)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)
))/(a^2*b^3) + 1/3*(b^3*c^3*x - 3*a*b^2*c^2*d*x + 3*a^2*b*c*d^2*x - a^3*d^3
*x)/((b*x^3 + a)*a*b^3) + 1/4*(b^6*d^3*x^4 + 12*b^6*c*d^2*x - 8*a*b^5*d^3*x
)/b^8
```

maple [B] time = 0.05, size = 529, normalized size = 2.26

$$\frac{d^3x^4}{4b^2}-\frac{a^2d^3x}{3\left(bx^3+a\right)b^3}+\frac{ac\,d^2x}{\left(bx^3+a\right)b^2}+\frac{c^3x}{3\left(bx^3+a\right)a}-\frac{c^2dx}{\left(bx^3+a\right)b}+\frac{7\sqrt{3}\,a^2d^3\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{9\left(\frac{a}{b}\right)^{\frac{2}{3}}b^4}+\frac{7a^2d^3\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\left(\frac{a}{b}\right)^{\frac{2}{3}}b^4}-\frac{7a^2d^3\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\left(\frac{a}{b}\right)^{\frac{2}{3}}b^4}-\frac{4\sqrt{3}\,ac\,d^2\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3\left(\frac{a}{b}\right)^{\frac{2}{3}}b^3}-\frac{4ac\,d^2\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{2}{3}}b^3}+\frac{2ac\,d^2\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{2}{3}}b^3}-\frac{2a\,d^3x}{b^3}+\frac{2\sqrt{3}\,c^3\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{9\left(\frac{a}{b}\right)^{\frac{2}{3}}ab}+\frac{2c^3\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\left(\frac{a}{b}\right)^{\frac{2}{3}}ab}-\frac{c^3\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{9\left(\frac{a}{b}\right)^{\frac{2}{3}}ab}+\frac{\sqrt{3}\,c^2d\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3\left(\frac{a}{b}\right)^{\frac{2}{3}}b^2}+\frac{c^2d\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{2}{3}}b^2}-\frac{c^2d\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(\frac{a}{b}\right)^{\frac{2}{3}}b^2}+\frac{3c\,d^2x}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c)^3/(b*x^3+a)^2,x)
```

```
[Out] 1/4*d^3*x^4/b^2-2*d^3/b^3*a*x+3*d^2/b^2*c*x-1/3/b^3*a^2*x/(b*x^3+a)*d^3+1/b
^2*a*x/(b*x^3+a)*c*d^2-1/b*x/(b*x^3+a)*c^2*d+1/3/a*x/(b*x^3+a)*c^3+7/9/b^4*
a^2/(a/b)^(2/3)*ln(x+(a/b)^(1/3))*d^3-4/3/b^3*a/(a/b)^(2/3)*ln(x+(a/b)^(1/3
))*c*d^2+1/3/b^2/(a/b)^(2/3)*ln(x+(a/b)^(1/3))*c^2*d+2/9/b/a/(a/b)^(2/3)*ln
(x+(a/b)^(1/3))*c^3-7/18/b^4*a^2/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/
3))*d^3+2/3/b^3*a/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*c*d^2-1/6/b
^2/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*c^2*d-1/9/b/a/(a/b)^(2/3)*
ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*c^3+7/9/b^4*a^2/(a/b)^(2/3)*3^(1/2)*arcta
n(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*d^3-4/3/b^3*a/(a/b)^(2/3)*3^(1/2)*arctan
(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*c*d^2+1/3/b^2/(a/b)^(2/3)*3^(1/2)*arctan(
1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*c^2*d+2/9/b/a/(a/b)^(2/3)*3^(1/2)*arctan(1
/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*c^3
```

maxima [A] time = 1.22, size = 306, normalized size = 1.31

$$\frac{\left(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3\right)x}{3\left(ab^4x^3+a^2b^3\right)}+\frac{bd^3x^4+4\left(3\,bcd^2-2\,ad^3\right)x}{4\,b^3}+\frac{\sqrt{3}\left(2\,b^3c^3+3\,ab^2c^2d-12\,a^2bcd^2+7\,a^3d^3\right)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,ab^4\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\left(2\,b^3c^3+3\,ab^2c^2d-12\,a^2bcd^2+7\,a^3d^3\right)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,ab^4\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\left(2\,b^3c^3+3\,ab^2c^2d-12\,a^2bcd^2+7\,a^3d^3\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\,ab^4\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^3/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] 1/3*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*x/(a*b^4*x^3 + a^2*
b^3) + 1/4*(b*d^3*x^4 + 4*(3*b*c*d^2 - 2*a*d^3)*x)/b^3 + 1/9*sqrt(3)*(2*b^3
*c^3 + 3*a*b^2*c^2*d - 12*a^2*b*c*d^2 + 7*a^3*d^3)*arctan(1/3*sqrt(3)*(2*x
- (a/b)^(1/3))/(a/b)^(1/3))/(a*b^4*(a/b)^(2/3)) - 1/18*(2*b^3*c^3 + 3*a*b^2
*c^2*d - 12*a^2*b*c*d^2 + 7*a^3*d^3)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))
/(a*b^4*(a/b)^(2/3)) + 1/9*(2*b^3*c^3 + 3*a*b^2*c^2*d - 12*a^2*b*c*d^2 + 7*
a^3*d^3)*log(x + (a/b)^(1/3))/(a*b^4*(a/b)^(2/3))
```

mupad [B] time = 0.30, size = 240, normalized size = 1.03

$$\frac{d^3\,x^4}{4\,b^2}-x\,\left(\frac{2\,a\,d^3}{b^3}-\frac{3\,c\,d^2}{b^2}\right)-\frac{x\,\left(a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b^3\,c^3\right)}{3\,a\,\left(b^4\,x^3+a\,b^3\right)}+\frac{\ln\left(b^{1/3}\,x+a^{1/3}\right)\,{\left(a\,d-b\,c\right)}^2\,\left(7\,a\,d+2\,b\,c\right)}{9\,a^{5/3}\,b^{10/3}}-\frac{\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1{}\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,{\left(a\,d-b\,c\right)}^2\,\left(7\,a\,d+2\,b\,c\right)}{9\,a^{5/3}\,b^{10/3}}+\frac{\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1{}\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,{\left(a\,d-b\,c\right)}^2\,\left(7\,a\,d+2\,b\,c\right)}{9\,a^{5/3}\,b^{10/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3)^3/(a + b*x^3)^2,x)
```

```
[Out] (d^3*x^4)/(4*b^2) - x*((2*a*d^3)/b^3 - (3*c*d^2)/b^2) - (x*(a^3*d^3 - b^3*c
^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2))/(3*a*(a*b^3 + b^4*x^3)) + (log(b^(1/3)
*x + a^(1/3))*(a*d - b*c)^2*(7*a*d + 2*b*c))/(9*a^(5/3)*b^(10/3)) - (log(3^
(1/2)*a^(1/3)*1i - 2*b^(1/3)*x + a^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(a*d - b*c
)^2*(7*a*d + 2*b*c))/(9*a^(5/3)*b^(10/3)) + (log(3^(1/2)*a^(1/3)*1i + 2*b^(
1/3)*x - a^(1/3))*((3^(1/2)*1i)/2 - 1/2)*(a*d - b*c)^2*(7*a*d + 2*b*c))/(9*
a^(5/3)*b^(10/3))
```

sympy [A] time = 4.33, size = 291, normalized size = 1.24

$$x\left(-\frac{2ad^3}{b^3}+\frac{3cd^2}{b^2}\right)+\frac{x\left(-a^3d^3+3a^2bcd^2-3ab^2c^2d+b^3c^3\right)}{3a^2b^3+3ab^4x^3}+\operatorname{RootSum}\left(729t^3a^5b^{10}-343a^9d^9+1764a^8bcd^8-3465a^7b^2c^2d^7+2946a^6b^3c^3d^6-477a^5b^4c^4d^5-792a^4b^5c^5d^4+321a^3b^6c^6d^3+90a^2b^7c^7d^2-36ab^8c^8d-8b^9c^9,\left(t\mapsto t\log\left(\frac{9ta^2b^3}{7a^3d^3-12a^2bcd^2+3ab^2c^2d+2b^3c^3}+x\right)\right)\right)+\frac{d^3x^4}{4b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c)**3/(b*x**3+a)**2,x)
```

```
[Out] x*(-2*a*d**3/b**3 + 3*c*d**2/b**2) + x*(-a**3*d**3 + 3*a**2*b*c*d**2 - 3*a*
b**2*c**2*d + b**3*c**3)/(3*a**2*b**3 + 3*a*b**4*x**3) + RootSum(729*_t**3*
a**5*b**10 - 343*a**9*d**9 + 1764*a**8*b*c*d**8 - 3465*a**7*b**2*c**2*d**7
+ 2946*a**6*b**3*c**3*d**6 - 477*a**5*b**4*c**4*d**5 - 792*a**4*b**5*c**5*d
**4 + 321*a**3*b**6*c**6*d**3 + 90*a**2*b**7*c**7*d**2 - 36*a*b**8*c**8*d -
 8*b**9*c**9, Lambda(_t, _t*log(9*_t*a**2*b**3/(7*a**3*d**3 - 12*a**2*b*c*d
**2 + 3*a*b**2*c**2*d + 2*b**3*c**3) + x))) + d**3*x**4/(4*b**2)
```

3.23 $\int \frac{(c+dx^3)^2}{(a+bx^3)^2}\,dx$

Optimal. Leaf size=203

$$-\frac{(bc-ad)(2ad+bc)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{9a^{5/3}b^{7/3}}+\frac{2(bc-ad)(2ad+bc)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{9a^{5/3}b^{7/3}}-\frac{2(bc-ad)(2ad+}{3\sqrt{3}}$$

Rubi [A] time = 0.23, antiderivative size = 203, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.421, Rules used = {390, 385, 200, 31, 634, 617, 204, 628}

$$-\frac{(bc-ad)(2ad+bc)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{9a^{5/3}b^{7/3}}+\frac{2(bc-ad)(2ad+bc)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{9a^{5/3}b^{7/3}}-\frac{2(bc-ad)(2ad+bc)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{5/3}b^{7/3}}+\frac{x(bc-ad)^2}{3ab^2\left(a+bx^3\right)}+\frac{d^2x}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3)^2/(a + b*x^3)^2,x]
```

```
[Out] (d^2*x)/b^2 + ((b*c - a*d)^2*x)/(3*a*b^2*(a + b*x^3)) - (2*(b*c - a*d)*(b*c
 + 2*a*d)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(
5/3)*b^(7/3)) + (2*(b*c - a*d)*(b*c + 2*a*d)*Log[a^(1/3) + b^(1/3)*x])/(9*a
^(5/3)*b^(7/3)) - ((b*c - a*d)*(b*c + 2*a*d)*Log[a^(2/3) - a^(1/3)*b^(1/3)*
x + b^(2/3)*x^2])/(9*a^(5/3)*b^(7/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{(c+dx^3)^2}{(a+bx^3)^2}\,dx = \int \left(\frac{d^2}{b^2}+\frac{b^2c^2-a^2d^2+2bd(bc-ad)x^3}{b^2(a+bx^3)^2}\right)dx$$

$$= \frac{d^2x}{b^2}+\frac{\int \frac{b^2c^2-a^2d^2+2bd(bc-ad)x^3}{(a+bx^3)^2}\,dx}{b^2}$$

$$= \frac{d^2x}{b^2}+\frac{(bc-ad)^2x}{3ab^2(a+bx^3)}+\frac{(2(bc-ad)(bc+2ad))\int \frac{1}{a+bx^3}\,dx}{3ab^2}$$

$$= \frac{d^2x}{b^2}+\frac{(bc-ad)^2x}{3ab^2(a+bx^3)}+\frac{(2(bc-ad)(bc+2ad))\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{9a^{5/3}b^2}+\frac{(2(bc-ad)(bc+2ad))\int \frac{2\cdots}{a^{2/3}-\cdots}}{9a^{5/3}b^2}$$

$$= \frac{d^2x}{b^2}+\frac{(bc-ad)^2x}{3ab^2(a+bx^3)}+\frac{2(bc-ad)(bc+2ad)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{5/3}b^{7/3}}-\frac{((bc-ad)(bc+2ad))\int \frac{-\cdots}{a^{2/3}-\cdots}}{9a^{5/3}b^{7/3}}$$

$$= \frac{d^2x}{b^2}+\frac{(bc-ad)^2x}{3ab^2(a+bx^3)}+\frac{2(bc-ad)(bc+2ad)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{5/3}b^{7/3}}-\frac{(bc-ad)(bc+2ad)\log\left(a^{2/\cdots}\right.}{9a^{5/3}b^{7/\cdots}}$$

$$= \frac{d^2x}{b^2}+\frac{(bc-ad)^2x}{3ab^2(a+bx^3)}-\frac{2(bc-ad)(bc+2ad)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{5/3}b^{7/3}}+\frac{2(bc-ad)(bc+2ad)\log\left(\sqrt[3]{\cdots}\right.}{9a^{5/3}b^{7/3}}$$

Mathematica [A] time = 0.20, size = 205, normalized size = 1.01

$$\frac{\frac{2(-2a^2d^2+abcd+b^2c^2)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{a^{5/3}}-\frac{2\sqrt{3}(-2a^2d^2+abcd+b^2c^2)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{a^{5/3}}-\frac{(-2a^2d^2+abcd+b^2c^2)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{a^{5/3}}+\frac{3\sqrt[3]{b}x(bc-ad)^2}{a(a+bx^3)}+9\sqrt[3]{b}d^2x}{9b^{7/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^3)^2/(a + b*x^3)^2,x]
```

```
[Out] (9*b^(1/3)*d^2*x + (3*b^(1/3)*(b*c - a*d)^2*x)/(a*(a + b*x^3)) - (2*Sqrt[3]
*(b^2*c^2 + a*b*c*d - 2*a^2*d^2)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]
])/a^(5/3) + (2*(b^2*c^2 + a*b*c*d - 2*a^2*d^2)*Log[a^(1/3) + b^(1/3)*x])/a
^(5/3) - ((b^2*c^2 + a*b*c*d - 2*a^2*d^2)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x +
 b^(2/3)*x^2])/a^(5/3))/(9*b^(7/3))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c+dx^3\right)^2}{\left(a+bx^3\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x^3)^2/(a + b*x^3)^2,x]
```

```
[Out] IntegrateAlgebraic[(c + d*x^3)^2/(a + b*x^3)^2, x]
```

fricas [B] time = 0.98, size = 768, normalized size = 3.78

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^2/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] [1/9*(9*a^3*b^2*d^2*x^4 - 3*sqrt(1/3)*(a^2*b^3*c^2 + a^3*b^2*c*d - 2*a^4*b*
d^2 + (a*b^4*c^2 + a^2*b^3*c*d - 2*a^3*b^2*d^2)*x^3)*sqrt((-a^2*b)^(1/3)/b)
*log((2*a*b*x^3 + 3*(-a^2*b)^(1/3)*a*x - a^2 - 3*sqrt(1/3)*(2*a*b*x^2 + (-a
^2*b)^(2/3)*x + (-a^2*b)^(1/3)*a)*sqrt((-a^2*b)^(1/3)/b))/(b*x^3 + a)) - (a
*b^2*c^2 + a^2*b*c*d - 2*a^3*d^2 + (b^3*c^2 + a*b^2*c*d - 2*a^2*b*d^2)*x^3)
*(-a^2*b)^(2/3)*log(a*b*x^2 - (-a^2*b)^(2/3)*x - (-a^2*b)^(1/3)*a) + 2*(a*b
^2*c^2 + a^2*b*c*d - 2*a^3*d^2 + (b^3*c^2 + a*b^2*c*d - 2*a^2*b*d^2)*x^3)*(
-a^2*b)^(2/3)*log(a*b*x + (-a^2*b)^(2/3)) + 3*(a^2*b^3*c^2 - 2*a^3*b^2*c*d
+ 4*a^4*b*d^2)*x)/(a^3*b^4*x^3 + a^4*b^3), 1/9*(9*a^3*b^2*d^2*x^4 + 6*sqrt(
1/3)*(a^2*b^3*c^2 + a^3*b^2*c*d - 2*a^4*b*d^2 + (a*b^4*c^2 + a^2*b^3*c*d -
2*a^3*b^2*d^2)*x^3)*sqrt(-(-a^2*b)^(1/3)/b)*arctan(sqrt(1/3)*(2*(-a^2*b)^(2
/3)*x + (-a^2*b)^(1/3)*a)*sqrt(-(-a^2*b)^(1/3)/b)/a^2) - (a*b^2*c^2 + a^2*b
*c*d - 2*a^3*d^2 + (b^3*c^2 + a*b^2*c*d - 2*a^2*b*d^2)*x^3)*(-a^2*b)^(2/3)*
log(a*b*x^2 - (-a^2*b)^(2/3)*x - (-a^2*b)^(1/3)*a) + 2*(a*b^2*c^2 + a^2*b*c
*d - 2*a^3*d^2 + (b^3*c^2 + a*b^2*c*d - 2*a^2*b*d^2)*x^3)*(-a^2*b)^(2/3)*lo
g(a*b*x + (-a^2*b)^(2/3)) + 3*(a^2*b^3*c^2 - 2*a^3*b^2*c*d + 4*a^4*b*d^2)*x
)/(a^3*b^4*x^3 + a^4*b^3)]
```

giac [A] time = 0.21, size = 227, normalized size = 1.12

$$\frac{d^2x}{b^2} - \frac{2\sqrt{3}\left(b^2c^2 + abcd - 2a^2d^2\right)\arctan\left(\frac{\sqrt{3}\left(2x + \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\left(-ab^2\right)^{\frac{2}{3}}ab} - \frac{\left(b^2c^2 + abcd - 2a^2d^2\right)\log\left(x^2 + x\left(-\frac{a}{b}\right)^{\frac{1}{3}} + \left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{9\left(-ab^2\right)^{\frac{2}{3}}ab} - \frac{2\left(b^2c^2 + abcd - 2a^2d^2\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9a^2b^2} + \frac{b^2c^2x - 2abcdx + a^2d^2x}{3\left(bx^3 + a\right)ab^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^2/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] d^2*x/b^2 - 2/9*sqrt(3)*(b^2*c^2 + a*b*c*d - 2*a^2*d^2)*arctan(1/3*sqrt(3)*
(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/((-a*b^2)^(2/3)*a*b) - 1/9*(b^2*c^2 + a*
b*c*d - 2*a^2*d^2)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/((-a*b^2)^(2/3)
*a*b) - 2/9*(b^2*c^2 + a*b*c*d - 2*a^2*d^2)*(-a/b)^(1/3)*log(abs(x - (-a/b)
^(1/3)))/(a^2*b^2) + 1/3*(b^2*c^2*x - 2*a*b*c*d*x + a^2*d^2*x)/((b*x^3 + a)
*a*b^2)
```

maple [B] time = 0.05, size = 367, normalized size = 1.81

$$\frac{a\,d^2 x}{3\left(b\,x^3+a\right)b^2}+\frac{c^2 x}{3\left(b\,x^3+a\right)a}-\frac{2cdx}{3\left(b\,x^3+a\right)b}-\frac{4\sqrt{3}\,a\,d^2\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{9\left(\frac{a}{b}\right)^{\frac{2}{3}}b^3}-\frac{4a\,d^2\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\left(\frac{a}{b}\right)^{\frac{2}{3}}b^3}+\frac{2a\,d^2\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{9\left(\frac{a}{b}\right)^{\frac{2}{3}}b^3}+\frac{2\sqrt{3}\,c^2\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{9\left(\frac{a}{b}\right)^{\frac{2}{3}}ab}+\frac{2c^2\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\left(\frac{a}{b}\right)^{\frac{2}{3}}ab}-\frac{c^2\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{9\left(\frac{a}{b}\right)^{\frac{2}{3}}ab}+\frac{2\sqrt{3}\,cd\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{9\left(\frac{a}{b}\right)^{\frac{2}{3}}b^2}+\frac{2cd\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\left(\frac{a}{b}\right)^{\frac{2}{3}}b^2}-\frac{cd\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{9\left(\frac{a}{b}\right)^{\frac{2}{3}}b^2}+\frac{d^2 x}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c)^2/(b*x^3+a)^2,x)
```

```
[Out] d^2*x/b^2+1/3/b^2*a*x/(b*x^3+a)*d^2-2/3/b*x/(b*x^3+a)*c*d+1/3/a*x/(b*x^3+a)
*c^2-4/9/b^3*a/(a/b)^(2/3)*ln(x+(a/b)^(1/3))*d^2+2/9/b^2/(a/b)^(2/3)*ln(x+(
a/b)^(1/3))*c*d+2/9/b/a/(a/b)^(2/3)*ln(x+(a/b)^(1/3))*c^2+2/9/b^3*a/(a/b)^(
2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*d^2-1/9/b^2/(a/b)^(2/3)*ln(x^2-(a/b)
^(1/3)*x+(a/b)^(2/3))*c*d-1/9/b/a/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2
/3))*c^2-4/9/b^3*a/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-
1))*d^2+2/9/b^2/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))
*c*d+2/9/b/a/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*c^
2
```

maxima [A] time = 1.39, size = 220, normalized size = 1.08

$$\frac{\left(b^2c^2-2\,abcd+a^2d^2\right)x}{3\left(ab^3x^3+a^2b^2\right)}+\frac{d^2x}{b^2}+\frac{2\sqrt{3}\left(b^2c^2+abcd-2\,a^2d^2\right)\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,ab^3\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\left(b^2c^2+abcd-2\,a^2d^2\right)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{9\,ab^3\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{2\left(b^2c^2+abcd-2\,a^2d^2\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\,ab^3\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^2/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] 1/3*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*x/(a*b^3*x^3 + a^2*b^2) + d^2*x/b^2 + 2
/9*sqrt(3)*(b^2*c^2 + a*b*c*d - 2*a^2*d^2)*arctan(1/3*sqrt(3)*(2*x - (a/b)^
(1/3))/(a/b)^(1/3))/(a*b^3*(a/b)^(2/3)) - 1/9*(b^2*c^2 + a*b*c*d - 2*a^2*d^
2)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a*b^3*(a/b)^(2/3)) + 2/9*(b^2*c^
2 + a*b*c*d - 2*a^2*d^2)*log(x + (a/b)^(1/3))/(a*b^3*(a/b)^(2/3))
```

mupad [B] time = 1.47, size = 191, normalized size = 0.94

$$\frac{d^2\,x}{b^2}+\frac{x\,\left(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2\right)}{3\,a\,\left(b^3\,x^3+a\,b^2\right)}-\frac{2\,\ln\left(b^{1/3}\,x+a^{1/3}\right)\,\left(a\,d-b\,c\right)\,\left(2\,a\,d+b\,c\right)}{9\,a^{5/3}\,b^{7/3}}-\frac{2\,\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1{}\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,\left(a\,d-b\,c\right)\,\left(2\,a\,d+b\,c\right)}{9\,a^{5/3}\,b^{7/3}}+\frac{2\,\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1{}\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,\left(a\,d-b\,c\right)\,\left(2\,a\,d+b\,c\right)}{9\,a^{5/3}\,b^{7/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3)^2/(a + b*x^3)^2,x)
```

```
[Out] (d^2*x)/b^2 + (x*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d))/(3*a*(a*b^2 + b^3*x^3)) -
 (2*log(b^(1/3)*x + a^(1/3))*(a*d - b*c)*(2*a*d + b*c))/(9*a^(5/3)*b^(7/3))
 - (2*log(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*x - a^(1/3))*((3^(1/2)*1i)/2 - 1/2
)*(a*d - b*c)*(2*a*d + b*c))/(9*a^(5/3)*b^(7/3)) + (2*log(3^(1/2)*a^(1/3)*1
i - 2*b^(1/3)*x + a^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(a*d - b*c)*(2*a*d + b*c)
)/(9*a^(5/3)*b^(7/3))
```

sympy [A] time = 2.56, size = 189, normalized size = 0.93

$$\frac{x\left(a^2d^2-2abcd+b^2c^2\right)}{3a^2b^2+3ab^3x^3}+\operatorname{RootSum}\left(729t^3a^5b^7+64a^6d^6-96a^5bcd^5-48a^4b^2c^2d^4+88a^3b^3c^3d^3+24a^2b^4c^4d^2-24ab^5c^5d-8b^6c^6,\left(t\mapsto t\log\left(-\frac{9ta^2b^2}{4a^2d^2-2abcd-2b^2c^2}+x\right)\right)\right)+\frac{d^2x}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c)**2/(b*x**3+a)**2,x)
```

```
[Out] x*(a**2*d**2 - 2*a*b*c*d + b**2*c**2)/(3*a**2*b**2 + 3*a*b**3*x**3) + RootS
um(729*_t**3*a**5*b**7 + 64*a**6*d**6 - 96*a**5*b*c*d**5 - 48*a**4*b**2*c**
2*d**4 + 88*a**3*b**3*c**3*d**3 + 24*a**2*b**4*c**4*d**2 - 24*a*b**5*c**5*d
 - 8*b**6*c**6, Lambda(_t, _t*log(-9*_t*a**2*b**2/(4*a**2*d**2 - 2*a*b*c*d
- 2*b**2*c**2) + x))) + d**2*x/b**2
```

3.24 $\int \frac{c+dx^3}{\left(a+bx^3\right)^2} \, dx$

Optimal. Leaf size=169

$$-\frac{(ad+2bc)\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{18a^{5/3}b^{4/3}}+\frac{(ad+2bc)\log\left(\sqrt[3]{a}\,+\,\sqrt[3]{b}\,x\right)}{9a^{5/3}b^{4/3}}-\frac{(ad+2bc)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{5/3}b^{4/3}}+\frac{x(bc}{3ab\,(a}$$

Rubi [A] time = 0.08, antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.412$, Rules used = {385, 200, 31, 634, 617, 204, 628}

$$-\frac{(ad+2bc)\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{18a^{5/3}b^{4/3}}+\frac{(ad+2bc)\log\left(\sqrt[3]{a}\,+\,\sqrt[3]{b}\,x\right)}{9a^{5/3}b^{4/3}}-\frac{(ad+2bc)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{5/3}b^{4/3}}+\frac{x(bc-ad)}{3ab\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3)/(a + b*x^3)^2,x]
```

```
[Out] ((b*c - a*d)*x)/(3*a*b*(a + b*x^3)) - ((2*b*c + a*d)*ArcTan[(a^(1/3) - 2*b^
(1/3)*x)/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(5/3)*b^(4/3)) + ((2*b*c + a*d)*L
og[a^(1/3) + b^(1/3)*x])/(9*a^(5/3)*b^(4/3)) - ((2*b*c + a*d)*Log[a^(2/3) -
 a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(18*a^(5/3)*b^(4/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{c+dx^3}{\left(a+bx^3\right)^2}\,dx = \frac{(bc-ad)x}{3ab\left(a+bx^3\right)}+\frac{(2bc+ad)\int \frac{1}{a+bx^3}\,dx}{3ab}$$

$$= \frac{(bc-ad)x}{3ab\left(a+bx^3\right)}+\frac{(2bc+ad)\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{9a^{5/3}b}+\frac{(2bc+ad)\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{9a^{5/3}b}$$

$$= \frac{(bc-ad)x}{3ab\left(a+bx^3\right)}+\frac{(2bc+ad)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{5/3}b^{4/3}}-\frac{(2bc+ad)\int \frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{18a^{5/3}b^{4/3}}+\frac{(2bc+ad}{}$$

$$= \frac{(bc-ad)x}{3ab\left(a+bx^3\right)}+\frac{(2bc+ad)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{5/3}b^{4/3}}-\frac{(2bc+ad)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18a^{5/3}b^{4/3}}+\frac{(2}{}$$

$$= \frac{(bc-ad)x}{3ab\left(a+bx^3\right)}-\frac{(2bc+ad)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{5/3}b^{4/3}}+\frac{(2bc+ad)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{5/3}b^{4/3}}-\frac{(2bc+ad)\log}{}$$

Mathematica [A] time = 0.09, size = 145, normalized size = 0.86

$$\frac{-(ad+2bc)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)-\frac{6a^{2/3}\sqrt[3]{b}x(ad-bc)}{a+bx^3}+2(ad+2bc)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)-2\sqrt{3}(ad+2bc)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{18a^{5/3}b^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^3)/(a + b*x^3)^2,x]
```

```
[Out] ((-6*a^(2/3)*b^(1/3)*(-(b*c) + a*d)*x)/(a + b*x^3) - 2*Sqrt[3]*(2*b*c + a*d
)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] + 2*(2*b*c + a*d)*Log[a^(1/3)
 + b^(1/3)*x] - (2*b*c + a*d)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2
])/(18*a^(5/3)*b^(4/3))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{c+dx^3}{\left(a+bx^3\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x^3)/(a + b*x^3)^2,x]
```

```
[Out] IntegrateAlgebraic[(c + d*x^3)/(a + b*x^3)^2, x]
```

fricas [A] time = 0.95, size = 537, normalized size = 3.18

$$\left[\frac{3\sqrt{\frac{1}{3}}\left(2a^2b^2c+a^3bd+\left(2ab^3c+a^2b^2d\right)x^3\right)\sqrt{-\frac{\left(a^2b\right)^{\frac{1}{3}}}{b}}\log\left(\frac{2abx^3-3\left(a^2b\right)^{\frac{1}{3}}ax-a^2+3\sqrt{\frac{1}{3}}\left(2abx^2+\left(a^2b\right)^{\frac{2}{3}}x-\left(a^2b\right)^{\frac{1}{3}}a\right)\sqrt{-\frac{\left(a^2b\right)^{\frac{1}{3}}}{b}}}{bx^3+a}\right)-\left(\left(2b^2c+abd\right)x^3+2abc+a^2d\right)\left(a^2b\right)^{\frac{2}{3}}\log\left(abx^2-\left(a^2b\right)^{\frac{2}{3}}x+\left(a^2b\right)^{\frac{1}{3}}a\right)+2\left(\left(2b^2c+abd\right)x^3+2abc+a^2d\right)\left(a^2b\right)^{\frac{2}{3}}\log\left(abx+\left(a^2b\right)^{\frac{2}{3}}\right)+6\left(a^2b^2c-a^3bd\right)x}{18\left(a^3b^3x^3+a^4b^2\right)}, \frac{6\sqrt{\frac{1}{3}}\left(2a^2b^2c+a^3bd+\left(2ab^3c+a^2b^2d\right)x^3\right)\sqrt{\frac{\left(a^2b\right)^{\frac{1}{3}}}{b}}\arctan\left(\frac{\sqrt{\frac{1}{3}}\left(2\left(a^2b\right)^{\frac{2}{3}}x-\left(a^2b\right)^{\frac{1}{3}}a\right)\sqrt{\frac{\left(a^2b\right)^{\frac{1}{3}}}{b}}}{a^2}\right)-\left(\left(2b^2c+abd\right)x^3+2abc+a^2d\right)\left(a^2b\right)^{\frac{2}{3}}\log\left(abx^2-\left(a^2b\right)^{\frac{2}{3}}x+\left(a^2b\right)^{\frac{1}{3}}a\right)+2\left(\left(2b^2c+abd\right)x^3+2abc+a^2d\right)\left(a^2b\right)^{\frac{2}{3}}\log\left(abx+\left(a^2b\right)^{\frac{2}{3}}\right)+6\left(a^2b^2c-a^3bd\right)x}{18\left(a^3b^3x^3+a^4b^2\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] [1/18*(3*sqrt(1/3)*(2*a^2*b^2*c + a^3*b*d + (2*a*b^3*c + a^2*b^2*d)*x^3)*sq
rt(-(a^2*b)^(1/3)/b)*log((2*a*b*x^3 - 3*(a^2*b)^(1/3)*a*x - a^2 + 3*sqrt(1/
3)*(2*a*b*x^2 + (a^2*b)^(2/3)*x - (a^2*b)^(1/3)*a)*sqrt(-(a^2*b)^(1/3)/b))/
(b*x^3 + a)) - ((2*b^2*c + a*b*d)*x^3 + 2*a*b*c + a^2*d)*(a^2*b)^(2/3)*log(
a*b*x^2 - (a^2*b)^(2/3)*x + (a^2*b)^(1/3)*a) + 2*((2*b^2*c + a*b*d)*x^3 + 2
*a*b*c + a^2*d)*(a^2*b)^(2/3)*log(a*b*x + (a^2*b)^(2/3)) + 6*(a^2*b^2*c - a
^3*b*d)*x)/(a^3*b^3*x^3 + a^4*b^2), 1/18*(6*sqrt(1/3)*(2*a^2*b^2*c + a^3*b*
d + (2*a*b^3*c + a^2*b^2*d)*x^3)*sqrt((a^2*b)^(1/3)/b)*arctan(sqrt(1/3)*(2*
(a^2*b)^(2/3)*x - (a^2*b)^(1/3)*a)*sqrt((a^2*b)^(1/3)/b)/a^2) - ((2*b^2*c +
 a*b*d)*x^3 + 2*a*b*c + a^2*d)*(a^2*b)^(2/3)*log(a*b*x^2 - (a^2*b)^(2/3)*x
+ (a^2*b)^(1/3)*a) + 2*((2*b^2*c + a*b*d)*x^3 + 2*a*b*c + a^2*d)*(a^2*b)^(2
/3)*log(a*b*x + (a^2*b)^(2/3)) + 6*(a^2*b^2*c - a^3*b*d)*x)/(a^3*b^3*x^3 +
a^4*b^2)]
```

giac [A] time = 0.17, size = 160, normalized size = 0.95

$$-\frac{\sqrt{3}\left(2\,bc+ad\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\left(-ab^2\right)^{\frac{2}{3}}a}-\frac{\left(2\,bc+ad\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\left(-ab^2\right)^{\frac{2}{3}}a}-\frac{\left(2\,bc+ad\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\,a^2b}+\frac{bcx-adx}{3\left(bx^3+a\right)ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] -1/9*sqrt(3)*(2*b*c + a*d)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(
1/3))/((-a*b^2)^(2/3)*a) - 1/18*(2*b*c + a*d)*log(x^2 + x*(-a/b)^(1/3) + (-
a/b)^(2/3))/((-a*b^2)^(2/3)*a) - 1/9*(2*b*c + a*d)*(-a/b)^(1/3)*log(abs(x -
 (-a/b)^(1/3)))/(a^2*b) + 1/3*(b*c*x - a*d*x)/((b*x^3 + a)*a*b)
```

maple [A] time = 0.05, size = 221, normalized size = 1.31

$$\frac{2\sqrt{3}\,c\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{9\left(\frac{a}{b}\right)^{\frac{2}{3}}ab}+\frac{2c\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\left(\frac{a}{b}\right)^{\frac{2}{3}}ab}-\frac{c\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{9\left(\frac{a}{b}\right)^{\frac{2}{3}}ab}+\frac{\sqrt{3}\,d\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{9\left(\frac{a}{b}\right)^{\frac{2}{3}}b^2}+\frac{d\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\left(\frac{a}{b}\right)^{\frac{2}{3}}b^2}-\frac{d\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\left(\frac{a}{b}\right)^{\frac{2}{3}}b^2}-\frac{\left(ad-bc\right)x}{3\left(b\,x^3+a\right)ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c)/(b*x^3+a)^2,x)
```

```
[Out] -1/3*(a*d-b*c)/a/b*x/(b*x^3+a)+1/9/b^2/(a/b)^(2/3)*ln(x+(a/b)^(1/3))*d+2/9/
b/a/(a/b)^(2/3)*ln(x+(a/b)^(1/3))*c-1/18/b^2/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)
*x+(a/b)^(2/3))*d-1/9/b/a/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*c+1
/9/b^2/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*d+2/9/b/
a/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*c
```

maxima [A] time = 1.44, size = 158, normalized size = 0.93

$$\frac{\left(bc-ad\right)x}{3\left(ab^2x^3+a^2b\right)}+\frac{\sqrt{3}\left(2\,bc+ad\right)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,ab^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\left(2\,bc+ad\right)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,ab^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\left(2\,bc+ad\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\,ab^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] 1/3*(b*c - a*d)*x/(a*b^2*x^3 + a^2*b) + 1/9*sqrt(3)*(2*b*c + a*d)*arctan(1/
3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a*b^2*(a/b)^(2/3)) - 1/18*(2*b*
c + a*d)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a*b^2*(a/b)^(2/3)) + 1/9*(
2*b*c + a*d)*log(x + (a/b)^(1/3))/(a*b^2*(a/b)^(2/3))
```

mupad [**B**] time = 1.43, size = 143, normalized size = 0.85

$$\frac{\ln\left(b^{1/3}\,x+a^{1/3}\right)\,\left(a\,d+2\,b\,c\right)}{9\,a^{5/3}\,b^{4/3}}-\frac{\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1{}\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,\left(a\,d+2\,b\,c\right)}{9\,a^{5/3}\,b^{4/3}}+\frac{\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1{}\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,\left(a\,d+2\,b\,c\right)}{9\,a^{5/3}\,b^{4/3}}-\frac{x\,\left(a\,d-b\,c\right)}{3\,a\,b\,\left(b\,x^3+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3)/(a + b*x^3)^2,x)
```

```
[Out] (log(b^(1/3)*x + a^(1/3))*(a*d + 2*b*c))/(9*a^(5/3)*b^(4/3)) - (log(3^(1/2)
*a^(1/3)*1i - 2*b^(1/3)*x + a^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(a*d + 2*b*c))/
(9*a^(5/3)*b^(4/3)) + (log(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*x - a^(1/3))*((3^
(1/2)*1i)/2 - 1/2)*(a*d + 2*b*c))/(9*a^(5/3)*b^(4/3)) - (x*(a*d - b*c))/(3*
a*b*(a + b*x^3))
```

sympy [**A**] time = 1.42, size = 97, normalized size = 0.57

$$\frac{x\left(-ad+bc\right)}{3a^{2}b+3ab^{2}x^{3}}+\operatorname{RootSum}\left(729t^{3}a^{5}b^{4}-a^{3}d^{3}-6a^{2}bcd^{2}-12ab^{2}c^{2}d-8b^{3}c^{3},\left(t\mapsto t\log\left(\frac{9ta^{2}b}{ad+2bc}+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c)/(b*x**3+a)**2,x)
```

```
[Out] x*(-a*d + b*c)/(3*a**2*b + 3*a*b**2*x**3) + RootSum(729*_t**3*a**5*b**4 - a
**3*d**3 - 6*a**2*b*c*d**2 - 12*a*b**2*c**2*d - 8*b**3*c**3, Lambda(_t, _t*
log(9*_t*a**2*b/(a*d + 2*b*c) + x)))
```

3.25 $\int \frac{1}{(a+bx^3)^2(c+dx^3)}\,dx$

Optimal. Leaf size=346

$$-\frac{b^{2/3}(2bc-5ad)\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{18a^{5/3}(bc-ad)^2}+\frac{b^{2/3}(2bc-5ad)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{9a^{5/3}(bc-ad)^2}-\frac{b^{2/3}(2bc-5ad)\tan^{-1}\left(\frac{\sqrt[3]{a}-}{\sqrt{3}}\right.}{3\sqrt{3}\,a^{5/3}(bc-ad)^2}$$

Rubi [A] time = 0.25, antiderivative size = 346, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.421, Rules used = {414, 522, 200, 31, 634, 617, 204, 628}

$$-\frac{b^{2/3}(2bc-5ad)\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{18a^{5/3}(bc-ad)^2}+\frac{b^{2/3}(2bc-5ad)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{9a^{5/3}(bc-ad)^2}-\frac{b^{2/3}(2bc-5ad)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{5/3}(bc-ad)^2}-\frac{d^{5/3}\log\left(c^{2/3}-\sqrt[3]{c}\,\sqrt[3]{d}\,x+d^{2/3}x^2\right)}{6c^{2/3}(bc-ad)^2}+\frac{d^{5/3}\log\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{3c^{2/3}(bc-ad)^2}-\frac{d^{5/3}\tan^{-1}\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}\,x}{\sqrt{3}\,\sqrt[3]{c}}\right)}{\sqrt{3}\,c^{2/3}(bc-ad)^2}+\frac{bx}{3a\left(a+bx^3\right)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^3)^2*(c + d*x^3)),x]
```

```
[Out] (b*x)/(3*a*(b*c - a*d)*(a + b*x^3)) - (b^(2/3)*(2*b*c - 5*a*d)*ArcTan[(a^(1
/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(5/3)*(b*c - a*d)^2) -
(d^(5/3)*ArcTan[(c^(1/3) - 2*d^(1/3)*x)/(Sqrt[3]*c^(1/3))])/(Sqrt[3]*c^(2/3
)*(b*c - a*d)^2) + (b^(2/3)*(2*b*c - 5*a*d)*Log[a^(1/3) + b^(1/3)*x])/(9*a^
(5/3)*(b*c - a*d)^2) + (d^(5/3)*Log[c^(1/3) + d^(1/3)*x])/(3*c^(2/3)*(b*c -
 a*d)^2) - (b^(2/3)*(2*b*c - 5*a*d)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/
3)*x^2])/(18*a^(5/3)*(b*c - a*d)^2) - (d^(5/3)*Log[c^(2/3) - c^(1/3)*d^(1/3
)*x + d^(2/3)*x^2])/(6*c^(2/3)*(b*c - a*d)^2)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
```

```
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{1}{\left(a+bx^3\right)^2\left(c+dx^3\right)}\,dx = \frac{bx}{3a(bc-ad)\left(a+bx^3\right)} - \frac{\int \frac{-2bc+3ad-2bdx^3}{\left(a+bx^3\right)\left(c+dx^3\right)}\,dx}{3a(bc-ad)}$$

$$= \frac{bx}{3a(bc-ad)\left(a+bx^3\right)} + \frac{d^2\int \frac{1}{c+dx^3}\,dx}{(bc-ad)^2} + \frac{(b(2bc-5ad))\int \frac{1}{a+bx^3}\,dx}{3a(bc-ad)^2}$$

$$= \frac{bx}{3a(bc-ad)\left(a+bx^3\right)} + \frac{d^2\int \frac{1}{\sqrt[3]{c}+\sqrt[3]{d}\,x}\,dx}{3c^{2/3}(bc-ad)^2} + \frac{d^2\int \frac{2\sqrt[3]{c}-\sqrt[3]{d}\,x}{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}\,x+d^{2/3}x^2}\,dx}{3c^{2/3}(bc-ad)^2} + \frac{(b(2bc-5ad}{9a^{5/3}(}$$

$$= \frac{bx}{3a(bc-ad)\left(a+bx^3\right)} + \frac{b^{2/3}(2bc-5ad)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{9a^{5/3}(bc-ad)^2} + \frac{d^{5/3}\log\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{3c^{2/3}(bc-ad)^2} -$$

$$= \frac{bx}{3a(bc-ad)\left(a+bx^3\right)} + \frac{b^{2/3}(2bc-5ad)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{9a^{5/3}(bc-ad)^2} + \frac{d^{5/3}\log\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{3c^{2/3}(bc-ad)^2} -$$

$$= \frac{bx}{3a(bc-ad)\left(a+bx^3\right)} - \frac{b^{2/3}(2bc-5ad)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{5/3}(bc-ad)^2} - \frac{d^{5/3}\tan^{-1}\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}\,x}{\sqrt{3}\sqrt[3]{c}}\right)}{\sqrt{3}\,c^{2/3}(bc-ad)^2} +$$

Mathematica [A] time = 0.20, size = 337, normalized size = 0.97

$$\frac{-b^{2/3}c^{2/3}\left(a+bx^3\right)(2bc-5ad)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)-3a^{5/3}d^{5/3}\left(a+bx^3\right)\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}\,x+d^{2/3}x^2\right)+6a^{2/3}bc^{2/3}x(bc-ad)+6a^{5/3}d^{5/3}\left(a+bx^3\right)\log\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)-6\sqrt{3}\,a^{5/3}d^{5/3}\left(a+bx^3\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\,x}{\sqrt[3]{c}}}{\sqrt{3}}\right)+2b^{2/3}c^{2/3}\left(a+bx^3\right)(2bc-5ad)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)-2\sqrt{3}\,b^{2/3}c^{2/3}\left(a+bx^3\right)(2bc-5ad)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{18a^{5/3}c^{2/3}\left(a+bx^3\right)(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x^3)^2*(c + d*x^3)),x]
```

```
[Out] (6*a^(2/3)*b*c^(2/3)*(b*c - a*d)*x - 2*Sqrt[3]*b^(2/3)*c^(2/3)*(2*b*c - 5*a
*d)*(a + b*x^3)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] - 6*Sqrt[3]*a^(
```

```
5/3)*d^(5/3)*(a + b*x^3)*ArcTan[(1 - (2*d^(1/3)*x)/c^(1/3))/Sqrt[3]] + 2*b^
(2/3)*c^(2/3)*(2*b*c - 5*a*d)*(a + b*x^3)*Log[a^(1/3) + b^(1/3)*x] + 6*a^(5
/3)*d^(5/3)*(a + b*x^3)*Log[c^(1/3) + d^(1/3)*x] - b^(2/3)*c^(2/3)*(2*b*c -
 5*a*d)*(a + b*x^3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2] - 3*a^(5
/3)*d^(5/3)*(a + b*x^3)*Log[c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2])/(18
*a^(5/3)*c^(2/3)*(b*c - a*d)^2*(a + b*x^3))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+bx^3\right)^2\left(c+dx^3\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/((a + b*x^3)^2*(c + d*x^3)),x]
```

```
[Out] IntegrateAlgebraic[1/((a + b*x^3)^2*(c + d*x^3)), x]
```

fricas [A] time = 18.53, size = 440, normalized size = 1.27

$$-\frac{2\sqrt{3}\left(\left(2b^2c-5abd\right)x^3+2abc-5a^2d\right)\left(-\frac{b^2}{a^2}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}ax\left(-\frac{b^2}{a^2}\right)^{\frac{2}{3}}-\sqrt{3}b}{3b}\right)-6\sqrt{3}\left(abdx^3+a^2d\right)\left(\frac{d^2}{c^2}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}cx\left(\frac{d^2}{c^2}\right)^{\frac{2}{3}}-\sqrt{3}d}{3d}\right)-\left(\left(2b^2c-5abd\right)x^3+2abc-5a^2d\right)\left(-\frac{b^2}{a^2}\right)^{\frac{1}{3}}\log\left(b^2x^2+abx\left(-\frac{b^2}{a^2}\right)^{\frac{1}{3}}+a^2\left(-\frac{b^2}{a^2}\right)^{\frac{2}{3}}\right)+3\left(abdx^3+a^2d\right)\left(\frac{d^2}{c^2}\right)^{\frac{1}{3}}\log\left(d^2x^2-cdx\left(\frac{d^2}{c^2}\right)^{\frac{1}{3}}+c^2\left(\frac{d^2}{c^2}\right)^{\frac{2}{3}}\right)+2\left(\left(2b^2c-5abd\right)x^3+2abc-5a^2d\right)\left(-\frac{b^2}{a^2}\right)^{\frac{1}{3}}\log\left(bx-a\left(-\frac{b^2}{a^2}\right)^{\frac{1}{3}}\right)-6\left(abdx^3+a^2d\right)\left(\frac{d^2}{c^2}\right)^{\frac{1}{3}}\log\left(dx+c\left(\frac{d^2}{c^2}\right)^{\frac{1}{3}}\right)-6\left(b^2c-abd\right)x}{18\left(a^2b^2c^2-2a^3bcd+a^4d^2+\left(ab^3c^2-2a^2b^2cd+a^3bd^2\right)x^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^2/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] -1/18*(2*sqrt(3)*((2*b^2*c - 5*a*b*d)*x^3 + 2*a*b*c - 5*a^2*d)*(-b^2/a^2)^(
1/3)*arctan(1/3*(2*sqrt(3)*a*x*(-b^2/a^2)^(2/3) - sqrt(3)*b)/b) - 6*sqrt(3)
*(a*b*d*x^3 + a^2*d)*(d^2/c^2)^(1/3)*arctan(1/3*(2*sqrt(3)*c*x*(d^2/c^2)^(2
/3) - sqrt(3)*d)/d) - ((2*b^2*c - 5*a*b*d)*x^3 + 2*a*b*c - 5*a^2*d)*(-b^2/a
^2)^(1/3)*log(b^2*x^2 + a*b*x*(-b^2/a^2)^(1/3) + a^2*(-b^2/a^2)^(2/3)) + 3*
(a*b*d*x^3 + a^2*d)*(d^2/c^2)^(1/3)*log(d^2*x^2 - c*d*x*(d^2/c^2)^(1/3) + c
^2*(d^2/c^2)^(2/3)) + 2*((2*b^2*c - 5*a*b*d)*x^3 + 2*a*b*c - 5*a^2*d)*(-b^2
/a^2)^(1/3)*log(b*x - a*(-b^2/a^2)^(1/3)) - 6*(a*b*d*x^3 + a^2*d)*(d^2/c^2)
^(1/3)*log(d*x + c*(d^2/c^2)^(1/3)) - 6*(b^2*c - a*b*d)*x)/(a^2*b^2*c^2 - 2
*a^3*b*c*d + a^4*d^2 + (a*b^3*c^2 - 2*a^2*b^2*c*d + a^3*b*d^2)*x^3)
```

giac [A] time = 0.20, size = 443, normalized size = 1.28

$$-\frac{d^2\left(-\frac{c}{d}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right|\right)}{3\left(b^2c^3-2abc^2d+a^2cd^2\right)}+\frac{\left(-cd^2\right)^{\frac{1}{3}}d\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}b^2c^3-2\sqrt{3}abc^2d+\sqrt{3}a^2cd^2}+\frac{\left(-cd^2\right)^{\frac{1}{3}}d\log\left(x^2+x\left(-\frac{c}{d}\right)^{\frac{1}{3}}+\left(-\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6\left(b^2c^3-2abc^2d+a^2cd^2\right)}-\frac{\left(2b^2c-5abd\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\left(a^2b^2c^2-2a^3bcd+a^4d^2\right)}+\frac{\left(2\left(-ab^2\right)^{\frac{1}{3}}bc-5\left(-ab^2\right)^{\frac{1}{3}}ad\right)\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\left(\sqrt{3}a^2b^2c^2-2\sqrt{3}a^3bcd+\sqrt{3}a^4d^2\right)}+\frac{\left(2\left(-ab^2\right)^{\frac{1}{3}}bc-5\left(-ab^2\right)^{\frac{1}{3}}ad\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\left(a^2b^2c^2-2a^3bcd+a^4d^2\right)}+\frac{bx}{3\left(bx^3+a\right)\left(abc-a^2d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^2/(d*x^3+c),x, algorithm="giac")
```

```
[Out] -1/3*d^2*(-c/d)^(1/3)*log(abs(x - (-c/d)^(1/3)))/(b^2*c^3 - 2*a*b*c^2*d + a
^2*c*d^2) + (-c*d^2)^(1/3)*d*arctan(1/3*sqrt(3)*(2*x + (-c/d)^(1/3))/(-c/d)
^(1/3))/(sqrt(3)*b^2*c^3 - 2*sqrt(3)*a*b*c^2*d + sqrt(3)*a^2*c*d^2) + 1/6*(
-c*d^2)^(1/3)*d*log(x^2 + x*(-c/d)^(1/3) + (-c/d)^(2/3))/(b^2*c^3 - 2*a*b*c
^2*d + a^2*c*d^2) - 1/9*(2*b^2*c - 5*a*b*d)*(-a/b)^(1/3)*log(abs(x - (-a/b)
^(1/3)))/(a^2*b^2*c^2 - 2*a^3*b*c*d + a^4*d^2) + 1/3*(2*(-a*b^2)^(1/3)*b*c
- 5*(-a*b^2)^(1/3)*a*d)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3
))/(sqrt(3)*a^2*b^2*c^2 - 2*sqrt(3)*a^3*b*c*d + sqrt(3)*a^4*d^2) + 1/18*(2*
(-a*b^2)^(1/3)*b*c - 5*(-a*b^2)^(1/3)*a*d)*log(x^2 + x*(-a/b)^(1/3) + (-a/b
)^(2/3))/(a^2*b^2*c^2 - 2*a^3*b*c*d + a^4*d^2) + 1/3*b*x/((b*x^3 + a)*(a*b*
c - a^2*d))
```

maple [A] time = 0.06, size = 406, normalized size = 1.17

$$\frac{b^2cx}{3\left(ad-bc\right)^2\left(bx^3+a\right)a}-\frac{bdx}{3\left(ad-bc\right)^2\left(bx^3+a\right)}+\frac{2\sqrt{3}\,bc\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{9\left(ad-bc\right)^2\left(\frac{a}{b}\right)^{\frac{2}{3}}a}+\frac{2bc\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\left(ad-bc\right)^2\left(\frac{a}{b}\right)^{\frac{2}{3}}a}-\frac{bc\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{9\left(ad-bc\right)^2\left(\frac{a}{b}\right)^{\frac{2}{3}}a}-\frac{5\sqrt{3}\,d\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{9\left(ad-bc\right)^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,d\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{c}{d}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3\left(ad-bc\right)^2\left(\frac{c}{d}\right)^{\frac{2}{3}}}-\frac{5d\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\left(ad-bc\right)^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{d\ln\left(x+\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(ad-bc\right)^2\left(\frac{c}{d}\right)^{\frac{2}{3}}}+\frac{5d\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\left(ad-bc\right)^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{d\ln\left(x^2-\left(\frac{c}{d}\right)^{\frac{1}{3}}x+\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6\left(ad-bc\right)^2\left(\frac{c}{d}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^3+a)^2/(d*x^3+c),x)
```

```
[Out] -1/3*b/(a*d-b*c)^2*x/(b*x^3+a)*d+1/3*b^2/(a*d-b*c)^2/a*x/(b*x^3+a)*c-5/9/(a
*d-b*c)^2/(a/b)^(2/3)*ln(x+(a/b)^(1/3))*d+2/9*b/(a*d-b*c)^2/a/(a/b)^(2/3)*l
n(x+(a/b)^(1/3))*c+5/18/(a*d-b*c)^2/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^
(2/3))*d-1/9*b/(a*d-b*c)^2/a/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*
c-5/9/(a*d-b*c)^2/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1
))*d+2/9*b/(a*d-b*c)^2/a/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1
/3)*x-1))*c+1/3*d/(a*d-b*c)^2/(c/d)^(2/3)*ln(x+(c/d)^(1/3))-1/6*d/(a*d-b*c)
^2/(c/d)^(2/3)*ln(x^2-(c/d)^(1/3)*x+(c/d)^(2/3))+1/3*d/(a*d-b*c)^2/(c/d)^(2
/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(c/d)^(1/3)*x-1))
```

maxima [A] time = 1.29, size = 489, normalized size = 1.41

$$\frac{\sqrt{3}\,(2\,bc-5\,ad)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\left(ab^2c^2\left(\frac{a}{b}\right)^{\frac{1}{3}}-2\,a^2bcd\left(\frac{a}{b}\right)^{\frac{1}{3}}+a^3d^2\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\sqrt{3}\,d\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{3\left(b^2c^2\left(\frac{c}{d}\right)^{\frac{1}{3}}-2\,abcd\left(\frac{c}{d}\right)^{\frac{1}{3}}+a^2d^2\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)\left(\frac{c}{d}\right)^{\frac{1}{3}}}+\frac{bx}{3\left(a^2bc-a^3d+\left(ab^2c-a^2bd\right)x^3\right)}-\frac{(2\,bc-5\,ad)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\left(ab^2c^2\left(\frac{a}{b}\right)^{\frac{2}{3}}-2\,a^2bcd\left(\frac{a}{b}\right)^{\frac{2}{3}}+a^3d^2\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}-\frac{d\log\left(x^2-x\left(\frac{c}{d}\right)^{\frac{1}{3}}+\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6\left(b^2c^2\left(\frac{c}{d}\right)^{\frac{2}{3}}-2\,abcd\left(\frac{c}{d}\right)^{\frac{2}{3}}+a^2d^2\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}+\frac{(2\,bc-5\,ad)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\left(ab^2c^2\left(\frac{a}{b}\right)^{\frac{2}{3}}-2\,a^2bcd\left(\frac{a}{b}\right)^{\frac{2}{3}}+a^3d^2\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}+\frac{d\log\left(x+\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(b^2c^2\left(\frac{c}{d}\right)^{\frac{2}{3}}-2\,abcd\left(\frac{c}{d}\right)^{\frac{2}{3}}+a^2d^2\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^2/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] 1/9*sqrt(3)*(2*b*c - 5*a*d)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1
/3))/((a*b^2*c^2*(a/b)^(1/3) - 2*a^2*b*c*d*(a/b)^(1/3) + a^3*d^2*(a/b)^(1/3
))*(a/b)^(1/3)) + 1/3*sqrt(3)*d*arctan(1/3*sqrt(3)*(2*x - (c/d)^(1/3))/(c/d
)^(1/3))/((b^2*c^2*(c/d)^(1/3) - 2*a*b*c*d*(c/d)^(1/3) + a^2*d^2*(c/d)^(1/3
))*(c/d)^(1/3)) + 1/3*b*x/(a^2*b*c - a^3*d + (a*b^2*c - a^2*b*d)*x^3) - 1/1
8*(2*b*c - 5*a*d)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a*b^2*c^2*(a/b)^(
2/3) - 2*a^2*b*c*d*(a/b)^(2/3) + a^3*d^2*(a/b)^(2/3)) - 1/6*d*log(x^2 - x*(
c/d)^(1/3) + (c/d)^(2/3))/(b^2*c^2*(c/d)^(2/3) - 2*a*b*c*d*(c/d)^(2/3) + a^
2*d^2*(c/d)^(2/3)) + 1/9*(2*b*c - 5*a*d)*log(x + (a/b)^(1/3))/(a*b^2*c^2*(a
/b)^(2/3) - 2*a^2*b*c*d*(a/b)^(2/3) + a^3*d^2*(a/b)^(2/3)) + 1/3*d*log(x +
(c/d)^(1/3))/(b^2*c^2*(c/d)^(2/3) - 2*a*b*c*d*(c/d)^(2/3) + a^2*d^2*(c/d)^(
2/3))
```

mupad [B] time = 15.93, size = 2492, normalized size = 7.20

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^3)^2*(c + d*x^3)),x)
```

```
[Out] log((((((27*b^3*d^3*x*(a*d - b*c)^3*(3*a^2*d^2 - 2*b^2*c^2 + 3*a*b*c*d))/a
+ 27*a*b^3*c*d^3*(a*d + b*c)*(a*d - b*c)^4*(-(b^2*(5*a*d - 2*b*c)^3)/(a^5*(
a*d - b*c)^6))^(1/3))*(-(b^2*(5*a*d - 2*b*c)^3)/(a^5*(a*d - b*c)^6))^(2/3))
/81 - (b^4*d^4*(27*a^3*d^3 - 8*b^3*c^3 + 52*a*b^2*c^2*d - 98*a^2*b*c*d^2))/
(3*a^4*d - 3*a^3*b*c))*(-(b^2*(5*a*d - 2*b*c)^3)/(a^5*(a*d - b*c)^6))^(1/3)
)/9 + (2*b^5*d^6*x*(85*a^3*d^3 - 4*b^3*c^3 + 30*a*b^2*c^2*d - 84*a^2*b*c*d^
2))/(9*a^3*(a*d - b*c)^4))*((8*b^5*c^3 - 125*a^3*b^2*d^3 + 150*a^2*b^3*c*d^
2 - 60*a*b^4*c^2*d)/(729*a^11*d^6 + 729*a^5*b^6*c^6 - 4374*a^6*b^5*c^5*d +
10935*a^7*b^4*c^4*d^2 - 14580*a^8*b^3*c^3*d^3 + 10935*a^9*b^2*c^2*d^4 - 437
4*a^10*b*c*d^5))^(1/3) + log((((((27*b^3*d^3*x*(a*d - b*c)^3*(3*a^2*d^2 - 2
*b^2*c^2 + 3*a*b*c*d))/a + 81*a*b^3*c*d^3*(a*d + b*c)*(a*d - b*c)^4*(d^5/(c
^2*(a*d - b*c)^6))^(1/3))*(d^5/(c^2*(a*d - b*c)^6))^(2/3))/9 - (b^4*d^4*(27
*a^3*d^3 - 8*b^3*c^3 + 52*a*b^2*c^2*d - 98*a^2*b*c*d^2))/(3*a^4*d - 3*a^3*b
*c))*(d^5/(c^2*(a*d - b*c)^6))^(1/3))/3 + (2*b^5*d^6*x*(85*a^3*d^3 - 4*b^3*
c^3 + 30*a*b^2*c^2*d - 84*a^2*b*c*d^2))/(9*a^3*(a*d - b*c)^4))*(d^5/(27*b^6
*c^8 + 27*a^6*c^2*d^6 - 162*a^5*b*c^3*d^5 + 405*a^2*b^4*c^6*d^2 - 540*a^3*b
^3*c^5*d^3 + 405*a^4*b^2*c^4*d^4 - 162*a*b^5*c^7*d))^(1/3) + (log(((3^(1/2)
*1i - 1)*(((3^(1/2)*1i - 1)^2*((27*b^3*d^3*x*(a*d - b*c)^3*(3*a^2*d^2 - 2*b
^2*c^2 + 3*a*b*c*d))/a + (27*a*b^3*c*d^3*(3^(1/2)*1i - 1)*(a*d + b*c)*(a*d
```

```
- b*c)^4*(-(b^2*(5*a*d - 2*b*c)^3)/(a^5*(a*d - b*c)^6))^(1/3))/2)*(-(b^2*(5
*a*d - 2*b*c)^3)/(a^5*(a*d - b*c)^6))^(2/3))/324 - (b^4*d^4*(27*a^3*d^3 - 8
*b^3*c^3 + 52*a*b^2*c^2*d - 98*a^2*b*c*d^2))/(3*a^4*d - 3*a^3*b*c))*(-(b^2*
(5*a*d - 2*b*c)^3)/(a^5*(a*d - b*c)^6))^(1/3))/18 + (2*b^5*d^6*x*(85*a^3*d^
3 - 4*b^3*c^3 + 30*a*b^2*c^2*d - 84*a^2*b*c*d^2))/(9*a^3*(a*d - b*c)^4))*(3
^(1/2)*1i - 1)*((8*b^5*c^3 - 125*a^3*b^2*d^3 + 150*a^2*b^3*c*d^2 - 60*a*b^4
*c^2*d)/(729*a^11*d^6 + 729*a^5*b^6*c^6 - 4374*a^6*b^5*c^5*d + 10935*a^7*b^
4*c^4*d^2 - 14580*a^8*b^3*c^3*d^3 + 10935*a^9*b^2*c^2*d^4 - 4374*a^10*b*c*d
^5))^(1/3))/2 - (log(((3^(1/2)*1i + 1)*(((3^(1/2)*1i + 1)^2*((27*b^3*d^3*x*
(a*d - b*c)^3*(3*a^2*d^2 - 2*b^2*c^2 + 3*a*b*c*d))/a - (27*a*b^3*c*d^3*(3^(
1/2)*1i + 1)*(a*d + b*c)*(a*d - b*c)^4*(-(b^2*(5*a*d - 2*b*c)^3)/(a^5*(a*d
- b*c)^6))^(1/3))/2)*(-(b^2*(5*a*d - 2*b*c)^3)/(a^5*(a*d - b*c)^6))^(2/3))/
324 - (b^4*d^4*(27*a^3*d^3 - 8*b^3*c^3 + 52*a*b^2*c^2*d - 98*a^2*b*c*d^2))/
(3*a^4*d - 3*a^3*b*c))*(-(b^2*(5*a*d - 2*b*c)^3)/(a^5*(a*d - b*c)^6))^(1/3)
)/18 - (2*b^5*d^6*x*(85*a^3*d^3 - 4*b^3*c^3 + 30*a*b^2*c^2*d - 84*a^2*b*c*d
^2))/(9*a^3*(a*d - b*c)^4))*(3^(1/2)*1i + 1)*((8*b^5*c^3 - 125*a^3*b^2*d^3
+ 150*a^2*b^3*c*d^2 - 60*a*b^4*c^2*d)/(729*a^11*d^6 + 729*a^5*b^6*c^6 - 437
4*a^6*b^5*c^5*d + 10935*a^7*b^4*c^4*d^2 - 14580*a^8*b^3*c^3*d^3 + 10935*a^9
*b^2*c^2*d^4 - 4374*a^10*b*c*d^5))^(1/3))/2 + (log(((3^(1/2)*1i - 1)*(((3^(
1/2)*1i - 1)^2*((27*b^3*d^3*x*(a*d - b*c)^3*(3*a^2*d^2 - 2*b^2*c^2 + 3*a*b*
c*d))/a + (81*a*b^3*c*d^3*(3^(1/2)*1i - 1)*(a*d + b*c)*(a*d - b*c)^4*(d^5/(
c^2*(a*d - b*c)^6))^(1/3))/2)*(d^5/(c^2*(a*d - b*c)^6))^(2/3))/36 - (b^4*d^
4*(27*a^3*d^3 - 8*b^3*c^3 + 52*a*b^2*c^2*d - 98*a^2*b*c*d^2))/(3*a^4*d - 3*
a^3*b*c))*(d^5/(c^2*(a*d - b*c)^6))^(1/3))/6 + (2*b^5*d^6*x*(85*a^3*d^3 - 4
*b^3*c^3 + 30*a*b^2*c^2*d - 84*a^2*b*c*d^2))/(9*a^3*(a*d - b*c)^4))*(3^(1/2
)*1i - 1)*(d^5/(27*b^6*c^8 + 27*a^6*c^2*d^6 - 162*a^5*b*c^3*d^5 + 405*a^2*b
^4*c^6*d^2 - 540*a^3*b^3*c^5*d^3 + 405*a^4*b^2*c^4*d^4 - 162*a*b^5*c^7*d))^
(1/3))/2 - (log(((3^(1/2)*1i + 1)*(((3^(1/2)*1i + 1)^2*((27*b^3*d^3*x*(a*d
- b*c)^3*(3*a^2*d^2 - 2*b^2*c^2 + 3*a*b*c*d))/a - (81*a*b^3*c*d^3*(3^(1/2)*
1i + 1)*(a*d + b*c)*(a*d - b*c)^4*(d^5/(c^2*(a*d - b*c)^6))^(1/3))/2)*(d^5/
(c^2*(a*d - b*c)^6))^(2/3))/36 - (b^4*d^4*(27*a^3*d^3 - 8*b^3*c^3 + 52*a*b^
2*c^2*d - 98*a^2*b*c*d^2))/(3*a^4*d - 3*a^3*b*c))*(d^5/(c^2*(a*d - b*c)^6))
^(1/3))/6 - (2*b^5*d^6*x*(85*a^3*d^3 - 4*b^3*c^3 + 30*a*b^2*c^2*d - 84*a^2*
b*c*d^2))/(9*a^3*(a*d - b*c)^4))*(3^(1/2)*1i + 1)*(d^5/(27*b^6*c^8 + 27*a^6
*c^2*d^6 - 162*a^5*b*c^3*d^5 + 405*a^2*b^4*c^6*d^2 - 540*a^3*b^3*c^5*d^3 +
405*a^4*b^2*c^4*d^4 - 162*a*b^5*c^7*d))^(1/3))/2 - (b*x)/(3*a*(a + b*x^3)*(
a*d - b*c))
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**3+a)**2/(d*x**3+c),x)
```

```
[Out] Timed out
```

3.26 $\int \frac{1}{\left(a+bx^3\right)^2\left(c+dx^3\right)^2}\,dx$

Optimal. Leaf size=419

$$-\frac{b^{5/3}(bc-4ad)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{9a^{5/3}(bc-ad)^3}+\frac{2b^{5/3}(bc-4ad)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{9a^{5/3}(bc-ad)^3}-\frac{2b^{5/3}(bc-4ad)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\sqrt[3]{a}}\right.}{3\sqrt{3}\,a^{5/3}(bc-ad)^3}$$

Rubi [A] time = 0.49, antiderivative size = 419, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 9, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.474, Rules used = {414, 527, 522, 200, 31, 634, 617, 204, 628}

$$-\frac{b^{5/3}(bc-4ad)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{9a^{5/3}(bc-ad)^3}+\frac{2b^{5/3}(bc-4ad)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{9a^{5/3}(bc-ad)^3}-\frac{2b^{5/3}(bc-4ad)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{5/3}(bc-ad)^3}-\frac{d^{5/3}(4bc-ad)\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}\,x+d^{2/3}x^2\right)}{9c^{5/3}(bc-ad)^3}+\frac{2d^{5/3}(4bc-ad)\log\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{9c^{5/3}(bc-ad)^3}-\frac{2d^{5/3}(4bc-ad)\tan^{-1}\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}\,x}{\sqrt{3}\sqrt[3]{c}}\right)}{3\sqrt{3}\,c^{5/3}(bc-ad)^3}+\frac{bx}{3a\left(a+bx^3\right)\left(c+dx^3\right)(bc-ad)}+\frac{dx(ad+bc)}{3ac\left(c+dx^3\right)(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^3)^2*(c + d*x^3)^2),x]
```

```
[Out] (d*(b*c + a*d)*x)/(3*a*c*(b*c - a*d)^2*(c + d*x^3)) + (b*x)/(3*a*(b*c - a*d
)*(a + b*x^3)*(c + d*x^3)) - (2*b^(5/3)*(b*c - 4*a*d)*ArcTan[(a^(1/3) - 2*b
^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(5/3)*(b*c - a*d)^3) - (2*d^(5/3
)*(4*b*c - a*d)*ArcTan[(c^(1/3) - 2*d^(1/3)*x)/(Sqrt[3]*c^(1/3))])/(3*Sqrt[
3]*c^(5/3)*(b*c - a*d)^3) + (2*b^(5/3)*(b*c - 4*a*d)*Log[a^(1/3) + b^(1/3)*
x])/(9*a^(5/3)*(b*c - a*d)^3) + (2*d^(5/3)*(4*b*c - a*d)*Log[c^(1/3) + d^(1
/3)*x])/(9*c^(5/3)*(b*c - a*d)^3) - (b^(5/3)*(b*c - 4*a*d)*Log[a^(2/3) - a^
(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(9*a^(5/3)*(b*c - a*d)^3) - (d^(5/3)*(4*b*c
 - a*d)*Log[c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2])/(9*c^(5/3)*(b*c - a
*d)^3)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{1}{\left(a+bx^3\right)^2\left(c+dx^3\right)^2}\,dx = \frac{bx}{3a(bc-ad)\left(a+bx^3\right)\left(c+dx^3\right)} - \frac{\int \frac{-2bc+3ad-5bdx^3}{\left(a+bx^3\right)\left(c+dx^3\right)^2}\,dx}{3a(bc-ad)}$$

$$= \frac{d(bc+ad)x}{3ac(bc-ad)^2\left(c+dx^3\right)} + \frac{bx}{3a(bc-ad)\left(a+bx^3\right)\left(c+dx^3\right)} - \frac{\int \frac{-6\left(b^2c^2-3abcd+a^2d^2\right)-6b}{\left(a+bx^3\right)\left(c+dx^3\right.}}{9ac(bc-ad}$$

$$= \frac{d(bc+ad)x}{3ac(bc-ad)^2\left(c+dx^3\right)} + \frac{bx}{3a(bc-ad)\left(a+bx^3\right)\left(c+dx^3\right)} + \frac{\left(2b^2(bc-4ad)\right)\int \frac{1}{a+bx}}{3a(bc-ad)^3}$$

$$= \frac{d(bc+ad)x}{3ac(bc-ad)^2\left(c+dx^3\right)} + \frac{bx}{3a(bc-ad)\left(a+bx^3\right)\left(c+dx^3\right)} + \frac{\left(2b^2(bc-4ad)\right)\int \frac{}{\sqrt[3]{a}-}}{9a^{5/3}(bc-ad)^3}$$

$$= \frac{d(bc+ad)x}{3ac(bc-ad)^2\left(c+dx^3\right)} + \frac{bx}{3a(bc-ad)\left(a+bx^3\right)\left(c+dx^3\right)} + \frac{2b^{5/3}(bc-4ad)\log\left(\sqrt[3]{\ }\right.}{9a^{5/3}(bc-ad}$$

$$= \frac{d(bc+ad)x}{3ac(bc-ad)^2\left(c+dx^3\right)} + \frac{bx}{3a(bc-ad)\left(a+bx^3\right)\left(c+dx^3\right)} + \frac{2b^{5/3}(bc-4ad)\log\left(\sqrt[3]{\ }\right.}{9a^{5/3}(bc-ad}$$

$$= \frac{d(bc+ad)x}{3ac(bc-ad)^2\left(c+dx^3\right)} + \frac{bx}{3a(bc-ad)\left(a+bx^3\right)\left(c+dx^3\right)} - \frac{2b^{5/3}(bc-4ad)\tan^{-1}\left(\right.}{3\sqrt{3}\,a^{5/3}(bc-}$$

Mathematica [A] time = 0.63, size = 381, normalized size = 0.91

$$\frac{1}{9}\left(\frac{b^{5/3}(bc-4ad)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{a^{5/3}(ad-bc)^3}+\frac{2b^{5/3}(4ad-bc)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{a^{5/3}(ad-bc)^3}+\frac{2\sqrt{3}\,b^{5/3}(bc-4ad)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{a^{5/3}(ad-bc)^3}+\frac{3b^2x}{a\left(a+bx^3\right)(bc-ad)^2}+\frac{d^{5/3}(ad-4bc)\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{c^{5/3}(bc-ad)^3}+\frac{2d^{5/3}(4bc-ad)\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{c^{5/3}(bc-ad)^3}+\frac{2\sqrt{3}\,d^{5/3}(ad-4bc)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}}{\sqrt{3}}\right)}{c^{5/3}(bc-ad)^3}+\frac{3d^2x}{c\left(c+dx^3\right)(bc-ad)^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x^3)^2*(c + d*x^3)^2),x]
```

```
[Out] ((3*b^2*x)/(a*(b*c - a*d)^2*(a + b*x^3)) + (3*d^2*x)/(c*(b*c - a*d)^2*(c +
d*x^3)) + (2*Sqrt[3]*b^(5/3)*(b*c - 4*a*d)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3
))/Sqrt[3]])/(a^(5/3)*(-(b*c) + a*d)^3) + (2*Sqrt[3]*d^(5/3)*(-4*b*c + a*d)
*ArcTan[(1 - (2*d^(1/3)*x)/c^(1/3))/Sqrt[3]])/(c^(5/3)*(b*c - a*d)^3) + (2*
b^(5/3)*(-(b*c) + 4*a*d)*Log[a^(1/3) + b^(1/3)*x])/(a^(5/3)*(-(b*c) + a*d)^
3) + (2*d^(5/3)*(4*b*c - a*d)*Log[c^(1/3) + d^(1/3)*x])/(c^(5/3)*(b*c - a*d
)^3) + (b^(5/3)*(b*c - 4*a*d)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2
])/(a^(5/3)*(-(b*c) + a*d)^3) + (d^(5/3)*(-4*b*c + a*d)*Log[c^(2/3) - c^(1/
3)*d^(1/3)*x + d^(2/3)*x^2])/(c^(5/3)*(b*c - a*d)^3))/9
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+bx^3\right)^2\left(c+dx^3\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/((a + b*x^3)^2*(c + d*x^3)^2),x]
```

```
[Out] IntegrateAlgebraic[1/((a + b*x^3)^2*(c + d*x^3)^2), x]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^2/(d*x^3+c)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [A] time = 0.22, size = 664, normalized size = 1.58

$$-\frac{2\left(b^3c-4ab^2d\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\left(a^2b^3c^3-3a^3b^2c^2d+3a^4bcd^2-a^5d^3\right)}-\frac{2\left(4bcd^2-ad^3\right)\left(-\frac{c}{d}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right|\right)}{9\left(b^3c^5-3ab^2c^4d+3a^2bc^3d^2-a^3c^2d^3\right)}+\frac{2\left(\left(-ab^2\right)^{\frac{1}{3}}b^2c-4\left(-ab^2\right)^{\frac{1}{3}}abd\right)\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\left(\sqrt{3}a^2b^3c^3-3\sqrt{3}a^3b^2c^2d+3\sqrt{3}a^4bcd^2-\sqrt{3}a^5d^3\right)}+\frac{2\left(4\left(-cd^2\right)^{\frac{1}{3}}bcd-\left(-cd^2\right)^{\frac{1}{3}}ad^2\right)\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{3\left(\sqrt{3}b^3c^5-3\sqrt{3}ab^2c^4d+3\sqrt{3}a^2bc^3d^2-\sqrt{3}a^3c^2d^3\right)}+\frac{\left(\left(-ab^2\right)^{\frac{1}{3}}b^2c-4\left(-ab^2\right)^{\frac{1}{3}}abd\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{9\left(a^2b^3c^3-3a^3b^2c^2d+3a^4bcd^2-a^5d^3\right)}+\frac{\left(4\left(-cd^2\right)^{\frac{1}{3}}bcd-\left(-cd^2\right)^{\frac{1}{3}}ad^2\right)\log\left(x^2+x\left(-\frac{c}{d}\right)^{\frac{1}{3}}+\left(-\frac{c}{d}\right)^{\frac{2}{3}}\right)}{9\left(b^3c^5-3ab^2c^4d+3a^2bc^3d^2-a^3c^2d^3\right)}+\frac{b^2cdx^4+abd^2x^4+b^2c^2x+a^2d^2x}{3\left(bdx^6+bcx^3+adx^3+ac\right)\left(ab^2c^3-2a^2bc^2d+a^3cd^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^2/(d*x^3+c)^2,x, algorithm="giac")
```

```
[Out] -2/9*(b^3*c - 4*a*b^2*d)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a^2*b^3*c
^3 - 3*a^3*b^2*c^2*d + 3*a^4*b*c*d^2 - a^5*d^3) - 2/9*(4*b*c*d^2 - a*d^3)*(
-c/d)^(1/3)*log(abs(x - (-c/d)^(1/3)))/(b^3*c^5 - 3*a*b^2*c^4*d + 3*a^2*b*c
^3*d^2 - a^3*c^2*d^3) + 2/3*((-a*b^2)^(1/3)*b^2*c - 4*(-a*b^2)^(1/3)*a*b*d)
*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(sqrt(3)*a^2*b^3*c^3
 - 3*sqrt(3)*a^3*b^2*c^2*d + 3*sqrt(3)*a^4*b*c*d^2 - sqrt(3)*a^5*d^3) + 2/3
*(4*(-c*d^2)^(1/3)*b*c*d - (-c*d^2)^(1/3)*a*d^2)*arctan(1/3*sqrt(3)*(2*x +
(-c/d)^(1/3))/(-c/d)^(1/3))/(sqrt(3)*b^3*c^5 - 3*sqrt(3)*a*b^2*c^4*d + 3*sq
rt(3)*a^2*b*c^3*d^2 - sqrt(3)*a^3*c^2*d^3) + 1/9*((-a*b^2)^(1/3)*b^2*c - 4*
(-a*b^2)^(1/3)*a*b*d)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a^2*b^3*c^3
 - 3*a^3*b^2*c^2*d + 3*a^4*b*c*d^2 - a^5*d^3) + 1/9*(4*(-c*d^2)^(1/3)*b*c*d
 - (-c*d^2)^(1/3)*a*d^2)*log(x^2 + x*(-c/d)^(1/3) + (-c/d)^(2/3))/(b^3*c^5
- 3*a*b^2*c^4*d + 3*a^2*b*c^3*d^2 - a^3*c^2*d^3) + 1/3*(b^2*c*d*x^4 + a*b*d
^2*x^4 + b^2*c^2*x + a^2*d^2*x)/((b*d*x^6 + b*c*x^3 + a*d*x^3 + a*c)*(a*b^2
*c^3 - 2*a^2*b*c^2*d + a^3*c*d^2))
```

maple [A] time = 0.06, size = 606, normalized size = 1.45

$$\frac{ad^3x}{3(ad-bc)^3(dx^3+c)c}-\frac{b^3cx}{3(ad-bc)^3(bx^3+a)a}+\frac{b^2dx}{3(ad-bc)^3(bx^3+a)}-\frac{bd^2x}{3(ad-bc)^3(dx^3+c)}+\frac{2\sqrt{3}\,ad^2\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{c}{d}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{9(ad-bc)^3\left(\frac{c}{d}\right)^{\frac{2}{3}}c}+\frac{2ad^2\ln\left(x+\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{9(ad-bc)^3\left(\frac{c}{d}\right)^{\frac{2}{3}}c}-\frac{ad^2\ln\left(x^2-\left(\frac{c}{d}\right)^{\frac{1}{3}}x+\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}{9(ad-bc)^3\left(\frac{c}{d}\right)^{\frac{2}{3}}c}-\frac{2\sqrt{3}\,b^2c\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{9(ad-bc)^3\left(\frac{a}{b}\right)^{\frac{2}{3}}a}-\frac{2b^2c\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9(ad-bc)^3\left(\frac{a}{b}\right)^{\frac{2}{3}}a}+\frac{b^2c\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{9(ad-bc)^3\left(\frac{a}{b}\right)^{\frac{2}{3}}a}+\frac{8\sqrt{3}\,bd\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{9(ad-bc)^3\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{8\sqrt{3}\,bd\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{c}{d}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{9(ad-bc)^3\left(\frac{c}{d}\right)^{\frac{2}{3}}}+\frac{8bd\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9(ad-bc)^3\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{8bd\ln\left(x+\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{9(ad-bc)^3\left(\frac{c}{d}\right)^{\frac{2}{3}}}-\frac{4bd\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{9(ad-bc)^3\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{4bd\ln\left(x^2-\left(\frac{c}{d}\right)^{\frac{1}{3}}x+\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}{9(ad-bc)^3\left(\frac{c}{d}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^3+a)^2/(d*x^3+c)^2,x)
```

```
[Out] 1/3*b^2/(a*d-b*c)^3*x/(b*x^3+a)*d-1/3*b^3/(a*d-b*c)^3/a*x/(b*x^3+a)*c+8/9*b
/(a*d-b*c)^3/(a/b)^(2/3)*ln(x+(a/b)^(1/3))*d-2/9*b^2/(a*d-b*c)^3/a/(a/b)^(2
/3)*ln(x+(a/b)^(1/3))*c-4/9*b/(a*d-b*c)^3/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+
(a/b)^(2/3))*d+1/9*b^2/(a*d-b*c)^3/a/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)
^(2/3))*c+8/9*b/(a*d-b*c)^3/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)
^(1/3)*x-1))*d-2/9*b^2/(a*d-b*c)^3/a/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)
*(2/(a/b)^(1/3)*x-1))*c+1/3*d^3/(a*d-b*c)^3/c*x/(d*x^3+c)*a-1/3*d^2/(a*d-b*
c)^3*x/(d*x^3+c)*b+2/9*d^2/(a*d-b*c)^3/c/(c/d)^(2/3)*ln(x+(c/d)^(1/3))*a-8/
9*d/(a*d-b*c)^3/(c/d)^(2/3)*ln(x+(c/d)^(1/3))*b-1/9*d^2/(a*d-b*c)^3/c/(c/d)
^(2/3)*ln(x^2-(c/d)^(1/3)*x+(c/d)^(2/3))*a+4/9*d/(a*d-b*c)^3/(c/d)^(2/3)*ln
(x^2-(c/d)^(1/3)*x+(c/d)^(2/3))*b+2/9*d^2/(a*d-b*c)^3/c/(c/d)^(2/3)*3^(1/2)
*arctan(1/3*3^(1/2)*(2/(c/d)^(1/3)*x-1))*a-8/9*d/(a*d-b*c)^3/(c/d)^(2/3)*3^
(1/2)*arctan(1/3*3^(1/2)*(2/(c/d)^(1/3)*x-1))*b
```

maxima [B] time = 1.26, size = 784, normalized size = 1.87

$$\frac{2\sqrt{3}\left(b^2c-4abd\right)\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\left(ab^3c^3\left(\frac{a}{b}\right)^{\frac{1}{3}}-3a^2b^2c^2d\left(\frac{a}{b}\right)^{\frac{1}{3}}+3a^3bcd^2\left(\frac{a}{b}\right)^{\frac{1}{3}}-a^4d^3\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{2\sqrt{3}\left(4bcd-ad^2\right)\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{9\left(b^3c^4\left(\frac{c}{d}\right)^{\frac{1}{3}}-3ab^2c^3d\left(\frac{c}{d}\right)^{\frac{1}{3}}+3a^2bc^2d^2\left(\frac{c}{d}\right)^{\frac{1}{3}}-a^3cd^3\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)\left(\frac{c}{d}\right)^{\frac{1}{3}}}-\frac{\left(b^2c-4abd\right)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{9\left(ab^3c^3\left(\frac{a}{b}\right)^{\frac{2}{3}}-3a^2b^2c^2d\left(\frac{a}{b}\right)^{\frac{2}{3}}+3a^3bcd^2\left(\frac{a}{b}\right)^{\frac{2}{3}}-a^4d^3\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}-\frac{\left(4bcd-ad^2\right)\log\left(x^2-x\left(\frac{c}{d}\right)^{\frac{1}{3}}+\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}{9\left(b^3c^4\left(\frac{c}{d}\right)^{\frac{2}{3}}-3ab^2c^3d\left(\frac{c}{d}\right)^{\frac{2}{3}}+3a^2bc^2d^2\left(\frac{c}{d}\right)^{\frac{2}{3}}-a^3cd^3\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}+\frac{2\left(b^2c-4abd\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\left(ab^3c^3\left(\frac{a}{b}\right)^{\frac{2}{3}}-3a^2b^2c^2d\left(\frac{a}{b}\right)^{\frac{2}{3}}+3a^3bcd^2\left(\frac{a}{b}\right)^{\frac{2}{3}}-a^4d^3\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}+\frac{2\left(4bcd-ad^2\right)\log\left(x+\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{9\left(b^3c^4\left(\frac{c}{d}\right)^{\frac{2}{3}}-3ab^2c^3d\left(\frac{c}{d}\right)^{\frac{2}{3}}+3a^2bc^2d^2\left(\frac{c}{d}\right)^{\frac{2}{3}}-a^3cd^3\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}+\frac{\left(b^2cd+abd^2\right)x^4+\left(b^2c^2+a^2d^2\right)x}{3\left(a^2b^2c^4-2a^3bc^3d+a^4c^2d^2+\left(ab^3c^3d-2a^2b^2c^2d^2+a^3bcd^3\right)x^6+\left(ab^3c^4-a^2b^2c^3d-a^3bc^2d^2+a^4cd^3\right)x^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^2/(d*x^3+c)^2,x, algorithm="maxima")
```

```
[Out] 2/9*sqrt(3)*(b^2*c - 4*a*b*d)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^
(1/3))/((a*b^3*c^3*(a/b)^(1/3) - 3*a^2*b^2*c^2*d*(a/b)^(1/3) + 3*a^3*b*c*d^
2*(a/b)^(1/3) - a^4*d^3*(a/b)^(1/3))*(a/b)^(1/3)) + 2/9*sqrt(3)*(4*b*c*d -
a*d^2)*arctan(1/3*sqrt(3)*(2*x - (c/d)^(1/3))/(c/d)^(1/3))/((b^3*c^4*(c/d)^
(1/3) - 3*a*b^2*c^3*d*(c/d)^(1/3) + 3*a^2*b*c^2*d^2*(c/d)^(1/3) - a^3*c*d^3
*(c/d)^(1/3))*(c/d)^(1/3)) - 1/9*(b^2*c - 4*a*b*d)*log(x^2 - x*(a/b)^(1/3)
+ (a/b)^(2/3))/(a*b^3*c^3*(a/b)^(2/3) - 3*a^2*b^2*c^2*d*(a/b)^(2/3) + 3*a^3
*b*c*d^2*(a/b)^(2/3) - a^4*d^3*(a/b)^(2/3)) - 1/9*(4*b*c*d - a*d^2)*log(x^2
 - x*(c/d)^(1/3) + (c/d)^(2/3))/(b^3*c^4*(c/d)^(2/3) - 3*a*b^2*c^3*d*(c/d)^
(2/3) + 3*a^2*b*c^2*d^2*(c/d)^(2/3) - a^3*c*d^3*(c/d)^(2/3)) + 2/9*(b^2*c -
 4*a*b*d)*log(x + (a/b)^(1/3))/(a*b^3*c^3*(a/b)^(2/3) - 3*a^2*b^2*c^2*d*(a/
b)^(2/3) + 3*a^3*b*c*d^2*(a/b)^(2/3) - a^4*d^3*(a/b)^(2/3)) + 2/9*(4*b*c*d
- a*d^2)*log(x + (c/d)^(1/3))/(b^3*c^4*(c/d)^(2/3) - 3*a*b^2*c^3*d*(c/d)^(2
/3) + 3*a^2*b*c^2*d^2*(c/d)^(2/3) - a^3*c*d^3*(c/d)^(2/3)) + 1/3*((b^2*c*d
+ a*b*d^2)*x^4 + (b^2*c^2 + a^2*d^2)*x)/(a^2*b^2*c^4 - 2*a^3*b*c^3*d + a^4*
c^2*d^2 + (a*b^3*c^3*d - 2*a^2*b^2*c^2*d^2 + a^3*b*c*d^3)*x^6 + (a*b^3*c^4
- a^2*b^2*c^3*d - a^3*b*c^2*d^2 + a^4*c*d^3)*x^3)
```

mupad [B] time = 24.31, size = 3637, normalized size = 8.68

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^3)^2*(c + d*x^3)^2),x)
```

```
[Out] ((x*(a^2*d^2 + b^2*c^2))/(3*a*c*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)) + (b*d*x^4
*(a*d + b*c))/(3*a*c*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)))/(a*c + x^3*(a*d + b*
c) + b*d*x^6) + log((2*((4*((54*b^3*d^3*x*(a*d - b*c)^2*(a^3*d^3 + b^3*c^3
- 3*a*b^2*c^2*d - 3*a^2*b*c*d^2))/(a*c) + 54*a*b^3*c*d^3*(a*d + b*c)*(a*d -
 b*c)^4*((b^5*(4*a*d - b*c)^3)/(a^5*(a*d - b*c)^9))^(1/3))*((b^5*(4*a*d - b
*c)^3)/(a^5*(a*d - b*c)^9))^(2/3))/81 - (8*b^4*d^4*(a^6*d^6 + b^6*c^6 + 37*
a^2*b^4*c^4*d^2 - 27*a^3*b^3*c^3*d^3 + 37*a^4*b^2*c^2*d^4 - 11*a*b^5*c^5*d
- 11*a^5*b*c*d^5))/(3*a^3*c^3*(a*d - b*c)^4))*((b^5*(4*a*d - b*c)^3)/(a^5*(
a*d - b*c)^9))^(1/3))/9 - (16*b^6*d^6*x*(4*a^6*d^6 + 4*b^6*c^6 + 268*a^2*b^
4*c^4*d^2 - 608*a^3*b^3*c^3*d^3 + 268*a^4*b^2*c^2*d^4 - 49*a*b^5*c^5*d - 49
*a^5*b*c*d^5))/(27*a^3*c^3*(a*d - b*c)^8))*(-(8*b^8*c^3 - 512*a^3*b^5*d^3 +
 384*a^2*b^6*c*d^2 - 96*a*b^7*c^2*d)/(729*a^14*d^9 - 729*a^5*b^9*c^9 + 6561
*a^6*b^8*c^8*d - 26244*a^7*b^7*c^7*d^2 + 61236*a^8*b^6*c^6*d^3 - 91854*a^9*
b^5*c^5*d^4 + 91854*a^10*b^4*c^4*d^5 - 61236*a^11*b^3*c^3*d^6 + 26244*a^12*
b^2*c^2*d^7 - 6561*a^13*b*c*d^8))^(1/3) + log((2*((4*((54*b^3*d^3*x*(a*d -
b*c)^2*(a^3*d^3 + b^3*c^3 - 3*a*b^2*c^2*d - 3*a^2*b*c*d^2))/(a*c) + 54*a*b^
3*c*d^3*(a*d + b*c)*(a*d - b*c)^4*((d^5*(a*d - 4*b*c)^3)/(c^5*(a*d - b*c)^9
))^(1/3))*((d^5*(a*d - 4*b*c)^3)/(c^5*(a*d - b*c)^9))^(2/3))/81 - (8*b^4*d^
4*(a^6*d^6 + b^6*c^6 + 37*a^2*b^4*c^4*d^2 - 27*a^3*b^3*c^3*d^3 + 37*a^4*b^2
*c^2*d^4 - 11*a*b^5*c^5*d - 11*a^5*b*c*d^5))/(3*a^3*c^3*(a*d - b*c)^4))*((d
^5*(a*d - 4*b*c)^3)/(c^5*(a*d - b*c)^9))^(1/3))/9 - (16*b^6*d^6*x*(4*a^6*d^
6 + 4*b^6*c^6 + 268*a^2*b^4*c^4*d^2 - 608*a^3*b^3*c^3*d^3 + 268*a^4*b^2*c^2
*d^4 - 49*a*b^5*c^5*d - 49*a^5*b*c*d^5))/(27*a^3*c^3*(a*d - b*c)^8))*(-(8*a
^3*d^8 - 512*b^3*c^3*d^5 + 384*a*b^2*c^2*d^6 - 96*a^2*b*c*d^7)/(729*b^9*c^1
4 - 729*a^9*c^5*d^9 + 6561*a^8*b*c^6*d^8 + 26244*a^2*b^7*c^12*d^2 - 61236*a
^3*b^6*c^11*d^3 + 91854*a^4*b^5*c^10*d^4 - 91854*a^5*b^4*c^9*d^5 + 61236*a^
6*b^3*c^8*d^6 - 26244*a^7*b^2*c^7*d^7 - 6561*a*b^8*c^13*d))^(1/3) + (log(((
3^(1/2)*1i - 1)*(((3^(1/2)*1i - 1)^2*((54*b^3*d^3*x*(a*d - b*c)^2*(a^3*d^3
+ b^3*c^3 - 3*a*b^2*c^2*d - 3*a^2*b*c*d^2))/(a*c) + 27*a*b^3*c*d^3*(3^(1/2)
*1i - 1)*(a*d + b*c)*(a*d - b*c)^4*((b^5*(4*a*d - b*c)^3)/(a^5*(a*d - b*c)^
9))^(1/3))*((b^5*(4*a*d - b*c)^3)/(a^5*(a*d - b*c)^9))^(2/3))/81 - (8*b^4*d
^4*(a^6*d^6 + b^6*c^6 + 37*a^2*b^4*c^4*d^2 - 27*a^3*b^3*c^3*d^3 + 37*a^4*b^
2*c^2*d^4 - 11*a*b^5*c^5*d - 11*a^5*b*c*d^5))/(3*a^3*c^3*(a*d - b*c)^4))*((
b^5*(4*a*d - b*c)^3)/(a^5*(a*d - b*c)^9))^(1/3))/9 - (16*b^6*d^6*x*(4*a^6*d
```

```
^6 + 4*b^6*c^6 + 268*a^2*b^4*c^4*d^2 - 608*a^3*b^3*c^3*d^3 + 268*a^4*b^2*c^
2*d^4 - 49*a*b^5*c^5*d - 49*a^5*b*c*d^5))/(27*a^3*c^3*(a*d - b*c)^8))*(3^(1
/2)*1i - 1)*(-(8*b^8*c^3 - 512*a^3*b^5*d^3 + 384*a^2*b^6*c*d^2 - 96*a*b^7*c
^2*d)/(729*a^14*d^9 - 729*a^5*b^9*c^9 + 6561*a^6*b^8*c^8*d - 26244*a^7*b^7*
c^7*d^2 + 61236*a^8*b^6*c^6*d^3 - 91854*a^9*b^5*c^5*d^4 + 91854*a^10*b^4*c^
4*d^5 - 61236*a^11*b^3*c^3*d^6 + 26244*a^12*b^2*c^2*d^7 - 6561*a^13*b*c*d^8
))^(1/3))/2 - (log(((3^(1/2)*1i + 1)*(((3^(1/2)*1i + 1)^2*((54*b^3*d^3*x*(a
*d - b*c)^2*(a^3*d^3 + b^3*c^3 - 3*a*b^2*c^2*d - 3*a^2*b*c*d^2))/(a*c) - 27
*a*b^3*c*d^3*(3^(1/2)*1i + 1)*(a*d + b*c)*(a*d - b*c)^4*((b^5*(4*a*d - b*c)
^3)/(a^5*(a*d - b*c)^9))^(1/3))*((b^5*(4*a*d - b*c)^3)/(a^5*(a*d - b*c)^9))
^(2/3))/81 - (8*b^4*d^4*(a^6*d^6 + b^6*c^6 + 37*a^2*b^4*c^4*d^2 - 27*a^3*b^
3*c^3*d^3 + 37*a^4*b^2*c^2*d^4 - 11*a*b^5*c^5*d - 11*a^5*b*c*d^5))/(3*a^3*c
^3*(a*d - b*c)^4))*((b^5*(4*a*d - b*c)^3)/(a^5*(a*d - b*c)^9))^(1/3))/9 + (
16*b^6*d^6*x*(4*a^6*d^6 + 4*b^6*c^6 + 268*a^2*b^4*c^4*d^2 - 608*a^3*b^3*c^3
*d^3 + 268*a^4*b^2*c^2*d^4 - 49*a*b^5*c^5*d - 49*a^5*b*c*d^5))/(27*a^3*c^3*
(a*d - b*c)^8))*(3^(1/2)*1i + 1)*(-(8*b^8*c^3 - 512*a^3*b^5*d^3 + 384*a^2*b
^6*c*d^2 - 96*a*b^7*c^2*d)/(729*a^14*d^9 - 729*a^5*b^9*c^9 + 6561*a^6*b^8*c
^8*d - 26244*a^7*b^7*c^7*d^2 + 61236*a^8*b^6*c^6*d^3 - 91854*a^9*b^5*c^5*d^
4 + 91854*a^10*b^4*c^4*d^5 - 61236*a^11*b^3*c^3*d^6 + 26244*a^12*b^2*c^2*d^
7 - 6561*a^13*b*c*d^8))^(1/3))/2 + (log(((3^(1/2)*1i - 1)*(((3^(1/2)*1i - 1
)^2*((54*b^3*d^3*x*(a*d - b*c)^2*(a^3*d^3 + b^3*c^3 - 3*a*b^2*c^2*d - 3*a^2
*b*c*d^2))/(a*c) + 27*a*b^3*c*d^3*(3^(1/2)*1i - 1)*(a*d + b*c)*(a*d - b*c)^
4*((d^5*(a*d - 4*b*c)^3)/(c^5*(a*d - b*c)^9))^(1/3))*((d^5*(a*d - 4*b*c)^3)
/(c^5*(a*d - b*c)^9))^(2/3))/81 - (8*b^4*d^4*(a^6*d^6 + b^6*c^6 + 37*a^2*b^
4*c^4*d^2 - 27*a^3*b^3*c^3*d^3 + 37*a^4*b^2*c^2*d^4 - 11*a*b^5*c^5*d - 11*a
^5*b*c*d^5))/(3*a^3*c^3*(a*d - b*c)^4))*((d^5*(a*d - 4*b*c)^3)/(c^5*(a*d -
b*c)^9))^(1/3))/9 - (16*b^6*d^6*x*(4*a^6*d^6 + 4*b^6*c^6 + 268*a^2*b^4*c^4*
d^2 - 608*a^3*b^3*c^3*d^3 + 268*a^4*b^2*c^2*d^4 - 49*a*b^5*c^5*d - 49*a^5*b
*c*d^5))/(27*a^3*c^3*(a*d - b*c)^8))*(3^(1/2)*1i - 1)*(-(8*a^3*d^8 - 512*b^
3*c^3*d^5 + 384*a*b^2*c^2*d^6 - 96*a^2*b*c*d^7)/(729*b^9*c^14 - 729*a^9*c^5
*d^9 + 6561*a^8*b*c^6*d^8 + 26244*a^2*b^7*c^12*d^2 - 61236*a^3*b^6*c^11*d^3
 + 91854*a^4*b^5*c^10*d^4 - 91854*a^5*b^4*c^9*d^5 + 61236*a^6*b^3*c^8*d^6 -
 26244*a^7*b^2*c^7*d^7 - 6561*a*b^8*c^13*d))^(1/3))/2 - (log(((3^(1/2)*1i +
 1)*(((3^(1/2)*1i + 1)^2*((54*b^3*d^3*x*(a*d - b*c)^2*(a^3*d^3 + b^3*c^3 -
3*a*b^2*c^2*d - 3*a^2*b*c*d^2))/(a*c) - 27*a*b^3*c*d^3*(3^(1/2)*1i + 1)*(a*
d + b*c)*(a*d - b*c)^4*((d^5*(a*d - 4*b*c)^3)/(c^5*(a*d - b*c)^9))^(1/3))*(
(d^5*(a*d - 4*b*c)^3)/(c^5*(a*d - b*c)^9))^(2/3))/81 - (8*b^4*d^4*(a^6*d^6
+ b^6*c^6 + 37*a^2*b^4*c^4*d^2 - 27*a^3*b^3*c^3*d^3 + 37*a^4*b^2*c^2*d^4 -
11*a*b^5*c^5*d - 11*a^5*b*c*d^5))/(3*a^3*c^3*(a*d - b*c)^4))*((d^5*(a*d - 4
*b*c)^3)/(c^5*(a*d - b*c)^9))^(1/3))/9 + (16*b^6*d^6*x*(4*a^6*d^6 + 4*b^6*c
^6 + 268*a^2*b^4*c^4*d^2 - 608*a^3*b^3*c^3*d^3 + 268*a^4*b^2*c^2*d^4 - 49*a
*b^5*c^5*d - 49*a^5*b*c*d^5))/(27*a^3*c^3*(a*d - b*c)^8))*(3^(1/2)*1i + 1)*
(-(8*a^3*d^8 - 512*b^3*c^3*d^5 + 384*a*b^2*c^2*d^6 - 96*a^2*b*c*d^7)/(729*b
^9*c^14 - 729*a^9*c^5*d^9 + 6561*a^8*b*c^6*d^8 + 26244*a^2*b^7*c^12*d^2 - 6
1236*a^3*b^6*c^11*d^3 + 91854*a^4*b^5*c^10*d^4 - 91854*a^5*b^4*c^9*d^5 + 61
236*a^6*b^3*c^8*d^6 - 26244*a^7*b^2*c^7*d^7 - 6561*a*b^8*c^13*d))^(1/3))/2
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**3+a)**2/(d*x**3+c)**2,x)
```

```
[Out] Timed out
```

3.27 $\int \left(a-bx^3\right)\left(a+bx^3\right)^{2/3}\,dx$

Optimal. Leaf size=112

$$-\frac{7a^2\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}\,x\right)}{18\sqrt[3]{b}}+\frac{7a^2\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{9\sqrt{3}\,\sqrt[3]{b}}+\frac{7}{18}ax\left(a+bx^3\right)^{2/3}-\frac{1}{6}x\left(a+bx^3\right)^{5/3}$$

Rubi [A] time = 0.03, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {388, 195, 239}

$$-\frac{7a^2\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}\,x\right)}{18\sqrt[3]{b}}+\frac{7a^2\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{9\sqrt{3}\,\sqrt[3]{b}}+\frac{7}{18}ax\left(a+bx^3\right)^{2/3}-\frac{1}{6}x\left(a+bx^3\right)^{5/3}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*x^3)*(a + b*x^3)^(2/3),x]

[Out] (7*a*x*(a + b*x^3)^(2/3))/18 - (x*(a + b*x^3)^(5/3))/6 + (7*a^2*ArcTan[(1 +
 (2*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]])/(9*Sqrt[3]*b^(1/3)) - (7*a^2*Lo
g[-(b^(1/3)*x) + (a + b*x^3)^(1/3)])/(18*b^(1/3))
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 239

```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + (2*Rt[b, 3]
*x)/(a + b*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rubi steps

$$\begin{aligned}\int \left(a-bx^3\right)\left(a+bx^3\right)^{2/3}\,dx &= -\frac{1}{6}x\left(a+bx^3\right)^{5/3}+\frac{1}{6}(7a)\int \left(a+bx^3\right)^{2/3}\,dx\\ &= \frac{7}{18}ax\left(a+bx^3\right)^{2/3}-\frac{1}{6}x\left(a+bx^3\right)^{5/3}+\frac{1}{9}\left(7a^2\right)\int \frac{1}{\sqrt[3]{a+bx^3}}\,dx\\ &= \frac{7}{18}ax\left(a+bx^3\right)^{2/3}-\frac{1}{6}x\left(a+bx^3\right)^{5/3}+\frac{7a^2\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{9\sqrt{3}\,\sqrt[3]{b}}-\frac{7a^2\log\left(-\sqrt[3]{b}\,x+\sqrt[3]{\right.}}{18\sqrt[3]{b}}\end{aligned}$$

Mathematica [C] time = 0.11, size = 62, normalized size = 0.55

$$\frac{1}{6}x\left(a+bx^3\right)^{2/3}\left(\frac{7a\,{}_2F_1\left(-\frac{2}{3},\frac{1}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{\left(\frac{bx^3}{a}+1\right)^{2/3}}-a-bx^3\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b*x^3)*(a + b*x^3)^(2/3),x]
```

```
[Out] (x*(a + b*x^3)^(2/3)*(-a - b*x^3 + (7*a*Hypergeometric2F1[-2/3, 1/3, 4/3, -
((b*x^3)/a)])/(1 + (b*x^3)/a)^(2/3)))/6
```

IntegrateAlgebraic [A] time = 0.39, size = 165, normalized size = 1.47

$$\frac{7a^2\log\left(\sqrt[3]{b}\,x\sqrt[3]{a+bx^3}+\left(a+bx^3\right)^{2/3}+b^{2/3}x^2\right)}{54\sqrt[3]{b}}-\frac{7a^2\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}\,x\right)}{27\sqrt[3]{b}}+\frac{7a^2\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{b}\,x}{2\sqrt[3]{a+bx^3}+\sqrt[3]{b}\,x}\right)}{9\sqrt{3}\sqrt[3]{b}}+\frac{1}{18}\left(a+bx^3\right)^{2/3}\left(4ax-3bx^4\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a - b*x^3)*(a + b*x^3)^(2/3),x]
```

```
[Out] ((a + b*x^3)^(2/3)*(4*a*x - 3*b*x^4))/18 + (7*a^2*ArcTan[(Sqrt[3]*b^(1/3)*x
)/(b^(1/3)*x + 2*(a + b*x^3)^(1/3))])/(9*Sqrt[3]*b^(1/3)) - (7*a^2*Log[-(b^
(1/3)*x) + (a + b*x^3)^(1/3)])/(27*b^(1/3)) + (7*a^2*Log[b^(2/3)*x^2 + b^(1
/3)*x*(a + b*x^3)^(1/3) + (a + b*x^3)^(2/3)])/(54*b^(1/3))
```

fricas [B] time = 1.06, size = 399, normalized size = 3.56

$$\left[\frac{21\sqrt{\frac{1}{3}}a^2b\sqrt{\frac{(-b)^{\frac{1}{3}}}{b}}\log\left(3bx^3-3\left(bx^3+a\right)^{\frac{1}{3}}(-b)^{\frac{2}{3}}x^2-3\sqrt{\frac{1}{3}}\left((-b)^{\frac{1}{3}}bx^3-\left(bx^3+a\right)^{\frac{1}{3}}bx^2+2\left(bx^3+a\right)^{\frac{2}{3}}(-b)^{\frac{2}{3}}x\right)\sqrt{\frac{(-b)^{\frac{1}{3}}}{b}}+2a\right)-14a^2(-b)^{\frac{2}{3}}\log\left(\frac{(-b)^{\frac{1}{3}}x+\left(bx^3+a\right)^{\frac{1}{3}}}{x}\right)+7a^2(-b)^{\frac{2}{3}}\log\left(\frac{(-b)^{\frac{2}{3}}x^2-\left(bx^3+a\right)^{\frac{1}{3}}(-b)^{\frac{1}{3}}x+\left(bx^3+a\right)^{\frac{2}{3}}}{x^2}\right)-3\left(3b^2x^4-4abx\right)\left(bx^3+a\right)^{\frac{2}{3}}}{54b},\ -\frac{42\sqrt{\frac{1}{3}}a^2b\sqrt{-\frac{(-b)^{\frac{1}{3}}}{b}}\arctan\left(-\frac{\sqrt{\frac{1}{3}}\left((-b)^{\frac{1}{3}}x-2\left(bx^3+a\right)^{\frac{1}{3}}\right)\sqrt{-\frac{(-b)^{\frac{1}{3}}}{b}}}{x}\right)+14a^2(-b)^{\frac{2}{3}}\log\left(\frac{(-b)^{\frac{1}{3}}x+\left(bx^3+a\right)^{\frac{1}{3}}}{x}\right)-7a^2(-b)^{\frac{2}{3}}\log\left(\frac{(-b)^{\frac{2}{3}}x^2-\left(bx^3+a\right)^{\frac{1}{3}}(-b)^{\frac{1}{3}}x+\left(bx^3+a\right)^{\frac{2}{3}}}{x^2}\right)+3\left(3b^2x^4-4abx\right)\left(bx^3+a\right)^{\frac{2}{3}}}{54b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^3+a)*(b*x^3+a)^(2/3),x, algorithm="fricas")
```

```
[Out] [1/54*(21*sqrt(1/3)*a^2*b*sqrt((-b)^(1/3)/b)*log(3*b*x^3 - 3*(b*x^3 + a)^(1
/3)*(-b)^(2/3)*x^2 - 3*sqrt(1/3)*((-b)^(1/3)*b*x^3 - (b*x^3 + a)^(1/3)*b*x^
2 + 2*(b*x^3 + a)^(2/3)*(-b)^(2/3)*x)*sqrt((-b)^(1/3)/b) + 2*a) - 14*a^2*(-
b)^(2/3)*log(((-b)^(1/3)*x + (b*x^3 + a)^(1/3))/x) + 7*a^2*(-b)^(2/3)*log((
(-b)^(2/3)*x^2 - (b*x^3 + a)^(1/3)*(-b)^(1/3)*x + (b*x^3 + a)^(2/3))/x^2) -
 3*(3*b^2*x^4 - 4*a*b*x)*(b*x^3 + a)^(2/3))/b, -1/54*(42*sqrt(1/3)*a^2*b*sq
rt(-(-b)^(1/3)/b)*arctan(-sqrt(1/3)*((-b)^(1/3)*x - 2*(b*x^3 + a)^(1/3))*sq
rt(-(-b)^(1/3)/b)/x) + 14*a^2*(-b)^(2/3)*log(((-b)^(1/3)*x + (b*x^3 + a)^(1
/3))/x) - 7*a^2*(-b)^(2/3)*log(((-b)^(2/3)*x^2 - (b*x^3 + a)^(1/3)*(-b)^(1/
3)*x + (b*x^3 + a)^(2/3))/x^2) + 3*(3*b^2*x^4 - 4*a*b*x)*(b*x^3 + a)^(2/3))
/b]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int-\left(bx^3+a\right)^{\frac{2}{3}}\left(bx^3-a\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^3+a)*(b*x^3+a)^(2/3),x, algorithm="giac")
```

```
[Out] integrate(-(b*x^3 + a)^(2/3)*(b*x^3 - a), x)
```

maple [F] time = 0.43, size = 0, normalized size = 0.00

$$\int\left(-b\,x^3+a\right)\left(b\,x^3+a\right)^{\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x^3+a)*(b*x^3+a)^(2/3),x)
```

```
[Out] int((-b*x^3+a)*(b*x^3+a)^(2/3),x)
```

maxima [B] time = 1.22, size = 322, normalized size = 2.88

$$-\frac{1}{9}\left(\frac{2\sqrt{3}\,a\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}}+\frac{2\,(bx^3+a)^{\frac{1}{3}}}{x}\right)}{3\,b^{\frac{1}{3}}}\right)}{b^{\frac{1}{3}}}-\frac{a\log\left(b^{\frac{2}{3}}+\frac{(bx^3+a)^{\frac{1}{3}}b^{\frac{1}{3}}}{x}+\frac{(bx^3+a)^{\frac{2}{3}}}{x^2}\right)}{b^{\frac{1}{3}}}+\frac{2\,a\log\left(-b^{\frac{1}{3}}+\frac{(bx^3+a)^{\frac{1}{3}}}{x}\right)}{b^{\frac{1}{3}}}+\frac{3\,(bx^3+a)^{\frac{2}{3}}a}{\left(b-\frac{bx^3+a}{x^3}\right)x^2}\right)a-\frac{1}{54}\left(\frac{2\sqrt{3}\,a^2\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}}+\frac{2\,(bx^3+a)^{\frac{1}{3}}}{x}\right)}{3\,b^{\frac{1}{3}}}\right)}{b^{\frac{4}{3}}}-\frac{a^2\log\left(b^{\frac{2}{3}}+\frac{(bx^3+a)^{\frac{1}{3}}b^{\frac{1}{3}}}{x}+\frac{(bx^3+a)^{\frac{2}{3}}}{x^2}\right)}{b^{\frac{4}{3}}}+\frac{2\,a^2\log\left(-b^{\frac{1}{3}}+\frac{(bx^3+a)^{\frac{1}{3}}}{x}\right)}{b^{\frac{4}{3}}}+\frac{3\left(\frac{(bx^3+a)^{\frac{2}{3}}a^2b}{x^2}+\frac{2\,(bx^3+a)^{\frac{5}{3}}a^2}{x^5}\right)}{b^3-\frac{2\,(bx^3+a)b^2}{x^3}+\frac{(bx^3+a)^2b}{x^6}}\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^3+a)*(b*x^3+a)^(2/3),x, algorithm="maxima")
```

```
[Out] -1/9*(2*sqrt(3)*a*arctan(1/3*sqrt(3)*(b^(1/3) + 2*(b*x^3 + a)^(1/3)/x)/b^(1
/3))/b^(1/3) - a*log(b^(2/3) + (b*x^3 + a)^(1/3)*b^(1/3)/x + (b*x^3 + a)^(2
/3)/x^2)/b^(1/3) + 2*a*log(-b^(1/3) + (b*x^3 + a)^(1/3)/x)/b^(1/3) + 3*(b*x
^3 + a)^(2/3)*a/((b - (b*x^3 + a)/x^3)*x^2))*a - 1/54*(2*sqrt(3)*a^2*arctan
(1/3*sqrt(3)*(b^(1/3) + 2*(b*x^3 + a)^(1/3)/x)/b^(1/3))/b^(4/3) - a^2*log(b
^(2/3) + (b*x^3 + a)^(1/3)*b^(1/3)/x + (b*x^3 + a)^(2/3)/x^2)/b^(4/3) + 2*a
^2*log(-b^(1/3) + (b*x^3 + a)^(1/3)/x)/b^(4/3) + 3*((b*x^3 + a)^(2/3)*a^2*b
/x^2 + 2*(b*x^3 + a)^(5/3)*a^2/x^5)/(b^3 - 2*(b*x^3 + a)*b^2/x^3 + (b*x^3 +
 a)^2*b/x^6))*b
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(b\,x^3+a\right)^{2/3}\,\left(a-b\,x^3\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^(2/3)*(a - b*x^3),x)
```

```
[Out] int((a + b*x^3)^(2/3)*(a - b*x^3), x)
```

sympy [C] time = 4.98, size = 80, normalized size = 0.71

$$\frac{a^{\frac{5}{3}}x\Gamma\left(\frac{1}{3}\right){}_{2}F_{1}\left(\left.\begin{matrix}-\frac{2}{3},\frac{1}{3}\\ \frac{4}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{4}{3}\right)}-\frac{a^{\frac{2}{3}}bx^4\Gamma\left(\frac{4}{3}\right){}_{2}F_{1}\left(\left.\begin{matrix}-\frac{2}{3},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{7}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x**3+a)*(b*x**3+a)**(2/3),x)
```

```
[Out] a**(5/3)*x*gamma(1/3)*hyper((-2/3, 1/3), (4/3,), b*x**3*exp_polar(I*pi)/a)/
(3*gamma(4/3)) - a**(2/3)*b*x**4*gamma(4/3)*hyper((-2/3, 4/3), (7/3,), b*x*
*3*exp_polar(I*pi)/a)/(3*gamma(7/3))
```

3.28 $\int \frac{a-bx^3}{\sqrt[3]{a+bx^3}}\,dx$

Optimal. Leaf size=91

$$-\frac{1}{3}x\left(a+bx^3\right)^{2/3}-\frac{2a\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}\,x\right)}{3\sqrt[3]{b}}+\frac{4a\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{3\sqrt{3}\sqrt[3]{b}}$$

Rubi [A] time = 0.02, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {388, 239}

$$-\frac{1}{3}x\left(a+bx^3\right)^{2/3}-\frac{2a\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}\,x\right)}{3\sqrt[3]{b}}+\frac{4a\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{3\sqrt{3}\sqrt[3]{b}}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*x^3)/(a + b*x^3)^(1/3),x]
```

```
[Out] -(x*(a + b*x^3)^(2/3))/3 + (4*a*ArcTan[(1 + (2*b^(1/3)*x)/(a + b*x^3)^(1/3)
)/Sqrt[3]])/(3*Sqrt[3]*b^(1/3)) - (2*a*Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)
])/(3*b^(1/3))
```

Rule 239

```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + (2*Rt[b, 3]
*x)/(a + b*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{a-bx^3}{\sqrt[3]{a+bx^3}}\,dx&=-\frac{1}{3}x\left(a+bx^3\right)^{2/3}+\frac{1}{3}(4a)\int \frac{1}{\sqrt[3]{a+bx^3}}\,dx\\&=-\frac{1}{3}x\left(a+bx^3\right)^{2/3}+\frac{4a\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{3\sqrt{3}\sqrt[3]{b}}-\frac{2a\log\left(-\sqrt[3]{b}\,x+\sqrt[3]{a+bx^3}\right)}{3\sqrt[3]{b}}\end{aligned}$$

Mathematica [A] time = 0.08, size = 134, normalized size = 1.47

$$\frac{2a\log\left(\frac{b^{2/3}x^2}{\left(a+bx^3\right)^{2/3}}+\frac{\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}+1\right)-3\sqrt[3]{b}\,x\left(a+bx^3\right)^{2/3}-4a\log\left(1-\frac{\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}\right)+4\sqrt{3}\,a\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{9\sqrt[3]{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b*x^3)/(a + b*x^3)^(1/3),x]
```

```
[Out] (-3*b^(1/3)*x*(a + b*x^3)^(2/3) + 4*Sqrt[3]*a*ArcTan[(1 + (2*b^(1/3)*x)/(a
+ b*x^3)^(1/3))/Sqrt[3]] - 4*a*Log[1 - (b^(1/3)*x)/(a + b*x^3)^(1/3)] + 2*a
*Log[1 + (b^(2/3)*x^2)/(a + b*x^3)^(2/3) + (b^(1/3)*x)/(a + b*x^3)^(1/3)])/
(9*b^(1/3))
```

IntegrateAlgebraic [A] time = 0.37, size = 149, normalized size = 1.64

$$\frac{2a\log\left(\sqrt[3]{b}\,x\sqrt[3]{a+bx^3}+\left(a+bx^3\right)^{2/3}+b^{2/3}x^2\right)}{9\sqrt[3]{b}}-\frac{1}{3}x\left(a+bx^3\right)^{2/3}-\frac{4a\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}\,x\right)}{9\sqrt[3]{b}}+\frac{4a\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{b}\,x}{2\sqrt[3]{a+bx^3}+\sqrt[3]{b}\,x}\right)}{3\sqrt{3}\sqrt[3]{b}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a - b*x^3)/(a + b*x^3)^(1/3),x]
```

```
[Out] -1/3*(x*(a + b*x^3)^(2/3)) + (4*a*ArcTan[(Sqrt[3]*b^(1/3)*x)/(b^(1/3)*x + 2
*(a + b*x^3)^(1/3))])/(3*Sqrt[3]*b^(1/3)) - (4*a*Log[-(b^(1/3)*x) + (a + b*
x^3)^(1/3)])/(9*b^(1/3)) + (2*a*Log[b^(2/3)*x^2 + b^(1/3)*x*(a + b*x^3)^(1/
3) + (a + b*x^3)^(2/3)])/(9*b^(1/3))
```

fricas [B] time = 1.02, size = 363, normalized size = 3.99

$$\left[\frac{6\sqrt{\frac{1}{3}}\,ab\sqrt{\frac{(-b)^{\frac{1}{3}}}{b}}\log\left(3\,bx^3-3\left(bx^3+a\right)^{\frac{1}{3}}(-b)^{\frac{2}{3}}x^2-3\sqrt{\frac{1}{3}}\left((-b)^{\frac{1}{3}}bx^3-\left(bx^3+a\right)^{\frac{1}{3}}bx^2+2\left(bx^3+a\right)^{\frac{2}{3}}(-b)^{\frac{2}{3}}x\right)\sqrt{\frac{(-b)^{\frac{1}{3}}}{b}}+2\,a\right)-3\left(bx^3+a\right)^{\frac{2}{3}}bx-4\,a\,(-b)^{\frac{2}{3}}\log\left(\frac{(-b)^{\frac{1}{3}}x+\left(bx^3+a\right)^{\frac{1}{3}}}{x}\right)+2\,a\,(-b)^{\frac{2}{3}}\log\left(\frac{(-b)^{\frac{2}{3}}x^2-\left(bx^3+a\right)^{\frac{1}{3}}(-b)^{\frac{1}{3}}x+\left(bx^3+a\right)^{\frac{2}{3}}}{x^2}\right)}{9\,b},-\frac{12\sqrt{\frac{1}{3}}\,ab\sqrt{-\frac{(-b)^{\frac{1}{3}}}{b}}\arctan\left(-\frac{\sqrt{\frac{1}{3}}\left((-b)^{\frac{1}{3}}x-2\left(bx^3+a\right)^{\frac{1}{3}}\right)\sqrt{-\frac{(-b)^{\frac{1}{3}}}{b}}}{x}\right)+3\left(bx^3+a\right)^{\frac{2}{3}}bx+4\,a\,(-b)^{\frac{2}{3}}\log\left(\frac{(-b)^{\frac{1}{3}}x+\left(bx^3+a\right)^{\frac{1}{3}}}{x}\right)-2\,a\,(-b)^{\frac{2}{3}}\log\left(\frac{(-b)^{\frac{2}{3}}x^2-\left(bx^3+a\right)^{\frac{1}{3}}(-b)^{\frac{1}{3}}x+\left(bx^3+a\right)^{\frac{2}{3}}}{x^2}\right)}{9\,b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^3+a)/(b*x^3+a)^(1/3),x, algorithm="fricas")
```

```
[Out] [1/9*(6*sqrt(1/3)*a*b*sqrt((-b)^(1/3)/b)*log(3*b*x^3 - 3*(b*x^3 + a)^(1/3)*
(-b)^(2/3)*x^2 - 3*sqrt(1/3)*((-b)^(1/3)*b*x^3 - (b*x^3 + a)^(1/3)*b*x^2 +
2*(b*x^3 + a)^(2/3)*(-b)^(2/3)*x)*sqrt((-b)^(1/3)/b) + 2*a) - 3*(b*x^3 + a)
^(2/3)*b*x - 4*a*(-b)^(2/3)*log(((-b)^(1/3)*x + (b*x^3 + a)^(1/3))/x) + 2*a
*(-b)^(2/3)*log(((-b)^(2/3)*x^2 - (b*x^3 + a)^(1/3)*(-b)^(1/3)*x + (b*x^3 +
 a)^(2/3))/x^2))/b, -1/9*(12*sqrt(1/3)*a*b*sqrt(-(-b)^(1/3)/b)*arctan(-sqrt
(1/3)*((-b)^(1/3)*x - 2*(b*x^3 + a)^(1/3))*sqrt(-(-b)^(1/3)/b)/x) + 3*(b*x^
3 + a)^(2/3)*b*x + 4*a*(-b)^(2/3)*log(((-b)^(1/3)*x + (b*x^3 + a)^(1/3))/x)
 - 2*a*(-b)^(2/3)*log(((-b)^(2/3)*x^2 - (b*x^3 + a)^(1/3)*(-b)^(1/3)*x + (b
*x^3 + a)^(2/3))/x^2))/b]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{bx^3-a}{\left(bx^3+a\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^3+a)/(b*x^3+a)^(1/3),x, algorithm="giac")
```

```
[Out] integrate(-(b*x^3 - a)/(b*x^3 + a)^(1/3), x)
```

maple [F] time = 0.37, size = 0, normalized size = 0.00

$$\int \frac{-b\,x^3+a}{\left(b\,x^3+a\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x^3+a)/(b*x^3+a)^(1/3),x)
```

```
[Out] int((-b*x^3+a)/(b*x^3+a)^(1/3),x)
```

maxima [B] time = 1.22, size = 244, normalized size = 2.68

$$-\frac{1}{6}\left(\frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}}+\frac{2\left(bx^3+a\right)^{\frac{1}{3}}}{x}\right)}{3b^{\frac{1}{3}}}\right)}{b^{\frac{1}{3}}}-\frac{\log\left(b^{\frac{2}{3}}+\frac{\left(bx^3+a\right)^{\frac{1}{3}}b^{\frac{1}{3}}}{x}+\frac{\left(bx^3+a\right)^{\frac{2}{3}}}{x^2}\right)}{b^{\frac{1}{3}}}+\frac{2\log\left(-b^{\frac{1}{3}}+\frac{\left(bx^3+a\right)^{\frac{1}{3}}}{x}\right)}{b^{\frac{1}{3}}}\right)a-\frac{1}{18}\left(\frac{2\sqrt{3}a\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}}+\frac{2\left(bx^3+a\right)^{\frac{1}{3}}}{x}\right)}{3b^{\frac{1}{3}}}\right)}{b^{\frac{4}{3}}}-\frac{a\log\left(b^{\frac{2}{3}}+\frac{\left(bx^3+a\right)^{\frac{1}{3}}b^{\frac{1}{3}}}{x}+\frac{\left(bx^3+a\right)^{\frac{2}{3}}}{x^2}\right)}{b^{\frac{4}{3}}}+\frac{2a\log\left(-b^{\frac{1}{3}}+\frac{\left(bx^3+a\right)^{\frac{1}{3}}}{x}\right)}{b^{\frac{4}{3}}}-\frac{6\left(bx^3+a\right)^{\frac{2}{3}}a}{\left(b^2-\frac{\left(bx^3+a\right)b}{x^3}\right)x^2}\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^3+a)/(b*x^3+a)^(1/3),x, algorithm="maxima")
```

```
[Out] -1/6*(2*sqrt(3)*arctan(1/3*sqrt(3)*(b^(1/3) + 2*(b*x^3 + a)^(1/3)/x)/b^(1/3
))/b^(1/3) - log(b^(2/3) + (b*x^3 + a)^(1/3)*b^(1/3)/x + (b*x^3 + a)^(2/3)/
x^2)/b^(1/3) + 2*log(-b^(1/3) + (b*x^3 + a)^(1/3)/x)/b^(1/3))*a - 1/18*(2*s
qrt(3)*a*arctan(1/3*sqrt(3)*(b^(1/3) + 2*(b*x^3 + a)^(1/3)/x)/b^(1/3))/b^(4
/3) - a*log(b^(2/3) + (b*x^3 + a)^(1/3)*b^(1/3)/x + (b*x^3 + a)^(2/3)/x^2)/
b^(4/3) + 2*a*log(-b^(1/3) + (b*x^3 + a)^(1/3)/x)/b^(4/3) - 6*(b*x^3 + a)^(
2/3)*a/((b^2 - (b*x^3 + a)*b/x^3)*x^2))*b
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a-b\,x^3}{{\left(b\,x^3+a\right)}^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - b*x^3)/(a + b*x^3)^(1/3),x)
```

```
[Out] int((a - b*x^3)/(a + b*x^3)^(1/3), x)
```

sympy [C] time = 5.53, size = 76, normalized size = 0.84

$$\frac{a^{\frac{2}{3}}x\Gamma\left(\frac{1}{3}\right){}_{2}F_{1}\left(\left.\begin{matrix}\frac{1}{3},\frac{1}{3}\\\frac{4}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{4}{3}\right)}-\frac{bx^4\Gamma\left(\frac{4}{3}\right){}_{2}F_{1}\left(\left.\begin{matrix}\frac{1}{3},\frac{4}{3}\\\frac{7}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt[3]{a}\,\Gamma\left(\frac{7}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x**3+a)/(b*x**3+a)**(1/3),x)
```

```
[Out] a**(2/3)*x*gamma(1/3)*hyper((1/3, 1/3), (4/3,), b*x**3*exp_polar(I*pi)/a)/(
3*gamma(4/3)) - b*x**4*gamma(4/3)*hyper((1/3, 4/3), (7/3,), b*x**3*exp_pola
r(I*pi)/a)/(3*a**(1/3)*gamma(7/3))
```

3.29 $\int \frac{a-bx^3}{\left(a+bx^3\right)^{4/3}}\,dx$

Optimal. Leaf size=85

$$\frac{2x}{\sqrt[3]{a+bx^3}}+\frac{\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}\,x\right)}{2\sqrt[3]{b}}-\frac{\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\,\sqrt[3]{b}}$$

Rubi [A] time = 0.01, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {385, 239}

$$\frac{2x}{\sqrt[3]{a+bx^3}}+\frac{\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}\,x\right)}{2\sqrt[3]{b}}-\frac{\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\,\sqrt[3]{b}}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*x^3)/(a + b*x^3)^(4/3),x]
```

```
[Out] (2*x)/(a + b*x^3)^(1/3) - ArcTan[(1 + (2*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt
[3]]/(Sqrt[3]*b^(1/3)) + Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)]/(2*b^(1/3))
```

Rule 239

```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + (2*Rt[b, 3]
*x)/(a + b*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{a-bx^3}{\left(a+bx^3\right)^{4/3}}\,dx&=\frac{2x}{\sqrt[3]{a+bx^3}}-\int \frac{1}{\sqrt[3]{a+bx^3}}\,dx\\&=\frac{2x}{\sqrt[3]{a+bx^3}}-\frac{\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}\,\sqrt[3]{b}}+\frac{\log\left(-\sqrt[3]{b}\,x+\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{b}}\end{aligned}$$

Mathematica [C] time = 0.04, size = 62, normalized size = 0.73

$$\frac{4ax-bx^4\sqrt[3]{\frac{bx^3}{a}+1}\,{}_2F_1\left(\frac{4}{3},\frac{4}{3};\frac{7}{3};-\frac{bx^3}{a}\right)}{4a\sqrt[3]{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b*x^3)/(a + b*x^3)^(4/3),x]
```

```
[Out] (4*a*x - b*x^4*(1 + (b*x^3)/a)^(1/3)*Hypergeometric2F1[4/3, 4/3, 7/3, -((b*
x^3)/a)])/(4*a*(a + b*x^3)^(1/3))
```

IntegrateAlgebraic [A] time = 0.29, size = 142, normalized size = 1.67

$$-\frac{\log\left(\sqrt[3]{b}\,x\sqrt[3]{a+bx^3}+\left(a+bx^3\right)^{2/3}+b^{2/3}x^2\right)}{6\sqrt[3]{b}}+\frac{2x}{\sqrt[3]{a+bx^3}}+\frac{\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}\,x\right)}{3\sqrt[3]{b}}-\frac{\tan^{-1}\left(\frac{\sqrt{3}\,\sqrt[3]{b}\,x}{2\sqrt[3]{a+bx^3}+\sqrt[3]{b}\,x}\right)}{\sqrt{3}\,\sqrt[3]{b}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a - b*x^3)/(a + b*x^3)^(4/3),x]
```

```
[Out] (2*x)/(a + b*x^3)^(1/3) - ArcTan[(Sqrt[3]*b^(1/3)*x)/(b^(1/3)*x + 2*(a + b*
x^3)^(1/3))]/(Sqrt[3]*b^(1/3)) + Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)]/(3*b
^(1/3)) - Log[b^(2/3)*x^2 + b^(1/3)*x*(a + b*x^3)^(1/3) + (a + b*x^3)^(2/3)
]/(6*b^(1/3))
```

fricas [B] time = 0.79, size = 372, normalized size = 4.38

$$\left[\frac{3\sqrt{\frac{1}{3}}\left(b^2x^3+ab\right)\sqrt{-\frac{1}{b^{\frac{2}{3}}}}\log\left(3bx^3-3\left(bx^3+a\right)^{\frac{1}{3}}b^{\frac{2}{3}}x^2-3\sqrt{\frac{1}{3}}\left(b^{\frac{4}{3}}x^3+\left(bx^3+a\right)^{\frac{1}{3}}bx^2-2\left(bx^3+a\right)^{\frac{2}{3}}b^{\frac{2}{3}}x\right)\sqrt{-\frac{1}{b^{\frac{2}{3}}}}+2a\right)+12\left(bx^3+a\right)^{\frac{2}{3}}bx+2\left(bx^3+a\right)b^{\frac{2}{3}}\log\left(-\frac{b^{\frac{1}{3}}x-\left(bx^3+a\right)^{\frac{1}{3}}}{x}\right)-\left(bx^3+a\right)b^{\frac{2}{3}}\log\left(\frac{b^{\frac{2}{3}}x^2+\left(bx^3+a\right)^{\frac{1}{3}}b^{\frac{1}{3}}x+\left(bx^3+a\right)^{\frac{2}{3}}}{x^2}\right)}{6\left(b^2x^3+ab\right)},\ \frac{12\left(bx^3+a\right)^{\frac{2}{3}}bx+2\left(bx^3+a\right)b^{\frac{2}{3}}\log\left(-\frac{b^{\frac{1}{3}}x-\left(bx^3+a\right)^{\frac{1}{3}}}{x}\right)-\left(bx^3+a\right)b^{\frac{2}{3}}\log\left(\frac{b^{\frac{2}{3}}x^2+\left(bx^3+a\right)^{\frac{1}{3}}b^{\frac{1}{3}}x+\left(bx^3+a\right)^{\frac{2}{3}}}{x^2}\right)+\frac{6\sqrt{\frac{1}{3}}\left(b^2x^3+ab\right)\arctan\left(\frac{\sqrt{\frac{1}{3}}\left(b^{\frac{1}{3}}x+2\left(bx^3+a\right)^{\frac{1}{3}}\right)}{b^{\frac{1}{3}}x}\right)}{b^{\frac{1}{3}}}}{6\left(b^2x^3+ab\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^3+a)/(b*x^3+a)^(4/3),x, algorithm="fricas")
```

```
[Out] [1/6*(3*sqrt(1/3)*(b^2*x^3 + a*b)*sqrt(-1/b^(2/3))*log(3*b*x^3 - 3*(b*x^3 +
 a)^(1/3)*b^(2/3)*x^2 - 3*sqrt(1/3)*(b^(4/3)*x^3 + (b*x^3 + a)^(1/3)*b*x^2
- 2*(b*x^3 + a)^(2/3)*b^(2/3)*x)*sqrt(-1/b^(2/3)) + 2*a) + 12*(b*x^3 + a)^(
2/3)*b*x + 2*(b*x^3 + a)*b^(2/3)*log(-(b^(1/3)*x - (b*x^3 + a)^(1/3))/x) -
(b*x^3 + a)*b^(2/3)*log((b^(2/3)*x^2 + (b*x^3 + a)^(1/3)*b^(1/3)*x + (b*x^3
 + a)^(2/3))/x^2))/(b^2*x^3 + a*b), 1/6*(12*(b*x^3 + a)^(2/3)*b*x + 2*(b*x^
3 + a)*b^(2/3)*log(-(b^(1/3)*x - (b*x^3 + a)^(1/3))/x) - (b*x^3 + a)*b^(2/3
)*log((b^(2/3)*x^2 + (b*x^3 + a)^(1/3)*b^(1/3)*x + (b*x^3 + a)^(2/3))/x^2)
+ 6*sqrt(1/3)*(b^2*x^3 + a*b)*arctan(sqrt(1/3)*(b^(1/3)*x + 2*(b*x^3 + a)^(
1/3))/(b^(1/3)*x))/b^(1/3))/(b^2*x^3 + a*b)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{bx^3-a}{\left(bx^3+a\right)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^3+a)/(b*x^3+a)^(4/3),x, algorithm="giac")
```

```
[Out] integrate(-(b*x^3 - a)/(b*x^3 + a)^(4/3), x)
```

maple [F] time = 0.39, size = 0, normalized size = 0.00

$$\int \frac{-b\,x^3+a}{\left(b\,x^3+a\right)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x^3+a)/(b*x^3+a)^(4/3),x)
```

```
[Out] int((-b*x^3+a)/(b*x^3+a)^(4/3),x)
```

maxima [A] time = 1.25, size = 130, normalized size = 1.53

$$\frac{1}{6}\,b\left(\frac{2\,\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}}+\frac{2\,\left(bx^3+a\right)^{\frac{1}{3}}}{x}\right)}{3\,b^{\frac{1}{3}}}\right)}{b^{\frac{4}{3}}}+\frac{6\,x}{\left(bx^3+a\right)^{\frac{1}{3}}b}-\frac{\log\left(b^{\frac{2}{3}}+\frac{\left(bx^3+a\right)^{\frac{1}{3}}b^{\frac{1}{3}}}{x}+\frac{\left(bx^3+a\right)^{\frac{2}{3}}}{x^2}\right)}{b^{\frac{4}{3}}}+\frac{2\,\log\left(-b^{\frac{1}{3}}+\frac{\left(bx^3+a\right)^{\frac{1}{3}}}{x}\right)}{b^{\frac{4}{3}}}\right)+\frac{x}{\left(bx^3+a\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^3+a)/(b*x^3+a)^(4/3),x, algorithm="maxima")
```

```
[Out] 1/6*b*(2*sqrt(3)*arctan(1/3*sqrt(3)*(b^(1/3) + 2*(b*x^3 + a)^(1/3)/x)/b^(1/
3))/b^(4/3) + 6*x/((b*x^3 + a)^(1/3)*b) - log(b^(2/3) + (b*x^3 + a)^(1/3)*b
^(1/3)/x + (b*x^3 + a)^(2/3)/x^2)/b^(4/3) + 2*log(-b^(1/3) + (b*x^3 + a)^(1
/3)/x)/b^(4/3)) + x/(b*x^3 + a)^(1/3)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a-b\,x^3}{{\left(b\,x^3+a\right)}^{4/3}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - b*x^3)/(a + b*x^3)^(4/3),x)
```

```
[Out] int((a - b*x^3)/(a + b*x^3)^(4/3), x)
```

sympy [C] time = 16.14, size = 70, normalized size = 0.82

$$\frac{x\Gamma\left(\frac{1}{3}\right)}{3\sqrt[3]{a}\,\sqrt[3]{1+\frac{bx^3}{a}}\,\Gamma\left(\frac{4}{3}\right)}-\frac{bx^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}\frac{4}{3},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{4}{3}}\Gamma\left(\frac{7}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x**3+a)/(b*x**3+a)**(4/3),x)
```

```
[Out] x*gamma(1/3)/(3*a**(1/3)*(1 + b*x**3/a)**(1/3)*gamma(4/3)) - b*x**4*gamma(4
/3)*hyper((4/3, 4/3), (7/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(4/3)*gamma(7
/3))
```

3.30 $\int \frac{a-bx^3}{\left(a+bx^3\right)^{7/3}} \, dx$

Optimal. Leaf size=47

$$\frac{3x}{4a\sqrt[3]{a+bx^3}}+\frac{x\left(a-bx^3\right)}{4a\left(a+bx^3\right)^{4/3}}$$

Rubi [A] time = 0.01, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {378, 191}

$$\frac{3x}{4a\sqrt[3]{a+bx^3}}+\frac{x\left(a-bx^3\right)}{4a\left(a+bx^3\right)^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*x^3)/(a + b*x^3)^(7/3),x]
[Out] (x*(a - b*x^3))/(4*a*(a + b*x^3)^(4/3)) + (3*x)/(4*a*(a + b*x^3)^(1/3))
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 378

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> -Simp[(x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*n*(p + 1)), x] - Dist[(c
*q)/(a*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1, 0]
 && GtQ[q, 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{a-bx^3}{\left(a+bx^3\right)^{7/3}}\,dx &= \frac{x\left(a-bx^3\right)}{4a\left(a+bx^3\right)^{4/3}}+\frac{3}{4}\int \frac{1}{\left(a+bx^3\right)^{4/3}}\,dx\\ &= \frac{x\left(a-bx^3\right)}{4a\left(a+bx^3\right)^{4/3}}+\frac{3x}{4a\sqrt[3]{a+bx^3}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 28, normalized size = 0.60

$$\frac{x\left(2a+bx^3\right)}{2a\left(a+bx^3\right)^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b*x^3)/(a + b*x^3)^(7/3),x]
[Out] (x*(2*a + b*x^3))/(2*a*(a + b*x^3)^(4/3))
```

IntegrateAlgebraic [A] time = 0.22, size = 28, normalized size = 0.60

$$\frac{x(2a+bx^3)}{2a(a+bx^3)^{4/3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a - b*x^3)/(a + b*x^3)^(7/3),x]
[Out] (x*(2*a + b*x^3))/(2*a*(a + b*x^3)^(4/3))
```

fricas [A] time = 1.06, size = 44, normalized size = 0.94

$$\frac{(bx^4+2\,ax)(bx^3+a)^{\frac{2}{3}}}{2\,(ab^2x^6+2\,a^2bx^3+a^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^3+a)/(b*x^3+a)^(7/3),x, algorithm="fricas")
[Out] 1/2*(b*x^4 + 2*a*x)*(b*x^3 + a)^(2/3)/(a*b^2*x^6 + 2*a^2*b*x^3 + a^3)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{bx^3-a}{(bx^3+a)^{\frac{7}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^3+a)/(b*x^3+a)^(7/3),x, algorithm="giac")
[Out] integrate(-(b*x^3 - a)/(b*x^3 + a)^(7/3), x)
```

maple [A] time = 0.05, size = 25, normalized size = 0.53

$$\frac{(b\,x^3+2a)\,x}{2\,(b\,x^3+a)^{\frac{4}{3}}\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x^3+a)/(b*x^3+a)^(7/3),x)
[Out] 1/2*x*(b*x^3+2*a)/(b*x^3+a)^(4/3)/a
```

maxima [A] time = 0.52, size = 50, normalized size = 1.06

$$-\frac{\left(b-\frac{4\,(bx^3+a)}{x^3}\right)x^4}{4\,(bx^3+a)^{\frac{4}{3}}a}-\frac{bx^4}{4\,(bx^3+a)^{\frac{4}{3}}a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^3+a)/(b*x^3+a)^(7/3),x, algorithm="maxima")
[Out] -1/4*(b - 4*(b*x^3 + a)/x^3)*x^4/((b*x^3 + a)^(4/3)*a) - 1/4*b*x^4/((b*x^3
+ a)^(4/3)*a)
```

mupad [B] time = 1.35, size = 27, normalized size = 0.57

$$\frac{x\left(b\,x^3+a\right)+a\,x}{2\,a\,{\left(b\,x^3+a\right)}^{4/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - b*x^3)/(a + b*x^3)^(7/3),x)
```

```
[Out] (x*(a + b*x^3) + a*x)/(2*a*(a + b*x^3)^(4/3))
```

sympy [B] time = 91.68, size = 190, normalized size = 4.04

$$a\left(\frac{4ax\Gamma\left(\frac{1}{3}\right)}{9a^{\frac{10}{3}}\sqrt[3]{1+\frac{bx^3}{a}}\Gamma\left(\frac{7}{3}\right)+9a^{\frac{7}{3}}bx^3\sqrt[3]{1+\frac{bx^3}{a}}\Gamma\left(\frac{7}{3}\right)}+\frac{3bx^4\Gamma\left(\frac{1}{3}\right)}{9a^{\frac{10}{3}}\sqrt[3]{1+\frac{bx^3}{a}}\Gamma\left(\frac{7}{3}\right)+9a^{\frac{7}{3}}bx^3\sqrt[3]{1+\frac{bx^3}{a}}\Gamma\left(\frac{7}{3}\right)}\right)-\frac{bx^4\Gamma\left(\frac{4}{3}\right)}{3a^{\frac{7}{3}}\sqrt[3]{1+\frac{bx^3}{a}}\Gamma\left(\frac{7}{3}\right)+3a^{\frac{4}{3}}bx^3\sqrt[3]{1+\frac{bx^3}{a}}\Gamma\left(\frac{7}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x**3+a)/(b*x**3+a)**(7/3),x)
```

```
[Out] a*(4*a*x*gamma(1/3)/(9*a**(10/3)*(1 + b*x**3/a)**(1/3)*gamma(7/3) + 9*a**(7
/3)*b*x**3*(1 + b*x**3/a)**(1/3)*gamma(7/3)) + 3*b*x**4*gamma(1/3)/(9*a**(1
0/3)*(1 + b*x**3/a)**(1/3)*gamma(7/3) + 9*a**(7/3)*b*x**3*(1 + b*x**3/a)**(
1/3)*gamma(7/3))) - b*x**4*gamma(4/3)/(3*a**(7/3)*(1 + b*x**3/a)**(1/3)*gam
ma(7/3) + 3*a**(4/3)*b*x**3*(1 + b*x**3/a)**(1/3)*gamma(7/3))
```

3.31 $\int \frac{a-bx^3}{\left(a+bx^3\right)^{10/3}}\,dx$

Optimal. Leaf size=55

$$\frac{15x}{28a^2\sqrt[3]{a+bx^3}}+\frac{5x}{28a\left(a+bx^3\right)^{4/3}}+\frac{2x}{7\left(a+bx^3\right)^{7/3}}$$

Rubi [A] time = 0.01, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {385, 192, 191}

$$\frac{15x}{28a^2\sqrt[3]{a+bx^3}}+\frac{5x}{28a\left(a+bx^3\right)^{4/3}}+\frac{2x}{7\left(a+bx^3\right)^{7/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*x^3)/(a + b*x^3)^(10/3),x]
```

```
[Out] (2*x)/(7*(a + b*x^3)^(7/3)) + (5*x)/(28*a*(a + b*x^3)^(4/3)) + (15*x)/(28*a
^2*(a + b*x^3)^(1/3))
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{a-bx^3}{\left(a+bx^3\right)^{10/3}}\,dx&=\frac{2x}{7\left(a+bx^3\right)^{7/3}}+\frac{5}{7}\int \frac{1}{\left(a+bx^3\right)^{7/3}}\,dx\\&=\frac{2x}{7\left(a+bx^3\right)^{7/3}}+\frac{5x}{28a\left(a+bx^3\right)^{4/3}}+\frac{15\int \frac{1}{\left(a+bx^3\right)^{4/3}}\,dx}{28a}\\&=\frac{2x}{7\left(a+bx^3\right)^{7/3}}+\frac{5x}{28a\left(a+bx^3\right)^{4/3}}+\frac{15x}{28a^2\sqrt[3]{a+bx^3}}\end{aligned}$$

Mathematica [A] time = 0.02, size = 40, normalized size = 0.73

$$\frac{x\left(28a^2+35abx^3+15b^2x^6\right)}{28a^2\left(a+bx^3\right)^{7/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b*x^3)/(a + b*x^3)^(10/3),x]
[Out] (x*(28*a^2 + 35*a*b*x^3 + 15*b^2*x^6))/(28*a^2*(a + b*x^3)^(7/3))
```

IntegrateAlgebraic [A] time = 0.29, size = 40, normalized size = 0.73

$$\frac{x\left(28a^2+35abx^3+15b^2x^6\right)}{28a^2\left(a+bx^3\right)^{7/3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a - b*x^3)/(a + b*x^3)^(10/3),x]
[Out] (x*(28*a^2 + 35*a*b*x^3 + 15*b^2*x^6))/(28*a^2*(a + b*x^3)^(7/3))
```

fricas [A] time = 0.67, size = 69, normalized size = 1.25

$$\frac{\left(15\,b^2x^7+35\,abx^4+28\,a^2x\right)\left(bx^3+a\right)^{\frac{2}{3}}}{28\left(a^2b^3x^9+3\,a^3b^2x^6+3\,a^4bx^3+a^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^3+a)/(b*x^3+a)^(10/3),x, algorithm="fricas")
[Out] 1/28*(15*b^2*x^7 + 35*a*b*x^4 + 28*a^2*x)*(b*x^3 + a)^(2/3)/(a^2*b^3*x^9 +
3*a^3*b^2*x^6 + 3*a^4*b*x^3 + a^5)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{bx^3-a}{\left(bx^3+a\right)^{\frac{10}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^3+a)/(b*x^3+a)^(10/3),x, algorithm="giac")
[Out] integrate(-(b*x^3 - a)/(b*x^3 + a)^(10/3), x)
```

maple [A] time = 0.04, size = 37, normalized size = 0.67

$$\frac{\left(15b^2x^6+35ab\,x^3+28a^2\right)x}{28\left(b\,x^3+a\right)^{\frac{7}{3}}a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x^3+a)/(b*x^3+a)^(10/3),x)
[Out] 1/28*x*(15*b^2*x^6+35*a*b*x^3+28*a^2)/(b*x^3+a)^(7/3)/a^2
```

maxima [A] time = 0.62, size = 85, normalized size = 1.55

$$\frac{\left(4\,b-\frac{7\left(bx^3+a\right)}{x^3}\right)bx^7}{28\left(bx^3+a\right)^{\frac{7}{3}}a^2}+\frac{\left(2\,b^2-\frac{7\left(bx^3+a\right)b}{x^3}+\frac{14\left(bx^3+a\right)^2}{x^6}\right)x^7}{14\left(bx^3+a\right)^{\frac{7}{3}}a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^3+a)/(b*x^3+a)^(10/3),x, algorithm="maxima")
```

```
[Out] 1/28*(4*b - 7*(b*x^3 + a)/x^3)*b*x^7/((b*x^3 + a)^(7/3)*a^2) + 1/14*(2*b^2
- 7*(b*x^3 + a)*b/x^3 + 14*(b*x^3 + a)^2/x^6)*x^7/((b*x^3 + a)^(7/3)*a^2)
```

mupad [B] time = 1.42, size = 44, normalized size = 0.80

$$\frac{15\,x\,\left(b\,x^3+a\right)^2+8\,a^2\,x+5\,a\,x\,\left(b\,x^3+a\right)}{28\,a^2\,\left(b\,x^3+a\right)^{7/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - b*x^3)/(a + b*x^3)^(10/3),x)
```

```
[Out] (15*x*(a + b*x^3)^2 + 8*a^2*x + 5*a*x*(a + b*x^3))/(28*a^2*(a + b*x^3)^(7/3
))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x**3+a)/(b*x**3+a)**(10/3),x)
```

```
[Out] Timed out
```

3.32 $\int \frac{a-bx^3}{\left(a+bx^3\right)^{13/3}} \, dx$

Optimal. Leaf size=74

$$\frac{18x}{35a^3\sqrt[3]{a+bx^3}}+\frac{6x}{35a^2\left(a+bx^3\right)^{4/3}}+\frac{4x}{35a\left(a+bx^3\right)^{7/3}}+\frac{x}{5\left(a+bx^3\right)^{10/3}}$$

Rubi [A] time = 0.02, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {385, 192, 191}

$$\frac{18x}{35a^3\sqrt[3]{a+bx^3}}+\frac{6x}{35a^2\left(a+bx^3\right)^{4/3}}+\frac{4x}{35a\left(a+bx^3\right)^{7/3}}+\frac{x}{5\left(a+bx^3\right)^{10/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*x^3)/(a + b*x^3)^(13/3),x]
```

```
[Out] x/(5*(a + b*x^3)^(10/3)) + (4*x)/(35*a*(a + b*x^3)^(7/3)) + (6*x)/(35*a^2*(
a + b*x^3)^(4/3)) + (18*x)/(35*a^3*(a + b*x^3)^(1/3))
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{a-bx^3}{\left(a+bx^3\right)^{13/3}} \, dx &= \frac{x}{5\left(a+bx^3\right)^{10/3}}+\frac{4}{5}\int \frac{1}{\left(a+bx^3\right)^{10/3}} \, dx\\
&= \frac{x}{5\left(a+bx^3\right)^{10/3}}+\frac{4x}{35a\left(a+bx^3\right)^{7/3}}+\frac{24\int \frac{1}{\left(a+bx^3\right)^{7/3}} \, dx}{35a}\\
&= \frac{x}{5\left(a+bx^3\right)^{10/3}}+\frac{4x}{35a\left(a+bx^3\right)^{7/3}}+\frac{6x}{35a^2\left(a+bx^3\right)^{4/3}}+\frac{18\int \frac{1}{\left(a+bx^3\right)^{4/3}} \, dx}{35a^2}\\
&= \frac{x}{5\left(a+bx^3\right)^{10/3}}+\frac{4x}{35a\left(a+bx^3\right)^{7/3}}+\frac{6x}{35a^2\left(a+bx^3\right)^{4/3}}+\frac{18x}{35a^3\sqrt[3]{a+bx^3}}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 51, normalized size = 0.69

$$\frac{x\left(35a^3+70a^2bx^3+60ab^2x^6+18b^3x^9\right)}{35a^3\left(a+bx^3\right)^{10/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b*x^3)/(a + b*x^3)^(13/3),x]
```

```
[Out] (x*(35*a^3 + 70*a^2*b*x^3 + 60*a*b^2*x^6 + 18*b^3*x^9))/(35*a^3*(a + b*x^3)
^(10/3))
```

IntegrateAlgebraic [A] time = 0.43, size = 51, normalized size = 0.69

$$\frac{x\left(35a^3+70a^2bx^3+60ab^2x^6+18b^3x^9\right)}{35a^3\left(a+bx^3\right)^{10/3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a - b*x^3)/(a + b*x^3)^(13/3),x]
```

```
[Out] (x*(35*a^3 + 70*a^2*b*x^3 + 60*a*b^2*x^6 + 18*b^3*x^9))/(35*a^3*(a + b*x^3)
^(10/3))
```

fricas [A] time = 1.02, size = 91, normalized size = 1.23

$$\frac{\left(18\,b^3x^{10}+60\,ab^2x^7+70\,a^2bx^4+35\,a^3x\right)\left(bx^3+a\right)^{\frac{2}{3}}}{35\left(a^3b^4x^{12}+4\,a^4b^3x^9+6\,a^5b^2x^6+4\,a^6bx^3+a^7\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^3+a)/(b*x^3+a)^(13/3),x, algorithm="fricas")
```

```
[Out] 1/35*(18*b^3*x^10 + 60*a*b^2*x^7 + 70*a^2*b*x^4 + 35*a^3*x)*(b*x^3 + a)^(2/
3)/(a^3*b^4*x^12 + 4*a^4*b^3*x^9 + 6*a^5*b^2*x^6 + 4*a^6*b*x^3 + a^7)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{bx^3-a}{\left(bx^3+a\right)^{\frac{13}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^3+a)/(b*x^3+a)^(13/3),x, algorithm="giac")
```

```
[Out] integrate(-(b*x^3 - a)/(b*x^3 + a)^(13/3), x)
```

maple [A] time = 0.05, size = 48, normalized size = 0.65

$$\frac{\left(18b^3x^9+60a\,b^2x^6+70a^2b\,x^3+35a^3\right)x}{35\left(b\,x^3+a\right)^{\frac{10}{3}}a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x^3+a)/(b*x^3+a)^(13/3),x)
```

```
[Out] 1/35*x*(18*b^3*x^9+60*a*b^2*x^6+70*a^2*b*x^3+35*a^3)/(b*x^3+a)^(10/3)/a^3
```

maxima [B] time = 0.58, size = 119, normalized size = 1.61

$$-\frac{\left(14\,b^{2}-\frac{40\,\left(bx^{3}+a\right)b}{x^{3}}+\frac{35\,\left(bx^{3}+a\right)^{2}}{x^{6}}\right)bx^{10}}{140\,\left(bx^{3}+a\right)^{\frac{10}{3}}a^{3}}-\frac{\left(14\,b^{3}-\frac{60\,\left(bx^{3}+a\right)b^{2}}{x^{3}}+\frac{105\,\left(bx^{3}+a\right)^{2}b}{x^{6}}-\frac{140\,\left(bx^{3}+a\right)^{3}}{x^{9}}\right)x^{10}}{140\,\left(bx^{3}+a\right)^{\frac{10}{3}}a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^3+a)/(b*x^3+a)^(13/3),x, algorithm="maxima")
```

```
[Out] -1/140*(14*b^2 - 40*(b*x^3 + a)*b/x^3 + 35*(b*x^3 + a)^2/x^6)*b*x^10/((b*x^
3 + a)^(10/3)*a^3) - 1/140*(14*b^3 - 60*(b*x^3 + a)*b^2/x^3 + 105*(b*x^3 +
a)^2*b/x^6 - 140*(b*x^3 + a)^3/x^9)*x^10/((b*x^3 + a)^(10/3)*a^3)
```

mupad [B] time = 1.39, size = 58, normalized size = 0.78

$$\frac{x}{5\,{\left(b\,x^3+a\right)}^{10/3}}+\frac{18\,x}{35\,a^3\,{\left(b\,x^3+a\right)}^{1/3}}+\frac{6\,x}{35\,a^2\,{\left(b\,x^3+a\right)}^{4/3}}+\frac{4\,x}{35\,a\,{\left(b\,x^3+a\right)}^{7/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - b*x^3)/(a + b*x^3)^(13/3),x)
```

```
[Out] x/(5*(a + b*x^3)^(10/3)) + (18*x)/(35*a^3*(a + b*x^3)^(1/3)) + (6*x)/(35*a^
2*(a + b*x^3)^(4/3)) + (4*x)/(35*a*(a + b*x^3)^(7/3))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x**3+a)/(b*x**3+a)**(13/3),x)
```

```
[Out] Timed out
```

3.33 $\int \frac{a-bx^3}{\left(a+bx^3\right)^{16/3}}\,dx$

Optimal. Leaf size=93

$$\frac{891x}{1820a^4\sqrt[3]{a+bx^3}}+\frac{297x}{1820a^3\left(a+bx^3\right)^{4/3}}+\frac{99x}{910a^2\left(a+bx^3\right)^{7/3}}+\frac{11x}{130a\left(a+bx^3\right)^{10/3}}+\frac{2x}{13\left(a+bx^3\right)^{13/3}}$$

Rubi [A] time = 0.03, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {385, 192, 191}

$$\frac{891x}{1820a^4\sqrt[3]{a+bx^3}}+\frac{297x}{1820a^3\left(a+bx^3\right)^{4/3}}+\frac{99x}{910a^2\left(a+bx^3\right)^{7/3}}+\frac{11x}{130a\left(a+bx^3\right)^{10/3}}+\frac{2x}{13\left(a+bx^3\right)^{13/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*x^3)/(a + b*x^3)^(16/3),x]
```

```
[Out] (2*x)/(13*(a + b*x^3)^(13/3)) + (11*x)/(130*a*(a + b*x^3)^(10/3)) + (99*x)/
(910*a^2*(a + b*x^3)^(7/3)) + (297*x)/(1820*a^3*(a + b*x^3)^(4/3)) + (891*x
)/(1820*a^4*(a + b*x^3)^(1/3))
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rubi steps

$$\int \frac{a-bx^3}{\left(a+bx^3\right)^{16/3}}\,dx = \frac{2x}{13\left(a+bx^3\right)^{13/3}} + \frac{11}{13}\int \frac{1}{\left(a+bx^3\right)^{13/3}}\,dx$$

$$= \frac{2x}{13\left(a+bx^3\right)^{13/3}} + \frac{11x}{130a\left(a+bx^3\right)^{10/3}} + \frac{99\int \frac{1}{\left(a+bx^3\right)^{10/3}}\,dx}{130a}$$

$$= \frac{2x}{13\left(a+bx^3\right)^{13/3}} + \frac{11x}{130a\left(a+bx^3\right)^{10/3}} + \frac{99x}{910a^2\left(a+bx^3\right)^{7/3}} + \frac{297\int \frac{1}{\left(a+bx^3\right)^{7/3}}\,dx}{455a^2}$$

$$= \frac{2x}{13\left(a+bx^3\right)^{13/3}} + \frac{11x}{130a\left(a+bx^3\right)^{10/3}} + \frac{99x}{910a^2\left(a+bx^3\right)^{7/3}} + \frac{297x}{1820a^3\left(a+bx^3\right)^{4/3}} + \frac{891}{\quad}$$

$$= \frac{2x}{13\left(a+bx^3\right)^{13/3}} + \frac{11x}{130a\left(a+bx^3\right)^{10/3}} + \frac{99x}{910a^2\left(a+bx^3\right)^{7/3}} + \frac{297x}{1820a^3\left(a+bx^3\right)^{4/3}} + \frac{\quad}{182}$$

Mathematica [A] time = 0.03, size = 62, normalized size = 0.67

$$\frac{x\left(1820a^4+5005a^3bx^3+6435a^2b^2x^6+3861ab^3x^9+891b^4x^{12}\right)}{1820a^4\left(a+bx^3\right)^{13/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b*x^3)/(a + b*x^3)^(16/3),x]
```

```
[Out] (x*(1820*a^4 + 5005*a^3*b*x^3 + 6435*a^2*b^2*x^6 + 3861*a*b^3*x^9 + 891*b^4
*x^12))/(1820*a^4*(a + b*x^3)^(13/3))
```

IntegrateAlgebraic [A] time = 0.63, size = 62, normalized size = 0.67

$$\frac{x\left(1820a^4+5005a^3bx^3+6435a^2b^2x^6+3861ab^3x^9+891b^4x^{12}\right)}{1820a^4\left(a+bx^3\right)^{13/3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a - b*x^3)/(a + b*x^3)^(16/3),x]
```

```
[Out] (x*(1820*a^4 + 5005*a^3*b*x^3 + 6435*a^2*b^2*x^6 + 3861*a*b^3*x^9 + 891*b^4
*x^12))/(1820*a^4*(a + b*x^3)^(13/3))
```

fricas [A] time = 0.97, size = 113, normalized size = 1.22

$$\frac{\left(891\,b^4x^{13}+3861\,ab^3x^{10}+6435\,a^2b^2x^7+5005\,a^3bx^4+1820\,a^4x\right)\left(bx^3+a\right)^{\frac{2}{3}}}{1820\left(a^4b^5x^{15}+5\,a^5b^4x^{12}+10\,a^6b^3x^9+10\,a^7b^2x^6+5\,a^8bx^3+a^9\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^3+a)/(b*x^3+a)^(16/3),x, algorithm="fricas")
```

```
[Out] 1/1820*(891*b^4*x^13 + 3861*a*b^3*x^10 + 6435*a^2*b^2*x^7 + 5005*a^3*b*x^4
+ 1820*a^4*x)*(b*x^3 + a)^(2/3)/(a^4*b^5*x^15 + 5*a^5*b^4*x^12 + 10*a^6*b^3
*x^9 + 10*a^7*b^2*x^6 + 5*a^8*b*x^3 + a^9)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{bx^3 - a}{\left(bx^3 + a\right)^{\frac{16}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^3+a)/(b*x^3+a)^(16/3),x, algorithm="giac")
```

```
[Out] integrate(-(b*x^3 - a)/(b*x^3 + a)^(16/3), x)
```

maple [A] time = 0.05, size = 59, normalized size = 0.63

$$\frac{\left(891b^4x^{12} + 3861b^3x^9a + 6435b^2x^6a^2 + 5005b\,x^3a^3 + 1820a^4\right)x}{1820\left(b\,x^3 + a\right)^{\frac{13}{3}}a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x^3+a)/(b*x^3+a)^(16/3),x)
```

```
[Out] 1/1820*x*(891*b^4*x^12+3861*a*b^3*x^9+6435*a^2*b^2*x^6+5005*a^3*b*x^3+1820*
a^4)/(b*x^3+a)^(13/3)/a^4
```

maxima [B] time = 0.54, size = 153, normalized size = 1.65

$$\frac{\left(140\,b^3 - \frac{546\,(bx^3+a)b^2}{x^3} + \frac{780\,(bx^3+a)^2b}{x^6} - \frac{455\,(bx^3+a)^3}{x^9}\right)bx^{13}}{1820\left(bx^3 + a\right)^{\frac{13}{3}}a^4} + \frac{\left(35\,b^4 - \frac{182\,(bx^3+a)b^3}{x^3} + \frac{390\,(bx^3+a)^2b^2}{x^6} - \frac{455\,(bx^3+a)^3b}{x^9} + \frac{455\,(bx^3+a)^4}{x^{12}}\right)x^{13}}{455\left(bx^3 + a\right)^{\frac{13}{3}}a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^3+a)/(b*x^3+a)^(16/3),x, algorithm="maxima")
```

```
[Out] 1/1820*(140*b^3 - 546*(b*x^3 + a)*b^2/x^3 + 780*(b*x^3 + a)^2*b/x^6 - 455*(
b*x^3 + a)^3/x^9)*b*x^13/((b*x^3 + a)^(13/3)*a^4) + 1/455*(35*b^4 - 182*(b*
x^3 + a)*b^3/x^3 + 390*(b*x^3 + a)^2*b^2/x^6 - 455*(b*x^3 + a)^3*b/x^9 + 45
5*(b*x^3 + a)^4/x^12)*x^13/((b*x^3 + a)^(13/3)*a^4)
```

mupad [B] time = 1.37, size = 73, normalized size = 0.78

$$\frac{2\,x}{13\left(b\,x^3 + a\right)^{13/3}} + \frac{891\,x}{1820\,a^4\left(b\,x^3 + a\right)^{1/3}} + \frac{297\,x}{1820\,a^3\left(b\,x^3 + a\right)^{4/3}} + \frac{99\,x}{910\,a^2\left(b\,x^3 + a\right)^{7/3}} + \frac{11\,x}{130\,a\left(b\,x^3 + a\right)^{10/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - b*x^3)/(a + b*x^3)^(16/3),x)
```

```
[Out] (2*x)/(13*(a + b*x^3)^(13/3)) + (891*x)/(1820*a^4*(a + b*x^3)^(1/3)) + (297
*x)/(1820*a^3*(a + b*x^3)^(4/3)) + (99*x)/(910*a^2*(a + b*x^3)^(7/3)) + (11
*x)/(130*a*(a + b*x^3)^(10/3))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x**3+a)/(b*x**3+a)**(16/3),x)
```

```
[Out] Timed out
```

3.34 $\int \frac{\sqrt[3]{a+bx^3}}{a-bx^3}\,dx$

Optimal. Leaf size=398

$$-\frac{\log\left(2^{2/3}-\frac{\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}\right)}{3\,2^{2/3}\sqrt[3]{a}\sqrt[3]{b}}+\frac{\log\left(\frac{2^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{\left(a+bx^3\right)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}+1\right)}{3\,2^{2/3}\sqrt[3]{a}\sqrt[3]{b}}-\frac{\sqrt[3]{2}\log\left(\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}+1\right)}{3\sqrt[3]{a}\sqrt[3]{b}}+\frac{\log\left(\frac{\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{\left(a+bx^3\right)^{2/3}}\right.}{6\,2}$$

Rubi [C] time = 0.03, antiderivative size = 58, normalized size of antiderivative = 0.15, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {430, 429}

$$\frac{x\sqrt[3]{a+bx^3}\,F_1\left(\frac{1}{3};1,-\frac{1}{3};\frac{4}{3};\frac{bx^3}{a},-\frac{bx^3}{a}\right)}{a\sqrt[3]{\frac{bx^3}{a}+1}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(a + b*x^3)^(1/3)/(a - b*x^3),x]
```

```
[Out] (x*(a + b*x^3)^(1/3)*AppellF1[1/3, 1, -1/3, 4/3, (b*x^3)/a, -((b*x^3)/a)])/
(a*(1 + (b*x^3)/a)^(1/3))
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt[3]{a+bx^3}}{a-bx^3}\,dx &= \frac{\sqrt[3]{a+bx^3}\int \frac{\sqrt[3]{1+\frac{bx^3}{a}}}{a-bx^3}\,dx}{\sqrt[3]{1+\frac{bx^3}{a}}}\\ &= \frac{x\sqrt[3]{a+bx^3}\,F_1\left(\frac{1}{3};1,-\frac{1}{3};\frac{4}{3};\frac{bx^3}{a},-\frac{bx^3}{a}\right)}{a\sqrt[3]{1+\frac{bx^3}{a}}}\end{aligned}$$

Mathematica [C] time = 0.13, size = 151, normalized size = 0.38

$$\frac{4ax\sqrt[3]{a+bx^3}\,F_1\left(\frac{1}{3};-\frac{1}{3},1;\frac{4}{3};-\frac{bx^3}{a},\frac{bx^3}{a}\right)}{\left(a-bx^3\right)\left(bx^3\left(3F_1\left(\frac{4}{3};-\frac{1}{3},2;\frac{7}{3};-\frac{bx^3}{a},\frac{bx^3}{a}\right)+F_1\left(\frac{4}{3};\frac{2}{3},1;\frac{7}{3};-\frac{bx^3}{a},\frac{bx^3}{a}\right)\right)+4aF_1\left(\frac{1}{3};-\frac{1}{3},1;\frac{4}{3};-\frac{bx^3}{a},\frac{bx^3}{a}\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^3)^(1/3)/(a - b*x^3),x]
```

```
[Out] (4*a*x*(a + b*x^3)^(1/3)*AppellF1[1/3, -1/3, 1, 4/3, -((b*x^3)/a), (b*x^3)/
a])/((a - b*x^3)*(4*a*AppellF1[1/3, -1/3, 1, 4/3, -((b*x^3)/a), (b*x^3)/a]
+ b*x^3*(3*AppellF1[4/3, -1/3, 2, 7/3, -((b*x^3)/a), (b*x^3)/a] + AppellF1[
4/3, 2/3, 1, 7/3, -((b*x^3)/a), (b*x^3)/a])))
```

IntegrateAlgebraic [A] time = 3.12, size = 515, normalized size = 1.29

$$\frac{\log\left(2^{2/3}a^{2/3}-\sqrt[3]{2}\sqrt[3]{b}x\sqrt[3]{a+bx^3}+\left(a+bx^3\right)^{2/3}-\sqrt[3]{2}\sqrt[3]{a}\sqrt[3]{a+bx^3}+2\,2^{2/3}\sqrt[3]{a}\sqrt[3]{b}x+2^{2/3}b^{2/3}x^2\right)}{3\,2^{2/3}\sqrt[3]{a}\sqrt[3]{b}}+\frac{\log\left(2^{2/3}a^{2/3}+2\sqrt[3]{2}\sqrt[3]{b}x\sqrt[3]{a+bx^3}+4\left(a+bx^3\right)^{2/3}+2\sqrt[3]{2}\sqrt[3]{a}\sqrt[3]{a+bx^3}+2\,2^{2/3}\sqrt[3]{a}\sqrt[3]{b}x+2^{2/3}b^{2/3}x^2\right)}{6\,2^{2/3}\sqrt[3]{a}\sqrt[3]{b}}-\frac{\sqrt[3]{2}\log\left(\sqrt[3]{a+bx^3}+\sqrt[3]{2}\sqrt[3]{a}+\sqrt[3]{2}\sqrt[3]{b}x\right)}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(2\sqrt[3]{a+bx^3}-\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{b}x\right)}{3\,2^{2/3}\sqrt[3]{a}\sqrt[3]{b}}+\frac{\sqrt[3]{2}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{a+bx^3}}{\sqrt[3]{a+bx^3}-2\sqrt[3]{2}\sqrt[3]{a}-2\sqrt[3]{2}\sqrt[3]{b}x}\right)}{\sqrt{3}\sqrt[3]{a}\sqrt[3]{b}}+\frac{\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{a+bx^3}}{\sqrt[3]{a+bx^3}+\sqrt[3]{2}\sqrt[3]{a}+\sqrt[3]{2}\sqrt[3]{b}x}\right)}{2^{2/3}\sqrt{3}\sqrt[3]{a}\sqrt[3]{b}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x^3)^(1/3)/(a - b*x^3),x]
```

```
[Out] (2^(1/3)*ArcTan[(Sqrt[3]*(a + b*x^3)^(1/3))/(-2*2^(1/3)*a^(1/3) - 2*2^(1/3)
*b^(1/3)*x + (a + b*x^3)^(1/3))])/(Sqrt[3]*a^(1/3)*b^(1/3)) + ArcTan[(Sqrt[
3]*(a + b*x^3)^(1/3))/(2^(1/3)*a^(1/3) + 2^(1/3)*b^(1/3)*x + (a + b*x^3)^(1
/3))]/(2^(2/3)*Sqrt[3]*a^(1/3)*b^(1/3)) - (2^(1/3)*Log[2^(1/3)*a^(1/3) + 2^
(1/3)*b^(1/3)*x + (a + b*x^3)^(1/3)])/(3*a^(1/3)*b^(1/3)) - Log[-(2^(1/3)*a
^(1/3)) - 2^(1/3)*b^(1/3)*x + 2*(a + b*x^3)^(1/3)]/(3*2^(2/3)*a^(1/3)*b^(1/
3)) + Log[2^(2/3)*a^(2/3) + 2*2^(2/3)*a^(1/3)*b^(1/3)*x + 2^(2/3)*b^(2/3)*x
^2 - 2^(1/3)*a^(1/3)*(a + b*x^3)^(1/3) - 2^(1/3)*b^(1/3)*x*(a + b*x^3)^(1/3
) + (a + b*x^3)^(2/3)]/(3*2^(2/3)*a^(1/3)*b^(1/3)) + Log[2^(2/3)*a^(2/3) +
2*2^(2/3)*a^(1/3)*b^(1/3)*x + 2^(2/3)*b^(2/3)*x^2 + 2*2^(1/3)*a^(1/3)*(a +
b*x^3)^(1/3) + 2*2^(1/3)*b^(1/3)*x*(a + b*x^3)^(1/3) + 4*(a + b*x^3)^(2/3)]
/(6*2^(2/3)*a^(1/3)*b^(1/3))
```

fricas [B] time = 54.97, size = 644, normalized size = 1.62

$$-\frac{1}{18}\sqrt{3}\,2^{\frac{1}{3}}\left(-\frac{1}{ab}\right)^{\frac{1}{3}}\arctan\left(\frac{6\sqrt{3}\,2^{\frac{2}{3}}\left(ab^6x^{16}+33\,a^2b^5x^{13}+110\,a^3b^4x^{10}+110\,a^4b^3x^7+33\,a^5b^2x^4+a^6bx\right)\left(bx^3+a\right)^{\frac{1}{3}}\left(-\frac{1}{ab}\right)^{\frac{2}{3}}+24\sqrt{3}\,2^{\frac{1}{3}}\left(ab^5x^{14}+2\,a^2b^4x^{11}-6\,a^3b^3x^8+2\,a^4b^2x^5+a^5bx^2\right)\left(bx^3+a\right)^{\frac{2}{3}}\left(-\frac{1}{ab}\right)^{\frac{1}{3}}-\sqrt{3}\left(b^6x^{18}-42\,ab^5x^{15}-417\,a^2b^4x^{12}-812\,a^3b^3x^9-417\,a^4b^2x^6-42\,a^5bx^3+a^6\right)}{3\left(b^6x^{18}+102\,ab^5x^{15}+447\,a^2b^4x^{12}+628\,a^3b^3x^9+447\,a^4b^2x^6+102\,a^5bx^3+a^6\right)}\right)-\frac{1}{36}\cdot 2^{\frac{1}{3}}\left(-\frac{1}{ab}\right)^{\frac{1}{3}}\log\left(\frac{12\cdot 2^{\frac{2}{3}}\left(ab^3x^8+4\,a^2b^2x^5+a^3bx^2\right)\left(bx^3+a\right)^{\frac{2}{3}}\left(-\frac{1}{ab}\right)^{\frac{2}{3}}-2^{\frac{1}{3}}\left(b^4x^{12}+32\,ab^3x^9+78\,a^2b^2x^6+32\,a^3bx^3+a^4\right)\left(-\frac{1}{ab}\right)^{\frac{1}{3}}+6\left(b^3x^{10}+11\,ab^2x^7+11\,a^2bx^4+a^3x\right)\left(bx^3+a\right)^{\frac{1}{3}}}{b^4x^{12}-4\,ab^3x^9+6\,a^2b^2x^6-4\,a^3bx^3+a^4}\right)+\frac{1}{18}\cdot 2^{\frac{1}{3}}\left(-\frac{1}{ab}\right)^{\frac{1}{3}}\log\left(-\frac{12\left(bx^3+a\right)^{\frac{2}{3}}x^2+2^{\frac{2}{3}}\left(b^2x^6-2\,abx^3+a^2\right)\left(-\frac{1}{ab}\right)^{\frac{2}{3}}+6\cdot 2^{\frac{1}{3}}\left(bx^4+ax\right)\left(bx^3+a\right)^{\frac{1}{3}}\left(-\frac{1}{ab}\right)^{\frac{1}{3}}}{b^2x^6-2\,abx^3+a^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)/(-b*x^3+a),x, algorithm="fricas")
```

```
[Out] -1/18*sqrt(3)*2^(1/3)*(-1/(a*b))^(1/3)*arctan(1/3*(6*sqrt(3)*2^(2/3)*(a*b^6
*x^16 + 33*a^2*b^5*x^13 + 110*a^3*b^4*x^10 + 110*a^4*b^3*x^7 + 33*a^5*b^2*x
^4 + a^6*b*x)*(b*x^3 + a)^(1/3)*(-1/(a*b))^(2/3) + 24*sqrt(3)*2^(1/3)*(a*b^
5*x^14 + 2*a^2*b^4*x^11 - 6*a^3*b^3*x^8 + 2*a^4*b^2*x^5 + a^5*b*x^2)*(b*x^3
 + a)^(2/3)*(-1/(a*b))^(1/3) - sqrt(3)*(b^6*x^18 - 42*a*b^5*x^15 - 417*a^2*
b^4*x^12 - 812*a^3*b^3*x^9 - 417*a^4*b^2*x^6 - 42*a^5*b*x^3 + a^6))/(b^6*x^
18 + 102*a*b^5*x^15 + 447*a^2*b^4*x^12 + 628*a^3*b^3*x^9 + 447*a^4*b^2*x^6
+ 102*a^5*b*x^3 + a^6)) - 1/36*2^(1/3)*(-1/(a*b))^(1/3)*log((12*2^(2/3)*(a*
b^3*x^8 + 4*a^2*b^2*x^5 + a^3*b*x^2)*(b*x^3 + a)^(2/3)*(-1/(a*b))^(2/3) - 2
^(1/3)*(b^4*x^12 + 32*a*b^3*x^9 + 78*a^2*b^2*x^6 + 32*a^3*b*x^3 + a^4)*(-1/
(a*b))^(1/3) + 6*(b^3*x^10 + 11*a*b^2*x^7 + 11*a^2*b*x^4 + a^3*x)*(b*x^3 +
a)^(1/3))/(b^4*x^12 - 4*a*b^3*x^9 + 6*a^2*b^2*x^6 - 4*a^3*b*x^3 + a^4)) + 1
/18*2^(1/3)*(-1/(a*b))^(1/3)*log(-(12*(b*x^3 + a)^(2/3)*x^2 + 2^(2/3)*(b^2*
x^6 - 2*a*b*x^3 + a^2)*(-1/(a*b))^(2/3) + 6*2^(1/3)*(b*x^4 + a*x)*(b*x^3 +
a)^(1/3)*(-1/(a*b))^(1/3))/(b^2*x^6 - 2*a*b*x^3 + a^2))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{\left(bx^3+a\right)^{\frac{1}{3}}}{bx^3-a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)/(-b*x^3+a),x, algorithm="giac")
```

```
[Out] integrate(-(b*x^3 + a)^(1/3)/(b*x^3 - a), x)
```

maple [F] time = 0.70, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^{3}+a\right)^{\frac{1}{3}}}{-b\,x^{3}+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(1/3)/(-b*x^3+a),x)
[Out] int((b*x^3+a)^(1/3)/(-b*x^3+a),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\left(bx^{3}+a\right)^{\frac{1}{3}}}{bx^{3}-a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)/(-b*x^3+a),x, algorithm="maxima")
[Out] -integrate((b*x^3 + a)^(1/3)/(b*x^3 - a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(b\,x^{3}+a\right)^{1/3}}{a-b\,x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^(1/3)/(a - b*x^3),x)
[Out] int((a + b*x^3)^(1/3)/(a - b*x^3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\sqrt[3]{a+bx^{3}}}{-a+bx^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(1/3)/(-b*x**3+a),x)
[Out] -Integral((a + b*x**3)**(1/3)/(-a + b*x**3), x)
```

3.35 $\int \left(a-bx^3\right)^2 \left(a+bx^3\right)^{2/3} \, dx$

Optimal. Leaf size=139

$$-\frac{38a^3 \log \left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}\,x\right)}{81\sqrt[3]{b}}+\frac{76a^3 \tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{81\sqrt{3}\sqrt[3]{b}}+\frac{38}{81}a^2x\left(a+bx^3\right)^{2/3}-\frac{8}{27}ax\left(a+bx^3\right)^{5/3}-\frac{1}{9}x\left(a-bx^3\right)\left(a+\right.$$

Rubi [A] time = 0.06, antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {416, 388, 195, 239}

$$\frac{38}{81}a^2x\left(a+bx^3\right)^{2/3}-\frac{38a^3 \log \left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}\,x\right)}{81\sqrt[3]{b}}+\frac{76a^3 \tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{81\sqrt{3}\sqrt[3]{b}}-\frac{8}{27}ax\left(a+bx^3\right)^{5/3}-\frac{1}{9}x\left(a-bx^3\right)\left(a+bx^3\right)^{5/3}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*x^3)^2*(a + b*x^3)^(2/3),x]
```

```
[Out] (38*a^2*x*(a + b*x^3)^(2/3))/81 - (8*a*x*(a + b*x^3)^(5/3))/27 - (x*(a - b*
x^3)*(a + b*x^3)^(5/3))/9 + (76*a^3*ArcTan[(1 + (2*b^(1/3)*x)/(a + b*x^3)^(
1/3))/Sqrt[3]])/(81*Sqrt[3]*b^(1/3)) - (38*a^3*Log[-(b^(1/3)*x) + (a + b*x^
3)^(1/3)])/(81*b^(1/3))
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 239

```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + (2*Rt[b, 3]
*x)/(a + b*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 416

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(d*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(b*(n*(p + q) + 1)),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rubi steps

$$\begin{aligned}
\int \left(a-bx^3\right)^2 \left(a+bx^3\right)^{2/3}\, dx &= -\frac{1}{9}x\left(a-bx^3\right)\left(a+bx^3\right)^{5/3} + \frac{\int \left(a+bx^3\right)^{2/3}\left(10a^2b-16ab^2x^3\right) dx}{9b} \\
&= -\frac{8}{27}ax\left(a+bx^3\right)^{5/3} - \frac{1}{9}x\left(a-bx^3\right)\left(a+bx^3\right)^{5/3} + \frac{1}{27}\left(38a^2\right)\int \left(a+bx^3\right)^{2/3} dx \\
&= \frac{38}{81}a^2x\left(a+bx^3\right)^{2/3} - \frac{8}{27}ax\left(a+bx^3\right)^{5/3} - \frac{1}{9}x\left(a-bx^3\right)\left(a+bx^3\right)^{5/3} + \frac{1}{81}\left(76a^3\right. \\
&= \frac{38}{81}a^2x\left(a+bx^3\right)^{2/3} - \frac{8}{27}ax\left(a+bx^3\right)^{5/3} - \frac{1}{9}x\left(a-bx^3\right)\left(a+bx^3\right)^{5/3} + \frac{76a^3\tan}{81}
\end{aligned}$$

Mathematica [A] time = 0.14, size = 151, normalized size = 1.09

$$\frac{1}{243}\left(\frac{38a^3\left(\log\left(\frac{b^{2/3}x^2}{\left(a+bx^3\right)^{2/3}}+\frac{\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1\right)-2\log\left(1-\frac{\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}\right)+2\sqrt{3}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)\right)}{\sqrt[3]{b}}+3\left(a+bx^3\right)^{2/3}\left(5a^2x-24abx^4+9b^2x^7\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b*x^3)^2*(a + b*x^3)^(2/3),x]
```

```
[Out] (3*(a + b*x^3)^(2/3)*(5*a^2*x - 24*a*b*x^4 + 9*b^2*x^7) + (38*a^3*(2*Sqrt[3
]*ArcTan[(1 + (2*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]] - 2*Log[1 - (b^(1/3
)*x)/(a + b*x^3)^(1/3)] + Log[1 + (b^(2/3)*x^2)/(a + b*x^3)^(2/3) + (b^(1/3
)*x)/(a + b*x^3)^(1/3)]))/b^(1/3))/243
```

IntegrateAlgebraic [A] time = 0.57, size = 176, normalized size = 1.27

$$\frac{38a^3\log\left(\sqrt[3]{b}x\sqrt[3]{a+bx^3}+\left(a+bx^3\right)^{2/3}+b^{2/3}x^2\right)}{243\sqrt[3]{b}}-\frac{76a^3\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}x\right)}{243\sqrt[3]{b}}+\frac{76a^3\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{b}x}{2\sqrt[3]{a+bx^3}+\sqrt[3]{b}x}\right)}{81\sqrt{3}\sqrt[3]{b}}+\frac{1}{81}\left(a+bx^3\right)^{2/3}\left(5a^2x-24abx^4+9b^2x^7\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a - b*x^3)^2*(a + b*x^3)^(2/3),x]
```

```
[Out] ((a + b*x^3)^(2/3)*(5*a^2*x - 24*a*b*x^4 + 9*b^2*x^7))/81 + (76*a^3*ArcTan[
(Sqrt[3]*b^(1/3)*x)/(b^(1/3)*x + 2*(a + b*x^3)^(1/3))])/(81*Sqrt[3]*b^(1/3)
) - (76*a^3*Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)])/(243*b^(1/3)) + (38*a^3*
Log[b^(2/3)*x^2 + b^(1/3)*x*(a + b*x^3)^(1/3) + (a + b*x^3)^(2/3)])/(243*b^
(1/3))
```

fricas [A] time = 1.25, size = 421, normalized size = 3.03

$$\left[\frac{114\sqrt{\frac{1}{3}}a^3b\sqrt{\frac{(-b)^{\frac{1}{3}}}{b}}\log\left(3bx^3-3\left(bx^3+a\right)^{\frac{1}{3}}(-b)^{\frac{2}{3}}x^2-3\sqrt{\frac{1}{3}}\left((-b)^{\frac{1}{3}}bx^3-\left(bx^3+a\right)^{\frac{1}{3}}bx^2+2\left(bx^3+a\right)^{\frac{2}{3}}(-b)^{\frac{2}{3}}x\right)\sqrt{\frac{(-b)^{\frac{1}{3}}}{b}}+2a\right)-76a^3(-b)^{\frac{2}{3}}\log\left(\frac{(-b)^{\frac{1}{3}}x+\left(bx^3+a\right)^{\frac{1}{3}}}{x}\right)+38a^3(-b)^{\frac{2}{3}}\log\left(\frac{(-b)^{\frac{2}{3}}x^2-\left(bx^3+a\right)^{\frac{1}{3}}(-b)^{\frac{1}{3}}x+\left(bx^3+a\right)^{\frac{2}{3}}}{x^2}\right)+3\left(9b^3x^7-24ab^2x^4+5a^2bx\right)\left(bx^3+a\right)^{\frac{2}{3}}}{243b}, -\frac{228\sqrt{\frac{1}{3}}a^3b\sqrt{-\frac{(-b)^{\frac{1}{3}}}{b}}\arctan\left(-\frac{\sqrt{\frac{1}{3}}\left((-b)^{\frac{1}{3}}x-2\left(bx^3+a\right)^{\frac{1}{3}}\right)\sqrt{-\frac{(-b)^{\frac{1}{3}}}{b}}}{x}\right)+76a^3(-b)^{\frac{2}{3}}\log\left(\frac{(-b)^{\frac{1}{3}}x+\left(bx^3+a\right)^{\frac{1}{3}}}{x}\right)-38a^3(-b)^{\frac{2}{3}}\log\left(\frac{(-b)^{\frac{2}{3}}x^2-\left(bx^3+a\right)^{\frac{1}{3}}(-b)^{\frac{1}{3}}x+\left(bx^3+a\right)^{\frac{2}{3}}}{x^2}\right)-3\left(9b^3x^7-24ab^2x^4+5a^2bx\right)\left(bx^3+a\right)^{\frac{2}{3}}}{243b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^3+a)^2*(b*x^3+a)^(2/3),x, algorithm="fricas")
```

```
[Out] [1/243*(114*sqrt(1/3)*a^3*b*sqrt((-b)^(1/3)/b)*log(3*b*x^3 - 3*(b*x^3 + a)^
(1/3)*(-b)^(2/3)*x^2 - 3*sqrt(1/3)*((-b)^(1/3)*b*x^3 - (b*x^3 + a)^(1/3)*b*
x^2 + 2*(b*x^3 + a)^(2/3)*(-b)^(2/3)*x)*sqrt((-b)^(1/3)/b) + 2*a) - 76*a^3*
(-b)^(2/3)*log(((-b)^(1/3)*x + (b*x^3 + a)^(1/3))/x) + 38*a^3*(-b)^(2/3)*lo
g(((-b)^(2/3)*x^2 - (b*x^3 + a)^(1/3)*(-b)^(1/3)*x + (b*x^3 + a)^(2/3))/x^2
```

```
) + 3*(9*b^3*x^7 - 24*a*b^2*x^4 + 5*a^2*b*x)*(b*x^3 + a)^(2/3))/b, -1/243*(
228*sqrt(1/3)*a^3*b*sqrt(-(-b)^(1/3)/b)*arctan(-sqrt(1/3)*((-b)^(1/3)*x - 2
*(b*x^3 + a)^(1/3))*sqrt(-(-b)^(1/3)/b)/x) + 76*a^3*(-b)^(2/3)*log(((-b)^(1
/3)*x + (b*x^3 + a)^(1/3))/x) - 38*a^3*(-b)^(2/3)*log(((-b)^(2/3)*x^2 - (b*
x^3 + a)^(1/3)*(-b)^(1/3)*x + (b*x^3 + a)^(2/3))/x^2) - 3*(9*b^3*x^7 - 24*a
*b^2*x^4 + 5*a^2*b*x)*(b*x^3 + a)^(2/3))/b]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx^3+a\right)^{\frac{2}{3}} \left(bx^3-a\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^3+a)^2*(b*x^3+a)^(2/3),x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a)^(2/3)*(b*x^3 - a)^2, x)
```

maple [F] time = 0.38, size = 0, normalized size = 0.00

$$\int \left(-b\,x^3+a\right)^2 \left(b\,x^3+a\right)^{\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x^3+a)^2*(b*x^3+a)^(2/3),x)
```

```
[Out] int((-b*x^3+a)^2*(b*x^3+a)^(2/3),x)
```

maxima [B] time = 1.53, size = 552, normalized size = 3.97

$$-\frac{1}{9}\left(\frac{2\sqrt{3}\,a\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}}+\frac{2\left(bx^3+a\right)^{\frac{1}{3}}}{x}\right)}{3\,b^{\frac{1}{3}}}\right)}{b^{\frac{1}{3}}}-\frac{a\log\left(b^{\frac{2}{3}}+\frac{\left(bx^3+a\right)^{\frac{1}{3}}b^{\frac{1}{3}}}{x}+\frac{\left(bx^3+a\right)^{\frac{2}{3}}}{x^2}\right)}{b^{\frac{1}{3}}}+\frac{2\,a\log\left(-b^{\frac{1}{3}}+\frac{\left(bx^3+a\right)^{\frac{1}{3}}}{x}\right)}{b^{\frac{1}{3}}}+\frac{3\left(bx^3+a\right)^{\frac{2}{3}}a}{\left(b-\frac{bx^3+a}{x^3}\right)x^2}\right)a^2-\frac{1}{27}\left(\frac{2\sqrt{3}\,a^2\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}}+\frac{2\left(bx^3+a\right)^{\frac{1}{3}}}{x}\right)}{3\,b^{\frac{1}{3}}}\right)}{b^{\frac{4}{3}}}-\frac{a^2\log\left(b^{\frac{2}{3}}+\frac{\left(bx^3+a\right)^{\frac{1}{3}}b^{\frac{1}{3}}}{x}+\frac{\left(bx^3+a\right)^{\frac{2}{3}}}{x^2}\right)}{b^{\frac{4}{3}}}+\frac{2\,a^2\log\left(-b^{\frac{1}{3}}+\frac{\left(bx^3+a\right)^{\frac{1}{3}}}{x}\right)}{b^{\frac{4}{3}}}+\frac{3\left(\frac{\left(bx^3+a\right)^{\frac{2}{3}}a^2b}{x^2}+\frac{2\left(bx^3+a\right)^{\frac{5}{3}}a^2}{x^5}\right)}{b^3-\frac{2\left(bx^3+a\right)b^2}{x^3}+\frac{\left(bx^3+a\right)^2b}{x^6}}\right)ab-\frac{1}{243}\left(\frac{4\sqrt{3}\,a^3\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}}+\frac{2\left(bx^3+a\right)^{\frac{1}{3}}}{x}\right)}{3\,b^{\frac{1}{3}}}\right)}{b^{\frac{7}{3}}}-\frac{2\,a^3\log\left(b^{\frac{2}{3}}+\frac{\left(bx^3+a\right)^{\frac{1}{3}}b^{\frac{1}{3}}}{x}+\frac{\left(bx^3+a\right)^{\frac{2}{3}}}{x^2}\right)}{b^{\frac{7}{3}}}+\frac{4\,a^3\log\left(-b^{\frac{1}{3}}+\frac{\left(bx^3+a\right)^{\frac{1}{3}}}{x}\right)}{b^{\frac{7}{3}}}+\frac{3\left(\frac{2\left(bx^3+a\right)^{\frac{2}{3}}a^3b^2}{x^2}+\frac{11\left(bx^3+a\right)^{\frac{5}{3}}a^3b}{x^5}-\frac{4\left(bx^3+a\right)^{\frac{8}{3}}a^3}{x^8}\right)}{b^5-\frac{3\left(bx^3+a\right)b^4}{x^3}+\frac{3\left(bx^3+a\right)^2b^3}{x^6}-\frac{\left(bx^3+a\right)^3b^2}{x^9}}\right)b^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^3+a)^2*(b*x^3+a)^(2/3),x, algorithm="maxima")
```

```
[Out] -1/9*(2*sqrt(3)*a*arctan(1/3*sqrt(3)*(b^(1/3) + 2*(b*x^3 + a)^(1/3)/x)/b^(1
/3))/b^(1/3) - a*log(b^(2/3) + (b*x^3 + a)^(1/3)*b^(1/3)/x + (b*x^3 + a)^(2
/3)/x^2)/b^(1/3) + 2*a*log(-b^(1/3) + (b*x^3 + a)^(1/3)/x)/b^(1/3) + 3*(b*x
^3 + a)^(2/3)*a/((b - (b*x^3 + a)/x^3)*x^2))*a^2 - 1/27*(2*sqrt(3)*a^2*arct
an(1/3*sqrt(3)*(b^(1/3) + 2*(b*x^3 + a)^(1/3)/x)/b^(1/3))/b^(4/3) - a^2*log
(b^(2/3) + (b*x^3 + a)^(1/3)*b^(1/3)/x + (b*x^3 + a)^(2/3)/x^2)/b^(4/3) + 2
*a^2*log(-b^(1/3) + (b*x^3 + a)^(1/3)/x)/b^(4/3) + 3*((b*x^3 + a)^(2/3)*a^2
*b/x^2 + 2*(b*x^3 + a)^(5/3)*a^2/x^5)/(b^3 - 2*(b*x^3 + a)*b^2/x^3 + (b*x^3
 + a)^2*b/x^6))*a*b - 1/243*(4*sqrt(3)*a^3*arctan(1/3*sqrt(3)*(b^(1/3) + 2*
(b*x^3 + a)^(1/3)/x)/b^(1/3))/b^(7/3) - 2*a^3*log(b^(2/3) + (b*x^3 + a)^(1/
3)*b^(1/3)/x + (b*x^3 + a)^(2/3)/x^2)/b^(7/3) + 4*a^3*log(-b^(1/3) + (b*x^3
 + a)^(1/3)/x)/b^(7/3) + 3*(2*(b*x^3 + a)^(2/3)*a^3*b^2/x^2 + 11*(b*x^3 + a
)^(5/3)*a^3*b/x^5 - 4*(b*x^3 + a)^(8/3)*a^3/x^8)/(b^5 - 3*(b*x^3 + a)*b^4/x
^3 + 3*(b*x^3 + a)^2*b^3/x^6 - (b*x^3 + a)^3*b^2/x^9))*b^2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(b\,x^3+a\right)}^{2/3}\,{\left(a-b\,x^3\right)}^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^(2/3)*(a - b*x^3)^2,x)
```

```
[Out] int((a + b*x^3)^(2/3)*(a - b*x^3)^2, x)
```

sympy [C] time = 9.17, size = 126, normalized size = 0.91

$$\frac{a^{\frac{8}{3}}x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{2}{3},\frac{1}{3}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{4}{3}\right)}-\frac{2a^{\frac{5}{3}}bx^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}-\frac{2}{3},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{7}{3}\right)}+\frac{a^{\frac{2}{3}}b^2x^7\Gamma\left(\frac{7}{3}\right){}_2F_1\left(\begin{matrix}-\frac{2}{3},\frac{7}{3}\\ \frac{10}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{10}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x**3+a)**2*(b*x**3+a)**(2/3),x)
```

```
[Out] a**(8/3)*x*gamma(1/3)*hyper((-2/3, 1/3), (4/3,), b*x**3*exp_polar(I*pi)/a)/
(3*gamma(4/3)) - 2*a**(5/3)*b*x**4*gamma(4/3)*hyper((-2/3, 4/3), (7/3,), b*
x**3*exp_polar(I*pi)/a)/(3*gamma(7/3)) + a**(2/3)*b**2*x**7*gamma(7/3)*hype
r((-2/3, 7/3), (10/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(10/3))
```

3.36 $\int \frac{\left(a-bx^3\right)^2}{\sqrt[3]{a+bx^3}}\,dx$

Optimal. Leaf size=120

$$-\frac{17a^2\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}\,x\right)}{18\sqrt[3]{b}}+\frac{17a^2\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{9\sqrt{3}\,\sqrt[3]{b}}-\frac{13}{18}ax\left(a+bx^3\right)^{2/3}-\frac{1}{6}x\left(a-bx^3\right)\left(a+bx^3\right)^{2/3}$$

Rubi [A] time = 0.04, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {416, 388, 239}

$$-\frac{17a^2\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}\,x\right)}{18\sqrt[3]{b}}+\frac{17a^2\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{9\sqrt{3}\,\sqrt[3]{b}}-\frac{13}{18}ax\left(a+bx^3\right)^{2/3}-\frac{1}{6}x\left(a-bx^3\right)\left(a+bx^3\right)^{2/3}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*x^3)^2/(a + b*x^3)^(1/3),x]
```

```
[Out] (-13*a*x*(a + b*x^3)^(2/3))/18 - (x*(a - b*x^3)*(a + b*x^3)^(2/3))/6 + (17*
a^2*ArcTan[(1 + (2*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]])/(9*Sqrt[3]*b^(1/
3)) - (17*a^2*Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)])/(18*b^(1/3))
```

Rule 239

```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + (2*Rt[b, 3]
*x)/(a + b*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 416

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(d*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(b*(n*(p + q) + 1)),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rubi steps

$$\int \frac{\left(a-bx^3\right)^2}{\sqrt[3]{a+bx^3}}\,dx = -\frac{1}{6}x\left(a-bx^3\right)\left(a+bx^3\right)^{2/3} + \frac{\int \frac{7a^2b-13ab^2x^3}{\sqrt[3]{a+bx^3}}\,dx}{6b}$$

$$= -\frac{13}{18}ax\left(a+bx^3\right)^{2/3} - \frac{1}{6}x\left(a-bx^3\right)\left(a+bx^3\right)^{2/3} + \frac{1}{9}\left(17a^2\right)\int \frac{1}{\sqrt[3]{a+bx^3}}\,dx$$

$$= -\frac{13}{18}ax\left(a+bx^3\right)^{2/3} - \frac{1}{6}x\left(a-bx^3\right)\left(a+bx^3\right)^{2/3} + \frac{17a^2\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{9\sqrt{3}\sqrt[3]{b}} - \frac{17a^2\log\left(-\sqrt[3]{b}\,x\right.}{18\sqrt[3]{}}$$

Mathematica [A] time = 0.08, size = 141, normalized size = 1.18

$$\frac{17a^2\left(\log\left(\frac{b^{2/3}x^2}{\left(a+bx^3\right)^{2/3}}+\frac{\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1\right)-2\log\left(1-\frac{\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}\right)+2\sqrt{3}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)\right)}{54\sqrt[3]{b}}+\left(a+bx^3\right)^{2/3}\left(\frac{bx^4}{6}-\frac{8ax}{9}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b*x^3)^2/(a + b*x^3)^(1/3),x]
```

```
[Out] (a + b*x^3)^(2/3)*((-8*a*x)/9 + (b*x^4)/6) + (17*a^2*(2*Sqrt[3]*ArcTan[(1 +
 (2*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]] - 2*Log[1 - (b^(1/3)*x)/(a + b*x
^3)^(1/3)] + Log[1 + (b^(2/3)*x^2)/(a + b*x^3)^(2/3) + (b^(1/3)*x)/(a + b*x
^3)^(1/3)]))/(54*b^(1/3))
```

IntegrateAlgebraic [A] time = 0.48, size = 165, normalized size = 1.38

$$\frac{17a^2\log\left(\sqrt[3]{b}x\sqrt[3]{a+bx^3}+\left(a+bx^3\right)^{2/3}+b^{2/3}x^2\right)}{54\sqrt[3]{b}}-\frac{17a^2\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}x\right)}{27\sqrt[3]{b}}+\frac{17a^2\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{b}x}{2\sqrt[3]{a+bx^3}+\sqrt[3]{b}x}\right)}{9\sqrt{3}\sqrt[3]{b}}+\frac{1}{18}\left(a+bx^3\right)^{2/3}\left(3bx^4-16ax\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a - b*x^3)^2/(a + b*x^3)^(1/3),x]
```

```
[Out] ((a + b*x^3)^(2/3)*(-16*a*x + 3*b*x^4))/18 + (17*a^2*ArcTan[(Sqrt[3]*b^(1/3
)*x)/(b^(1/3)*x + 2*(a + b*x^3)^(1/3))])/(9*Sqrt[3]*b^(1/3)) - (17*a^2*Log[
-(b^(1/3)*x) + (a + b*x^3)^(1/3)])/(27*b^(1/3)) + (17*a^2*Log[b^(2/3)*x^2 +
 b^(1/3)*x*(a + b*x^3)^(1/3) + (a + b*x^3)^(2/3)])/(54*b^(1/3))
```

fricas [A] time = 1.66, size = 399, normalized size = 3.32

$$\left[\frac{51\sqrt{\frac{1}{3}}a^2b\sqrt{\frac{(-b)^{\frac{1}{3}}}{b}}\log\left(3bx^3-3\left(bx^3+a\right)^{\frac{1}{3}}(-b)^{\frac{2}{3}}x^2-3\sqrt{\frac{1}{3}}\left((-b)^{\frac{1}{3}}bx^3-\left(bx^3+a\right)^{\frac{1}{3}}bx^2+2\left(bx^3+a\right)^{\frac{2}{3}}(-b)^{\frac{2}{3}}x\right)\sqrt{\frac{(-b)^{\frac{1}{3}}}{b}}+2a\right)-34a^2(-b)^{\frac{2}{3}}\log\left(\frac{(-b)^{\frac{1}{3}}x+\left(bx^3+a\right)^{\frac{1}{3}}}{x}\right)+17a^2(-b)^{\frac{2}{3}}\log\left(\frac{(-b)^{\frac{2}{3}}x^2-\left(bx^3+a\right)^{\frac{1}{3}}(-b)^{\frac{1}{3}}x+\left(bx^3+a\right)^{\frac{2}{3}}}{x^2}\right)+3\left(3b^2x^4-16abx\right)\left(bx^3+a\right)^{\frac{2}{3}}}{54b}, -\frac{102\sqrt{\frac{1}{3}}a^2b\sqrt{-\frac{(-b)^{\frac{1}{3}}}{b}}\arctan\left(-\frac{\sqrt{\frac{1}{3}}\left((-b)^{\frac{1}{3}}x-2\left(bx^3+a\right)^{\frac{1}{3}}\right)\sqrt{-\frac{(-b)^{\frac{1}{3}}}{b}}}{x}\right)+34a^2(-b)^{\frac{2}{3}}\log\left(\frac{(-b)^{\frac{1}{3}}x+\left(bx^3+a\right)^{\frac{1}{3}}}{x}\right)-17a^2(-b)^{\frac{2}{3}}\log\left(\frac{(-b)^{\frac{2}{3}}x^2-\left(bx^3+a\right)^{\frac{1}{3}}(-b)^{\frac{1}{3}}x+\left(bx^3+a\right)^{\frac{2}{3}}}{x^2}\right)-3\left(3b^2x^4-16abx\right)\left(bx^3+a\right)^{\frac{2}{3}}}{54b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^3+a)^2/(b*x^3+a)^(1/3),x, algorithm="fricas")
```

```
[Out] [1/54*(51*sqrt(1/3)*a^2*b*sqrt((-b)^(1/3)/b)*log(3*b*x^3 - 3*(b*x^3 + a)^(1
/3)*(-b)^(2/3)*x^2 - 3*sqrt(1/3)*((-b)^(1/3)*b*x^3 - (b*x^3 + a)^(1/3)*b*x^
2 + 2*(b*x^3 + a)^(2/3)*(-b)^(2/3)*x)*sqrt((-b)^(1/3)/b) + 2*a) - 34*a^2*(-
b)^(2/3)*log(((-b)^(1/3)*x + (b*x^3 + a)^(1/3))/x) + 17*a^2*(-b)^(2/3)*log(
((-b)^(2/3)*x^2 - (b*x^3 + a)^(1/3)*(-b)^(1/3)*x + (b*x^3 + a)^(2/3))/x^2)
+ 3*(3*b^2*x^4 - 16*a*b*x)*(b*x^3 + a)^(2/3))/b, -1/54*(102*sqrt(1/3)*a^2*b
*sqrt(-(-b)^(1/3)/b)*arctan(-sqrt(1/3)*((-b)^(1/3)*x - 2*(b*x^3 + a)^(1/3))
*sqrt(-(-b)^(1/3)/b)/x) + 34*a^2*(-b)^(2/3)*log(((-b)^(1/3)*x + (b*x^3 + a)
```

```
^(1/3))/x) - 17*a^2*(-b)^(2/3)*log(((-b)^(2/3)*x^2 - (b*x^3 + a)^(1/3)*(-b)
^(1/3)*x + (b*x^3 + a)^(2/3))/x^2) - 3*(3*b^2*x^4 - 16*a*b*x)*(b*x^3 + a)^(
2/3))/b]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^3 - a\right)^2}{\left(bx^3 + a\right)^{\frac{1}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^3+a)^2/(b*x^3+a)^(1/3),x, algorithm="giac")
```

```
[Out] integrate((b*x^3 - a)^2/(b*x^3 + a)^(1/3), x)
```

maple [F] time = 0.38, size = 0, normalized size = 0.00

$$\int \frac{\left(-b\,x^3 + a\right)^2}{\left(b\,x^3 + a\right)^{\frac{1}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x^3+a)^2/(b*x^3+a)^(1/3),x)
```

```
[Out] int((-b*x^3+a)^2/(b*x^3+a)^(1/3),x)
```

maxima [B] time = 1.40, size = 436, normalized size = 3.63

$$-\frac{1}{6}\left(\frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}}+\frac{2\left(bx^3+a\right)^{\frac{1}{3}}}{x}\right)}{3b^{\frac{1}{3}}}\right)}{b^{\frac{1}{3}}} - \frac{\log\left(b^{\frac{2}{3}}+\frac{\left(bx^3+a\right)^{\frac{1}{3}}b^{\frac{1}{3}}}{x}+\frac{\left(bx^3+a\right)^{\frac{2}{3}}}{x^2}\right)}{b^{\frac{1}{3}}} + \frac{2\log\left(-b^{\frac{1}{3}}+\frac{\left(bx^3+a\right)^{\frac{1}{3}}}{x}\right)}{b^{\frac{1}{3}}}\right)a^2 - \frac{1}{9}\left(\frac{2\sqrt{3}\,a\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}}+\frac{2\left(bx^3+a\right)^{\frac{1}{3}}}{x}\right)}{3b^{\frac{1}{3}}}\right)}{b^{\frac{4}{3}}} - \frac{a\log\left(b^{\frac{2}{3}}+\frac{\left(bx^3+a\right)^{\frac{1}{3}}b^{\frac{1}{3}}}{x}+\frac{\left(bx^3+a\right)^{\frac{2}{3}}}{x^2}\right)}{b^{\frac{4}{3}}} + \frac{2a\log\left(-b^{\frac{1}{3}}+\frac{\left(bx^3+a\right)^{\frac{1}{3}}}{x}\right)}{b^{\frac{4}{3}}} - \frac{6\left(bx^3+a\right)^{\frac{2}{3}}a}{\left(b^2-\frac{\left(bx^3+a\right)b}{x^3}\right)x^2}\right)ab - \frac{1}{54}\left(\frac{4\sqrt{3}\,a^2\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}}+\frac{2\left(bx^3+a\right)^{\frac{1}{3}}}{x}\right)}{3b^{\frac{1}{3}}}\right)}{b^{\frac{7}{3}}} - \frac{2a^2\log\left(b^{\frac{2}{3}}+\frac{\left(bx^3+a\right)^{\frac{1}{3}}b^{\frac{1}{3}}}{x}+\frac{\left(bx^3+a\right)^{\frac{2}{3}}}{x^2}\right)}{b^{\frac{7}{3}}} + \frac{4a^2\log\left(-b^{\frac{1}{3}}+\frac{\left(bx^3+a\right)^{\frac{1}{3}}}{x}\right)}{b^{\frac{7}{3}}} - \frac{3\left(\frac{7\left(bx^3+a\right)^{\frac{2}{3}}a^2b}{x^2}-\frac{4\left(bx^3+a\right)^{\frac{5}{3}}a^2}{x^5}\right)}{b^4-\frac{2\left(bx^3+a\right)b^3}{x^3}+\frac{\left(bx^3+a\right)^2b^2}{x^6}}\right)b^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^3+a)^2/(b*x^3+a)^(1/3),x, algorithm="maxima")
```

```
[Out] -1/6*(2*sqrt(3)*arctan(1/3*sqrt(3)*(b^(1/3) + 2*(b*x^3 + a)^(1/3)/x)/b^(1/3
))/b^(1/3) - log(b^(2/3) + (b*x^3 + a)^(1/3)*b^(1/3)/x + (b*x^3 + a)^(2/3)/
x^2)/b^(1/3) + 2*log(-b^(1/3) + (b*x^3 + a)^(1/3)/x)/b^(1/3))*a^2 - 1/9*(2*
sqrt(3)*a*arctan(1/3*sqrt(3)*(b^(1/3) + 2*(b*x^3 + a)^(1/3)/x)/b^(1/3))/b^(
4/3) - a*log(b^(2/3) + (b*x^3 + a)^(1/3)*b^(1/3)/x + (b*x^3 + a)^(2/3)/x^2)
/b^(4/3) + 2*a*log(-b^(1/3) + (b*x^3 + a)^(1/3)/x)/b^(4/3) - 6*(b*x^3 + a)^
(2/3)*a/((b^2 - (b*x^3 + a)*b/x^3)*x^2))*a*b - 1/54*(4*sqrt(3)*a^2*arctan(1
/3*sqrt(3)*(b^(1/3) + 2*(b*x^3 + a)^(1/3)/x)/b^(1/3))/b^(7/3) - 2*a^2*log(b
^(2/3) + (b*x^3 + a)^(1/3)*b^(1/3)/x + (b*x^3 + a)^(2/3)/x^2)/b^(7/3) + 4*a
^2*log(-b^(1/3) + (b*x^3 + a)^(1/3)/x)/b^(7/3) - 3*(7*(b*x^3 + a)^(2/3)*a^2
*b/x^2 - 4*(b*x^3 + a)^(5/3)*a^2/x^5)/(b^4 - 2*(b*x^3 + a)*b^3/x^3 + (b*x^3
 + a)^2*b^2/x^6))*b^2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a - b\,x^3\right)^2}{\left(b\,x^3 + a\right)^{1/3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - b*x^3)^2/(a + b*x^3)^(1/3),x)
```

```
[Out] int((a - b*x^3)^2/(a + b*x^3)^(1/3), x)
```

sympy [C]   time = 7.60, size = 121, normalized size = 1.01

$$\frac{a^{\frac{5}{3}}x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{3}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{4}{3}\right)}-\frac{2a^{\frac{2}{3}}bx^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{7}{3}\right)}+\frac{b^2x^7\Gamma\left(\frac{7}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{7}{3}\\ \frac{10}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt[3]{a}\,\Gamma\left(\frac{10}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x**3+a)**2/(b*x**3+a)**(1/3),x)
```

```
[Out] a**(5/3)*x*gamma(1/3)*hyper((1/3, 1/3), (4/3,), b*x**3*exp_polar(I*pi)/a)/(
3*gamma(4/3)) - 2*a**(2/3)*b*x**4*gamma(4/3)*hyper((1/3, 4/3), (7/3,), b*x*
*3*exp_polar(I*pi)/a)/(3*gamma(7/3)) + b**2*x**7*gamma(7/3)*hyper((1/3, 7/3
), (10/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(1/3)*gamma(10/3))
```

3.37 $\int \frac{(a-bx^3)^2}{(a+bx^3)^{4/3}}\,dx$

Optimal. Leaf size=113

$$\frac{7}{3}x\left(a+bx^3\right)^{2/3}+\frac{2x\left(a-bx^3\right)}{\sqrt[3]{a+bx^3}}+\frac{5a\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}\,x\right)}{3\sqrt[3]{b}}-\frac{10a\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{3\sqrt{3}\,\sqrt[3]{b}}$$

Rubi [A] time = 0.04, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {413, 388, 239}

$$\frac{7}{3}x\left(a+bx^3\right)^{2/3}+\frac{2x\left(a-bx^3\right)}{\sqrt[3]{a+bx^3}}+\frac{5a\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}\,x\right)}{3\sqrt[3]{b}}-\frac{10a\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{3\sqrt{3}\,\sqrt[3]{b}}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*x^3)^2/(a + b*x^3)^(4/3),x]
```

```
[Out] (2*x*(a - b*x^3))/(a + b*x^3)^(1/3) + (7*x*(a + b*x^3)^(2/3))/3 - (10*a*Arc
Tan[(1 + (2*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]])/(3*Sqrt[3]*b^(1/3)) + (
5*a*Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)])/(3*b^(1/3))
```

Rule 239

```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + (2*Rt[b, 3]
*x)/(a + b*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 413

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[((a*d - c*b)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*n*(p +
 1)), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rubi steps

$$\int \frac{\left(a-bx^3\right)^2}{\left(a+bx^3\right)^{4/3}}\,dx = \frac{2x\left(a-bx^3\right)}{\sqrt[3]{a+bx^3}} + \frac{\int \frac{-a^2b+7ab^2x^3}{\sqrt[3]{a+bx^3}}\,dx}{ab}$$

$$= \frac{2x\left(a-bx^3\right)}{\sqrt[3]{a+bx^3}} + \frac{7}{3}x\left(a+bx^3\right)^{2/3} - \frac{1}{3}(10a)\int \frac{1}{\sqrt[3]{a+bx^3}}\,dx$$

$$= \frac{2x\left(a-bx^3\right)}{\sqrt[3]{a+bx^3}} + \frac{7}{3}x\left(a+bx^3\right)^{2/3} - \frac{10a\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{3\sqrt{3}\sqrt[3]{b}} + \frac{5a\log\left(-\sqrt[3]{b}x+\sqrt[3]{a+bx^3}\right)}{3\sqrt[3]{b}}$$

Mathematica [A] time = 0.08, size = 137, normalized size = 1.21

$$\frac{x\left(13a+bx^3\right)}{3\sqrt[3]{a+bx^3}} - \frac{5a\left(\log\left(\frac{b^{2/3}x^2}{\left(a+bx^3\right)^{2/3}}+\frac{\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1\right)-2\log\left(1-\frac{\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}\right)+2\sqrt{3}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)\right)}{9\sqrt[3]{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b*x^3)^2/(a + b*x^3)^(4/3),x]
```

```
[Out] (x*(13*a + b*x^3))/(3*(a + b*x^3)^(1/3)) - (5*a*(2*Sqrt[3]*ArcTan[(1 + (2*b
^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]] - 2*Log[1 - (b^(1/3)*x)/(a + b*x^3)^(
1/3)] + Log[1 + (b^(2/3)*x^2)/(a + b*x^3)^(2/3) + (b^(1/3)*x)/(a + b*x^3)^(
1/3)]))/(9*b^(1/3))
```

IntegrateAlgebraic [A] time = 0.50, size = 158, normalized size = 1.40

$$-\frac{5a\log\left(\sqrt[3]{b}x\sqrt[3]{a+bx^3}+\left(a+bx^3\right)^{2/3}+b^{2/3}x^2\right)}{9\sqrt[3]{b}} + \frac{10a\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}x\right)}{9\sqrt[3]{b}} - \frac{10a\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{b}x}{2\sqrt[3]{a+bx^3}+\sqrt[3]{b}x}\right)}{3\sqrt{3}\sqrt[3]{b}} + \frac{13ax+bx^4}{3\sqrt[3]{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a - b*x^3)^2/(a + b*x^3)^(4/3),x]
```

```
[Out] (13*a*x + b*x^4)/(3*(a + b*x^3)^(1/3)) - (10*a*ArcTan[(Sqrt[3]*b^(1/3)*x)/(
b^(1/3)*x + 2*(a + b*x^3)^(1/3))])/(3*Sqrt[3]*b^(1/3)) + (10*a*Log[-(b^(1/3
)*x) + (a + b*x^3)^(1/3)])/(9*b^(1/3)) - (5*a*Log[b^(2/3)*x^2 + b^(1/3)*x*(
a + b*x^3)^(1/3) + (a + b*x^3)^(2/3)])/(9*b^(1/3))
```

fricas [B] time = 1.73, size = 412, normalized size = 3.65

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^3+a)^2/(b*x^3+a)^(4/3),x, algorithm="fricas")
```

```
[Out] [1/9*(15*sqrt(1/3)*(a*b^2*x^3 + a^2*b)*sqrt(-1/b^(2/3))*log(3*b*x^3 - 3*(b*
x^3 + a)^(1/3)*b^(2/3)*x^2 - 3*sqrt(1/3)*(b^(4/3)*x^3 + (b*x^3 + a)^(1/3)*b
*x^2 - 2*(b*x^3 + a)^(2/3)*b^(2/3)*x)*sqrt(-1/b^(2/3)) + 2*a) + 10*(a*b*x^3
 + a^2)*b^(2/3)*log(-(b^(1/3)*x - (b*x^3 + a)^(1/3))/x) - 5*(a*b*x^3 + a^2)
*b^(2/3)*log((b^(2/3)*x^2 + (b*x^3 + a)^(1/3)*b^(1/3)*x + (b*x^3 + a)^(2/3)
)/x^2) + 3*(b^2*x^4 + 13*a*b*x)*(b*x^3 + a)^(2/3))/(b^2*x^3 + a*b), 1/9*(10
```

```
*(a*b*x^3 + a^2)*b^(2/3)*log(-(b^(1/3)*x - (b*x^3 + a)^(1/3))/x) - 5*(a*b*x
^3 + a^2)*b^(2/3)*log((b^(2/3)*x^2 + (b*x^3 + a)^(1/3)*b^(1/3)*x + (b*x^3 +
 a)^(2/3))/x^2) + 30*sqrt(1/3)*(a*b^2*x^3 + a^2*b)*arctan(sqrt(1/3)*(b^(1/3
)*x + 2*(b*x^3 + a)^(1/3))/(b^(1/3)*x))/b^(1/3) + 3*(b^2*x^4 + 13*a*b*x)*(b
*x^3 + a)^(2/3))/(b^2*x^3 + a*b)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^3 - a\right)^2}{\left(bx^3 + a\right)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^3+a)^2/(b*x^3+a)^(4/3),x, algorithm="giac")
```

```
[Out] integrate((b*x^3 - a)^2/(b*x^3 + a)^(4/3), x)
```

maple [F] time = 0.53, size = 0, normalized size = 0.00

$$\int \frac{\left(-b\,x^3 + a\right)^2}{\left(b\,x^3 + a\right)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x^3+a)^2/(b*x^3+a)^(4/3),x)
```

```
[Out] int((-b*x^3+a)^2/(b*x^3+a)^(4/3),x)
```

maxima [B] time = 1.37, size = 296, normalized size = 2.62

$$\frac{1}{9}\,b^2\left(\frac{4\,\sqrt{3}\,a\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}}+\frac{2\,(bx^3+a)^{\frac{1}{3}}}{x}\right)}{3\,b^{\frac{1}{3}}}\right)}{b^{\frac{7}{3}}}+\frac{3\left(3\,ab-\frac{4\,(bx^3+a)a}{x^3}\right)}{\frac{(bx^3+a)^{\frac{1}{3}}b^3}{x}-\frac{(bx^3+a)^{\frac{4}{3}}b^2}{x^4}}-\frac{2\,a\log\left(b^{\frac{2}{3}}+\frac{(bx^3+a)^{\frac{1}{3}}b^{\frac{1}{3}}}{x}+\frac{(bx^3+a)^{\frac{2}{3}}}{x^2}\right)}{b^{\frac{7}{3}}}+\frac{4\,a\log\left(-b^{\frac{1}{3}}+\frac{(bx^3+a)^{\frac{1}{3}}}{x}\right)}{b^{\frac{7}{3}}}\right)+\frac{1}{3}\,ab\left(\frac{2\,\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}}+\frac{2\,(bx^3+a)^{\frac{1}{3}}}{x}\right)}{3\,b^{\frac{1}{3}}}\right)}{b^{\frac{4}{3}}}+\frac{6\,x}{(bx^3+a)^{\frac{1}{3}}b}-\frac{\log\left(b^{\frac{2}{3}}+\frac{(bx^3+a)^{\frac{1}{3}}b^{\frac{1}{3}}}{x}+\frac{(bx^3+a)^{\frac{2}{3}}}{x^2}\right)}{b^{\frac{4}{3}}}+\frac{2\log\left(-b^{\frac{1}{3}}+\frac{(bx^3+a)^{\frac{1}{3}}}{x}\right)}{b^{\frac{4}{3}}}\right)+\frac{ax}{(bx^3+a)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^3+a)^2/(b*x^3+a)^(4/3),x, algorithm="maxima")
```

```
[Out] 1/9*b^2*(4*sqrt(3)*a*arctan(1/3*sqrt(3)*(b^(1/3) + 2*(b*x^3 + a)^(1/3)/x)/b
^(1/3))/b^(7/3) + 3*(3*a*b - 4*(b*x^3 + a)*a/x^3)/((b*x^3 + a)^(1/3)*b^3/x
- (b*x^3 + a)^(4/3)*b^2/x^4) - 2*a*log(b^(2/3) + (b*x^3 + a)^(1/3)*b^(1/3)/
x + (b*x^3 + a)^(2/3)/x^2)/b^(7/3) + 4*a*log(-b^(1/3) + (b*x^3 + a)^(1/3)/x
)/b^(7/3)) + 1/3*a*b*(2*sqrt(3)*arctan(1/3*sqrt(3)*(b^(1/3) + 2*(b*x^3 + a)
^(1/3)/x)/b^(1/3))/b^(4/3) + 6*x/((b*x^3 + a)^(1/3)*b) - log(b^(2/3) + (b*x
^3 + a)^(1/3)*b^(1/3)/x + (b*x^3 + a)^(2/3)/x^2)/b^(4/3) + 2*log(-b^(1/3) +
 (b*x^3 + a)^(1/3)/x)/b^(4/3)) + a*x/(b*x^3 + a)^(1/3)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a - b\,x^3\right)^2}{\left(b\,x^3 + a\right)^{4/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - b*x^3)^2/(a + b*x^3)^(4/3),x)
```

```
[Out] int((a - b*x^3)^2/(a + b*x^3)^(4/3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-a+bx^3\right)^2}{\left(a+bx^3\right)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x**3+a)**2/(b*x**3+a)**(4/3),x)
```

```
[Out] Integral((-a + b*x**3)**2/(a + b*x**3)**(4/3), x)
```

3.38 $\int \frac{\left(a-bx^3\right)^2}{\left(a+bx^3\right)^{7/3}} \, dx$

Optimal. Leaf size=110

$$-\frac{x}{2\sqrt[3]{a+bx^3}}+\frac{x\left(a-bx^3\right)}{2\left(a+bx^3\right)^{4/3}}-\frac{\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}\,x\right)}{2\sqrt[3]{b}}+\frac{\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\,\sqrt[3]{b}}$$

Rubi [A] time = 0.04, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {413, 385, 239}

$$-\frac{x}{2\sqrt[3]{a+bx^3}}+\frac{x\left(a-bx^3\right)}{2\left(a+bx^3\right)^{4/3}}-\frac{\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}\,x\right)}{2\sqrt[3]{b}}+\frac{\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\,\sqrt[3]{b}}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*x^3)^2/(a + b*x^3)^(7/3),x]
```

```
[Out] (x*(a - b*x^3))/(2*(a + b*x^3)^(4/3)) - x/(2*(a + b*x^3)^(1/3)) + ArcTan[(1
 + (2*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]*b^(1/3)) - Log[-(b^(1
/3)*x) + (a + b*x^3)^(1/3)]/(2*b^(1/3))
```

Rule 239

```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + (2*Rt[b, 3]
*x)/(a + b*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 413

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[((a*d - c*b)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*n*(p +
 1)), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rubi steps

$$\int \frac{\left(a-bx^3\right)^2}{\left(a+bx^3\right)^{7/3}}\,dx = \frac{x\left(a-bx^3\right)}{2\left(a+bx^3\right)^{4/3}} + \frac{\int \frac{2a^2b+4ab^2x^3}{\left(a+bx^3\right)^{4/3}}\,dx}{4ab}$$

$$= \frac{x\left(a-bx^3\right)}{2\left(a+bx^3\right)^{4/3}} - \frac{x}{2\sqrt[3]{a+bx^3}} + \int \frac{1}{\sqrt[3]{a+bx^3}}\,dx$$

$$= \frac{x\left(a-bx^3\right)}{2\left(a+bx^3\right)^{4/3}} - \frac{x}{2\sqrt[3]{a+bx^3}} + \frac{\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}\,\sqrt[3]{b}} - \frac{\log\left(-\sqrt[3]{b}\,x+\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{b}}$$

Mathematica [A] time = 0.06, size = 131, normalized size = 1.19

$$\frac{-\frac{6b^{4/3}x^4}{\left(a+bx^3\right)^{4/3}} + \log\left(\frac{b^{2/3}x^2}{\left(a+bx^3\right)^{2/3}} + \frac{\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}} + 1\right) - 2\log\left(1-\frac{\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}\right) + 2\sqrt{3}\,\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{6\sqrt[3]{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b*x^3)^2/(a + b*x^3)^(7/3),x]
```

```
[Out] ((-6*b^(4/3)*x^4)/(a + b*x^3)^(4/3) + 2*Sqrt[3]*ArcTan[(1 + (2*b^(1/3)*x)/(
a + b*x^3)^(1/3))/Sqrt[3]] - 2*Log[1 - (b^(1/3)*x)/(a + b*x^3)^(1/3)] + Log
[1 + (b^(2/3)*x^2)/(a + b*x^3)^(2/3) + (b^(1/3)*x)/(a + b*x^3)^(1/3)])/(6*b
^(1/3))
```

IntegrateAlgebraic [A] time = 0.39, size = 144, normalized size = 1.31

$$\frac{\log\left(\sqrt[3]{b}\,x\sqrt[3]{a+bx^3}+\left(a+bx^3\right)^{2/3}+b^{2/3}x^2\right)}{6\sqrt[3]{b}} - \frac{\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}\,x\right)}{3\sqrt[3]{b}} + \frac{\tan^{-1}\left(\frac{\sqrt{3}\,\sqrt[3]{b}\,x}{2\sqrt[3]{a+bx^3}+\sqrt[3]{b}\,x}\right)}{\sqrt{3}\,\sqrt[3]{b}} - \frac{bx^4}{\left(a+bx^3\right)^{4/3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a - b*x^3)^2/(a + b*x^3)^(7/3),x]
```

```
[Out] -((b*x^4)/(a + b*x^3)^(4/3)) + ArcTan[(Sqrt[3]*b^(1/3)*x)/(b^(1/3)*x + 2*(a
 + b*x^3)^(1/3))]/(Sqrt[3]*b^(1/3)) - Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)]
/(3*b^(1/3)) + Log[b^(2/3)*x^2 + b^(1/3)*x*(a + b*x^3)^(1/3) + (a + b*x^3)^
(2/3)]/(6*b^(1/3))
```

fricas [B] time = 1.16, size = 521, normalized size = 4.74

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^3+a)^2/(b*x^3+a)^(7/3),x, algorithm="fricas")
```

```
[Out] [-1/6*(6*(b*x^3 + a)^(2/3)*b^2*x^4 - 3*sqrt(1/3)*(b^3*x^6 + 2*a*b^2*x^3 + a
^2*b)*sqrt((-b)^(1/3)/b)*log(3*b*x^3 - 3*(b*x^3 + a)^(1/3)*(-b)^(2/3)*x^2 -
 3*sqrt(1/3)*((-b)^(1/3)*b*x^3 - (b*x^3 + a)^(1/3)*b*x^2 + 2*(b*x^3 + a)^(2
/3)*(-b)^(2/3)*x)*sqrt((-b)^(1/3)/b) + 2*a) + 2*(b^2*x^6 + 2*a*b*x^3 + a^2)
*(-b)^(2/3)*log(((-b)^(1/3)*x + (b*x^3 + a)^(1/3))/x) - (b^2*x^6 + 2*a*b*x^
```

```
3 + a^2)*(-b)^(2/3)*log(((-b)^(2/3)*x^2 - (b*x^3 + a)^(1/3)*(-b)^(1/3)*x +
(b*x^3 + a)^(2/3))/x^2))/(b^3*x^6 + 2*a*b^2*x^3 + a^2*b), -1/6*(6*(b*x^3 +
a)^(2/3)*b^2*x^4 + 6*sqrt(1/3)*(b^3*x^6 + 2*a*b^2*x^3 + a^2*b)*sqrt(-(-b)^(
1/3)/b)*arctan(-sqrt(1/3)*((-b)^(1/3)*x - 2*(b*x^3 + a)^(1/3))*sqrt(-(-b)^(
1/3)/b)/x) + 2*(b^2*x^6 + 2*a*b*x^3 + a^2)*(-b)^(2/3)*log(((-b)^(1/3)*x + (
b*x^3 + a)^(1/3))/x) - (b^2*x^6 + 2*a*b*x^3 + a^2)*(-b)^(2/3)*log(((-b)^(2/
3)*x^2 - (b*x^3 + a)^(1/3)*(-b)^(1/3)*x + (b*x^3 + a)^(2/3))/x^2))/(b^3*x^6
 + 2*a*b^2*x^3 + a^2*b)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^3 - a\right)^2}{\left(bx^3 + a\right)^{\frac{7}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^3+a)^2/(b*x^3+a)^(7/3),x, algorithm="giac")
```

```
[Out] integrate((b*x^3 - a)^2/(b*x^3 + a)^(7/3), x)
```

maple [F] time = 0.53, size = 0, normalized size = 0.00

$$\int \frac{\left(-b\,x^3 + a\right)^2}{\left(b\,x^3 + a\right)^{\frac{7}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x^3+a)^2/(b*x^3+a)^(7/3),x)
```

```
[Out] int((-b*x^3+a)^2/(b*x^3+a)^(7/3),x)
```

maxima [B] time = 1.35, size = 180, normalized size = 1.64

$$-\frac{\left(b - \frac{4\left(bx^3+a\right)}{x^3}\right)x^4}{4\left(bx^3+a\right)^{\frac{4}{3}}} - \frac{bx^4}{2\left(bx^3+a\right)^{\frac{4}{3}}} - \frac{1}{12}\left(\frac{3\left(b + \frac{4\left(bx^3+a\right)}{x^3}\right)x^4}{\left(bx^3+a\right)^{\frac{4}{3}}b^2} + \frac{4\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}} + \frac{2\left(bx^3+a\right)^{\frac{1}{3}}}{x}\right)}{3b^{\frac{1}{3}}}\right)}{b^{\frac{7}{3}}} - \frac{2\log\left(b^{\frac{2}{3}} + \frac{\left(bx^3+a\right)^{\frac{1}{3}}b^{\frac{1}{3}}}{x} + \frac{\left(bx^3+a\right)^{\frac{2}{3}}}{x^2}\right)}{b^{\frac{7}{3}}} + \frac{4\log\left(-b^{\frac{1}{3}} + \frac{\left(bx^3+a\right)^{\frac{1}{3}}}{x}\right)}{b^{\frac{7}{3}}}\right)b^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^3+a)^2/(b*x^3+a)^(7/3),x, algorithm="maxima")
```

```
[Out] -1/4*(b - 4*(b*x^3 + a)/x^3)*x^4/(b*x^3 + a)^(4/3) - 1/2*b*x^4/(b*x^3 + a)^
(4/3) - 1/12*(3*(b + 4*(b*x^3 + a)/x^3)*x^4/((b*x^3 + a)^(4/3)*b^2) + 4*sqr
t(3)*arctan(1/3*sqrt(3)*(b^(1/3) + 2*(b*x^3 + a)^(1/3)/x)/b^(1/3))/b^(7/3)
- 2*log(b^(2/3) + (b*x^3 + a)^(1/3)*b^(1/3)/x + (b*x^3 + a)^(2/3)/x^2)/b^(7
/3) + 4*log(-b^(1/3) + (b*x^3 + a)^(1/3)/x)/b^(7/3))*b^2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(a - b\,x^3\right)}^2}{{\left(b\,x^3 + a\right)}^{7/3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - b*x^3)^2/(a + b*x^3)^(7/3),x)
```

```
[Out] int((a - b*x^3)^2/(a + b*x^3)^(7/3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-a + bx^3\right)^2}{\left(a + bx^3\right)^{\frac{7}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x**3+a)**2/(b*x**3+a)**(7/3),x)
```

```
[Out] Integral((-a + b*x**3)**2/(a + b*x**3)**(7/3), x)
```

3.39 $\int \frac{(a-bx^3)^2}{(a+bx^3)^{10/3}}\,dx$

Optimal. Leaf size=76

$$\frac{x(a-bx^3)^2}{7a(a+bx^3)^{7/3}}+\frac{3x(a-bx^3)}{14a(a+bx^3)^{4/3}}+\frac{9x}{14a\sqrt[3]{a+bx^3}}$$

Rubi [A] time = 0.02, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {378, 191}

$$\frac{x(a-bx^3)^2}{7a(a+bx^3)^{7/3}}+\frac{3x(a-bx^3)}{14a(a+bx^3)^{4/3}}+\frac{9x}{14a\sqrt[3]{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*x^3)^2/(a + b*x^3)^(10/3),x]
```

```
[Out] (x*(a - b*x^3)^2)/(7*a*(a + b*x^3)^(7/3)) + (3*x*(a - b*x^3))/(14*a*(a + b*
x^3)^(4/3)) + (9*x)/(14*a*(a + b*x^3)^(1/3))
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 378

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> -Simp[(x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*n*(p + 1)), x] - Dist[(c
*q)/(a*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1, 0]
 && GtQ[q, 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{(a-bx^3)^2}{(a+bx^3)^{10/3}}\,dx &= \frac{x(a-bx^3)^2}{7a(a+bx^3)^{7/3}}+\frac{6}{7}\int \frac{a-bx^3}{(a+bx^3)^{7/3}}\,dx\\ &= \frac{x(a-bx^3)^2}{7a(a+bx^3)^{7/3}}+\frac{3x(a-bx^3)}{14a(a+bx^3)^{4/3}}+\frac{9}{14}\int \frac{1}{(a+bx^3)^{4/3}}\,dx\\ &= \frac{x(a-bx^3)^2}{7a(a+bx^3)^{7/3}}+\frac{3x(a-bx^3)}{14a(a+bx^3)^{4/3}}+\frac{9x}{14a\sqrt[3]{a+bx^3}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 40, normalized size = 0.53

$$\frac{x(7a^2+7abx^3+4b^2x^6)}{7a(a+bx^3)^{7/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b*x^3)^2/(a + b*x^3)^(10/3),x]
[Out] (x*(7*a^2 + 7*a*b*x^3 + 4*b^2*x^6))/(7*a*(a + b*x^3)^(7/3))
```

IntegrateAlgebraic [A] time = 0.35, size = 40, normalized size = 0.53

$$\frac{x\left(7a^2+7abx^3+4b^2x^6\right)}{7a\left(a+bx^3\right)^{7/3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a - b*x^3)^2/(a + b*x^3)^(10/3),x]
[Out] (x*(7*a^2 + 7*a*b*x^3 + 4*b^2*x^6))/(7*a*(a + b*x^3)^(7/3))
```

fricas [A] time = 0.89, size = 67, normalized size = 0.88

$$\frac{\left(4\,b^2x^7+7\,abx^4+7\,a^2x\right)\left(bx^3+a\right)^{\frac{2}{3}}}{7\left(ab^3x^9+3\,a^2b^2x^6+3\,a^3bx^3+a^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^3+a)^2/(b*x^3+a)^(10/3),x, algorithm="fricas")
[Out] 1/7*(4*b^2*x^7 + 7*a*b*x^4 + 7*a^2*x)*(b*x^3 + a)^(2/3)/(a*b^3*x^9 + 3*a^2*
b^2*x^6 + 3*a^3*b*x^3 + a^4)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^3-a\right)^2}{\left(bx^3+a\right)^{\frac{10}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^3+a)^2/(b*x^3+a)^(10/3),x, algorithm="giac")
[Out] integrate((b*x^3 - a)^2/(b*x^3 + a)^(10/3), x)
```

maple [A] time = 0.04, size = 37, normalized size = 0.49

$$\frac{\left(4b^2x^6+7ab\,x^3+7a^2\right)x}{7\left(b\,x^3+a\right)^{\frac{7}{3}}a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x^3+a)^2/(b*x^3+a)^(10/3),x)
[Out] 1/7*x*(4*b^2*x^6+7*a*b*x^3+7*a^2)/(b*x^3+a)^(7/3)/a
```

maxima [A] time = 0.50, size = 105, normalized size = 1.38

$$\frac{\left(4\,b-\frac{7\left(bx^3+a\right)}{x^3}\right)bx^7}{14\left(bx^3+a\right)^{\frac{7}{3}}a}+\frac{b^2x^7}{7\left(bx^3+a\right)^{\frac{7}{3}}a}+\frac{\left(2\,b^2-\frac{7\left(bx^3+a\right)b}{x^3}+\frac{14\left(bx^3+a\right)^2}{x^6}\right)x^7}{14\left(bx^3+a\right)^{\frac{7}{3}}a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^3+a)^2/(b*x^3+a)^(10/3),x, algorithm="maxima")
```

```
[Out] 1/14*(4*b - 7*(b*x^3 + a)/x^3)*b*x^7/((b*x^3 + a)^(7/3)*a) + 1/7*b^2*x^7/((
b*x^3 + a)^(7/3)*a) + 1/14*(2*b^2 - 7*(b*x^3 + a)*b/x^3 + 14*(b*x^3 + a)^2/
x^6)*x^7/((b*x^3 + a)^(7/3)*a)
```

mupad [B] time = 1.43, size = 44, normalized size = 0.58

$$\frac{4\,x\,\left(b\,x^3+a\right)^2+4\,a^2\,x-a\,x\,\left(b\,x^3+a\right)}{7\,a\,\left(b\,x^3+a\right)^{7/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - b*x^3)^2/(a + b*x^3)^(10/3),x)
[Out] (4*x*(a + b*x^3)^2 + 4*a^2*x - a*x*(a + b*x^3))/(7*a*(a + b*x^3)^(7/3))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x**3+a)**2/(b*x**3+a)**(10/3),x)
[Out] Timed out
```

3.40 $\int \frac{\left(a-bx^3\right)^2}{\left(a+bx^3\right)^{13/3}}\,dx$

Optimal. Leaf size=105

$$\frac{x\left(a-bx^3\right)^3}{20a^2\left(a+bx^3\right)^{10/3}}+\frac{19x\left(a-bx^3\right)^2}{140a^2\left(a+bx^3\right)^{7/3}}+\frac{57x\left(a-bx^3\right)}{280a^2\left(a+bx^3\right)^{4/3}}+\frac{171x}{280a^2\sqrt[3]{a+bx^3}}$$

Rubi [A] time = 0.04, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {382, 378, 191}

$$\frac{x\left(a-bx^3\right)^3}{20a^2\left(a+bx^3\right)^{10/3}}+\frac{19x\left(a-bx^3\right)^2}{140a^2\left(a+bx^3\right)^{7/3}}+\frac{57x\left(a-bx^3\right)}{280a^2\left(a+bx^3\right)^{4/3}}+\frac{171x}{280a^2\sqrt[3]{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*x^3)^2/(a + b*x^3)^(13/3),x]
```

```
[Out] (x*(a - b*x^3)^3)/(20*a^2*(a + b*x^3)^(10/3)) + (19*x*(a - b*x^3)^2)/(140*a
^2*(a + b*x^3)^(7/3)) + (57*x*(a - b*x^3))/(280*a^2*(a + b*x^3)^(4/3)) + (1
71*x)/(280*a^2*(a + b*x^3)^(1/3))
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 378

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> -Simp[(x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*n*(p + 1)), x] - Dist[(c
*q)/(a*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1, 0]
 && GtQ[q, 0] && NeQ[p, -1]
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)), I
nt[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q}, x
] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !LtQ
[q, -1]) && NeQ[p, -1]
```

Rubi steps

$$\int \frac{(a-bx^3)^2}{(a+bx^3)^{13/3}}\,dx = \frac{x(a-bx^3)^3}{20a^2(a+bx^3)^{10/3}} + \frac{19\int \frac{(a-bx^3)^2}{(a+bx^3)^{10/3}}\,dx}{20a}$$

$$= \frac{x(a-bx^3)^3}{20a^2(a+bx^3)^{10/3}} + \frac{19x(a-bx^3)^2}{140a^2(a+bx^3)^{7/3}} + \frac{57\int \frac{a-bx^3}{(a+bx^3)^{7/3}}\,dx}{70a}$$

$$= \frac{x(a-bx^3)^3}{20a^2(a+bx^3)^{10/3}} + \frac{19x(a-bx^3)^2}{140a^2(a+bx^3)^{7/3}} + \frac{57x(a-bx^3)}{280a^2(a+bx^3)^{4/3}} + \frac{171\int \frac{1}{(a+bx^3)^{4/3}}\,dx}{280a}$$

$$= \frac{x(a-bx^3)^3}{20a^2(a+bx^3)^{10/3}} + \frac{19x(a-bx^3)^2}{140a^2(a+bx^3)^{7/3}} + \frac{57x(a-bx^3)}{280a^2(a+bx^3)^{4/3}} + \frac{171x}{280a^2\sqrt[3]{a+bx^3}}$$

Mathematica [A] time = 0.03, size = 51, normalized size = 0.49

$$\frac{x(140a^3+245a^2bx^3+230ab^2x^6+69b^3x^9)}{140a^2(a+bx^3)^{10/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b*x^3)^2/(a + b*x^3)^(13/3),x]
```

```
[Out] (x*(140*a^3 + 245*a^2*b*x^3 + 230*a*b^2*x^6 + 69*b^3*x^9))/(140*a^2*(a + b*
x^3)^(10/3))
```

IntegrateAlgebraic [A] time = 0.53, size = 51, normalized size = 0.49

$$\frac{x(140a^3+245a^2bx^3+230ab^2x^6+69b^3x^9)}{140a^2(a+bx^3)^{10/3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a - b*x^3)^2/(a + b*x^3)^(13/3),x]
```

```
[Out] (x*(140*a^3 + 245*a^2*b*x^3 + 230*a*b^2*x^6 + 69*b^3*x^9))/(140*a^2*(a + b*
x^3)^(10/3))
```

fricas [A] time = 1.08, size = 91, normalized size = 0.87

$$\frac{(69\,b^3x^{10}+230\,ab^2x^7+245\,a^2bx^4+140\,a^3x)(bx^3+a)^{\frac{2}{3}}}{140\,(a^2b^4x^{12}+4\,a^3b^3x^9+6\,a^4b^2x^6+4\,a^5bx^3+a^6)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^3+a)^2/(b*x^3+a)^(13/3),x, algorithm="fricas")
```

```
[Out] 1/140*(69*b^3*x^10 + 230*a*b^2*x^7 + 245*a^2*b*x^4 + 140*a^3*x)*(b*x^3 + a)
^(2/3)/(a^2*b^4*x^12 + 4*a^3*b^3*x^9 + 6*a^4*b^2*x^6 + 4*a^5*b*x^3 + a^6)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx^3-a)^2}{(bx^3+a)^{\frac{13}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^3+a)^2/(b*x^3+a)^(13/3),x, algorithm="giac")
```

```
[Out] integrate((b*x^3 - a)^2/(b*x^3 + a)^(13/3), x)
```

maple [A] time = 0.05, size = 48, normalized size = 0.46

$$\frac{\left(69b^{3}x^{9}+230a\,b^{2}x^{6}+245a^{2}b\,x^{3}+140a^{3}\right)x}{140\left(b\,x^{3}+a\right)^{\frac{10}{3}}a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x^3+a)^2/(b*x^3+a)^(13/3),x)
```

```
[Out] 1/140*x*(69*b^3*x^9+230*a*b^2*x^6+245*a^2*b*x^3+140*a^3)/(b*x^3+a)^(10/3)/a
^2
```

maxima [A] time = 0.59, size = 155, normalized size = 1.48

$$-\frac{\left(7\,b-\frac{10\left(bx^{3}+a\right)}{x^{3}}\right)b^{2}x^{10}}{70\left(bx^{3}+a\right)^{\frac{10}{3}}a^{2}}-\frac{\left(14\,b^{2}-\frac{40\left(bx^{3}+a\right)b}{x^{3}}+\frac{35\left(bx^{3}+a\right)^{2}}{x^{6}}\right)bx^{10}}{70\left(bx^{3}+a\right)^{\frac{10}{3}}a^{2}}-\frac{\left(14\,b^{3}-\frac{60\left(bx^{3}+a\right)b^{2}}{x^{3}}+\frac{105\left(bx^{3}+a\right)^{2}b}{x^{6}}-\frac{140\left(bx^{3}+a\right)^{3}}{x^{9}}\right)x^{10}}{140\left(bx^{3}+a\right)^{\frac{10}{3}}a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^3+a)^2/(b*x^3+a)^(13/3),x, algorithm="maxima")
```

```
[Out] -1/70*(7*b - 10*(b*x^3 + a)/x^3)*b^2*x^10/((b*x^3 + a)^(10/3)*a^2) - 1/70*(
14*b^2 - 40*(b*x^3 + a)*b/x^3 + 35*(b*x^3 + a)^2/x^6)*b*x^10/((b*x^3 + a)^(
10/3)*a^2) - 1/140*(14*b^3 - 60*(b*x^3 + a)*b^2/x^3 + 105*(b*x^3 + a)^2*b/x
^6 - 140*(b*x^3 + a)^3/x^9)*x^10/((b*x^3 + a)^(10/3)*a^2)
```

mupad [B] time = 1.39, size = 56, normalized size = 0.53

$$\frac{69\,x}{140\,a^{2}\left(b\,x^{3}+a\right)^{1/3}}-\frac{2\,x}{35\left(b\,x^{3}+a\right)^{7/3}}+\frac{23\,x}{140\,a\left(b\,x^{3}+a\right)^{4/3}}+\frac{2\,a\,x}{5\left(b\,x^{3}+a\right)^{10/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - b*x^3)^2/(a + b*x^3)^(13/3),x)
```

```
[Out] (69*x)/(140*a^2*(a + b*x^3)^(1/3)) - (2*x)/(35*(a + b*x^3)^(7/3)) + (23*x)/
(140*a*(a + b*x^3)^(4/3)) + (2*a*x)/(5*(a + b*x^3)^(10/3))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x**3+a)**2/(b*x**3+a)**(13/3),x)
```

```
[Out] Timed out
```

3.41 $\int \frac{\left(a-bx^3\right)^2}{\left(a+bx^3\right)^{16/3}}\,dx$

Optimal. Leaf size=98

$$\frac{423x}{910a^3\sqrt[3]{a+bx^3}}+\frac{141x}{910a^2\left(a+bx^3\right)^{4/3}}+\frac{47x}{455a\left(a+bx^3\right)^{7/3}}+\frac{8x}{65\left(a+bx^3\right)^{10/3}}+\frac{2x\left(a-bx^3\right)}{13\left(a+bx^3\right)^{13/3}}$$

Rubi [A] time = 0.04, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {413, 385, 192, 191}

$$\frac{423x}{910a^3\sqrt[3]{a+bx^3}}+\frac{141x}{910a^2\left(a+bx^3\right)^{4/3}}+\frac{47x}{455a\left(a+bx^3\right)^{7/3}}+\frac{8x}{65\left(a+bx^3\right)^{10/3}}+\frac{2x\left(a-bx^3\right)}{13\left(a+bx^3\right)^{13/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*x^3)^2/(a + b*x^3)^(16/3),x]

[Out] (2*x*(a - b*x^3))/(13*(a + b*x^3)^(13/3)) + (8*x)/(65*(a + b*x^3)^(10/3)) +
 (47*x)/(455*a*(a + b*x^3)^(7/3)) + (141*x)/(910*a^2*(a + b*x^3)^(4/3)) + (
423*x)/(910*a^3*(a + b*x^3)^(1/3))
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 413

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[((a*d - c*b)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*n*(p +
 1)), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rubi steps

$$\int \frac{\left(a-bx^3\right)^2}{\left(a+bx^3\right)^{16/3}}\,dx = \frac{2x\left(a-bx^3\right)}{13\left(a+bx^3\right)^{13/3}} + \frac{\int \frac{11a^2b-5ab^2x^3}{\left(a+bx^3\right)^{13/3}}\,dx}{13ab}$$

$$= \frac{2x\left(a-bx^3\right)}{13\left(a+bx^3\right)^{13/3}} + \frac{8x}{65\left(a+bx^3\right)^{10/3}} + \frac{47}{65}\int \frac{1}{\left(a+bx^3\right)^{10/3}}\,dx$$

$$= \frac{2x\left(a-bx^3\right)}{13\left(a+bx^3\right)^{13/3}} + \frac{8x}{65\left(a+bx^3\right)^{10/3}} + \frac{47x}{455a\left(a+bx^3\right)^{7/3}} + \frac{282\int \frac{1}{\left(a+bx^3\right)^{7/3}}\,dx}{455a}$$

$$= \frac{2x\left(a-bx^3\right)}{13\left(a+bx^3\right)^{13/3}} + \frac{8x}{65\left(a+bx^3\right)^{10/3}} + \frac{47x}{455a\left(a+bx^3\right)^{7/3}} + \frac{141x}{910a^2\left(a+bx^3\right)^{4/3}} + \frac{423\int \frac{}{(a}}{91}$$

$$= \frac{2x\left(a-bx^3\right)}{13\left(a+bx^3\right)^{13/3}} + \frac{8x}{65\left(a+bx^3\right)^{10/3}} + \frac{47x}{455a\left(a+bx^3\right)^{7/3}} + \frac{141x}{910a^2\left(a+bx^3\right)^{4/3}} + \frac{42}{910a^3\sqrt[3]{}}$$

Mathematica [A] time = 0.04, size = 62, normalized size = 0.63

$$\frac{x\left(910a^4+2275a^3bx^3+3055a^2b^2x^6+1833ab^3x^9+423b^4x^{12}\right)}{910a^3\left(a+bx^3\right)^{13/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b*x^3)^2/(a + b*x^3)^(16/3),x]
```

```
[Out] (x*(910*a^4 + 2275*a^3*b*x^3 + 3055*a^2*b^2*x^6 + 1833*a*b^3*x^9 + 423*b^4*
x^12))/(910*a^3*(a + b*x^3)^(13/3))
```

IntegrateAlgebraic [A] time = 0.76, size = 62, normalized size = 0.63

$$\frac{x\left(910a^4+2275a^3bx^3+3055a^2b^2x^6+1833ab^3x^9+423b^4x^{12}\right)}{910a^3\left(a+bx^3\right)^{13/3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a - b*x^3)^2/(a + b*x^3)^(16/3),x]
```

```
[Out] (x*(910*a^4 + 2275*a^3*b*x^3 + 3055*a^2*b^2*x^6 + 1833*a*b^3*x^9 + 423*b^4*
x^12))/(910*a^3*(a + b*x^3)^(13/3))
```

fricas [A] time = 1.25, size = 113, normalized size = 1.15

$$\frac{\left(423\,b^4x^{13}+1833\,ab^3x^{10}+3055\,a^2b^2x^7+2275\,a^3bx^4+910\,a^4x\right)\left(bx^3+a\right)^{\frac{2}{3}}}{910\left(a^3b^5x^{15}+5\,a^4b^4x^{12}+10\,a^5b^3x^9+10\,a^6b^2x^6+5\,a^7bx^3+a^8\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^3+a)^2/(b*x^3+a)^(16/3),x, algorithm="fricas")
```

```
[Out] 1/910*(423*b^4*x^13 + 1833*a*b^3*x^10 + 3055*a^2*b^2*x^7 + 2275*a^3*b*x^4 +
 910*a^4*x)*(b*x^3 + a)^(2/3)/(a^3*b^5*x^15 + 5*a^4*b^4*x^12 + 10*a^5*b^3*x
^9 + 10*a^6*b^2*x^6 + 5*a^7*b*x^3 + a^8)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^3-a\right)^2}{\left(bx^3+a\right)^{\frac{16}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^3+a)^2/(b*x^3+a)^(16/3),x, algorithm="giac")
```

```
[Out] integrate((b*x^3 - a)^2/(b*x^3 + a)^(16/3), x)
```

maple [A] time = 0.05, size = 59, normalized size = 0.60

$$\frac{\left(423b^4x^{12}+1833b^3x^9a+3055b^2x^6a^2+2275b\,x^3a^3+910a^4\right)x}{910\left(b\,x^3+a\right)^{\frac{13}{3}}\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x^3+a)^2/(b*x^3+a)^(16/3),x)
```

```
[Out] 1/910*x*(423*b^4*x^12+1833*a*b^3*x^9+3055*a^2*b^2*x^6+2275*a^3*b*x^3+910*a^
4)/(b*x^3+a)^(13/3)/a^3
```

maxima [B] time = 0.57, size = 206, normalized size = 2.10

$$\frac{\left(35\,b^2-\frac{91\,(bx^3+a)b}{x^3}+\frac{65\,(bx^3+a)^2}{x^6}\right)b^2x^{13}}{455\left(bx^3+a\right)^{\frac{13}{3}}a^3}+\frac{\left(140\,b^3-\frac{546\,(bx^3+a)b^2}{x^3}+\frac{780\,(bx^3+a)^2b}{x^6}-\frac{455\,(bx^3+a)^3}{x^9}\right)bx^{13}}{910\left(bx^3+a\right)^{\frac{13}{3}}a^3}+\frac{\left(35\,b^4-\frac{182\,(bx^3+a)b^3}{x^3}+\frac{390\,(bx^3+a)^2b^2}{x^6}-\frac{455\,(bx^3+a)^3b}{x^9}+\frac{455\,(bx^3+a)^4}{x^{12}}\right)x^{13}}{455\left(bx^3+a\right)^{\frac{13}{3}}a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^3+a)^2/(b*x^3+a)^(16/3),x, algorithm="maxima")
```

```
[Out] 1/455*(35*b^2 - 91*(b*x^3 + a)*b/x^3 + 65*(b*x^3 + a)^2/x^6)*b^2*x^13/((b*x
^3 + a)^(13/3)*a^3) + 1/910*(140*b^3 - 546*(b*x^3 + a)*b^2/x^3 + 780*(b*x^3
 + a)^2*b/x^6 - 455*(b*x^3 + a)^3/x^9)*b*x^13/((b*x^3 + a)^(13/3)*a^3) + 1/
455*(35*b^4 - 182*(b*x^3 + a)*b^3/x^3 + 390*(b*x^3 + a)^2*b^2/x^6 - 455*(b*
x^3 + a)^3*b/x^9 + 455*(b*x^3 + a)^4/x^12)*x^13/((b*x^3 + a)^(13/3)*a^3)
```

mupad [B] time = 1.44, size = 71, normalized size = 0.72

$$\frac{423\,x}{910\,a^3\left(b\,x^3+a\right)^{1/3}}-\frac{2\,x}{65\left(b\,x^3+a\right)^{10/3}}+\frac{141\,x}{910\,a^2\left(b\,x^3+a\right)^{4/3}}+\frac{47\,x}{455\,a\left(b\,x^3+a\right)^{7/3}}+\frac{4\,a\,x}{13\left(b\,x^3+a\right)^{13/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - b*x^3)^2/(a + b*x^3)^(16/3),x)
```

```
[Out] (423*x)/(910*a^3*(a + b*x^3)^(1/3)) - (2*x)/(65*(a + b*x^3)^(10/3)) + (141*
x)/(910*a^2*(a + b*x^3)^(4/3)) + (47*x)/(455*a*(a + b*x^3)^(7/3)) + (4*a*x)
/(13*(a + b*x^3)^(13/3))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x**3+a)**2/(b*x**3+a)**(16/3),x)
```

```
[Out] Timed out
```

3.42 $\int \frac{\left(a-bx^3\right)^2}{\left(a+bx^3\right)^{19/3}}\,dx$

Optimal. Leaf size=117

$$\frac{81x}{182a^4\sqrt[3]{a+bx^3}}+\frac{27x}{182a^3\left(a+bx^3\right)^{4/3}}+\frac{9x}{91a^2\left(a+bx^3\right)^{7/3}}+\frac{x}{13a\left(a+bx^3\right)^{10/3}}+\frac{11x}{104\left(a+bx^3\right)^{13/3}}+\frac{x\left(a-bx^3\right)}{8\left(a+bx^3\right)^{16/3}}$$

Rubi [A] time = 0.04, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {413, 385, 192, 191}

$$\frac{81x}{182a^4\sqrt[3]{a+bx^3}}+\frac{27x}{182a^3\left(a+bx^3\right)^{4/3}}+\frac{9x}{91a^2\left(a+bx^3\right)^{7/3}}+\frac{x}{13a\left(a+bx^3\right)^{10/3}}+\frac{11x}{104\left(a+bx^3\right)^{13/3}}+\frac{x\left(a-bx^3\right)}{8\left(a+bx^3\right)^{16/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*x^3)^2/(a + b*x^3)^(19/3),x]
```

```
[Out] (x*(a - b*x^3))/(8*(a + b*x^3)^(16/3)) + (11*x)/(104*(a + b*x^3)^(13/3)) +
x/(13*a*(a + b*x^3)^(10/3)) + (9*x)/(91*a^2*(a + b*x^3)^(7/3)) + (27*x)/(18
2*a^3*(a + b*x^3)^(4/3)) + (81*x)/(182*a^4*(a + b*x^3)^(1/3))
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 413

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[((a*d - c*b)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*n*(p +
 1)), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rubi steps

$$\int \frac{(a-bx^3)^2}{(a+bx^3)^{19/3}}\,dx = \frac{x(a-bx^3)}{8(a+bx^3)^{16/3}} + \frac{\int \frac{14a^2b-8ab^2x^3}{(a+bx^3)^{16/3}}\,dx}{16ab}$$

$$= \frac{x(a-bx^3)}{8(a+bx^3)^{16/3}} + \frac{11x}{104(a+bx^3)^{13/3}} + \frac{10}{13}\int \frac{1}{(a+bx^3)^{13/3}}\,dx$$

$$= \frac{x(a-bx^3)}{8(a+bx^3)^{16/3}} + \frac{11x}{104(a+bx^3)^{13/3}} + \frac{x}{13a(a+bx^3)^{10/3}} + \frac{9\int \frac{1}{(a+bx^3)^{10/3}}\,dx}{13a}$$

$$= \frac{x(a-bx^3)}{8(a+bx^3)^{16/3}} + \frac{11x}{104(a+bx^3)^{13/3}} + \frac{x}{13a(a+bx^3)^{10/3}} + \frac{9x}{91a^2(a+bx^3)^{7/3}} + \frac{54\int \frac{1}{(a+bx^3)^{7/}}}{91a^2}$$

$$= \frac{x(a-bx^3)}{8(a+bx^3)^{16/3}} + \frac{11x}{104(a+bx^3)^{13/3}} + \frac{x}{13a(a+bx^3)^{10/3}} + \frac{9x}{91a^2(a+bx^3)^{7/3}} + \frac{27x}{182a^3(a+b}$$

$$= \frac{x(a-bx^3)}{8(a+bx^3)^{16/3}} + \frac{11x}{104(a+bx^3)^{13/3}} + \frac{x}{13a(a+bx^3)^{10/3}} + \frac{9x}{91a^2(a+bx^3)^{7/3}} + \frac{27x}{182a^3(a+b}$$

Mathematica [A] time = 0.04, size = 73, normalized size = 0.62

$$\frac{x(364a^5+1183a^4bx^3+2080a^3b^2x^6+1872a^2b^3x^9+864ab^4x^{12}+162b^5x^{15})}{364a^4(a+bx^3)^{16/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b*x^3)^2/(a + b*x^3)^(19/3),x]
```

```
[Out] (x*(364*a^5 + 1183*a^4*b*x^3 + 2080*a^3*b^2*x^6 + 1872*a^2*b^3*x^9 + 864*a*
b^4*x^12 + 162*b^5*x^15))/(364*a^4*(a + b*x^3)^(16/3))
```

IntegrateAlgebraic [A] time = 1.19, size = 73, normalized size = 0.62

$$\frac{x(364a^5+1183a^4bx^3+2080a^3b^2x^6+1872a^2b^3x^9+864ab^4x^{12}+162b^5x^{15})}{364a^4(a+bx^3)^{16/3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a - b*x^3)^2/(a + b*x^3)^(19/3),x]
```

```
[Out] (x*(364*a^5 + 1183*a^4*b*x^3 + 2080*a^3*b^2*x^6 + 1872*a^2*b^3*x^9 + 864*a*
b^4*x^12 + 162*b^5*x^15))/(364*a^4*(a + b*x^3)^(16/3))
```

fricas [A] time = 0.80, size = 135, normalized size = 1.15

$$\frac{\left(162\,b^5x^{16}+864\,ab^4x^{13}+1872\,a^2b^3x^{10}+2080\,a^3b^2x^7+1183\,a^4bx^4+364\,a^5x\right)\left(bx^3+a\right)^{\frac{2}{3}}}{364\left(a^4b^6x^{18}+6\,a^5b^5x^{15}+15\,a^6b^4x^{12}+20\,a^7b^3x^9+15\,a^8b^2x^6+6\,a^9bx^3+a^{10}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^3+a)^2/(b*x^3+a)^(19/3),x, algorithm="fricas")
```

```
[Out] 1/364*(162*b^5*x^16 + 864*a*b^4*x^13 + 1872*a^2*b^3*x^10 + 2080*a^3*b^2*x^7
 + 1183*a^4*b*x^4 + 364*a^5*x)*(b*x^3 + a)^(2/3)/(a^4*b^6*x^18 + 6*a^5*b^5*
x^15 + 15*a^6*b^4*x^12 + 20*a^7*b^3*x^9 + 15*a^8*b^2*x^6 + 6*a^9*b*x^3 + a^
10)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^3-a\right)^2}{\left(bx^3+a\right)^{\frac{19}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^3+a)^2/(b*x^3+a)^(19/3),x, algorithm="giac")
```

```
[Out] integrate((b*x^3 - a)^2/(b*x^3 + a)^(19/3), x)
```

maple [A] time = 0.05, size = 70, normalized size = 0.60

$$\frac{\left(162b^5x^{15}+864a\,b^4x^{12}+1872a^2b^3x^9+2080a^3b^2x^6+1183a^4b\,x^3+364a^5\right)x}{364\left(b\,x^3+a\right)^{\frac{16}{3}}a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x^3+a)^2/(b*x^3+a)^(19/3),x)
```

```
[Out] 1/364*x*(162*b^5*x^15+864*a*b^4*x^12+1872*a^2*b^3*x^9+2080*a^3*b^2*x^6+1183
*a^4*b*x^3+364*a^5)/(b*x^3+a)^(16/3)/a^4
```

maxima [B] time = 0.51, size = 257, normalized size = 2.20

$$-\frac{\left(455\,b^3-\frac{1680\,(bx^3+a)b^2}{x^3}+\frac{2184\,(bx^3+a)^2b}{x^6}-\frac{1040\,(bx^3+a)^3}{x^9}\right)b^2x^{16}}{7280\,(bx^3+a)^{\frac{16}{3}}a^4}-\frac{\left(455\,b^4-\frac{2240\,(bx^3+a)b^3}{x^3}+\frac{4368\,(bx^3+a)^2b^2}{x^6}-\frac{4160\,(bx^3+a)^3b}{x^9}+\frac{1820\,(bx^3+a)^4}{x^{12}}\right)bx^{16}}{3640\,(bx^3+a)^{\frac{16}{3}}a^4}-\frac{\left(91\,b^5-\frac{560\,(bx^3+a)b^4}{x^3}+\frac{1456\,(bx^3+a)^2b^3}{x^6}-\frac{2080\,(bx^3+a)^3b^2}{x^9}+\frac{1820\,(bx^3+a)^4b}{x^{12}}-\frac{1456\,(bx^3+a)^5}{x^{15}}\right)x^{16}}{1456\,(bx^3+a)^{\frac{16}{3}}a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^3+a)^2/(b*x^3+a)^(19/3),x, algorithm="maxima")
```

```
[Out] -1/7280*(455*b^3 - 1680*(b*x^3 + a)*b^2/x^3 + 2184*(b*x^3 + a)^2*b/x^6 - 10
40*(b*x^3 + a)^3/x^9)*b^2*x^16/((b*x^3 + a)^(16/3)*a^4) - 1/3640*(455*b^4 -
 2240*(b*x^3 + a)*b^3/x^3 + 4368*(b*x^3 + a)^2*b^2/x^6 - 4160*(b*x^3 + a)^3
*b/x^9 + 1820*(b*x^3 + a)^4/x^12)*b*x^16/((b*x^3 + a)^(16/3)*a^4) - 1/1456*
(91*b^5 - 560*(b*x^3 + a)*b^4/x^3 + 1456*(b*x^3 + a)^2*b^3/x^6 - 2080*(b*x^
3 + a)^3*b^2/x^9 + 1820*(b*x^3 + a)^4*b/x^12 - 1456*(b*x^3 + a)^5/x^15)*x^1
6/((b*x^3 + a)^(16/3)*a^4)
```

mupad [B] time = 1.43, size = 86, normalized size = 0.74

$$\frac{81\,x}{182\,a^4\left(b\,x^3+a\right)^{1/3}}-\frac{x}{52\left(b\,x^3+a\right)^{13/3}}+\frac{27\,x}{182\,a^3\left(b\,x^3+a\right)^{4/3}}+\frac{9\,x}{91\,a^2\left(b\,x^3+a\right)^{7/3}}+\frac{x}{13\,a\left(b\,x^3+a\right)^{10/3}}+\frac{a\,x}{4\left(b\,x^3+a\right)^{16/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - b*x^3)^2/(a + b*x^3)^(19/3),x)
```

```
[Out] (81*x)/(182*a^4*(a + b*x^3)^(1/3)) - x/(52*(a + b*x^3)^(13/3)) + (27*x)/(18
2*a^3*(a + b*x^3)^(4/3)) + (9*x)/(91*a^2*(a + b*x^3)^(7/3)) + x/(13*a*(a +
b*x^3)^(10/3)) + (a*x)/(4*(a + b*x^3)^(16/3))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x**3+a)**2/(b*x**3+a)**(19/3),x)
```

```
[Out] Timed out
```

3.43 $\int \left(a+bx^3\right)^{5/3} \left(c+dx^3\right) dx$

Optimal. Leaf size=174

$$-\frac{5a^2(9bc-ad)\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}\,x\right)}{162b^{4/3}}+\frac{5a^2(9bc-ad)\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{81\sqrt{3}\,b^{4/3}}+\frac{x\left(a+bx^3\right)^{5/3}(9bc-ad)}{54b}+\frac{5ax\left(a+\right.}{}$$

Rubi [A] time = 0.06, antiderivative size = 174, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {388, 195, 239}

$$-\frac{5a^2(9bc-ad)\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}\,x\right)}{162b^{4/3}}+\frac{5a^2(9bc-ad)\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{81\sqrt{3}\,b^{4/3}}+\frac{x\left(a+bx^3\right)^{5/3}(9bc-ad)}{54b}+\frac{5ax\left(a+bx^3\right)^{2/3}(9bc-ad)}{162b}+\frac{dx\left(a+bx^3\right)^{8/3}}{9b}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^(5/3)*(c + d*x^3),x]
```

```
[Out] (5*a*(9*b*c - a*d)*x*(a + b*x^3)^(2/3))/(162*b) + ((9*b*c - a*d)*x*(a + b*x
^3)^(5/3))/(54*b) + (d*x*(a + b*x^3)^(8/3))/(9*b) + (5*a^2*(9*b*c - a*d)*Ar
cTan[(1 + (2*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]])/(81*Sqrt[3]*b^(4/3)) -
 (5*a^2*(9*b*c - a*d)*Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)])/(162*b^(4/3))
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 239

```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + (2*Rt[b, 3]
*x)/(a + b*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rubi steps

$$\int \left(a+bx^3\right)^{5/3}\left(c+dx^3\right)dx = \frac{dx\left(a+bx^3\right)^{8/3}}{9b} - \frac{(-9bc+ad)\int \left(a+bx^3\right)^{5/3}dx}{9b}$$

$$= \frac{(9bc-ad)x\left(a+bx^3\right)^{5/3}}{54b} + \frac{dx\left(a+bx^3\right)^{8/3}}{9b} + \frac{(5a(9bc-ad))\int \left(a+bx^3\right)^{2/3}dx}{54b}$$

$$= \frac{5a(9bc-ad)x\left(a+bx^3\right)^{2/3}}{162b} + \frac{(9bc-ad)x\left(a+bx^3\right)^{5/3}}{54b} + \frac{dx\left(a+bx^3\right)^{8/3}}{9b} + \frac{\left(5a^2(9b \ldots\right.}{\ldots}$$

$$= \frac{5a(9bc-ad)x\left(a+bx^3\right)^{2/3}}{162b} + \frac{(9bc-ad)x\left(a+bx^3\right)^{5/3}}{54b} + \frac{dx\left(a+bx^3\right)^{8/3}}{9b} + \frac{5a^2(9bc \ldots}{\ldots}$$

Mathematica [C] time = 0.07, size = 75, normalized size = 0.43

$$\frac{x\left(a+bx^3\right)^{2/3}\left(d\left(a+bx^3\right)^2-\frac{a(ad-9bc)\,{}_2F_1\left(-\frac{5}{3},\frac{1}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{\left(\frac{bx^3}{a}+1\right)^{2/3}}\right)}{9b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^(5/3)*(c + d*x^3),x]
```

```
[Out] (x*(a + b*x^3)^(2/3)*(d*(a + b*x^3)^2 - (a*(-9*b*c + a*d)*Hypergeometric2F1
[-5/3, 1/3, 4/3, -((b*x^3)/a)])/(1 + (b*x^3)/a)^(2/3)))/(9*b)
```

IntegrateAlgebraic [A] time = 0.71, size = 227, normalized size = 1.30

$$\frac{\left(a+bx^3\right)^{2/3}\left(10a^2dx+72abcx+33abdx^4+27b^2cx^4+18b^2dx^7\right)}{162b}+\frac{5\left(a^3d-9a^2bc\right)\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}x\right)}{243b^{4/3}}-\frac{5\left(a^3d-9a^2bc\right)\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{b}x}{2\sqrt[3]{a+bx^3}+\sqrt[3]{b}x}\right)}{81\sqrt{3}b^{4/3}}-\frac{5\left(a^3d-9a^2bc\right)\log\left(\sqrt[3]{b}x\sqrt[3]{a+bx^3}+\left(a+bx^3\right)^{2/3}+b^{2/3}x^2\right)}{486b^{4/3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x^3)^(5/3)*(c + d*x^3),x]
```

```
[Out] ((a + b*x^3)^(2/3)*(72*a*b*c*x + 10*a^2*d*x + 27*b^2*c*x^4 + 33*a*b*d*x^4 +
 18*b^2*d*x^7))/(162*b) - (5*(-9*a^2*b*c + a^3*d)*ArcTan[(Sqrt[3]*b^(1/3)*x
)/(b^(1/3)*x + 2*(a + b*x^3)^(1/3))])/(81*Sqrt[3]*b^(4/3)) + (5*(-9*a^2*b*c
 + a^3*d)*Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)])/(243*b^(4/3)) - (5*(-9*a^2
*b*c + a^3*d)*Log[b^(2/3)*x^2 + b^(1/3)*x*(a + b*x^3)^(1/3) + (a + b*x^3)^(
2/3)])/(486*b^(4/3))
```

fricas [A] time = 1.21, size = 482, normalized size = 2.77

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(5/3)*(d*x^3+c),x, algorithm="fricas")
```

```
[Out] [-1/486*(15*sqrt(1/3)*(9*a^2*b^2*c - a^3*b*d)*sqrt(-1/b^(2/3))*log(3*b*x^3
- 3*(b*x^3 + a)^(1/3)*b^(2/3)*x^2 - 3*sqrt(1/3)*(b^(4/3)*x^3 + (b*x^3 + a)^
(1/3)*b*x^2 - 2*(b*x^3 + a)^(2/3)*b^(2/3)*x)*sqrt(-1/b^(2/3)) + 2*a) + 10*(
9*a^2*b*c - a^3*d)*b^(2/3)*log(-(b^(1/3)*x - (b*x^3 + a)^(1/3))/x) - 5*(9*a
^2*b*c - a^3*d)*b^(2/3)*log((b^(2/3)*x^2 + (b*x^3 + a)^(1/3)*b^(1/3)*x + (b
*x^3 + a)^(2/3))/x^2) - 3*(18*b^3*d*x^7 + 3*(9*b^3*c + 11*a*b^2*d)*x^4 + 2*
```

```
(36*a*b^2*c + 5*a^2*b*d)*x)*(b*x^3 + a)^(2/3))/b^2, -1/486*(10*(9*a^2*b*c -
 a^3*d)*b^(2/3)*log(-(b^(1/3)*x - (b*x^3 + a)^(1/3))/x) - 5*(9*a^2*b*c - a^
3*d)*b^(2/3)*log((b^(2/3)*x^2 + (b*x^3 + a)^(1/3)*b^(1/3)*x + (b*x^3 + a)^(
2/3))/x^2) + 30*sqrt(1/3)*(9*a^2*b^2*c - a^3*b*d)*arctan(sqrt(1/3)*(b^(1/3)
*x + 2*(b*x^3 + a)^(1/3))/(b^(1/3)*x))/b^(1/3) - 3*(18*b^3*d*x^7 + 3*(9*b^3
*c + 11*a*b^2*d)*x^4 + 2*(36*a*b^2*c + 5*a^2*b*d)*x)*(b*x^3 + a)^(2/3))/b^2
]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx^3+a\right)^{\frac{5}{3}}\left(dx^3+c\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(5/3)*(d*x^3+c),x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a)^(5/3)*(d*x^3 + c), x)
```

maple [F] time = 0.37, size = 0, normalized size = 0.00

$$\int \left(b\,x^3+a\right)^{\frac{5}{3}}\left(d\,x^3+c\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(5/3)*(d*x^3+c),x)
```

```
[Out] int((b*x^3+a)^(5/3)*(d*x^3+c),x)
```

maxima [B] time = 1.68, size = 406, normalized size = 2.33

$$-\frac{1}{54}\left(\frac{10\sqrt{3}a^2\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}}+\frac{2\left(bx^3+a\right)^{\frac{1}{3}}}{x}\right)}{3b^{\frac{1}{3}}}\right)}{b^{\frac{1}{3}}}-\frac{5a^2\log\left(b^{\frac{2}{3}}+\frac{\left(bx^3+a\right)^{\frac{1}{3}}b^{\frac{1}{3}}}{x}+\frac{\left(bx^3+a\right)^{\frac{2}{3}}}{x^2}\right)}{b^{\frac{1}{3}}}+\frac{10a^2\log\left(-b^{\frac{1}{3}}+\frac{\left(bx^3+a\right)^{\frac{1}{3}}}{x}\right)}{b^{\frac{1}{3}}}+\frac{3\left(\frac{5\left(bx^3+a\right)^{\frac{2}{3}}a^2b}{x^2}-\frac{8\left(bx^3+a\right)^{\frac{5}{3}}a^2}{x^5}\right)}{b^2-\frac{2\left(bx^3+a\right)b}{x^3}+\frac{\left(bx^3+a\right)^2}{x^6}}\right)c+\frac{1}{486}\left(\frac{10\sqrt{3}a^3\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}}+\frac{2\left(bx^3+a\right)^{\frac{1}{3}}}{x}\right)}{3b^{\frac{1}{3}}}\right)}{b^{\frac{4}{3}}}-\frac{5a^3\log\left(b^{\frac{2}{3}}+\frac{\left(bx^3+a\right)^{\frac{1}{3}}b^{\frac{1}{3}}}{x}+\frac{\left(bx^3+a\right)^{\frac{2}{3}}}{x^2}\right)}{b^{\frac{4}{3}}}+\frac{10a^3\log\left(-b^{\frac{1}{3}}+\frac{\left(bx^3+a\right)^{\frac{1}{3}}}{x}\right)}{b^{\frac{4}{3}}}+\frac{3\left(\frac{5\left(bx^3+a\right)^{\frac{2}{3}}a^3b^2}{x^2}-\frac{13\left(bx^3+a\right)^{\frac{5}{3}}a^3b}{x^5}-\frac{10\left(bx^3+a\right)^{\frac{8}{3}}a^3}{x^8}\right)}{b^4-\frac{3\left(bx^3+a\right)b^3}{x^3}+\frac{3\left(bx^3+a\right)^2b^2}{x^6}-\frac{\left(bx^3+a\right)^3b}{x^9}}\right)d$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(5/3)*(d*x^3+c),x, algorithm="maxima")
```

```
[Out] -1/54*(10*sqrt(3)*a^2*arctan(1/3*sqrt(3)*(b^(1/3) + 2*(b*x^3 + a)^(1/3)/x)/
b^(1/3))/b^(1/3) - 5*a^2*log(b^(2/3) + (b*x^3 + a)^(1/3)*b^(1/3)/x + (b*x^3
 + a)^(2/3)/x^2)/b^(1/3) + 10*a^2*log(-b^(1/3) + (b*x^3 + a)^(1/3)/x)/b^(1/
3) + 3*(5*(b*x^3 + a)^(2/3)*a^2*b/x^2 - 8*(b*x^3 + a)^(5/3)*a^2/x^5)/(b^2 -
 2*(b*x^3 + a)*b/x^3 + (b*x^3 + a)^2/x^6))*c + 1/486*(10*sqrt(3)*a^3*arctan
(1/3*sqrt(3)*(b^(1/3) + 2*(b*x^3 + a)^(1/3)/x)/b^(1/3))/b^(4/3) - 5*a^3*log
(b^(2/3) + (b*x^3 + a)^(1/3)*b^(1/3)/x + (b*x^3 + a)^(2/3)/x^2)/b^(4/3) + 1
0*a^3*log(-b^(1/3) + (b*x^3 + a)^(1/3)/x)/b^(4/3) + 3*(5*(b*x^3 + a)^(2/3)*
a^3*b^2/x^2 - 13*(b*x^3 + a)^(5/3)*a^3*b/x^5 - 10*(b*x^3 + a)^(8/3)*a^3/x^8
)/(b^4 - 3*(b*x^3 + a)*b^3/x^3 + 3*(b*x^3 + a)^2*b^2/x^6 - (b*x^3 + a)^3*b/
x^9))*d
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(b\,x^3+a\right)}^{5/3}\,\left(d\,x^3+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^(5/3)*(c + d*x^3),x)
```

```
[Out] int((a + b*x^3)^(5/3)*(c + d*x^3), x)
```

sympy [C] time = 10.42, size = 170, normalized size = 0.98

$$\frac{a^{\frac{5}{3}}cx\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{2}{3},\frac{1}{3}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{4}{3}\right)}+\frac{a^{\frac{5}{3}}dx^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}-\frac{2}{3},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{7}{3}\right)}+\frac{a^{\frac{2}{3}}bcx^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}-\frac{2}{3},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{7}{3}\right)}+\frac{a^{\frac{2}{3}}bdx^7\Gamma\left(\frac{7}{3}\right){}_2F_1\left(\begin{matrix}-\frac{2}{3},\frac{7}{3}\\ \frac{10}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{10}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(5/3)*(d*x**3+c),x)
```

```
[Out] a**(5/3)*c*x*gamma(1/3)*hyper((-2/3, 1/3), (4/3,), b*x**3*exp_polar(I*pi)/a
)/(3*gamma(4/3)) + a**(5/3)*d*x**4*gamma(4/3)*hyper((-2/3, 4/3), (7/3,), b*
x**3*exp_polar(I*pi)/a)/(3*gamma(7/3)) + a**(2/3)*b*c*x**4*gamma(4/3)*hyper
((-2/3, 4/3), (7/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(7/3)) + a**(2/3)*b
*d*x**7*gamma(7/3)*hyper((-2/3, 7/3), (10/3,), b*x**3*exp_polar(I*pi)/a)/(3
*gamma(10/3))
```

3.44 $\int \left(a+bx^3\right)^{2/3} \left(c+dx^3\right) dx$

Optimal. Leaf size=141

$$-\frac{a(6bc-ad)\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}\,x\right)}{18b^{4/3}}+\frac{a(6bc-ad)\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{9\sqrt{3}\,b^{4/3}}+\frac{x\left(a+bx^3\right)^{2/3}(6bc-ad)}{18b}+\frac{dx\left(a+bx^3\right)^{5/3}}{6b}$$

Rubi [A] time = 0.05, antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {388, 195, 239}

$$-\frac{a(6bc-ad)\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}\,x\right)}{18b^{4/3}}+\frac{a(6bc-ad)\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{9\sqrt{3}\,b^{4/3}}+\frac{x\left(a+bx^3\right)^{2/3}(6bc-ad)}{18b}+\frac{dx\left(a+bx^3\right)^{5/3}}{6b}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^(2/3)*(c + d*x^3),x]
```

```
[Out] ((6*b*c - a*d)*x*(a + b*x^3)^(2/3))/(18*b) + (d*x*(a + b*x^3)^(5/3))/(6*b)
+ (a*(6*b*c - a*d)*ArcTan[(1 + (2*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]])/(
9*Sqrt[3]*b^(4/3)) - (a*(6*b*c - a*d)*Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)]
)/(18*b^(4/3))
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 239

```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + (2*Rt[b, 3]
*x)/(a + b*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rubi steps

$$\int \left(a+bx^3\right)^{2/3}\left(c+dx^3\right)dx = \frac{dx\left(a+bx^3\right)^{5/3}}{6b} - \frac{(-6bc+ad)\int \left(a+bx^3\right)^{2/3}dx}{6b}$$
$$= \frac{(6bc-ad)x\left(a+bx^3\right)^{2/3}}{18b} + \frac{dx\left(a+bx^3\right)^{5/3}}{6b} + \frac{(a(6bc-ad))\int \frac{1}{\sqrt[3]{a+bx^3}}dx}{9b}$$
$$= \frac{(6bc-ad)x\left(a+bx^3\right)^{2/3}}{18b} + \frac{dx\left(a+bx^3\right)^{5/3}}{6b} + \frac{a(6bc-ad)\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{9\sqrt{3}b^{4/3}} - \frac{a(6bc}{}$$

Mathematica [C] time = 0.10, size = 72, normalized size = 0.51

$$\frac{x\left(a+bx^3\right)^{2/3}\left(\frac{(6bc-ad)\,{}_2F_1\left(-\frac{2}{3},\frac{1}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{\left(\frac{bx^3}{a}+1\right)^{2/3}}+d\left(a+bx^3\right)\right)}{6b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^(2/3)*(c + d*x^3),x]
```

```
[Out] (x*(a + b*x^3)^(2/3)*(d*(a + b*x^3) + ((6*b*c - a*d)*Hypergeometric2F1[-2/3
, 1/3, 4/3, -((b*x^3)/a)])/(1 + (b*x^3)/a)^(2/3)))/(6*b)
```

IntegrateAlgebraic [A] time = 0.67, size = 200, normalized size = 1.42

$$\frac{\left(a^2d-6abc\right)\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}x\right)}{27b^{4/3}} - \frac{\left(a^2d-6abc\right)\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{b}x}{2\sqrt[3]{a+bx^3}+\sqrt[3]{b}x}\right)}{9\sqrt{3}b^{4/3}} + \frac{\left(6abc-a^2d\right)\log\left(\sqrt[3]{b}x\sqrt[3]{a+bx^3}+\left(a+bx^3\right)^{2/3}+b^{2/3}x^2\right)}{54b^{4/3}} + \frac{\left(a+bx^3\right)^{2/3}\left(2adx+6bcx+3bdx^4\right)}{18b}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x^3)^(2/3)*(c + d*x^3),x]
```

```
[Out] ((a + b*x^3)^(2/3)*(6*b*c*x + 2*a*d*x + 3*b*d*x^4))/(18*b) - ((-6*a*b*c + a
^2*d)*ArcTan[(Sqrt[3]*b^(1/3)*x)/(b^(1/3)*x + 2*(a + b*x^3)^(1/3))])/(9*Sqr
t[3]*b^(4/3)) + ((-6*a*b*c + a^2*d)*Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)])/
(27*b^(4/3)) + ((6*a*b*c - a^2*d)*Log[b^(2/3)*x^2 + b^(1/3)*x*(a + b*x^3)^(
1/3) + (a + b*x^3)^(2/3)])/(54*b^(4/3))
```

fricas [A] time = 1.10, size = 424, normalized size = 3.01

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)*(d*x^3+c),x, algorithm="fricas")
```

```
[Out] [-1/54*(3*sqrt(1/3)*(6*a*b^2*c - a^2*b*d)*sqrt(-1/b^(2/3))*log(3*b*x^3 - 3*
(b*x^3 + a)^(1/3)*b^(2/3)*x^2 - 3*sqrt(1/3)*(b^(4/3)*x^3 + (b*x^3 + a)^(1/3
)*b*x^2 - 2*(b*x^3 + a)^(2/3)*b^(2/3)*x)*sqrt(-1/b^(2/3)) + 2*a) + 2*(6*a*b
*c - a^2*d)*b^(2/3)*log(-(b^(1/3)*x - (b*x^3 + a)^(1/3))/x) - (6*a*b*c - a^
2*d)*b^(2/3)*log((b^(2/3)*x^2 + (b*x^3 + a)^(1/3)*b^(1/3)*x + (b*x^3 + a)^(
2/3))/x^2) - 3*(3*b^2*d*x^4 + 2*(3*b^2*c + a*b*d)*x)*(b*x^3 + a)^(2/3))/b^2
, -1/54*(2*(6*a*b*c - a^2*d)*b^(2/3)*log(-(b^(1/3)*x - (b*x^3 + a)^(1/3))/x
) - (6*a*b*c - a^2*d)*b^(2/3)*log((b^(2/3)*x^2 + (b*x^3 + a)^(1/3)*b^(1/3)*
x + (b*x^3 + a)^(2/3))/x^2) + 6*sqrt(1/3)*(6*a*b^2*c - a^2*b*d)*arctan(sqrt
```

```
(1/3)*(b^(1/3)*x + 2*(b*x^3 + a)^(1/3))/(b^(1/3)*x))/b^(1/3) - 3*(3*b^2*d*x
^4 + 2*(3*b^2*c + a*b*d)*x)*(b*x^3 + a)^(2/3))/b^2]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx^3 + a\right)^{\frac{2}{3}} \left(dx^3 + c\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)*(d*x^3+c),x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a)^(2/3)*(d*x^3 + c), x)
```

maple [F] time = 0.38, size = 0, normalized size = 0.00

$$\int \left(b\,x^3 + a\right)^{\frac{2}{3}} \left(d\,x^3 + c\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(2/3)*(d*x^3+c),x)
```

```
[Out] int((b*x^3+a)^(2/3)*(d*x^3+c),x)
```

maxima [B] time = 1.44, size = 322, normalized size = 2.28

$$-\frac{1}{9}\left(\frac{2\,\sqrt{3}\,a\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}}+\frac{2\,(bx^3+a)^{\frac{1}{3}}}{x}\right)}{3\,b^{\frac{1}{3}}}\right)}{b^{\frac{1}{3}}} - \frac{a\log\left(b^{\frac{2}{3}} + \frac{(bx^3+a)^{\frac{1}{3}}b^{\frac{1}{3}}}{x} + \frac{(bx^3+a)^{\frac{2}{3}}}{x^2}\right)}{b^{\frac{1}{3}}} + \frac{2\,a\log\left(-b^{\frac{1}{3}} + \frac{(bx^3+a)^{\frac{1}{3}}}{x}\right)}{b^{\frac{1}{3}}} + \frac{3\left(bx^3+a\right)^{\frac{2}{3}}a}{\left(b - \frac{bx^3+a}{x^3}\right)x^2}\right)c + \frac{1}{54}\left(\frac{2\,\sqrt{3}\,a^2\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}}+\frac{2\,(bx^3+a)^{\frac{1}{3}}}{x}\right)}{3\,b^{\frac{1}{3}}}\right)}{b^{\frac{4}{3}}} - \frac{a^2\log\left(b^{\frac{2}{3}} + \frac{(bx^3+a)^{\frac{1}{3}}b^{\frac{1}{3}}}{x} + \frac{(bx^3+a)^{\frac{2}{3}}}{x^2}\right)}{b^{\frac{4}{3}}} + \frac{2\,a^2\log\left(-b^{\frac{1}{3}} + \frac{(bx^3+a)^{\frac{1}{3}}}{x}\right)}{b^{\frac{4}{3}}} + \frac{3\left(\frac{(bx^3+a)^{\frac{2}{3}}a^2b}{x^2} + \frac{2\,(bx^3+a)^{\frac{5}{3}}a^2}{x^5}\right)}{b^3 - \frac{2\,(bx^3+a)b^2}{x^3} + \frac{(bx^3+a)^2b}{x^6}}\right)d$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)*(d*x^3+c),x, algorithm="maxima")
```

```
[Out] -1/9*(2*sqrt(3)*a*arctan(1/3*sqrt(3)*(b^(1/3) + 2*(b*x^3 + a)^(1/3)/x)/b^(1
/3))/b^(1/3) - a*log(b^(2/3) + (b*x^3 + a)^(1/3)*b^(1/3)/x + (b*x^3 + a)^(2
/3)/x^2)/b^(1/3) + 2*a*log(-b^(1/3) + (b*x^3 + a)^(1/3)/x)/b^(1/3) + 3*(b*x
^3 + a)^(2/3)*a/((b - (b*x^3 + a)/x^3)*x^2))*c + 1/54*(2*sqrt(3)*a^2*arctan
(1/3*sqrt(3)*(b^(1/3) + 2*(b*x^3 + a)^(1/3)/x)/b^(1/3))/b^(4/3) - a^2*log(b
^(2/3) + (b*x^3 + a)^(1/3)*b^(1/3)/x + (b*x^3 + a)^(2/3)/x^2)/b^(4/3) + 2*a
^2*log(-b^(1/3) + (b*x^3 + a)^(1/3)/x)/b^(4/3) + 3*((b*x^3 + a)^(2/3)*a^2*b
/x^2 + 2*(b*x^3 + a)^(5/3)*a^2/x^5)/(b^3 - 2*(b*x^3 + a)*b^2/x^3 + (b*x^3 +
 a)^2*b/x^6))*d
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(b\,x^3+a\right)}^{2/3}\,\left(d\,x^3+c\right) \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^(2/3)*(c + d*x^3),x)
```

```
[Out] int((a + b*x^3)^(2/3)*(c + d*x^3), x)
```

sympy [C] time = 5.35, size = 82, normalized size = 0.58

$$\frac{a^{\frac{2}{3}} c x \Gamma\left(\frac{1}{3}\right) {}_{2}F_{1}\left(\begin{matrix} -\frac{2}{3}, \frac{1}{3} \\ \frac{4}{3} \end{matrix}\middle| {\frac{b x^{3} e^{i \pi}}{a}} \right)}{3 \Gamma\left(\frac{4}{3}\right)} + \frac{a^{\frac{2}{3}} d x^{4} \Gamma\left(\frac{4}{3}\right) {}_{2}F_{1}\left(\begin{matrix} -\frac{2}{3}, \frac{4}{3} \\ \frac{7}{3} \end{matrix}\middle| {\frac{b x^{3} e^{i \pi}}{a}} \right)}{3 \Gamma\left(\frac{7}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(2/3)*(d*x**3+c),x)
```

```
[Out] a**(2/3)*c*x*gamma(1/3)*hyper((-2/3, 1/3), (4/3,), b*x**3*exp_polar(I*pi)/a
)/(3*gamma(4/3)) + a**(2/3)*d*x**4*gamma(4/3)*hyper((-2/3, 4/3), (7/3,), b*
x**3*exp_polar(I*pi)/a)/(3*gamma(7/3))
```

3.45 $\int \frac{c+dx^3}{\sqrt[3]{a+bx^3}}\,dx$

Optimal. Leaf size=111

$$-\frac{(3bc-ad)\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}\,x\right)}{6b^{4/3}}+\frac{(3bc-ad)\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{3\sqrt{3}\,b^{4/3}}+\frac{dx\left(a+bx^3\right)^{2/3}}{3b}$$

Rubi [A] time = 0.03, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {388, 239}

$$-\frac{(3bc-ad)\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}\,x\right)}{6b^{4/3}}+\frac{(3bc-ad)\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{3\sqrt{3}\,b^{4/3}}+\frac{dx\left(a+bx^3\right)^{2/3}}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3)/(a + b*x^3)^(1/3),x]
```

```
[Out] (d*x*(a + b*x^3)^(2/3))/(3*b) + ((3*b*c - a*d)*ArcTan[(1 + (2*b^(1/3)*x)/(a
 + b*x^3)^(1/3))/Sqrt[3]])/(3*Sqrt[3]*b^(4/3)) - ((3*b*c - a*d)*Log[-(b^(1/
3)*x) + (a + b*x^3)^(1/3)])/(6*b^(4/3))
```

Rule 239

```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + (2*Rt[b, 3]
*x)/(a + b*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{c+dx^3}{\sqrt[3]{a+bx^3}}\,dx &= \frac{dx\left(a+bx^3\right)^{2/3}}{3b}-\frac{(-3bc+ad)\int \frac{1}{\sqrt[3]{a+bx^3}}\,dx}{3b}\\ &= \frac{dx\left(a+bx^3\right)^{2/3}}{3b}+\frac{(3bc-ad)\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{3\sqrt{3}\,b^{4/3}}-\frac{(3bc-ad)\log\left(-\sqrt[3]{b}\,x+\sqrt[3]{a+bx^3}\right)}{6b^{4/3}}\end{aligned}$$

Mathematica [A] time = 0.15, size = 141, normalized size = 1.27

$$\frac{\frac{(3bc-ad)\left(\log\left(\frac{b^{2/3}x^2}{\left(a+bx^3\right)^{2/3}}+\frac{\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}+1\right)-2\log\left(1-\frac{\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}\right)+2\sqrt{3}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)\right)}{6\sqrt[3]{b}}+dx\left(a+bx^3\right)^{2/3}}{3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^3)/(a + b*x^3)^(1/3),x]
```

```
[Out] (d*x*(a + b*x^3)^(2/3) + ((3*b*c - a*d)*(2*Sqrt[3]*ArcTan[(1 + (2*b^(1/3)*x
)/(a + b*x^3)^(1/3))/Sqrt[3]] - 2*Log[1 - (b^(1/3)*x)/(a + b*x^3)^(1/3)] +
Log[1 + (b^(2/3)*x^2)/(a + b*x^3)^(2/3) + (b^(1/3)*x)/(a + b*x^3)^(1/3)]))/
(6*b^(1/3)))/(3*b)
```

IntegrateAlgebraic [A] time = 0.51, size = 176, normalized size = 1.59

$$\frac{(ad-3bc)\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}\,x\right)}{9b^{4/3}}+\frac{(3bc-ad)\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{b}\,x}{2\sqrt[3]{a+bx^3}+\sqrt[3]{b}\,x}\right)}{3\sqrt{3}\,b^{4/3}}+\frac{(3bc-ad)\log\left(\sqrt[3]{b}\,x\sqrt[3]{a+bx^3}+\left(a+bx^3\right)^{2/3}+b^{2/3}x^2\right)}{18b^{4/3}}+\frac{dx\left(a+bx^3\right)^{2/3}}{3b}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c + d*x^3)/(a + b*x^3)^(1/3),x]
```

```
[Out] (d*x*(a + b*x^3)^(2/3))/(3*b) + ((3*b*c - a*d)*ArcTan[(Sqrt[3]*b^(1/3)*x)/(
b^(1/3)*x + 2*(a + b*x^3)^(1/3))])/(3*Sqrt[3]*b^(4/3)) + ((-3*b*c + a*d)*Lo
g[-(b^(1/3)*x) + (a + b*x^3)^(1/3)])/(9*b^(4/3)) + ((3*b*c - a*d)*Log[b^(2/
3)*x^2 + b^(1/3)*x*(a + b*x^3)^(1/3) + (a + b*x^3)^(2/3)])/(18*b^(4/3))
```

fricas [A] time = 1.31, size = 362, normalized size = 3.26

$$\left[\frac{6\left(bx^3+a\right)^{\frac{2}{3}}bdx-3\sqrt{\frac{1}{3}}\left(3b^2c-abd\right)\sqrt{-\frac{1}{b^{\frac{2}{3}}}}\log\left(3bx^3-3\left(bx^3+a\right)^{\frac{1}{3}}b^{\frac{2}{3}}x^2-3\sqrt{\frac{1}{3}}\left(b^{\frac{4}{3}}x^3+\left(bx^3+a\right)^{\frac{1}{3}}bx^2-2\left(bx^3+a\right)^{\frac{2}{3}}b^{\frac{2}{3}}x\right)\sqrt{-\frac{1}{b^{\frac{2}{3}}}}+2a\right)-2\left(3bc-ad\right)b^{\frac{2}{3}}\log\left(-\frac{b^{\frac{1}{3}}x-\left(bx^3+a\right)^{\frac{1}{3}}}{x}\right)+\left(3bc-ad\right)b^{\frac{2}{3}}\log\left(\frac{b^{\frac{2}{3}}x^2+\left(bx^3+a\right)^{\frac{1}{3}}b^{\frac{1}{3}}x+\left(bx^3+a\right)^{\frac{2}{3}}}{x^2}\right)}{18b^2},\ \frac{6\left(bx^3+a\right)^{\frac{2}{3}}bdx-2\left(3bc-ad\right)b^{\frac{2}{3}}\log\left(-\frac{b^{\frac{1}{3}}x-\left(bx^3+a\right)^{\frac{1}{3}}}{x}\right)+\left(3bc-ad\right)b^{\frac{2}{3}}\log\left(\frac{b^{\frac{2}{3}}x^2+\left(bx^3+a\right)^{\frac{1}{3}}b^{\frac{1}{3}}x+\left(bx^3+a\right)^{\frac{2}{3}}}{x^2}\right)-\frac{6\sqrt{\frac{1}{3}}\left(3b^2c-abd\right)\arctan\left(\frac{\sqrt{\frac{1}{3}}\left(b^{\frac{1}{3}}x+2\left(bx^3+a\right)^{\frac{1}{3}}\right)}{b^{\frac{1}{3}}x}\right)}{b^{\frac{1}{3}}}}{18b^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)/(b*x^3+a)^(1/3),x, algorithm="fricas")
```

```
[Out] [1/18*(6*(b*x^3 + a)^(2/3)*b*d*x - 3*sqrt(1/3)*(3*b^2*c - a*b*d)*sqrt(-1/b^
(2/3))*log(3*b*x^3 - 3*(b*x^3 + a)^(1/3)*b^(2/3)*x^2 - 3*sqrt(1/3)*(b^(4/3)
*x^3 + (b*x^3 + a)^(1/3)*b*x^2 - 2*(b*x^3 + a)^(2/3)*b^(2/3)*x)*sqrt(-1/b^(
2/3)) + 2*a) - 2*(3*b*c - a*d)*b^(2/3)*log(-(b^(1/3)*x - (b*x^3 + a)^(1/3))
/x) + (3*b*c - a*d)*b^(2/3)*log((b^(2/3)*x^2 + (b*x^3 + a)^(1/3)*b^(1/3)*x
+ (b*x^3 + a)^(2/3))/x^2))/b^2, 1/18*(6*(b*x^3 + a)^(2/3)*b*d*x - 2*(3*b*c
- a*d)*b^(2/3)*log(-(b^(1/3)*x - (b*x^3 + a)^(1/3))/x) + (3*b*c - a*d)*b^(2
/3)*log((b^(2/3)*x^2 + (b*x^3 + a)^(1/3)*b^(1/3)*x + (b*x^3 + a)^(2/3))/x^2
) - 6*sqrt(1/3)*(3*b^2*c - a*b*d)*arctan(sqrt(1/3)*(b^(1/3)*x + 2*(b*x^3 +
a)^(1/3))/(b^(1/3)*x))/b^(1/3))/b^2]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{dx^3+c}{\left(bx^3+a\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)/(b*x^3+a)^(1/3),x, algorithm="giac")
```

```
[Out] integrate((d*x^3 + c)/(b*x^3 + a)^(1/3), x)
```

maple [F] time = 0.40, size = 0, normalized size = 0.00

$$\int \frac{d\,x^3+c}{\left(b\,x^3+a\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c)/(b*x^3+a)^(1/3),x)
```

```
[Out] int((d*x^3+c)/(b*x^3+a)^(1/3),x)
```

maxima [B] time = 1.20, size = 244, normalized size = 2.20

$$-\frac{1}{6}\left(\frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}}+\frac{2(bx^3+a)^{\frac{1}{3}}}{x}\right)}{3b^{\frac{1}{3}}}\right)}{b^{\frac{1}{3}}}-\frac{\log\left(b^{\frac{2}{3}}+\frac{(bx^3+a)^{\frac{1}{3}}b^{\frac{1}{3}}}{x}+\frac{(bx^3+a)^{\frac{2}{3}}}{x^2}\right)}{b^{\frac{1}{3}}}+\frac{2\log\left(-b^{\frac{1}{3}}+\frac{(bx^3+a)^{\frac{1}{3}}}{x}\right)}{b^{\frac{1}{3}}}\right)c+\frac{1}{18}\left(\frac{2\sqrt{3}a\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}}+\frac{2(bx^3+a)^{\frac{1}{3}}}{x}\right)}{3b^{\frac{1}{3}}}\right)}{b^{\frac{4}{3}}}-\frac{a\log\left(b^{\frac{2}{3}}+\frac{(bx^3+a)^{\frac{1}{3}}b^{\frac{1}{3}}}{x}+\frac{(bx^3+a)^{\frac{2}{3}}}{x^2}\right)}{b^{\frac{4}{3}}}+\frac{2a\log\left(-b^{\frac{1}{3}}+\frac{(bx^3+a)^{\frac{1}{3}}}{x}\right)}{b^{\frac{4}{3}}}-\frac{6(bx^3+a)^{\frac{2}{3}}a}{\left(b^2-\frac{(bx^3+a)b}{x^3}\right)x^2}\right)d$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)/(b*x^3+a)^(1/3),x, algorithm="maxima")
```

```
[Out] -1/6*(2*sqrt(3)*arctan(1/3*sqrt(3)*(b^(1/3) + 2*(b*x^3 + a)^(1/3)/x)/b^(1/3
))/b^(1/3) - log(b^(2/3) + (b*x^3 + a)^(1/3)*b^(1/3)/x + (b*x^3 + a)^(2/3)/
x^2)/b^(1/3) + 2*log(-b^(1/3) + (b*x^3 + a)^(1/3)/x)/b^(1/3))*c + 1/18*(2*s
qrt(3)*a*arctan(1/3*sqrt(3)*(b^(1/3) + 2*(b*x^3 + a)^(1/3)/x)/b^(1/3))/b^(4
/3) - a*log(b^(2/3) + (b*x^3 + a)^(1/3)*b^(1/3)/x + (b*x^3 + a)^(2/3)/x^2)/
b^(4/3) + 2*a*log(-b^(1/3) + (b*x^3 + a)^(1/3)/x)/b^(4/3) - 6*(b*x^3 + a)^(
2/3)*a/((b^2 - (b*x^3 + a)*b/x^3)*x^2))*d
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{d\,x^3+c}{{\left(b\,x^3+a\right)}^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3)/(a + b*x^3)^(1/3),x)
```

```
[Out] int((c + d*x^3)/(a + b*x^3)^(1/3), x)
```

sympy [C] time = 4.44, size = 78, normalized size = 0.70

$$\frac{cx\Gamma\left(\frac{1}{3}\right){}_{2}F_{1}\left(\left.\begin{matrix}\frac{1}{3},\frac{1}{3}\\ \frac{4}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt[3]{a}\,\Gamma\left(\frac{4}{3}\right)}+\frac{dx^4\Gamma\left(\frac{4}{3}\right){}_{2}F_{1}\left(\left.\begin{matrix}\frac{1}{3},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt[3]{a}\,\Gamma\left(\frac{7}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c)/(b*x**3+a)**(1/3),x)
```

```
[Out] c*x*gamma(1/3)*hyper((1/3, 1/3), (4/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(1
/3)*gamma(4/3)) + d*x**4*gamma(4/3)*hyper((1/3, 4/3), (7/3,), b*x**3*exp_po
lar(I*pi)/a)/(3*a**(1/3)*gamma(7/3))
```

3.46 $\int \frac{c+dx^3}{\left(a+bx^3\right)^{4/3}}\,dx$

Optimal. Leaf size=99

$$-\frac{d\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}\,x\right)}{2b^{4/3}}+\frac{d\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\,b^{4/3}}+\frac{x(bc-ad)}{ab\sqrt[3]{a+bx^3}}$$

Rubi [A] time = 0.02, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {385, 239}

$$-\frac{d\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}\,x\right)}{2b^{4/3}}+\frac{d\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\,b^{4/3}}+\frac{x(bc-ad)}{ab\sqrt[3]{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3)/(a + b*x^3)^(4/3),x]
```

```
[Out] ((b*c - a*d)*x)/(a*b*(a + b*x^3)^(1/3)) + (d*ArcTan[(1 + (2*b^(1/3)*x)/(a +
 b*x^3)^(1/3))/Sqrt[3]])/(Sqrt[3]*b^(4/3)) - (d*Log[-(b^(1/3)*x) + (a + b*x
^3)^(1/3)])/(2*b^(4/3))
```

Rule 239

```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + (2*Rt[b, 3]
*x)/(a + b*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{c+dx^3}{\left(a+bx^3\right)^{4/3}}\,dx&=\frac{(bc-ad)x}{ab\sqrt[3]{a+bx^3}}+\frac{d\int \frac{1}{\sqrt[3]{a+bx^3}}\,dx}{b}\\&=\frac{(bc-ad)x}{ab\sqrt[3]{a+bx^3}}+\frac{d\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}\,b^{4/3}}-\frac{d\log\left(-\sqrt[3]{b}\,x+\sqrt[3]{a+bx^3}\right)}{2b^{4/3}}\end{aligned}$$

Mathematica [C] time = 0.05, size = 61, normalized size = 0.62

$$\frac{dx^4\sqrt[3]{\frac{bx^3}{a}+1}\,{}_2F_1\left(\frac{4}{3},\frac{4}{3};\frac{7}{3};-\frac{bx^3}{a}\right)+4cx}{4a\sqrt[3]{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^3)/(a + b*x^3)^(4/3),x]
```

```
[Out] (4*c*x + d*x^4*(1 + (b*x^3)/a)^(1/3)*Hypergeometric2F1[4/3, 4/3, 7/3, -((b*
x^3)/a)])/(4*a*(a + b*x^3)^(1/3))
```

IntegrateAlgebraic [A] time = 0.37, size = 158, normalized size = 1.60

$$-\frac{d\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}\,x\right)}{3b^{4/3}}+\frac{d\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{b}\,x}{2\sqrt[3]{a+bx^3}+\sqrt[3]{b}\,x}\right)}{\sqrt{3}\,b^{4/3}}+\frac{d\log\left(\sqrt[3]{b}\,x\sqrt[3]{a+bx^3}+\left(a+bx^3\right)^{2/3}+b^{2/3}x^2\right)}{6b^{4/3}}-\frac{x(ad-bc)}{ab\sqrt[3]{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c + d*x^3)/(a + b*x^3)^(4/3),x]
```

```
[Out] -(((-(b*c) + a*d)*x)/(a*b*(a + b*x^3)^(1/3))) + (d*ArcTan[(Sqrt[3]*b^(1/3)*
x)/(b^(1/3)*x + 2*(a + b*x^3)^(1/3))])/(Sqrt[3]*b^(4/3)) - (d*Log[-(b^(1/3)
*x) + (a + b*x^3)^(1/3)])/(3*b^(4/3)) + (d*Log[b^(2/3)*x^2 + b^(1/3)*x*(a +
 b*x^3)^(1/3) + (a + b*x^3)^(2/3)])/(6*b^(4/3))
```

fricas [B] time = 1.06, size = 488, normalized size = 4.93

$$\left[\frac{3\sqrt{\frac{1}{3}}\left(ab^2dx^3+a^2bd\right)\sqrt{\frac{(-b)^{\frac{1}{3}}}{b}}\log\left(3bx^3-3\left(bx^3+a\right)^{\frac{1}{3}}(-b)^{\frac{2}{3}}x^2-3\sqrt{\frac{1}{3}}\left((-b)^{\frac{1}{3}}bx^3-\left(bx^3+a\right)^{\frac{1}{3}}bx^2+2\left(bx^3+a\right)^{\frac{2}{3}}(-b)^{\frac{2}{3}}x\right)\sqrt{\frac{(-b)^{\frac{1}{3}}}{b}}+2a\right)+6\left(bx^3+a\right)^{\frac{2}{3}}\left(b^2c-abd\right)x-2\left(abdx^3+a^2d\right)(-b)^{\frac{2}{3}}\log\left(\frac{(-b)^{\frac{1}{3}}x+\left(bx^3+a\right)^{\frac{1}{3}}}{x}\right)+\left(abdx^3+a^2d\right)(-b)^{\frac{2}{3}}\log\left(\frac{(-b)^{\frac{2}{3}}x^2-\left(bx^3+a\right)^{\frac{1}{3}}(-b)^{\frac{1}{3}}x+\left(bx^3+a\right)^{\frac{2}{3}}}{x^2}\right)}{6\left(ab^3x^3+a^2b^2\right)},\ -\frac{6\sqrt{\frac{1}{3}}\left(ab^2dx^3+a^2bd\right)\sqrt{-\frac{(-b)^{\frac{1}{3}}}{b}}\arctan\left(-\frac{\sqrt{\frac{1}{3}}\left((-b)^{\frac{1}{3}}x-2\left(bx^3+a\right)^{\frac{1}{3}}\right)\sqrt{-\frac{(-b)^{\frac{1}{3}}}{b}}}{x}\right)-6\left(bx^3+a\right)^{\frac{2}{3}}\left(b^2c-abd\right)x+2\left(abdx^3+a^2d\right)(-b)^{\frac{2}{3}}\log\left(\frac{(-b)^{\frac{1}{3}}x+\left(bx^3+a\right)^{\frac{1}{3}}}{x}\right)-\left(abdx^3+a^2d\right)(-b)^{\frac{2}{3}}\log\left(\frac{(-b)^{\frac{2}{3}}x^2-\left(bx^3+a\right)^{\frac{1}{3}}(-b)^{\frac{1}{3}}x+\left(bx^3+a\right)^{\frac{2}{3}}}{x^2}\right)}{6\left(ab^3x^3+a^2b^2\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)/(b*x^3+a)^(4/3),x, algorithm="fricas")
```

```
[Out] [1/6*(3*sqrt(1/3)*(a*b^2*d*x^3 + a^2*b*d)*sqrt((-b)^(1/3)/b)*log(3*b*x^3 -
3*(b*x^3 + a)^(1/3)*(-b)^(2/3)*x^2 - 3*sqrt(1/3)*((-b)^(1/3)*b*x^3 - (b*x^3
 + a)^(1/3)*b*x^2 + 2*(b*x^3 + a)^(2/3)*(-b)^(2/3)*x)*sqrt((-b)^(1/3)/b) +
2*a) + 6*(b*x^3 + a)^(2/3)*(b^2*c - a*b*d)*x - 2*(a*b*d*x^3 + a^2*d)*(-b)^(
2/3)*log(((-b)^(1/3)*x + (b*x^3 + a)^(1/3))/x) + (a*b*d*x^3 + a^2*d)*(-b)^(
2/3)*log(((-b)^(2/3)*x^2 - (b*x^3 + a)^(1/3)*(-b)^(1/3)*x + (b*x^3 + a)^(2/
3))/x^2))/(a*b^3*x^3 + a^2*b^2), -1/6*(6*sqrt(1/3)*(a*b^2*d*x^3 + a^2*b*d)*
sqrt(-(-b)^(1/3)/b)*arctan(-sqrt(1/3)*((-b)^(1/3)*x - 2*(b*x^3 + a)^(1/3))*
sqrt(-(-b)^(1/3)/b)/x) - 6*(b*x^3 + a)^(2/3)*(b^2*c - a*b*d)*x + 2*(a*b*d*x
^3 + a^2*d)*(-b)^(2/3)*log(((-b)^(1/3)*x + (b*x^3 + a)^(1/3))/x) - (a*b*d*x
^3 + a^2*d)*(-b)^(2/3)*log(((-b)^(2/3)*x^2 - (b*x^3 + a)^(1/3)*(-b)^(1/3)*x
 + (b*x^3 + a)^(2/3))/x^2))/(a*b^3*x^3 + a^2*b^2)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{dx^3+c}{\left(bx^3+a\right)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)/(b*x^3+a)^(4/3),x, algorithm="giac")
```

```
[Out] integrate((d*x^3 + c)/(b*x^3 + a)^(4/3), x)
```

maple [F] time = 0.38, size = 0, normalized size = 0.00

$$\int \frac{d\,x^3+c}{\left(b\,x^3+a\right)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c)/(b*x^3+a)^(4/3),x)
```

```
[Out] int((d*x^3+c)/(b*x^3+a)^(4/3),x)
```

maxima [A] time = 1.22, size = 134, normalized size = 1.35

$$-\frac{1}{6}d\left(\frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}}+\frac{2(bx^3+a)^{\frac{1}{3}}}{x}\right)}{3b^{\frac{1}{3}}}\right)}{b^{\frac{4}{3}}}+\frac{6x}{(bx^3+a)^{\frac{1}{3}}b}-\frac{\log\left(b^{\frac{2}{3}}+\frac{(bx^3+a)^{\frac{1}{3}}b^{\frac{1}{3}}}{x}+\frac{(bx^3+a)^{\frac{2}{3}}}{x^2}\right)}{b^{\frac{4}{3}}}+\frac{2\log\left(-b^{\frac{1}{3}}+\frac{(bx^3+a)^{\frac{1}{3}}}{x}\right)}{b^{\frac{4}{3}}}\right)+\frac{cx}{(bx^3+a)^{\frac{1}{3}}a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)/(b*x^3+a)^(4/3),x, algorithm="maxima")
```

```
[Out] -1/6*d*(2*sqrt(3)*arctan(1/3*sqrt(3)*(b^(1/3) + 2*(b*x^3 + a)^(1/3)/x)/b^(1
/3))/b^(4/3) + 6*x/((b*x^3 + a)^(1/3)*b) - log(b^(2/3) + (b*x^3 + a)^(1/3)*
b^(1/3)/x + (b*x^3 + a)^(2/3)/x^2)/b^(4/3) + 2*log(-b^(1/3) + (b*x^3 + a)^(
1/3)/x)/b^(4/3)) + c*x/((b*x^3 + a)^(1/3)*a)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{d\,x^3+c}{\left(b\,x^3+a\right)^{4/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3)/(a + b*x^3)^(4/3),x)
```

```
[Out] int((c + d*x^3)/(a + b*x^3)^(4/3), x)
```

sympy [C] time = 12.83, size = 71, normalized size = 0.72

$$\frac{cx\Gamma\left(\frac{1}{3}\right)}{3a^{\frac{4}{3}}\sqrt[3]{1+\frac{bx^3}{a}}\,\Gamma\left(\frac{4}{3}\right)}+\frac{dx^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{4}{3},\frac{4}{3}\\\frac{7}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{4}{3}}\Gamma\left(\frac{7}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c)/(b*x**3+a)**(4/3),x)
```

```
[Out] c*x*gamma(1/3)/(3*a**(4/3)*(1 + b*x**3/a)**(1/3)*gamma(4/3)) + d*x**4*gamma
(4/3)*hyper((4/3, 4/3), (7/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(4/3)*gamma
(7/3))
```

3.47 $\int \frac{c+dx^3}{\left(a+bx^3\right)^{7/3}}\,dx$

Optimal. Leaf size=47

$$\frac{3cx}{4a^2\sqrt[3]{a+bx^3}}+\frac{x\left(c+dx^3\right)}{4a\left(a+bx^3\right)^{4/3}}$$

Rubi [A] time = 0.01, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {378, 191}

$$\frac{3cx}{4a^2\sqrt[3]{a+bx^3}}+\frac{x\left(c+dx^3\right)}{4a\left(a+bx^3\right)^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3)/(a + b*x^3)^(7/3),x]
[Out] (3*c*x)/(4*a^2*(a + b*x^3)^(1/3)) + (x*(c + d*x^3))/(4*a*(a + b*x^3)^(4/3))
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 378

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> -Simp[(x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*n*(p + 1)), x] - Dist[(c
*q)/(a*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1, 0]
 && GtQ[q, 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{c+dx^3}{\left(a+bx^3\right)^{7/3}}\,dx &= \frac{x\left(c+dx^3\right)}{4a\left(a+bx^3\right)^{4/3}}+\frac{(3c)\int \frac{1}{\left(a+bx^3\right)^{4/3}}\,dx}{4a}\\ &= \frac{3cx}{4a^2\sqrt[3]{a+bx^3}}+\frac{x\left(c+dx^3\right)}{4a\left(a+bx^3\right)^{4/3}}\end{aligned}$$

Mathematica [A] time = 0.04, size = 37, normalized size = 0.79

$$\frac{x\left(4ac+adx^3+3bcx^3\right)}{4a^2\left(a+bx^3\right)^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^3)/(a + b*x^3)^(7/3),x]
[Out] (x*(4*a*c + 3*b*c*x^3 + a*d*x^3))/(4*a^2*(a + b*x^3)^(4/3))
```

IntegrateAlgebraic [A] time = 0.30, size = 37, normalized size = 0.79

$$\frac{x\left(4ac+adx^3+3bcx^3\right)}{4a^2\left(a+bx^3\right)^{4/3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c + d*x^3)/(a + b*x^3)^(7/3),x]
```

```
[Out] (x*(4*a*c + 3*b*c*x^3 + a*d*x^3))/(4*a^2*(a + b*x^3)^(4/3))
```

fricas [A] time = 1.23, size = 54, normalized size = 1.15

$$\frac{\left((3\,bc+ad)x^4+4\,acx\right)\left(bx^3+a\right)^{\frac{2}{3}}}{4\left(a^2b^2x^6+2\,a^3bx^3+a^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)/(b*x^3+a)^(7/3),x, algorithm="fricas")
```

```
[Out] 1/4*((3*b*c + a*d)*x^4 + 4*a*c*x)*(b*x^3 + a)^(2/3)/(a^2*b^2*x^6 + 2*a^3*b*
x^3 + a^4)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{dx^3+c}{\left(bx^3+a\right)^{\frac{7}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)/(b*x^3+a)^(7/3),x, algorithm="giac")
```

```
[Out] integrate((d*x^3 + c)/(b*x^3 + a)^(7/3), x)
```

maple [A] time = 0.05, size = 34, normalized size = 0.72

$$\frac{\left(ad\,x^3+3bc\,x^3+4ac\right)x}{4\left(b\,x^3+a\right)^{\frac{4}{3}}a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c)/(b*x^3+a)^(7/3),x)
```

```
[Out] 1/4*x*(a*d*x^3+3*b*c*x^3+4*a*c)/(b*x^3+a)^(4/3)/a^2
```

maxima [A] time = 0.47, size = 51, normalized size = 1.09

$$-\frac{\left(b-\frac{4\left(bx^3+a\right)}{x^3}\right)cx^4}{4\left(bx^3+a\right)^{\frac{4}{3}}a^2}+\frac{dx^4}{4\left(bx^3+a\right)^{\frac{4}{3}}a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)/(b*x^3+a)^(7/3),x, algorithm="maxima")
```

```
[Out] -1/4*(b - 4*(b*x^3 + a)/x^3)*c*x^4/((b*x^3 + a)^(4/3)*a^2) + 1/4*d*x^4/((b*
x^3 + a)^(4/3)*a)
```

mupad [B] time = 1.37, size = 33, normalized size = 0.70

$$\frac{4\,a\,c\,x+a\,d\,x^4+3\,b\,c\,x^4}{4\,a^2\,{\left(b\,x^3+a\right)}^{4/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3)/(a + b*x^3)^(7/3),x)
```

```
[Out] (4*a*c*x + a*d*x^4 + 3*b*c*x^4)/(4*a^2*(a + b*x^3)^(4/3))
```

sympy [B] time = 82.05, size = 190, normalized size = 4.04

$$c\left(\frac{4ax\Gamma\left(\frac{1}{3}\right)}{9a^{\frac{10}{3}}\sqrt[3]{1+\frac{bx^3}{a}}\Gamma\left(\frac{7}{3}\right)+9a^{\frac{7}{3}}bx^3\sqrt[3]{1+\frac{bx^3}{a}}\Gamma\left(\frac{7}{3}\right)}+\frac{3bx^4\Gamma\left(\frac{1}{3}\right)}{9a^{\frac{10}{3}}\sqrt[3]{1+\frac{bx^3}{a}}\Gamma\left(\frac{7}{3}\right)+9a^{\frac{7}{3}}bx^3\sqrt[3]{1+\frac{bx^3}{a}}\Gamma\left(\frac{7}{3}\right)}\right)+\frac{dx^4\Gamma\left(\frac{4}{3}\right)}{3a^{\frac{7}{3}}\sqrt[3]{1+\frac{bx^3}{a}}\Gamma\left(\frac{7}{3}\right)+3a^{\frac{4}{3}}bx^3\sqrt[3]{1+\frac{bx^3}{a}}\Gamma\left(\frac{7}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c)/(b*x**3+a)**(7/3),x)
```

```
[Out] c*(4*a*x*gamma(1/3)/(9*a**(10/3)*(1 + b*x**3/a)**(1/3)*gamma(7/3) + 9*a**(7
/3)*b*x**3*(1 + b*x**3/a)**(1/3)*gamma(7/3)) + 3*b*x**4*gamma(1/3)/(9*a**(1
0/3)*(1 + b*x**3/a)**(1/3)*gamma(7/3) + 9*a**(7/3)*b*x**3*(1 + b*x**3/a)**(
1/3)*gamma(7/3))) + d*x**4*gamma(4/3)/(3*a**(7/3)*(1 + b*x**3/a)**(1/3)*gam
ma(7/3) + 3*a**(4/3)*b*x**3*(1 + b*x**3/a)**(1/3)*gamma(7/3))
```

3.48 $\int \frac{c+dx^3}{\left(a+bx^3\right)^{10/3}}\,dx$

Optimal. Leaf size=91

$$\frac{3x(ad+6bc)}{28a^3b\sqrt[3]{a+bx^3}}+\frac{x(ad+6bc)}{28a^2b\left(a+bx^3\right)^{4/3}}+\frac{x(bc-ad)}{7ab\left(a+bx^3\right)^{7/3}}$$

Rubi [A] time = 0.03, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {385, 192, 191}

$$\frac{3x(ad+6bc)}{28a^3b\sqrt[3]{a+bx^3}}+\frac{x(ad+6bc)}{28a^2b\left(a+bx^3\right)^{4/3}}+\frac{x(bc-ad)}{7ab\left(a+bx^3\right)^{7/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3)/(a + b*x^3)^(10/3),x]
```

```
[Out] ((b*c - a*d)*x)/(7*a*b*(a + b*x^3)^(7/3)) + ((6*b*c + a*d)*x)/(28*a^2*b*(a
+ b*x^3)^(4/3)) + (3*(6*b*c + a*d)*x)/(28*a^3*b*(a + b*x^3)^(1/3))
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{c+dx^3}{\left(a+bx^3\right)^{10/3}}\,dx &= \frac{(bc-ad)x}{7ab\left(a+bx^3\right)^{7/3}}+\frac{(6bc+ad)\int \frac{1}{\left(a+bx^3\right)^{7/3}}\,dx}{7ab}\\ &= \frac{(bc-ad)x}{7ab\left(a+bx^3\right)^{7/3}}+\frac{(6bc+ad)x}{28a^2b\left(a+bx^3\right)^{4/3}}+\frac{(3(6bc+ad))\int \frac{1}{\left(a+bx^3\right)^{4/3}}\,dx}{28a^2b}\\ &= \frac{(bc-ad)x}{7ab\left(a+bx^3\right)^{7/3}}+\frac{(6bc+ad)x}{28a^2b\left(a+bx^3\right)^{4/3}}+\frac{3(6bc+ad)x}{28a^3b\sqrt[3]{a+bx^3}}\end{aligned}$$

Mathematica [A] time = 0.04, size = 59, normalized size = 0.65

$$\frac{7a^2\left(4cx+dx^4\right)+3abx^4\left(14c+dx^3\right)+18b^2cx^7}{28a^3\left(a+bx^3\right)^{7/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^3)/(a + b*x^3)^(10/3),x]
```

```
[Out] (18*b^2*c*x^7 + 3*a*b*x^4*(14*c + d*x^3) + 7*a^2*(4*c*x + d*x^4))/(28*a^3*(
a + b*x^3)^(7/3))
```

IntegrateAlgebraic [A] time = 0.39, size = 60, normalized size = 0.66

$$\frac{x\left(28a^2c+7a^2dx^3+42abcx^3+3abdx^6+18b^2cx^6\right)}{28a^3\left(a+bx^3\right)^{7/3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c + d*x^3)/(a + b*x^3)^(10/3),x]
```

```
[Out] (x*(28*a^2*c + 42*a*b*c*x^3 + 7*a^2*d*x^3 + 18*b^2*c*x^6 + 3*a*b*d*x^6))/(2
8*a^3*(a + b*x^3)^(7/3))
```

fricas [A] time = 0.72, size = 87, normalized size = 0.96

$$\frac{\left(3\left(6\,b^2c+abd\right)x^7+7\left(6\,abc+a^2d\right)x^4+28\,a^2cx\right)\left(bx^3+a\right)^{\frac{2}{3}}}{28\left(a^3b^3x^9+3\,a^4b^2x^6+3\,a^5bx^3+a^6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)/(b*x^3+a)^(10/3),x, algorithm="fricas")
```

```
[Out] 1/28*(3*(6*b^2*c + a*b*d)*x^7 + 7*(6*a*b*c + a^2*d)*x^4 + 28*a^2*c*x)*(b*x^
3 + a)^(2/3)/(a^3*b^3*x^9 + 3*a^4*b^2*x^6 + 3*a^5*b*x^3 + a^6)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{dx^3+c}{\left(bx^3+a\right)^{\frac{10}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)/(b*x^3+a)^(10/3),x, algorithm="giac")
```

```
[Out] integrate((d*x^3 + c)/(b*x^3 + a)^(10/3), x)
```

maple [A] time = 0.04, size = 57, normalized size = 0.63

$$\frac{\left(3abd\,x^6+18b^2c\,x^6+7a^2d\,x^3+42abc\,x^3+28a^2c\right)x}{28\left(b\,x^3+a\right)^{\frac{7}{3}}a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c)/(b*x^3+a)^(10/3),x)
```

```
[Out] 1/28*x*(3*a*b*d*x^6+18*b^2*c*x^6+7*a^2*d*x^3+42*a*b*c*x^3+28*a^2*c)/(b*x^3+
a)^(7/3)/a^3
```

maxima [A] time = 0.61, size = 86, normalized size = 0.95

$$-\frac{\left(4\,b-\frac{7\,\left(bx^3+a\right)}{x^3}\right)dx^7}{28\,\left(bx^3+a\right)^{\frac{7}{3}}a^2}+\frac{\left(2\,b^2-\frac{7\,\left(bx^3+a\right)b}{x^3}+\frac{14\,\left(bx^3+a\right)^2}{x^6}\right)cx^7}{14\,\left(bx^3+a\right)^{\frac{7}{3}}a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)/(b*x^3+a)^(10/3),x, algorithm="maxima")
```

```
[Out] -1/28*(4*b - 7*(b*x^3 + a)/x^3)*d*x^7/((b*x^3 + a)^(7/3)*a^2) + 1/14*(2*b^2
 - 7*(b*x^3 + a)*b/x^3 + 14*(b*x^3 + a)^2/x^6)*c*x^7/((b*x^3 + a)^(7/3)*a^3
)
```

mupad [B] time = 1.42, size = 87, normalized size = 0.96

$$\frac{3\,a\,d\,x\,\left(b\,x^3+a\right)^2-4\,a^3\,d\,x+18\,b\,c\,x\,\left(b\,x^3+a\right)^2+a^2\,d\,x\,\left(b\,x^3+a\right)+4\,a^2\,b\,c\,x+6\,a\,b\,c\,x\,\left(b\,x^3+a\right)}{28\,a^3\,b\,\left(b\,x^3+a\right)^{7/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3)/(a + b*x^3)^(10/3),x)
```

```
[Out] (3*a*d*x*(a + b*x^3)^2 - 4*a^3*d*x + 18*b*c*x*(a + b*x^3)^2 + a^2*d*x*(a +
b*x^3) + 4*a^2*b*c*x + 6*a*b*c*x*(a + b*x^3))/(28*a^3*b*(a + b*x^3)^(7/3))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c)/(b*x**3+a)**(10/3),x)
```

```
[Out] Timed out
```

3.49 $\int \frac{c+dx^3}{\left(a+bx^3\right)^{13/3}}\,dx$

Optimal. Leaf size=121

$$\frac{9x(ad+9bc)}{140a^4b\sqrt[3]{a+bx^3}}+\frac{3x(ad+9bc)}{140a^3b\left(a+bx^3\right)^{4/3}}+\frac{x(ad+9bc)}{70a^2b\left(a+bx^3\right)^{7/3}}+\frac{x(bc-ad)}{10ab\left(a+bx^3\right)^{10/3}}$$

Rubi [A] time = 0.04, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {385, 192, 191}

$$\frac{9x(ad+9bc)}{140a^4b\sqrt[3]{a+bx^3}}+\frac{3x(ad+9bc)}{140a^3b\left(a+bx^3\right)^{4/3}}+\frac{x(ad+9bc)}{70a^2b\left(a+bx^3\right)^{7/3}}+\frac{x(bc-ad)}{10ab\left(a+bx^3\right)^{10/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3)/(a + b*x^3)^(13/3),x]
```

```
[Out] ((b*c - a*d)*x)/(10*a*b*(a + b*x^3)^(10/3)) + ((9*b*c + a*d)*x)/(70*a^2*b*(
a + b*x^3)^(7/3)) + (3*(9*b*c + a*d)*x)/(140*a^3*b*(a + b*x^3)^(4/3)) + (9*
(9*b*c + a*d)*x)/(140*a^4*b*(a + b*x^3)^(1/3))
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rubi steps

$$\int \frac{c+dx^3}{\left(a+bx^3\right)^{13/3}}\,dx=\frac{(bc-ad)x}{10ab\left(a+bx^3\right)^{10/3}}+\frac{(9bc+ad)\int \frac{1}{\left(a+bx^3\right)^{10/3}}\,dx}{10ab}$$
$$=\frac{(bc-ad)x}{10ab\left(a+bx^3\right)^{10/3}}+\frac{(9bc+ad)x}{70a^2b\left(a+bx^3\right)^{7/3}}+\frac{(3(9bc+ad))\int \frac{1}{\left(a+bx^3\right)^{7/3}}\,dx}{35a^2b}$$
$$=\frac{(bc-ad)x}{10ab\left(a+bx^3\right)^{10/3}}+\frac{(9bc+ad)x}{70a^2b\left(a+bx^3\right)^{7/3}}+\frac{3(9bc+ad)x}{140a^3b\left(a+bx^3\right)^{4/3}}+\frac{(9(9bc+ad))\int \frac{1}{\left(a+bx^3\right)^{4/3}}\,dx}{140a^3b}$$
$$=\frac{(bc-ad)x}{10ab\left(a+bx^3\right)^{10/3}}+\frac{(9bc+ad)x}{70a^2b\left(a+bx^3\right)^{7/3}}+\frac{3(9bc+ad)x}{140a^3b\left(a+bx^3\right)^{4/3}}+\frac{9(9bc+ad)x}{140a^4b\sqrt[3]{a+bx^3}}$$

Mathematica [A] time = 0.03, size = 80, normalized size = 0.66

$$\frac{x\left(35a^3\left(4c+dx^3\right)+15a^2bx^3\left(21c+2dx^3\right)+9ab^2x^6\left(30c+dx^3\right)+81b^3cx^9\right)}{140a^4\left(a+bx^3\right)^{10/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^3)/(a + b*x^3)^(13/3),x]
```

```
[Out] (x*(81*b^3*c*x^9 + 35*a^3*(4*c + d*x^3) + 9*a*b^2*x^6*(30*c + d*x^3) + 15*a
^2*b*x^3*(21*c + 2*d*x^3)))/(140*a^4*(a + b*x^3)^(10/3))
```

IntegrateAlgebraic [A] time = 0.57, size = 84, normalized size = 0.69

$$\frac{x\left(140a^3c+35a^3dx^3+315a^2bcx^3+30a^2bdx^6+270ab^2cx^6+9ab^2dx^9+81b^3cx^9\right)}{140a^4\left(a+bx^3\right)^{10/3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c + d*x^3)/(a + b*x^3)^(13/3),x]
```

```
[Out] (x*(140*a^3*c + 315*a^2*b*c*x^3 + 35*a^3*d*x^3 + 270*a*b^2*c*x^6 + 30*a^2*b
*d*x^6 + 81*b^3*c*x^9 + 9*a*b^2*d*x^9))/(140*a^4*(a + b*x^3)^(10/3))
```

fricas [A] time = 0.91, size = 121, normalized size = 1.00

$$\frac{\left(9\left(9\,b^3c+ab^2d\right)x^{10}+30\left(9\,ab^2c+a^2bd\right)x^7+140\,a^3cx+35\left(9\,a^2bc+a^3d\right)x^4\right)\left(bx^3+a\right)^{\frac{2}{3}}}{140\left(a^4b^4x^{12}+4\,a^5b^3x^9+6\,a^6b^2x^6+4\,a^7bx^3+a^8\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)/(b*x^3+a)^(13/3),x, algorithm="fricas")
```

```
[Out] 1/140*(9*(9*b^3*c + a*b^2*d)*x^10 + 30*(9*a*b^2*c + a^2*b*d)*x^7 + 140*a^3*
c*x + 35*(9*a^2*b*c + a^3*d)*x^4)*(b*x^3 + a)^(2/3)/(a^4*b^4*x^12 + 4*a^5*b
^3*x^9 + 6*a^6*b^2*x^6 + 4*a^7*b*x^3 + a^8)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{dx^3+c}{\left(bx^3+a\right)^{\frac{13}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)/(b*x^3+a)^(13/3),x, algorithm="giac")
```

```
[Out] integrate((d*x^3 + c)/(b*x^3 + a)^(13/3), x)
```

maple [A] time = 0.04, size = 81, normalized size = 0.67

$$\frac{\left(9a\,b^{2}d\,x^{9}+81b^{3}c\,x^{9}+30a^{2}bd\,x^{6}+270a\,b^{2}c\,x^{6}+35a^{3}d\,x^{3}+315a^{2}bc\,x^{3}+140c\,a^{3}\right)x}{140\left(b\,x^{3}+a\right)^{\frac{10}{3}}a^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c)/(b*x^3+a)^(13/3),x)
```

```
[Out] 1/140*x*(9*a*b^2*d*x^9+81*b^3*c*x^9+30*a^2*b*d*x^6+270*a*b^2*c*x^6+35*a^3*d
*x^3+315*a^2*b*c*x^3+140*a^3*c)/(b*x^3+a)^(10/3)/a^4
```

maxima [A] time = 0.50, size = 120, normalized size = 0.99

$$\frac{\left(14\,b^{2}-\frac{40\left(bx^{3}+a\right)b}{x^{3}}+\frac{35\left(bx^{3}+a\right)^{2}}{x^{6}}\right)dx^{10}}{140\left(bx^{3}+a\right)^{\frac{10}{3}}a^{3}}-\frac{\left(14\,b^{3}-\frac{60\left(bx^{3}+a\right)b^{2}}{x^{3}}+\frac{105\left(bx^{3}+a\right)^{2}b}{x^{6}}-\frac{140\left(bx^{3}+a\right)^{3}}{x^{9}}\right)cx^{10}}{140\left(bx^{3}+a\right)^{\frac{10}{3}}a^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)/(b*x^3+a)^(13/3),x, algorithm="maxima")
```

```
[Out] 1/140*(14*b^2 - 40*(b*x^3 + a)*b/x^3 + 35*(b*x^3 + a)^2/x^6)*d*x^10/((b*x^3
 + a)^(10/3)*a^3) - 1/140*(14*b^3 - 60*(b*x^3 + a)*b^2/x^3 + 105*(b*x^3 + a
)^2*b/x^6 - 140*(b*x^3 + a)^3/x^9)*c*x^10/((b*x^3 + a)^(10/3)*a^4)
```

mupad [B] time = 1.46, size = 105, normalized size = 0.87

$$\frac{x\left(\frac{c}{10\,a}-\frac{d}{10\,b}\right)}{\left(b\,x^{3}+a\right)^{10/3}}+\frac{x\,\left(a\,d+9\,b\,c\right)}{70\,a^{2}\,b\left(b\,x^{3}+a\right)^{7/3}}+\frac{x\,\left(3\,a\,d+27\,b\,c\right)}{140\,a^{3}\,b\left(b\,x^{3}+a\right)^{4/3}}+\frac{x\,\left(9\,a\,d+81\,b\,c\right)}{140\,a^{4}\,b\left(b\,x^{3}+a\right)^{1/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3)/(a + b*x^3)^(13/3),x)
```

```
[Out] (x*(c/(10*a) - d/(10*b)))/(a + b*x^3)^(10/3) + (x*(a*d + 9*b*c))/(70*a^2*b*
(a + b*x^3)^(7/3)) + (x*(3*a*d + 27*b*c))/(140*a^3*b*(a + b*x^3)^(4/3)) + (
x*(9*a*d + 81*b*c))/(140*a^4*b*(a + b*x^3)^(1/3))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c)/(b*x**3+a)**(13/3),x)
```

```
[Out] Timed out
```

3.50 $\int \frac{c+dx^3}{\left(a+bx^3\right)^{16/3}}\,dx$

Optimal. Leaf size=151

$$\frac{81x(ad+12bc)}{1820a^5b\sqrt[3]{a+bx^3}}+\frac{27x(ad+12bc)}{1820a^4b\left(a+bx^3\right)^{4/3}}+\frac{9x(ad+12bc)}{910a^3b\left(a+bx^3\right)^{7/3}}+\frac{x(ad+12bc)}{130a^2b\left(a+bx^3\right)^{10/3}}+\frac{x(bc-ad)}{13ab\left(a+bx^3\right)^{13/3}}$$

Rubi [A] time = 0.05, antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {385, 192, 191}

$$\frac{81x(ad+12bc)}{1820a^5b\sqrt[3]{a+bx^3}}+\frac{27x(ad+12bc)}{1820a^4b\left(a+bx^3\right)^{4/3}}+\frac{9x(ad+12bc)}{910a^3b\left(a+bx^3\right)^{7/3}}+\frac{x(ad+12bc)}{130a^2b\left(a+bx^3\right)^{10/3}}+\frac{x(bc-ad)}{13ab\left(a+bx^3\right)^{13/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3)/(a + b*x^3)^(16/3),x]
```

```
[Out] ((b*c - a*d)*x)/(13*a*b*(a + b*x^3)^(13/3)) + ((12*b*c + a*d)*x)/(130*a^2*b
*(a + b*x^3)^(10/3)) + (9*(12*b*c + a*d)*x)/(910*a^3*b*(a + b*x^3)^(7/3)) +
 (27*(12*b*c + a*d)*x)/(1820*a^4*b*(a + b*x^3)^(4/3)) + (81*(12*b*c + a*d)*
x)/(1820*a^5*b*(a + b*x^3)^(1/3))
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rubi steps

$$\int \frac{c+dx^3}{\left(a+bx^3\right)^{16/3}}\,dx = \frac{(bc-ad)x}{13ab\left(a+bx^3\right)^{13/3}} + \frac{(12bc+ad)\int \frac{1}{\left(a+bx^3\right)^{13/3}}\,dx}{13ab}$$

$$= \frac{(bc-ad)x}{13ab\left(a+bx^3\right)^{13/3}} + \frac{(12bc+ad)x}{130a^2b\left(a+bx^3\right)^{10/3}} + \frac{(9(12bc+ad))\int \frac{1}{\left(a+bx^3\right)^{10/3}}\,dx}{130a^2b}$$

$$= \frac{(bc-ad)x}{13ab\left(a+bx^3\right)^{13/3}} + \frac{(12bc+ad)x}{130a^2b\left(a+bx^3\right)^{10/3}} + \frac{9(12bc+ad)x}{910a^3b\left(a+bx^3\right)^{7/3}} + \frac{(27(12bc+ad))\int \frac{}{(a}}{455a^3b}$$

$$= \frac{(bc-ad)x}{13ab\left(a+bx^3\right)^{13/3}} + \frac{(12bc+ad)x}{130a^2b\left(a+bx^3\right)^{10/3}} + \frac{9(12bc+ad)x}{910a^3b\left(a+bx^3\right)^{7/3}} + \frac{27(12bc+ad)x}{1820a^4b\left(a+bx^3\right)^{4/3}}$$

$$= \frac{(bc-ad)x}{13ab\left(a+bx^3\right)^{13/3}} + \frac{(12bc+ad)x}{130a^2b\left(a+bx^3\right)^{10/3}} + \frac{9(12bc+ad)x}{910a^3b\left(a+bx^3\right)^{7/3}} + \frac{27(12bc+ad)x}{1820a^4b\left(a+bx^3\right)^{4/3}}$$

Mathematica [A] time = 0.06, size = 100, normalized size = 0.66

$$\frac{x\left(455a^4\left(4c+dx^3\right)+195a^3bx^3\left(28c+3dx^3\right)+351a^2b^2x^6\left(20c+dx^3\right)+81ab^3x^9\left(52c+dx^3\right)+972b^4cx^{12}\right)}{1820a^5\left(a+bx^3\right)^{13/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^3)/(a + b*x^3)^(16/3),x]
```

```
[Out] (x*(972*b^4*c*x^12 + 455*a^4*(4*c + d*x^3) + 351*a^2*b^2*x^6*(20*c + d*x^3)
 + 81*a*b^3*x^9*(52*c + d*x^3) + 195*a^3*b*x^3*(28*c + 3*d*x^3)))/(1820*a^5
*(a + b*x^3)^(13/3))
```

IntegrateAlgebraic [A] time = 0.87, size = 108, normalized size = 0.72

$$\frac{x\left(1820a^4c+455a^4dx^3+5460a^3bcx^3+585a^3bdx^6+7020a^2b^2cx^6+351a^2b^2dx^9+4212ab^3cx^9+81ab^3dx^{12}+972b^4cx^{12}\right)}{1820a^5\left(a+bx^3\right)^{13/3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c + d*x^3)/(a + b*x^3)^(16/3),x]
```

```
[Out] (x*(1820*a^4*c + 5460*a^3*b*c*x^3 + 455*a^4*d*x^3 + 7020*a^2*b^2*c*x^6 + 58
5*a^3*b*d*x^6 + 4212*a*b^3*c*x^9 + 351*a^2*b^2*d*x^9 + 972*b^4*c*x^12 + 81*
a*b^3*d*x^12))/(1820*a^5*(a + b*x^3)^(13/3))
```

fricas [A] time = 1.30, size = 155, normalized size = 1.03

$$\frac{\left(81\left(12\,b^4c+ab^3d\right)x^{13}+351\left(12\,ab^3c+a^2b^2d\right)x^{10}+585\left(12\,a^2b^2c+a^3bd\right)x^7+1820\,a^4cx+455\left(12\,a^3bc+a^4d\right)x^4\right)\left(bx^3+a\right)^{\frac{2}{3}}}{1820\left(a^5b^5x^{15}+5\,a^6b^4x^{12}+10\,a^7b^3x^9+10\,a^8b^2x^6+5\,a^9bx^3+a^{10}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)/(b*x^3+a)^(16/3),x, algorithm="fricas")
```

```
[Out] 1/1820*(81*(12*b^4*c + a*b^3*d)*x^13 + 351*(12*a*b^3*c + a^2*b^2*d)*x^10 +
585*(12*a^2*b^2*c + a^3*b*d)*x^7 + 1820*a^4*c*x + 455*(12*a^3*b*c + a^4*d)*
x^4)*(b*x^3 + a)^(2/3)/(a^5*b^5*x^15 + 5*a^6*b^4*x^12 + 10*a^7*b^3*x^9 + 10
*a^8*b^2*x^6 + 5*a^9*b*x^3 + a^10)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{dx^3 + c}{\left(bx^3 + a\right)^{\frac{16}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)/(b*x^3+a)^(16/3),x, algorithm="giac")
```

```
[Out] integrate((d*x^3 + c)/(b*x^3 + a)^(16/3), x)
```

maple [A] time = 0.05, size = 105, normalized size = 0.70

$$\frac{\left(81 a\, b^{3} d\, x^{12}+972 b^{4} c\, x^{12}+351 a^{2} b^{2} d\, x^{9}+4212 a\, b^{3} c\, x^{9}+585 a^{3} b d\, x^{6}+7020 a^{2} b^{2} c\, x^{6}+455 a^{4} d\, x^{3}+5460 a^{3} b c\, x^{3}+1820 c\, a^{4}\right) x}{1820 \left(b\, x^{3}+a\right)^{\frac{13}{3}} a^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c)/(b*x^3+a)^(16/3),x)
```

```
[Out] 1/1820*x*(81*a*b^3*d*x^12+972*b^4*c*x^12+351*a^2*b^2*d*x^9+4212*a*b^3*c*x^9
+585*a^3*b*d*x^6+7020*a^2*b^2*c*x^6+455*a^4*d*x^3+5460*a^3*b*c*x^3+1820*a^4
*c)/(b*x^3+a)^(13/3)/a^5
```

maxima [A] time = 0.66, size = 154, normalized size = 1.02

$$-\frac{\left(140\, b^{3}-\frac{546\left(bx^{3}+a\right)b^{2}}{x^{3}}+\frac{780\left(bx^{3}+a\right)^{2} b}{x^{6}}-\frac{455\left(bx^{3}+a\right)^{3}}{x^{9}}\right)dx^{13}}{1820\left(bx^{3}+a\right)^{\frac{13}{3}} a^{4}}+\frac{\left(35\, b^{4}-\frac{182\left(bx^{3}+a\right)b^{3}}{x^{3}}+\frac{390\left(bx^{3}+a\right)^{2} b^{2}}{x^{6}}-\frac{455\left(bx^{3}+a\right)^{3} b}{x^{9}}+\frac{455\left(bx^{3}+a\right)^{4}}{x^{12}}\right)cx^{13}}{455\left(bx^{3}+a\right)^{\frac{13}{3}} a^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)/(b*x^3+a)^(16/3),x, algorithm="maxima")
```

```
[Out] -1/1820*(140*b^3 - 546*(b*x^3 + a)*b^2/x^3 + 780*(b*x^3 + a)^2*b/x^6 - 455*
(b*x^3 + a)^3/x^9)*d*x^13/((b*x^3 + a)^(13/3)*a^4) + 1/455*(35*b^4 - 182*(b
*x^3 + a)*b^3/x^3 + 390*(b*x^3 + a)^2*b^2/x^6 - 455*(b*x^3 + a)^3*b/x^9 + 4
55*(b*x^3 + a)^4/x^12)*c*x^13/((b*x^3 + a)^(13/3)*a^5)
```

mupad [B] time = 1.45, size = 132, normalized size = 0.87

$$\frac{x\left(\frac{c}{13\, a}-\frac{d}{13\, b}\right)}{\left(b\, x^{3}+a\right)^{13/3}}+\frac{x\, \left(a\, d+12\, b\, c\right)}{130\, a^{2}\, b\, \left(b\, x^{3}+a\right)^{10/3}}+\frac{x\, \left(9\, a\, d+108\, b\, c\right)}{910\, a^{3}\, b\, \left(b\, x^{3}+a\right)^{7/3}}+\frac{x\, \left(27\, a\, d+324\, b\, c\right)}{1820\, a^{4}\, b\, \left(b\, x^{3}+a\right)^{4/3}}+\frac{x\, \left(81\, a\, d+972\, b\, c\right)}{1820\, a^{5}\, b\, \left(b\, x^{3}+a\right)^{1/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3)/(a + b*x^3)^(16/3),x)
```

```
[Out] (x*(c/(13*a) - d/(13*b)))/(a + b*x^3)^(13/3) + (x*(a*d + 12*b*c))/(130*a^2*
b*(a + b*x^3)^(10/3)) + (x*(9*a*d + 108*b*c))/(910*a^3*b*(a + b*x^3)^(7/3))
 + (x*(27*a*d + 324*b*c))/(1820*a^4*b*(a + b*x^3)^(4/3)) + (x*(81*a*d + 972
*b*c))/(1820*a^5*b*(a + b*x^3)^(1/3))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c)/(b*x**3+a)**(16/3),x)
```

```
[Out] Timed out
```

3.51 $\int \left(a+bx^3\right)^{5/3} \left(c+dx^3\right)^2 \, dx$

Optimal. Leaf size=262

$$\frac{x\left(a+bx^3\right)^{5/3}\left(a^2d^2-6abcd+27b^2c^2\right)}{162b^2}+\frac{5ax\left(a+bx^3\right)^{2/3}\left(a^2d^2-6abcd+27b^2c^2\right)}{486b^2}-\frac{5a^2\left(a^2d^2-6abcd+27b^2c\right.}{486b}$$

Rubi [A] time = 0.16, antiderivative size = 262, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {416, 388, 195, 239}

$$\frac{x\left(a+bx^3\right)^{5/3}\left(a^2d^2-6abcd+27b^2c^2\right)}{162b^2}+\frac{5ax\left(a+bx^3\right)^{2/3}\left(a^2d^2-6abcd+27b^2c^2\right)}{486b^2}-\frac{5a^2\left(a^2d^2-6abcd+27b^2c^2\right)\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}x\right)}{486b^{7/3}}+\frac{5a^2\left(a^2d^2-6abcd+27b^2c^2\right)\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{243\sqrt{3}b^{7/3}}+\frac{dx\left(a+bx^3\right)^{8/3}(15bc-4ad)}{108b^2}+\frac{dx\left(a+bx^3\right)^{8/3}\left(c+dx^3\right)}{12b}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^(5/3)*(c + d*x^3)^2,x]
```

```
[Out] (5*a*(27*b^2*c^2 - 6*a*b*c*d + a^2*d^2)*x*(a + b*x^3)^(2/3))/(486*b^2) + ((
27*b^2*c^2 - 6*a*b*c*d + a^2*d^2)*x*(a + b*x^3)^(5/3))/(162*b^2) + (d*(15*b
*c - 4*a*d)*x*(a + b*x^3)^(8/3))/(108*b^2) + (d*x*(a + b*x^3)^(8/3)*(c + d*
x^3))/(12*b) + (5*a^2*(27*b^2*c^2 - 6*a*b*c*d + a^2*d^2)*ArcTan[(1 + (2*b^(
1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]])/(243*Sqrt[3]*b^(7/3)) - (5*a^2*(27*b^2
*c^2 - 6*a*b*c*d + a^2*d^2)*Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)])/(486*b^(
7/3))
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 239

```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + (2*Rt[b, 3]
*x)/(a + b*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 416

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(d*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(b*(n*(p + q) + 1)),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rubi steps

$$
\begin{aligned}
\int \left(a+bx^3\right)^{5/3}\left(c+dx^3\right)^2\,dx &= \frac{dx\left(a+bx^3\right)^{8/3}\left(c+dx^3\right)}{12b}+\frac{\int \left(a+bx^3\right)^{5/3}\left(c(12bc-ad)+d(15bc-4ad)x^3\right)dx}{12b}\\
&= \frac{d(15bc-4ad)x\left(a+bx^3\right)^{8/3}}{108b^2}+\frac{dx\left(a+bx^3\right)^{8/3}\left(c+dx^3\right)}{12b}+\frac{\left(27b^2c^2-6abcd+a^2d\right.}{27b}\\
&= \frac{\left(27b^2c^2-6abcd+a^2d^2\right)x\left(a+bx^3\right)^{5/3}}{162b^2}+\frac{d(15bc-4ad)x\left(a+bx^3\right)^{8/3}}{108b^2}+\frac{dx\left(a+b\right.}{}\\
&= \frac{5a\left(27b^2c^2-6abcd+a^2d^2\right)x\left(a+bx^3\right)^{2/3}}{486b^2}+\frac{\left(27b^2c^2-6abcd+a^2d^2\right)x\left(a+bx^3\right)^{5}}{162b^2}\\
&= \frac{5a\left(27b^2c^2-6abcd+a^2d^2\right)x\left(a+bx^3\right)^{2/3}}{486b^2}+\frac{\left(27b^2c^2-6abcd+a^2d^2\right)x\left(a+bx^3\right)^{5}}{162b^2}
\end{aligned}
$$

Mathematica [A] time = 5.19, size = 238, normalized size = 0.91

$$
\frac{10a^2\left(a^2d^2-6abcd+27b^2c^2\right)\left(\log\left(\frac{b^{2/3}x^2}{\left(a+bx^3\right)^{2/3}}+\frac{\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1\right)-2\log\left(1-\frac{\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}\right)+2\sqrt{3}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)\right)+3\sqrt[3]{b}x\left(a+bx^3\right)^{2/3}\left(-20a^3d^2+15a^2bd\left(8c+dx^3\right)+18ab^2\left(24c^2+22cdx^3+7d^2x^6\right)+27b^3x^3\left(6c^2+8cdx^3+3d^2x^6\right)\right)}{2916b^{7/3}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^(5/3)*(c + d*x^3)^2,x]
```

```
[Out] (3*b^(1/3)*x*(a + b*x^3)^(2/3)*(-20*a^3*d^2 + 15*a^2*b*d*(8*c + d*x^3) + 27
*b^3*x^3*(6*c^2 + 8*c*d*x^3 + 3*d^2*x^6) + 18*a*b^2*(24*c^2 + 22*c*d*x^3 +
7*d^2*x^6)) + 10*a^2*(27*b^2*c^2 - 6*a*b*c*d + a^2*d^2)*(2*Sqrt[3]*ArcTan[(
1 + (2*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]] - 2*Log[1 - (b^(1/3)*x)/(a +
b*x^3)^(1/3)] + Log[1 + (b^(2/3)*x^2)/(a + b*x^3)^(2/3) + (b^(1/3)*x)/(a +
b*x^3)^(1/3)]))/(2916*b^(7/3))
```

IntegrateAlgebraic [A] time = 1.16, size = 325, normalized size = 1.24

$$
\frac{\left(a+bx^3\right)^{2/3}\left(-20a^3d^2x+120a^2bcdx+15a^2bd^2x^4+432ab^2c^2x+396ab^2cdx^4+126ab^2d^2x^7+162b^3c^2x^4+216b^3cdx^7+81b^3d^2x^{10}\right)}{972b^2}-\frac{5\left(a^4d^2-6a^3bcd+27a^2b^2c^2\right)\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}x\right)}{729b^{7/3}}+\frac{5\left(a^4d^2-6a^3bcd+27a^2b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{b}x}{2\sqrt[3]{a+bx^3}+\sqrt[3]{b}x}\right)}{243\sqrt{3}b^{7/3}}+\frac{5\left(a^4d^2-6a^3bcd+27a^2b^2c^2\right)\log\left(\sqrt[3]{b}x\sqrt[3]{a+bx^3}+\left(a+bx^3\right)^{2/3}+b^{2/3}x^2\right)}{1458b^{7/3}}
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x^3)^(5/3)*(c + d*x^3)^2,x]
```

```
[Out] ((a + b*x^3)^(2/3)*(432*a*b^2*c^2*x + 120*a^2*b*c*d*x - 20*a^3*d^2*x + 162*
b^3*c^2*x^4 + 396*a*b^2*c*d*x^4 + 15*a^2*b*d^2*x^4 + 216*b^3*c*d*x^7 + 126*
a*b^2*d^2*x^7 + 81*b^3*d^2*x^10))/(972*b^2) + (5*(27*a^2*b^2*c^2 - 6*a^3*b*
c*d + a^4*d^2)*ArcTan[(Sqrt[3]*b^(1/3)*x)/(b^(1/3)*x + 2*(a + b*x^3)^(1/3))
])/(243*Sqrt[3]*b^(7/3)) - (5*(27*a^2*b^2*c^2 - 6*a^3*b*c*d + a^4*d^2)*Log[
-(b^(1/3)*x) + (a + b*x^3)^(1/3)])/(729*b^(7/3)) + (5*(27*a^2*b^2*c^2 - 6*a
^3*b*c*d + a^4*d^2)*Log[b^(2/3)*x^2 + b^(1/3)*x*(a + b*x^3)^(1/3) + (a + b*
x^3)^(2/3)])/(1458*b^(7/3))
```

fricas [A] time = 1.22, size = 717, normalized size = 2.74

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(5/3)*(d*x^3+c)^2,x, algorithm="fricas")
```

```
[Out] [1/2916*(30*sqrt(1/3)*(27*a^2*b^3*c^2 - 6*a^3*b^2*c*d + a^4*b*d^2)*sqrt((-b
)^(1/3)/b)*log(3*b*x^3 - 3*(b*x^3 + a)^(1/3)*(-b)^(2/3)*x^2 - 3*sqrt(1/3)*(
(-b)^(1/3)*b*x^3 - (b*x^3 + a)^(1/3)*b*x^2 + 2*(b*x^3 + a)^(2/3)*(-b)^(2/3)
*x)*sqrt((-b)^(1/3)/b) + 2*a) - 20*(27*a^2*b^2*c^2 - 6*a^3*b*c*d + a^4*d^2)
*(-b)^(2/3)*log(((-b)^(1/3)*x + (b*x^3 + a)^(1/3))/x) + 10*(27*a^2*b^2*c^2
- 6*a^3*b*c*d + a^4*d^2)*(-b)^(2/3)*log(((-b)^(2/3)*x^2 - (b*x^3 + a)^(1/3)
*(-b)^(1/3)*x + (b*x^3 + a)^(2/3))/x^2) + 3*(81*b^4*d^2*x^10 + 18*(12*b^4*c
*d + 7*a*b^3*d^2)*x^7 + 3*(54*b^4*c^2 + 132*a*b^3*c*d + 5*a^2*b^2*d^2)*x^4
+ 4*(108*a*b^3*c^2 + 30*a^2*b^2*c*d - 5*a^3*b*d^2)*x)*(b*x^3 + a)^(2/3))/b^
3, -1/2916*(60*sqrt(1/3)*(27*a^2*b^3*c^2 - 6*a^3*b^2*c*d + a^4*b*d^2)*sqrt(
-(-b)^(1/3)/b)*arctan(-sqrt(1/3)*((-b)^(1/3)*x - 2*(b*x^3 + a)^(1/3))*sqrt(
-(-b)^(1/3)/b)/x) + 20*(27*a^2*b^2*c^2 - 6*a^3*b*c*d + a^4*d^2)*(-b)^(2/3)*
log(((-b)^(1/3)*x + (b*x^3 + a)^(1/3))/x) - 10*(27*a^2*b^2*c^2 - 6*a^3*b*c*
d + a^4*d^2)*(-b)^(2/3)*log(((-b)^(2/3)*x^2 - (b*x^3 + a)^(1/3)*(-b)^(1/3)*
x + (b*x^3 + a)^(2/3))/x^2) - 3*(81*b^4*d^2*x^10 + 18*(12*b^4*c*d + 7*a*b^3
*d^2)*x^7 + 3*(54*b^4*c^2 + 132*a*b^3*c*d + 5*a^2*b^2*d^2)*x^4 + 4*(108*a*b
^3*c^2 + 30*a^2*b^2*c*d - 5*a^3*b*d^2)*x)*(b*x^3 + a)^(2/3))/b^3]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx^3 + a\right)^{\frac{5}{3}} \left(dx^3 + c\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(5/3)*(d*x^3+c)^2,x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a)^(5/3)*(d*x^3 + c)^2, x)
```

maple [F] time = 0.38, size = 0, normalized size = 0.00

$$\int \left(b\,x^3 + a\right)^{\frac{5}{3}} \left(d\,x^3 + c\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(5/3)*(d*x^3+c)^2,x)
```

```
[Out] int((b*x^3+a)^(5/3)*(d*x^3+c)^2,x)
```

maxima [B] time = 1.29, size = 672, normalized size = 2.56

$$-\frac{1}{54}\left(\frac{10\sqrt{3}a^2\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}}+\frac{2(bx^3+a)^{\frac{1}{3}}}{x}\right)}{3b^{\frac{1}{3}}}\right)}{b^{\frac{1}{3}}}-\frac{5a^2\log\left(b^{\frac{2}{3}}+\frac{(bx^3+a)^{\frac{1}{3}}b^{\frac{1}{3}}}{x}+\frac{(bx^3+a)^{\frac{2}{3}}}{x^2}\right)}{b^{\frac{1}{3}}}+\frac{10a^2\log\left(-b^{\frac{1}{3}}+\frac{(bx^3+a)^{\frac{1}{3}}}{x}\right)}{b^{\frac{1}{3}}}+\frac{3\left(\frac{5(bx^3+a)^{\frac{2}{3}}a^2b}{x^2}-\frac{8(bx^3+a)^{\frac{5}{3}}a^2}{x^5}\right)}{b^2-\frac{2(bx^3+a)b}{x^3}+\frac{(bx^3+a)^2}{x^6}}\right)c^2+\frac{1}{243}\left(\frac{10\sqrt{3}a^3\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}}+\frac{2(bx^3+a)^{\frac{1}{3}}}{x}\right)}{3b^{\frac{1}{3}}}\right)}{b^{\frac{4}{3}}}-\frac{5a^3\log\left(b^{\frac{2}{3}}+\frac{(bx^3+a)^{\frac{1}{3}}b^{\frac{1}{3}}}{x}+\frac{(bx^3+a)^{\frac{2}{3}}}{x^2}\right)}{b^{\frac{4}{3}}}+\frac{10a^3\log\left(-b^{\frac{1}{3}}+\frac{(bx^3+a)^{\frac{1}{3}}}{x}\right)}{b^{\frac{4}{3}}}+\frac{3\left(\frac{5(bx^3+a)^{\frac{2}{3}}a^3b^2}{x^2}-\frac{13(bx^3+a)^{\frac{5}{3}}a^3b}{x^5}-\frac{10(bx^3+a)^{\frac{8}{3}}a^3}{x^8}\right)}{b^4-\frac{3(bx^3+a)b^3}{x^3}+\frac{3(bx^3+a)^2b^2}{x^6}-\frac{(bx^3+a)^3b}{x^9}}\right)cd-\frac{1}{2916}\left(\frac{20\sqrt{3}a^4\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}}+\frac{2(bx^3+a)^{\frac{1}{3}}}{x}\right)}{3b^{\frac{1}{3}}}\right)}{b^{\frac{7}{3}}}-\frac{10a^4\log\left(b^{\frac{2}{3}}+\frac{(bx^3+a)^{\frac{1}{3}}b^{\frac{1}{3}}}{x}+\frac{(bx^3+a)^{\frac{2}{3}}}{x^2}\right)}{b^{\frac{7}{3}}}+\frac{20a^4\log\left(-b^{\frac{1}{3}}+\frac{(bx^3+a)^{\frac{1}{3}}}{x}\right)}{b^{\frac{7}{3}}}+\frac{3\left(\frac{10(bx^3+a)^{\frac{2}{3}}a^4b^3}{x^2}-\frac{36(bx^3+a)^{\frac{5}{3}}a^4b^2}{x^5}-\frac{75(bx^3+a)^{\frac{8}{3}}a^4b}{x^8}+\frac{20(bx^3+a)^{\frac{11}{3}}a^4}{x^{11}}\right)}{b^6-\frac{4(bx^3+a)b^5}{x^3}+\frac{6(bx^3+a)^2b^4}{x^6}-\frac{4(bx^3+a)^3b^3}{x^9}+\frac{(bx^3+a)^4b^2}{x^{12}}}\right)d^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(5/3)*(d*x^3+c)^2,x, algorithm="maxima")
```

```
[Out] -1/54*(10*sqrt(3)*a^2*arctan(1/3*sqrt(3)*(b^(1/3) + 2*(b*x^3 + a)^(1/3)/x)/
b^(1/3))/b^(1/3) - 5*a^2*log(b^(2/3) + (b*x^3 + a)^(1/3)*b^(1/3)/x + (b*x^3
 + a)^(2/3)/x^2)/b^(1/3) + 10*a^2*log(-b^(1/3) + (b*x^3 + a)^(1/3)/x)/b^(1/
3) + 3*(5*(b*x^3 + a)^(2/3)*a^2*b/x^2 - 8*(b*x^3 + a)^(5/3)*a^2/x^5)/(b^2 -
 2*(b*x^3 + a)*b/x^3 + (b*x^3 + a)^2/x^6))*c^2 + 1/243*(10*sqrt(3)*a^3*arct
an(1/3*sqrt(3)*(b^(1/3) + 2*(b*x^3 + a)^(1/3)/x)/b^(1/3))/b^(4/3) - 5*a^3*l
og(b^(2/3) + (b*x^3 + a)^(1/3)*b^(1/3)/x + (b*x^3 + a)^(2/3)/x^2)/b^(4/3) +
 10*a^3*log(-b^(1/3) + (b*x^3 + a)^(1/3)/x)/b^(4/3) + 3*(5*(b*x^3 + a)^(2/3
)*a^3*b^2/x^2 - 13*(b*x^3 + a)^(5/3)*a^3*b/x^5 - 10*(b*x^3 + a)^(8/3)*a^3/x
^8)/(b^4 - 3*(b*x^3 + a)*b^3/x^3 + 3*(b*x^3 + a)^2*b^2/x^6 - (b*x^3 + a)^3*
b/x^9))*c*d - 1/2916*(20*sqrt(3)*a^4*arctan(1/3*sqrt(3)*(b^(1/3) + 2*(b*x^3
 + a)^(1/3)/x)/b^(1/3))/b^(7/3) - 10*a^4*log(b^(2/3) + (b*x^3 + a)^(1/3)*b^
(1/3)/x + (b*x^3 + a)^(2/3)/x^2)/b^(7/3) + 20*a^4*log(-b^(1/3) + (b*x^3 + a
```

```
)^(1/3)/x)/b^(7/3) + 3*(10*(b*x^3 + a)^(2/3)*a^4*b^3/x^2 - 36*(b*x^3 + a)^(
5/3)*a^4*b^2/x^5 - 75*(b*x^3 + a)^(8/3)*a^4*b/x^8 + 20*(b*x^3 + a)^(11/3)*a
^4/x^11)/(b^6 - 4*(b*x^3 + a)*b^5/x^3 + 6*(b*x^3 + a)^2*b^4/x^6 - 4*(b*x^3
+ a)^3*b^3/x^9 + (b*x^3 + a)^4*b^2/x^12))*d^2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \left(b\,x^3+a\right)^{5/3}\,\left(d\,x^3+c\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^(5/3)*(c + d*x^3)^2,x)
```

```
[Out] int((a + b*x^3)^(5/3)*(c + d*x^3)^2, x)
```

sympy [C] time = 13.13, size = 270, normalized size = 1.03

$$\frac{a^{\frac{5}{3}}c^{2}x\Gamma\left(\frac{1}{3}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{2}{3},\frac{1}{3}\\\frac{4}{3}\end{matrix}\middle|\frac{bx^{3}e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{4}{3}\right)}+\frac{2a^{\frac{5}{3}}cdx^{4}\Gamma\left(\frac{4}{3}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{2}{3},\frac{4}{3}\\\frac{7}{3}\end{matrix}\middle|\frac{bx^{3}e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{7}{3}\right)}+\frac{a^{\frac{5}{3}}d^{2}x^{7}\Gamma\left(\frac{7}{3}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{2}{3},\frac{7}{3}\\\frac{10}{3}\end{matrix}\middle|\frac{bx^{3}e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{10}{3}\right)}+\frac{a^{\frac{2}{3}}bc^{2}x^{4}\Gamma\left(\frac{4}{3}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{2}{3},\frac{4}{3}\\\frac{7}{3}\end{matrix}\middle|\frac{bx^{3}e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{7}{3}\right)}+\frac{2a^{\frac{2}{3}}bcdx^{7}\Gamma\left(\frac{7}{3}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{2}{3},\frac{7}{3}\\\frac{10}{3}\end{matrix}\middle|\frac{bx^{3}e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{10}{3}\right)}+\frac{a^{\frac{2}{3}}bd^{2}x^{10}\Gamma\left(\frac{10}{3}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{2}{3},\frac{10}{3}\\\frac{13}{3}\end{matrix}\middle|\frac{bx^{3}e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{13}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(5/3)*(d*x**3+c)**2,x)
```

```
[Out] a**(5/3)*c**2*x*gamma(1/3)*hyper((-2/3, 1/3), (4/3,), b*x**3*exp_polar(I*pi
)/a)/(3*gamma(4/3)) + 2*a**(5/3)*c*d*x**4*gamma(4/3)*hyper((-2/3, 4/3), (7/
3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(7/3)) + a**(5/3)*d**2*x**7*gamma(7/
3)*hyper((-2/3, 7/3), (10/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(10/3)) +
a**(2/3)*b*c**2*x**4*gamma(4/3)*hyper((-2/3, 4/3), (7/3,), b*x**3*exp_polar
(I*pi)/a)/(3*gamma(7/3)) + 2*a**(2/3)*b*c*d*x**7*gamma(7/3)*hyper((-2/3, 7/
3), (10/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(10/3)) + a**(2/3)*b*d**2*x*
*10*gamma(10/3)*hyper((-2/3, 10/3), (13/3,), b*x**3*exp_polar(I*pi)/a)/(3*g
amma(13/3))
```

3.52 $\int \left(a+bx^3\right)^{2/3} \left(c+dx^3\right)^2 \, dx$

Optimal. Leaf size=219

$$\frac{x\left(a+bx^3\right)^{2/3}\left(2a^2d^2-9abcd+27b^2c^2\right)}{81b^2}-\frac{a\left(2a^2d^2-9abcd+27b^2c^2\right)\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}\,x\right)}{81b^{7/3}}+\frac{2a\left(2a^2d^2-9al\right.}{}$$

Rubi [A] time = 0.17, antiderivative size = 219, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {416, 388, 195, 239}

$$\frac{x\left(a+bx^3\right)^{2/3}\left(2a^2d^2-9abcd+27b^2c^2\right)}{81b^2}-\frac{a\left(2a^2d^2-9abcd+27b^2c^2\right)\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}\,x\right)}{81b^{7/3}}+\frac{2a\left(2a^2d^2-9abcd+27b^2c^2\right)\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{81\sqrt{3}\,b^{7/3}}+\frac{2dx\left(a+bx^3\right)^{5/3}(3bc-ad)}{27b^2}+\frac{dx\left(a+bx^3\right)^{5/3}\left(c+dx^3\right)}{9b}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^(2/3)*(c + d*x^3)^2,x]
```

```
[Out] ((27*b^2*c^2 - 9*a*b*c*d + 2*a^2*d^2)*x*(a + b*x^3)^(2/3))/(81*b^2) + (2*d*
(3*b*c - a*d)*x*(a + b*x^3)^(5/3))/(27*b^2) + (d*x*(a + b*x^3)^(5/3)*(c + d
*x^3))/(9*b) + (2*a*(27*b^2*c^2 - 9*a*b*c*d + 2*a^2*d^2)*ArcTan[(1 + (2*b^(
1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]])/(81*Sqrt[3]*b^(7/3)) - (a*(27*b^2*c^2
- 9*a*b*c*d + 2*a^2*d^2)*Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)])/(81*b^(7/3)
)
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 239

```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + (2*Rt[b, 3]
*x)/(a + b*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 416

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(d*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(b*(n*(p + q) + 1)),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rubi steps

$$\int \left(a+bx^3\right)^{2/3}\left(c+dx^3\right)^2\,dx = \frac{dx\left(a+bx^3\right)^{5/3}\left(c+dx^3\right)}{9b}+\frac{\int \left(a+bx^3\right)^{2/3}\left(c(9bc-ad)+4d(3bc-ad)x^3\right)dx}{9b}$$

$$= \frac{2d(3bc-ad)x\left(a+bx^3\right)^{5/3}}{27b^2}+\frac{dx\left(a+bx^3\right)^{5/3}\left(c+dx^3\right)}{9b}-\frac{(4ad(3bc-ad)-6bc(9b}{5}$$

$$= \frac{\left(27b^2c^2-9abcd+2a^2d^2\right)x\left(a+bx^3\right)^{2/3}}{81b^2}+\frac{2d(3bc-ad)x\left(a+bx^3\right)^{5/3}}{27b^2}+\frac{dx\left(a+b\right.}{}$$

$$= \frac{\left(27b^2c^2-9abcd+2a^2d^2\right)x\left(a+bx^3\right)^{2/3}}{81b^2}+\frac{2d(3bc-ad)x\left(a+bx^3\right)^{5/3}}{27b^2}+\frac{dx\left(a+b\right.}{}$$

Mathematica [A] time = 5.17, size = 203, normalized size = 0.93

$$\frac{3\sqrt[3]{b}x\left(a+bx^3\right)^{2/3}\left(-4a^2d^2+3abd\left(6c+dx^3\right)+9b^2\left(3c^2+3cdx^3+d^2x^6\right)\right)+a\left(2a^2d^2-9abcd+27b^2c^2\right)\left(\log\left(\frac{b^{2/3}x^2}{\left(a+bx^3\right)^{2/3}}+\frac{\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1\right)-2\log\left(1-\frac{\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}\right)+2\sqrt{3}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)\right)}{243b^{7/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^(2/3)*(c + d*x^3)^2,x]
```

```
[Out] (3*b^(1/3)*x*(a + b*x^3)^(2/3)*(-4*a^2*d^2 + 3*a*b*d*(6*c + d*x^3) + 9*b^2*
(3*c^2 + 3*c*d*x^3 + d^2*x^6)) + a*(27*b^2*c^2 - 9*a*b*c*d + 2*a^2*d^2)*(2*
Sqrt[3]*ArcTan[(1 + (2*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]] - 2*Log[1 - (
b^(1/3)*x)/(a + b*x^3)^(1/3)] + Log[1 + (b^(2/3)*x^2)/(a + b*x^3)^(2/3) + (
b^(1/3)*x)/(a + b*x^3)^(1/3)]))/(243*b^(7/3))
```

IntegrateAlgebraic [A] time = 0.90, size = 283, normalized size = 1.29

$$\frac{\left(a+bx^3\right)^{2/3}\left(-4a^2d^2x+18abcdx+3abd^2x^4+27b^2c^2x+27b^2cdx^4+9b^2d^2x^7\right)}{81b^2}-\frac{2\left(2a^3d^2-9a^2bcd+27ab^2c^2\right)\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}x\right)}{243b^{7/3}}+\frac{2\left(2a^3d^2-9a^2bcd+27ab^2c^2\right)\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{b}x}{2\sqrt[3]{a+bx^3}+\sqrt[3]{b}x}\right)}{81\sqrt{3}b^{7/3}}+\frac{\left(2a^3d^2-9a^2bcd+27ab^2c^2\right)\log\left(\sqrt[3]{b}x\sqrt[3]{a+bx^3}+\left(a+bx^3\right)^{2/3}+b^{2/3}x^2\right)}{243b^{7/3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x^3)^(2/3)*(c + d*x^3)^2,x]
```

```
[Out] ((a + b*x^3)^(2/3)*(27*b^2*c^2*x + 18*a*b*c*d*x - 4*a^2*d^2*x + 27*b^2*c*d*
x^4 + 3*a*b*d^2*x^4 + 9*b^2*d^2*x^7))/(81*b^2) + (2*(27*a*b^2*c^2 - 9*a^2*b
*c*d + 2*a^3*d^2)*ArcTan[(Sqrt[3]*b^(1/3)*x)/(b^(1/3)*x + 2*(a + b*x^3)^(1/
3))])/(81*Sqrt[3]*b^(7/3)) - (2*(27*a*b^2*c^2 - 9*a^2*b*c*d + 2*a^3*d^2)*Lo
g[-(b^(1/3)*x) + (a + b*x^3)^(1/3)])/(243*b^(7/3)) + ((27*a*b^2*c^2 - 9*a^2
*b*c*d + 2*a^3*d^2)*Log[b^(2/3)*x^2 + b^(1/3)*x*(a + b*x^3)^(1/3) + (a + b*
x^3)^(2/3)])/(243*b^(7/3))
```

fricas [A] time = 1.25, size = 634, normalized size = 2.89

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)*(d*x^3+c)^2,x, algorithm="fricas")
```

```
[Out] [1/243*(3*sqrt(1/3)*(27*a*b^3*c^2 - 9*a^2*b^2*c*d + 2*a^3*b*d^2)*sqrt((-b)^
(1/3)/b)*log(3*b*x^3 - 3*(b*x^3 + a)^(1/3)*(-b)^(2/3)*x^2 - 3*sqrt(1/3)*((-
b)^(1/3)*b*x^3 - (b*x^3 + a)^(1/3)*b*x^2 + 2*(b*x^3 + a)^(2/3)*(-b)^(2/3)*x
)*sqrt((-b)^(1/3)/b) + 2*a) - 2*(27*a*b^2*c^2 - 9*a^2*b*c*d + 2*a^3*d^2)*(-
b)^(2/3)*log(((-b)^(1/3)*x + (b*x^3 + a)^(1/3))/x) + (27*a*b^2*c^2 - 9*a^2*
```

```
b*c*d + 2*a^3*d^2)*(-b)^(2/3)*log(((-b)^(2/3)*x^2 - (b*x^3 + a)^(1/3)*(-b)^
(1/3)*x + (b*x^3 + a)^(2/3))/x^2) + 3*(9*b^3*d^2*x^7 + 3*(9*b^3*c*d + a*b^2
*d^2)*x^4 + (27*b^3*c^2 + 18*a*b^2*c*d - 4*a^2*b*d^2)*x)*(b*x^3 + a)^(2/3))
/b^3, -1/243*(6*sqrt(1/3)*(27*a*b^3*c^2 - 9*a^2*b^2*c*d + 2*a^3*b*d^2)*sqrt
(-(-b)^(1/3)/b)*arctan(-sqrt(1/3)*((-b)^(1/3)*x - 2*(b*x^3 + a)^(1/3))*sqrt
(-(-b)^(1/3)/b)/x) + 2*(27*a*b^2*c^2 - 9*a^2*b*c*d + 2*a^3*d^2)*(-b)^(2/3)*
log(((-b)^(1/3)*x + (b*x^3 + a)^(1/3))/x) - (27*a*b^2*c^2 - 9*a^2*b*c*d + 2
*a^3*d^2)*(-b)^(2/3)*log(((-b)^(2/3)*x^2 - (b*x^3 + a)^(1/3)*(-b)^(1/3)*x +
 (b*x^3 + a)^(2/3))/x^2) - 3*(9*b^3*d^2*x^7 + 3*(9*b^3*c*d + a*b^2*d^2)*x^4
 + (27*b^3*c^2 + 18*a*b^2*c*d - 4*a^2*b*d^2)*x)*(b*x^3 + a)^(2/3))/b^3]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx^3+a\right)^{\frac{2}{3}}\left(dx^3+c\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)*(d*x^3+c)^2,x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a)^(2/3)*(d*x^3 + c)^2, x)
```

maple [F] time = 0.38, size = 0, normalized size = 0.00

$$\int \left(b\,x^{3}+a\right)^{\frac{2}{3}}\left(d\,x^{3}+c\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(2/3)*(d*x^3+c)^2,x)
```

```
[Out] int((b*x^3+a)^(2/3)*(d*x^3+c)^2,x)
```

maxima [B] time = 1.48, size = 552, normalized size = 2.52

$$-\frac{1}{9}\left(\frac{2\sqrt{3}a\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}}+\frac{2(bx^3+a)^{\frac{1}{3}}}{x}\right)}{3b^{\frac{1}{3}}}\right)}{b^{\frac{1}{3}}}-\frac{a\log\left(b^{\frac{2}{3}}+\frac{(bx^3+a)^{\frac{1}{3}}b^{\frac{1}{3}}}{x}+\frac{(bx^3+a)^{\frac{2}{3}}}{x^2}\right)}{b^{\frac{1}{3}}}+\frac{2a\log\left(-b^{\frac{1}{3}}+\frac{(bx^3+a)^{\frac{1}{3}}}{x}\right)}{b^{\frac{1}{3}}}+\frac{3(bx^3+a)^{\frac{2}{3}}a}{\left(b-\frac{bx^3+a}{x^3}\right)x^2}\right)c^2+\frac{1}{27}\left(\frac{2\sqrt{3}a^2\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}}+\frac{2(bx^3+a)^{\frac{1}{3}}}{x}\right)}{3b^{\frac{1}{3}}}\right)}{b^{\frac{4}{3}}}-\frac{a^2\log\left(b^{\frac{2}{3}}+\frac{(bx^3+a)^{\frac{1}{3}}b^{\frac{1}{3}}}{x}+\frac{(bx^3+a)^{\frac{2}{3}}}{x^2}\right)}{b^{\frac{4}{3}}}+\frac{2a^2\log\left(-b^{\frac{1}{3}}+\frac{(bx^3+a)^{\frac{1}{3}}}{x}\right)}{b^{\frac{4}{3}}}+\frac{3\left(\frac{(bx^3+a)^{\frac{2}{3}}a^2b}{x^2}+\frac{2(bx^3+a)^{\frac{5}{3}}a^2}{x^5}\right)}{b^3-\frac{2(bx^3+a)b^2}{x^3}+\frac{(bx^3+a)^2b}{x^6}}\right)cd-\frac{1}{243}\left(\frac{4\sqrt{3}a^3\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}}+\frac{2(bx^3+a)^{\frac{1}{3}}}{x}\right)}{3b^{\frac{1}{3}}}\right)}{b^{\frac{7}{3}}}-\frac{2a^3\log\left(b^{\frac{2}{3}}+\frac{(bx^3+a)^{\frac{1}{3}}b^{\frac{1}{3}}}{x}+\frac{(bx^3+a)^{\frac{2}{3}}}{x^2}\right)}{b^{\frac{7}{3}}}+\frac{4a^3\log\left(-b^{\frac{1}{3}}+\frac{(bx^3+a)^{\frac{1}{3}}}{x}\right)}{b^{\frac{7}{3}}}+\frac{3\left(\frac{2(bx^3+a)^{\frac{2}{3}}a^3b^2}{x^2}+\frac{11(bx^3+a)^{\frac{5}{3}}a^3b}{x^5}-\frac{4(bx^3+a)^{\frac{8}{3}}a^3}{x^8}\right)}{b^5-\frac{3(bx^3+a)b^4}{x^3}+\frac{3(bx^3+a)^2b^3}{x^6}-\frac{(bx^3+a)^3b^2}{x^9}}\right)d^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)*(d*x^3+c)^2,x, algorithm="maxima")
```

```
[Out] -1/9*(2*sqrt(3)*a*arctan(1/3*sqrt(3)*(b^(1/3) + 2*(b*x^3 + a)^(1/3)/x)/b^(1
/3))/b^(1/3) - a*log(b^(2/3) + (b*x^3 + a)^(1/3)*b^(1/3)/x + (b*x^3 + a)^(2
/3)/x^2)/b^(1/3) + 2*a*log(-b^(1/3) + (b*x^3 + a)^(1/3)/x)/b^(1/3) + 3*(b*x
^3 + a)^(2/3)*a/((b - (b*x^3 + a)/x^3)*x^2))*c^2 + 1/27*(2*sqrt(3)*a^2*arct
an(1/3*sqrt(3)*(b^(1/3) + 2*(b*x^3 + a)^(1/3)/x)/b^(1/3))/b^(4/3) - a^2*log
(b^(2/3) + (b*x^3 + a)^(1/3)*b^(1/3)/x + (b*x^3 + a)^(2/3)/x^2)/b^(4/3) + 2
*a^2*log(-b^(1/3) + (b*x^3 + a)^(1/3)/x)/b^(4/3) + 3*((b*x^3 + a)^(2/3)*a^2
*b/x^2 + 2*(b*x^3 + a)^(5/3)*a^2/x^5)/(b^3 - 2*(b*x^3 + a)*b^2/x^3 + (b*x^3
 + a)^2*b/x^6))*c*d - 1/243*(4*sqrt(3)*a^3*arctan(1/3*sqrt(3)*(b^(1/3) + 2*
(b*x^3 + a)^(1/3)/x)/b^(1/3))/b^(7/3) - 2*a^3*log(b^(2/3) + (b*x^3 + a)^(1/
3)*b^(1/3)/x + (b*x^3 + a)^(2/3)/x^2)/b^(7/3) + 4*a^3*log(-b^(1/3) + (b*x^3
 + a)^(1/3)/x)/b^(7/3) + 3*(2*(b*x^3 + a)^(2/3)*a^3*b^2/x^2 + 11*(b*x^3 + a
)^(5/3)*a^3*b/x^5 - 4*(b*x^3 + a)^(8/3)*a^3/x^8)/(b^5 - 3*(b*x^3 + a)*b^4/x
^3 + 3*(b*x^3 + a)^2*b^3/x^6 - (b*x^3 + a)^3*b^2/x^9))*d^2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int {\left(b\,x^3+a\right)}^{2/3}\,{\left(d\,x^3+c\right)}^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^(2/3)*(c + d*x^3)^2,x)
```

```
[Out] int((a + b*x^3)^(2/3)*(c + d*x^3)^2, x)
```

sympy [C] time = 7.27, size = 131, normalized size = 0.60

$$\frac{a^{\frac{2}{3}}c^2x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{2}{3},\frac{1}{3}\\\frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{4}{3}\right)}+\frac{2a^{\frac{2}{3}}cdx^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}-\frac{2}{3},\frac{4}{3}\\\frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{7}{3}\right)}+\frac{a^{\frac{2}{3}}d^2x^7\Gamma\left(\frac{7}{3}\right){}_2F_1\left(\begin{matrix}-\frac{2}{3},\frac{7}{3}\\\frac{10}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{10}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(2/3)*(d*x**3+c)**2,x)
```

```
[Out] a**(2/3)*c**2*x*gamma(1/3)*hyper((-2/3, 1/3), (4/3,), b*x**3*exp_polar(I*pi
)/a)/(3*gamma(4/3)) + 2*a**(2/3)*c*d*x**4*gamma(4/3)*hyper((-2/3, 4/3), (7/
3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(7/3)) + a**(2/3)*d**2*x**7*gamma(7/
3)*hyper((-2/3, 7/3), (10/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(10/3))
```

3.53 $\int \frac{(c+dx^3)^2}{\sqrt[3]{a+bx^3}}\,dx$

Optimal. Leaf size=175

$$-\frac{(2a^2d^2-6abcd+9b^2c^2)\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}\,x\right)}{18b^{7/3}}+\frac{(2a^2d^2-6abcd+9b^2c^2)\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{9\sqrt{3}\,b^{7/3}}+\frac{dx\,(a+bx^3)^{2/3}\cdots}{18b\cdots}$$

Rubi [A] time = 0.10, antiderivative size = 175, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {416, 388, 239}

$$-\frac{(2a^2d^2-6abcd+9b^2c^2)\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}\,x\right)}{18b^{7/3}}+\frac{(2a^2d^2-6abcd+9b^2c^2)\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{9\sqrt{3}\,b^{7/3}}+\frac{dx\,(a+bx^3)^{2/3}(9bc-4ad)}{18b^2}+\frac{dx\,(a+bx^3)^{2/3}(c+dx^3)}{6b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3)^2/(a + b*x^3)^(1/3),x]
```

```
[Out] (d*(9*b*c - 4*a*d)*x*(a + b*x^3)^(2/3))/(18*b^2) + (d*x*(a + b*x^3)^(2/3)*(
c + d*x^3))/(6*b) + ((9*b^2*c^2 - 6*a*b*c*d + 2*a^2*d^2)*ArcTan[(1 + (2*b^(
1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]])/(9*Sqrt[3]*b^(7/3)) - ((9*b^2*c^2 - 6*
a*b*c*d + 2*a^2*d^2)*Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)])/(18*b^(7/3))
```

Rule 239

```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + (2*Rt[b, 3]
*x)/(a + b*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 416

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(d*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(b*(n*(p + q) + 1)),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rubi steps

$$\int \frac{(c+dx^3)^2}{\sqrt[3]{a+bx^3}}\,dx = \frac{dx\left(a+bx^3\right)^{2/3}\left(c+dx^3\right)}{6b} + \frac{\int \frac{c(6bc-ad)+d(9bc-4ad)x^3}{\sqrt[3]{a+bx^3}}\,dx}{6b}$$

$$= \frac{d(9bc-4ad)x\left(a+bx^3\right)^{2/3}}{18b^2} + \frac{dx\left(a+bx^3\right)^{2/3}\left(c+dx^3\right)}{6b} + \frac{\left(9b^2c^2-6abcd+2a^2d^2\right)\int \frac{1}{\sqrt[3]{a+bx^3}}\,d\ldots}{9b^2}$$

$$= \frac{d(9bc-4ad)x\left(a+bx^3\right)^{2/3}}{18b^2} + \frac{dx\left(a+bx^3\right)^{2/3}\left(c+dx^3\right)}{6b} + \frac{\left(9b^2c^2-6abcd+2a^2d^2\right)\tan^{-1}\left(\ldots\right)}{9\sqrt{3}\,b^{7/3}}$$

Mathematica [A] time = 5.15, size = 172, normalized size = 0.98

$$\frac{\left(2a^2d^2-6abcd+9b^2c^2\right)\left(\log\left(\frac{b^{2/3}x^2}{\left(a+bx^3\right)^{2/3}}+\frac{\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1\right)-2\log\left(1-\frac{\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}\right)+2\sqrt{3}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)\right)+3\sqrt[3]{b}\,dx\left(a+bx^3\right)^{2/3}\left(3b\left(4c+dx^3\right)-4ad\right)}{54b^{7/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^3)^2/(a + b*x^3)^(1/3),x]
```

```
[Out] (3*b^(1/3)*d*x*(a + b*x^3)^(2/3)*(-4*a*d + 3*b*(4*c + d*x^3)) + (9*b^2*c^2
- 6*a*b*c*d + 2*a^2*d^2)*(2*Sqrt[3]*ArcTan[(1 + (2*b^(1/3)*x)/(a + b*x^3)^(
1/3))/Sqrt[3]] - 2*Log[1 - (b^(1/3)*x)/(a + b*x^3)^(1/3)] + Log[1 + (b^(2/3
)*x^2)/(a + b*x^3)^(2/3) + (b^(1/3)*x)/(a + b*x^3)^(1/3)]))/(54*b^(7/3))
```

IntegrateAlgebraic [A] time = 0.77, size = 240, normalized size = 1.37

$$\frac{\left(-2a^2d^2+6abcd-9b^2c^2\right)\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}x\right)}{27b^{7/3}}+\frac{\left(2a^2d^2-6abcd+9b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{b}x}{2\sqrt[3]{a+bx^3}+\sqrt[3]{b}x}\right)}{9\sqrt{3}\,b^{7/3}}+\frac{\left(2a^2d^2-6abcd+9b^2c^2\right)\log\left(\sqrt[3]{b}x\sqrt[3]{a+bx^3}+\left(a+bx^3\right)^{2/3}+b^{2/3}x^2\right)}{54b^{7/3}}+\frac{\left(a+bx^3\right)^{2/3}\left(-4ad^2x+12bcdx+3bd^2x^4\right)}{18b^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c + d*x^3)^2/(a + b*x^3)^(1/3),x]
```

```
[Out] ((a + b*x^3)^(2/3)*(12*b*c*d*x - 4*a*d^2*x + 3*b*d^2*x^4))/(18*b^2) + ((9*b
^2*c^2 - 6*a*b*c*d + 2*a^2*d^2)*ArcTan[(Sqrt[3]*b^(1/3)*x)/(b^(1/3)*x + 2*(
a + b*x^3)^(1/3))])/(9*Sqrt[3]*b^(7/3)) + ((-9*b^2*c^2 + 6*a*b*c*d - 2*a^2*
d^2)*Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)])/(27*b^(7/3)) + ((9*b^2*c^2 - 6*
a*b*c*d + 2*a^2*d^2)*Log[b^(2/3)*x^2 + b^(1/3)*x*(a + b*x^3)^(1/3) + (a + b
*x^3)^(2/3)])/(54*b^(7/3))
```

fricas [A] time = 1.26, size = 554, normalized size = 3.17

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^2/(b*x^3+a)^(1/3),x, algorithm="fricas")
```

```
[Out] [1/54*(3*sqrt(1/3)*(9*b^3*c^2 - 6*a*b^2*c*d + 2*a^2*b*d^2)*sqrt((-b)^(1/3)/
b)*log(3*b*x^3 - 3*(b*x^3 + a)^(1/3)*(-b)^(2/3)*x^2 - 3*sqrt(1/3)*((-b)^(1/
3)*b*x^3 - (b*x^3 + a)^(1/3)*b*x^2 + 2*(b*x^3 + a)^(2/3)*(-b)^(2/3)*x)*sqrt
((-b)^(1/3)/b) + 2*a) - 2*(9*b^2*c^2 - 6*a*b*c*d + 2*a^2*d^2)*(-b)^(2/3)*lo
g(((-b)^(1/3)*x + (b*x^3 + a)^(1/3))/x) + (9*b^2*c^2 - 6*a*b*c*d + 2*a^2*d^
2)*(-b)^(2/3)*log(((-b)^(2/3)*x^2 - (b*x^3 + a)^(1/3)*(-b)^(1/3)*x + (b*x^3
 + a)^(2/3))/x^2) + 3*(3*b^2*d^2*x^4 + 4*(3*b^2*c*d - a*b*d^2)*x)*(b*x^3 +
a)^(2/3))/b^3, -1/54*(6*sqrt(1/3)*(9*b^3*c^2 - 6*a*b^2*c*d + 2*a^2*b*d^2)*s
```

```
qrt(-(-b)^(1/3)/b)*arctan(-sqrt(1/3)*((-b)^(1/3)*x - 2*(b*x^3 + a)^(1/3))*s
qrt(-(-b)^(1/3)/b)/x) + 2*(9*b^2*c^2 - 6*a*b*c*d + 2*a^2*d^2)*(-b)^(2/3)*lo
g(((-b)^(1/3)*x + (b*x^3 + a)^(1/3))/x) - (9*b^2*c^2 - 6*a*b*c*d + 2*a^2*d^
2)*(-b)^(2/3)*log(((-b)^(2/3)*x^2 - (b*x^3 + a)^(1/3)*(-b)^(1/3)*x + (b*x^3
 + a)^(2/3))/x^2) - 3*(3*b^2*d^2*x^4 + 4*(3*b^2*c*d - a*b*d^2)*x)*(b*x^3 +
a)^(2/3))/b^3]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(dx^3+c\right)^2}{\left(bx^3+a\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^2/(b*x^3+a)^(1/3),x, algorithm="giac")
```

```
[Out] integrate((d*x^3 + c)^2/(b*x^3 + a)^(1/3), x)
```

maple [F] time = 0.38, size = 0, normalized size = 0.00

$$\int \frac{\left(d\,x^3+c\right)^2}{\left(b\,x^3+a\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c)^2/(b*x^3+a)^(1/3),x)
```

```
[Out] int((d*x^3+c)^2/(b*x^3+a)^(1/3),x)
```

maxima [B] time = 1.28, size = 436, normalized size = 2.49

$$-\frac{1}{6}\left(\frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}}+\frac{2(bx^3+a)^{\frac{1}{3}}}{x}\right)}{3b^{\frac{1}{3}}}\right)}{b^{\frac{1}{3}}}-\frac{\log\left(b^{\frac{2}{3}}+\frac{(bx^3+a)^{\frac{1}{3}}b^{\frac{1}{3}}}{x}+\frac{(bx^3+a)^{\frac{2}{3}}}{x^2}\right)}{b^{\frac{1}{3}}}+\frac{2\log\left(-b^{\frac{1}{3}}+\frac{(bx^3+a)^{\frac{1}{3}}}{x}\right)}{b^{\frac{1}{3}}}\right)c^2+\frac{1}{9}\left(\frac{2\sqrt{3}a\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}}+\frac{2(bx^3+a)^{\frac{1}{3}}}{x}\right)}{3b^{\frac{1}{3}}}\right)}{b^{\frac{4}{3}}}-\frac{a\log\left(b^{\frac{2}{3}}+\frac{(bx^3+a)^{\frac{1}{3}}b^{\frac{1}{3}}}{x}+\frac{(bx^3+a)^{\frac{2}{3}}}{x^2}\right)}{b^{\frac{4}{3}}}+\frac{2a\log\left(-b^{\frac{1}{3}}+\frac{(bx^3+a)^{\frac{1}{3}}}{x}\right)}{b^{\frac{4}{3}}}-\frac{6(bx^3+a)^{\frac{2}{3}}a}{\left(b^2-\frac{(bx^3+a)b}{x^3}\right)x^2}\right)cd-\frac{1}{54}\left(\frac{4\sqrt{3}a^2\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}}+\frac{2(bx^3+a)^{\frac{1}{3}}}{x}\right)}{3b^{\frac{1}{3}}}\right)}{b^{\frac{7}{3}}}-\frac{2a^2\log\left(b^{\frac{2}{3}}+\frac{(bx^3+a)^{\frac{1}{3}}b^{\frac{1}{3}}}{x}+\frac{(bx^3+a)^{\frac{2}{3}}}{x^2}\right)}{b^{\frac{7}{3}}}+\frac{4a^2\log\left(-b^{\frac{1}{3}}+\frac{(bx^3+a)^{\frac{1}{3}}}{x}\right)}{b^{\frac{7}{3}}}-\frac{3\left(\frac{7(bx^3+a)^{\frac{2}{3}}a^2b}{x^2}-\frac{4(bx^3+a)^{\frac{5}{3}}a^2}{x^5}\right)}{b^4-\frac{2(bx^3+a)b^3}{x^3}+\frac{(bx^3+a)^2b^2}{x^6}}\right)d^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^2/(b*x^3+a)^(1/3),x, algorithm="maxima")
```

```
[Out] -1/6*(2*sqrt(3)*arctan(1/3*sqrt(3)*(b^(1/3) + 2*(b*x^3 + a)^(1/3)/x)/b^(1/3
))/b^(1/3) - log(b^(2/3) + (b*x^3 + a)^(1/3)*b^(1/3)/x + (b*x^3 + a)^(2/3)/
x^2)/b^(1/3) + 2*log(-b^(1/3) + (b*x^3 + a)^(1/3)/x)/b^(1/3))*c^2 + 1/9*(2*
sqrt(3)*a*arctan(1/3*sqrt(3)*(b^(1/3) + 2*(b*x^3 + a)^(1/3)/x)/b^(1/3))/b^(
4/3) - a*log(b^(2/3) + (b*x^3 + a)^(1/3)*b^(1/3)/x + (b*x^3 + a)^(2/3)/x^2)
/b^(4/3) + 2*a*log(-b^(1/3) + (b*x^3 + a)^(1/3)/x)/b^(4/3) - 6*(b*x^3 + a)^
(2/3)*a/((b^2 - (b*x^3 + a)*b/x^3)*x^2))*c*d - 1/54*(4*sqrt(3)*a^2*arctan(1
/3*sqrt(3)*(b^(1/3) + 2*(b*x^3 + a)^(1/3)/x)/b^(1/3))/b^(7/3) - 2*a^2*log(b
^(2/3) + (b*x^3 + a)^(1/3)*b^(1/3)/x + (b*x^3 + a)^(2/3)/x^2)/b^(7/3) + 4*a
^2*log(-b^(1/3) + (b*x^3 + a)^(1/3)/x)/b^(7/3) - 3*(7*(b*x^3 + a)^(2/3)*a^2
*b/x^2 - 4*(b*x^3 + a)^(5/3)*a^2/x^5)/(b^4 - 2*(b*x^3 + a)*b^3/x^3 + (b*x^3
 + a)^2*b^2/x^6))*d^2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(d\,x^3+c\right)^2}{\left(b\,x^3+a\right)^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3)^2/(a + b*x^3)^(1/3),x)
```

```
[Out] int((c + d*x^3)^2/(a + b*x^3)^(1/3), x)
```

sympy [C] time = 6.44, size = 126, normalized size = 0.72

$$\frac{c^2 x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{3}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt[3]{a}\,\Gamma\left(\frac{4}{3}\right)}+\frac{2cdx^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt[3]{a}\,\Gamma\left(\frac{7}{3}\right)}+\frac{d^2x^7\Gamma\left(\frac{7}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{7}{3}\\ \frac{10}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt[3]{a}\,\Gamma\left(\frac{10}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c)**2/(b*x**3+a)**(1/3),x)
```

```
[Out] c**2*x*gamma(1/3)*hyper((1/3, 1/3), (4/3,), b*x**3*exp_polar(I*pi)/a)/(3*a*
*(1/3)*gamma(4/3)) + 2*c*d*x**4*gamma(4/3)*hyper((1/3, 4/3), (7/3,), b*x**3
*exp_polar(I*pi)/a)/(3*a**(1/3)*gamma(7/3)) + d**2*x**7*gamma(7/3)*hyper((1
/3, 7/3), (10/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(1/3)*gamma(10/3))
```

3.54 $\int \frac{\left(c+dx^3\right)^2}{\left(a+bx^3\right)^{4/3}}\,dx$

Optimal. Leaf size=159

$$-\frac{d(3bc-2ad)\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}\,x\right)}{3b^{7/3}}+\frac{2d(3bc-2ad)\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{3\sqrt{3}\,b^{7/3}}-\frac{dx\left(a+bx^3\right)^{2/3}(3bc-4ad)}{3ab^2}+\frac{x\left(c+d\right.}{ab\sqrt[3]{\ }}$$

Rubi [A] time = 0.10, antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {413, 388, 239}

$$-\frac{dx\left(a+bx^3\right)^{2/3}(3bc-4ad)}{3ab^2}-\frac{d(3bc-2ad)\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}\,x\right)}{3b^{7/3}}+\frac{2d(3bc-2ad)\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{3\sqrt{3}\,b^{7/3}}+\frac{x\left(c+dx^3\right)(bc-ad)}{ab\sqrt[3]{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3)^2/(a + b*x^3)^(4/3),x]
```

```
[Out] -(d*(3*b*c - 4*a*d)*x*(a + b*x^3)^(2/3))/(3*a*b^2) + ((b*c - a*d)*x*(c + d*
x^3))/(a*b*(a + b*x^3)^(1/3)) + (2*d*(3*b*c - 2*a*d)*ArcTan[(1 + (2*b^(1/3)
*x)/(a + b*x^3)^(1/3))/Sqrt[3]])/(3*Sqrt[3]*b^(7/3)) - (d*(3*b*c - 2*a*d)*L
og[-(b^(1/3)*x) + (a + b*x^3)^(1/3)])/(3*b^(7/3))
```

Rule 239

```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + (2*Rt[b, 3]
*x)/(a + b*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 413

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[((a*d - c*b)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*n*(p +
 1)), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rubi steps

$$\int \frac{\left(c+dx^3\right)^2}{\left(a+bx^3\right)^{4/3}}\,dx = \frac{(bc-ad)x\left(c+dx^3\right)}{ab\sqrt[3]{a+bx^3}} + \frac{\int \frac{acd-d(3bc-4ad)x^3}{\sqrt[3]{a+bx^3}}\,dx}{ab}$$

$$= -\frac{d(3bc-4ad)x\left(a+bx^3\right)^{2/3}}{3ab^2} + \frac{(bc-ad)x\left(c+dx^3\right)}{ab\sqrt[3]{a+bx^3}} + \frac{(2d(3bc-2ad))\int \frac{1}{\sqrt[3]{a+bx^3}}\,dx}{3b^2}$$

$$= -\frac{d(3bc-4ad)x\left(a+bx^3\right)^{2/3}}{3ab^2} + \frac{(bc-ad)x\left(c+dx^3\right)}{ab\sqrt[3]{a+bx^3}} + \frac{2d(3bc-2ad)\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{3\sqrt{3}\,b^{7/3}} - \frac{d(3 \ldots}{}$$

Mathematica [A] time = 5.16, size = 168, normalized size = 1.06

$$\frac{d(3bc-2ad)\left(\log\left(\frac{b^{2/3}x^2}{\left(a+bx^3\right)^{2/3}}+\frac{\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1\right)-2\log\left(1-\frac{\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}\right)+2\sqrt{3}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)\right)}{9b^{7/3}}+\frac{x\left(a+bx^3\right)^{2/3}\left(\frac{3(bc-ad)^2}{a\left(a+bx^3\right)}+d^2\right)}{3b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^3)^2/(a + b*x^3)^(4/3),x]
```

```
[Out] (x*(a + b*x^3)^(2/3)*(d^2 + (3*(b*c - a*d)^2)/(a*(a + b*x^3))))/(3*b^2) + (
d*(3*b*c - 2*a*d)*(2*Sqrt[3]*ArcTan[(1 + (2*b^(1/3)*x)/(a + b*x^3)^(1/3))/S
qrt[3]] - 2*Log[1 - (b^(1/3)*x)/(a + b*x^3)^(1/3)] + Log[1 + (b^(2/3)*x^2)/
(a + b*x^3)^(2/3) + (b^(1/3)*x)/(a + b*x^3)^(1/3)]))/(9*b^(7/3))
```

IntegrateAlgebraic [A] time = 0.78, size = 222, normalized size = 1.40

$$\frac{4a^2d^2x-6abcdx+abd^2x^4+3b^2c^2x}{3ab^2\sqrt[3]{a+bx^3}}-\frac{2\left(3bcd-2ad^2\right)\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}x\right)}{9b^{7/3}}+\frac{2\left(3bcd-2ad^2\right)\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{b}x}{2\sqrt[3]{a+bx^3}+\sqrt[3]{b}x}\right)}{3\sqrt{3}\,b^{7/3}}+\frac{\left(3bcd-2ad^2\right)\log\left(\sqrt[3]{b}x\sqrt[3]{a+bx^3}+\left(a+bx^3\right)^{2/3}+b^{2/3}x^2\right)}{9b^{7/3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c + d*x^3)^2/(a + b*x^3)^(4/3),x]
```

```
[Out] (3*b^2*c^2*x - 6*a*b*c*d*x + 4*a^2*d^2*x + a*b*d^2*x^4)/(3*a*b^2*(a + b*x^3
)^(1/3)) + (2*(3*b*c*d - 2*a*d^2)*ArcTan[(Sqrt[3]*b^(1/3)*x)/(b^(1/3)*x + 2
*(a + b*x^3)^(1/3))])/(3*Sqrt[3]*b^(7/3)) - (2*(3*b*c*d - 2*a*d^2)*Log[-(b^
(1/3)*x) + (a + b*x^3)^(1/3)])/(9*b^(7/3)) + ((3*b*c*d - 2*a*d^2)*Log[b^(2/
3)*x^2 + b^(1/3)*x*(a + b*x^3)^(1/3) + (a + b*x^3)^(2/3)])/(9*b^(7/3))
```

fricas [B] time = 1.17, size = 652, normalized size = 4.10

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^2/(b*x^3+a)^(4/3),x, algorithm="fricas")
```

```
[Out] [-1/9*(3*sqrt(1/3)*(3*a^2*b^2*c*d - 2*a^3*b*d^2 + (3*a*b^3*c*d - 2*a^2*b^2*
d^2)*x^3)*sqrt(-1/b^(2/3))*log(3*b*x^3 - 3*(b*x^3 + a)^(1/3)*b^(2/3)*x^2 -
3*sqrt(1/3)*(b^(4/3)*x^3 + (b*x^3 + a)^(1/3)*b*x^2 - 2*(b*x^3 + a)^(2/3)*b^
(2/3)*x)*sqrt(-1/b^(2/3)) + 2*a) + 2*(3*a^2*b*c*d - 2*a^3*d^2 + (3*a*b^2*c*
d - 2*a^2*b*d^2)*x^3)*b^(2/3)*log(-(b^(1/3)*x - (b*x^3 + a)^(1/3))/x) - (3*
a^2*b*c*d - 2*a^3*d^2 + (3*a*b^2*c*d - 2*a^2*b*d^2)*x^3)*b^(2/3)*log((b^(2/
3)*x^2 + (b*x^3 + a)^(1/3)*b^(1/3)*x + (b*x^3 + a)^(2/3))/x^2) - 3*(a*b^2*d
^2*x^4 + (3*b^3*c^2 - 6*a*b^2*c*d + 4*a^2*b*d^2)*x)*(b*x^3 + a)^(2/3))/(a*b
```

```
^4*x^3 + a^2*b^3), -1/9*(2*(3*a^2*b*c*d - 2*a^3*d^2 + (3*a*b^2*c*d - 2*a^2*
b*d^2)*x^3)*b^(2/3)*log(-(b^(1/3)*x - (b*x^3 + a)^(1/3))/x) - (3*a^2*b*c*d
- 2*a^3*d^2 + (3*a*b^2*c*d - 2*a^2*b*d^2)*x^3)*b^(2/3)*log((b^(2/3)*x^2 + (
b*x^3 + a)^(1/3)*b^(1/3)*x + (b*x^3 + a)^(2/3))/x^2) + 6*sqrt(1/3)*(3*a^2*b
^2*c*d - 2*a^3*b*d^2 + (3*a*b^3*c*d - 2*a^2*b^2*d^2)*x^3)*arctan(sqrt(1/3)*
(b^(1/3)*x + 2*(b*x^3 + a)^(1/3))/(b^(1/3)*x))/b^(1/3) - 3*(a*b^2*d^2*x^4 +
 (3*b^3*c^2 - 6*a*b^2*c*d + 4*a^2*b*d^2)*x)*(b*x^3 + a)^(2/3))/(a*b^4*x^3 +
 a^2*b^3)]
```

giac [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(dx^3+c\right)^2}{\left(bx^3+a\right)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^2/(b*x^3+a)^(4/3),x, algorithm="giac")
```

```
[Out] integrate((d*x^3 + c)^2/(b*x^3 + a)^(4/3), x)
```

maple [**F**] time = 0.56, size = 0, normalized size = 0.00

$$\int \frac{\left(d\,x^3+c\right)^2}{\left(b\,x^3+a\right)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c)^2/(b*x^3+a)^(4/3),x)
```

```
[Out] int((d*x^3+c)^2/(b*x^3+a)^(4/3),x)
```

maxima [**B**] time = 1.14, size = 301, normalized size = 1.89

$$\frac{1}{9}\,d^2\left(\frac{4\,\sqrt{3}\,a\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}}+\frac{2\left(bx^3+a\right)^{\frac{1}{3}}}{x}\right)}{3\,b^{\frac{1}{3}}}\right)}{b^{\frac{7}{3}}}+\frac{3\left(3\,ab-\frac{4\left(bx^3+a\right)a}{x^3}\right)}{\frac{\left(bx^3+a\right)^{\frac{1}{3}}b^3}{x}-\frac{\left(bx^3+a\right)^{\frac{4}{3}}b^2}{x^4}}-\frac{2\,a\log\left(b^{\frac{2}{3}}+\frac{\left(bx^3+a\right)^{\frac{1}{3}}b^{\frac{1}{3}}}{x}+\frac{\left(bx^3+a\right)^{\frac{2}{3}}}{x^2}\right)}{b^{\frac{7}{3}}}+\frac{4\,a\log\left(-b^{\frac{1}{3}}+\frac{\left(bx^3+a\right)^{\frac{1}{3}}}{x}\right)}{b^{\frac{7}{3}}}\right)-\frac{1}{3}\,cd\left(\frac{2\,\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}}+\frac{2\left(bx^3+a\right)^{\frac{1}{3}}}{x}\right)}{3\,b^{\frac{1}{3}}}\right)}{b^{\frac{4}{3}}}+\frac{6\,x}{\left(bx^3+a\right)^{\frac{1}{3}}b}-\frac{\log\left(b^{\frac{2}{3}}+\frac{\left(bx^3+a\right)^{\frac{1}{3}}b^{\frac{1}{3}}}{x}+\frac{\left(bx^3+a\right)^{\frac{2}{3}}}{x^2}\right)}{b^{\frac{4}{3}}}+\frac{2\log\left(-b^{\frac{1}{3}}+\frac{\left(bx^3+a\right)^{\frac{1}{3}}}{x}\right)}{b^{\frac{4}{3}}}\right)+\frac{c^2x}{\left(bx^3+a\right)^{\frac{1}{3}}a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^2/(b*x^3+a)^(4/3),x, algorithm="maxima")
```

```
[Out] 1/9*d^2*(4*sqrt(3)*a*arctan(1/3*sqrt(3)*(b^(1/3) + 2*(b*x^3 + a)^(1/3)/x)/b
^(1/3))/b^(7/3) + 3*(3*a*b - 4*(b*x^3 + a)*a/x^3)/((b*x^3 + a)^(1/3)*b^3/x
- (b*x^3 + a)^(4/3)*b^2/x^4) - 2*a*log(b^(2/3) + (b*x^3 + a)^(1/3)*b^(1/3)/
x + (b*x^3 + a)^(2/3)/x^2)/b^(7/3) + 4*a*log(-b^(1/3) + (b*x^3 + a)^(1/3)/x
)/b^(7/3)) - 1/3*c*d*(2*sqrt(3)*arctan(1/3*sqrt(3)*(b^(1/3) + 2*(b*x^3 + a)
^(1/3)/x)/b^(1/3))/b^(4/3) + 6*x/((b*x^3 + a)^(1/3)*b) - log(b^(2/3) + (b*x
^3 + a)^(1/3)*b^(1/3)/x + (b*x^3 + a)^(2/3)/x^2)/b^(4/3) + 2*log(-b^(1/3) +
 (b*x^3 + a)^(1/3)/x)/b^(4/3)) + c^2*x/((b*x^3 + a)^(1/3)*a)
```

mupad [**F**] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(d\,x^3+c\right)}^2}{{\left(b\,x^3+a\right)}^{4/3}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3)^2/(a + b*x^3)^(4/3),x)
```

```
[Out] int((c + d*x^3)^2/(a + b*x^3)^(4/3), x)
```

sympy [F]   time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c+dx^3\right)^2}{\left(a+bx^3\right)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c)**2/(b*x**3+a)**(4/3),x)
[Out] Integral((c + d*x**3)**2/(a + b*x**3)**(4/3), x)
```

3.55 $\int \frac{\left(c+dx^3\right)^2}{\left(a+bx^3\right)^{7/3}}\,dx$

Optimal. Leaf size=152

$$\frac{x(bc-ad)(4ad+3bc)}{4a^2b^2\sqrt[3]{a+bx^3}}-\frac{d^2\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}\,x\right)}{2b^{7/3}}+\frac{d^2\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\,b^{7/3}}+\frac{x\left(c+dx^3\right)(bc-ad)}{4ab\left(a+bx^3\right)^{4/3}}$$

Rubi [A] time = 0.07, antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {413, 385, 239}

$$\frac{x(bc-ad)(4ad+3bc)}{4a^2b^2\sqrt[3]{a+bx^3}}-\frac{d^2\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}\,x\right)}{2b^{7/3}}+\frac{d^2\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\,b^{7/3}}+\frac{x\left(c+dx^3\right)(bc-ad)}{4ab\left(a+bx^3\right)^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3)^2/(a + b*x^3)^(7/3),x]
```

```
[Out] ((b*c - a*d)*(3*b*c + 4*a*d)*x)/(4*a^2*b^2*(a + b*x^3)^(1/3)) + ((b*c - a*d
)*x*(c + d*x^3))/(4*a*b*(a + b*x^3)^(4/3)) + (d^2*ArcTan[(1 + (2*b^(1/3)*x)
/(a + b*x^3)^(1/3))/Sqrt[3]])/(Sqrt[3]*b^(7/3)) - (d^2*Log[-(b^(1/3)*x) + (
a + b*x^3)^(1/3)])/(2*b^(7/3))
```

Rule 239

```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + (2*Rt[b, 3]
*x)/(a + b*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 413

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[((a*d - c*b)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*n*(p +
 1)), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rubi steps

$$\int \frac{\left(c+dx^3\right)^2}{\left(a+bx^3\right)^{7/3}}\,dx = \frac{(bc-ad)x\left(c+dx^3\right)}{4ab\left(a+bx^3\right)^{4/3}} + \frac{\int \frac{c(3bc+ad)+4ad^2x^3}{\left(a+bx^3\right)^{4/3}}\,dx}{4ab}$$

$$= \frac{(bc-ad)(3bc+4ad)x}{4a^2b^2\sqrt[3]{a+bx^3}} + \frac{(bc-ad)x\left(c+dx^3\right)}{4ab\left(a+bx^3\right)^{4/3}} + \frac{d^2\int \frac{1}{\sqrt[3]{a+bx^3}}\,dx}{b^2}$$

$$= \frac{(bc-ad)(3bc+4ad)x}{4a^2b^2\sqrt[3]{a+bx^3}} + \frac{(bc-ad)x\left(c+dx^3\right)}{4ab\left(a+bx^3\right)^{4/3}} + \frac{d^2\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}\,b^{7/3}} - \frac{d^2\log\left(-\sqrt[3]{b}\,x+\sqrt[3]{a\right.}}{2b^{7/3}}$$

Mathematica [A] time = 5.25, size = 180, normalized size = 1.18

$$\frac{x\left(\left(a+bx^3\right)\left(-5a^2d^2+2abcd+3b^2c^2\right)+a(bc-ad)^2\right)}{4a^2b^2\left(a+bx^3\right)^{4/3}} + \frac{d^2\left(\log\left(\frac{b^{2/3}x^2}{\left(a+bx^3\right)^{2/3}}+\frac{\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1\right)-2\log\left(1-\frac{\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}\right)+2\sqrt{3}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)\right)}{6b^{7/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^3)^2/(a + b*x^3)^(7/3),x]
```

```
[Out] (x*(a*(b*c - a*d)^2 + (3*b^2*c^2 + 2*a*b*c*d - 5*a^2*d^2)*(a + b*x^3)))/(4*
a^2*b^2*(a + b*x^3)^(4/3)) + (d^2*(2*Sqrt[3]*ArcTan[(1 + (2*b^(1/3)*x)/(a +
 b*x^3)^(1/3))/Sqrt[3]] - 2*Log[1 - (b^(1/3)*x)/(a + b*x^3)^(1/3)] + Log[1
+ (b^(2/3)*x^2)/(a + b*x^3)^(2/3) + (b^(1/3)*x)/(a + b*x^3)^(1/3)]))/(6*b^(
7/3))
```

IntegrateAlgebraic [A] time = 0.69, size = 211, normalized size = 1.39

$$\frac{-4a^3d^2x-5a^2bd^2x^4+4ab^2c^2x+2ab^2cdx^4+3b^3c^2x^4}{4a^2b^2\left(a+bx^3\right)^{4/3}} - \frac{d^2\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}x\right)}{3b^{7/3}} + \frac{d^2\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{b}x}{2\sqrt[3]{a+bx^3}+\sqrt[3]{b}x}\right)}{\sqrt{3}\,b^{7/3}} + \frac{d^2\log\left(\sqrt[3]{b}x\sqrt[3]{a+bx^3}+\left(a+bx^3\right)^{2/3}+b^{2/3}x^2\right)}{6b^{7/3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c + d*x^3)^2/(a + b*x^3)^(7/3),x]
```

```
[Out] (4*a*b^2*c^2*x - 4*a^3*d^2*x + 3*b^3*c^2*x^4 + 2*a*b^2*c*d*x^4 - 5*a^2*b*d^
2*x^4)/(4*a^2*b^2*(a + b*x^3)^(4/3)) + (d^2*ArcTan[(Sqrt[3]*b^(1/3)*x)/(b^(
1/3)*x + 2*(a + b*x^3)^(1/3))])/(Sqrt[3]*b^(7/3)) - (d^2*Log[-(b^(1/3)*x) +
 (a + b*x^3)^(1/3)])/(3*b^(7/3)) + (d^2*Log[b^(2/3)*x^2 + b^(1/3)*x*(a + b*
x^3)^(1/3) + (a + b*x^3)^(2/3)])/(6*b^(7/3))
```

fricas [B] time = 1.30, size = 719, normalized size = 4.73

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^2/(b*x^3+a)^(7/3),x, algorithm="fricas")
```

```
[Out] [1/12*(6*sqrt(1/3)*(a^2*b^3*d^2*x^6 + 2*a^3*b^2*d^2*x^3 + a^4*b*d^2)*sqrt((
-b)^(1/3)/b)*log(3*b*x^3 - 3*(b*x^3 + a)^(1/3)*(-b)^(2/3)*x^2 - 3*sqrt(1/3)
*((-b)^(1/3)*b*x^3 - (b*x^3 + a)^(1/3)*b*x^2 + 2*(b*x^3 + a)^(2/3)*(-b)^(2/
3)*x)*sqrt((-b)^(1/3)/b) + 2*a) - 4*(a^2*b^2*d^2*x^6 + 2*a^3*b*d^2*x^3 + a^
4*d^2)*(-b)^(2/3)*log(((-b)^(1/3)*x + (b*x^3 + a)^(1/3))/x) + 2*(a^2*b^2*d^
```

```
2*x^6 + 2*a^3*b*d^2*x^3 + a^4*d^2)*(-b)^(2/3)*log(((-b)^(2/3)*x^2 - (b*x^3
+ a)^(1/3)*(-b)^(1/3)*x + (b*x^3 + a)^(2/3))/x^2) + 3*((3*b^4*c^2 + 2*a*b^3
*c*d - 5*a^2*b^2*d^2)*x^4 + 4*(a*b^3*c^2 - a^3*b*d^2)*x)*(b*x^3 + a)^(2/3))
/(a^2*b^5*x^6 + 2*a^3*b^4*x^3 + a^4*b^3), -1/12*(12*sqrt(1/3)*(a^2*b^3*d^2*
x^6 + 2*a^3*b^2*d^2*x^3 + a^4*b*d^2)*sqrt(-(-b)^(1/3)/b)*arctan(-sqrt(1/3)*
((-b)^(1/3)*x - 2*(b*x^3 + a)^(1/3))*sqrt(-(-b)^(1/3)/b)/x) + 4*(a^2*b^2*d^
2*x^6 + 2*a^3*b*d^2*x^3 + a^4*d^2)*(-b)^(2/3)*log(((-b)^(1/3)*x + (b*x^3 +
a)^(1/3))/x) - 2*(a^2*b^2*d^2*x^6 + 2*a^3*b*d^2*x^3 + a^4*d^2)*(-b)^(2/3)*l
og(((-b)^(2/3)*x^2 - (b*x^3 + a)^(1/3)*(-b)^(1/3)*x + (b*x^3 + a)^(2/3))/x^
2) - 3*((3*b^4*c^2 + 2*a*b^3*c*d - 5*a^2*b^2*d^2)*x^4 + 4*(a*b^3*c^2 - a^3*
b*d^2)*x)*(b*x^3 + a)^(2/3))/(a^2*b^5*x^6 + 2*a^3*b^4*x^3 + a^4*b^3)]
```

giac [F]   time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(dx^3+c\right)^2}{\left(bx^3+a\right)^{\frac{7}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^2/(b*x^3+a)^(7/3),x, algorithm="giac")
```

```
[Out] integrate((d*x^3 + c)^2/(b*x^3 + a)^(7/3), x)
```

maple [F]   time = 0.55, size = 0, normalized size = 0.00

$$\int \frac{\left(d\,x^3+c\right)^2}{\left(b\,x^3+a\right)^{\frac{7}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c)^2/(b*x^3+a)^(7/3),x)
```

```
[Out] int((d*x^3+c)^2/(b*x^3+a)^(7/3),x)
```

maxima [A]   time = 1.19, size = 190, normalized size = 1.25

$$-\frac{\left(b-\frac{4\left(bx^3+a\right)}{x^3}\right)c^2x^4}{4\left(bx^3+a\right)^{\frac{4}{3}}a^2}+\frac{cdx^4}{2\left(bx^3+a\right)^{\frac{4}{3}}a}-\frac{1}{12}\left(\frac{3\left(b+\frac{4\left(bx^3+a\right)}{x^3}\right)x^4}{\left(bx^3+a\right)^{\frac{4}{3}}b^2}+\frac{4\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}}+\frac{2\left(bx^3+a\right)^{\frac{1}{3}}}{x}\right)}{3b^{\frac{1}{3}}}\right)}{b^{\frac{7}{3}}}-\frac{2\log\left(b^{\frac{2}{3}}+\frac{\left(bx^3+a\right)^{\frac{1}{3}}b^{\frac{1}{3}}}{x}+\frac{\left(bx^3+a\right)^{\frac{2}{3}}}{x^2}\right)}{b^{\frac{7}{3}}}+\frac{4\log\left(-b^{\frac{1}{3}}+\frac{\left(bx^3+a\right)^{\frac{1}{3}}}{x}\right)}{b^{\frac{7}{3}}}\right)d^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^2/(b*x^3+a)^(7/3),x, algorithm="maxima")
```

```
[Out] -1/4*(b - 4*(b*x^3 + a)/x^3)*c^2*x^4/((b*x^3 + a)^(4/3)*a^2) + 1/2*c*d*x^4/
((b*x^3 + a)^(4/3)*a) - 1/12*(3*(b + 4*(b*x^3 + a)/x^3)*x^4/((b*x^3 + a)^(4
/3)*b^2) + 4*sqrt(3)*arctan(1/3*sqrt(3)*(b^(1/3) + 2*(b*x^3 + a)^(1/3)/x)/b
^(1/3))/b^(7/3) - 2*log(b^(2/3) + (b*x^3 + a)^(1/3)*b^(1/3)/x + (b*x^3 + a)
^(2/3)/x^2)/b^(7/3) + 4*log(-b^(1/3) + (b*x^3 + a)^(1/3)/x)/b^(7/3))*d^2
```

mupad [F]   time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(d\,x^3+c\right)^2}{\left(b\,x^3+a\right)^{7/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3)^2/(a + b*x^3)^(7/3),x)
```

```
[Out] int((c + d*x^3)^2/(a + b*x^3)^(7/3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c + dx^{3}\right)^{2}}{\left(a + bx^{3}\right)^{\frac{7}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c)**2/(b*x**3+a)**(7/3),x)
```

```
[Out] Integral((c + d*x**3)**2/(a + b*x**3)**(7/3), x)
```

3.56 $\int \frac{(c+dx^3)^2}{(a+bx^3)^{10/3}}\,dx$

Optimal. Leaf size=78

$$\frac{9c^2x}{14a^3\sqrt[3]{a+bx^3}}+\frac{3cx\left(c+dx^3\right)}{14a^2\left(a+bx^3\right)^{4/3}}+\frac{x\left(c+dx^3\right)^2}{7a\left(a+bx^3\right)^{7/3}}$$

Rubi [A] time = 0.02, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {378, 191}

$$\frac{9c^2x}{14a^3\sqrt[3]{a+bx^3}}+\frac{3cx\left(c+dx^3\right)}{14a^2\left(a+bx^3\right)^{4/3}}+\frac{x\left(c+dx^3\right)^2}{7a\left(a+bx^3\right)^{7/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3)^2/(a + b*x^3)^(10/3),x]
```

```
[Out] (9*c^2*x)/(14*a^3*(a + b*x^3)^(1/3)) + (3*c*x*(c + d*x^3))/(14*a^2*(a + b*x
^3)^(4/3)) + (x*(c + d*x^3)^2)/(7*a*(a + b*x^3)^(7/3))
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 378

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> -Simp[(x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*n*(p + 1)), x] - Dist[(c
*q)/(a*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1, 0]
 && GtQ[q, 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(c+dx^3\right)^2}{\left(a+bx^3\right)^{10/3}}\,dx &= \frac{x\left(c+dx^3\right)^2}{7a\left(a+bx^3\right)^{7/3}}+\frac{(6c)\int \frac{c+dx^3}{\left(a+bx^3\right)^{7/3}}\,dx}{7a}\\
&= \frac{3cx\left(c+dx^3\right)}{14a^2\left(a+bx^3\right)^{4/3}}+\frac{x\left(c+dx^3\right)^2}{7a\left(a+bx^3\right)^{7/3}}+\frac{\left(9c^2\right)\int \frac{1}{\left(a+bx^3\right)^{4/3}}\,dx}{14a^2}\\
&= \frac{9c^2x}{14a^3\sqrt[3]{a+bx^3}}+\frac{3cx\left(c+dx^3\right)}{14a^2\left(a+bx^3\right)^{4/3}}+\frac{x\left(c+dx^3\right)^2}{7a\left(a+bx^3\right)^{7/3}}
\end{aligned}$$

Mathematica [A] time = 0.10, size = 126, normalized size = 1.62

$$\frac{x\sqrt[3]{\frac{bx^3}{a}+1}\left(a^2\left(14c^2+7cdx^3+2d^2x^6\right)+3abcx^3\left(7c+dx^3\right)+9b^2c^2x^6\right)}{14a^3\left(a+bx^3\right)^{7/3}\sqrt[3]{\frac{dx^3}{c}+1}\sqrt[3]{\frac{c\left(a+bx^3\right)}{a\left(c+dx^3\right)}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x^3)^2/(a + b*x^3)^(10/3),x]
```

```
[Out] (x*(1 + (b*x^3)/a)^(1/3)*(9*b^2*c^2*x^6 + 3*a*b*c*x^3*(7*c + d*x^3) + a^2*(
14*c^2 + 7*c*d*x^3 + 2*d^2*x^6)))/(14*a^3*(a + b*x^3)^(7/3)*((c*(a + b*x^3)
)/(a*(c + d*x^3)))^(1/3)*(1 + (d*x^3)/c)^(1/3))
```

IntegrateAlgebraic [A] time = 0.57, size = 79, normalized size = 1.01

$$\frac{x\left(14a^2c^2+7a^2cdx^3+2a^2d^2x^6+21abc^2x^3+3abcdx^6+9b^2c^2x^6\right)}{14a^3\left(a+bx^3\right)^{7/3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c + d*x^3)^2/(a + b*x^3)^(10/3),x]
```

```
[Out] (x*(14*a^2*c^2 + 21*a*b*c^2*x^3 + 7*a^2*c*d*x^3 + 9*b^2*c^2*x^6 + 3*a*b*c*d
*x^6 + 2*a^2*d^2*x^6))/(14*a^3*(a + b*x^3)^(7/3))
```

fricas [A] time = 1.19, size = 103, normalized size = 1.32

$$\frac{\left(\left(9\,b^2c^2+3\,abcd+2\,a^2d^2\right)x^7+14\,a^2c^2x+7\left(3\,abc^2+a^2cd\right)x^4\right)\left(bx^3+a\right)^{\frac{2}{3}}}{14\left(a^3b^3x^9+3\,a^4b^2x^6+3\,a^5bx^3+a^6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^2/(b*x^3+a)^(10/3),x, algorithm="fricas")
```

```
[Out] 1/14*((9*b^2*c^2 + 3*a*b*c*d + 2*a^2*d^2)*x^7 + 14*a^2*c^2*x + 7*(3*a*b*c^2
 + a^2*c*d)*x^4)*(b*x^3 + a)^(2/3)/(a^3*b^3*x^9 + 3*a^4*b^2*x^6 + 3*a^5*b*x
^3 + a^6)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(dx^3+c\right)^2}{\left(bx^3+a\right)^{\frac{10}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^2/(b*x^3+a)^(10/3),x, algorithm="giac")
```

```
[Out] integrate((d*x^3 + c)^2/(b*x^3 + a)^(10/3), x)
```

maple [A] time = 0.05, size = 76, normalized size = 0.97

$$\frac{\left(2a^2d^2x^6+3abcd\,x^6+9b^2c^2x^6+7a^2cd\,x^3+21ab\,c^2x^3+14a^2c^2\right)x}{14\left(b\,x^3+a\right)^{\frac{7}{3}}a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c)^2/(b*x^3+a)^(10/3),x)
```

```
[Out] 1/14*x*(2*a^2*d^2*x^6+3*a*b*c*d*x^6+9*b^2*c^2*x^6+7*a^2*c*d*x^3+21*a*b*c^2*
x^3+14*a^2*c^2)/(b*x^3+a)^(7/3)/a^3
```

maxima [A] time = 0.51, size = 109, normalized size = 1.40

$$-\frac{\left(4\,b-\frac{7\left(bx^3+a\right)}{x^3}\right)cdx^7}{14\left(bx^3+a\right)^{\frac{7}{3}}a^2}+\frac{d^2x^7}{7\left(bx^3+a\right)^{\frac{7}{3}}a}+\frac{\left(2\,b^2-\frac{7\left(bx^3+a\right)b}{x^3}+\frac{14\left(bx^3+a\right)^2}{x^6}\right)c^2x^7}{14\left(bx^3+a\right)^{\frac{7}{3}}a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^2/(b*x^3+a)^(10/3),x, algorithm="maxima")
```

```
[Out] -1/14*(4*b - 7*(b*x^3 + a)/x^3)*c*d*x^7/((b*x^3 + a)^(7/3)*a^2) + 1/7*d^2*x
^7/((b*x^3 + a)^(7/3)*a) + 1/14*(2*b^2 - 7*(b*x^3 + a)*b/x^3 + 14*(b*x^3 +
a)^2/x^6)*c^2*x^7/((b*x^3 + a)^(7/3)*a^3)
```

mupad [B] time = 1.43, size = 148, normalized size = 1.90

$$\frac{2\,a^4\,d^2\,x+2\,a^2\,d^2\,x\,{\left(b\,x^3+a\right)}^2+9\,b^2\,c^2\,x\,{\left(b\,x^3+a\right)}^2+2\,a^2\,b^2\,c^2\,x-4\,a^3\,d^2\,x\,\left(b\,x^3+a\right)+3\,a\,b^2\,c^2\,x\,\left(b\,x^3+a\right)-4\,a^3\,b\,c\,d\,x+3\,a\,b\,c\,d\,x\,{\left(b\,x^3+a\right)}^2+a^2\,b\,c\,d\,x\,\left(b\,x^3+a\right)}{14\,a^3\,b^2\,{\left(b\,x^3+a\right)}^{7/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3)^2/(a + b*x^3)^(10/3),x)
```

```
[Out] (2*a^4*d^2*x + 2*a^2*d^2*x*(a + b*x^3)^2 + 9*b^2*c^2*x*(a + b*x^3)^2 + 2*a^
2*b^2*c^2*x - 4*a^3*d^2*x*(a + b*x^3) + 3*a*b^2*c^2*x*(a + b*x^3) - 4*a^3*b
*c*d*x + 3*a*b*c*d*x*(a + b*x^3)^2 + a^2*b*c*d*x*(a + b*x^3))/(14*a^3*b^2*(
a + b*x^3)^(7/3))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c)**2/(b*x**3+a)**(10/3),x)
```

```
[Out] Timed out
```

3.57 $\int \frac{(c+dx^3)^2}{(a+bx^3)^{13/3}} \, dx$

Optimal. Leaf size=174

$$\frac{9c^2x(9bc-10ad)}{140a^4\sqrt[3]{a+bx^3}\,(bc-ad)}+\frac{3cx\left(c+dx^3\right)(9bc-10ad)}{140a^3\left(a+bx^3\right)^{4/3}(bc-ad)}+\frac{x\left(c+dx^3\right)^2(9bc-10ad)}{70a^2\left(a+bx^3\right)^{7/3}(bc-ad)}+\frac{bx\left(c+dx^3\right)^3}{10a\left(a+bx^3\right)^{10/3}(bc-ad)}$$

Rubi [A] time = 0.07, antiderivative size = 174, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {382, 378, 191}

$$\frac{9c^2x(9bc-10ad)}{140a^4\sqrt[3]{a+bx^3}\,(bc-ad)}+\frac{x\left(c+dx^3\right)^2(9bc-10ad)}{70a^2\left(a+bx^3\right)^{7/3}(bc-ad)}+\frac{3cx\left(c+dx^3\right)(9bc-10ad)}{140a^3\left(a+bx^3\right)^{4/3}(bc-ad)}+\frac{bx\left(c+dx^3\right)^3}{10a\left(a+bx^3\right)^{10/3}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3)^2/(a + b*x^3)^(13/3),x]
```

```
[Out] (9*c^2*(9*b*c - 10*a*d)*x)/(140*a^4*(b*c - a*d)*(a + b*x^3)^(1/3)) + (3*c*(
9*b*c - 10*a*d)*x*(c + d*x^3))/(140*a^3*(b*c - a*d)*(a + b*x^3)^(4/3)) + ((
9*b*c - 10*a*d)*x*(c + d*x^3)^2)/(70*a^2*(b*c - a*d)*(a + b*x^3)^(7/3)) + (
b*x*(c + d*x^3)^3)/(10*a*(b*c - a*d)*(a + b*x^3)^(10/3))
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 378

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> -Simp[(x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*n*(p + 1)), x] - Dist[(c
*q)/(a*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1, 0]
 && GtQ[q, 0] && NeQ[p, -1]
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)), I
nt[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q}, x
] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !LtQ
[q, -1]) && NeQ[p, -1]
```

Rubi steps

$$\int \frac{\left(c+dx^3\right)^2}{\left(a+bx^3\right)^{13/3}}\,dx = \frac{bx\left(c+dx^3\right)^3}{10a(bc-ad)\left(a+bx^3\right)^{10/3}} + \frac{(9bc-10ad)\int \frac{\left(c+dx^3\right)^2}{\left(a+bx^3\right)^{10/3}}dx}{10a(bc-ad)}$$

$$= \frac{(9bc-10ad)x\left(c+dx^3\right)^2}{70a^2(bc-ad)\left(a+bx^3\right)^{7/3}} + \frac{bx\left(c+dx^3\right)^3}{10a(bc-ad)\left(a+bx^3\right)^{10/3}} + \frac{(3c(9bc-10ad))\int \frac{c+dx^3}{\left(a+bx^3\right)^{7/3}}d}{35a^2(bc-ad)}$$

$$= \frac{3c(9bc-10ad)x\left(c+dx^3\right)}{140a^3(bc-ad)\left(a+bx^3\right)^{4/3}} + \frac{(9bc-10ad)x\left(c+dx^3\right)^2}{70a^2(bc-ad)\left(a+bx^3\right)^{7/3}} + \frac{bx\left(c+dx^3\right)^3}{10a(bc-ad)\left(a+bx^3\right)^{10/3}} +$$

$$= \frac{9c^2(9bc-10ad)x}{140a^4(bc-ad)\sqrt[3]{a+bx^3}} + \frac{3c(9bc-10ad)x\left(c+dx^3\right)}{140a^3(bc-ad)\left(a+bx^3\right)^{4/3}} + \frac{(9bc-10ad)x\left(c+dx^3\right)^2}{70a^2(bc-ad)\left(a+bx^3\right)^{7/3}} +$$

Mathematica [A] time = 5.11, size = 106, normalized size = 0.61

$$\frac{x\left(10a^3\left(14c^2+7cdx^3+2d^2x^6\right)+3a^2bx^3\left(105c^2+20cdx^3+2d^2x^6\right)+18ab^2cx^6\left(15c+dx^3\right)+81b^3c^2x^9\right)}{140a^4\left(a+bx^3\right)^{10/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^3)^2/(a + b*x^3)^(13/3),x]
```

```
[Out] (x*(81*b^3*c^2*x^9 + 18*a*b^2*c*x^6*(15*c + d*x^3) + 10*a^3*(14*c^2 + 7*c*d
*x^3 + 2*d^2*x^6) + 3*a^2*b*x^3*(105*c^2 + 20*c*d*x^3 + 2*d^2*x^6)))/(140*a
^4*(a + b*x^3)^(10/3))
```

IntegrateAlgebraic [A] time = 0.83, size = 118, normalized size = 0.68

$$\frac{x\left(140a^3c^2+70a^3cdx^3+20a^3d^2x^6+315a^2bc^2x^3+60a^2bcdx^6+6a^2bd^2x^9+270ab^2c^2x^6+18ab^2cdx^9+81b^3c^2x^9\right)}{140a^4\left(a+bx^3\right)^{10/3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c + d*x^3)^2/(a + b*x^3)^(13/3),x]
```

```
[Out] (x*(140*a^3*c^2 + 315*a^2*b*c^2*x^3 + 70*a^3*c*d*x^3 + 270*a*b^2*c^2*x^6 +
60*a^2*b*c*d*x^6 + 20*a^3*d^2*x^6 + 81*b^3*c^2*x^9 + 18*a*b^2*c*d*x^9 + 6*a
^2*b*d^2*x^9))/(140*a^4*(a + b*x^3)^(10/3))
```

fricas [A] time = 0.99, size = 152, normalized size = 0.87

$$\frac{\left(3\left(27\,b^3c^2+6\,ab^2cd+2\,a^2bd^2\right)x^{10}+10\left(27\,ab^2c^2+6\,a^2bcd+2\,a^3d^2\right)x^7+140\,a^3c^2x+35\left(9\,a^2bc^2+2\,a^3cd\right)x^4\right)\left(bx^3+a\right)^{\frac{2}{3}}}{140\left(a^4b^4x^{12}+4\,a^5b^3x^9+6\,a^6b^2x^6+4\,a^7bx^3+a^8\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^2/(b*x^3+a)^(13/3),x, algorithm="fricas")
```

```
[Out] 1/140*(3*(27*b^3*c^2 + 6*a*b^2*c*d + 2*a^2*b*d^2)*x^10 + 10*(27*a*b^2*c^2 +
 6*a^2*b*c*d + 2*a^3*d^2)*x^7 + 140*a^3*c^2*x + 35*(9*a^2*b*c^2 + 2*a^3*c*d
)*x^4)*(b*x^3 + a)^(2/3)/(a^4*b^4*x^12 + 4*a^5*b^3*x^9 + 6*a^6*b^2*x^6 + 4*
a^7*b*x^3 + a^8)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(dx^3+c\right)^2}{\left(bx^3+a\right)^{\frac{13}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^2/(b*x^3+a)^(13/3),x, algorithm="giac")
```

```
[Out] integrate((d*x^3 + c)^2/(b*x^3 + a)^(13/3), x)
```

maple [A] time = 0.05, size = 115, normalized size = 0.66

$$\frac{\left(6a^2b\,d^2x^9+18a\,b^2cd\,x^9+81b^3c^2x^9+20a^3d^2x^6+60a^2bcd\,x^6+270a\,b^2c^2x^6+70a^3cd\,x^3+315a^2b\,c^2x^3+140c^2a^3\right)x}{140\left(b\,x^3+a\right)^{\frac{10}{3}}a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c)^2/(b*x^3+a)^(13/3),x)
```

```
[Out] 1/140*x*(6*a^2*b*d^2*x^9+18*a*b^2*c*d*x^9+81*b^3*c^2*x^9+20*a^3*d^2*x^6+60*
a^2*b*c*d*x^6+270*a*b^2*c^2*x^6+70*a^3*c*d*x^3+315*a^2*b*c^2*x^3+140*a^3*c^
2)/(b*x^3+a)^(10/3)/a^4
```

maxima [A] time = 0.71, size = 159, normalized size = 0.91

$$-\frac{\left(7\,b-\frac{10\left(bx^3+a\right)}{x^3}\right)d^2x^{10}}{70\left(bx^3+a\right)^{\frac{10}{3}}a^2}+\frac{\left(14\,b^2-\frac{40\left(bx^3+a\right)b}{x^3}+\frac{35\left(bx^3+a\right)^2}{x^6}\right)cdx^{10}}{70\left(bx^3+a\right)^{\frac{10}{3}}a^3}-\frac{\left(14\,b^3-\frac{60\left(bx^3+a\right)b^2}{x^3}+\frac{105\left(bx^3+a\right)^2b}{x^6}-\frac{140\left(bx^3+a\right)^3}{x^9}\right)c^2x^{10}}{140\left(bx^3+a\right)^{\frac{10}{3}}a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^2/(b*x^3+a)^(13/3),x, algorithm="maxima")
```

```
[Out] -1/70*(7*b - 10*(b*x^3 + a)/x^3)*d^2*x^10/((b*x^3 + a)^(10/3)*a^2) + 1/70*(
14*b^2 - 40*(b*x^3 + a)*b/x^3 + 35*(b*x^3 + a)^2/x^6)*c*d*x^10/((b*x^3 + a)
^(10/3)*a^3) - 1/140*(14*b^3 - 60*(b*x^3 + a)*b^2/x^3 + 105*(b*x^3 + a)^2*b
/x^6 - 140*(b*x^3 + a)^3/x^9)*c^2*x^10/((b*x^3 + a)^(10/3)*a^4)
```

mupad [B] time = 1.45, size = 176, normalized size = 1.01

$$\frac{x\left(\frac{c^2}{10\,a}+\frac{a\left(\frac{d^2}{10\,b}-\frac{c\,d}{5\,a}\right)}{b}\right)}{\left(b\,x^3+a\right)^{10/3}}-\frac{x\left(\frac{d^2}{7\,b^2}-\frac{-a^2\,d^2+2\,a\,b\,c\,d+9\,b^2\,c^2}{70\,a^2\,b^2}\right)}{\left(b\,x^3+a\right)^{7/3}}+\frac{x\left(2\,a^2\,d^2+6\,a\,b\,c\,d+27\,b^2\,c^2\right)}{140\,a^3\,b^2\left(b\,x^3+a\right)^{4/3}}+\frac{x\left(6\,a^2\,d^2+18\,a\,b\,c\,d+81\,b^2\,c^2\right)}{140\,a^4\,b^2\left(b\,x^3+a\right)^{1/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3)^2/(a + b*x^3)^(13/3),x)
```

```
[Out] (x*(c^2/(10*a) + (a*(d^2/(10*b) - (c*d)/(5*a)))/b))/(a + b*x^3)^(10/3) - (x
*(d^2/(7*b^2) - (9*b^2*c^2 - a^2*d^2 + 2*a*b*c*d)/(70*a^2*b^2)))/(a + b*x^3
)^(7/3) + (x*(2*a^2*d^2 + 27*b^2*c^2 + 6*a*b*c*d))/(140*a^3*b^2*(a + b*x^3)
^(4/3)) + (x*(6*a^2*d^2 + 81*b^2*c^2 + 18*a*b*c*d))/(140*a^4*b^2*(a + b*x^3
)^(1/3))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c)**2/(b*x**3+a)**(13/3),x)
```

```
[Out] Timed out
```

3.58 $\int \frac{\left(c+dx^3\right)^2}{\left(a+bx^3\right)^{16/3}}\,dx$

Optimal. Leaf size=211

$$\frac{2x(bc-ad)(ad+3bc)}{65a^2b^2\left(a+bx^3\right)^{10/3}}+\frac{9x\left(2a^2d^2+9abcd+54b^2c^2\right)}{910a^5b^2\sqrt[3]{a+bx^3}}+\frac{3x\left(2a^2d^2+9abcd+54b^2c^2\right)}{910a^4b^2\left(a+bx^3\right)^{4/3}}+\frac{x\left(2a^2d^2+9abcd+54b^2\right.}{455a^3b^2\left(a+bx^3\right)^{7/3}}$$

Rubi [A] time = 0.13, antiderivative size = 211, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {413, 385, 192, 191}

$$\frac{9x\left(2a^2d^2+9abcd+54b^2c^2\right)}{910a^5b^2\sqrt[3]{a+bx^3}}+\frac{3x\left(2a^2d^2+9abcd+54b^2c^2\right)}{910a^4b^2\left(a+bx^3\right)^{4/3}}+\frac{x\left(2a^2d^2+9abcd+54b^2c^2\right)}{455a^3b^2\left(a+bx^3\right)^{7/3}}+\frac{2x(bc-ad)(ad+3bc)}{65a^2b^2\left(a+bx^3\right)^{10/3}}+\frac{x\left(c+dx^3\right)(bc-ad)}{13ab\left(a+bx^3\right)^{13/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3)^2/(a + b*x^3)^(16/3),x]
```

```
[Out] (2*(b*c - a*d)*(3*b*c + a*d)*x)/(65*a^2*b^2*(a + b*x^3)^(10/3)) + ((54*b^2*
c^2 + 9*a*b*c*d + 2*a^2*d^2)*x)/(455*a^3*b^2*(a + b*x^3)^(7/3)) + (3*(54*b^
2*c^2 + 9*a*b*c*d + 2*a^2*d^2)*x)/(910*a^4*b^2*(a + b*x^3)^(4/3)) + (9*(54*
b^2*c^2 + 9*a*b*c*d + 2*a^2*d^2)*x)/(910*a^5*b^2*(a + b*x^3)^(1/3)) + ((b*c
 - a*d)*x*(c + d*x^3))/(13*a*b*(a + b*x^3)^(13/3))
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 413

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[((a*d - c*b)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*n*(p +
 1)), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(c+dx^3\right)^2}{\left(a+bx^3\right)^{16/3}}\,dx &= \frac{(bc-ad)x\left(c+dx^3\right)}{13ab\left(a+bx^3\right)^{13/3}} + \frac{\int \frac{c(12bc+ad)+d(9bc+4ad)x^3}{\left(a+bx^3\right)^{13/3}}\,dx}{13ab} \\
&= \frac{2(bc-ad)(3bc+ad)x}{65a^2b^2\left(a+bx^3\right)^{10/3}} + \frac{(bc-ad)x\left(c+dx^3\right)}{13ab\left(a+bx^3\right)^{13/3}} + \frac{\left(54b^2c^2+9abcd+2a^2d^2\right)\int \frac{1}{\left(a+bx^3\right)^{10/3}}\,dx}{65a^2b^2} \\
&= \frac{2(bc-ad)(3bc+ad)x}{65a^2b^2\left(a+bx^3\right)^{10/3}} + \frac{\left(54b^2c^2+9abcd+2a^2d^2\right)x}{455a^3b^2\left(a+bx^3\right)^{7/3}} + \frac{(bc-ad)x\left(c+dx^3\right)}{13ab\left(a+bx^3\right)^{13/3}} + \frac{\left(6\left(54b^2c^2 \right.\right.}{} \\
&= \frac{2(bc-ad)(3bc+ad)x}{65a^2b^2\left(a+bx^3\right)^{10/3}} + \frac{\left(54b^2c^2+9abcd+2a^2d^2\right)x}{455a^3b^2\left(a+bx^3\right)^{7/3}} + \frac{3\left(54b^2c^2+9abcd+2a^2d^2\right)x}{910a^4b^2\left(a+bx^3\right)^{4/3}} + \frac{(bc}{13} \\
&= \frac{2(bc-ad)(3bc+ad)x}{65a^2b^2\left(a+bx^3\right)^{10/3}} + \frac{\left(54b^2c^2+9abcd+2a^2d^2\right)x}{455a^3b^2\left(a+bx^3\right)^{7/3}} + \frac{3\left(54b^2c^2+9abcd+2a^2d^2\right)x}{910a^4b^2\left(a+bx^3\right)^{4/3}} + \frac{9\left(\right.}{}
\end{aligned}
$$

Mathematica [A] time = 5.18, size = 138, normalized size = 0.65

$$
\frac{x\left(65a^4\left(14c^2+7cdx^3+2d^2x^6\right)+39a^3bx^3\left(70c^2+15cdx^3+2d^2x^6\right)+9a^2b^2x^6\left(390c^2+39cdx^3+2d^2x^6\right)+81ab^3cx^9\left(26c+dx^3\right)+486b^4c^2x^{12}\right)}{910a^5\left(a+bx^3\right)^{13/3}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^3)^2/(a + b*x^3)^(16/3),x]
```

```
[Out] (x*(486*b^4*c^2*x^12 + 81*a*b^3*c*x^9*(26*c + d*x^3) + 65*a^4*(14*c^2 + 7*c
*d*x^3 + 2*d^2*x^6) + 39*a^3*b*x^3*(70*c^2 + 15*c*d*x^3 + 2*d^2*x^6) + 9*a^
2*b^2*x^6*(390*c^2 + 39*c*d*x^3 + 2*d^2*x^6)))/(910*a^5*(a + b*x^3)^(13/3))
```

IntegrateAlgebraic [A] time = 1.26, size = 159, normalized size = 0.75

$$
\frac{x\left(910a^4c^2+455a^4cdx^3+130a^4d^2x^6+2730a^3bc^2x^3+585a^3bcdx^6+78a^3bd^2x^9+3510a^2b^2c^2x^6+351a^2b^2cdx^9+18a^2b^2d^2x^{12}+2106ab^3c^2x^9+81ab^3cdx^{12}+486b^4c^2x^{12}\right)}{910a^5\left(a+bx^3\right)^{13/3}}
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c + d*x^3)^2/(a + b*x^3)^(16/3),x]
```

```
[Out] (x*(910*a^4*c^2 + 2730*a^3*b*c^2*x^3 + 455*a^4*c*d*x^3 + 3510*a^2*b^2*c^2*x
^6 + 585*a^3*b*c*d*x^6 + 130*a^4*d^2*x^6 + 2106*a*b^3*c^2*x^9 + 351*a^2*b^2
*c*d*x^9 + 78*a^3*b*d^2*x^9 + 486*b^4*c^2*x^12 + 81*a*b^3*c*d*x^12 + 18*a^2
*b^2*d^2*x^12))/(910*a^5*(a + b*x^3)^(13/3))
```

fricas [A] time = 1.07, size = 200, normalized size = 0.95

$$
\frac{\left(9\left(54b^4c^2+9ab^3cd+2a^2b^2d^2\right)x^{13}+39\left(54ab^3c^2+9a^2b^2cd+2a^3bd^2\right)x^{10}+65\left(54a^2b^2c^2+9a^3bcd+2a^4d^2\right)x^7+910a^4c^2x+455\left(6a^3bc^2+a^4cd\right)x^4\right)\left(bx^3+a\right)^{\frac{2}{3}}}{910\left(a^5b^5x^{15}+5a^6b^4x^{12}+10a^7b^3x^9+10a^8b^2x^6+5a^9bx^3+a^{10}\right)}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^2/(b*x^3+a)^(16/3),x, algorithm="fricas")
```

```
[Out] 1/910*(9*(54*b^4*c^2 + 9*a*b^3*c*d + 2*a^2*b^2*d^2)*x^13 + 39*(54*a*b^3*c^2
 + 9*a^2*b^2*c*d + 2*a^3*b*d^2)*x^10 + 65*(54*a^2*b^2*c^2 + 9*a^3*b*c*d + 2
*a^4*d^2)*x^7 + 910*a^4*c^2*x + 455*(6*a^3*b*c^2 + a^4*c*d)*x^4)*(b*x^3 + a
```

```
)^(2/3)/(a^5*b^5*x^15 + 5*a^6*b^4*x^12 + 10*a^7*b^3*x^9 + 10*a^8*b^2*x^6 +
5*a^9*b*x^3 + a^10)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(dx^3+c\right)^2}{\left(bx^3+a\right)^{\frac{16}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^2/(b*x^3+a)^(16/3),x, algorithm="giac")
```

```
[Out] integrate((d*x^3 + c)^2/(b*x^3 + a)^(16/3), x)
```

maple [A] time = 0.05, size = 156, normalized size = 0.74

$$\frac{\left(18a^2b^2d^2x^{12}+81a\,b^3cd\,x^{12}+486b^4c^2x^{12}+78a^3b\,d^2x^9+351a^2b^2cd\,x^9+2106a\,b^3c^2x^9+130a^4d^2x^6+585a^3bcd\,x^6+3510a^2b^2c^2x^6+455a^4cd\,x^3+2730a^3b\,c^2x^3+910c^2a^4\right)x}{910\left(b\,x^3+a\right)^{\frac{13}{3}}a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c)^2/(b*x^3+a)^(16/3),x)
```

```
[Out] 1/910*x*(18*a^2*b^2*d^2*x^12+81*a*b^3*c*d*x^12+486*b^4*c^2*x^12+78*a^3*b*d^
2*x^9+351*a^2*b^2*c*d*x^9+2106*a*b^3*c^2*x^9+130*a^4*d^2*x^6+585*a^3*b*c*d*
x^6+3510*a^2*b^2*c^2*x^6+455*a^4*c*d*x^3+2730*a^3*b*c^2*x^3+910*a^4*c^2)/(b
*x^3+a)^(13/3)/a^5
```

maxima [A] time = 0.50, size = 210, normalized size = 1.00

$$\frac{\left(35\,b^2-\frac{91\left(bx^3+a\right)b}{x^3}+\frac{65\left(bx^3+a\right)^2}{x^6}\right)d^2x^{13}}{455\left(bx^3+a\right)^{\frac{13}{3}}a^3}-\frac{\left(140\,b^3-\frac{546\left(bx^3+a\right)b^2}{x^3}+\frac{780\left(bx^3+a\right)^2b}{x^6}-\frac{455\left(bx^3+a\right)^3}{x^9}\right)cdx^{13}}{910\left(bx^3+a\right)^{\frac{13}{3}}a^4}+\frac{\left(35\,b^4-\frac{182\left(bx^3+a\right)b^3}{x^3}+\frac{390\left(bx^3+a\right)^2b^2}{x^6}-\frac{455\left(bx^3+a\right)^3b}{x^9}+\frac{455\left(bx^3+a\right)^4}{x^{12}}\right)c^2x^{13}}{455\left(bx^3+a\right)^{\frac{13}{3}}a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^2/(b*x^3+a)^(16/3),x, algorithm="maxima")
```

```
[Out] 1/455*(35*b^2 - 91*(b*x^3 + a)*b/x^3 + 65*(b*x^3 + a)^2/x^6)*d^2*x^13/((b*x
^3 + a)^(13/3)*a^3) - 1/910*(140*b^3 - 546*(b*x^3 + a)*b^2/x^3 + 780*(b*x^3
 + a)^2*b/x^6 - 455*(b*x^3 + a)^3/x^9)*c*d*x^13/((b*x^3 + a)^(13/3)*a^4) +
1/455*(35*b^4 - 182*(b*x^3 + a)*b^3/x^3 + 390*(b*x^3 + a)^2*b^2/x^6 - 455*(
b*x^3 + a)^3*b/x^9 + 455*(b*x^3 + a)^4/x^12)*c^2*x^13/((b*x^3 + a)^(13/3)*a
^5)
```

mupad [B] time = 1.43, size = 217, normalized size = 1.03

$$\frac{x\left(\frac{c^2}{13\,a}+\frac{a\left(\frac{d^2}{13\,b}-\frac{2\,c\,d}{13\,a}\right)}{b}\right)}{\left(b\,x^3+a\right)^{13/3}}-\frac{x\left(\frac{d^2}{10\,b^2}-\frac{-a^2\,d^2+2\,a\,b\,c\,d+12\,b^2\,c^2}{130\,a^2\,b^2}\right)}{\left(b\,x^3+a\right)^{10/3}}+\frac{x\left(2\,a^2\,d^2+9\,a\,b\,c\,d+54\,b^2\,c^2\right)}{455\,a^3\,b^2\left(b\,x^3+a\right)^{7/3}}+\frac{x\left(6\,a^2\,d^2+27\,a\,b\,c\,d+162\,b^2\,c^2\right)}{910\,a^4\,b^2\left(b\,x^3+a\right)^{4/3}}+\frac{x\left(18\,a^2\,d^2+81\,a\,b\,c\,d+486\,b^2\,c^2\right)}{910\,a^5\,b^2\left(b\,x^3+a\right)^{1/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3)^2/(a + b*x^3)^(16/3),x)
```

```
[Out] (x*(c^2/(13*a) + (a*(d^2/(13*b) - (2*c*d)/(13*a)))/b))/(a + b*x^3)^(13/3) -
 (x*(d^2/(10*b^2) - (12*b^2*c^2 - a^2*d^2 + 2*a*b*c*d)/(130*a^2*b^2)))/(a +
 b*x^3)^(10/3) + (x*(2*a^2*d^2 + 54*b^2*c^2 + 9*a*b*c*d))/(455*a^3*b^2*(a +
 b*x^3)^(7/3)) + (x*(6*a^2*d^2 + 162*b^2*c^2 + 27*a*b*c*d))/(910*a^4*b^2*(a
 + b*x^3)^(4/3)) + (x*(18*a^2*d^2 + 486*b^2*c^2 + 81*a*b*c*d))/(910*a^5*b^2
*(a + b*x^3)^(1/3))
```

<u>**sympy**</u> **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c)**2/(b*x**3+a)**(16/3),x)
```

```
[Out] Timed out
```

3.59 $\int \frac{(c+dx^3)^2}{(a+bx^3)^{19/3}}\,dx$

Optimal. Leaf size=253

$$\frac{x(bc-ad)(4ad+15bc)}{208a^2b^2\left(a+bx^3\right)^{13/3}}+\frac{81x\left(a^2d^2+6abcd+45b^2c^2\right)}{7280a^6b^2\sqrt[3]{a+bx^3}}+\frac{27x\left(a^2d^2+6abcd+45b^2c^2\right)}{7280a^5b^2\left(a+bx^3\right)^{4/3}}+\frac{9x\left(a^2d^2+6abcd+45b\right.}{3640a^4b^2\left(a+bx^3\right)^{7}}$$

Rubi [A] time = 0.21, antiderivative size = 253, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {413, 385, 192, 191}

$$\frac{81x\left(a^2d^2+6abcd+45b^2c^2\right)}{7280a^6b^2\sqrt[3]{a+bx^3}}+\frac{27x\left(a^2d^2+6abcd+45b^2c^2\right)}{7280a^5b^2\left(a+bx^3\right)^{4/3}}+\frac{9x\left(a^2d^2+6abcd+45b^2c^2\right)}{3640a^4b^2\left(a+bx^3\right)^{7/3}}+\frac{x\left(a^2d^2+6abcd+45b^2c^2\right)}{520a^3b^2\left(a+bx^3\right)^{10/3}}+\frac{x(bc-ad)(4ad+15bc)}{208a^2b^2\left(a+bx^3\right)^{13/3}}+\frac{x\left(c+dx^3\right)(bc-ad)}{16ab\left(a+bx^3\right)^{16/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3)^2/(a + b*x^3)^(19/3),x]
```

```
[Out] ((b*c - a*d)*(15*b*c + 4*a*d)*x)/(208*a^2*b^2*(a + b*x^3)^(13/3)) + ((45*b^
2*c^2 + 6*a*b*c*d + a^2*d^2)*x)/(520*a^3*b^2*(a + b*x^3)^(10/3)) + (9*(45*b
^2*c^2 + 6*a*b*c*d + a^2*d^2)*x)/(3640*a^4*b^2*(a + b*x^3)^(7/3)) + (27*(45
*b^2*c^2 + 6*a*b*c*d + a^2*d^2)*x)/(7280*a^5*b^2*(a + b*x^3)^(4/3)) + (81*(
45*b^2*c^2 + 6*a*b*c*d + a^2*d^2)*x)/(7280*a^6*b^2*(a + b*x^3)^(1/3)) + ((b
*c - a*d)*x*(c + d*x^3))/(16*a*b*(a + b*x^3)^(16/3))
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 413

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[((a*d - c*b)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*n*(p +
 1)), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rubi steps

$$\int \frac{\left(c+dx^3\right)^2}{\left(a+bx^3\right)^{19/3}}\,dx = \frac{(bc-ad)x\left(c+dx^3\right)}{16ab\left(a+bx^3\right)^{16/3}} + \frac{\int \frac{c(15bc+ad)+4d(3bc+ad)x^3}{\left(a+bx^3\right)^{16/3}}\,dx}{16ab}$$

$$= \frac{(bc-ad)(15bc+4ad)x}{208a^2b^2\left(a+bx^3\right)^{13/3}} + \frac{(bc-ad)x\left(c+dx^3\right)}{16ab\left(a+bx^3\right)^{16/3}} + \frac{\left(45b^2c^2+6abcd+a^2d^2\right)\int \frac{1}{\left(a+bx^3\right)^{13/3}}\,dx}{52a^2b^2}$$

$$= \frac{(bc-ad)(15bc+4ad)x}{208a^2b^2\left(a+bx^3\right)^{13/3}} + \frac{\left(45b^2c^2+6abcd+a^2d^2\right)x}{520a^3b^2\left(a+bx^3\right)^{10/3}} + \frac{(bc-ad)x\left(c+dx^3\right)}{16ab\left(a+bx^3\right)^{16/3}} + \frac{\left(9\left(45b^2c^2+\right.\right.}{}$$

$$= \frac{(bc-ad)(15bc+4ad)x}{208a^2b^2\left(a+bx^3\right)^{13/3}} + \frac{\left(45b^2c^2+6abcd+a^2d^2\right)x}{520a^3b^2\left(a+bx^3\right)^{10/3}} + \frac{9\left(45b^2c^2+6abcd+a^2d^2\right)x}{3640a^4b^2\left(a+bx^3\right)^{7/3}} + \frac{(bc}{16a}$$

$$= \frac{(bc-ad)(15bc+4ad)x}{208a^2b^2\left(a+bx^3\right)^{13/3}} + \frac{\left(45b^2c^2+6abcd+a^2d^2\right)x}{520a^3b^2\left(a+bx^3\right)^{10/3}} + \frac{9\left(45b^2c^2+6abcd+a^2d^2\right)x}{3640a^4b^2\left(a+bx^3\right)^{7/3}} + \frac{27(}{}$$

$$= \frac{(bc-ad)(15bc+4ad)x}{208a^2b^2\left(a+bx^3\right)^{13/3}} + \frac{\left(45b^2c^2+6abcd+a^2d^2\right)x}{520a^3b^2\left(a+bx^3\right)^{10/3}} + \frac{9\left(45b^2c^2+6abcd+a^2d^2\right)x}{3640a^4b^2\left(a+bx^3\right)^{7/3}} + \frac{27(}{}$$

Mathematica [A] time = 5.15, size = 169, normalized size = 0.67

$$\frac{x\left(520a^5\left(14c^2+7cdx^3+2d^2x^6\right)+156a^4bx^3\left(175c^2+40cdx^3+6d^2x^6\right)+144a^3b^2x^6\left(325c^2+39cdx^3+3d^2x^6\right)+81a^2b^3x^9\left(520c^2+32cdx^3+d^2x^6\right)+486ab^4cx^{12}\left(40c+dx^3\right)+3645b^5c^2x^{15}\right)}{7280a^6\left(a+bx^3\right)^{16/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^3)^2/(a + b*x^3)^(19/3),x]
```

```
[Out] (x*(3645*b^5*c^2*x^15 + 486*a*b^4*c*x^12*(40*c + d*x^3) + 81*a^2*b^3*x^9*(5
20*c^2 + 32*c*d*x^3 + d^2*x^6) + 520*a^5*(14*c^2 + 7*c*d*x^3 + 2*d^2*x^6) +
 144*a^3*b^2*x^6*(325*c^2 + 39*c*d*x^3 + 3*d^2*x^6) + 156*a^4*b*x^3*(175*c^
2 + 40*c*d*x^3 + 6*d^2*x^6)))/(7280*a^6*(a + b*x^3)^(16/3))
```

IntegrateAlgebraic [A] time = 1.87, size = 200, normalized size = 0.79

$$\frac{x\left(7280a^5c^2+3640a^5cdx^3+1040a^5d^2x^6+27300a^4bc^2x^3+6240a^4bcdx^6+936a^4bd^2x^9+46800a^3b^2c^2x^6+5616a^3b^2cdx^9+432a^3b^2d^2x^{12}+42120a^2b^3c^2x^9+2592a^2b^3cdx^{12}+81a^2b^3d^2x^{15}+19440ab^4c^2x^{12}+486ab^4cdx^{15}+3645b^5c^2x^{15}\right)}{7280a^6\left(a+bx^3\right)^{16/3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c + d*x^3)^2/(a + b*x^3)^(19/3),x]
```

```
[Out] (x*(7280*a^5*c^2 + 27300*a^4*b*c^2*x^3 + 3640*a^5*c*d*x^3 + 46800*a^3*b^2*c
^2*x^6 + 6240*a^4*b*c*d*x^6 + 1040*a^5*d^2*x^6 + 42120*a^2*b^3*c^2*x^9 + 56
16*a^3*b^2*c*d*x^9 + 936*a^4*b*d^2*x^9 + 19440*a*b^4*c^2*x^12 + 2592*a^2*b^
3*c*d*x^12 + 432*a^3*b^2*d^2*x^12 + 3645*b^5*c^2*x^15 + 486*a*b^4*c*d*x^15
+ 81*a^2*b^3*d^2*x^15))/(7280*a^6*(a + b*x^3)^(16/3))
```

fricas [A] time = 1.41, size = 246, normalized size = 0.97

$$\frac{\left(81\left(45\,b^5c^2+6\,ab^4cd+a^2b^3d^2\right)x^{16}+432\left(45\,ab^4c^2+6\,a^2b^3cd+a^3b^2d^2\right)x^{13}+936\left(45\,a^2b^3c^2+6\,a^3b^2cd+a^4bd^2\right)x^{10}+7280\,a^5c^2x+1040\left(45\,a^3b^2c^2+6\,a^4bcd+a^5d^2\right)x^7+1820\left(15\,a^4bc^2+2\,a^5cd\right)x^4\right)\left(bx^3+a\right)^{\frac{2}{3}}}{7280\left(a^6b^6x^{18}+6\,a^7b^5x^{15}+15\,a^8b^4x^{12}+20\,a^9b^3x^9+15\,a^{10}b^2x^6+6\,a^{11}bx^3+a^{12}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^2/(b*x^3+a)^(19/3),x, algorithm="fricas")
```

```
[Out] 1/7280*(81*(45*b^5*c^2 + 6*a*b^4*c*d + a^2*b^3*d^2)*x^16 + 432*(45*a*b^4*c^
2 + 6*a^2*b^3*c*d + a^3*b^2*d^2)*x^13 + 936*(45*a^2*b^3*c^2 + 6*a^3*b^2*c*d
 + a^4*b*d^2)*x^10 + 7280*a^5*c^2*x + 1040*(45*a^3*b^2*c^2 + 6*a^4*b*c*d +
a^5*d^2)*x^7 + 1820*(15*a^4*b*c^2 + 2*a^5*c*d)*x^4)*(b*x^3 + a)^(2/3)/(a^6*
b^6*x^18 + 6*a^7*b^5*x^15 + 15*a^8*b^4*x^12 + 20*a^9*b^3*x^9 + 15*a^10*b^2*
x^6 + 6*a^11*b*x^3 + a^12)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(dx^3 + c\right)^2}{\left(bx^3 + a\right)^{\frac{19}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^2/(b*x^3+a)^(19/3),x, algorithm="giac")
```

```
[Out] integrate((d*x^3 + c)^2/(b*x^3 + a)^(19/3), x)
```

maple [A] time = 0.05, size = 197, normalized size = 0.78

$$\frac{\left(81a^2b^3d^2x^{15} + 486a\,b^4cd\,x^{15} + 3645b^5c^2x^{15} + 432a^3b^2d^2x^{12} + 2592a^2b^3cd\,x^{12} + 19440a\,b^4c^2x^{12} + 936a^4b\,d^2x^9 + 5616a^3b^2cd\,x^9 + 42120a^2b^3c^2x^9 + 1040a^5d^2x^6 + 6240a^4bcd\,x^6 + 46800a^3b^2c^2x^6 + 3640a^5cd\,x^3 + 27300a^4b\,c^2x^3 + 7280c^2a^5\right)x}{7280\left(b\,x^3 + a\right)^{\frac{16}{3}}a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c)^2/(b*x^3+a)^(19/3),x)
```

```
[Out] 1/7280*x*(81*a^2*b^3*d^2*x^15+486*a*b^4*c*d*x^15+3645*b^5*c^2*x^15+432*a^3*
b^2*d^2*x^12+2592*a^2*b^3*c*d*x^12+19440*a*b^4*c^2*x^12+936*a^4*b*d^2*x^9+5
616*a^3*b^2*c*d*x^9+42120*a^2*b^3*c^2*x^9+1040*a^5*d^2*x^6+6240*a^4*b*c*d*x
^6+46800*a^3*b^2*c^2*x^6+3640*a^5*c*d*x^3+27300*a^4*b*c^2*x^3+7280*a^5*c^2)
/(b*x^3+a)^(16/3)/a^6
```

maxima [A] time = 0.64, size = 261, normalized size = 1.03

$$-\frac{\left(455\,b^3 - \frac{1680\left(bx^3+a\right)b^2}{x^3} + \frac{2184\left(bx^3+a\right)^2 b}{x^6} - \frac{1040\left(bx^3+a\right)^3}{x^9}\right)d^2x^{16}}{7280\left(bx^3 + a\right)^{\frac{16}{3}}a^4} + \frac{\left(455\,b^4 - \frac{2240\left(bx^3+a\right)b^3}{x^3} + \frac{4368\left(bx^3+a\right)^2 b^2}{x^6} - \frac{4160\left(bx^3+a\right)^3 b}{x^9} + \frac{1820\left(bx^3+a\right)^4}{x^{12}}\right)cdx^{16}}{3640\left(bx^3 + a\right)^{\frac{16}{3}}a^5} - \frac{\left(91\,b^5 - \frac{560\left(bx^3+a\right)b^4}{x^3} + \frac{1456\left(bx^3+a\right)^2 b^3}{x^6} - \frac{2080\left(bx^3+a\right)^3 b^2}{x^9} + \frac{1820\left(bx^3+a\right)^4 b}{x^{12}} - \frac{1456\left(bx^3+a\right)^5}{x^{15}}\right)c^2x^{16}}{1456\left(bx^3 + a\right)^{\frac{16}{3}}a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^2/(b*x^3+a)^(19/3),x, algorithm="maxima")
```

```
[Out] -1/7280*(455*b^3 - 1680*(b*x^3 + a)*b^2/x^3 + 2184*(b*x^3 + a)^2*b/x^6 - 10
40*(b*x^3 + a)^3/x^9)*d^2*x^16/((b*x^3 + a)^(16/3)*a^4) + 1/3640*(455*b^4 -
 2240*(b*x^3 + a)*b^3/x^3 + 4368*(b*x^3 + a)^2*b^2/x^6 - 4160*(b*x^3 + a)^3
*b/x^9 + 1820*(b*x^3 + a)^4/x^12)*c*d*x^16/((b*x^3 + a)^(16/3)*a^5) - 1/145
6*(91*b^5 - 560*(b*x^3 + a)*b^4/x^3 + 1456*(b*x^3 + a)^2*b^3/x^6 - 2080*(b*
x^3 + a)^3*b^2/x^9 + 1820*(b*x^3 + a)^4*b/x^12 - 1456*(b*x^3 + a)^5/x^15)*c
^2*x^16/((b*x^3 + a)^(16/3)*a^6)
```

mupad [B] time = 1.48, size = 257, normalized size = 1.02

$$\frac{x\left(\frac{c^2}{16\,a} + \frac{a\left(\frac{d^2}{16\,b} - \frac{c\,d}{8\,a}\right)}{b}\right)}{\left(b\,x^3 + a\right)^{16/3}} - \frac{x\left(\frac{d^2}{13\,b^2} - \frac{-a^2\,d^2 + 2\,a\,b\,c\,d + 15\,b^2\,c^2}{208\,a^2\,b^2}\right)}{\left(b\,x^3 + a\right)^{13/3}} + \frac{x\left(a^2\,d^2 + 6\,a\,b\,c\,d + 45\,b^2\,c^2\right)}{520\,a^3\,b^2\left(b\,x^3 + a\right)^{10/3}} + \frac{x\left(9\,a^2\,d^2 + 54\,a\,b\,c\,d + 405\,b^2\,c^2\right)}{3640\,a^4\,b^2\left(b\,x^3 + a\right)^{7/3}} + \frac{x\left(27\,a^2\,d^2 + 162\,a\,b\,c\,d + 1215\,b^2\,c^2\right)}{7280\,a^5\,b^2\left(b\,x^3 + a\right)^{4/3}} + \frac{x\left(81\,a^2\,d^2 + 486\,a\,b\,c\,d + 3645\,b^2\,c^2\right)}{7280\,a^6\,b^2\left(b\,x^3 + a\right)^{1/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3)^2/(a + b*x^3)^(19/3),x)
```

```
[Out] (x*(c^2/(16*a) + (a*(d^2/(16*b) - (c*d)/(8*a)))/b))/(a + b*x^3)^(16/3) - (x
*(d^2/(13*b^2) - (15*b^2*c^2 - a^2*d^2 + 2*a*b*c*d)/(208*a^2*b^2)))/(a + b*
x^3)^(13/3) + (x*(a^2*d^2 + 45*b^2*c^2 + 6*a*b*c*d))/(520*a^3*b^2*(a + b*x^
```

```
3)^(10/3)) + (x*(9*a^2*d^2 + 405*b^2*c^2 + 54*a*b*c*d))/(3640*a^4*b^2*(a +
b*x^3)^(7/3)) + (x*(27*a^2*d^2 + 1215*b^2*c^2 + 162*a*b*c*d))/(7280*a^5*b^2
*(a + b*x^3)^(4/3)) + (x*(81*a^2*d^2 + 3645*b^2*c^2 + 486*a*b*c*d))/(7280*a
^6*b^2*(a + b*x^3)^(1/3))
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c)**2/(b*x**3+a)**(19/3),x)
```

```
[Out] Timed out
```

3.60 $\int \frac{\left(a+bx^3\right)^3}{\left(c+dx^3\right)^{13/3}}\,dx$

Optimal. Leaf size=109

$$\frac{81a^3x}{140c^4\sqrt[3]{c+dx^3}}+\frac{27a^2x\left(a+bx^3\right)}{140c^3\left(c+dx^3\right)^{4/3}}+\frac{9ax\left(a+bx^3\right)^2}{70c^2\left(c+dx^3\right)^{7/3}}+\frac{x\left(a+bx^3\right)^3}{10c\left(c+dx^3\right)^{10/3}}$$

Rubi [A] time = 0.04, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {378, 191}

$$\frac{27a^2x\left(a+bx^3\right)}{140c^3\left(c+dx^3\right)^{4/3}}+\frac{81a^3x}{140c^4\sqrt[3]{c+dx^3}}+\frac{9ax\left(a+bx^3\right)^2}{70c^2\left(c+dx^3\right)^{7/3}}+\frac{x\left(a+bx^3\right)^3}{10c\left(c+dx^3\right)^{10/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^3/(c + d*x^3)^(13/3),x]
```

```
[Out] (x*(a + b*x^3)^3)/(10*c*(c + d*x^3)^(10/3)) + (9*a*x*(a + b*x^3)^2)/(70*c^2
*(c + d*x^3)^(7/3)) + (27*a^2*x*(a + b*x^3))/(140*c^3*(c + d*x^3)^(4/3)) +
(81*a^3*x)/(140*c^4*(c + d*x^3)^(1/3))
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 378

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> -Simp[(x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*n*(p + 1)), x] - Dist[(c
*q)/(a*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1, 0]
 && GtQ[q, 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a+bx^3\right)^3}{\left(c+dx^3\right)^{13/3}}\,dx &= \frac{x\left(a+bx^3\right)^3}{10c\left(c+dx^3\right)^{10/3}}+\frac{(9a)\int \frac{\left(a+bx^3\right)^2}{\left(c+dx^3\right)^{10/3}}\,dx}{10c}\\
&= \frac{x\left(a+bx^3\right)^3}{10c\left(c+dx^3\right)^{10/3}}+\frac{9ax\left(a+bx^3\right)^2}{70c^2\left(c+dx^3\right)^{7/3}}+\frac{\left(27a^2\right)\int \frac{a+bx^3}{\left(c+dx^3\right)^{7/3}}\,dx}{35c^2}\\
&= \frac{x\left(a+bx^3\right)^3}{10c\left(c+dx^3\right)^{10/3}}+\frac{9ax\left(a+bx^3\right)^2}{70c^2\left(c+dx^3\right)^{7/3}}+\frac{27a^2x\left(a+bx^3\right)}{140c^3\left(c+dx^3\right)^{4/3}}+\frac{\left(81a^3\right)\int \frac{1}{\left(c+dx^3\right)^{4/3}}\,dx}{140c^3}\\
&= \frac{x\left(a+bx^3\right)^3}{10c\left(c+dx^3\right)^{10/3}}+\frac{9ax\left(a+bx^3\right)^2}{70c^2\left(c+dx^3\right)^{7/3}}+\frac{27a^2x\left(a+bx^3\right)}{140c^3\left(c+dx^3\right)^{4/3}}+\frac{81a^3x}{140c^4\sqrt[3]{c+dx^3}}
\end{aligned}$$

Mathematica [A] time = 0.04, size = 120, normalized size = 1.10

$$\frac{x\left(a^3\left(140c^3+315c^2dx^3+270cd^2x^6+81d^3x^9\right)+3a^2bcx^3\left(35c^2+30cdx^3+9d^2x^6\right)+6ab^2c^2x^6\left(10c+3dx^3\right)+14b^3c^3x^9\right)}{140c^4\left(c+dx^3\right)^{10/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^3/(c + d*x^3)^(13/3),x]
```

```
[Out] (x*(14*b^3*c^3*x^9 + 6*a*b^2*c^2*x^6*(10*c + 3*d*x^3) + 3*a^2*b*c*x^3*(35*c
^2 + 30*c*d*x^3 + 9*d^2*x^6) + a^3*(140*c^3 + 315*c^2*d*x^3 + 270*c*d^2*x^6
 + 81*d^3*x^9)))/(140*c^4*(c + d*x^3)^(10/3))
```

IntegrateAlgebraic [A] time = 1.14, size = 137, normalized size = 1.26

$$\frac{x\left(140a^3c^3+315a^3c^2dx^3+270a^3cd^2x^6+81a^3d^3x^9+105a^2bc^3x^3+90a^2bc^2dx^6+27a^2bcd^2x^9+60ab^2c^3x^6+18ab^2c^2dx^9+14b^3c^3x^9\right)}{140c^4\left(c+dx^3\right)^{10/3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x^3)^3/(c + d*x^3)^(13/3),x]
```

```
[Out] (x*(140*a^3*c^3 + 105*a^2*b*c^3*x^3 + 315*a^3*c^2*d*x^3 + 60*a*b^2*c^3*x^6
+ 90*a^2*b*c^2*d*x^6 + 270*a^3*c*d^2*x^6 + 14*b^3*c^3*x^9 + 18*a*b^2*c^2*d*
x^9 + 27*a^2*b*c*d^2*x^9 + 81*a^3*d^3*x^9))/(140*c^4*(c + d*x^3)^(10/3))
```

fricas [A] time = 0.89, size = 166, normalized size = 1.52

$$\frac{\left(\left(14\,b^3c^3+18\,ab^2c^2d+27\,a^2bcd^2+81\,a^3d^3\right)x^{10}+30\left(2\,ab^2c^3+3\,a^2bc^2d+9\,a^3cd^2\right)x^7+140\,a^3c^3x+105\left(a^2bc^3+3\,a^3c^2d\right)x^4\right)\left(dx^3+c\right)^{\frac{2}{3}}}{140\left(c^4d^4x^{12}+4\,c^5d^3x^9+6\,c^6d^2x^6+4\,c^7dx^3+c^8\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^3/(d*x^3+c)^(13/3),x, algorithm="fricas")
```

```
[Out] 1/140*((14*b^3*c^3 + 18*a*b^2*c^2*d + 27*a^2*b*c*d^2 + 81*a^3*d^3)*x^10 + 3
0*(2*a*b^2*c^3 + 3*a^2*b*c^2*d + 9*a^3*c*d^2)*x^7 + 140*a^3*c^3*x + 105*(a^
2*b*c^3 + 3*a^3*c^2*d)*x^4)*(d*x^3 + c)^(2/3)/(c^4*d^4*x^12 + 4*c^5*d^3*x^9
 + 6*c^6*d^2*x^6 + 4*c^7*d*x^3 + c^8)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^3+a\right)^3}{\left(dx^3+c\right)^{\frac{13}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^3/(d*x^3+c)^(13/3),x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a)^3/(d*x^3 + c)^(13/3), x)
```

maple [A] time = 0.05, size = 134, normalized size = 1.23

$$\frac{\left(81a^3d^3x^9+27a^2bc\,d^2x^9+18a\,b^2c^2d\,x^9+14b^3c^3x^9+270a^3c\,d^2x^6+90a^2b\,c^2d\,x^6+60a\,b^2c^3x^6+315a^3c^2d\,x^3+105a^2b\,c^3x^3+140a^3c^3\right)x}{140\left(d\,x^3+c\right)^{\frac{10}{3}}c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^3/(d*x^3+c)^(13/3),x)
```

```
[Out] 1/140*x*(81*a^3*d^3*x^9+27*a^2*b*c*d^2*x^9+18*a*b^2*c^2*d*x^9+14*b^3*c^3*x^
9+270*a^3*c*d^2*x^6+90*a^2*b*c^2*d*x^6+60*a*b^2*c^3*x^6+315*a^3*c^2*d*x^3+1
05*a^2*b*c^3*x^3+140*a^3*c^3)/(d*x^3+c)^(10/3)/c^4
```

maxima [A] time = 0.63, size = 182, normalized size = 1.67

$$\frac{b^3x^{10}}{10\left(dx^3+c\right)^{\frac{10}{3}}c}-\frac{3\,ab^2\left(7\,d-\frac{10\left(dx^3+c\right)}{x^3}\right)x^{10}}{70\left(dx^3+c\right)^{\frac{10}{3}}c^2}+\frac{3\left(14\,d^2-\frac{40\left(dx^3+c\right)d}{x^3}+\frac{35\left(dx^3+c\right)^2}{x^6}\right)a^2bx^{10}}{140\left(dx^3+c\right)^{\frac{10}{3}}c^3}-\frac{\left(14\,d^3-\frac{60\left(dx^3+c\right)d^2}{x^3}+\frac{105\left(dx^3+c\right)^2d}{x^6}-\frac{140\left(dx^3+c\right)^3}{x^9}\right)a^3x^{10}}{140\left(dx^3+c\right)^{\frac{10}{3}}c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^3/(d*x^3+c)^(13/3),x, algorithm="maxima")
```

```
[Out] 1/10*b^3*x^10/((d*x^3 + c)^(10/3)*c) - 3/70*a*b^2*(7*d - 10*(d*x^3 + c)/x^3
)*x^10/((d*x^3 + c)^(10/3)*c^2) + 3/140*(14*d^2 - 40*(d*x^3 + c)*d/x^3 + 35
*(d*x^3 + c)^2/x^6)*a^2*b*x^10/((d*x^3 + c)^(10/3)*c^3) - 1/140*(14*d^3 - 6
0*(d*x^3 + c)*d^2/x^3 + 105*(d*x^3 + c)^2*d/x^6 - 140*(d*x^3 + c)^3/x^9)*a^
3*x^10/((d*x^3 + c)^(10/3)*c^4)
```

mupad [B] time = 1.56, size = 271, normalized size = 2.49

$$\frac{x\left(\frac{a^3}{10\,c}-\frac{c\left(\frac{c\left(\frac{b^3}{10\,d}-\frac{3\,a\,b^2}{10\,c}\right)}{d}+\frac{3\,a^2\,b}{10\,c}\right)}{d}\right)}{\left(d\,x^3+c\right)^{10/3}}-\frac{x\left(\frac{b^3}{4\,d^3}-\frac{27\,a^3\,d^3+9\,a^2\,b\,c\,d^2+6\,a\,b^2\,c^2\,d-7\,b^3\,c^3}{140\,c^3\,d^3}\right)}{\left(d\,x^3+c\right)^{4/3}}+\frac{x\left(\frac{c\left(\frac{b^3}{7\,d^2}-\frac{b^2\,(3\,a\,d-b\,c)}{7\,c\,d^2}\right)}{d}+\frac{9\,a^3\,d^3+3\,a^2\,b\,c\,d^2-3\,a\,b^2\,c^2\,d+b^3\,c^3}{70\,c^2\,d^3}\right)}{\left(d\,x^3+c\right)^{7/3}}+\frac{x\left(81\,a^3\,d^3+27\,a^2\,b\,c\,d^2+18\,a\,b^2\,c^2\,d+14\,b^3\,c^3\right)}{140\,c^4\,d^3\left(d\,x^3+c\right)^{1/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^3/(c + d*x^3)^(13/3),x)
```

```
[Out] (x*(a^3/(10*c) - (c*((c*(b^3/(10*d) - (3*a*b^2)/(10*c)))/d + (3*a^2*b)/(10*
c)))/d))/(c + d*x^3)^(10/3) - (x*(b^3/(4*d^3) - (27*a^3*d^3 - 7*b^3*c^3 + 6
*a*b^2*c^2*d + 9*a^2*b*c*d^2)/(140*c^3*d^3)))/(c + d*x^3)^(4/3) + (x*((c*(b
^3/(7*d^2) - (b^2*(3*a*d - b*c))/(7*c*d^2)))/d + (9*a^3*d^3 + b^3*c^3 - 3*a
*b^2*c^2*d + 3*a^2*b*c*d^2)/(70*c^2*d^3)))/(c + d*x^3)^(7/3) + (x*(81*a^3*d
^3 + 14*b^3*c^3 + 18*a*b^2*c^2*d + 27*a^2*b*c*d^2))/(140*c^4*d^3*(c + d*x^3
)^(1/3))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**3/(d*x**3+c)**(13/3),x)
```

```
[Out] Timed out
```

3.61 $\int \frac{\left(a+bx^3\right)^{8/3}}{c+dx^3}\,dx$

Optimal. Leaf size=331

$$-\frac{b^{2/3}\left(20a^2d^2-24abcd+9b^2c^2\right)\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}\,x\right)}{18d^3}+\frac{b^{2/3}\left(20a^2d^2-24abcd+9b^2c^2\right)\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{9\sqrt{3}\,d^3}-\frac{(bc-}{}$$

Rubi [C] time = 0.03, antiderivative size = 62, normalized size of antiderivative = 0.19, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {430, 429}

$$\frac{a^2x\left(a+bx^3\right)^{2/3}F_1\left(\frac{1}{3};-\frac{8}{3},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c\left(\frac{bx^3}{a}+1\right)^{2/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(a + b*x^3)^(8/3)/(c + d*x^3),x]
```

```
[Out] (a^2*x*(a + b*x^3)^(2/3)*AppellF1[1/3, -8/3, 1, 4/3, -((b*x^3)/a), -((d*x^3
)/c)])/(c*(1 + (b*x^3)/a)^(2/3))
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^3\right)^{8/3}}{c+dx^3}\,dx &= \frac{\left(a^2\left(a+bx^3\right)^{2/3}\right)\int \frac{\left(1+\frac{bx^3}{a}\right)^{8/3}}{c+dx^3}\,dx}{\left(1+\frac{bx^3}{a}\right)^{2/3}}\\ &= \frac{a^2x\left(a+bx^3\right)^{2/3}F_1\left(\frac{1}{3};-\frac{8}{3},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c\left(1+\frac{bx^3}{a}\right)^{2/3}}\end{aligned}$$

Mathematica [C] time = 1.23, size = 655, normalized size = 1.98

$$\frac{3bx^4\sqrt[3]{\frac{bx^3}{a}+1}\,\sqrt[3]{bc-ad}\left(20a^2d^2-24abcd+9b^2c^2\right)F_1\left(\frac{4}{3};\frac{1}{3},1;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+2\sqrt[3]{c}\left(9a^3d^2\sqrt[3]{a+bx^3}\log\left(\frac{\sqrt[3]{c}\,x\sqrt[3]{bc-ad}}{\sqrt[3]{ax^3+b}}+\frac{x^2(bc-ad)^{2/3}}{\left(ax^3+b\right)^{2/3}}+c^{2/3}\right)-2a\sqrt[3]{a+bx^3}\left(9a^2d^2-7abcd+3b^2c^2\right)\log\left(\sqrt[3]{c}-\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{ax^3+b}}\right)+2\sqrt{3}\,a\sqrt[3]{a+bx^3}\left(9a^2d^2-7abcd+3b^2c^2\right)\tan^{-1}\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\sqrt[3]{ax^3+b}}+1}{\sqrt{3}}\right)-7a^2bcd\sqrt[3]{a+bx^3}\log\left(\frac{\sqrt[3]{c}\,x\sqrt[3]{bc-ad}}{\sqrt[3]{ax^3+b}}+\frac{x^2(bc-ad)^{2/3}}{\left(ax^3+b\right)^{2/3}}+c^{2/3}\right)+42a^2bc^{2/3}dx\sqrt[3]{bc-ad}+9b^3c^{2/3}dx^7\sqrt[3]{bc-ad}-18b^3c^{5/3}x^4\sqrt[3]{bc-ad}+51ab^2c^{2/3}dx^4\sqrt[3]{bc-ad}-18ab^2c^{5/3}x\sqrt[3]{bc-ad}+3ab^2c^2\sqrt[3]{a+bx^3}\log\left(\frac{\sqrt[3]{c}\,x\sqrt[3]{bc-ad}}{\sqrt[3]{ax^3+b}}+\frac{x^2(bc-ad)^{2/3}}{\left(ax^3+b\right)^{2/3}}+c^{2/3}\right)\right)}{108cd^2\sqrt[3]{a+bx^3}\,\sqrt[3]{bc-ad}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^3)^(8/3)/(c + d*x^3),x]
```

```
[Out] (3*b*(b*c - a*d)^(1/3)*(9*b^2*c^2 - 24*a*b*c*d + 20*a^2*d^2)*x^4*(1 + (b*x^
3)/a)^(1/3)*AppellF1[4/3, 1/3, 1, 7/3, -((b*x^3)/a), -((d*x^3)/c)] + 2*c^(1
/3)*(-18*a*b^2*c^(5/3)*(b*c - a*d)^(1/3)*x + 42*a^2*b*c^(2/3)*d*(b*c - a*d)
^(1/3)*x - 18*b^3*c^(5/3)*(b*c - a*d)^(1/3)*x^4 + 51*a*b^2*c^(2/3)*d*(b*c -
 a*d)^(1/3)*x^4 + 9*b^3*c^(2/3)*d*(b*c - a*d)^(1/3)*x^7 + 2*Sqrt[3]*a*(3*b^
2*c^2 - 7*a*b*c*d + 9*a^2*d^2)*(a + b*x^3)^(1/3)*ArcTan[(1 + (2*(b*c - a*d)
^(1/3)*x)/(c^(1/3)*(b + a*x^3)^(1/3)))/Sqrt[3]] - 2*a*(3*b^2*c^2 - 7*a*b*c*
d + 9*a^2*d^2)*(a + b*x^3)^(1/3)*Log[c^(1/3) - ((b*c - a*d)^(1/3)*x)/(b + a
*x^3)^(1/3)] + 3*a*b^2*c^2*(a + b*x^3)^(1/3)*Log[c^(2/3) + ((b*c - a*d)^(2/
3)*x^2)/(b + a*x^3)^(2/3) + (c^(1/3)*(b*c - a*d)^(1/3)*x)/(b + a*x^3)^(1/3)
] - 7*a^2*b*c*d*(a + b*x^3)^(1/3)*Log[c^(2/3) + ((b*c - a*d)^(2/3)*x^2)/(b
+ a*x^3)^(2/3) + (c^(1/3)*(b*c - a*d)^(1/3)*x)/(b + a*x^3)^(1/3)] + 9*a^3*d
^2*(a + b*x^3)^(1/3)*Log[c^(2/3) + ((b*c - a*d)^(2/3)*x^2)/(b + a*x^3)^(2/3
) + (c^(1/3)*(b*c - a*d)^(1/3)*x)/(b + a*x^3)^(1/3)]))/(108*c*d^2*(b*c - a*
d)^(1/3)*(a + b*x^3)^(1/3))
```

IntegrateAlgebraic [C] time = 14.45, size = 618, normalized size = 1.87

$$\frac{\left(-20a^2b^{2/3}d^2+24ab^{5/3}cd-9b^{8/3}c^2\right)\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}x\right)}{27d^3}+\frac{\left(20a^2b^{2/3}d^2-24ab^{5/3}cd+9b^{8/3}c^2\right)\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{b}x}{2\sqrt[3]{a+bx^3}+\sqrt[3]{b}x}\right)}{9\sqrt{3}d^3}+\frac{\left(20a^2b^{2/3}d^2-24ab^{5/3}cd+9b^{8/3}c^2\right)\log\left(\sqrt[3]{b}x\sqrt[3]{a+bx^3}+\left(a+bx^3\right)^{2/3}+b^{2/3}x^2\right)}{54d^3}+\frac{\left(a+bx^3\right)^{2/3}\left(14abdx-6b^2cx+3b^2dx^4\right)}{18d^2}-\frac{i\left(\sqrt{3}(bc-ad)^{8/3}-i(bc-ad)^{8/3}\right)\log\left(2x\sqrt[3]{bc-ad}+\left(1+i\sqrt{3}\right)\sqrt[3]{c}\sqrt[3]{a+bx^3}\right)}{6c^{2/3}d^3}+\frac{\sqrt{\frac{1}{6}\left(-1+i\sqrt{3}\right)}(bc-ad)^{8/3}\tan^{-1}\left(\frac{3x\sqrt[3]{bc-ad}}{\sqrt{3}x\sqrt[3]{bc-ad}-\sqrt{3}\sqrt[3]{c}\sqrt[3]{a+bx^3}-3i\sqrt[3]{c}\sqrt[3]{a+bx^3}}\right)}{c^{2/3}d^3}+\frac{\left((bc-ad)^{8/3}+i\sqrt{3}(bc-ad)^{8/3}\right)\log\left(\left(\sqrt{3}+i\right)c^{2/3}\left(a+bx^3\right)^{2/3}+\sqrt[3]{c}\left(-\sqrt{3}x+ix\right)\sqrt[3]{a+bx^3}\sqrt[3]{bc-ad}-2ix^2(bc-ad)^{2/3}\right)}{12c^{2/3}d^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x^3)^(8/3)/(c + d*x^3),x]
```

```
[Out] ((a + b*x^3)^(2/3)*(-6*b^2*c*x + 14*a*b*d*x + 3*b^2*d*x^4))/(18*d^2) + ((9*
b^(8/3)*c^2 - 24*a*b^(5/3)*c*d + 20*a^2*b^(2/3)*d^2)*ArcTan[(Sqrt[3]*b^(1/3
)*x)/(b^(1/3)*x + 2*(a + b*x^3)^(1/3))])/(9*Sqrt[3]*d^3) + (Sqrt[(-1 + I*Sq
rt[3])/6]*(b*c - a*d)^(8/3)*ArcTan[(3*(b*c - a*d)^(1/3)*x)/(Sqrt[3]*(b*c -
a*d)^(1/3)*x - (3*I)*c^(1/3)*(a + b*x^3)^(1/3) - Sqrt[3]*c^(1/3)*(a + b*x^3
)^(1/3))])/(c^(2/3)*d^3) + ((-9*b^(8/3)*c^2 + 24*a*b^(5/3)*c*d - 20*a^2*b^(
2/3)*d^2)*Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)])/(27*d^3) - ((I/6)*((-I)*(b
*c - a*d)^(8/3) + Sqrt[3]*(b*c - a*d)^(8/3))*Log[2*(b*c - a*d)^(1/3)*x + (1
 + I*Sqrt[3])*c^(1/3)*(a + b*x^3)^(1/3)])/(c^(2/3)*d^3) + ((9*b^(8/3)*c^2 -
 24*a*b^(5/3)*c*d + 20*a^2*b^(2/3)*d^2)*Log[b^(2/3)*x^2 + b^(1/3)*x*(a + b*
x^3)^(1/3) + (a + b*x^3)^(2/3)])/(54*d^3) + (((b*c - a*d)^(8/3) + I*Sqrt[3]
*(b*c - a*d)^(8/3))*Log[(-2*I)*(b*c - a*d)^(2/3)*x^2 + c^(1/3)*(b*c - a*d)^
(1/3)*(I*x - Sqrt[3]*x)*(a + b*x^3)^(1/3) + (I + Sqrt[3])*c^(2/3)*(a + b*x^
3)^(2/3)])/(12*c^(2/3)*d^3)
```

fricas [B] time = 36.58, size = 643, normalized size = 1.94

$$-\frac{18\sqrt{3}\left(b^2c^2-2abcd+a^2d^2\right)\left(\frac{b^2c^2-2abcd+a^2d^2}{c^2}\right)^{1/3}\arctan\left(-\frac{\sqrt{3}(bc-ad)x+2\sqrt{3}\left(bx^3+a\right)^{1/3}c\left(\frac{b^2c^2-2abcd+a^2d^2}{c^2}\right)^{1/3}}{3(bc-ad)x}\right)+2\sqrt{3}\left(9b^2c^2-24abcd+20a^2d^2\right)\left(-b^2\right)^{1/3}\arctan\left(-\frac{\sqrt{3}bx-2\sqrt{3}\left(bx^3+a\right)^{1/3}\left(-b^2\right)^{1/3}}{3bx}\right)-18\left(b^2c^2-2abcd+a^2d^2\right)\left(\frac{b^2c^2-2abcd+a^2d^2}{c^2}\right)^{1/3}\log\left(\frac{cx\left(\frac{b^2c^2-2abcd+a^2d^2}{c^2}\right)^{2/3}-\left(bx^3+a\right)^{1/3}(bc-ad)}{x}\right)-2\left(9b^2c^2-24abcd+20a^2d^2\right)\left(-b^2\right)^{1/3}\log\left(-\frac{\left(-b^2\right)^{2/3}x-\left(bx^3+a\right)^{1/3}b}{x}\right)+\left(9b^2c^2-24abcd+20a^2d^2\right)\left(-b^2\right)^{1/3}\log\left(-\frac{\left(-b^2\right)^{1/3}bx^2-\left(bx^3+a\right)^{1/3}\left(-b^2\right)^{2/3}x-\left(bx^3+a\right)^{2/3}b}{x^2}\right)+9\left(b^2c^2-2abcd+a^2d^2\right)\left(\frac{b^2c^2-2abcd+a^2d^2}{c^2}\right)^{1/3}\log\left(-\frac{(bc-ad)x^2\left(\frac{b^2c^2-2abcd+a^2d^2}{c^2}\right)^{1/3}+\left(bx^3+a\right)^{1/3}cx\left(\frac{b^2c^2-2abcd+a^2d^2}{c^2}\right)^{2/3}-\left(bx^3+a\right)^{2/3}(bc-ad)}{x^2}\right)-3\left(3b^2d^2x^4-2\left(3b^2cd-7abd^2\right)x\right)\left(bx^3+a\right)^{2/3}}{54d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(8/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] -1/54*(18*sqrt(3)*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*((b^2*c^2 - 2*a*b*c*d + a
^2*d^2)/c^2)^(1/3)*arctan(-1/3*(sqrt(3)*(b*c - a*d)*x + 2*sqrt(3)*(b*x^3 +
a)^(1/3)*c*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)/c^2)^(1/3))/((b*c - a*d)*x)) +
2*sqrt(3)*(9*b^2*c^2 - 24*a*b*c*d + 20*a^2*d^2)*(-b^2)^(1/3)*arctan(-1/3*(s
qrt(3)*b*x - 2*sqrt(3)*(b*x^3 + a)^(1/3)*(-b^2)^(1/3))/(b*x)) - 18*(b^2*c^2
 - 2*a*b*c*d + a^2*d^2)*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)/c^2)^(1/3)*log((c*
x*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)/c^2)^(2/3) - (b*x^3 + a)^(1/3)*(b*c - a*
d))/x) - 2*(9*b^2*c^2 - 24*a*b*c*d + 20*a^2*d^2)*(-b^2)^(1/3)*log(-((-b^2)^
(2/3)*x - (b*x^3 + a)^(1/3)*b)/x) + (9*b^2*c^2 - 24*a*b*c*d + 20*a^2*d^2)*(
-b^2)^(1/3)*log(-((-b^2)^(1/3)*b*x^2 - (b*x^3 + a)^(1/3)*(-b^2)^(2/3)*x - (
b*x^3 + a)^(2/3)*b)/x^2) + 9*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*((b^2*c^2 - 2*
a*b*c*d + a^2*d^2)/c^2)^(1/3)*log(-((b*c - a*d)*x^2*((b^2*c^2 - 2*a*b*c*d +
 a^2*d^2)/c^2)^(1/3) + (b*x^3 + a)^(1/3)*c*x*((b^2*c^2 - 2*a*b*c*d + a^2*d^
```

```
2)/c^2)^(2/3) + (b*x^3 + a)^(2/3)*(b*c - a*d))/x^2) - 3*(3*b^2*d^2*x^4 - 2*
(3*b^2*c*d - 7*a*b*d^2)*x)*(b*x^3 + a)^(2/3))/d^3
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^3+a\right)^{\frac{8}{3}}}{dx^3+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(8/3)/(d*x^3+c),x, algorithm="giac")
[Out] integrate((b*x^3 + a)^(8/3)/(d*x^3 + c), x)
```

maple [F] time = 0.70, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^3+a\right)^{\frac{8}{3}}}{d\,x^3+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(8/3)/(d*x^3+c),x)
[Out] int((b*x^3+a)^(8/3)/(d*x^3+c),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^3+a\right)^{\frac{8}{3}}}{dx^3+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(8/3)/(d*x^3+c),x, algorithm="maxima")
[Out] integrate((b*x^3 + a)^(8/3)/(d*x^3 + c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(b\,x^3+a\right)^{8/3}}{d\,x^3+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^(8/3)/(c + d*x^3),x)
[Out] int((a + b*x^3)^(8/3)/(c + d*x^3), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(8/3)/(d*x**3+c),x)
[Out] Timed out
```

3.62 $\int \frac{(a+bx^3)^{5/3}}{c+dx^3}\,dx$

Optimal. Leaf size=273

$$\frac{b^{2/3}(3bc-5ad)\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}\,x\right)}{6d^2}-\frac{b^{2/3}(3bc-5ad)\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{3\sqrt{3}\,d^2}+\frac{(bc-ad)^{5/3}\log\left(c+dx^3\right)}{6c^{2/3}d^2}-\frac{(bc-ad}{}$$

Rubi [C] time = 0.03, antiderivative size = 60, normalized size of antiderivative = 0.22, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {430, 429}

$$\frac{ax\left(a+bx^3\right)^{2/3}F_1\left(\frac{1}{3};-\frac{5}{3},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c\left(\frac{bx^3}{a}+1\right)^{2/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(a + b*x^3)^(5/3)/(c + d*x^3),x]
```

```
[Out] (a*x*(a + b*x^3)^(2/3)*AppellF1[1/3, -5/3, 1, 4/3, -((b*x^3)/a), -((d*x^3)/
c)])/(c*(1 + (b*x^3)/a)^(2/3))
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^3\right)^{5/3}}{c+dx^3}\,dx &= \frac{\left(a\left(a+bx^3\right)^{2/3}\right)\int \frac{\left(1+\frac{bx^3}{a}\right)^{5/3}}{c+dx^3}\,dx}{\left(1+\frac{bx^3}{a}\right)^{2/3}}\\ &= \frac{ax\left(a+bx^3\right)^{2/3}F_1\left(\frac{1}{3};-\frac{5}{3},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c\left(1+\frac{bx^3}{a}\right)^{2/3}}\end{aligned}$$

Mathematica [C] time = 0.71, size = 443, normalized size = 1.62

$$\frac{2\sqrt[3]{c}\left(3a^2d\sqrt[3]{a+bx^3}\log\left(\frac{\sqrt[3]{c}\,x\sqrt[3]{bc-ad}}{\sqrt[3]{ax^3+b}}+\frac{x^2(bc-ad)^{2/3}}{\left(ax^3+b\right)^{2/3}}+c^{2/3}\right)+6b^2c^{2/3}x^4\sqrt[3]{bc-ad}-abc\sqrt[3]{a+bx^3}\log\left(\frac{\sqrt[3]{c}\,x\sqrt[3]{bc-ad}}{\sqrt[3]{ax^3+b}}+\frac{x^2(bc-ad)^{2/3}}{\left(ax^3+b\right)^{2/3}}+c^{2/3}\right)+6abc^{2/3}x\sqrt[3]{bc-ad}+2a\sqrt[3]{a+bx^3}(bc-3ad)\log\left(\sqrt[3]{c}-\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{ax^3+b}}\right)+2\sqrt{3}\,a\sqrt[3]{a+bx^3}(3ad-bc)\tan^{-1}\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\sqrt[3]{ax^3+b}}+1}{\sqrt{3}}\right)\right)+3bx^4\sqrt[3]{\frac{bx^3}{a}+1}\,\sqrt[3]{bc-ad}\,(5ad-3bc)F_1\left(\frac{4}{3};\frac{1}{3},1;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{36cd\sqrt[3]{a+bx^3}\,\sqrt[3]{bc-ad}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^3)^(5/3)/(c + d*x^3),x]
```

```
[Out] (3*b*(b*c - a*d)^(1/3)*(-3*b*c + 5*a*d)*x^4*(1 + (b*x^3)/a)^(1/3)*AppellF1[
4/3, 1/3, 1, 7/3, -((b*x^3)/a), -((d*x^3)/c)] + 2*c^(1/3)*(6*a*b*c^(2/3)*(b
*c - a*d)^(1/3)*x + 6*b^2*c^(2/3)*(b*c - a*d)^(1/3)*x^4 + 2*Sqrt[3]*a*(-(b*
c) + 3*a*d)*(a + b*x^3)^(1/3)*ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c^(1/3)*
(b + a*x^3)^(1/3)))/Sqrt[3]] + 2*a*(b*c - 3*a*d)*(a + b*x^3)^(1/3)*Log[c^(1
/3) - ((b*c - a*d)^(1/3)*x)/(b + a*x^3)^(1/3)] - a*b*c*(a + b*x^3)^(1/3)*Lo
g[c^(2/3) + ((b*c - a*d)^(2/3)*x^2)/(b + a*x^3)^(2/3) + (c^(1/3)*(b*c - a*d
)^(1/3)*x)/(b + a*x^3)^(1/3)] + 3*a^2*d*(a + b*x^3)^(1/3)*Log[c^(2/3) + ((b
*c - a*d)^(2/3)*x^2)/(b + a*x^3)^(2/3) + (c^(1/3)*(b*c - a*d)^(1/3)*x)/(b +
 a*x^3)^(1/3)]))/(36*c*d*(b*c - a*d)^(1/3)*(a + b*x^3)^(1/3))
```

IntegrateAlgebraic [C] time = 7.77, size = 550, normalized size = 2.01

$$\frac{\left(3b^{5/3}c-5ab^{2/3}d\right)\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}x\right)}{9d^2}-\frac{\left(3b^{5/3}c-5ab^{2/3}d\right)\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{b}x}{2\sqrt[3]{a+bx^3}+\sqrt[3]{b}x}\right)}{3\sqrt{3}d^2}+\frac{\left(5ab^{2/3}d-3b^{5/3}c\right)\log\left(\sqrt[3]{b}x\sqrt[3]{a+bx^3}+\left(a+bx^3\right)^{2/3}+b^{2/3}x^2\right)}{18d^2}+\frac{\left((bc-ad)^{5/3}+i\sqrt{3}(bc-ad)^{5/3}\right)\log\left(2x\sqrt[3]{bc-ad}+\left(1+i\sqrt{3}\right)\sqrt[3]{c}\sqrt[3]{a+bx^3}\right)}{6c^{2/3}d^2}-\frac{\sqrt{\frac{1}{6}\left(-1+i\sqrt{3}\right)}(bc-ad)^{5/3}\tan^{-1}\left(\frac{3x\sqrt[3]{bc-ad}}{\sqrt{3}x\sqrt[3]{bc-ad}-\sqrt{3}\sqrt[3]{c}\sqrt[3]{a+bx^3}-3i\sqrt[3]{c}\sqrt[3]{a+bx^3}}\right)}{c^{2/3}d^2}-\frac{i\left(\sqrt{3}(bc-ad)^{5/3}-i(bc-ad)^{5/3}\right)\log\left(\left(\sqrt{3}+i\right)c^{2/3}\left(a+bx^3\right)^{2/3}+\sqrt[3]{c}\left(-\sqrt{3}x+ix\right)\sqrt[3]{a+bx^3}\sqrt[3]{bc-ad}-2ix^2(bc-ad)^{2/3}\right)}{12c^{2/3}d^2}+\frac{bx\left(a+bx^3\right)^{2/3}}{3d}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x^3)^(5/3)/(c + d*x^3),x]
```

```
[Out] (b*x*(a + b*x^3)^(2/3))/(3*d) - ((3*b^(5/3)*c - 5*a*b^(2/3)*d)*ArcTan[(Sqrt
[3]*b^(1/3)*x)/(b^(1/3)*x + 2*(a + b*x^3)^(1/3))])/(3*Sqrt[3]*d^2) - (Sqrt[
(-1 + I*Sqrt[3])/6]*(b*c - a*d)^(5/3)*ArcTan[(3*(b*c - a*d)^(1/3)*x)/(Sqrt[
3]*(b*c - a*d)^(1/3)*x - (3*I)*c^(1/3)*(a + b*x^3)^(1/3) - Sqrt[3]*c^(1/3)*
(a + b*x^3)^(1/3))])/(c^(2/3)*d^2) + ((3*b^(5/3)*c - 5*a*b^(2/3)*d)*Log[-(b
^(1/3)*x) + (a + b*x^3)^(1/3)])/(9*d^2) + (((b*c - a*d)^(5/3) + I*Sqrt[3]*(
b*c - a*d)^(5/3))*Log[2*(b*c - a*d)^(1/3)*x + (1 + I*Sqrt[3])*c^(1/3)*(a +
b*x^3)^(1/3)])/(6*c^(2/3)*d^2) + ((-3*b^(5/3)*c + 5*a*b^(2/3)*d)*Log[b^(2/3
)*x^2 + b^(1/3)*x*(a + b*x^3)^(1/3) + (a + b*x^3)^(2/3)])/(18*d^2) - ((I/12
)*((-I)*(b*c - a*d)^(5/3) + Sqrt[3]*(b*c - a*d)^(5/3))*Log[(-2*I)*(b*c - a*
d)^(2/3)*x^2 + c^(1/3)*(b*c - a*d)^(1/3)*(I*x - Sqrt[3]*x)*(a + b*x^3)^(1/3
) + (I + Sqrt[3])*c^(2/3)*(a + b*x^3)^(2/3)])/(c^(2/3)*d^2)
```

fricas [B] time = 3.51, size = 535, normalized size = 1.96

$$\frac{6\left(bx^3+a\right)^{\frac{2}{3}}bdx+6\sqrt{3}(bc-ad)\left(\frac{b^2c^2-2abcd+a^2d^2}{c^2}\right)^{\frac{1}{3}}\arctan\left(-\frac{\sqrt{3}(bc-ad)x+2\sqrt{3}\left(bx^3+a\right)^{\frac{1}{3}}c\left(\frac{b^2c^2-2abcd+a^2d^2}{c^2}\right)^{\frac{1}{3}}}{3(bc-ad)x}\right)+2\sqrt{3}\left(-b^2\right)^{\frac{1}{3}}(3bc-5ad)\arctan\left(-\frac{\sqrt{3}bx-2\sqrt{3}\left(bx^3+a\right)^{\frac{1}{3}}\left(-b^2\right)^{\frac{1}{3}}}{3bx}\right)-6(bc-ad)\left(\frac{b^2c^2-2abcd+a^2d^2}{c^2}\right)^{\frac{1}{3}}\log\left(\frac{cx\left(\frac{b^2c^2-2abcd+a^2d^2}{c^2}\right)^{\frac{2}{3}}-\left(bx^3+a\right)^{\frac{1}{3}}(bc-ad)}{x}\right)-2\left(-b^2\right)^{\frac{1}{3}}(3bc-5ad)\log\left(-\frac{\left(-b^2\right)^{\frac{2}{3}}x-\left(bx^3+a\right)^{\frac{1}{3}}b}{x}\right)+\left(-b^2\right)^{\frac{1}{3}}(3bc-5ad)\log\left(-\frac{\left(-b^2\right)^{\frac{1}{3}}bx^2-\left(bx^3+a\right)^{\frac{1}{3}}\left(-b^2\right)^{\frac{2}{3}}x-\left(bx^3+a\right)^{\frac{2}{3}}b}{x^2}\right)+3(bc-ad)\left(\frac{b^2c^2-2abcd+a^2d^2}{c^2}\right)^{\frac{1}{3}}\log\left(-\frac{(bc-ad)x^2\left(\frac{b^2c^2-2abcd+a^2d^2}{c^2}\right)^{\frac{1}{3}}+\left(bx^3+a\right)^{\frac{1}{3}}cx\left(\frac{b^2c^2-2abcd+a^2d^2}{c^2}\right)^{\frac{2}{3}}+\left(bx^3+a\right)^{\frac{2}{3}}(bc-ad)}{x^2}\right)}{18d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(5/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] 1/18*(6*(b*x^3 + a)^(2/3)*b*d*x + 6*sqrt(3)*(b*c - a*d)*((b^2*c^2 - 2*a*b*c
*d + a^2*d^2)/c^2)^(1/3)*arctan(-1/3*(sqrt(3)*(b*c - a*d)*x + 2*sqrt(3)*(b*
x^3 + a)^(1/3)*c*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)/c^2)^(1/3))/((b*c - a*d)*
x)) + 2*sqrt(3)*(-b^2)^(1/3)*(3*b*c - 5*a*d)*arctan(-1/3*(sqrt(3)*b*x - 2*s
qrt(3)*(b*x^3 + a)^(1/3)*(-b^2)^(1/3))/(b*x)) - 6*(b*c - a*d)*((b^2*c^2 - 2
*a*b*c*d + a^2*d^2)/c^2)^(1/3)*log((c*x*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)/c^
2)^(2/3) - (b*x^3 + a)^(1/3)*(b*c - a*d))/x) - 2*(-b^2)^(1/3)*(3*b*c - 5*a*
d)*log(-((-b^2)^(2/3)*x - (b*x^3 + a)^(1/3)*b)/x) + (-b^2)^(1/3)*(3*b*c - 5
*a*d)*log(-((-b^2)^(1/3)*b*x^2 - (b*x^3 + a)^(1/3)*(-b^2)^(2/3)*x - (b*x^3
+ a)^(2/3)*b)/x^2) + 3*(b*c - a*d)*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)/c^2)^(1
/3)*log(-((b*c - a*d)*x^2*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)/c^2)^(1/3) + (b*
x^3 + a)^(1/3)*c*x*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)/c^2)^(2/3) + (b*x^3 + a
)^(2/3)*(b*c - a*d))/x^2))/d^2
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^3+a\right)^{\frac{5}{3}}}{dx^3+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(5/3)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a)^(5/3)/(d*x^3 + c), x)
```

maple [F] time = 0.56, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^{3}+a\right)^{\frac{5}{3}}}{d\,x^{3}+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(5/3)/(d*x^3+c),x)
```

```
[Out] int((b*x^3+a)^(5/3)/(d*x^3+c),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^{3}+a\right)^{\frac{5}{3}}}{dx^{3}+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(5/3)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a)^(5/3)/(d*x^3 + c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(b\,x^{3}+a\right)^{5/3}}{d\,x^{3}+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^(5/3)/(c + d*x^3),x)
```

```
[Out] int((a + b*x^3)^(5/3)/(c + d*x^3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+bx^{3}\right)^{\frac{5}{3}}}{c+dx^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(5/3)/(d*x**3+c),x)
```

```
[Out] Integral((a + b*x**3)**(5/3)/(c + d*x**3), x)
```

3.63 $\int \frac{(a+bx^3)^{2/3}}{c+dx^3}\,dx$

Optimal. Leaf size=233

$$-\frac{b^{2/3}\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}\,x\right)}{2d}+\frac{b^{2/3}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\,d}-\frac{(bc-ad)^{2/3}\log\left(c+dx^3\right)}{6c^{2/3}d}+\frac{(bc-ad)^{2/3}\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a}\right.}{2c^{2/3}d}$$

Rubi [C] time = 0.03, antiderivative size = 59, normalized size of antiderivative = 0.25, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {430, 429}

$$\frac{x\left(a+bx^3\right)^{2/3}F_1\left(\frac{1}{3};-\frac{2}{3},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c\left(\frac{bx^3}{a}+1\right)^{2/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(a + b*x^3)^(2/3)/(c + d*x^3),x]
```

```
[Out] (x*(a + b*x^3)^(2/3)*AppellF1[1/3, -2/3, 1, 4/3, -((b*x^3)/a), -((d*x^3)/c)
])/(c*(1 + (b*x^3)/a)^(2/3))
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^3\right)^{2/3}}{c+dx^3}\,dx &= \frac{\left(a+bx^3\right)^{2/3}\int \frac{\left(1+\frac{bx^3}{a}\right)^{2/3}}{c+dx^3}\,dx}{\left(1+\frac{bx^3}{a}\right)^{2/3}}\\ &= \frac{x\left(a+bx^3\right)^{2/3}F_1\left(\frac{1}{3};-\frac{2}{3},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c\left(1+\frac{bx^3}{a}\right)^{2/3}}\end{aligned}$$

Mathematica [C] time = 0.22, size = 161, normalized size = 0.69

$$\frac{4acx\left(a+bx^3\right)^{2/3}F_1\left(\frac{1}{3};-\frac{2}{3},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{\left(c+dx^3\right)\left(x^3\left(2bcF_1\left(\frac{4}{3};\frac{1}{3},1;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)-3adF_1\left(\frac{4}{3};-\frac{2}{3},2;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)\right)+4acF_1\left(\frac{1}{3};-\frac{2}{3},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^3)^(2/3)/(c + d*x^3),x]
```

```
[Out] (4*a*c*x*(a + b*x^3)^(2/3)*AppellF1[1/3, -2/3, 1, 4/3, -((b*x^3)/a), -((d*x
^3)/c)])/((c + d*x^3)*(4*a*c*AppellF1[1/3, -2/3, 1, 4/3, -((b*x^3)/a), -((d
*x^3)/c)] + x^3*(-3*a*d*AppellF1[4/3, -2/3, 2, 7/3, -((b*x^3)/a), -((d*x^3)
/c)] + 2*b*c*AppellF1[4/3, 1/3, 1, 7/3, -((b*x^3)/a), -((d*x^3)/c)])))
```

IntegrateAlgebraic [C] time = 4.35, size = 487, normalized size = 2.09

$$-\frac{b^{2/3}\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}\,x\right)}{3d}+\frac{b^{2/3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{b}\,x}{2\sqrt[3]{a+bx^3}+\sqrt[3]{b}\,x}\right)}{\sqrt{3}\,d}+\frac{b^{2/3}\log\left(\sqrt[3]{b}\,x\sqrt[3]{a+bx^3}+\left(a+bx^3\right)^{2/3}+b^{2/3}x^2\right)}{6d}-\frac{i\left(\sqrt{3}\,(bc-ad)^{2/3}-i(bc-ad)^{2/3}\right)\log\left(2x\sqrt[3]{bc-ad}+\left(1+i\sqrt{3}\right)\sqrt[3]{c}\,\sqrt[3]{a+bx^3}\right)}{6c^{2/3}d}+\frac{\sqrt{\frac{1}{6}\left(-1+i\sqrt{3}\right)}\,(bc-ad)^{2/3}\tan^{-1}\left(\frac{3x\sqrt[3]{bc-ad}}{\sqrt{3}x\sqrt[3]{bc-ad}-\sqrt{3}\sqrt[3]{c}\sqrt[3]{a+bx^3}-3i\sqrt[3]{c}\sqrt[3]{a+bx^3}}\right)}{c^{2/3}d}+\frac{\left((bc-ad)^{2/3}+i\sqrt{3}\,(bc-ad)^{2/3}\right)\log\left(\left(\sqrt{3}+i\right)c^{2/3}\left(a+bx^3\right)^{2/3}+\sqrt[3]{c}\left(-\sqrt{3}x+ix\right)\sqrt[3]{a+bx^3}\sqrt[3]{bc-ad}-2ix^2(bc-ad)^{2/3}\right)}{12c^{2/3}d}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x^3)^(2/3)/(c + d*x^3),x]
```

```
[Out] (b^(2/3)*ArcTan[(Sqrt[3]*b^(1/3)*x)/(b^(1/3)*x + 2*(a + b*x^3)^(1/3))])/(Sq
rt[3]*d) + (Sqrt[(-1 + I*Sqrt[3])/6]*(b*c - a*d)^(2/3)*ArcTan[(3*(b*c - a*d
)^(1/3)*x)/(Sqrt[3]*(b*c - a*d)^(1/3)*x - (3*I)*c^(1/3)*(a + b*x^3)^(1/3) -
 Sqrt[3]*c^(1/3)*(a + b*x^3)^(1/3))])/(c^(2/3)*d) - (b^(2/3)*Log[-(b^(1/3)*
x) + (a + b*x^3)^(1/3)])/(3*d) - ((I/6)*((-I)*(b*c - a*d)^(2/3) + Sqrt[3]*(
b*c - a*d)^(2/3))*Log[2*(b*c - a*d)^(1/3)*x + (1 + I*Sqrt[3])*c^(1/3)*(a +
b*x^3)^(1/3)])/(c^(2/3)*d) + (b^(2/3)*Log[b^(2/3)*x^2 + b^(1/3)*x*(a + b*x^
3)^(1/3) + (a + b*x^3)^(2/3)])/(6*d) + (((b*c - a*d)^(2/3) + I*Sqrt[3]*(b*c
 - a*d)^(2/3))*Log[(-2*I)*(b*c - a*d)^(2/3)*x^2 + c^(1/3)*(b*c - a*d)^(1/3)
*(I*x - Sqrt[3]*x)*(a + b*x^3)^(1/3) + (I + Sqrt[3])*c^(2/3)*(a + b*x^3)^(2
/3)])/(12*c^(2/3)*d)
```

fricas [B] time = 1.58, size = 469, normalized size = 2.01

$$-\frac{2\sqrt{3}\left(\frac{b^2c^2-2abcd+a^2d^2}{c^2}\right)^{\frac{1}{3}}\arctan\left(-\frac{\sqrt{3}(bc-ad)x+2\sqrt{3}\left(bx^3+a\right)^{\frac{1}{3}}c\left(\frac{b^2c^2-2abcd+a^2d^2}{c^2}\right)^{\frac{1}{3}}}{3(bc-ad)x}\right)+2\sqrt{3}\left(-b^2\right)^{\frac{1}{3}}\arctan\left(-\frac{\sqrt{3}bx-2\sqrt{3}\left(bx^3+a\right)^{\frac{1}{3}}\left(-b^2\right)^{\frac{1}{3}}}{3bx}\right)-2\left(\frac{b^2c^2-2abcd+a^2d^2}{c^2}\right)^{\frac{1}{3}}\log\left(\frac{cx\left(\frac{b^2c^2-2abcd+a^2d^2}{c^2}\right)^{\frac{2}{3}}-\left(bx^3+a\right)^{\frac{1}{3}}(bc-ad)}{x}\right)-2\left(-b^2\right)^{\frac{1}{3}}\log\left(-\frac{\left(-b^2\right)^{\frac{2}{3}}x-\left(bx^3+a\right)^{\frac{1}{3}}b}{x}\right)+\left(-b^2\right)^{\frac{1}{3}}\log\left(-\frac{\left(-b^2\right)^{\frac{1}{3}}bx^2-\left(bx^3+a\right)^{\frac{1}{3}}\left(-b^2\right)^{\frac{2}{3}}x-\left(bx^3+a\right)^{\frac{2}{3}}b}{x^2}\right)+\left(\frac{b^2c^2-2abcd+a^2d^2}{c^2}\right)^{\frac{1}{3}}\log\left(-\frac{(bc-ad)x^2\left(\frac{b^2c^2-2abcd+a^2d^2}{c^2}\right)^{\frac{1}{3}}+\left(bx^3+a\right)^{\frac{1}{3}}cx\left(\frac{b^2c^2-2abcd+a^2d^2}{c^2}\right)^{\frac{2}{3}}+\left(bx^3+a\right)^{\frac{2}{3}}(bc-ad)}{x^2}\right)}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] -1/6*(2*sqrt(3)*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)/c^2)^(1/3)*arctan(-1/3*(sq
rt(3)*(b*c - a*d)*x + 2*sqrt(3)*(b*x^3 + a)^(1/3)*c*((b^2*c^2 - 2*a*b*c*d +
 a^2*d^2)/c^2)^(1/3))/((b*c - a*d)*x)) + 2*sqrt(3)*(-b^2)^(1/3)*arctan(-1/3
*(sqrt(3)*b*x - 2*sqrt(3)*(b*x^3 + a)^(1/3)*(-b^2)^(1/3))/(b*x)) - 2*((b^2*
c^2 - 2*a*b*c*d + a^2*d^2)/c^2)^(1/3)*log((c*x*((b^2*c^2 - 2*a*b*c*d + a^2*
d^2)/c^2)^(2/3) - (b*x^3 + a)^(1/3)*(b*c - a*d))/x) - 2*(-b^2)^(1/3)*log(-(
(-b^2)^(2/3)*x - (b*x^3 + a)^(1/3)*b)/x) + (-b^2)^(1/3)*log(-((-b^2)^(1/3)*
b*x^2 - (b*x^3 + a)^(1/3)*(-b^2)^(2/3)*x - (b*x^3 + a)^(2/3)*b)/x^2) + ((b^
2*c^2 - 2*a*b*c*d + a^2*d^2)/c^2)^(1/3)*log(-((b*c - a*d)*x^2*((b^2*c^2 - 2
*a*b*c*d + a^2*d^2)/c^2)^(1/3) + (b*x^3 + a)^(1/3)*c*x*((b^2*c^2 - 2*a*b*c*
d + a^2*d^2)/c^2)^(2/3) + (b*x^3 + a)^(2/3)*(b*c - a*d))/x^2))/d
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^3+a\right)^{\frac{2}{3}}}{dx^3+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a)^(2/3)/(d*x^3 + c), x)
```

maple [F] time = 0.57, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^{3}+a\right)^{\frac{2}{3}}}{d\,x^{3}+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(2/3)/(d*x^3+c),x)
[Out] int((b*x^3+a)^(2/3)/(d*x^3+c),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^{3}+a\right)^{\frac{2}{3}}}{dx^{3}+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="maxima")
[Out] integrate((b*x^3 + a)^(2/3)/(d*x^3 + c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(b\,x^{3}+a\right)^{2/3}}{d\,x^{3}+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^(2/3)/(c + d*x^3),x)
[Out] int((a + b*x^3)^(2/3)/(c + d*x^3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+bx^{3}\right)^{\frac{2}{3}}}{c+dx^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(2/3)/(d*x**3+c),x)
[Out] Integral((a + b*x**3)**(2/3)/(c + d*x**3), x)
```

3.64 $\int \frac{1}{\sqrt[3]{a+bx^3}\left(c+dx^3\right)}\,dx$

Optimal. Leaf size=148

$$\frac{\log\left(c+dx^3\right)}{6c^{2/3}\sqrt[3]{bc-ad}}-\frac{\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2c^{2/3}\sqrt[3]{bc-ad}}+\frac{\tan^{-1}\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\,c^{2/3}\sqrt[3]{bc-ad}}$$

Rubi [A] time = 0.19, antiderivative size = 207, normalized size of antiderivative = 1.40, number of steps used = 7, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {377, 200, 31, 634, 617, 204, 628}

$$-\frac{\log\left(\sqrt[3]{c}-\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{a+bx^3}}\right)}{3c^{2/3}\sqrt[3]{bc-ad}}+\frac{\log\left(\frac{x^2(bc-ad)^{2/3}}{\left(a+bx^3\right)^{2/3}}+\frac{\sqrt[3]{c}\,x\sqrt[3]{bc-ad}}{\sqrt[3]{a+bx^3}}+c^{2/3}\right)}{6c^{2/3}\sqrt[3]{bc-ad}}+\frac{\tan^{-1}\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{a+bx^3}}+\sqrt[3]{c}}{\sqrt{3}\sqrt[3]{c}}\right)}{\sqrt{3}\,c^{2/3}\sqrt[3]{bc-ad}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^3)^(1/3)*(c + d*x^3)),x]
```

```
[Out] ArcTan[(c^(1/3) + (2*(b*c - a*d)^(1/3)*x)/(a + b*x^3)^(1/3))/(Sqrt[3]*c^(1/
3))]/(Sqrt[3]*c^(2/3)*(b*c - a*d)^(1/3)) - Log[c^(1/3) - ((b*c - a*d)^(1/3)
*x)/(a + b*x^3)^(1/3)]/(3*c^(2/3)*(b*c - a*d)^(1/3)) + Log[c^(2/3) + ((b*c
- a*d)^(2/3)*x^2)/(a + b*x^3)^(2/3) + (c^(1/3)*(b*c - a*d)^(1/3)*x)/(a + b*
x^3)^(1/3)]/(6*c^(2/3)*(b*c - a*d)^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
```

```
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{\sqrt[3]{a+bx^3}\left(c+dx^3\right)}\,dx &= \text{Subst}\left(\int \frac{1}{c-(bc-ad)x^3}\,dx, x, \frac{x}{\sqrt[3]{a+bx^3}}\right)\\
&= \frac{\text{Subst}\left(\int \frac{1}{\sqrt[3]{c}-\sqrt[3]{bc-ad}\,x}\,dx, x, \frac{x}{\sqrt[3]{a+bx^3}}\right)}{3c^{2/3}} + \frac{\text{Subst}\left(\int \frac{2\sqrt[3]{c}+\sqrt[3]{bc-ad}\,x}{c^{2/3}+\sqrt[3]{c}\,\sqrt[3]{bc-ad}\,x+(bc-ad)^{2/3}x^2}\,dx, x, \cdots\right)}{3c^{2/3}}\\
&= -\frac{\log\left(\sqrt[3]{c}-\frac{\sqrt[3]{bc-ad}\,x}{\sqrt[3]{a+bx^3}}\right)}{3c^{2/3}\sqrt[3]{bc-ad}} + \frac{\text{Subst}\left(\int \frac{1}{c^{2/3}+\sqrt[3]{c}\,\sqrt[3]{bc-ad}\,x+(bc-ad)^{2/3}x^2}\,dx, x, \frac{x}{\sqrt[3]{a+bx^3}}\right)}{2\sqrt[3]{c}} + \frac{\text{Subst}\cdots}{\cdots}\\
&= -\frac{\log\left(\sqrt[3]{c}-\frac{\sqrt[3]{bc-ad}\,x}{\sqrt[3]{a+bx^3}}\right)}{3c^{2/3}\sqrt[3]{bc-ad}} + \frac{\log\left(c^{2/3}+\frac{(bc-ad)^{2/3}x^2}{\left(a+bx^3\right)^{2/3}}+\frac{\sqrt[3]{c}\,\sqrt[3]{bc-ad}\,x}{\sqrt[3]{a+bx^3}}\right)}{6c^{2/3}\sqrt[3]{bc-ad}} - \frac{\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, \cdots\right)}{c^{2/3}\sqrt[3]{bc-\cdots}}\\
&= \frac{\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{bc-ad}\,x}{\sqrt[3]{c}\,\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}\,c^{2/3}\sqrt[3]{bc-ad}} - \frac{\log\left(\sqrt[3]{c}-\frac{\sqrt[3]{bc-ad}\,x}{\sqrt[3]{a+bx^3}}\right)}{3c^{2/3}\sqrt[3]{bc-ad}} + \frac{\log\left(c^{2/3}+\frac{(bc-ad)^{2/3}x^2}{\left(a+bx^3\right)^{2/3}}+\frac{\sqrt[3]{c}\,\sqrt[3]{bc-ad}\,x}{\sqrt[3]{a+bx^3}}\right)}{6c^{2/3}\sqrt[3]{bc-ad}}
\end{aligned}
$$

Mathematica [A] time = 0.15, size = 168, normalized size = 1.14

$$
\frac{\log\left(\frac{\sqrt[3]{c}\,x\sqrt[3]{bc-ad}}{\sqrt[3]{a+bx^3}}+\frac{x^2(bc-ad)^{2/3}}{\left(a+bx^3\right)^{2/3}}+c^{2/3}\right)-2\log\left(\sqrt[3]{c}-\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{a+bx^3}}\right)+2\sqrt{3}\,\tan^{-1}\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\,\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{6c^{2/3}\sqrt[3]{bc-ad}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x^3)^(1/3)*(c + d*x^3)),x]
```

```
[Out] (2*Sqrt[3]*ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c^(1/3)*(a + b*x^3)^(1/3)))
/Sqrt[3]] - 2*Log[c^(1/3) - ((b*c - a*d)^(1/3)*x)/(a + b*x^3)^(1/3)] + Log[
c^(2/3) + ((b*c - a*d)^(2/3)*x^2)/(a + b*x^3)^(2/3) + (c^(1/3)*(b*c - a*d)^
(1/3)*x)/(a + b*x^3)^(1/3)]])/(6*c^(2/3)*(b*c - a*d)^(1/3))
```

IntegrateAlgebraic [C] time = 1.91, size = 320, normalized size = 2.16

$$
\frac{\left(1+i\sqrt{3}\right)\log\left(2x\sqrt[3]{bc-ad}+\left(1+i\sqrt{3}\right)\sqrt[3]{c}\,\sqrt[3]{a+bx^3}\right)}{6c^{2/3}\sqrt[3]{bc-ad}} - \frac{\sqrt{-1+i\sqrt{3}}\,\tan^{-1}\left(\frac{3x\sqrt[3]{bc-ad}}{\sqrt{3}x\sqrt[3]{bc-ad}-\sqrt{3}\sqrt[3]{c}\,\sqrt[3]{a+bx^3}-3i\sqrt[3]{c}\,\sqrt[3]{a+bx^3}}\right)}{\sqrt{6}\,c^{2/3}\sqrt[3]{bc-ad}} - \frac{i\left(\sqrt{3}-i\right)\log\left(\left(\sqrt{3}+i\right)c^{2/3}\left(a+bx^3\right)^{2/3}+\sqrt[3]{c}\left(-\sqrt{3}x+ix\right)\sqrt[3]{a+bx^3}\,\sqrt[3]{bc-ad}-2ix^2(bc-ad)^{2/3}\right)}{12c^{2/3}\sqrt[3]{bc-ad}}
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((a + b*x^3)^(1/3)*(c + d*x^3)),x]
```

```
[Out] -((Sqrt[-1 + I*Sqrt[3]]*ArcTan[(3*(b*c - a*d)^(1/3)*x)/(Sqrt[3]*(b*c - a*d)
^(1/3)*x - (3*I)*c^(1/3)*(a + b*x^3)^(1/3) - Sqrt[3]*c^(1/3)*(a + b*x^3)^(1
/3))])/(Sqrt[6]*c^(2/3)*(b*c - a*d)^(1/3))) + ((1 + I*Sqrt[3])*Log[2*(b*c -
 a*d)^(1/3)*x + (1 + I*Sqrt[3])*c^(1/3)*(a + b*x^3)^(1/3)])/(6*c^(2/3)*(b*c
 - a*d)^(1/3)) - ((I/12)*(-I + Sqrt[3])*Log[(-2*I)*(b*c - a*d)^(2/3)*x^2 +
c^(1/3)*(b*c - a*d)^(1/3)*(I*x - Sqrt[3]*x)*(a + b*x^3)^(1/3) + (I + Sqrt[3
])*c^(2/3)*(a + b*x^3)^(2/3)])/(c^(2/3)*(b*c - a*d)^(1/3))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^3+a\right)^{\frac{1}{3}}\left(dx^3+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^3 + a)^(1/3)*(d*x^3 + c)), x)
```

maple [F] time = 0.53, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,x^3+a\right)^{\frac{1}{3}}\left(d\,x^3+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^3+a)^(1/3)/(d*x^3+c),x)
```

```
[Out] int(1/(b*x^3+a)^(1/3)/(d*x^3+c),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^3+a\right)^{\frac{1}{3}}\left(dx^3+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^3 + a)^(1/3)*(d*x^3 + c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(b\,x^3+a\right)}^{1/3}\,\left(d\,x^3+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^3)^(1/3)*(c + d*x^3)),x)
```

```
[Out] int(1/((a + b*x^3)^(1/3)*(c + d*x^3)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[3]{a+bx^3}\left(c+dx^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**3+a)**(1/3)/(d*x**3+c),x)
```

```
[Out] Integral(1/((a + b*x**3)**(1/3)*(c + d*x**3)), x)
```

3.65 $\int \frac{1}{\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)}\,dx$

Optimal. Leaf size=179

$$-\frac{d\log\left(c+dx^3\right)}{6c^{2/3}(bc-ad)^{4/3}}+\frac{d\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2c^{2/3}(bc-ad)^{4/3}}-\frac{d\tan^{-1}\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\,c^{2/3}(bc-ad)^{4/3}}+\frac{bx}{a\sqrt[3]{a+bx^3}\,(bc-ad)}$$

Rubi [A] time = 0.19, antiderivative size = 238, normalized size of antiderivative = 1.33, number of steps used = 8, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {382, 377, 200, 31, 634, 617, 204, 628}

$$\frac{d\log\left(\sqrt[3]{c}-\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{a+bx^3}}\right)}{3c^{2/3}(bc-ad)^{4/3}}-\frac{d\log\left(\frac{x^2(bc-ad)^{2/3}}{\left(a+bx^3\right)^{2/3}}+\frac{\sqrt[3]{c}\,x\sqrt[3]{bc-ad}}{\sqrt[3]{a+bx^3}}+c^{2/3}\right)}{6c^{2/3}(bc-ad)^{4/3}}-\frac{d\tan^{-1}\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{a+bx^3}}+\sqrt[3]{c}}{\sqrt{3}\sqrt[3]{c}}\right)}{\sqrt{3}\,c^{2/3}(bc-ad)^{4/3}}+\frac{bx}{a\sqrt[3]{a+bx^3}\,(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^3)^(4/3)*(c + d*x^3)),x]
```

```
[Out] (b*x)/(a*(b*c - a*d)*(a + b*x^3)^(1/3)) - (d*ArcTan[(c^(1/3) + (2*(b*c - a*
d)^(1/3)*x)/(a + b*x^3)^(1/3))/(Sqrt[3]*c^(1/3))])/(Sqrt[3]*c^(2/3)*(b*c -
a*d)^(4/3)) + (d*Log[c^(1/3) - ((b*c - a*d)^(1/3)*x)/(a + b*x^3)^(1/3)])/(3
*c^(2/3)*(b*c - a*d)^(4/3)) - (d*Log[c^(2/3) + ((b*c - a*d)^(2/3)*x^2)/(a +
 b*x^3)^(2/3) + (c^(1/3)*(b*c - a*d)^(1/3)*x)/(a + b*x^3)^(1/3)])/(6*c^(2/3
)*(b*c - a*d)^(4/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)), I
```

```
nt[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q}, x
] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !LtQ
[q, -1]) && NeQ[p, -1]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)}\,dx &= \frac{bx}{a(bc-ad)\sqrt[3]{a+bx^3}} - \frac{d\int \frac{1}{\sqrt[3]{a+bx^3}\left(c+dx^3\right)}\,dx}{bc-ad} \\
&= \frac{bx}{a(bc-ad)\sqrt[3]{a+bx^3}} - \frac{d\,\text{Subst}\left(\int \frac{1}{c-(bc-ad)x^3}\,dx, x, \frac{x}{\sqrt[3]{a+bx^3}}\right)}{bc-ad} \\
&= \frac{bx}{a(bc-ad)\sqrt[3]{a+bx^3}} - \frac{d\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{c}-\sqrt[3]{bc-ad}\,x}\,dx, x, \frac{x}{\sqrt[3]{a+bx^3}}\right)}{3c^{2/3}(bc-ad)} - \frac{d\,\text{Subst}\left(\int \frac{2\cdots}{c^{2/3}+\sqrt[3]{c}\cdots}\right.}{\cdots} \\
&= \frac{bx}{a(bc-ad)\sqrt[3]{a+bx^3}} + \frac{d\log\left(\sqrt[3]{c}-\frac{\sqrt[3]{bc-ad}\,x}{\sqrt[3]{a+bx^3}}\right)}{3c^{2/3}(bc-ad)^{4/3}} - \frac{d\,\text{Subst}\left(\int \frac{\sqrt[3]{c}\sqrt[3]{bc-ad}+2(bc-ad)^{2/3}x}{c^{2/3}+\sqrt[3]{c}\sqrt[3]{bc-ad}\,x+(bc-ad)^{2/3}\cdots}\right.}{6c^{2/3}(bc-ad)^{4/3}} \\
&= \frac{bx}{a(bc-ad)\sqrt[3]{a+bx^3}} + \frac{d\log\left(\sqrt[3]{c}-\frac{\sqrt[3]{bc-ad}\,x}{\sqrt[3]{a+bx^3}}\right)}{3c^{2/3}(bc-ad)^{4/3}} - \frac{d\log\left(c^{2/3}+\frac{(bc-ad)^{2/3}x^2}{\left(a+bx^3\right)^{2/3}}+\frac{\sqrt[3]{c}\sqrt[3]{bc-a\cdots}}{\sqrt[3]{a+bx^3}}\right.}{6c^{2/3}(bc-ad)^{4/3}} \\
&= \frac{bx}{a(bc-ad)\sqrt[3]{a+bx^3}} - \frac{d\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{bc-ad}\,x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}\,c^{2/3}(bc-ad)^{4/3}} + \frac{d\log\left(\sqrt[3]{c}-\frac{\sqrt[3]{bc-ad}\,x}{\sqrt[3]{a+bx^3}}\right)}{3c^{2/3}(bc-ad)^{4/3}} - \frac{d\log\left(c^{2/3}\cdots\right.}{\cdots}
\end{aligned}
$$

Mathematica [C] time = 0.80, size = 256, normalized size = 1.43

$$
-\frac{28c^3\left(a+bx^3\right)^2 {}_2F_1\left(\frac{1}{3},1;\frac{4}{3};\frac{(bc-ad)x^3}{c\left(bx^3+a\right)}\right)-28c^3\left(a+bx^3\right)^2+21c^2dx^3\left(a+bx^3\right)^2 {}_2F_1\left(\frac{1}{3},1;\frac{4}{3};\frac{(bc-ad)x^3}{c\left(bx^3+a\right)}\right)-21c^2dx^3\left(a+bx^3\right)^2+3dx^9(bc-ad)^2 {}_2F_1\left(2,\frac{7}{3};\frac{10}{3};\frac{(bc-ad)x^3}{c\left(bx^3+a\right)}\right)+3cx^6(bc-ad)^2 {}_2F_1\left(2,\frac{7}{3};\frac{10}{3};\frac{(bc-ad)x^3}{c\left(bx^3+a\right)}\right)}{7c^3x^2\left(a+bx^3\right)^{7/3}(ad-bc)}
$$

Warning: Unable to verify antiderivative.

[In] Integrate[1/((a + b*x^3)^(4/3)*(c + d*x^3)),x]

[Out] -1/7*(-28*c^3*(a + b*x^3)^2 - 21*c^2*d*x^3*(a + b*x^3)^2 + 28*c^3*(a + b*x^3)^2*Hypergeometric2F1[1/3, 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 21*c^2*d*x^3*(a + b*x^3)^2*Hypergeometric2F1[1/3, 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 3*c*(b*c - a*d)^2*x^6*Hypergeometric2F1[2, 7/3, 10/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 3*d*(b*c - a*d)^2*x^9*Hypergeometric2F1[2, 7/3, 10/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))])/(c^3*(-(b*c) + a*d)*x^2*(a + b*x^3)^(7/3))

IntegrateAlgebraic [C] time = 2.70, size = 352, normalized size = 1.97

$$-\frac{i\left(\sqrt{3}d-id\right)\log\left(2x\sqrt[3]{bc-ad}+\left(1+i\sqrt{3}\right)\sqrt[3]{c}\sqrt[3]{a+bx^3}\right)}{6c^{2/3}(bc-ad)^{4/3}}+\frac{\sqrt{\frac{1}{6}\left(-1+i\sqrt{3}\right)}\,d\tan^{-1}\left(\frac{3x\sqrt[3]{bc-ad}}{\sqrt{3}x\sqrt[3]{bc-ad}-\sqrt{3}\sqrt[3]{c}\sqrt[3]{a+bx^3}-3i\sqrt[3]{c}\sqrt[3]{a+bx^3}}\right)}{c^{2/3}(bc-ad)^{4/3}}+\frac{\left(d+i\sqrt{3}d\right)\log\left(\left(\sqrt{3}+i\right)c^{2/3}\left(a+bx^3\right)^{2/3}+\sqrt[3]{c}\left(-\sqrt{3}x+ix\right)\sqrt[3]{a+bx^3}\sqrt[3]{bc-ad}-2ix^2(bc-ad)^{2/3}\right)}{12c^{2/3}(bc-ad)^{4/3}}-\frac{bx}{a\sqrt[3]{a+bx^3}\,(ad-bc)}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[1/((a + b*x^3)^(4/3)*(c + d*x^3)),x]

[Out] -((b*x)/(a*(-(b*c) + a*d)*(a + b*x^3)^(1/3))) + (Sqrt[(-1 + I*Sqrt[3])/6]*d*ArcTan[(3*(b*c - a*d)^(1/3)*x)/(Sqrt[3]*(b*c - a*d)^(1/3)*x - (3*I)*c^(1/3)*(a + b*x^3)^(1/3) - Sqrt[3]*c^(1/3)*(a + b*x^3)^(1/3))]])/(c^(2/3)*(b*c - a*d)^(4/3)) - ((I/6)*((-I)*d + Sqrt[3]*d)*Log[2*(b*c - a*d)^(1/3)*x + (1 + I*Sqrt[3])*c^(1/3)*(a + b*x^3)^(1/3)])/(c^(2/3)*(b*c - a*d)^(4/3)) + ((d + I*Sqrt[3]*d)*Log[(-2*I)*(b*c - a*d)^(2/3)*x^2 + c^(1/3)*(b*c - a*d)^(1/3)*(I*x - Sqrt[3]*x)*(a + b*x^3)^(1/3) + (I + Sqrt[3])*c^(2/3)*(a + b*x^3)^(2/3)])/(12*c^(2/3)*(b*c - a*d)^(4/3))

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="fricas")

[Out] Timed out

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^3+a\right)^{\frac{4}{3}}\left(dx^3+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="giac")

[Out] integrate(1/((b*x^3 + a)^(4/3)*(d*x^3 + c)), x)

maple [F] time = 0.57, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,x^3+a\right)^{\frac{4}{3}}\left(d\,x^3+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b*x^3+a)^(4/3)/(d*x^3+c),x)

[Out] int(1/(b*x^3+a)^(4/3)/(d*x^3+c),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^3+a\right)^{\frac{4}{3}}\left(dx^3+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="maxima")
[Out] integrate(1/((b*x^3 + a)^(4/3)*(d*x^3 + c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(b\,x^3+a\right)^{4/3}\,\left(d\,x^3+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^3)^(4/3)*(c + d*x^3)),x)
[Out] int(1/((a + b*x^3)^(4/3)*(c + d*x^3)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+bx^{3}\right)^{\frac{4}{3}}\left(c+dx^{3}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**3+a)**(4/3)/(d*x**3+c),x)
[Out] Integral(1/((a + b*x**3)**(4/3)*(c + d*x**3)), x)
```

3.66 $\int \frac{1}{\left(a+bx^3\right)^{7/3}\left(c+dx^3\right)}\,dx$

Optimal. Leaf size=226

$$\frac{bx(3bc-7ad)}{4a^2\sqrt[3]{a+bx^3}\,(bc-ad)^2}+\frac{d^2\log\left(c+dx^3\right)}{6c^{2/3}(bc-ad)^{7/3}}-\frac{d^2\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2c^{2/3}(bc-ad)^{7/3}}+\frac{d^2\tan^{-1}\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\,c^{2/3}(bc-ad)^{7/3}}+\frac{\ }{4a\left(a+bx^3\right.}$$

Rubi [C] time = 2.58, antiderivative size = 621, normalized size of antiderivative = 2.75, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {430, 429}

$$-\frac{-9c^2x^9(bc-ad)^3\,{}_3F_2\left(2,2,\frac{10}{3};1,\frac{13}{3};\frac{(bc-ad)x^3}{c(bx^3+a)}\right)-9d^2x^{15}(bc-ad)^3\,{}_3F_2\left(2,2,\frac{10}{3};1,\frac{13}{3};\frac{(bc-ad)x^3}{c(bx^3+a)}\right)-18cdx^{12}(bc-ad)^3\,{}_3F_2\left(2,2,\frac{10}{3};1,\frac{13}{3};\frac{(bc-ad)x^3}{c(bx^3+a)}\right)-180c^3d^2x^6\left(a+bx^3\right)^3\,{}_2F_1\left(\frac{1}{3},1;\frac{4}{3};\frac{(bc-ad)x^3}{c(bx^3+a)}\right)+45c^2d^2x^9\left(a+bx^3\right)^2(bc-ad)+180c^3d^2x^6\left(a+bx^3\right)^3-33c^2x^9(bc-ad)^3\,{}_2F_1\left(2,\frac{10}{3};\frac{13}{3};\frac{(bc-ad)x^3}{c(bx^3+a)}\right)-420c^4dx^3\left(a+bx^3\right)^3\,{}_2F_1\left(\frac{1}{3},1;\frac{4}{3};\frac{(bc-ad)x^3}{c(bx^3+a)}\right)-280c^5\left(a+bx^3\right)^3\,{}_2F_1\left(\frac{1}{3},1;\frac{4}{3};\frac{(bc-ad)x^3}{c(bx^3+a)}\right)+105c^3dx^6\left(a+bx^3\right)^2(bc-ad)+420c^4dx^3\left(a+bx^3\right)^3+70c^4x^3\left(a+bx^3\right)^2(bc-ad)+280c^5\left(a+bx^3\right)^3-27d^2x^{15}(bc-ad)^3\,{}_2F_1\left(2,\frac{10}{3};\frac{13}{3};\frac{(bc-ad)x^3}{c(bx^3+a)}\right)-60cdx^{12}(bc-ad)^3\,{}_2F_1\left(2,\frac{10}{3};\frac{13}{3};\frac{(bc-ad)x^3}{c(bx^3+a)}\right)}{40c^4x^5\left(a+bx^3\right)^{10/3}(bc-ad)^2}$$

Warning: Unable to verify antiderivative.

```
[In] Int[1/((a + b*x^3)^(7/3)*(c + d*x^3)),x]
```

```
[Out] -(70*c^4*(b*c - a*d)*x^3*(a + b*x^3)^2 + 105*c^3*d*(b*c - a*d)*x^6*(a + b*x
^3)^2 + 45*c^2*d^2*(b*c - a*d)*x^9*(a + b*x^3)^2 + 280*c^5*(a + b*x^3)^3 +
420*c^4*d*x^3*(a + b*x^3)^3 + 180*c^3*d^2*x^6*(a + b*x^3)^3 - 280*c^5*(a +
b*x^3)^3*Hypergeometric2F1[1/3, 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))]
- 420*c^4*d*x^3*(a + b*x^3)^3*Hypergeometric2F1[1/3, 1, 4/3, ((b*c - a*d)*x
^3)/(c*(a + b*x^3))] - 180*c^3*d^2*x^6*(a + b*x^3)^3*Hypergeometric2F1[1/3,
 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 33*c^2*(b*c - a*d)^3*x^9*Hype
rgeometric2F1[2, 10/3, 13/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 60*c*d*(b
*c - a*d)^3*x^12*Hypergeometric2F1[2, 10/3, 13/3, ((b*c - a*d)*x^3)/(c*(a +
 b*x^3))] - 27*d^2*(b*c - a*d)^3*x^15*Hypergeometric2F1[2, 10/3, 13/3, ((b*
c - a*d)*x^3)/(c*(a + b*x^3))] - 9*c^2*(b*c - a*d)^3*x^9*HypergeometricPFQ[
{2, 2, 10/3}, {1, 13/3}, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 18*c*d*(b*c -
 a*d)^3*x^12*HypergeometricPFQ[{2, 2, 10/3}, {1, 13/3}, ((b*c - a*d)*x^3)/(
c*(a + b*x^3))] - 9*d^2*(b*c - a*d)^3*x^15*HypergeometricPFQ[{2, 2, 10/3},
{1, 13/3}, ((b*c - a*d)*x^3)/(c*(a + b*x^3))])/(40*c^4*(b*c - a*d)^2*x^5*(a
 + b*x^3)^(10/3))
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{1}{\left(a+bx^3\right)^{7/3}\left(c+dx^3\right)}\,dx = \frac{\sqrt[3]{1+\frac{bx^3}{a}}\int \frac{1}{\left(1+\frac{bx^3}{a}\right)^{7/3}\left(c+dx^3\right)}dx}{a^2\sqrt[3]{a+bx^3}}$$

$$= -\frac{70c^4(bc-ad)x^3\left(a+bx^3\right)^2+105c^3d(bc-ad)x^6\left(a+bx^3\right)^2+45c^2d^2(bc-ad)x^9\left(\right.}{}$$

Mathematica [C] time = 2.67, size = 621, normalized size = 2.75

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + b*x^3)^(7/3)*(c + d*x^3)),x]
```

```
[Out] (-70*c^4*(b*c - a*d)*x^3*(a + b*x^3)^2 - 105*c^3*d*(b*c - a*d)*x^6*(a + b*x
^3)^2 - 45*c^2*d^2*(b*c - a*d)*x^9*(a + b*x^3)^2 - 280*c^5*(a + b*x^3)^3 -
420*c^4*d*x^3*(a + b*x^3)^3 - 180*c^3*d^2*x^6*(a + b*x^3)^3 + 280*c^5*(a +
b*x^3)^3*Hypergeometric2F1[1/3, 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))]
+ 420*c^4*d*x^3*(a + b*x^3)^3*Hypergeometric2F1[1/3, 1, 4/3, ((b*c - a*d)*x
^3)/(c*(a + b*x^3))] + 180*c^3*d^2*x^6*(a + b*x^3)^3*Hypergeometric2F1[1/3,
 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 33*c^2*(b*c - a*d)^3*x^9*Hype
rgeometric2F1[2, 10/3, 13/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 60*c*d*(b
*c - a*d)^3*x^12*Hypergeometric2F1[2, 10/3, 13/3, ((b*c - a*d)*x^3)/(c*(a +
 b*x^3))] + 27*d^2*(b*c - a*d)^3*x^15*Hypergeometric2F1[2, 10/3, 13/3, ((b*
c - a*d)*x^3)/(c*(a + b*x^3))] + 9*c^2*(b*c - a*d)^3*x^9*HypergeometricPFQ[
{2, 2, 10/3}, {1, 13/3}, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 18*c*d*(b*c -
 a*d)^3*x^12*HypergeometricPFQ[{2, 2, 10/3}, {1, 13/3}, ((b*c - a*d)*x^3)/(
c*(a + b*x^3))] + 9*d^2*(b*c - a*d)^3*x^15*HypergeometricPFQ[{2, 2, 10/3},
{1, 13/3}, ((b*c - a*d)*x^3)/(c*(a + b*x^3))])/(40*c^4*(b*c - a*d)^2*x^5*(a
 + b*x^3)^(10/3))
```

IntegrateAlgebraic [C] time = 3.79, size = 399, normalized size = 1.77

$$\frac{-8a^2bdx+4ab^2cx-7ab^2dx^4+3b^3cx^4}{4a^2\left(a+bx^3\right)^{4/3}(ad-bc)^2}+\frac{\left(d^2+i\sqrt{3}d^2\right)\log\left(2x\sqrt[3]{bc-ad}+\left(1+i\sqrt{3}\right)\sqrt[3]{c}\sqrt[3]{a+bx^3}\right)}{6c^{2/3}(bc-ad)^{7/3}}-\frac{\sqrt{\frac{1}{6}\left(-1+i\sqrt{3}\right)}d^2\tan^{-1}\left(\frac{3x\sqrt[3]{bc-ad}}{\sqrt{3}x\sqrt[3]{bc-ad}-\sqrt{3}\sqrt[3]{c}\sqrt[3]{a+bx^3}-3i\sqrt[3]{c}\sqrt[3]{a+bx^3}}\right)}{c^{2/3}(bc-ad)^{7/3}}-\frac{i\left(\sqrt{3}d^2-id^2\right)\log\left(\left(\sqrt{3}+i\right)c^{2/3}\left(a+bx^3\right)^{2/3}+\sqrt[3]{c}\left(-\sqrt{3}x+ix\right)\sqrt[3]{a+bx^3}\sqrt[3]{bc-ad}-2ix^2(bc-ad)^{2/3}\right)}{12c^{2/3}(bc-ad)^{7/3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((a + b*x^3)^(7/3)*(c + d*x^3)),x]
```

```
[Out] (4*a*b^2*c*x - 8*a^2*b*d*x + 3*b^3*c*x^4 - 7*a*b^2*d*x^4)/(4*a^2*(-(b*c) +
a*d)^2*(a + b*x^3)^(4/3)) - (Sqrt[(-1 + I*Sqrt[3])/6]*d^2*ArcTan[(3*(b*c -
a*d)^(1/3)*x)/(Sqrt[3]*(b*c - a*d)^(1/3)*x - (3*I)*c^(1/3)*(a + b*x^3)^(1/3
) - Sqrt[3]*c^(1/3)*(a + b*x^3)^(1/3))])/(c^(2/3)*(b*c - a*d)^(7/3)) + ((d^
2 + I*Sqrt[3]*d^2)*Log[2*(b*c - a*d)^(1/3)*x + (1 + I*Sqrt[3])*c^(1/3)*(a +
 b*x^3)^(1/3)])/(6*c^(2/3)*(b*c - a*d)^(7/3)) - ((I/12)*((-I)*d^2 + Sqrt[3]
*d^2)*Log[(-2*I)*(b*c - a*d)^(2/3)*x^2 + c^(1/3)*(b*c - a*d)^(1/3)*(I*x - S
qrt[3]*x)*(a + b*x^3)^(1/3) + (I + Sqrt[3])*c^(2/3)*(a + b*x^3)^(2/3)])/(c^
(2/3)*(b*c - a*d)^(7/3))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^(7/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^3 + a\right)^{\frac{7}{3}} \left(dx^3 + c\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^(7/3)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^3 + a)^(7/3)*(d*x^3 + c)), x)
```

maple [F] time = 0.56, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,x^3 + a\right)^{\frac{7}{3}} \left(d\,x^3 + c\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^3+a)^(7/3)/(d*x^3+c),x)
```

```
[Out] int(1/(b*x^3+a)^(7/3)/(d*x^3+c),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^3 + a\right)^{\frac{7}{3}} \left(dx^3 + c\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^(7/3)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^3 + a)^(7/3)*(d*x^3 + c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(b\,x^3+a\right)}^{7/3}\,\left(d\,x^3+c\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^3)^(7/3)*(c + d*x^3)),x)
```

```
[Out] int(1/((a + b*x^3)^(7/3)*(c + d*x^3)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a + bx^{3}\right)^{\frac{7}{3}} \left(c + dx^{3}\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**3+a)**(7/3)/(d*x**3+c),x)
```

```
[Out] Integral(1/((a + b*x**3)**(7/3)*(c + d*x**3)), x)
```

3.67 $\int \frac{1}{\left(a+bx^3\right)^{10/3}\left(c+dx^3\right)}\,dx$

Optimal. Leaf size=280

$$\frac{bx(6bc-13ad)}{28a^2\left(a+bx^3\right)^{4/3}(bc-ad)^2}+\frac{bx\left(67a^2d^2-57abcd+18b^2c^2\right)}{28a^3\sqrt[3]{a+bx^3}\,(bc-ad)^3}-\frac{d^3\log\left(c+dx^3\right)}{6c^{2/3}(bc-ad)^{10/3}}+\frac{d^3\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2c^{2/3}(bc-ad)^{10/3}}-$$

Rubi [C] time = 6.64, antiderivative size = 1172, normalized size of antiderivative = 4.19, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {430, 429}

Warning: Unable to verify antiderivative.

```
[In] Int[1/((a + b*x^3)^(10/3)*(c + d*x^3)),x]
```

```
[Out] -(7280*c^5*(b*c - a*d)^2*x^6*(a + b*x^3)^2 + 16380*c^4*d*(b*c - a*d)^2*x^9*
(a + b*x^3)^2 + 14040*c^3*d^2*(b*c - a*d)^2*x^12*(a + b*x^3)^2 + 4212*c^2*d
^3*(b*c - a*d)^2*x^15*(a + b*x^3)^2 + 12740*c^6*(b*c - a*d)*x^3*(a + b*x^3)
^3 + 28665*c^5*d*(b*c - a*d)*x^6*(a + b*x^3)^3 + 24570*c^4*d^2*(b*c - a*d)*
x^9*(a + b*x^3)^3 + 7371*c^3*d^3*(b*c - a*d)*x^12*(a + b*x^3)^3 + 50960*c^7
*(a + b*x^3)^4 + 114660*c^6*d*x^3*(a + b*x^3)^4 + 98280*c^5*d^2*x^6*(a + b*
x^3)^4 + 29484*c^4*d^3*x^9*(a + b*x^3)^4 - 50960*c^7*(a + b*x^3)^4*Hypergeo
metric2F1[1/3, 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 114660*c^6*d*x^
3*(a + b*x^3)^4*Hypergeometric2F1[1/3, 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*
x^3))] - 98280*c^5*d^2*x^6*(a + b*x^3)^4*Hypergeometric2F1[1/3, 1, 4/3, ((b
*c - a*d)*x^3)/(c*(a + b*x^3))] - 29484*c^4*d^3*x^9*(a + b*x^3)^4*Hypergeom
etric2F1[1/3, 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 5796*c^3*(b*c -
a*d)^4*x^12*Hypergeometric2F1[2, 13/3, 16/3, ((b*c - a*d)*x^3)/(c*(a + b*x^
3))] - 15246*c^2*d*(b*c - a*d)^4*x^15*Hypergeometric2F1[2, 13/3, 16/3, ((b*
c - a*d)*x^3)/(c*(a + b*x^3))] - 13608*c*d^2*(b*c - a*d)^4*x^18*Hypergeomet
ric2F1[2, 13/3, 16/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 4158*d^3*(b*c -
a*d)^4*x^21*Hypergeometric2F1[2, 13/3, 16/3, ((b*c - a*d)*x^3)/(c*(a + b*x^
3))] - 2646*c^3*(b*c - a*d)^4*x^12*HypergeometricPFQ[{2, 2, 13/3}, {1, 16/3
}, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 7560*c^2*d*(b*c - a*d)^4*x^15*Hyper
geometricPFQ[{2, 2, 13/3}, {1, 16/3}, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] -
7182*c*d^2*(b*c - a*d)^4*x^18*HypergeometricPFQ[{2, 2, 13/3}, {1, 16/3}, ((
b*c - a*d)*x^3)/(c*(a + b*x^3))] - 2268*d^3*(b*c - a*d)^4*x^21*Hypergeometr
icPFQ[{2, 2, 13/3}, {1, 16/3}, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 378*c^3
*(b*c - a*d)^4*x^12*HypergeometricPFQ[{2, 2, 2, 13/3}, {1, 1, 16/3}, ((b*c
- a*d)*x^3)/(c*(a + b*x^3))] - 1134*c^2*d*(b*c - a*d)^4*x^15*Hypergeometric
PFQ[{2, 2, 2, 13/3}, {1, 1, 16/3}, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 113
4*c*d^2*(b*c - a*d)^4*x^18*HypergeometricPFQ[{2, 2, 2, 13/3}, {1, 1, 16/3},
 ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 378*d^3*(b*c - a*d)^4*x^21*Hypergeome
tricPFQ[{2, 2, 2, 13/3}, {1, 1, 16/3}, ((b*c - a*d)*x^3)/(c*(a + b*x^3))])/
(5096*c^5*(b*c - a*d)^3*x^8*(a + b*x^3)^(13/3))
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{1}{\left(a+bx^3\right)^{10/3}\left(c+dx^3\right)}\,dx = \frac{\sqrt[3]{1+\frac{bx^3}{a}}\int \frac{1}{\left(1+\frac{bx^3}{a}\right)^{10/3}\left(c+dx^3\right)}\,dx}{a^3\sqrt[3]{a+bx^3}}$$

$$= -\frac{7280c^5(bc-ad)^2x^6\left(a+bx^3\right)^2+16380c^4d(bc-ad)^2x^9\left(a+bx^3\right)^2+14040c^3\ldots}{\ldots}$$

Mathematica [A]   time = 5.75, size = 277, normalized size = 0.99

$$\frac{bx\left(\left(a+bx^3\right)^2\left(67a^2d^2-57abcd+18b^2c^2\right)+4a^2(bc-ad)^2+a\left(a+bx^3\right)(ad-bc)(13ad-6bc)\right)}{28a^3\left(a+bx^3\right)^{7/3}(bc-ad)^3}-\frac{d^3\left(\log\left(\frac{\sqrt[3]{c}\,x\sqrt[3]{bc-ad}}{\sqrt[3]{ax^3+b}}+\frac{x^2(bc-ad)^{2/3}}{\left(ax^3+b\right)^{2/3}}+c^{2/3}\right)-2\log\left(\sqrt[3]{c}-\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{ax^3+b}}\right)+2\sqrt{3}\tan^{-1}\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\sqrt[3]{ax^3+b}}+1}{\sqrt{3}}\right)\right)}{6c^{2/3}(bc-ad)^{10/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + b*x^3)^(10/3)*(c + d*x^3)),x]
```

```
[Out] (b*x*(4*a^2*(b*c - a*d)^2 + a*(-(b*c) + a*d)*(-6*b*c + 13*a*d)*(a + b*x^3)
+ (18*b^2*c^2 - 57*a*b*c*d + 67*a^2*d^2)*(a + b*x^3)^2))/(28*a^3*(b*c - a*d
)^3*(a + b*x^3)^(7/3)) - (d^3*(2*Sqrt[3]*ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x
)/(c^(1/3)*(b + a*x^3)^(1/3)))/Sqrt[3]] - 2*Log[c^(1/3) - ((b*c - a*d)^(1/3
)*x)/(b + a*x^3)^(1/3)] + Log[c^(2/3) + ((b*c - a*d)^(2/3)*x^2)/(b + a*x^3)
^(2/3) + (c^(1/3)*(b*c - a*d)^(1/3)*x)/(b + a*x^3)^(1/3)]))/(6*c^(2/3)*(b*c
 - a*d)^(10/3))
```

IntegrateAlgebraic [C]   time = 8.10, size = 471, normalized size = 1.68

$$\frac{-84a^4bd^2x+84a^3b^2cdx-147a^3b^2d^2x^4-28a^2b^3c^2x+133a^2b^3cdx^4-67a^2b^3d^2x^7-42ab^4c^2x^4+57ab^4cdx^7-18b^5c^2x^7}{28a^3\left(a+bx^3\right)^{7/3}(ad-bc)^3}-\frac{i\left(\sqrt{3}d^3-id^3\right)\log\left(2x\sqrt[3]{bc-ad}+\left(1+i\sqrt{3}\right)\sqrt[3]{c}\sqrt[3]{a+bx^3}\right)}{6c^{2/3}(bc-ad)^{10/3}}+\frac{\sqrt{\frac{1}{6}\left(-1+i\sqrt{3}\right)}d^3\tan^{-1}\left(\frac{3x\sqrt[3]{bc-ad}}{\sqrt{3}x\sqrt[3]{bc-ad}-\sqrt{3}\sqrt[3]{c}\sqrt[3]{a+bx^3}-3i\sqrt[3]{c}\sqrt[3]{a+bx^3}}\right)}{c^{2/3}(bc-ad)^{10/3}}+\frac{\left(d^3+i\sqrt{3}d^3\right)\log\left(\left(\sqrt{3}+i\right)c^{2/3}\left(a+bx^3\right)^{2/3}+\sqrt[3]{c}\left(-\sqrt{3}x+ix\right)\sqrt[3]{a+bx^3}\sqrt[3]{bc-ad}-2ix^2(bc-ad)^{2/3}\right)}{12c^{2/3}(bc-ad)^{10/3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((a + b*x^3)^(10/3)*(c + d*x^3)),x]
```

```
[Out] (-28*a^2*b^3*c^2*x + 84*a^3*b^2*c*d*x - 84*a^4*b*d^2*x - 42*a*b^4*c^2*x^4 +
 133*a^2*b^3*c*d*x^4 - 147*a^3*b^2*d^2*x^4 - 18*b^5*c^2*x^7 + 57*a*b^4*c*d*
x^7 - 67*a^2*b^3*d^2*x^7)/(28*a^3*(-(b*c) + a*d)^3*(a + b*x^3)^(7/3)) + (Sq
rt[(-1 + I*Sqrt[3])/6]*d^3*ArcTan[(3*(b*c - a*d)^(1/3)*x)/(Sqrt[3]*(b*c - a
*d)^(1/3)*x - (3*I)*c^(1/3)*(a + b*x^3)^(1/3) - Sqrt[3]*c^(1/3)*(a + b*x^3)
^(1/3))]])/(c^(2/3)*(b*c - a*d)^(10/3)) - ((I/6)*((-I)*d^3 + Sqrt[3]*d^3)*Lo
g[2*(b*c - a*d)^(1/3)*x + (1 + I*Sqrt[3])*c^(1/3)*(a + b*x^3)^(1/3)])/(c^(2
/3)*(b*c - a*d)^(10/3)) + ((d^3 + I*Sqrt[3]*d^3)*Log[(-2*I)*(b*c - a*d)^(2/
3)*x^2 + c^(1/3)*(b*c - a*d)^(1/3)*(I*x - Sqrt[3]*x)*(a + b*x^3)^(1/3) + (I
 + Sqrt[3])*c^(2/3)*(a + b*x^3)^(2/3)])/(12*c^(2/3)*(b*c - a*d)^(10/3))
```

fricas [F(-1)]   time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^(10/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^3+a\right)^{\frac{10}{3}}\left(dx^3+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^(10/3)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^3 + a)^(10/3)*(d*x^3 + c)), x)
```

maple [F] time = 0.60, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,x^3+a\right)^{\frac{10}{3}}\left(d\,x^3+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^3+a)^(10/3)/(d*x^3+c),x)
```

```
[Out] int(1/(b*x^3+a)^(10/3)/(d*x^3+c),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^3+a\right)^{\frac{10}{3}}\left(dx^3+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^(10/3)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^3 + a)^(10/3)*(d*x^3 + c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(b\,x^3+a\right)}^{10/3}\,\left(d\,x^3+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^3)^(10/3)*(c + d*x^3)),x)
```

```
[Out] int(1/((a + b*x^3)^(10/3)*(c + d*x^3)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+bx^3\right)^{\frac{10}{3}}\left(c+dx^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**3+a)**(10/3)/(d*x**3+c),x)
```

```
[Out] Integral(1/((a + b*x**3)**(10/3)*(c + d*x**3)), x)
```

3.68 $\int \frac{\left(a+bx^3\right)^{8/3}}{\left(c+dx^3\right)^2}\,dx$

Optimal. Leaf size=351

$$\frac{b^{5/3}(3bc-4ad)\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}\,x\right)}{3d^3}-\frac{2b^{5/3}(3bc-4ad)\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{3\sqrt{3}\,d^3}+\frac{(bc-ad)^{5/3}(ad+3bc)\log\left(c+dx\right.}{9c^{5/3}d^3}$$

Rubi [C] time = 0.03, antiderivative size = 62, normalized size of antiderivative = 0.18, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {430, 429}

$$\frac{a^2x\left(a+bx^3\right)^{2/3}F_1\left(\frac{1}{3};-\frac{8}{3},2;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c^2\left(\frac{bx^3}{a}+1\right)^{2/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(a + b*x^3)^(8/3)/(c + d*x^3)^2,x]
```

```
[Out] (a^2*x*(a + b*x^3)^(2/3)*AppellF1[1/3, -8/3, 2, 4/3, -((b*x^3)/a), -((d*x^3
)/c)])/(c^2*(1 + (b*x^3)/a)^(2/3))
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^3\right)^{8/3}}{\left(c+dx^3\right)^2}\,dx &= \frac{\left(a^2\left(a+bx^3\right)^{2/3}\right)\int \frac{\left(1+\frac{bx^3}{a}\right)^{8/3}}{\left(c+dx^3\right)^2}\,dx}{\left(1+\frac{bx^3}{a}\right)^{2/3}}\\ &= \frac{a^2x\left(a+bx^3\right)^{2/3}F_1\left(\frac{1}{3};-\frac{8}{3},2;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c^2\left(1+\frac{bx^3}{a}\right)^{2/3}}\end{aligned}$$

Mathematica [C] time = 1.08, size = 698, normalized size = 1.99

$$\frac{1}{18}\left(\frac{2a^3\left(\log\left(\frac{\sqrt[3]{c}\,x\sqrt[3]{bc-ad}}{\sqrt[3]{ax^3+b}}+\frac{x^2(bc-ad)^{2/3}}{\left(ax^3+b\right)^{2/3}}+c^{2/3}\right)-2\log\left(\sqrt[3]{c}-\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{ax^3+b}}\right)+2\sqrt{3}\tan^{-1}\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\sqrt[3]{ax^3+b}}+1}{\sqrt{3}}\right)\right)}{c^{5/3}\sqrt[3]{bc-ad}}+\frac{2a^2b\left(\log\left(\frac{\sqrt[3]{c}\,x\sqrt[3]{bc-ad}}{\sqrt[3]{ax^3+b}}+\frac{x^2(bc-ad)^{2/3}}{\left(ax^3+b\right)^{2/3}}+c^{2/3}\right)-2\log\left(\sqrt[3]{c}-\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{ax^3+b}}\right)+2\sqrt{3}\tan^{-1}\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\sqrt[3]{ax^3+b}}+1}{\sqrt{3}}\right)\right)}{c^{2/3}d\sqrt[3]{bc-ad}}-\frac{9b^3x^4\sqrt[3]{\frac{bx^3}{a}+1}\,F_1\left(\frac{4}{3};\frac{1}{3},1;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{d^2\sqrt[3]{a+bx^3}}+\frac{12ab^2x^4\sqrt[3]{\frac{bx^3}{a}+1}\,F_1\left(\frac{4}{3};\frac{1}{3},1;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{cd\sqrt[3]{a+bx^3}}-\frac{2ab^2\sqrt[3]{c}\left(\log\left(\frac{\sqrt[3]{c}\,x\sqrt[3]{bc-ad}}{\sqrt[3]{ax^3+b}}+\frac{x^2(bc-ad)^{2/3}}{\left(ax^3+b\right)^{2/3}}+c^{2/3}\right)-2\log\left(\sqrt[3]{c}-\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{ax^3+b}}\right)+2\sqrt{3}\tan^{-1}\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\sqrt[3]{ax^3+b}}+1}{\sqrt{3}}\right)\right)}{d^2\sqrt[3]{bc-ad}}+\frac{6x\left(a+bx^3\right)^{2/3}\left(\frac{(bc-ad)^2}{c\left(c+dx^3\right)}+b^2\right)}{d^2}\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^3)^(8/3)/(c + d*x^3)^2,x]
```

```
[Out] ((6*x*(a + b*x^3)^(2/3)*(b^2 + (b*c - a*d)^2/(c*(c + d*x^3)))))/d^2 - (9*b^3
*x^4*(1 + (b*x^3)/a)^(1/3)*AppellF1[4/3, 1/3, 1, 7/3, -((b*x^3)/a), -((d*x^
3)/c)])/(d^2*(a + b*x^3)^(1/3)) + (12*a*b^2*x^4*(1 + (b*x^3)/a)^(1/3)*Appel
lF1[4/3, 1/3, 1, 7/3, -((b*x^3)/a), -((d*x^3)/c)])/(c*d*(a + b*x^3)^(1/3))
+ (2*a^3*(2*Sqrt[3]*ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c^(1/3)*(b + a*x^3
)^(1/3)))/Sqrt[3]] - 2*Log[c^(1/3) - ((b*c - a*d)^(1/3)*x)/(b + a*x^3)^(1/3
)] + Log[c^(2/3) + ((b*c - a*d)^(2/3)*x^2)/(b + a*x^3)^(2/3) + (c^(1/3)*(b*
c - a*d)^(1/3)*x)/(b + a*x^3)^(1/3)]))/(c^(5/3)*(b*c - a*d)^(1/3)) - (2*a*b
^2*c^(1/3)*(2*Sqrt[3]*ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c^(1/3)*(b + a*x
^3)^(1/3)))/Sqrt[3]] - 2*Log[c^(1/3) - ((b*c - a*d)^(1/3)*x)/(b + a*x^3)^(1
/3)] + Log[c^(2/3) + ((b*c - a*d)^(2/3)*x^2)/(b + a*x^3)^(2/3) + (c^(1/3)*(
b*c - a*d)^(1/3)*x)/(b + a*x^3)^(1/3)]))/(d^2*(b*c - a*d)^(1/3)) + (2*a^2*b
*(2*Sqrt[3]*ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c^(1/3)*(b + a*x^3)^(1/3))
)/Sqrt[3]] - 2*Log[c^(1/3) - ((b*c - a*d)^(1/3)*x)/(b + a*x^3)^(1/3)] + Log
[c^(2/3) + ((b*c - a*d)^(2/3)*x^2)/(b + a*x^3)^(2/3) + (c^(1/3)*(b*c - a*d)
^(1/3)*x)/(b + a*x^3)^(1/3)]))/(c^(2/3)*d*(b*c - a*d)^(1/3)))/18
```

IntegrateAlgebraic [**C**] time = 14.09, size = 680, normalized size = 1.94

[illegible]

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x^3)^(8/3)/(c + d*x^3)^2,x]
```

```
[Out] ((a + b*x^3)^(2/3)*(2*b^2*c^2*x - 2*a*b*c*d*x + a^2*d^2*x + b^2*c*d*x^4))/(
3*c*d^2*(c + d*x^3)) - (2*(3*b^(8/3)*c - 4*a*b^(5/3)*d)*ArcTan[(Sqrt[3]*b^(
1/3)*x)/(b^(1/3)*x + 2*(a + b*x^3)^(1/3))])/(3*Sqrt[3]*d^3) + ((b*c - a*d)^
(5/3)*(9*b*c - (3*I)*Sqrt[3]*b*c + 3*a*d - I*Sqrt[3]*a*d)*ArcTanh[(I*(b*c -
 a*d)^(1/3)*x + (-I + Sqrt[3])*c^(1/3)*(a + b*x^3)^(1/3))/(Sqrt[3]*(b*c - a
*d)^(1/3)*x)])/(9*c^(5/3)*d^3) + (2*(3*b^(8/3)*c - 4*a*b^(5/3)*d)*Log[-(b^(
1/3)*x) + (a + b*x^3)^(1/3)])/(9*d^3) + ((3*b*c*(b*c - a*d)^(5/3) + (3*I)*S
qrt[3]*b*c*(b*c - a*d)^(5/3) + a*d*(b*c - a*d)^(5/3) + I*Sqrt[3]*a*d*(b*c -
 a*d)^(5/3))*Log[2*(b*c - a*d)^(1/3)*x + (1 + I*Sqrt[3])*c^(1/3)*(a + b*x^3
)^(1/3)])/(9*c^(5/3)*d^3) + ((-3*b^(8/3)*c + 4*a*b^(5/3)*d)*Log[b^(2/3)*x^2
 + b^(1/3)*x*(a + b*x^3)^(1/3) + (a + b*x^3)^(2/3)])/(9*d^3) - ((I/18)*((-3
*I)*b*c*(b*c - a*d)^(5/3) + 3*Sqrt[3]*b*c*(b*c - a*d)^(5/3) - I*a*d*(b*c -
a*d)^(5/3) + Sqrt[3]*a*d*(b*c - a*d)^(5/3))*Log[(-2*I)*(b*c - a*d)^(2/3)*x^
2 + c^(1/3)*(b*c - a*d)^(1/3)*(I*x - Sqrt[3]*x)*(a + b*x^3)^(1/3) + (I + Sq
rt[3])*c^(2/3)*(a + b*x^3)^(2/3)])/(c^(5/3)*d^3)
```

fricas [**B**] time = 19.12, size = 819, normalized size = 2.33

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(8/3)/(d*x^3+c)^2,x, algorithm="fricas")
```

```
[Out] 1/9*(2*sqrt(3)*(3*b^2*c^3 - 2*a*b*c^2*d - a^2*c*d^2 + (3*b^2*c^2*d - 2*a*b*
c*d^2 - a^2*d^3)*x^3)*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)/c^2)^(1/3)*arctan(-1
/3*(sqrt(3)*(b*c - a*d)*x + 2*sqrt(3)*(b*x^3 + a)^(1/3)*c*((b^2*c^2 - 2*a*b
*c*d + a^2*d^2)/c^2)^(1/3))/((b*c - a*d)*x)) + 2*sqrt(3)*(3*b^2*c^3 - 4*a*b
*c^2*d + (3*b^2*c^2*d - 4*a*b*c*d^2)*x^3)*(-b^2)^(1/3)*arctan(-1/3*(sqrt(3)
*b*x - 2*sqrt(3)*(b*x^3 + a)^(1/3)*(-b^2)^(1/3))/(b*x)) - 2*(3*b^2*c^3 - 2*
a*b*c^2*d - a^2*c*d^2 + (3*b^2*c^2*d - 2*a*b*c*d^2 - a^2*d^3)*x^3)*((b^2*c^
2 - 2*a*b*c*d + a^2*d^2)/c^2)^(1/3)*log((c*x*((b^2*c^2 - 2*a*b*c*d + a^2*d^
2)/c^2)^(2/3) - (b*x^3 + a)^(1/3)*(b*c - a*d))/x) - 2*(3*b^2*c^3 - 4*a*b*c^
```

```
2*d + (3*b^2*c^2*d - 4*a*b*c*d^2)*x^3)*(-b^2)^(1/3)*log(-((-b^2)^(2/3)*x -
(b*x^3 + a)^(1/3)*b)/x) + (3*b^2*c^3 - 4*a*b*c^2*d + (3*b^2*c^2*d - 4*a*b*c
*d^2)*x^3)*(-b^2)^(1/3)*log(-((-b^2)^(1/3)*b*x^2 - (b*x^3 + a)^(1/3)*(-b^2)
^(2/3)*x - (b*x^3 + a)^(2/3)*b)/x^2) + (3*b^2*c^3 - 2*a*b*c^2*d - a^2*c*d^2
 + (3*b^2*c^2*d - 2*a*b*c*d^2 - a^2*d^3)*x^3)*((b^2*c^2 - 2*a*b*c*d + a^2*d
^2)/c^2)^(1/3)*log(-((b*c - a*d)*x^2*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)/c^2)^
(1/3) + (b*x^3 + a)^(1/3)*c*x*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)/c^2)^(2/3) +
 (b*x^3 + a)^(2/3)*(b*c - a*d))/x^2) + 3*(b^2*c*d^2*x^4 + (2*b^2*c^2*d - 2*
a*b*c*d^2 + a^2*d^3)*x)*(b*x^3 + a)^(2/3))/(c*d^4*x^3 + c^2*d^3)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^3+a\right)^{\frac{8}{3}}}{\left(dx^3+c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(8/3)/(d*x^3+c)^2,x, algorithm="giac")
[Out] integrate((b*x^3 + a)^(8/3)/(d*x^3 + c)^2, x)
```

maple [F] time = 0.40, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^3+a\right)^{\frac{8}{3}}}{\left(d\,x^3+c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(8/3)/(d*x^3+c)^2,x)
[Out] int((b*x^3+a)^(8/3)/(d*x^3+c)^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^3+a\right)^{\frac{8}{3}}}{\left(dx^3+c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(8/3)/(d*x^3+c)^2,x, algorithm="maxima")
[Out] integrate((b*x^3 + a)^(8/3)/(d*x^3 + c)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(b\,x^3+a\right)}^{8/3}}{{\left(d\,x^3+c\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^(8/3)/(c + d*x^3)^2,x)
[Out] int((a + b*x^3)^(8/3)/(c + d*x^3)^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(8/3)/(d*x**3+c)**2,x)
```

```
[Out] Timed out
```

3.69 $\int \frac{\left(a+bx^3\right)^{5/3}}{\left(c+dx^3\right)^2} \, dx$

Optimal. Leaf size=301

$$-\frac{b^{5/3}\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}\,x\right)}{2d^2}+\frac{b^{5/3}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\,d^2}-\frac{(bc-ad)^{2/3}(2ad+3bc)\log\left(c+dx^3\right)}{18c^{5/3}d^2}+\frac{(bc-ad)^{2/3}(2ad\ \cdots}{}$$

Rubi [C] time = 0.03, antiderivative size = 60, normalized size of antiderivative = 0.20, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {430, 429}

$$\frac{ax\left(a+bx^3\right)^{2/3}F_1\left(\frac{1}{3};-\frac{5}{3},2;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c^2\left(\frac{bx^3}{a}+1\right)^{2/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(a + b*x^3)^(5/3)/(c + d*x^3)^2,x]
```

```
[Out] (a*x*(a + b*x^3)^(2/3)*AppellF1[1/3, -5/3, 2, 4/3, -((b*x^3)/a), -((d*x^3)/
c)])/(c^2*(1 + (b*x^3)/a)^(2/3))
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^3\right)^{5/3}}{\left(c+dx^3\right)^2} \, dx &= \frac{\left(a\left(a+bx^3\right)^{2/3}\right)\int \frac{\left(1+\frac{bx^3}{a}\right)^{5/3}}{\left(c+dx^3\right)^2} \, dx}{\left(1+\frac{bx^3}{a}\right)^{2/3}}\\ &= \frac{ax\left(a+bx^3\right)^{2/3}F_1\left(\frac{1}{3};-\frac{5}{3},2;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c^2\left(1+\frac{bx^3}{a}\right)^{2/3}}\end{aligned}$$

Mathematica [C] time = 0.64, size = 450, normalized size = 1.50

$$\frac{\frac{4a^2\left(\log\left(\frac{\sqrt[3]{c}\,x\sqrt[3]{bc-ad}}{\sqrt[3]{ax^3+b}}+\frac{x^2(bc-ad)^{2/3}}{\left(ax^3+b\right)^{2/3}}+c^{2/3}\right)-2\log\left(\sqrt[3]{c}-\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{ax^3+b}}\right)+2\sqrt{3}\tan^{-1}\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\sqrt[3]{ax^3+b}}+1}{\sqrt{3}}\right)\right)}{\sqrt[3]{bc-ad}}+\frac{9b^2c^{2/3}x^4\sqrt[3]{\frac{bx^3}{a}+1}\,F_1\left(\frac{4}{3};\frac{1}{3},1;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{d\sqrt[3]{a+bx^3}}-\frac{12c^{2/3}x\left(a+bx^3\right)^{2/3}(bc-ad)}{d\left(c+dx^3\right)}+\frac{2abc\left(\log\left(\frac{\sqrt[3]{c}\,x\sqrt[3]{bc-ad}}{\sqrt[3]{ax^3+b}}+\frac{x^2(bc-ad)^{2/3}}{\left(ax^3+b\right)^{2/3}}+c^{2/3}\right)-2\log\left(\sqrt[3]{c}-\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{ax^3+b}}\right)+2\sqrt{3}\tan^{-1}\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\sqrt[3]{ax^3+b}}+1}{\sqrt{3}}\right)\right)}{d\sqrt[3]{bc-ad}}}{36c^{5/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^3)^(5/3)/(c + d*x^3)^2,x]
```

```
[Out] ((-12*c^(2/3)*(b*c - a*d)*x*(a + b*x^3)^(2/3))/(d*(c + d*x^3)) + (9*b^2*c^(
2/3)*x^4*(1 + (b*x^3)/a)^(1/3)*AppellF1[4/3, 1/3, 1, 7/3, -((b*x^3)/a), -((
d*x^3)/c)])/(d*(a + b*x^3)^(1/3)) + (4*a^2*(2*Sqrt[3]*ArcTan[(1 + (2*(b*c -
 a*d)^(1/3)*x)/(c^(1/3)*(b + a*x^3)^(1/3)))/Sqrt[3]] - 2*Log[c^(1/3) - ((b*
c - a*d)^(1/3)*x)/(b + a*x^3)^(1/3)] + Log[c^(2/3) + ((b*c - a*d)^(2/3)*x^2
)/(b + a*x^3)^(2/3) + (c^(1/3)*(b*c - a*d)^(1/3)*x)/(b + a*x^3)^(1/3)]))/(b
*c - a*d)^(1/3) + (2*a*b*c*(2*Sqrt[3]*ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(
c^(1/3)*(b + a*x^3)^(1/3)))/Sqrt[3]] - 2*Log[c^(1/3) - ((b*c - a*d)^(1/3)*x
)/(b + a*x^3)^(1/3)] + Log[c^(2/3) + ((b*c - a*d)^(2/3)*x^2)/(b + a*x^3)^(2
/3) + (c^(1/3)*(b*c - a*d)^(1/3)*x)/(b + a*x^3)^(1/3)]))/(d*(b*c - a*d)^(1/
3)))/(36*c^(5/3))
```

IntegrateAlgebraic [C] time = 7.33, size = 645, normalized size = 2.14

$$\frac{\left(2i\sqrt{3}\,a^2d^2+2a^2d^2+i\sqrt{3}\,abcd+abcd-3i\sqrt{3}\,b^2c^2-3b^2c^2\right)\log\left(2x\sqrt[3]{bc-ad}+\left(1+i\sqrt{3}\right)\sqrt[3]{c}\sqrt[3]{a+bx^3}\right)}{18c^{5/3}d^2\sqrt[3]{bc-ad}}+\frac{\left(-2i\sqrt{3}\,a^2d^2+6a^2d^2-i\sqrt{3}\,abcd+3abcd+3i\sqrt{3}\,b^2c^2-9b^2c^2\right)\tanh^{-1}\left(\frac{ix\sqrt[3]{bc-ad}+\left(\sqrt{3}-i\right)\sqrt[3]{c}\sqrt[3]{a+bx^3}}{\sqrt{3}\,x\sqrt[3]{bc-ad}}\right)}{18c^{5/3}d^2\sqrt[3]{bc-ad}}+\frac{\left(-2i\sqrt{3}\,a^2d^2-2a^2d^2-i\sqrt{3}\,abcd-abcd+3i\sqrt{3}\,b^2c^2+3b^2c^2\right)\log\left(\left(\sqrt{3}+i\right)c^{2/3}\left(a+bx^3\right)^{2/3}+\sqrt[3]{c}\left(-\sqrt{3}\,x+ix\right)\sqrt[3]{a+bx^3}\sqrt[3]{bc-ad}-2ix^2(bc-ad)^{2/3}\right)}{36c^{5/3}d^2\sqrt[3]{bc-ad}}-\frac{b^{5/3}\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}\,x\right)}{3d^2}+\frac{b^{5/3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{b}\,x}{2\sqrt[3]{a+bx^3}+\sqrt[3]{b}\,x}\right)}{\sqrt{3}\,d^2}+\frac{b^{5/3}\log\left(\sqrt[3]{b}\,x\sqrt[3]{a+bx^3}+\left(a+bx^3\right)^{2/3}+b^{2/3}x^2\right)}{6d^2}+\frac{x\left(a+bx^3\right)^{2/3}(ad-bc)}{3cd\left(c+dx^3\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x^3)^(5/3)/(c + d*x^3)^2,x]
```

```
[Out] ((-(b*c) + a*d)*x*(a + b*x^3)^(2/3))/(3*c*d*(c + d*x^3)) + (b^(5/3)*ArcTan[
(Sqrt[3]*b^(1/3)*x)/(b^(1/3)*x + 2*(a + b*x^3)^(1/3))])/(Sqrt[3]*d^2) + ((-
9*b^2*c^2 + (3*I)*Sqrt[3]*b^2*c^2 + 3*a*b*c*d - I*Sqrt[3]*a*b*c*d + 6*a^2*d
^2 - (2*I)*Sqrt[3]*a^2*d^2)*ArcTanh[(I*(b*c - a*d)^(1/3)*x + (-I + Sqrt[3])
*c^(1/3)*(a + b*x^3)^(1/3))/(Sqrt[3]*(b*c - a*d)^(1/3)*x)])/(18*c^(5/3)*d^2
*(b*c - a*d)^(1/3)) - (b^(5/3)*Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)])/(3*d^
2) + ((-3*b^2*c^2 - (3*I)*Sqrt[3]*b^2*c^2 + a*b*c*d + I*Sqrt[3]*a*b*c*d + 2
*a^2*d^2 + (2*I)*Sqrt[3]*a^2*d^2)*Log[2*(b*c - a*d)^(1/3)*x + (1 + I*Sqrt[3
])*c^(1/3)*(a + b*x^3)^(1/3)])/(18*c^(5/3)*d^2*(b*c - a*d)^(1/3)) + (b^(5/3
)*Log[b^(2/3)*x^2 + b^(1/3)*x*(a + b*x^3)^(1/3) + (a + b*x^3)^(2/3)])/(6*d^
2) + ((3*b^2*c^2 + (3*I)*Sqrt[3]*b^2*c^2 - a*b*c*d - I*Sqrt[3]*a*b*c*d - 2*
a^2*d^2 - (2*I)*Sqrt[3]*a^2*d^2)*Log[(-2*I)*(b*c - a*d)^(2/3)*x^2 + c^(1/3)
*(b*c - a*d)^(1/3)*(I*x - Sqrt[3]*x)*(a + b*x^3)^(1/3) + (I + Sqrt[3])*c^(2
/3)*(a + b*x^3)^(2/3)])/(36*c^(5/3)*d^2*(b*c - a*d)^(1/3))
```

fricas [B] time = 2.32, size = 631, normalized size = 2.10

$$-\frac{2\sqrt{3}\left(\left(3bcd+2ad^2\right)x^3+3bc^2+2acd\right)\left(\frac{b^2c^2-2abcd+a^2d^2}{c^2}\right)^{\frac{1}{3}}\arctan\left(-\frac{\sqrt{3}(bc-ad)x+2\sqrt{3}\left(bx^3+a\right)^{\frac{1}{3}}c\left(\frac{b^2c^2-2abcd+a^2d^2}{c^2}\right)^{\frac{1}{3}}}{3(bc-ad)x}\right)+6\sqrt{3}\left(bcdx^3+bc^2\right)\left(-b^2\right)^{\frac{1}{3}}\arctan\left(-\frac{\sqrt{3}bx-2\sqrt{3}\left(bx^3+a\right)^{\frac{1}{3}}\left(-b^2\right)^{\frac{1}{3}}}{3bx}\right)+6\left(bx^3+a\right)^{\frac{2}{3}}\left(bcd-ad^2\right)x-2\left(\left(3bcd+2ad^2\right)x^3+3bc^2+2acd\right)\left(\frac{b^2c^2-2abcd+a^2d^2}{c^2}\right)^{\frac{1}{3}}\log\left(\frac{cx\left(\frac{b^2c^2-2abcd+a^2d^2}{c^2}\right)^{\frac{2}{3}}-\left(bx^3+a\right)^{\frac{1}{3}}(bc-ad)}{x}\right)-6\left(bcdx^3+bc^2\right)\left(-b^2\right)^{\frac{1}{3}}\log\left(-\frac{\left(-b^2\right)^{\frac{2}{3}}x-\left(bx^3+a\right)^{\frac{1}{3}}b}{x}\right)+3\left(bcdx^3+bc^2\right)\left(-b^2\right)^{\frac{1}{3}}\log\left(-\frac{\left(-b^2\right)^{\frac{1}{3}}bx^2-\left(bx^3+a\right)^{\frac{1}{3}}\left(-b^2\right)^{\frac{2}{3}}x-\left(bx^3+a\right)^{\frac{2}{3}}b}{x^2}\right)+\left(\left(3bcd+2ad^2\right)x^3+3bc^2+2acd\right)\left(\frac{b^2c^2-2abcd+a^2d^2}{c^2}\right)^{\frac{1}{3}}\log\left(-\frac{(bc-ad)x^2\left(\frac{b^2c^2-2abcd+a^2d^2}{c^2}\right)^{\frac{1}{3}}+\left(bx^3+a\right)^{\frac{1}{3}}cx\left(\frac{b^2c^2-2abcd+a^2d^2}{c^2}\right)^{\frac{2}{3}}+\left(bx^3+a\right)^{\frac{2}{3}}(bc-ad)}{x^2}\right)}{18\left(cd^3x^3+c^2d^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(5/3)/(d*x^3+c)^2,x, algorithm="fricas")
```

```
[Out] -1/18*(2*sqrt(3)*((3*b*c*d + 2*a*d^2)*x^3 + 3*b*c^2 + 2*a*c*d)*((b^2*c^2 -
2*a*b*c*d + a^2*d^2)/c^2)^(1/3)*arctan(-1/3*(sqrt(3)*(b*c - a*d)*x + 2*sqrt
(3)*(b*x^3 + a)^(1/3)*c*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)/c^2)^(1/3))/((b*c
- a*d)*x)) + 6*sqrt(3)*(b*c*d*x^3 + b*c^2)*(-b^2)^(1/3)*arctan(-1/3*(sqrt(3
)*b*x - 2*sqrt(3)*(b*x^3 + a)^(1/3)*(-b^2)^(1/3))/(b*x)) + 6*(b*x^3 + a)^(2
/3)*(b*c*d - a*d^2)*x - 2*((3*b*c*d + 2*a*d^2)*x^3 + 3*b*c^2 + 2*a*c*d)*((b
^2*c^2 - 2*a*b*c*d + a^2*d^2)/c^2)^(1/3)*log((c*x*((b^2*c^2 - 2*a*b*c*d + a
^2*d^2)/c^2)^(2/3) - (b*x^3 + a)^(1/3)*(b*c - a*d))/x) - 6*(b*c*d*x^3 + b*c
^2)*(-b^2)^(1/3)*log(-((-b^2)^(2/3)*x - (b*x^3 + a)^(1/3)*b)/x) + 3*(b*c*d*
x^3 + b*c^2)*(-b^2)^(1/3)*log(-((-b^2)^(1/3)*b*x^2 - (b*x^3 + a)^(1/3)*(-b^
2)^(2/3)*x - (b*x^3 + a)^(2/3)*b)/x^2) + ((3*b*c*d + 2*a*d^2)*x^3 + 3*b*c^2
 + 2*a*c*d)*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)/c^2)^(1/3)*log(-((b*c - a*d)*x
^2*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)/c^2)^(1/3) + (b*x^3 + a)^(1/3)*c*x*((b^
2*c^2 - 2*a*b*c*d + a^2*d^2)/c^2)^(2/3) + (b*x^3 + a)^(2/3)*(b*c - a*d))/x^
2))/(c*d^3*x^3 + c^2*d^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^3+a\right)^{\frac{5}{3}}}{\left(dx^3+c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(5/3)/(d*x^3+c)^2,x, algorithm="giac")
[Out] integrate((b*x^3 + a)^(5/3)/(d*x^3 + c)^2, x)
```

maple [F] time = 0.58, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^3+a\right)^{\frac{5}{3}}}{\left(d\,x^3+c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(5/3)/(d*x^3+c)^2,x)
[Out] int((b*x^3+a)^(5/3)/(d*x^3+c)^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^3+a\right)^{\frac{5}{3}}}{\left(dx^3+c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(5/3)/(d*x^3+c)^2,x, algorithm="maxima")
[Out] integrate((b*x^3 + a)^(5/3)/(d*x^3 + c)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(b\,x^3+a\right)^{5/3}}{\left(d\,x^3+c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^(5/3)/(c + d*x^3)^2,x)
[Out] int((a + b*x^3)^(5/3)/(c + d*x^3)^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(5/3)/(d*x**3+c)**2,x)
[Out] Timed out
```

3.70 $\int \frac{\left(a+bx^3\right)^{2/3}}{\left(c+dx^3\right)^2} \, dx$

Optimal. Leaf size=182

$$\frac{a\log\left(c+dx^3\right)}{9c^{5/3}\sqrt[3]{bc-ad}}-\frac{a\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{3c^{5/3}\sqrt[3]{bc-ad}}+\frac{2a\tan^{-1}\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{3\sqrt{3}\,c^{5/3}\sqrt[3]{bc-ad}}+\frac{x\left(a+bx^3\right)^{2/3}}{3c\left(c+dx^3\right)}$$

Rubi [A] time = 0.21, antiderivative size = 241, normalized size of antiderivative = 1.32, number of steps used = 8, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {378, 377, 200, 31, 634, 617, 204, 628}

$$-\frac{2a\log\left(\sqrt[3]{c}-\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{a+bx^3}}\right)}{9c^{5/3}\sqrt[3]{bc-ad}}+\frac{a\log\left(\frac{x^2(bc-ad)^{2/3}}{\left(a+bx^3\right)^{2/3}}+\frac{\sqrt[3]{c}\,x\sqrt[3]{bc-ad}}{\sqrt[3]{a+bx^3}}+c^{2/3}\right)}{9c^{5/3}\sqrt[3]{bc-ad}}+\frac{2a\tan^{-1}\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{a+bx^3}}+\sqrt[3]{c}}{\sqrt{3}\sqrt[3]{c}}\right)}{3\sqrt{3}\,c^{5/3}\sqrt[3]{bc-ad}}+\frac{x\left(a+bx^3\right)^{2/3}}{3c\left(c+dx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^(2/3)/(c + d*x^3)^2,x]
```

```
[Out] (x*(a + b*x^3)^(2/3))/(3*c*(c + d*x^3)) + (2*a*ArcTan[(c^(1/3) + (2*(b*c -
a*d)^(1/3)*x)/(a + b*x^3)^(1/3))/(Sqrt[3]*c^(1/3))])/(3*Sqrt[3]*c^(5/3)*(b*
c - a*d)^(1/3)) - (2*a*Log[c^(1/3) - ((b*c - a*d)^(1/3)*x)/(a + b*x^3)^(1/3
)])/(9*c^(5/3)*(b*c - a*d)^(1/3)) + (a*Log[c^(2/3) + ((b*c - a*d)^(2/3)*x^2
)/(a + b*x^3)^(2/3) + (c^(1/3)*(b*c - a*d)^(1/3)*x)/(a + b*x^3)^(1/3)])/(9*
c^(5/3)*(b*c - a*d)^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 378

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> -Simp[(x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*n*(p + 1)), x] - Dist[(c
```

```
*q)/(a*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1, 0]
 && GtQ[q, 0] && NeQ[p, -1]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{\left(a+bx^3\right)^{2/3}}{\left(c+dx^3\right)^2}\,dx = \frac{x\left(a+bx^3\right)^{2/3}}{3c\left(c+dx^3\right)} + \frac{(2a)\int \frac{1}{\sqrt[3]{a+bx^3}\left(c+dx^3\right)}\,dx}{3c}$$

$$= \frac{x\left(a+bx^3\right)^{2/3}}{3c\left(c+dx^3\right)} + \frac{(2a)\,\text{Subst}\left(\int \frac{1}{c-(bc-ad)x^3}\,dx, x, \frac{x}{\sqrt[3]{a+bx^3}}\right)}{3c}$$

$$= \frac{x\left(a+bx^3\right)^{2/3}}{3c\left(c+dx^3\right)} + \frac{(2a)\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{c}-\sqrt[3]{bc-ad}\,x}\,dx, x, \frac{x}{\sqrt[3]{a+bx^3}}\right)}{9c^{5/3}} + \frac{(2a)\,\text{Subst}\left(\int \frac{2\sqrt[3]{c}+\sqrt[3]{b\cdots}}{c^{2/3}+\sqrt[3]{c}\,\sqrt[3]{bc-ad}\,x\cdots}\right.}{9c^{5/\cdots}}$$

$$= \frac{x\left(a+bx^3\right)^{2/3}}{3c\left(c+dx^3\right)} - \frac{2a\log\left(\sqrt[3]{c}-\frac{\sqrt[3]{bc-ad}\,x}{\sqrt[3]{a+bx^3}}\right)}{9c^{5/3}\sqrt[3]{bc-ad}} + \frac{a\,\text{Subst}\left(\int \frac{1}{c^{2/3}+\sqrt[3]{c}\,\sqrt[3]{bc-ad}\,x+(bc-ad)^{2/3}x^2}\,dx, x, \frac{\cdots}{\sqrt[3]{a\cdots}}\right.}{3c^{4/3}}$$

$$= \frac{x\left(a+bx^3\right)^{2/3}}{3c\left(c+dx^3\right)} - \frac{2a\log\left(\sqrt[3]{c}-\frac{\sqrt[3]{bc-ad}\,x}{\sqrt[3]{a+bx^3}}\right)}{9c^{5/3}\sqrt[3]{bc-ad}} + \frac{a\log\left(c^{2/3}+\frac{(bc-ad)^{2/3}x^2}{\left(a+bx^3\right)^{2/3}}+\frac{\sqrt[3]{c}\,\sqrt[3]{bc-ad}\,x}{\sqrt[3]{a+bx^3}}\right)}{9c^{5/3}\sqrt[3]{bc-ad}} - \frac{(2a)\,\text{S}\cdots}{\cdots}$$

$$= \frac{x\left(a+bx^3\right)^{2/3}}{3c\left(c+dx^3\right)} + \frac{2a\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{bc-ad}\,x}{\sqrt[3]{c}\,\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{3\sqrt{3}\,c^{5/3}\sqrt[3]{bc-ad}} - \frac{2a\log\left(\sqrt[3]{c}-\frac{\sqrt[3]{bc-ad}\,x}{\sqrt[3]{a+bx^3}}\right)}{9c^{5/3}\sqrt[3]{bc-ad}} + \frac{a\log\left(c^{2/3}+\frac{(bc-ad)^{2/3}\cdots}{\left(a+bx^3\right)^{2}\cdots}\right.}{9c^{5/3}\sqrt[3]{bc\cdots}}$$

Mathematica [C] time = 0.04, size = 78, normalized size = 0.43

$$\frac{x\left(a+bx^3\right)^{2/3}\,{}_2F_1\left(-\frac{2}{3},\frac{1}{3};\frac{4}{3};\frac{(ad-bc)x^3}{a\left(dx^3+c\right)}\right)}{c^2\left(\frac{bx^3}{a}+1\right)^{2/3}\sqrt[3]{\frac{dx^3}{c}+1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^3)^(2/3)/(c + d*x^3)^2,x]
```

```
[Out] (x*(a + b*x^3)^(2/3)*Hypergeometric2F1[-2/3, 1/3, 4/3, ((-(b*c) + a*d)*x^3)
/(a*(c + d*x^3))])/(c^2*(1 + (b*x^3)/a)^(2/3)*(1 + (d*x^3)/c)^(1/3))
```

IntegrateAlgebraic [C] time = 2.40, size = 356, normalized size = 1.96

$$\frac{\left(a+i\sqrt{3}\,a\right)\log\left(2x\sqrt[3]{bc-ad}+\left(1+i\sqrt{3}\right)\sqrt[3]{c}\,\sqrt[3]{a+bx^3}\right)}{9c^{5/3}\sqrt[3]{bc-ad}}-\frac{\sqrt{-2+2i\sqrt{3}}\,a\tan^{-1}\left(\frac{3x\sqrt[3]{bc-ad}}{\sqrt{3}\,x\sqrt[3]{bc-ad}-\sqrt{3}\,\sqrt[3]{c}\,\sqrt[3]{a+bx^3}-3i\sqrt[3]{c}\,\sqrt[3]{a+bx^3}}\right)}{3\sqrt{3}\,c^{5/3}\sqrt[3]{bc-ad}}-\frac{i\left(\sqrt{3}\,a-ia\right)\log\left(\left(\sqrt{3}+i\right)c^{2/3}\left(a+bx^3\right)^{2/3}+\sqrt[3]{c}\left(-\sqrt{3}\,x+ix\right)\sqrt[3]{a+bx^3}\,\sqrt[3]{bc-ad}-2ix^2(bc-ad)^{2/3}\right)}{18c^{5/3}\sqrt[3]{bc-ad}}+\frac{x\left(a+bx^3\right)^{2/3}}{3c\left(c+dx^3\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x^3)^(2/3)/(c + d*x^3)^2,x]
```

```
[Out] (x*(a + b*x^3)^(2/3))/(3*c*(c + d*x^3)) - (Sqrt[-2 + (2*I)*Sqrt[3]]*a*ArcTa
n[(3*(b*c - a*d)^(1/3)*x)/(Sqrt[3]*(b*c - a*d)^(1/3)*x - (3*I)*c^(1/3)*(a +
 b*x^3)^(1/3) - Sqrt[3]*c^(1/3)*(a + b*x^3)^(1/3))])/(3*Sqrt[3]*c^(5/3)*(b*
c - a*d)^(1/3)) + ((a + I*Sqrt[3]*a)*Log[2*(b*c - a*d)^(1/3)*x + (1 + I*Sqr
t[3])*c^(1/3)*(a + b*x^3)^(1/3)])/(9*c^(5/3)*(b*c - a*d)^(1/3)) - ((I/18)*(
(-I)*a + Sqrt[3]*a)*Log[(-2*I)*(b*c - a*d)^(2/3)*x^2 + c^(1/3)*(b*c - a*d)^
(1/3)*(I*x - Sqrt[3]*x)*(a + b*x^3)^(1/3) + (I + Sqrt[3])*c^(2/3)*(a + b*x^
3)^(2/3)])/(c^(5/3)*(b*c - a*d)^(1/3))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)/(d*x^3+c)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^3+a\right)^{\frac{2}{3}}}{\left(dx^3+c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)/(d*x^3+c)^2,x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a)^(2/3)/(d*x^3 + c)^2, x)
```

maple [F] time = 0.54, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^3+a\right)^{\frac{2}{3}}}{\left(d\,x^3+c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(2/3)/(d*x^3+c)^2,x)
```

```
[Out] int((b*x^3+a)^(2/3)/(d*x^3+c)^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^3+a\right)^{\frac{2}{3}}}{\left(dx^3+c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)/(d*x^3+c)^2,x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a)^(2/3)/(d*x^3 + c)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(b\,x^3+a\right)^{2/3}}{\left(d\,x^3+c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^(2/3)/(c + d*x^3)^2,x)
```

```
[Out] int((a + b*x^3)^(2/3)/(c + d*x^3)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+bx^3\right)^{\frac{2}{3}}}{\left(c+dx^3\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(2/3)/(d*x**3+c)**2,x)
```

```
[Out] Integral((a + b*x**3)**(2/3)/(c + d*x**3)**2, x)
```

3.71 $\int \frac{1}{\sqrt[3]{a+bx^3}\left(c+dx^3\right)^2} \, dx$

Optimal. Leaf size=217

$$\frac{(3bc-2ad)\log\left(c+dx^3\right)}{18c^{5/3}(bc-ad)^{4/3}}-\frac{(3bc-2ad)\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{6c^{5/3}(bc-ad)^{4/3}}+\frac{(3bc-2ad)\tan^{-1}\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{3\sqrt{3}c^{5/3}(bc-ad)^{4/3}}-\frac{dx\left(a+b\right.}{3c\left(c+dx^3\right)}$$

Rubi [A] time = 0.20, antiderivative size = 276, normalized size of antiderivative = 1.27, number of steps used = 8, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {382, 377, 200, 31, 634, 617, 204, 628}

$$-\frac{(3bc-2ad)\log\left(\sqrt[3]{c}-\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{a+bx^3}}\right)}{9c^{5/3}(bc-ad)^{4/3}}+\frac{(3bc-2ad)\log\left(\frac{x^2(bc-ad)^{2/3}}{\left(a+bx^3\right)^{2/3}}+\frac{\sqrt[3]{c}x\sqrt[3]{bc-ad}}{\sqrt[3]{a+bx^3}}+c^{2/3}\right)}{18c^{5/3}(bc-ad)^{4/3}}+\frac{(3bc-2ad)\tan^{-1}\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{a+bx^3}}+\sqrt[3]{c}}{\sqrt{3}\sqrt[3]{c}}\right)}{3\sqrt{3}c^{5/3}(bc-ad)^{4/3}}-\frac{dx\left(a+bx^3\right)^{2/3}}{3c\left(c+dx^3\right)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^3)^(1/3)*(c + d*x^3)^2),x]
```

```
[Out] -(d*x*(a + b*x^3)^(2/3))/(3*c*(b*c - a*d)*(c + d*x^3)) + ((3*b*c - 2*a*d)*A
rcTan[(c^(1/3) + (2*(b*c - a*d)^(1/3)*x)/(a + b*x^3)^(1/3))/(Sqrt[3]*c^(1/3
))])/(3*Sqrt[3]*c^(5/3)*(b*c - a*d)^(4/3)) - ((3*b*c - 2*a*d)*Log[c^(1/3) -
 ((b*c - a*d)^(1/3)*x)/(a + b*x^3)^(1/3)])/(9*c^(5/3)*(b*c - a*d)^(4/3)) +
((3*b*c - 2*a*d)*Log[c^(2/3) + ((b*c - a*d)^(2/3)*x^2)/(a + b*x^3)^(2/3) +
(c^(1/3)*(b*c - a*d)^(1/3)*x)/(a + b*x^3)^(1/3)])/(18*c^(5/3)*(b*c - a*d)^(
4/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)), I
```

```
nt[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q}, x
] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !LtQ
[q, -1]) && NeQ[p, -1]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{1}{\sqrt[3]{a+bx^3}\left(c+dx^3\right)^2}\,dx = -\frac{dx\left(a+bx^3\right)^{2/3}}{3c(bc-ad)\left(c+dx^3\right)}+\frac{(3bc-2ad)\int \frac{1}{\sqrt[3]{a+bx^3}\left(c+dx^3\right)}\,dx}{3c(bc-ad)}$$

$$= -\frac{dx\left(a+bx^3\right)^{2/3}}{3c(bc-ad)\left(c+dx^3\right)}+\frac{(3bc-2ad)\,\mathrm{Subst}\left(\int \frac{1}{c-(bc-ad)x^3}\,dx, x, \frac{x}{\sqrt[3]{a+bx^3}}\right)}{3c(bc-ad)}$$

$$= -\frac{dx\left(a+bx^3\right)^{2/3}}{3c(bc-ad)\left(c+dx^3\right)}+\frac{(3bc-2ad)\,\mathrm{Subst}\left(\int \frac{1}{\sqrt[3]{c}-\sqrt[3]{bc-ad}\,x}\,dx, x, \frac{x}{\sqrt[3]{a+bx^3}}\right)}{9c^{5/3}(bc-ad)}+\frac{(3b}{}$$

$$= -\frac{dx\left(a+bx^3\right)^{2/3}}{3c(bc-ad)\left(c+dx^3\right)}-\frac{(3bc-2ad)\log\left(\sqrt[3]{c}-\frac{\sqrt[3]{bc-ad}\,x}{\sqrt[3]{a+bx^3}}\right)}{9c^{5/3}(bc-ad)^{4/3}}+\frac{(3bc-2ad)\,\mathrm{Subst}\Big(}{}$$

$$= -\frac{dx\left(a+bx^3\right)^{2/3}}{3c(bc-ad)\left(c+dx^3\right)}-\frac{(3bc-2ad)\log\left(\sqrt[3]{c}-\frac{\sqrt[3]{bc-ad}\,x}{\sqrt[3]{a+bx^3}}\right)}{9c^{5/3}(bc-ad)^{4/3}}+\frac{(3bc-2ad)\log\Big(c^{2/}}{18c^5}$$

$$= -\frac{dx\left(a+bx^3\right)^{2/3}}{3c(bc-ad)\left(c+dx^3\right)}+\frac{(3bc-2ad)\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{bc-ad}\,x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{3\sqrt{3}\,c^{5/3}(bc-ad)^{4/3}}-\frac{(3bc-2ad)\log\Big(\sqrt[3]{c}}{9c^{5/3}(bc-a}$$

Mathematica [C] time = 0.14, size = 99, normalized size = 0.46

$$\frac{x\left(\left(c+dx^3\right)(3bc-2ad)\,{}_2F_1\left(\frac{1}{3},1;\frac{4}{3};\frac{(bc-ad)x^3}{c\left(bx^3+a\right)}\right)-cd\left(a+bx^3\right)\right)}{3c^2\sqrt[3]{a+bx^3}\left(c+dx^3\right)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x^3)^(1/3)*(c + d*x^3)^2),x]
```

```
[Out] (x*(-(c*d*(a + b*x^3)) + (3*b*c - 2*a*d)*(c + d*x^3)*Hypergeometric2F1[1/3,
 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))]))/(3*c^2*(b*c - a*d)*(a + b*x^3
)^(1/3)*(c + d*x^3))
```

IntegrateAlgebraic [C] time = 2.67, size = 397, normalized size = 1.83

$$\frac{\left(-2i\sqrt{3}ad-2ad+3i\sqrt{3}bc+3bc\right)\log\left(2x\sqrt[3]{bc-ad}+\left(1+i\sqrt{3}\right)\sqrt[3]{c}\sqrt[3]{a+bx^3}\right)}{18c^{5/3}(bc-ad)^{4/3}}+\frac{\left(2i\sqrt{3}ad-6ad-3i\sqrt{3}bc+9bc\right)\tanh^{-1}\left(\frac{ix\sqrt[3]{bc-ad}+\left(\sqrt{3}-i\right)\sqrt[3]{c}\sqrt[3]{a+bx^3}}{\sqrt{3}x\sqrt[3]{bc-ad}}\right)}{18c^{5/3}(bc-ad)^{4/3}}+\frac{\left(2i\sqrt{3}ad+2ad-3i\sqrt{3}bc-3bc\right)\log\left(\left(\sqrt{3}+i\right)c^{2/3}\left(a+bx^3\right)^{2/3}+\sqrt[3]{c}\left(-\sqrt{3}x+ix\right)\sqrt[3]{a+bx^3}\sqrt[3]{bc-ad}-2ix^2(bc-ad)^{2/3}\right)}{36c^{5/3}(bc-ad)^{4/3}}-\frac{dx\left(a+bx^3\right)^{2/3}}{3c\left(c+dx^3\right)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((a + b*x^3)^(1/3)*(c + d*x^3)^2),x]
```

```
[Out] -1/3*(d*x*(a + b*x^3)^(2/3))/(c*(b*c - a*d)*(c + d*x^3)) + ((9*b*c - (3*I)*
Sqrt[3]*b*c - 6*a*d + (2*I)*Sqrt[3]*a*d)*ArcTanh[(I*(b*c - a*d)^(1/3)*x + (
-I + Sqrt[3])*c^(1/3)*(a + b*x^3)^(1/3))/(Sqrt[3]*(b*c - a*d)^(1/3)*x)])/(1
8*c^(5/3)*(b*c - a*d)^(4/3)) + ((3*b*c + (3*I)*Sqrt[3]*b*c - 2*a*d - (2*I)*
Sqrt[3]*a*d)*Log[2*(b*c - a*d)^(1/3)*x + (1 + I*Sqrt[3])*c^(1/3)*(a + b*x^3
)^(1/3)])/(18*c^(5/3)*(b*c - a*d)^(4/3)) + ((-3*b*c - (3*I)*Sqrt[3]*b*c + 2
*a*d + (2*I)*Sqrt[3]*a*d)*Log[(-2*I)*(b*c - a*d)^(2/3)*x^2 + c^(1/3)*(b*c -
 a*d)^(1/3)*(I*x - Sqrt[3]*x)*(a + b*x^3)^(1/3) + (I + Sqrt[3])*c^(2/3)*(a
+ b*x^3)^(2/3)])/(36*c^(5/3)*(b*c - a*d)^(4/3))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^(1/3)/(d*x^3+c)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^3+a\right)^{\frac{1}{3}}\left(dx^3+c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^(1/3)/(d*x^3+c)^2,x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^3 + a)^(1/3)*(d*x^3 + c)^2), x)
```

maple [F] time = 0.56, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,x^3+a\right)^{\frac{1}{3}}\left(d\,x^3+c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^3+a)^(1/3)/(d*x^3+c)^2,x)
```

```
[Out] int(1/(b*x^3+a)^(1/3)/(d*x^3+c)^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^3+a\right)^{\frac{1}{3}}\left(dx^3+c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^(1/3)/(d*x^3+c)^2,x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^3 + a)^(1/3)*(d*x^3 + c)^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(b\,x^3+a\right)^{1/3}\,\left(d\,x^3+c\right)^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^3)^(1/3)*(c + d*x^3)^2),x)
```

```
[Out] int(1/((a + b*x^3)^(1/3)*(c + d*x^3)^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[3]{a + bx^{3}} \left(c + dx^{3}\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**3+a)**(1/3)/(d*x**3+c)**2,x)
```

```
[Out] Integral(1/((a + b*x**3)**(1/3)*(c + d*x**3)**2), x)
```

3.72 $\int \frac{1}{\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)^2}\,dx$

Optimal. Leaf size=261

$$-\frac{d(3bc-ad)\log\left(c+dx^3\right)}{9c^{5/3}(bc-ad)^{7/3}}+\frac{d(3bc-ad)\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{3c^{5/3}(bc-ad)^{7/3}}-\frac{2d(3bc-ad)\tan^{-1}\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{3\sqrt{3}\,c^{5/3}(bc-ad)^{7/3}}+\frac{bx(a}{3ac\sqrt[3]{a+}$$

Rubi [C] time = 1.93, antiderivative size = 625, normalized size of antiderivative = 2.39, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {430, 429}

$$-\frac{c\left(a+bx^3\right)^{2/3}\left(-\frac{54d^2x^{15}(bc-ad)^3\,{}_3F_2\left(2,2,\frac{7}{3};1,\frac{13}{3};\frac{(bc-ad)x^3}{c\left(bx^3+a\right)}\right)}{c^5\left(a+bx^3\right)^3}-\frac{108dx^{12}(bc-ad)^3\,{}_3F_2\left(2,2,\frac{7}{3};1,\frac{13}{3};\frac{(bc-ad)x^3}{c\left(bx^3+a\right)}\right)}{c^4\left(a+bx^3\right)^3}-\frac{54x^9(bc-ad)^3\,{}_3F_2\left(2,2,\frac{7}{3};1,\frac{13}{3};\frac{(bc-ad)x^3}{c\left(bx^3+a\right)}\right)}{c^3\left(a+bx^3\right)^3}+\frac{2520d^2x^9(bc-ad)\,{}_2F_1\left(\frac{1}{3},1;\frac{4}{3};\frac{(bc-ad)x^3}{c\left(bx^3+a\right)}\right)}{c^3\left(a+bx^3\right)}-\frac{6300d^2x^6\,{}_2F_1\left(\frac{1}{3},1;\frac{4}{3};\frac{(bc-ad)x^3}{c\left(bx^3+a\right)}\right)}{c^2}-\frac{945d^2x^9(bc-ad)}{c^3\left(a+bx^3\right)}+\frac{5320dx^6(bc-ad)\,{}_2F_1\left(\frac{1}{3},1;\frac{4}{3};\frac{(bc-ad)x^3}{c\left(bx^3+a\right)}\right)}{c^2\left(a+bx^3\right)}-\frac{1890dx^6(bc-ad)}{c^2\left(a+bx^3\right)}-\frac{13720dx^3\,{}_2F_1\left(\frac{1}{3},1;\frac{4}{3};\frac{(bc-ad)x^3}{c\left(bx^3+a\right)}\right)}{c}+\frac{2240x^3(bc-ad)\,{}_2F_1\left(\frac{1}{3},1;\frac{4}{3};\frac{(bc-ad)x^3}{c\left(bx^3+a\right)}\right)}{c\left(a+bx^3\right)}-6860\,{}_2F_1\left(\frac{1}{3},1;\frac{4}{3};\frac{(bc-ad)x^3}{c\left(bx^3+a\right)}\right)-\frac{525x^3(bc-ad)}{c\left(a+bx^3\right)}+\frac{6300d^2x^6}{c^2}+\frac{13720dx^3}{c}+6860\right)}{420x^5\left(c+dx^3\right)(bc-ad)^2}$$

Warning: Unable to verify antiderivative.

```
[In] Int[1/((a + b*x^3)^(4/3)*(c + d*x^3)^2),x]
```

```
[Out] -(c*(a + b*x^3)^(2/3)*(6860 + (13720*d*x^3)/c + (6300*d^2*x^6)/c^2 - (525*(
b*c - a*d)*x^3)/(c*(a + b*x^3)) - (1890*d*(b*c - a*d)*x^6)/(c^2*(a + b*x^3)
) - (945*d^2*(b*c - a*d)*x^9)/(c^3*(a + b*x^3)) - 6860*Hypergeometric2F1[1/
3, 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - (13720*d*x^3*Hypergeometric
2F1[1/3, 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))])/c - (6300*d^2*x^6*Hype
rgeometric2F1[1/3, 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))])/c^2 + (2240*
(b*c - a*d)*x^3*Hypergeometric2F1[1/3, 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*
x^3))])/(c*(a + b*x^3)) + (5320*d*(b*c - a*d)*x^6*Hypergeometric2F1[1/3, 1,
 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))])/(c^2*(a + b*x^3)) + (2520*d^2*(b*
c - a*d)*x^9*Hypergeometric2F1[1/3, 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3
))])/(c^3*(a + b*x^3)) - (54*(b*c - a*d)^3*x^9*HypergeometricPFQ[{2, 2, 7/3
}, {1, 13/3}, ((b*c - a*d)*x^3)/(c*(a + b*x^3))])/(c^3*(a + b*x^3)^3) - (10
8*d*(b*c - a*d)^3*x^12*HypergeometricPFQ[{2, 2, 7/3}, {1, 13/3}, ((b*c - a*
d)*x^3)/(c*(a + b*x^3))])/(c^4*(a + b*x^3)^3) - (54*d^2*(b*c - a*d)^3*x^15*
HypergeometricPFQ[{2, 2, 7/3}, {1, 13/3}, ((b*c - a*d)*x^3)/(c*(a + b*x^3))
])/(c^5*(a + b*x^3)^3)))/(420*(b*c - a*d)^2*x^5*(c + d*x^3))
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{1}{\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)^2}\,dx = \frac{\sqrt[3]{1+\frac{bx^3}{a}}\int \frac{1}{\left(1+\frac{bx^3}{a}\right)^{4/3}\left(c+dx^3\right)^2}\,dx}{a\sqrt[3]{a+bx^3}}$$

$$= -\frac{c\left(a+bx^3\right)^{2/3}\left(6860+\frac{13720dx^3}{c}+\frac{6300d^2x^6}{c^2}-\frac{525(bc-ad)x^3}{c\left(a+bx^3\right)}-\frac{1890d(bc-ad)x^6}{c^2\left(a+bx^3\right)}-\frac{945d^2}{c^3(}\right.}{}$$

Mathematica [C] time = 2.03, size = 625, normalized size = 2.39

$$\frac{c\left(a+bx^3\right)^{2/3}\left(\frac{54d^2x^{15}(bc-ad)^3\,{}_3F_2\left(2,2,\frac{7}{3};1,\frac{13}{3};\frac{(bc-ad)x^3}{c(bx^3+a)}\right)}{c^5\left(a+bx^3\right)^3}+\frac{108dx^{12}(bc-ad)^3\,{}_3F_2\left(2,2,\frac{7}{3};1,\frac{13}{3};\frac{(bc-ad)x^3}{c(bx^3+a)}\right)}{c^4\left(a+bx^3\right)^3}+\frac{54x^9(bc-ad)^3\,{}_3F_2\left(2,2,\frac{7}{3};1,\frac{13}{3};\frac{(bc-ad)x^3}{c(bx^3+a)}\right)}{c^3\left(a+bx^3\right)^3}+\frac{2520d^2x^9(ad-bc)\,{}_2F_1\left(\frac{1}{3},1;\frac{4}{3};\frac{(bc-ad)x^3}{c(bx^3+a)}\right)}{c^3\left(a+bx^3\right)}+\frac{945d^2x^9(bc-ad)}{c^3\left(a+bx^3\right)}+\frac{6300d^2x^6\,{}_2F_1\left(\frac{1}{3},1;\frac{4}{3};\frac{(bc-ad)x^3}{c(bx^3+a)}\right)}{c^2}+\frac{5320dx^6(ad-bc)\,{}_2F_1\left(\frac{1}{3},1;\frac{4}{3};\frac{(bc-ad)x^3}{c(bx^3+a)}\right)}{c^2\left(a+bx^3\right)}+\frac{1890dx^6(bc-ad)}{c^2\left(a+bx^3\right)}+\frac{13720dx^3\,{}_2F_1\left(\frac{1}{3},1;\frac{4}{3};\frac{(bc-ad)x^3}{c(bx^3+a)}\right)}{c}-\frac{2240x^3(bc-ad)\,{}_2F_1\left(\frac{1}{3},1;\frac{4}{3};\frac{(bc-ad)x^3}{c(bx^3+a)}\right)}{c\left(a+bx^3\right)}+6860\,{}_2F_1\left(\frac{1}{3},1;\frac{4}{3};\frac{(bc-ad)x^3}{c(bx^3+a)}\right)+\frac{525x^3(bc-ad)}{c\left(a+bx^3\right)}-\frac{6300d^2x^6}{c^2}-\frac{13720dx^3}{c}-6860\right)}{420x^5\left(c+dx^3\right)(bc-ad)^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + b*x^3)^(4/3)*(c + d*x^3)^2),x]
```

```
[Out] (c*(a + b*x^3)^(2/3)*(-6860 - (13720*d*x^3)/c - (6300*d^2*x^6)/c^2 + (525*(
b*c - a*d)*x^3)/(c*(a + b*x^3)) + (1890*d*(b*c - a*d)*x^6)/(c^2*(a + b*x^3)
) + (945*d^2*(b*c - a*d)*x^9)/(c^3*(a + b*x^3)) + 6860*Hypergeometric2F1[1/
3, 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + (13720*d*x^3*Hypergeometric
2F1[1/3, 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))])/c + (6300*d^2*x^6*Hype
rgeometric2F1[1/3, 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))])/c^2 - (2240*
(b*c - a*d)*x^3*Hypergeometric2F1[1/3, 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*
x^3))])/(c*(a + b*x^3)) + (5320*d*(-(b*c) + a*d)*x^6*Hypergeometric2F1[1/3,
 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))])/(c^2*(a + b*x^3)) + (2520*d^2*
(-(b*c) + a*d)*x^9*Hypergeometric2F1[1/3, 1, 4/3, ((b*c - a*d)*x^3)/(c*(a +
 b*x^3))])/(c^3*(a + b*x^3)) + (54*(b*c - a*d)^3*x^9*HypergeometricPFQ[{2,
2, 7/3}, {1, 13/3}, ((b*c - a*d)*x^3)/(c*(a + b*x^3))])/(c^3*(a + b*x^3)^3)
 + (108*d*(b*c - a*d)^3*x^12*HypergeometricPFQ[{2, 2, 7/3}, {1, 13/3}, ((b*
c - a*d)*x^3)/(c*(a + b*x^3))])/(c^4*(a + b*x^3)^3) + (54*d^2*(b*c - a*d)^3
*x^15*HypergeometricPFQ[{2, 2, 7/3}, {1, 13/3}, ((b*c - a*d)*x^3)/(c*(a + b
*x^3))])/(c^5*(a + b*x^3)^3)))/(420*(b*c - a*d)^2*x^5*(c + d*x^3))
```

IntegrateAlgebraic [C] time = 4.08, size = 452, normalized size = 1.73

$$\frac{a^2d^2x+abd^2x^4+3b^2c^2x+3b^2cdx^4}{3ac\sqrt[3]{a+bx^3}\left(c+dx^3\right)(ad-bc)^2}+\frac{\left(i\sqrt{3}\,ad^2+ad^2-3i\sqrt{3}\,bcd-3bcd\right)\log\left(2x\sqrt[3]{bc-ad}+\left(1+i\sqrt{3}\right)\sqrt[3]{c}\,\sqrt[3]{a+bx^3}\right)}{9c^{5/3}(bc-ad)^{7/3}}+\frac{\left(-i\sqrt{3}\,ad^2+3ad^2+3i\sqrt{3}\,bcd-9bcd\right)\tanh^{-1}\left(\frac{ix\sqrt[3]{bc-ad}+\left(\sqrt{3}-i\right)\sqrt[3]{c}\,\sqrt[3]{a+bx^3}}{\sqrt{3}\,x\sqrt[3]{bc-ad}}\right)}{9c^{5/3}(bc-ad)^{7/3}}+\frac{\left(-i\sqrt{3}\,ad^2-ad^2+3i\sqrt{3}\,bcd+3bcd\right)\log\left(\left(\sqrt{3}+i\right)c^{2/3}\left(a+bx^3\right)^{2/3}+\sqrt[3]{c}\left(-\sqrt{3}\,x+ix\right)\sqrt[3]{a+bx^3}\,\sqrt[3]{bc-ad}-2ix^2(bc-ad)^{2/3}\right)}{18c^{5/3}(bc-ad)^{7/3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((a + b*x^3)^(4/3)*(c + d*x^3)^2),x]
```

```
[Out] (3*b^2*c^2*x + a^2*d^2*x + 3*b^2*c*d*x^4 + a*b*d^2*x^4)/(3*a*c*(-(b*c) + a*
d)^2*(a + b*x^3)^(1/3)*(c + d*x^3)) + ((-9*b*c*d + (3*I)*Sqrt[3]*b*c*d + 3*
a*d^2 - I*Sqrt[3]*a*d^2)*ArcTanh[(I*(b*c - a*d)^(1/3)*x + (-I + Sqrt[3])*c^
(1/3)*(a + b*x^3)^(1/3))/(Sqrt[3]*(b*c - a*d)^(1/3)*x)])/(9*c^(5/3)*(b*c -
a*d)^(7/3)) + ((-3*b*c*d - (3*I)*Sqrt[3]*b*c*d + a*d^2 + I*Sqrt[3]*a*d^2)*L
og[2*(b*c - a*d)^(1/3)*x + (1 + I*Sqrt[3])*c^(1/3)*(a + b*x^3)^(1/3)]])/(9*c
^(5/3)*(b*c - a*d)^(7/3)) + ((3*b*c*d + (3*I)*Sqrt[3]*b*c*d - a*d^2 - I*Sqr
t[3]*a*d^2)*Log[(-2*I)*(b*c - a*d)^(2/3)*x^2 + c^(1/3)*(b*c - a*d)^(1/3)*(I
*x - Sqrt[3]*x)*(a + b*x^3)^(1/3) + (I + Sqrt[3])*c^(2/3)*(a + b*x^3)^(2/3)
])/(18*c^(5/3)*(b*c - a*d)^(7/3))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^(4/3)/(d*x^3+c)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^3+a\right)^{\frac{4}{3}}\left(dx^3+c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^(4/3)/(d*x^3+c)^2,x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^3 + a)^(4/3)*(d*x^3 + c)^2), x)
```

maple [F] time = 0.60, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,x^3+a\right)^{\frac{4}{3}}\left(d\,x^3+c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^3+a)^(4/3)/(d*x^3+c)^2,x)
```

```
[Out] int(1/(b*x^3+a)^(4/3)/(d*x^3+c)^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^3+a\right)^{\frac{4}{3}}\left(dx^3+c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^(4/3)/(d*x^3+c)^2,x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^3 + a)^(4/3)*(d*x^3 + c)^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(b\,x^3+a\right)}^{4/3}\,{\left(d\,x^3+c\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^3)^(4/3)*(c + d*x^3)^2),x)
```

```
[Out] int(1/((a + b*x^3)^(4/3)*(c + d*x^3)^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+bx^3\right)^{\frac{4}{3}}\left(c+dx^3\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**3+a)**(4/3)/(d*x**3+c)**2,x)
```

```
[Out] Integral(1/((a + b*x**3)**(4/3)*(c + d*x**3)**2), x)
```

3.73 $\int \frac{1}{(a+bx^3)^{7/3}(c+dx^3)^2}\,dx$

Optimal. Leaf size=324

$$\frac{bx\left(-4a^2d^2-33abcd+9b^2c^2\right)}{12a^2c\sqrt[3]{a+bx^3}\,(bc-ad)^3}+\frac{d^2(9bc-2ad)\log\left(c+dx^3\right)}{18c^{5/3}(bc-ad)^{10/3}}-\frac{d^2(9bc-2ad)\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{6c^{5/3}(bc-ad)^{10/3}}+\frac{d^2(9b\ldots}{\ldots}$$

Rubi [C] time = 5.69, antiderivative size = 1214, normalized size of antiderivative = 3.75, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {430, 429}

Warning: Unable to verify antiderivative.

```
[In] Int[1/((a + b*x^3)^(7/3)*(c + d*x^3)^2),x]
```

```
[Out] (26130*c^5*(b*c - a*d)^2*x^6*(a + b*x^3)^2 + 89505*c^4*d*(b*c - a*d)^2*x^9*
(a + b*x^3)^2 + 84240*c^3*d^2*(b*c - a*d)^2*x^12*(a + b*x^3)^2 + 26325*c^2*
d^3*(b*c - a*d)^2*x^15*(a + b*x^3)^2 + 748020*c^6*(b*c - a*d)*x^3*(a + b*x^
3)^3 + 2113020*c^5*d*(b*c - a*d)*x^6*(a + b*x^3)^3 + 1916460*c^4*d^2*(b*c -
 a*d)*x^9*(a + b*x^3)^3 + 589680*c^3*d^3*(b*c - a*d)*x^12*(a + b*x^3)^3 - 2
002000*c^7*(a + b*x^3)^4 - 5460000*c^6*d*x^3*(a + b*x^3)^4 - 4914000*c^5*d^
2*x^6*(a + b*x^3)^4 - 1506960*c^4*d^3*x^9*(a + b*x^3)^4 - 1248520*c^6*(b*c
- a*d)*x^3*(a + b*x^3)^3*Hypergeometric2F1[1/3, 1, 4/3, ((b*c - a*d)*x^3)/(
c*(a + b*x^3))] - 3478020*c^5*d*(b*c - a*d)*x^6*(a + b*x^3)^3*Hypergeometri
c2F1[1/3, 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 3144960*c^4*d^2*(b*c
 - a*d)*x^9*(a + b*x^3)^3*Hypergeometric2F1[1/3, 1, 4/3, ((b*c - a*d)*x^3)/
(c*(a + b*x^3))] - 966420*c^3*d^3*(b*c - a*d)*x^12*(a + b*x^3)^3*Hypergeome
tric2F1[1/3, 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 2002000*c^7*(a +
b*x^3)^4*Hypergeometric2F1[1/3, 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))]
+ 5460000*c^6*d*x^3*(a + b*x^3)^4*Hypergeometric2F1[1/3, 1, 4/3, ((b*c - a*
d)*x^3)/(c*(a + b*x^3))] + 4914000*c^5*d^2*x^6*(a + b*x^3)^4*Hypergeometric
2F1[1/3, 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 1506960*c^4*d^3*x^9*(
a + b*x^3)^4*Hypergeometric2F1[1/3, 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3
))] + 7938*c^3*(b*c - a*d)^4*x^12*HypergeometricPFQ[{2, 2, 10/3}, {1, 16/3}
, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 22680*c^2*d*(b*c - a*d)^4*x^15*Hyper
geometricPFQ[{2, 2, 10/3}, {1, 16/3}, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] +
21546*c*d^2*(b*c - a*d)^4*x^18*HypergeometricPFQ[{2, 2, 10/3}, {1, 16/3}, (
(b*c - a*d)*x^3)/(c*(a + b*x^3))] + 6804*d^3*(b*c - a*d)^4*x^21*Hypergeomet
ricPFQ[{2, 2, 10/3}, {1, 16/3}, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 1134*c
^3*(b*c - a*d)^4*x^12*HypergeometricPFQ[{2, 2, 2, 10/3}, {1, 1, 16/3}, ((b*
c - a*d)*x^3)/(c*(a + b*x^3))] + 3402*c^2*d*(b*c - a*d)^4*x^15*Hypergeometr
icPFQ[{2, 2, 2, 10/3}, {1, 1, 16/3}, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 3
402*c*d^2*(b*c - a*d)^4*x^18*HypergeometricPFQ[{2, 2, 2, 10/3}, {1, 1, 16/3
}, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 1134*d^3*(b*c - a*d)^4*x^21*Hyperge
ometricPFQ[{2, 2, 2, 10/3}, {1, 1, 16/3}, ((b*c - a*d)*x^3)/(c*(a + b*x^3))
])/(21840*c^5*(b*c - a*d)^3*x^8*(a + b*x^3)^(10/3)*(c + d*x^3))
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{1}{\left(a+bx^3\right)^{7/3}\left(c+dx^3\right)^2}\,dx = \frac{\sqrt[3]{1+\frac{bx^3}{a}}\int \frac{1}{\left(1+\frac{bx^3}{a}\right)^{7/3}\left(c+dx^3\right)^2}\,dx}{a^2\sqrt[3]{a+bx^3}}$$

$$= \frac{26130c^5(bc-ad)^2x^6\left(a+bx^3\right)^2+89505c^4d(bc-ad)^2x^9\left(a+bx^3\right)^2+84240c^3d^2(}{}$$

Mathematica [A] time = 5.94, size = 288, normalized size = 0.89

$$\frac{1}{36}\left(3x\left(a+bx^3\right)^{2/3}\left(\frac{3b^2(11ad-3bc)}{a^2\left(a+bx^3\right)(ad-bc)^3}+\frac{3b^2}{a\left(a+bx^3\right)^2(bc-ad)^2}-\frac{4d^3}{c\left(c+dx^3\right)(bc-ad)^3}\right)+\frac{2d^2(9bc-2ad)\left(\log\left(\frac{\sqrt[3]{c}\,x\sqrt[3]{bc-ad}}{\sqrt[3]{ax^3+b}}+\frac{x^2(bc-ad)^{2/3}}{\left(ax^3+b\right)^{2/3}}+c^{2/3}\right)-2\log\left(\sqrt[3]{c}-\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{ax^3+b}}\right)+2\sqrt{3}\tan^{-1}\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\sqrt[3]{ax^3+b}}+1}{\sqrt{3}}\right)\right)}{c^{5/3}(bc-ad)^{10/3}}\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + b*x^3)^(7/3)*(c + d*x^3)^2),x]
```

```
[Out] (3*x*(a + b*x^3)^(2/3)*((3*b^2)/(a*(b*c - a*d)^2*(a + b*x^3)^2) + (3*b^2*(-
3*b*c + 11*a*d))/(a^2*(-(b*c) + a*d)^3*(a + b*x^3)) - (4*d^3)/(c*(b*c - a*d
)^3*(c + d*x^3))) + (2*d^2*(9*b*c - 2*a*d)*(2*Sqrt[3]*ArcTan[(1 + (2*(b*c -
 a*d)^(1/3)*x)/(c^(1/3)*(b + a*x^3)^(1/3)))/Sqrt[3]] - 2*Log[c^(1/3) - ((b*
c - a*d)^(1/3)*x)/(b + a*x^3)^(1/3)] + Log[c^(2/3) + ((b*c - a*d)^(2/3)*x^2
)/(b + a*x^3)^(2/3) + (c^(1/3)*(b*c - a*d)^(1/3)*x)/(b + a*x^3)^(1/3)]))/(c
^(5/3)*(b*c - a*d)^(10/3)))/36
```

IntegrateAlgebraic [C] time = 9.70, size = 551, normalized size = 1.70

$$\frac{4a^4d^3x+8a^3bd^3x^4+36a^2b^2c^2dx+36a^2b^2cd^2x^4+4a^2b^2d^3x^7-12ab^3c^3x+21ab^3c^2dx^4+33ab^3cd^2x^7-9b^4c^3x^4-9b^4c^2dx^7}{12a^2c\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)(ad-bc)^3}+\frac{\left(-2i\sqrt{3}\,ad^3-2ad^3+9i\sqrt{3}\,bcd^2+9bcd^2\right)\log\left(2x\sqrt[3]{bc-ad}+\left(1+i\sqrt{3}\right)\sqrt[3]{c}\sqrt[3]{a+bx^3}\right)}{18c^{5/3}(bc-ad)^{10/3}}+\frac{\left(2i\sqrt{3}\,ad^3-6ad^3-9i\sqrt{3}\,bcd^2+27bcd^2\right)\tanh^{-1}\left(\frac{ix\sqrt[3]{bc-ad}+\left(\sqrt{3}-i\right)\sqrt[3]{c}\sqrt[3]{a+bx^3}}{\sqrt{3}\,x\sqrt[3]{bc-ad}}\right)}{18c^{5/3}(bc-ad)^{10/3}}+\frac{\left(2i\sqrt{3}\,ad^3+2ad^3-9i\sqrt{3}\,bcd^2-9bcd^2\right)\log\left(\left(\sqrt{3}+i\right)c^{2/3}\left(a+bx^3\right)^{2/3}+\sqrt[3]{c}\left(-\sqrt{3}\,x+ix\right)\sqrt[3]{a+bx^3}\sqrt[3]{bc-ad}-2ix^2(bc-ad)^{2/3}\right)}{36c^{5/3}(bc-ad)^{10/3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((a + b*x^3)^(7/3)*(c + d*x^3)^2),x]
```

```
[Out] (-12*a*b^3*c^3*x + 36*a^2*b^2*c^2*d*x + 4*a^4*d^3*x - 9*b^4*c^3*x^4 + 21*a*
b^3*c^2*d*x^4 + 36*a^2*b^2*c*d^2*x^4 + 8*a^3*b*d^3*x^4 - 9*b^4*c^2*d*x^7 +
33*a*b^3*c*d^2*x^7 + 4*a^2*b^2*d^3*x^7)/(12*a^2*c*(-(b*c) + a*d)^3*(a + b*x
^3)^(4/3)*(c + d*x^3)) + ((27*b*c*d^2 - (9*I)*Sqrt[3]*b*c*d^2 - 6*a*d^3 + (
2*I)*Sqrt[3]*a*d^3)*ArcTanh[(I*(b*c - a*d)^(1/3)*x + (-I + Sqrt[3])*c^(1/3)
*(a + b*x^3)^(1/3))/(Sqrt[3]*(b*c - a*d)^(1/3)*x)])/(18*c^(5/3)*(b*c - a*d)
^(10/3)) + ((9*b*c*d^2 + (9*I)*Sqrt[3]*b*c*d^2 - 2*a*d^3 - (2*I)*Sqrt[3]*a*
d^3)*Log[2*(b*c - a*d)^(1/3)*x + (1 + I*Sqrt[3])*c^(1/3)*(a + b*x^3)^(1/3)]
)/(18*c^(5/3)*(b*c - a*d)^(10/3)) + ((-9*b*c*d^2 - (9*I)*Sqrt[3]*b*c*d^2 +
2*a*d^3 + (2*I)*Sqrt[3]*a*d^3)*Log[(-2*I)*(b*c - a*d)^(2/3)*x^2 + c^(1/3)*(
b*c - a*d)^(1/3)*(I*x - Sqrt[3]*x)*(a + b*x^3)^(1/3) + (I + Sqrt[3])*c^(2/3
)*(a + b*x^3)^(2/3)])/(36*c^(5/3)*(b*c - a*d)^(10/3))
```

fricas [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^(7/3)/(d*x^3+c)^2,x, algorithm="fricas")
[Out] Timed out
```

giac [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^3+a\right)^{\frac{7}{3}}\left(dx^3+c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^(7/3)/(d*x^3+c)^2,x, algorithm="giac")
[Out] integrate(1/((b*x^3 + a)^(7/3)*(d*x^3 + c)^2), x)
```

maple [**F**] time = 0.59, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,x^3+a\right)^{\frac{7}{3}}\left(d\,x^3+c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^3+a)^(7/3)/(d*x^3+c)^2,x)
[Out] int(1/(b*x^3+a)^(7/3)/(d*x^3+c)^2,x)
```

maxima [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^3+a\right)^{\frac{7}{3}}\left(dx^3+c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^(7/3)/(d*x^3+c)^2,x, algorithm="maxima")
[Out] integrate(1/((b*x^3 + a)^(7/3)*(d*x^3 + c)^2), x)
```

mupad [**F**] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(b\,x^3+a\right)^{7/3}\,\left(d\,x^3+c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^3)^(7/3)*(c + d*x^3)^2),x)
[Out] int(1/((a + b*x^3)^(7/3)*(c + d*x^3)^2), x)
```

sympy [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+bx^3\right)^{\frac{7}{3}}\left(c+dx^3\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**3+a)**(7/3)/(d*x**3+c)**2,x)
[Out] Integral(1/((a + b*x**3)**(7/3)*(c + d*x**3)**2), x)
```

3.74 $\int \frac{\left(a+bx^3\right)^{14/3}}{\left(c+dx^3\right)^3}\,dx$

Optimal. Leaf size=541

$$-\frac{bx\left(a+bx^3\right)^{2/3}(2bc-ad)\left(-5a^2d^2-18abcd+18b^2c^2\right)}{18c^2d^4}+\frac{bx\left(a+bx^3\right)^{5/3}\left(-5a^2d^2-10abcd+18b^2c^2\right)}{18c^2d^3}-\frac{(bc-ad)^{8/\ldots}}{\ldots}$$

Rubi [C] time = 0.03, antiderivative size = 62, normalized size of antiderivative = 0.11, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {430, 429}

$$\frac{a^4x\left(a+bx^3\right)^{2/3}F_1\left(\frac{1}{3};-\frac{14}{3},3;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c^3\left(\frac{bx^3}{a}+1\right)^{2/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(a + b*x^3)^(14/3)/(c + d*x^3)^3,x]
```

```
[Out] (a^4*x*(a + b*x^3)^(2/3)*AppellF1[1/3, -14/3, 3, 4/3, -((b*x^3)/a), -((d*x^
3)/c)])/(c^3*(1 + (b*x^3)/a)^(2/3))
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{\left(a+bx^3\right)^{14/3}}{\left(c+dx^3\right)^3}\,dx=\frac{\left(a^4\left(a+bx^3\right)^{2/3}\right)\int \frac{\left(1+\frac{bx^3}{a}\right)^{14/3}}{\left(c+dx^3\right)^3}\,dx}{\left(1+\frac{bx^3}{a}\right)^{2/3}}$$
$$=\frac{a^4x\left(a+bx^3\right)^{2/3}F_1\left(\frac{1}{3};-\frac{14}{3},3;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c^3\left(1+\frac{bx^3}{a}\right)^{2/3}}$$

Mathematica [C] time = 2.34, size = 1171, normalized size = 2.16

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^3)^(14/3)/(c + d*x^3)^3,x]
```

```
[Out] ((6*x*(a + b*x^3)^(2/3)*(-2*b^3*(9*b*c - 13*a*d) + 3*b^4*d*x^3 + (3*(b*c -
a*d)^4)/(c*(c + d*x^3)^2) - ((b*c - a*d)^3*(21*b*c + 5*a*d))/(c^2*(c + d*x^
3))))/d^4 + (162*b^5*c*x^4*(1 + (b*x^3)/a)^(1/3)*AppellF1[4/3, 1/3, 1, 7/3,
 -((b*x^3)/a), -((d*x^3)/c)])/(d^4*(a + b*x^3)^(1/3)) - (378*a*b^4*x^4*(1 +
 (b*x^3)/a)^(1/3)*AppellF1[4/3, 1/3, 1, 7/3, -((b*x^3)/a), -((d*x^3)/c)])/(
d^3*(a + b*x^3)^(1/3)) + (231*a^2*b^3*x^4*(1 + (b*x^3)/a)^(1/3)*AppellF1[4/
3, 1/3, 1, 7/3, -((b*x^3)/a), -((d*x^3)/c)])/(c*d^2*(a + b*x^3)^(1/3)) + (1
0*a^5*(2*Sqrt[3]*ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c^(1/3)*(b + a*x^3)^(
1/3)))/Sqrt[3]] - 2*Log[c^(1/3) - ((b*c - a*d)^(1/3)*x)/(b + a*x^3)^(1/3)]
+ Log[c^(2/3) + ((b*c - a*d)^(2/3)*x^2)/(b + a*x^3)^(2/3) + (c^(1/3)*(b*c -
 a*d)^(1/3)*x)/(b + a*x^3)^(1/3)]))/(c^(8/3)*(b*c - a*d)^(1/3)) + (36*a*b^4
*c^(4/3)*(2*Sqrt[3]*ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c^(1/3)*(b + a*x^3
)^(1/3)))/Sqrt[3]] - 2*Log[c^(1/3) - ((b*c - a*d)^(1/3)*x)/(b + a*x^3)^(1/3
)] + Log[c^(2/3) + ((b*c - a*d)^(2/3)*x^2)/(b + a*x^3)^(2/3) + (c^(1/3)*(b*
c - a*d)^(1/3)*x)/(b + a*x^3)^(1/3)]))/(d^4*(b*c - a*d)^(1/3)) - (72*a^2*b^
3*c^(1/3)*(2*Sqrt[3]*ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c^(1/3)*(b + a*x^
3)^(1/3)))/Sqrt[3]] - 2*Log[c^(1/3) - ((b*c - a*d)^(1/3)*x)/(b + a*x^3)^(1/
3)] + Log[c^(2/3) + ((b*c - a*d)^(2/3)*x^2)/(b + a*x^3)^(2/3) + (c^(1/3)*(b
*c - a*d)^(1/3)*x)/(b + a*x^3)^(1/3)]))/(d^3*(b*c - a*d)^(1/3)) + (30*a^3*b
^2*(2*Sqrt[3]*ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c^(1/3)*(b + a*x^3)^(1/3
)))/Sqrt[3]] - 2*Log[c^(1/3) - ((b*c - a*d)^(1/3)*x)/(b + a*x^3)^(1/3)] + L
og[c^(2/3) + ((b*c - a*d)^(2/3)*x^2)/(b + a*x^3)^(2/3) + (c^(1/3)*(b*c - a*
d)^(1/3)*x)/(b + a*x^3)^(1/3)]))/(c^(2/3)*d^2*(b*c - a*d)^(1/3)) + (6*a^4*b
*(2*Sqrt[3]*ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c^(1/3)*(b + a*x^3)^(1/3))
)/Sqrt[3]] - 2*Log[c^(1/3) - ((b*c - a*d)^(1/3)*x)/(b + a*x^3)^(1/3)] + Log
[c^(2/3) + ((b*c - a*d)^(2/3)*x^2)/(b + a*x^3)^(2/3) + (c^(1/3)*(b*c - a*d)
^(1/3)*x)/(b + a*x^3)^(1/3)]))/(c^(5/3)*d*(b*c - a*d)^(1/3)))/108
```

IntegrateAlgebraic [C] time = 102.41, size = 1081, normalized size = 2.00

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x^3)^(14/3)/(c + d*x^3)^3,x]
```

```
[Out] ((a + b*x^3)^(2/3)*(-36*b^4*c^5*x + 72*a*b^3*c^4*d*x - 30*a^2*b^2*c^3*d^2*x
 - 6*a^3*b*c^2*d^3*x + 8*a^4*c*d^4*x - 54*b^4*c^4*d*x^4 + 110*a*b^3*c^3*d^2
*x^4 - 48*a^2*b^2*c^2*d^3*x^4 + 6*a^3*b*c*d^4*x^4 + 5*a^4*d^5*x^4 - 12*b^4*
c^3*d^2*x^7 + 26*a*b^3*c^2*d^3*x^7 + 3*b^4*c^2*d^3*x^10))/(18*c^2*d^4*(c +
d*x^3)^2) + ((54*b^(14/3)*c^2 - 126*a*b^(11/3)*c*d + 77*a^2*b^(8/3)*d^2)*Ar
cTan[(Sqrt[3]*b^(1/3)*x)/(b^(1/3)*x + 2*(a + b*x^3)^(1/3))])/(9*Sqrt[3]*d^5
) + ((I/54)*((162*I)*b^2*c^2*(b*c - a*d)^(8/3) + 54*Sqrt[3]*b^2*c^2*(b*c -
a*d)^(8/3) + (54*I)*a*b*c*d*(b*c - a*d)^(8/3) + 18*Sqrt[3]*a*b*c*d*(b*c - a
*d)^(8/3) + (15*I)*a^2*d^2*(b*c - a*d)^(8/3) + 5*Sqrt[3]*a^2*d^2*(b*c - a*d
)^(8/3))*ArcTanh[(I*(b*c - a*d)^(1/3)*x + (-I + Sqrt[3])*c^(1/3)*(a + b*x^3
)^(1/3))/(Sqrt[3]*(b*c - a*d)^(1/3)*x)])/(c^(8/3)*d^5) + ((-54*b^(14/3)*c^2
 + 126*a*b^(11/3)*c*d - 77*a^2*b^(8/3)*d^2)*Log[-(b^(1/3)*x) + (a + b*x^3)^
(1/3)])/(27*d^5) - ((I/54)*((-54*I)*b^2*c^2*(b*c - a*d)^(8/3) + 54*Sqrt[3]*
b^2*c^2*(b*c - a*d)^(8/3) - (18*I)*a*b*c*d*(b*c - a*d)^(8/3) + 18*Sqrt[3]*a
*b*c*d*(b*c - a*d)^(8/3) - (5*I)*a^2*d^2*(b*c - a*d)^(8/3) + 5*Sqrt[3]*a^2*
d^2*(b*c - a*d)^(8/3))*Log[2*(b*c - a*d)^(1/3)*x + (1 + I*Sqrt[3])*c^(1/3)*
(a + b*x^3)^(1/3)])/(c^(8/3)*d^5) + ((54*b^(14/3)*c^2 - 126*a*b^(11/3)*c*d
+ 77*a^2*b^(8/3)*d^2)*Log[b^(2/3)*x^2 + b^(1/3)*x*(a + b*x^3)^(1/3) + (a +
b*x^3)^(2/3)])/(54*d^5) + ((54*b^2*c^2*(b*c - a*d)^(8/3) + (54*I)*Sqrt[3]*b
^2*c^2*(b*c - a*d)^(8/3) + 18*a*b*c*d*(b*c - a*d)^(8/3) + (18*I)*Sqrt[3]*a*
b*c*d*(b*c - a*d)^(8/3) + 5*a^2*d^2*(b*c - a*d)^(8/3) + (5*I)*Sqrt[3]*a^2*d
```

```
^2*(b*c - a*d)^(8/3))*Log[(-2*I)*(b*c - a*d)^(2/3)*x^2 + c^(1/3)*(b*c - a*d
)^(1/3)*(I*x - Sqrt[3]*x)*(a + b*x^3)^(1/3) + (I + Sqrt[3])*c^(2/3)*(a + b*
x^3)^(2/3)])/(108*c^(8/3)*d^5)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(14/3)/(d*x^3+c)^3,x, algorithm="fricas")
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^3+a\right)^{\frac{14}{3}}}{\left(dx^3+c\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(14/3)/(d*x^3+c)^3,x, algorithm="giac")
[Out] integrate((b*x^3 + a)^(14/3)/(d*x^3 + c)^3, x)
```

maple [F] time = 0.40, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^3+a\right)^{\frac{14}{3}}}{\left(d\,x^3+c\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(14/3)/(d*x^3+c)^3,x)
[Out] int((b*x^3+a)^(14/3)/(d*x^3+c)^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^3+a\right)^{\frac{14}{3}}}{\left(dx^3+c\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(14/3)/(d*x^3+c)^3,x, algorithm="maxima")
[Out] integrate((b*x^3 + a)^(14/3)/(d*x^3 + c)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(b\,x^3+a\right)^{14/3}}{\left(d\,x^3+c\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^(14/3)/(c + d*x^3)^3,x)
[Out] int((a + b*x^3)^(14/3)/(c + d*x^3)^3, x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(14/3)/(d*x**3+c)**3,x)
[Out] Timed out
```

3.75 $\int \frac{\left(a+bx^3\right)^{11/3}}{\left(c+dx^3\right)^3}\,dx$

Optimal. Leaf size=458

$$\frac{bx\left(a+bx^3\right)^{2/3}\left(-5a^2d^2-7abcd+18b^2c^2\right)}{18c^2d^3}+\frac{(bc-ad)^{5/3}\left(5a^2d^2+12abcd+27b^2c^2\right)\log\left(c+dx^3\right)}{54c^{8/3}d^4}-\frac{(bc-ad)^{5/3}\left(5\right.}{}$$

Rubi [C] time = 0.03, antiderivative size = 62, normalized size of antiderivative = 0.14, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {430, 429}

$$\frac{a^3x\left(a+bx^3\right)^{2/3}F_1\left(\frac{1}{3};-\frac{11}{3},3;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c^3\left(\frac{bx^3}{a}+1\right)^{2/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(a + b*x^3)^(11/3)/(c + d*x^3)^3,x]
```

```
[Out] (a^3*x*(a + b*x^3)^(2/3)*AppellF1[1/3, -11/3, 3, 4/3, -((b*x^3)/a), -((d*x^
3)/c)])/(c^3*(1 + (b*x^3)/a)^(2/3))
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^3\right)^{11/3}}{\left(c+dx^3\right)^3}\,dx &= \frac{\left(a^3\left(a+bx^3\right)^{2/3}\right)\int \frac{\left(1+\frac{bx^3}{a}\right)^{11/3}}{\left(c+dx^3\right)^3}\,dx}{\left(1+\frac{bx^3}{a}\right)^{2/3}}\\ &= \frac{a^3x\left(a+bx^3\right)^{2/3}F_1\left(\frac{1}{3};-\frac{11}{3},3;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c^3\left(1+\frac{bx^3}{a}\right)^{2/3}}\end{aligned}$$

Mathematica [C] time = 1.84, size = 908, normalized size = 1.98

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^3)^(11/3)/(c + d*x^3)^3,x]
```

```
[Out] ((6*x*(a + b*x^3)^(2/3)*(6*b^3 - (3*(b*c - a*d)^3)/(c*(c + d*x^3)^2) + (5*(
b*c - a*d)^2*(3*b*c + a*d))/(c^2*(c + d*x^3))))/d^3 - (81*b^4*x^4*(1 + (b*x
^3)/a)^(1/3)*AppellF1[4/3, 1/3, 1, 7/3, -((b*x^3)/a), -((d*x^3)/c)])/(d^3*(
a + b*x^3)^(1/3)) + (99*a*b^3*x^4*(1 + (b*x^3)/a)^(1/3)*AppellF1[4/3, 1/3,
1, 7/3, -((b*x^3)/a), -((d*x^3)/c)])/(c*d^2*(a + b*x^3)^(1/3)) + (10*a^4*(2
*Sqrt[3]*ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c^(1/3)*(b + a*x^3)^(1/3)))/S
qrt[3]] - 2*Log[c^(1/3) - ((b*c - a*d)^(1/3)*x)/(b + a*x^3)^(1/3)] + Log[c^
(2/3) + ((b*c - a*d)^(2/3)*x^2)/(b + a*x^3)^(2/3) + (c^(1/3)*(b*c - a*d)^(1
/3)*x)/(b + a*x^3)^(1/3)]))/(c^(8/3)*(b*c - a*d)^(1/3)) - (18*a*b^3*c^(1/3)
*(2*Sqrt[3]*ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c^(1/3)*(b + a*x^3)^(1/3))
)/Sqrt[3]] - 2*Log[c^(1/3) - ((b*c - a*d)^(1/3)*x)/(b + a*x^3)^(1/3)] + Log
[c^(2/3) + ((b*c - a*d)^(2/3)*x^2)/(b + a*x^3)^(2/3) + (c^(1/3)*(b*c - a*d)
^(1/3)*x)/(b + a*x^3)^(1/3)]))/(d^3*(b*c - a*d)^(1/3)) + (16*a^2*b^2*(2*Sqr
t[3]*ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c^(1/3)*(b + a*x^3)^(1/3)))/Sqrt[
3]] - 2*Log[c^(1/3) - ((b*c - a*d)^(1/3)*x)/(b + a*x^3)^(1/3)] + Log[c^(2/3
) + ((b*c - a*d)^(2/3)*x^2)/(b + a*x^3)^(2/3) + (c^(1/3)*(b*c - a*d)^(1/3)*
x)/(b + a*x^3)^(1/3)]))/(c^(2/3)*d^2*(b*c - a*d)^(1/3)) + (4*a^3*b*(2*Sqrt[
3]*ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c^(1/3)*(b + a*x^3)^(1/3)))/Sqrt[3]
] - 2*Log[c^(1/3) - ((b*c - a*d)^(1/3)*x)/(b + a*x^3)^(1/3)] + Log[c^(2/3)
+ ((b*c - a*d)^(2/3)*x^2)/(b + a*x^3)^(2/3) + (c^(1/3)*(b*c - a*d)^(1/3)*x)
/(b + a*x^3)^(1/3)]))/(c^(5/3)*d*(b*c - a*d)^(1/3)))/108
```

IntegrateAlgebraic [C] time = 76.86, size = 970, normalized size = 2.12

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x^3)^(11/3)/(c + d*x^3)^3,x]
```

```
[Out] ((a + b*x^3)^(2/3)*(18*b^3*c^4*x - 16*a*b^2*c^3*d*x - 4*a^2*b*c^2*d^2*x + 8
*a^3*c*d^3*x + 27*b^3*c^3*d*x^4 - 25*a*b^2*c^2*d^2*x^4 + 5*a^2*b*c*d^3*x^4
+ 5*a^3*d^4*x^4 + 6*b^3*c^2*d^2*x^7))/(18*c^2*d^3*(c + d*x^3)^2) - ((9*b^(1
1/3)*c - 11*a*b^(8/3)*d)*ArcTan[(Sqrt[3]*b^(1/3)*x)/(b^(1/3)*x + 2*(a + b*x
^3)^(1/3))])/(3*Sqrt[3]*d^4) + ((81*b^2*c^2*(b*c - a*d)^(5/3) - (27*I)*Sqrt
[3]*b^2*c^2*(b*c - a*d)^(5/3) + 36*a*b*c*d*(b*c - a*d)^(5/3) - (12*I)*Sqrt[
3]*a*b*c*d*(b*c - a*d)^(5/3) + 15*a^2*d^2*(b*c - a*d)^(5/3) - (5*I)*Sqrt[3]
*a^2*d^2*(b*c - a*d)^(5/3))*ArcTanh[(I*(b*c - a*d)^(1/3)*x + (-I + Sqrt[3])
*c^(1/3)*(a + b*x^3)^(1/3))/(Sqrt[3]*(b*c - a*d)^(1/3)*x)])/(54*c^(8/3)*d^4
) + ((9*b^(11/3)*c - 11*a*b^(8/3)*d)*Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)])
/(9*d^4) + ((27*b^2*c^2*(b*c - a*d)^(5/3) + (27*I)*Sqrt[3]*b^2*c^2*(b*c - a
*d)^(5/3) + 12*a*b*c*d*(b*c - a*d)^(5/3) + (12*I)*Sqrt[3]*a*b*c*d*(b*c - a*
d)^(5/3) + 5*a^2*d^2*(b*c - a*d)^(5/3) + (5*I)*Sqrt[3]*a^2*d^2*(b*c - a*d)^
(5/3))*Log[2*(b*c - a*d)^(1/3)*x + (1 + I*Sqrt[3])*c^(1/3)*(a + b*x^3)^(1/3
)])/(54*c^(8/3)*d^4) + ((-9*b^(11/3)*c + 11*a*b^(8/3)*d)*Log[b^(2/3)*x^2 +
b^(1/3)*x*(a + b*x^3)^(1/3) + (a + b*x^3)^(2/3)])/(18*d^4) - ((I/108)*((-27
*I)*b^2*c^2*(b*c - a*d)^(5/3) + 27*Sqrt[3]*b^2*c^2*(b*c - a*d)^(5/3) - (12*
I)*a*b*c*d*(b*c - a*d)^(5/3) + 12*Sqrt[3]*a*b*c*d*(b*c - a*d)^(5/3) - (5*I)
*a^2*d^2*(b*c - a*d)^(5/3) + 5*Sqrt[3]*a^2*d^2*(b*c - a*d)^(5/3))*Log[(-2*I
)*(b*c - a*d)^(2/3)*x^2 + c^(1/3)*(b*c - a*d)^(1/3)*(I*x - Sqrt[3]*x)*(a +
b*x^3)^(1/3) + (I + Sqrt[3])*c^(2/3)*(a + b*x^3)^(2/3)])/(c^(8/3)*d^4)
```

fricas [B] time = 114.55, size = 1246, normalized size = 2.72

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(11/3)/(d*x^3+c)^3,x, algorithm="fricas")
```

```
[Out] 1/54*(2*sqrt(3)*(27*b^3*c^5 - 15*a*b^2*c^4*d - 7*a^2*b*c^3*d^2 - 5*a^3*c^2*
d^3 + (27*b^3*c^3*d^2 - 15*a*b^2*c^2*d^3 - 7*a^2*b*c*d^4 - 5*a^3*d^5)*x^6 +
 2*(27*b^3*c^4*d - 15*a*b^2*c^3*d^2 - 7*a^2*b*c^2*d^3 - 5*a^3*c*d^4)*x^3)*(
(b^2*c^2 - 2*a*b*c*d + a^2*d^2)/c^2)^(1/3)*arctan(-1/3*(sqrt(3)*(b*c - a*d)
*x + 2*sqrt(3)*(b*x^3 + a)^(1/3)*c*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)/c^2)^(1
/3))/((b*c - a*d)*x)) + 6*sqrt(3)*(9*b^3*c^5 - 11*a*b^2*c^4*d + (9*b^3*c^3*
d^2 - 11*a*b^2*c^2*d^3)*x^6 + 2*(9*b^3*c^4*d - 11*a*b^2*c^3*d^2)*x^3)*(-b^2
)^(1/3)*arctan(-1/3*(sqrt(3)*b*x - 2*sqrt(3)*(b*x^3 + a)^(1/3)*(-b^2)^(1/3)
)/(b*x)) - 2*(27*b^3*c^5 - 15*a*b^2*c^4*d - 7*a^2*b*c^3*d^2 - 5*a^3*c^2*d^3
 + (27*b^3*c^3*d^2 - 15*a*b^2*c^2*d^3 - 7*a^2*b*c*d^4 - 5*a^3*d^5)*x^6 + 2*
(27*b^3*c^4*d - 15*a*b^2*c^3*d^2 - 7*a^2*b*c^2*d^3 - 5*a^3*c*d^4)*x^3)*((b^
2*c^2 - 2*a*b*c*d + a^2*d^2)/c^2)^(1/3)*log((c*x*((b^2*c^2 - 2*a*b*c*d + a^
2*d^2)/c^2)^(2/3) - (b*x^3 + a)^(1/3)*(b*c - a*d))/x) - 6*(9*b^3*c^5 - 11*a
*b^2*c^4*d + (9*b^3*c^3*d^2 - 11*a*b^2*c^2*d^3)*x^6 + 2*(9*b^3*c^4*d - 11*a
*b^2*c^3*d^2)*x^3)*(-b^2)^(1/3)*log(-((-b^2)^(2/3)*x - (b*x^3 + a)^(1/3)*b)
/x) + 3*(9*b^3*c^5 - 11*a*b^2*c^4*d + (9*b^3*c^3*d^2 - 11*a*b^2*c^2*d^3)*x^
6 + 2*(9*b^3*c^4*d - 11*a*b^2*c^3*d^2)*x^3)*(-b^2)^(1/3)*log(-((-b^2)^(1/3)
*b*x^2 - (b*x^3 + a)^(1/3)*(-b^2)^(2/3)*x - (b*x^3 + a)^(2/3)*b)/x^2) + (27
*b^3*c^5 - 15*a*b^2*c^4*d - 7*a^2*b*c^3*d^2 - 5*a^3*c^2*d^3 + (27*b^3*c^3*d
^2 - 15*a*b^2*c^2*d^3 - 7*a^2*b*c*d^4 - 5*a^3*d^5)*x^6 + 2*(27*b^3*c^4*d -
15*a*b^2*c^3*d^2 - 7*a^2*b*c^2*d^3 - 5*a^3*c*d^4)*x^3)*((b^2*c^2 - 2*a*b*c*
d + a^2*d^2)/c^2)^(1/3)*log(-((b*c - a*d)*x^2*((b^2*c^2 - 2*a*b*c*d + a^2*d
^2)/c^2)^(1/3) + (b*x^3 + a)^(1/3)*c*x*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)/c^2
)^(2/3) + (b*x^3 + a)^(2/3)*(b*c - a*d))/x^2) + 3*(6*b^3*c^2*d^3*x^7 + (27*
b^3*c^3*d^2 - 25*a*b^2*c^2*d^3 + 5*a^2*b*c*d^4 + 5*a^3*d^5)*x^4 + 2*(9*b^3*
c^4*d - 8*a*b^2*c^3*d^2 - 2*a^2*b*c^2*d^3 + 4*a^3*c*d^4)*x)*(b*x^3 + a)^(2/
3))/(c^2*d^6*x^6 + 2*c^3*d^5*x^3 + c^4*d^4)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^3 + a\right)^{\frac{11}{3}}}{\left(dx^3 + c\right)^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(11/3)/(d*x^3+c)^3,x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a)^(11/3)/(d*x^3 + c)^3, x)
```

maple [F] time = 0.41, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^3 + a\right)^{\frac{11}{3}}}{\left(d\,x^3 + c\right)^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(11/3)/(d*x^3+c)^3,x)
```

```
[Out] int((b*x^3+a)^(11/3)/(d*x^3+c)^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^3 + a\right)^{\frac{11}{3}}}{\left(dx^3 + c\right)^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(11/3)/(d*x^3+c)^3,x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a)^(11/3)/(d*x^3 + c)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(b\,x^3+a\right)^{11/3}}{\left(d\,x^3+c\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^(11/3)/(c + d*x^3)^3,x)
[Out] int((a + b*x^3)^(11/3)/(c + d*x^3)^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(11/3)/(d*x**3+c)**3,x)
[Out] Timed out
```

3.76 $\int \frac{\left(a+bx^3\right)^{8/3}}{\left(c+dx^3\right)^3}\,dx$

Optimal. Leaf size=391

$$-\frac{(bc-ad)^{2/3}\left(5a^2d^2+6abcd+9b^2c^2\right)\log\left(c+dx^3\right)}{54c^{8/3}d^3}+\frac{(bc-ad)^{2/3}\left(5a^2d^2+6abcd+9b^2c^2\right)\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+b}\cdots\right.}{18c^{8/3}d^3}$$

Rubi [C] time = 0.03, antiderivative size = 62, normalized size of antiderivative = 0.16, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {430, 429}

$$\frac{a^2x\left(a+bx^3\right)^{2/3}F_1\left(\frac{1}{3};-\frac{8}{3},3;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c^3\left(\frac{bx^3}{a}+1\right)^{2/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(a + b*x^3)^(8/3)/(c + d*x^3)^3,x]
```

```
[Out] (a^2*x*(a + b*x^3)^(2/3)*AppellF1[1/3, -8/3, 3, 4/3, -((b*x^3)/a), -((d*x^3
)/c)])/(c^3*(1 + (b*x^3)/a)^(2/3))
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^3\right)^{8/3}}{\left(c+dx^3\right)^3}\,dx &= \frac{\left(a^2\left(a+bx^3\right)^{2/3}\right)\int \frac{\left(1+\frac{bx^3}{a}\right)^{8/3}}{\left(c+dx^3\right)^3}\,dx}{\left(1+\frac{bx^3}{a}\right)^{2/3}}\\ &= \frac{a^2x\left(a+bx^3\right)^{2/3}F_1\left(\frac{1}{3};-\frac{8}{3},3;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c^3\left(1+\frac{bx^3}{a}\right)^{2/3}}\end{aligned}$$

Mathematica [C] time = 1.16, size = 651, normalized size = 1.66

$$\frac{\frac{10a^3\left(\log\left(\frac{\sqrt[3]{c}\,x\sqrt[3]{bc-ad}}{\sqrt[3]{ax^3+b}}+\frac{x^2(bc-ad)^{2/3}}{\left(ax^3+b\right)^{2/3}}+c^{2/3}\right)-2\log\left(\sqrt[3]{c}-\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{ax^3+b}}\right)+2\sqrt{3}\tan^{-1}\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\sqrt[3]{ax^3+b}}+1}{\sqrt{3}}\right)\right)}{\sqrt[3]{bc-ad}}+\frac{2a^2bc\left(\log\left(\frac{\sqrt[3]{c}\,x\sqrt[3]{bc-ad}}{\sqrt[3]{ax^3+b}}+\frac{x^2(bc-ad)^{2/3}}{\left(ax^3+b\right)^{2/3}}+c^{2/3}\right)-2\log\left(\sqrt[3]{c}-\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{ax^3+b}}\right)+2\sqrt{3}\tan^{-1}\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\sqrt[3]{ax^3+b}}+1}{\sqrt{3}}\right)\right)}{d\sqrt[3]{bc-ad}}+\frac{27b^3c^{5/3}x^4\sqrt[3]{\frac{bx^3}{a}+1}\,F_1\left(\frac{4}{3};\frac{1}{3},1;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{d^2\sqrt[3]{a+bx^3}}+\frac{6ab^2c^2\left(\log\left(\frac{\sqrt[3]{c}\,x\sqrt[3]{bc-ad}}{\sqrt[3]{ax^3+b}}+\frac{x^2(bc-ad)^{2/3}}{\left(ax^3+b\right)^{2/3}}+c^{2/3}\right)-2\log\left(\sqrt[3]{c}-\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{ax^3+b}}\right)+2\sqrt{3}\tan^{-1}\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\sqrt[3]{ax^3+b}}+1}{\sqrt{3}}\right)\right)}{d^2\sqrt[3]{bc-ad}}+\frac{6c^{2/3}x\left(a+bx^3\right)^{2/3}(ad-bc)\left(ad\left(8c+5dx^3\right)+3bc\left(2c+3dx^3\right)\right)}{d^2\left(c+dx^3\right)^2}}{108c^{8/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^3)^(8/3)/(c + d*x^3)^3,x]
```

```
[Out] ((6*c^(2/3)*(-(b*c) + a*d)*x*(a + b*x^3)^(2/3)*(3*b*c*(2*c + 3*d*x^3) + a*d
*(8*c + 5*d*x^3)))/(d^2*(c + d*x^3)^2) + (27*b^3*c^(5/3)*x^4*(1 + (b*x^3)/a
)^(1/3)*AppellF1[4/3, 1/3, 1, 7/3, -((b*x^3)/a), -((d*x^3)/c)])/(d^2*(a + b
*x^3)^(1/3)) + (10*a^3*(2*Sqrt[3]*ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c^(1
/3)*(b + a*x^3)^(1/3)))/Sqrt[3]] - 2*Log[c^(1/3) - ((b*c - a*d)^(1/3)*x)/(b
 + a*x^3)^(1/3)] + Log[c^(2/3) + ((b*c - a*d)^(2/3)*x^2)/(b + a*x^3)^(2/3)
+ (c^(1/3)*(b*c - a*d)^(1/3)*x)/(b + a*x^3)^(1/3)]))/(b*c - a*d)^(1/3) + (6
*a*b^2*c^2*(2*Sqrt[3]*ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c^(1/3)*(b + a*x
^3)^(1/3)))/Sqrt[3]] - 2*Log[c^(1/3) - ((b*c - a*d)^(1/3)*x)/(b + a*x^3)^(1
/3)] + Log[c^(2/3) + ((b*c - a*d)^(2/3)*x^2)/(b + a*x^3)^(2/3) + (c^(1/3)*(
b*c - a*d)^(1/3)*x)/(b + a*x^3)^(1/3)]))/(d^2*(b*c - a*d)^(1/3)) + (2*a^2*b
*c*(2*Sqrt[3]*ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c^(1/3)*(b + a*x^3)^(1/3
)))/Sqrt[3]] - 2*Log[c^(1/3) - ((b*c - a*d)^(1/3)*x)/(b + a*x^3)^(1/3)] + L
og[c^(2/3) + ((b*c - a*d)^(2/3)*x^2)/(b + a*x^3)^(2/3) + (c^(1/3)*(b*c - a*
d)^(1/3)*x)/(b + a*x^3)^(1/3)]))/(d*(b*c - a*d)^(1/3)))/(108*c^(8/3))
```

IntegrateAlgebraic [C] time = 28.13, size = 804, normalized size = 2.06

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x^3)^(8/3)/(c + d*x^3)^3,x]
```

```
[Out] ((a + b*x^3)^(2/3)*(-6*b^2*c^3*x - 2*a*b*c^2*d*x + 8*a^2*c*d^2*x - 9*b^2*c^
2*d*x^4 + 4*a*b*c*d^2*x^4 + 5*a^2*d^3*x^4))/(18*c^2*d^2*(c + d*x^3)^2) + (b
^(8/3)*ArcTan[(Sqrt[3]*b^(1/3)*x)/(b^(1/3)*x + 2*(a + b*x^3)^(1/3))])/(Sqrt
[3]*d^3) + ((-27*b^3*c^3 + (9*I)*Sqrt[3]*b^3*c^3 + 9*a*b^2*c^2*d - (3*I)*Sq
rt[3]*a*b^2*c^2*d + 3*a^2*b*c*d^2 - I*Sqrt[3]*a^2*b*c*d^2 + 15*a^3*d^3 - (5
*I)*Sqrt[3]*a^3*d^3)*ArcTanh[(I*(b*c - a*d)^(1/3)*x + (-I + Sqrt[3])*c^(1/3
)*(a + b*x^3)^(1/3))/(Sqrt[3]*(b*c - a*d)^(1/3)*x)])/(54*c^(8/3)*d^3*(b*c -
 a*d)^(1/3)) - (b^(8/3)*Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)])/(3*d^3) + ((
-9*b^3*c^3 - (9*I)*Sqrt[3]*b^3*c^3 + 3*a*b^2*c^2*d + (3*I)*Sqrt[3]*a*b^2*c^
2*d + a^2*b*c*d^2 + I*Sqrt[3]*a^2*b*c*d^2 + 5*a^3*d^3 + (5*I)*Sqrt[3]*a^3*d
^3)*Log[2*(b*c - a*d)^(1/3)*x + (1 + I*Sqrt[3])*c^(1/3)*(a + b*x^3)^(1/3)])
/(54*c^(8/3)*d^3*(b*c - a*d)^(1/3)) + (b^(8/3)*Log[b^(2/3)*x^2 + b^(1/3)*x*
(a + b*x^3)^(1/3) + (a + b*x^3)^(2/3)])/(6*d^3) + ((9*b^3*c^3 + (9*I)*Sqrt[
3]*b^3*c^3 - 3*a*b^2*c^2*d - (3*I)*Sqrt[3]*a*b^2*c^2*d - a^2*b*c*d^2 - I*Sq
rt[3]*a^2*b*c*d^2 - 5*a^3*d^3 - (5*I)*Sqrt[3]*a^3*d^3)*Log[(-2*I)*(b*c - a*
d)^(2/3)*x^2 + c^(1/3)*(b*c - a*d)^(1/3)*(I*x - Sqrt[3]*x)*(a + b*x^3)^(1/3
) + (I + Sqrt[3])*c^(2/3)*(a + b*x^3)^(2/3)])/(108*c^(8/3)*d^3*(b*c - a*d)^
(1/3))
```

fricas [B] time = 12.61, size = 954, normalized size = 2.44

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(8/3)/(d*x^3+c)^3,x, algorithm="fricas")
```

```
[Out] -1/54*(2*sqrt(3)*((9*b^2*c^2*d^2 + 6*a*b*c*d^3 + 5*a^2*d^4)*x^6 + 9*b^2*c^4
 + 6*a*b*c^3*d + 5*a^2*c^2*d^2 + 2*(9*b^2*c^3*d + 6*a*b*c^2*d^2 + 5*a^2*c*d
^3)*x^3)*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)/c^2)^(1/3)*arctan(-1/3*(sqrt(3)*(
b*c - a*d)*x + 2*sqrt(3)*(b*x^3 + a)^(1/3)*c*((b^2*c^2 - 2*a*b*c*d + a^2*d^
2)/c^2)^(1/3))/((b*c - a*d)*x)) + 18*sqrt(3)*(b^2*c^2*d^2*x^6 + 2*b^2*c^3*d
*x^3 + b^2*c^4)*(-b^2)^(1/3)*arctan(-1/3*(sqrt(3)*b*x - 2*sqrt(3)*(b*x^3 +
a)^(1/3)*(-b^2)^(1/3))/(b*x)) - 2*((9*b^2*c^2*d^2 + 6*a*b*c*d^3 + 5*a^2*d^4
```

```
)*x^6 + 9*b^2*c^4 + 6*a*b*c^3*d + 5*a^2*c^2*d^2 + 2*(9*b^2*c^3*d + 6*a*b*c^
2*d^2 + 5*a^2*c*d^3)*x^3)*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)/c^2)^(1/3)*log((
c*x*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)/c^2)^(2/3) - (b*x^3 + a)^(1/3)*(b*c -
a*d))/x) - 18*(b^2*c^2*d^2*x^6 + 2*b^2*c^3*d*x^3 + b^2*c^4)*(-b^2)^(1/3)*lo
g(-((-b^2)^(2/3)*x - (b*x^3 + a)^(1/3)*b)/x) + 9*(b^2*c^2*d^2*x^6 + 2*b^2*c
^3*d*x^3 + b^2*c^4)*(-b^2)^(1/3)*log(-((-b^2)^(1/3)*b*x^2 - (b*x^3 + a)^(1/
3)*(-b^2)^(2/3)*x - (b*x^3 + a)^(2/3)*b)/x^2) + ((9*b^2*c^2*d^2 + 6*a*b*c*d
^3 + 5*a^2*d^4)*x^6 + 9*b^2*c^4 + 6*a*b*c^3*d + 5*a^2*c^2*d^2 + 2*(9*b^2*c^
3*d + 6*a*b*c^2*d^2 + 5*a^2*c*d^3)*x^3)*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)/c^
2)^(1/3)*log(-((b*c - a*d)*x^2*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)/c^2)^(1/3)
+ (b*x^3 + a)^(1/3)*c*x*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)/c^2)^(2/3) + (b*x^
3 + a)^(2/3)*(b*c - a*d))/x^2) + 3*((9*b^2*c^2*d^2 - 4*a*b*c*d^3 - 5*a^2*d^
4)*x^4 + 2*(3*b^2*c^3*d + a*b*c^2*d^2 - 4*a^2*c*d^3)*x)*(b*x^3 + a)^(2/3))/
(c^2*d^5*x^6 + 2*c^3*d^4*x^3 + c^4*d^3)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^3+a\right)^{\frac{8}{3}}}{\left(dx^3+c\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(8/3)/(d*x^3+c)^3,x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a)^(8/3)/(d*x^3 + c)^3, x)
```

maple [F] time = 0.61, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^3+a\right)^{\frac{8}{3}}}{\left(d\,x^3+c\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(8/3)/(d*x^3+c)^3,x)
```

```
[Out] int((b*x^3+a)^(8/3)/(d*x^3+c)^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^3+a\right)^{\frac{8}{3}}}{\left(dx^3+c\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(8/3)/(d*x^3+c)^3,x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a)^(8/3)/(d*x^3 + c)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(b\,x^3+a\right)}^{8/3}}{{\left(d\,x^3+c\right)}^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^(8/3)/(c + d*x^3)^3,x)
```

```
[Out] int((a + b*x^3)^(8/3)/(c + d*x^3)^3, x)
```

<u>**sympy**</u> **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(8/3)/(d*x**3+c)**3,x)
```

```
[Out] Timed out
```

3.77 $\int \frac{\left(a+bx^3\right)^{5/3}}{\left(c+dx^3\right)^3}\,dx$

Optimal. Leaf size=217

$$\frac{5a^2\log\left(c+dx^3\right)}{54c^{8/3}\sqrt[3]{bc-ad}}-\frac{5a^2\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{18c^{8/3}\sqrt[3]{bc-ad}}+\frac{5a^2\tan^{-1}\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{9\sqrt{3}\,c^{8/3}\sqrt[3]{bc-ad}}+\frac{5ax\left(a+bx^3\right)^{2/3}}{18c^2\left(c+dx^3\right)}+\frac{x\left(a+bx^3\right)^{5/3}}{6c\left(c+dx^3\right)^2}$$

Rubi [A] time = 0.24, antiderivative size = 276, normalized size of antiderivative = 1.27, number of steps used = 9, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {378, 377, 200, 31, 634, 617, 204, 628}

$$-\frac{5a^2\log\left(\sqrt[3]{c}-\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{a+bx^3}}\right)}{27c^{8/3}\sqrt[3]{bc-ad}}+\frac{5a^2\log\left(\frac{x^2(bc-ad)^{2/3}}{\left(a+bx^3\right)^{2/3}}+\frac{\sqrt[3]{c}\,x\sqrt[3]{bc-ad}}{\sqrt[3]{a+bx^3}}+c^{2/3}\right)}{54c^{8/3}\sqrt[3]{bc-ad}}+\frac{5a^2\tan^{-1}\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{a+bx^3}}+\sqrt[3]{c}}{\sqrt{3}\sqrt[3]{c}}\right)}{9\sqrt{3}\,c^{8/3}\sqrt[3]{bc-ad}}+\frac{5ax\left(a+bx^3\right)^{2/3}}{18c^2\left(c+dx^3\right)}+\frac{x\left(a+bx^3\right)^{5/3}}{6c\left(c+dx^3\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^(5/3)/(c + d*x^3)^3,x]
```

```
[Out] (x*(a + b*x^3)^(5/3))/(6*c*(c + d*x^3)^2) + (5*a*x*(a + b*x^3)^(2/3))/(18*c
^2*(c + d*x^3)) + (5*a^2*ArcTan[(c^(1/3) + (2*(b*c - a*d)^(1/3)*x)/(a + b*x
^3)^(1/3))/(Sqrt[3]*c^(1/3))])/(9*Sqrt[3]*c^(8/3)*(b*c - a*d)^(1/3)) - (5*a
^2*Log[c^(1/3) - ((b*c - a*d)^(1/3)*x)/(a + b*x^3)^(1/3)])/(27*c^(8/3)*(b*c
 - a*d)^(1/3)) + (5*a^2*Log[c^(2/3) + ((b*c - a*d)^(2/3)*x^2)/(a + b*x^3)^(
2/3) + (c^(1/3)*(b*c - a*d)^(1/3)*x)/(a + b*x^3)^(1/3)])/(54*c^(8/3)*(b*c -
 a*d)^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 378

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> -Simp[(x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*n*(p + 1)), x] - Dist[(c
```

```
*q)/(a*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1, 0]
 && GtQ[q, 0] && NeQ[p, -1]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{\left(a+bx^3\right)^{5/3}}{\left(c+dx^3\right)^3}\,dx = \frac{x\left(a+bx^3\right)^{5/3}}{6c\left(c+dx^3\right)^2}+\frac{(5a)\int \frac{\left(a+bx^3\right)^{2/3}}{\left(c+dx^3\right)^2}\,dx}{6c}$$

$$= \frac{x\left(a+bx^3\right)^{5/3}}{6c\left(c+dx^3\right)^2}+\frac{5ax\left(a+bx^3\right)^{2/3}}{18c^2\left(c+dx^3\right)}+\frac{\left(5a^2\right)\int \frac{1}{\sqrt[3]{a+bx^3}\left(c+dx^3\right)}\,dx}{9c^2}$$

$$= \frac{x\left(a+bx^3\right)^{5/3}}{6c\left(c+dx^3\right)^2}+\frac{5ax\left(a+bx^3\right)^{2/3}}{18c^2\left(c+dx^3\right)}+\frac{\left(5a^2\right)\mathrm{Subst}\left(\int \frac{1}{c-(bc-ad)x^3}\,dx,x,\frac{x}{\sqrt[3]{a+bx^3}}\right)}{9c^2}$$

$$= \frac{x\left(a+bx^3\right)^{5/3}}{6c\left(c+dx^3\right)^2}+\frac{5ax\left(a+bx^3\right)^{2/3}}{18c^2\left(c+dx^3\right)}+\frac{\left(5a^2\right)\mathrm{Subst}\left(\int \frac{1}{\sqrt[3]{c}-\sqrt[3]{bc-ad}\,x}\,dx,x,\frac{x}{\sqrt[3]{a+bx^3}}\right)}{27c^{8/3}}+\frac{\left(5a^2\right)\mathrm{Su}\ldots}{\ldots}$$

$$= \frac{x\left(a+bx^3\right)^{5/3}}{6c\left(c+dx^3\right)^2}+\frac{5ax\left(a+bx^3\right)^{2/3}}{18c^2\left(c+dx^3\right)}-\frac{5a^2\log\left(\sqrt[3]{c}-\frac{\sqrt[3]{bc-ad}\,x}{\sqrt[3]{a+bx^3}}\right)}{27c^{8/3}\sqrt[3]{bc-ad}}+\frac{\left(5a^2\right)\mathrm{Subst}\left(\int \frac{\ldots}{c^{2/3}+\sqrt[3]{c}\,\sqrt[3]{bc\ldots}}\right.}{1\ldots}$$

$$= \frac{x\left(a+bx^3\right)^{5/3}}{6c\left(c+dx^3\right)^2}+\frac{5ax\left(a+bx^3\right)^{2/3}}{18c^2\left(c+dx^3\right)}-\frac{5a^2\log\left(\sqrt[3]{c}-\frac{\sqrt[3]{bc-ad}\,x}{\sqrt[3]{a+bx^3}}\right)}{27c^{8/3}\sqrt[3]{bc-ad}}+\frac{5a^2\log\left(c^{2/3}+\frac{(bc-ad)^{2/3}x^2}{\left(a+bx^3\right)^{2/3}}\ldots\right.}{54c^{8/3}\sqrt[3]{bc-a\ldots}}$$

$$= \frac{x\left(a+bx^3\right)^{5/3}}{6c\left(c+dx^3\right)^2}+\frac{5ax\left(a+bx^3\right)^{2/3}}{18c^2\left(c+dx^3\right)}+\frac{5a^2\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{bc-ad}\,x}{\sqrt[3]{c}\,\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{9\sqrt{3}\,c^{8/3}\sqrt[3]{bc-ad}}-\frac{5a^2\log\left(\sqrt[3]{c}-\frac{\sqrt[3]{bc-ad}\,x}{\sqrt[3]{a+bx^3}}\right)}{27c^{8/3}\sqrt[3]{bc-ad}}+\ldots$$

Mathematica [C] time = 0.03, size = 79, normalized size = 0.36

$$\frac{ax\left(a+bx^3\right)^{2/3}\,{}_2F_1\left(-\frac{5}{3},\frac{1}{3};\frac{4}{3};\frac{(ad-bc)x^3}{a\left(dx^3+c\right)}\right)}{c^3\left(\frac{bx^3}{a}+1\right)^{2/3}\sqrt[3]{\frac{dx^3}{c}+1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^3)^(5/3)/(c + d*x^3)^3,x]
```

```
[Out] (a*x*(a + b*x^3)^(2/3)*Hypergeometric2F1[-5/3, 1/3, 4/3, ((-(b*c) + a*d)*x^
3)/(a*(c + d*x^3))])/(c^3*(1 + (b*x^3)/a)^(2/3)*(1 + (d*x^3)/c)^(1/3))
```

IntegrateAlgebraic [C] time = 3.31, size = 385, normalized size = 1.77

$$\frac{5\left(a^2+i\sqrt{3}a^2\right)\log\left(2x\sqrt[3]{bc-ad}+\left(1+i\sqrt{3}\right)\sqrt[3]{c}\sqrt[3]{a+bx^3}\right)}{54c^{8/3}\sqrt[3]{bc-ad}}-\frac{5\sqrt{-1+i\sqrt{3}}\,a^2\tan^{-1}\left(\frac{3x\sqrt[3]{bc-ad}}{\sqrt{3}x\sqrt[3]{bc-ad}-\sqrt{3}\sqrt[3]{c}\sqrt[3]{a+bx^3}-3i\sqrt[3]{c}\sqrt[3]{a+bx^3}}\right)}{9\sqrt{6}c^{8/3}\sqrt[3]{bc-ad}}-\frac{5i\left(\sqrt{3}a^2-ia^2\right)\log\left(\left(\sqrt{3}+i\right)c^{2/3}\left(a+bx^3\right)^{2/3}+\sqrt[3]{c}\left(-\sqrt{3}x+ix\right)\sqrt[3]{a+bx^3}\sqrt[3]{bc-ad}-2ix^2(bc-ad)^{2/3}\right)}{108c^{8/3}\sqrt[3]{bc-ad}}+\frac{\left(a+bx^3\right)^{2/3}\left(8acx+5adx^4+3bcx^4\right)}{18c^2\left(c+dx^3\right)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x^3)^(5/3)/(c + d*x^3)^3,x]
```

```
[Out] ((a + b*x^3)^(2/3)*(8*a*c*x + 3*b*c*x^4 + 5*a*d*x^4))/(18*c^2*(c + d*x^3)^2
) - (5*Sqrt[-1 + I*Sqrt[3]]*a^2*ArcTan[(3*(b*c - a*d)^(1/3)*x)/(Sqrt[3]*(b*
c - a*d)^(1/3)*x - (3*I)*c^(1/3)*(a + b*x^3)^(1/3) - Sqrt[3]*c^(1/3)*(a + b
*x^3)^(1/3))])/(9*Sqrt[6]*c^(8/3)*(b*c - a*d)^(1/3)) + (5*(a^2 + I*Sqrt[3]*
a^2)*Log[2*(b*c - a*d)^(1/3)*x + (1 + I*Sqrt[3])*c^(1/3)*(a + b*x^3)^(1/3)]
)/(54*c^(8/3)*(b*c - a*d)^(1/3)) - (((5*I)/108)*((-I)*a^2 + Sqrt[3]*a^2)*Lo
g[(-2*I)*(b*c - a*d)^(2/3)*x^2 + c^(1/3)*(b*c - a*d)^(1/3)*(I*x - Sqrt[3]*x
)*(a + b*x^3)^(1/3) + (I + Sqrt[3])*c^(2/3)*(a + b*x^3)^(2/3)])/(c^(8/3)*(b
*c - a*d)^(1/3))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(5/3)/(d*x^3+c)^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(bx^3+a\right)^{\frac{5}{3}}}{\left(dx^3+c\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(5/3)/(d*x^3+c)^3,x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a)^(5/3)/(d*x^3 + c)^3, x)
```

maple [F] time = 0.57, size = 0, normalized size = 0.00

$$\int\frac{\left(b\,x^3+a\right)^{\frac{5}{3}}}{\left(d\,x^3+c\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(5/3)/(d*x^3+c)^3,x)
```

```
[Out] int((b*x^3+a)^(5/3)/(d*x^3+c)^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^3 + a\right)^{\frac{5}{3}}}{\left(dx^3 + c\right)^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(5/3)/(d*x^3+c)^3,x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a)^(5/3)/(d*x^3 + c)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(b\,x^3+a\right)^{5/3}}{\left(d\,x^3+c\right)^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^(5/3)/(c + d*x^3)^3,x)
```

```
[Out] int((a + b*x^3)^(5/3)/(c + d*x^3)^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(5/3)/(d*x**3+c)**3,x)
```

```
[Out] Timed out
```

3.78 $\int \frac{(a+bx^3)^{2/3}}{(c+dx^3)^3}\,dx$

Optimal. Leaf size=267

$$\frac{a(6bc-5ad)\log\left(c+dx^3\right)}{54c^{8/3}(bc-ad)^{4/3}}-\frac{a(6bc-5ad)\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{18c^{8/3}(bc-ad)^{4/3}}+\frac{a(6bc-5ad)\tan^{-1}\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{9\sqrt{3}\,c^{8/3}(bc-ad)^{4/3}}+\frac{x\left(a+bx\right.}{18c^2\left(c\right.}$$

Rubi [A] time = 0.24, antiderivative size = 326, normalized size of antiderivative = 1.22, number of steps used = 9, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {382, 378, 377, 200, 31, 634, 617, 204, 628}

$$\frac{x\left(a+bx^3\right)^{2/3}(6bc-5ad)}{18c^2\left(c+dx^3\right)(bc-ad)}-\frac{a(6bc-5ad)\log\left(\sqrt[3]{c}-\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{a+bx^3}}\right)}{27c^{8/3}(bc-ad)^{4/3}}+\frac{a(6bc-5ad)\log\left(\frac{x^2(bc-ad)^{2/3}}{\left(a+bx^3\right)^{2/3}}+\frac{\sqrt[3]{c}\,x\sqrt[3]{bc-ad}}{\sqrt[3]{a+bx^3}}+c^{2/3}\right)}{54c^{8/3}(bc-ad)^{4/3}}+\frac{a(6bc-5ad)\tan^{-1}\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{a+bx^3}}+\sqrt[3]{c}}{\sqrt{3}\sqrt[3]{c}}\right)}{9\sqrt{3}\,c^{8/3}(bc-ad)^{4/3}}-\frac{dx\left(a+bx^3\right)^{5/3}}{6c\left(c+dx^3\right)^2(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^(2/3)/(c + d*x^3)^3,x]
```

```
[Out] -(d*x*(a + b*x^3)^(5/3))/(6*c*(b*c - a*d)*(c + d*x^3)^2) + ((6*b*c - 5*a*d)
*x*(a + b*x^3)^(2/3))/(18*c^2*(b*c - a*d)*(c + d*x^3)) + (a*(6*b*c - 5*a*d)
*ArcTan[(c^(1/3) + (2*(b*c - a*d)^(1/3)*x)/(a + b*x^3)^(1/3))/(Sqrt[3]*c^(1
/3))])/(9*Sqrt[3]*c^(8/3)*(b*c - a*d)^(4/3)) - (a*(6*b*c - 5*a*d)*Log[c^(1/
3) - ((b*c - a*d)^(1/3)*x)/(a + b*x^3)^(1/3)])/(27*c^(8/3)*(b*c - a*d)^(4/3
)) + (a*(6*b*c - 5*a*d)*Log[c^(2/3) + ((b*c - a*d)^(2/3)*x^2)/(a + b*x^3)^(
2/3) + (c^(1/3)*(b*c - a*d)^(1/3)*x)/(a + b*x^3)^(1/3)])/(54*c^(8/3)*(b*c -
 a*d)^(4/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 378

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> -Simp[(x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*n*(p + 1)), x] - Dist[(c
```

```
*q)/(a*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1, 0]
 && GtQ[q, 0] && NeQ[p, -1]
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)), I
nt[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q}, x
] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !LtQ
[q, -1]) && NeQ[p, -1]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{\left(a+bx^3\right)^{2/3}}{\left(c+dx^3\right)^3}\,dx = -\frac{dx\left(a+bx^3\right)^{5/3}}{6c(bc-ad)\left(c+dx^3\right)^2} + \frac{(6bc-5ad)\int \frac{\left(a+bx^3\right)^{2/3}}{\left(c+dx^3\right)^2}\,dx}{6c(bc-ad)}$$

$$= -\frac{dx\left(a+bx^3\right)^{5/3}}{6c(bc-ad)\left(c+dx^3\right)^2} + \frac{(6bc-5ad)x\left(a+bx^3\right)^{2/3}}{18c^2(bc-ad)\left(c+dx^3\right)} + \frac{(a(6bc-5ad))\int \frac{1}{\sqrt[3]{a+bx^3}\left(c+dx^3\right)}\,dx}{9c^2(bc-ad)}$$

$$= -\frac{dx\left(a+bx^3\right)^{5/3}}{6c(bc-ad)\left(c+dx^3\right)^2} + \frac{(6bc-5ad)x\left(a+bx^3\right)^{2/3}}{18c^2(bc-ad)\left(c+dx^3\right)} + \frac{(a(6bc-5ad))\,\text{Subst}\left(\int \frac{1}{c-(bc-ad)x^3}\,dx\right.}{9c^2(bc-ad)}$$

$$= -\frac{dx\left(a+bx^3\right)^{5/3}}{6c(bc-ad)\left(c+dx^3\right)^2} + \frac{(6bc-5ad)x\left(a+bx^3\right)^{2/3}}{18c^2(bc-ad)\left(c+dx^3\right)} + \frac{(a(6bc-5ad))\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{c}-\sqrt[3]{bc-ad}\,x}\right.}{27c^{8/3}(bc-ad)}$$

$$= -\frac{dx\left(a+bx^3\right)^{5/3}}{6c(bc-ad)\left(c+dx^3\right)^2} + \frac{(6bc-5ad)x\left(a+bx^3\right)^{2/3}}{18c^2(bc-ad)\left(c+dx^3\right)} - \frac{a(6bc-5ad)\log\left(\sqrt[3]{c}-\frac{\sqrt[3]{bc-ad}\,x}{\sqrt[3]{a+bx^3}}\right)}{27c^{8/3}(bc-ad)^{4/3}} + $$

$$= -\frac{dx\left(a+bx^3\right)^{5/3}}{6c(bc-ad)\left(c+dx^3\right)^2} + \frac{(6bc-5ad)x\left(a+bx^3\right)^{2/3}}{18c^2(bc-ad)\left(c+dx^3\right)} - \frac{a(6bc-5ad)\log\left(\sqrt[3]{c}-\frac{\sqrt[3]{bc-ad}\,x}{\sqrt[3]{a+bx^3}}\right)}{27c^{8/3}(bc-ad)^{4/3}} + $$

$$= -\frac{dx\left(a+bx^3\right)^{5/3}}{6c(bc-ad)\left(c+dx^3\right)^2} + \frac{(6bc-5ad)x\left(a+bx^3\right)^{2/3}}{18c^2(bc-ad)\left(c+dx^3\right)} + \frac{a(6bc-5ad)\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{bc-ad}\,x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{9\sqrt{3}\,c^{8/3}(bc-ad)^{4/3}} - $$

Mathematica [C] time = 0.22, size = 153, normalized size = 0.57

$$\frac{x\left(c\left(-a^2d\left(8c+5dx^3\right)+ab\left(6c^2-5cdx^3-5d^2x^6\right)+3b^2cx^3\left(2c+dx^3\right)\right)-2a\left(c+dx^3\right)^2(5ad-6bc)\,{}_2F_1\left(\frac{1}{3},1;\frac{4}{3};\frac{(bc-ad)x^3}{c\left(bx^3+a\right)}\right)\right)}{18c^3\sqrt[3]{a+bx^3}\left(c+dx^3\right)^2(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^(2/3)/(c + d*x^3)^3,x]
```

```
[Out] (x*(c*(3*b^2*c*x^3*(2*c + d*x^3) - a^2*d*(8*c + 5*d*x^3) + a*b*(6*c^2 - 5*c
*d*x^3 - 5*d^2*x^6)) - 2*a*(-6*b*c + 5*a*d)*(c + d*x^3)^2*Hypergeometric2F1
[1/3, 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))]))/(18*c^3*(b*c - a*d)*(a +
 b*x^3)^(1/3)*(c + d*x^3)^2)
```

IntegrateAlgebraic [C] time = 3.66, size = 444, normalized size = 1.66

$$\frac{\left(-5i\sqrt{3}a^2d-5a^2d+6i\sqrt{3}abc+6abc\right)\log\left(2x\sqrt[3]{bc-ad}+\left(1+i\sqrt{3}\right)\sqrt[3]{c}\sqrt[3]{a+bx^3}\right)}{54c^{8/3}(bc-ad)^{4/3}}+\frac{\left(5i\sqrt{3}a^2d-15a^2d-6i\sqrt{3}abc+18abc\right)\tanh^{-1}\left(\frac{ix\sqrt[3]{bc-ad}+\left(\sqrt{3}-i\right)\sqrt[3]{c}\sqrt[3]{a+bx^3}}{\sqrt{3}x\sqrt[3]{bc-ad}}\right)}{54c^{8/3}(bc-ad)^{4/3}}+\frac{\left(5i\sqrt{3}a^2d+5a^2d-6i\sqrt{3}abc-6abc\right)\log\left(\left(\sqrt{3}+i\right)c^{2/3}\left(a+bx^3\right)^{2/3}+\sqrt[3]{c}\left(-\sqrt{3}x+ix\right)\sqrt[3]{a+bx^3}\sqrt[3]{bc-ad}-2ix^2(bc-ad)^{2/3}\right)}{108c^{8/3}(bc-ad)^{4/3}}+\frac{\left(a+bx^3\right)^{2/3}\left(-8acdx-5ad^2x^4+6bc^2x+3bcdx^4\right)}{18c^2\left(c+dx^3\right)^2(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x^3)^(2/3)/(c + d*x^3)^3,x]
```

```
[Out] ((a + b*x^3)^(2/3)*(6*b*c^2*x - 8*a*c*d*x + 3*b*c*d*x^4 - 5*a*d^2*x^4))/(18
*c^2*(b*c - a*d)*(c + d*x^3)^2) + ((18*a*b*c - (6*I)*Sqrt[3]*a*b*c - 15*a^2
*d + (5*I)*Sqrt[3]*a^2*d)*ArcTanh[(I*(b*c - a*d)^(1/3)*x + (-I + Sqrt[3])*c
^(1/3)*(a + b*x^3)^(1/3))/(Sqrt[3]*(b*c - a*d)^(1/3)*x)])/(54*c^(8/3)*(b*c
```

```
- a*d)^(4/3)) + ((6*a*b*c + (6*I)*Sqrt[3]*a*b*c - 5*a^2*d - (5*I)*Sqrt[3]*a
^2*d)*Log[2*(b*c - a*d)^(1/3)*x + (1 + I*Sqrt[3])*c^(1/3)*(a + b*x^3)^(1/3)
])/(54*c^(8/3)*(b*c - a*d)^(4/3)) + ((-6*a*b*c - (6*I)*Sqrt[3]*a*b*c + 5*a^
2*d + (5*I)*Sqrt[3]*a^2*d)*Log[(-2*I)*(b*c - a*d)^(2/3)*x^2 + c^(1/3)*(b*c
- a*d)^(1/3)*(I*x - Sqrt[3]*x)*(a + b*x^3)^(1/3) + (I + Sqrt[3])*c^(2/3)*(a
 + b*x^3)^(2/3)])/(108*c^(8/3)*(b*c - a*d)^(4/3))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)/(d*x^3+c)^3,x, algorithm="fricas")
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^3+a\right)^{\frac{2}{3}}}{\left(dx^3+c\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)/(d*x^3+c)^3,x, algorithm="giac")
[Out] integrate((b*x^3 + a)^(2/3)/(d*x^3 + c)^3, x)
```

maple [F] time = 0.55, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^3+a\right)^{\frac{2}{3}}}{\left(d\,x^3+c\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(2/3)/(d*x^3+c)^3,x)
[Out] int((b*x^3+a)^(2/3)/(d*x^3+c)^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^3+a\right)^{\frac{2}{3}}}{\left(dx^3+c\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)/(d*x^3+c)^3,x, algorithm="maxima")
[Out] integrate((b*x^3 + a)^(2/3)/(d*x^3 + c)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(b\,x^3+a\right)}^{2/3}}{{\left(d\,x^3+c\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^(2/3)/(c + d*x^3)^3,x)
```

```
[Out] int((a + b*x^3)^(2/3)/(c + d*x^3)^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(2/3)/(d*x**3+c)**3,x)
```

```
[Out] Timed out
```

3.79 $\int \frac{1}{\sqrt[3]{a+bx^3}\left(c+dx^3\right)^3}\,dx$

Optimal. Leaf size=307

$$\frac{\left(5a^2d^2-12abcd+9b^2c^2\right)\log\left(c+dx^3\right)}{54c^{8/3}(bc-ad)^{7/3}}-\frac{\left(5a^2d^2-12abcd+9b^2c^2\right)\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{18c^{8/3}(bc-ad)^{7/3}}+\frac{\left(5a^2d^2-12ab\right.}{9}$$

Rubi [C] time = 0.31, antiderivative size = 167, normalized size of antiderivative = 0.54, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {430, 429}

$$-\frac{x\left(cd\left(-a^2d\left(8c+5dx^3\right)+ab\left(12c^2+cdx^3-5d^2x^6\right)+3b^2cx^3\left(4c+3dx^3\right)\right)-2\left(c+dx^3\right)^2\left(5a^2d^2-12abcd+9b^2c^2\right){}_2F_1\left(\frac{1}{3},1;\frac{4}{3};\frac{(bc-ad)x^3}{c\left(bx^3+a\right)}\right)\right)}{18c^3\sqrt[3]{a+bx^3}\left(c+dx^3\right)^2(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^3)^(1/3)*(c + d*x^3)^3),x]
```

```
[Out] -(x*(c*d*(3*b^2*c*x^3*(4*c + 3*d*x^3) - a^2*d*(8*c + 5*d*x^3) + a*b*(12*c^2
 + c*d*x^3 - 5*d^2*x^6)) - 2*(9*b^2*c^2 - 12*a*b*c*d + 5*a^2*d^2)*(c + d*x^
3)^2*Hypergeometric2F1[1/3, 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))]))/(1
8*c^3*(b*c - a*d)^2*(a + b*x^3)^(1/3)*(c + d*x^3)^2)
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] && !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{1}{\sqrt[3]{a+bx^3}\left(c+dx^3\right)^3}\,dx=\frac{\sqrt[3]{1+\frac{bx^3}{a}}\int \frac{1}{\sqrt[3]{1+\frac{bx^3}{a}}\left(c+dx^3\right)^3}\,dx}{\sqrt[3]{a+bx^3}}$$

$$=-\frac{x\left(cd\left(3b^2cx^3\left(4c+3dx^3\right)-a^2d\left(8c+5dx^3\right)+ab\left(12c^2+cdx^3-5d^2x^6\right)\right)-2\left(9\right.\right.}{18c^3(bc-ad)^2\sqrt[3]{a+bx^3}\left(c\right.}$$

Mathematica [C] time = 0.38, size = 168, normalized size = 0.55

$$\frac{x\left(2\left(c+dx^3\right)^2\left(5a^2d^2-12abcd+9b^2c^2\right){}_2F_1\left(\frac{1}{3},1;\frac{4}{3};\frac{(bc-ad)x^3}{c\left(bx^3+a\right)}\right)-cd\left(-a^2d\left(8c+5dx^3\right)+ab\left(12c^2+cdx^3-5d^2x^6\right)+3b^2cx^3\left(4c+3dx^3\right)\right)\right)}{18c^3\sqrt[3]{a+bx^3}\left(c+dx^3\right)^2(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x^3)^(1/3)*(c + d*x^3)^3),x]
```

```
[Out] (x*(-(c*d*(3*b^2*c*x^3*(4*c + 3*d*x^3) - a^2*d*(8*c + 5*d*x^3) + a*b*(12*c^
2 + c*d*x^3 - 5*d^2*x^6))) + 2*(9*b^2*c^2 - 12*a*b*c*d + 5*a^2*d^2)*(c + d*
x^3)^2*Hypergeometric2F1[1/3, 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))]))/
(18*c^3*(b*c - a*d)^2*(a + b*x^3)^(1/3)*(c + d*x^3)^2)
```

IntegrateAlgebraic [C] time = 5.65, size = 536, normalized size = 1.75

$$\frac{\left(5i\sqrt{3}\,a^2d^2+5a^2d^2-12i\sqrt{3}\,abcd-12abcd+9i\sqrt{3}\,b^2c^2+9b^2c^2\right)\log\left(2x\sqrt[3]{bc-ad}+\left(1+i\sqrt{3}\right)\sqrt[3]{c}\,\sqrt[3]{a+bx^3}\right)}{54c^{8/3}(bc-ad)^{7/3}}+\frac{\left(-5i\sqrt{3}\,a^2d^2+15a^2d^2+12i\sqrt{3}\,abcd-36abcd-9i\sqrt{3}\,b^2c^2+27b^2c^2\right)\tanh^{-1}\left(\frac{ix\sqrt[3]{bc-ad}+\left(\sqrt{3}-i\right)\sqrt[3]{c}\,\sqrt[3]{a+bx^3}}{\sqrt{3}\,x\sqrt[3]{bc-ad}}\right)}{54c^{8/3}(bc-ad)^{7/3}}+\frac{\left(-5i\sqrt{3}\,a^2d^2-5a^2d^2+12i\sqrt{3}\,abcd+12abcd-9i\sqrt{3}\,b^2c^2-9b^2c^2\right)\log\left(\left(\sqrt{3}+i\right)c^{2/3}\left(a+bx^3\right)^{2/3}+\sqrt[3]{c}\left(-\sqrt{3}\,x+ix\right)\sqrt[3]{a+bx^3}\,\sqrt[3]{bc-ad}-2ix^2(bc-ad)^{2/3}\right)}{108c^{8/3}(bc-ad)^{7/3}}+\frac{\left(a+bx^3\right)^{2/3}\left(8acd^2x+5ad^3x^4-12bc^2dx-9bcd^2x^4\right)}{18c^2\left(c+dx^3\right)^2(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((a + b*x^3)^(1/3)*(c + d*x^3)^3),x]
```

```
[Out] ((a + b*x^3)^(2/3)*(-12*b*c^2*d*x + 8*a*c*d^2*x - 9*b*c*d^2*x^4 + 5*a*d^3*x
^4))/(18*c^2*(b*c - a*d)^2*(c + d*x^3)^2) + ((27*b^2*c^2 - (9*I)*Sqrt[3]*b^
2*c^2 - 36*a*b*c*d + (12*I)*Sqrt[3]*a*b*c*d + 15*a^2*d^2 - (5*I)*Sqrt[3]*a^
2*d^2)*ArcTanh[(I*(b*c - a*d)^(1/3)*x + (-I + Sqrt[3])*c^(1/3)*(a + b*x^3)^
(1/3))/(Sqrt[3]*(b*c - a*d)^(1/3)*x)])/(54*c^(8/3)*(b*c - a*d)^(7/3)) + ((9
*b^2*c^2 + (9*I)*Sqrt[3]*b^2*c^2 - 12*a*b*c*d - (12*I)*Sqrt[3]*a*b*c*d + 5*
a^2*d^2 + (5*I)*Sqrt[3]*a^2*d^2)*Log[2*(b*c - a*d)^(1/3)*x + (1 + I*Sqrt[3]
)*c^(1/3)*(a + b*x^3)^(1/3)])/(54*c^(8/3)*(b*c - a*d)^(7/3)) + ((-9*b^2*c^2
 - (9*I)*Sqrt[3]*b^2*c^2 + 12*a*b*c*d + (12*I)*Sqrt[3]*a*b*c*d - 5*a^2*d^2
- (5*I)*Sqrt[3]*a^2*d^2)*Log[(-2*I)*(b*c - a*d)^(2/3)*x^2 + c^(1/3)*(b*c -
a*d)^(1/3)*(I*x - Sqrt[3]*x)*(a + b*x^3)^(1/3) + (I + Sqrt[3])*c^(2/3)*(a +
 b*x^3)^(2/3)])/(108*c^(8/3)*(b*c - a*d)^(7/3))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^(1/3)/(d*x^3+c)^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^3+a\right)^{\frac{1}{3}}\left(dx^3+c\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^(1/3)/(d*x^3+c)^3,x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^3 + a)^(1/3)*(d*x^3 + c)^3), x)
```

maple [F] time = 0.58, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,x^3+a\right)^{\frac{1}{3}}\left(d\,x^3+c\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^3+a)^(1/3)/(d*x^3+c)^3,x)
```

```
[Out] int(1/(b*x^3+a)^(1/3)/(d*x^3+c)^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^3+a\right)^{\frac{1}{3}}\left(dx^3+c\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^(1/3)/(d*x^3+c)^3,x, algorithm="maxima")
[Out] integrate(1/((b*x^3 + a)^(1/3)*(d*x^3 + c)^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(b\,x^3+a\right)}^{1/3}\,{\left(d\,x^3+c\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^3)^(1/3)*(c + d*x^3)^3),x)
[Out] int(1/((a + b*x^3)^(1/3)*(c + d*x^3)^3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[3]{a+bx^3}\left(c+dx^3\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**3+a)**(1/3)/(d*x**3+c)**3,x)
[Out] Integral(1/((a + b*x**3)**(1/3)*(c + d*x**3)**3), x)
```

3.80 $\int \frac{1}{\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)^3}\,dx$

Optimal. Leaf size=377

$$\frac{dx\left(a+bx^3\right)^{2/3}\left(-5a^2d^2+15abcd+18b^2c^2\right)}{18ac^2\left(c+dx^3\right)(bc-ad)^3}-\frac{d\left(5a^2d^2-18abcd+27b^2c^2\right)\log\left(c+dx^3\right)}{54c^{8/3}(bc-ad)^{10/3}}+\frac{d\left(5a^2d^2-18abcd+27\right.}{18c^{8/}}$$

Rubi [C] time = 2.74, antiderivative size = 428, normalized size of antiderivative = 1.14, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {430, 429}

$$-\frac{65c^2\left(a+bx^3\right)^2\left(-28\left(c+dx^3\right)^2\left(a^2\left(843c^2dx^3+500c^3+375cd^2x^6+27d^3x^9\right)+9abcx^3\left(73c^2+104cdx^3+33d^2x^6\right)+27b^2c^2x^6\left(7c+6dx^3\right)\right){}_2F_1\left(\frac{1}{3},1;\frac{4}{3};\frac{(bc-ad)x^3}{c\left(bx^3+a\right)}\right)+33657a^2c^2d^3x^9+60807a^2c^3d^2x^6+48104a^2c^4dx^3+14000a^2c^5+7155a^2cd^4x^{12}+243a^2d^5x^{15}+38652abc^2d^3x^{12}+81534abc^3d^2x^9+70802abc^4dx^6+21896abc^5x^3+5940abcd^4x^{15}+7425b^2c^2d^3x^{15}+23409b^2c^3d^2x^{12}+24417b^2c^4dx^9+8391b^2c^5x^6\right)-486x^{12}\left(c+dx^3\right)^3(bc-ad)^4\,{}_4F_3\left(2,2,2,\frac{7}{3};1,1,\frac{16}{3};\frac{(bc-ad)x^3}{c\left(bx^3+a\right)}\right)}{16380c^5x^8\left(a+bx^3\right)^{7/3}\left(c+dx^3\right)^2(bc-ad)^3}$$

Warning: Unable to verify antiderivative.

```
[In] Int[1/((a + b*x^3)^(4/3)*(c + d*x^3)^3),x]
```

```
[Out] -(65*c^2*(a + b*x^3)^2*(14000*a^2*c^5 + 21896*a*b*c^5*x^3 + 48104*a^2*c^4*d
*x^3 + 8391*b^2*c^5*x^6 + 70802*a*b*c^4*d*x^6 + 60807*a^2*c^3*d^2*x^6 + 244
17*b^2*c^4*d*x^9 + 81534*a*b*c^3*d^2*x^9 + 33657*a^2*c^2*d^3*x^9 + 23409*b^
2*c^3*d^2*x^12 + 38652*a*b*c^2*d^3*x^12 + 7155*a^2*c*d^4*x^12 + 7425*b^2*c^
2*d^3*x^15 + 5940*a*b*c*d^4*x^15 + 243*a^2*d^5*x^15 - 28*(c + d*x^3)^2*(27*
b^2*c^2*x^6*(7*c + 6*d*x^3) + 9*a*b*c*x^3*(73*c^2 + 104*c*d*x^3 + 33*d^2*x^
6) + a^2*(500*c^3 + 843*c^2*d*x^3 + 375*c*d^2*x^6 + 27*d^3*x^9))*Hypergeome
tric2F1[1/3, 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))]) - 486*(b*c - a*d)^
4*x^12*(c + d*x^3)^3*HypergeometricPFQ[{2, 2, 2, 7/3}, {1, 1, 16/3}, ((b*c
- a*d)*x^3)/(c*(a + b*x^3))])/(16380*c^5*(b*c - a*d)^3*x^8*(a + b*x^3)^(7/3
)*(c + d*x^3)^2)
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] && !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{1}{\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)^3}\,dx=\frac{\sqrt[3]{1+\frac{bx^3}{a}}\int \frac{1}{\left(1+\frac{bx^3}{a}\right)^{4/3}\left(c+dx^3\right)^3}\,dx}{a\sqrt[3]{a+bx^3}}$$

$$=-\frac{65c^2\left(a+bx^3\right)^2\left(14000a^2c^5+21896abc^5x^3+48104a^2c^4dx^3+8391b^2c^5x^6+708\right.}{}$$

Mathematica [C] time = 3.08, size = 428, normalized size = 1.14

$$-\frac{486x^{12}\left(c+dx^3\right)^3(bc-ad)^4\,{}_4F_3\left(2,2,2,\frac{7}{3};1,1,\frac{16}{3};\frac{(bc-ad)x^3}{c\left(bx^3+a\right)}\right)+65c^2\left(a+bx^3\right)^2\left(28\left(c+dx^3\right)^2\left(a^2\left(500c^3+843c^2dx^3+375cd^2x^6+27d^3x^9\right)+9abcx^3\left(73c^2+104cdx^3+33d^2x^6\right)+27b^2c^2x^6\left(7c+6dx^3\right)\right){}_2F_1\left(\frac{1}{3},1;\frac{4}{3};\frac{(bc-ad)x^3}{c\left(bx^3+a\right)}\right)-14000a^2c^5-48104a^2c^4dx^3-60807a^2c^3d^2x^6-33657a^2c^2d^3x^9-7155a^2cd^4x^{12}-243a^2d^5x^{15}-21896abc^5x^3-70802abc^4dx^6-81534abc^3d^2x^9-38652abc^2d^3x^{12}-5940abcd^4x^{15}-8391b^2c^5x^6-24417b^2c^4dx^9-23409b^2c^3d^2x^{12}-7425b^2c^2d^3x^{15}\right)}{16380c^5x^8\left(a+bx^3\right)^{7/3}\left(c+dx^3\right)^2(ad-bc)^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + b*x^3)^(4/3)*(c + d*x^3)^3),x]
```

```
[Out] -1/16380*(65*c^2*(a + b*x^3)^2*(-14000*a^2*c^5 - 21896*a*b*c^5*x^3 - 48104*
a^2*c^4*d*x^3 - 8391*b^2*c^5*x^6 - 70802*a*b*c^4*d*x^6 - 60807*a^2*c^3*d^2*
x^6 - 24417*b^2*c^4*d*x^9 - 81534*a*b*c^3*d^2*x^9 - 33657*a^2*c^2*d^3*x^9 -
 23409*b^2*c^3*d^2*x^12 - 38652*a*b*c^2*d^3*x^12 - 7155*a^2*c*d^4*x^12 - 74
25*b^2*c^2*d^3*x^15 - 5940*a*b*c*d^4*x^15 - 243*a^2*d^5*x^15 + 28*(c + d*x^
3)^2*(27*b^2*c^2*x^6*(7*c + 6*d*x^3) + 9*a*b*c*x^3*(73*c^2 + 104*c*d*x^3 +
33*d^2*x^6) + a^2*(500*c^3 + 843*c^2*d*x^3 + 375*c*d^2*x^6 + 27*d^3*x^9))*H
ypergeometric2F1[1/3, 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))]) + 486*(b*
c - a*d)^4*x^12*(c + d*x^3)^3*HypergeometricPFQ[{2, 2, 2, 7/3}, {1, 1, 16/3
}, ((b*c - a*d)*x^3)/(c*(a + b*x^3))])/(c^5*(-(b*c) + a*d)^3*x^8*(a + b*x^3
)^(7/3)*(c + d*x^3)^2)
```

IntegrateAlgebraic [C] time = 8.92, size = 644, normalized size = 1.71

$$\frac{\left(-5i\sqrt{3}\,a^2d^3-5a^2d^3+18i\sqrt{3}\,abcd^2+18abcd^2-27i\sqrt{3}\,b^2c^2d-27b^2c^2d\right)\log\left(2x\sqrt[3]{bc-ad}+\left(1+i\sqrt{3}\right)\sqrt[3]{c}\,\sqrt[3]{a+bx^3}\right)}{54c^{8/3}(bc-ad)^{10/3}}+\frac{\left(5i\sqrt{3}\,a^2d^3-15a^2d^3-18i\sqrt{3}\,abcd^2+54abcd^2+27i\sqrt{3}\,b^2c^2d-81b^2c^2d\right)\tanh^{-1}\left(\frac{ix\sqrt[3]{bc-ad}+\left(\sqrt{3}-i\right)\sqrt[3]{c}\,\sqrt[3]{a+bx^3}}{\sqrt{3}\,x\sqrt[3]{bc-ad}}\right)}{54c^{8/3}(bc-ad)^{10/3}}+\frac{\left(5i\sqrt{3}\,a^2d^3+5a^2d^3-18i\sqrt{3}\,abcd^2-18abcd^2+27i\sqrt{3}\,b^2c^2d+27b^2c^2d\right)\log\left(\left(\sqrt{3}+i\right)c^{2/3}\left(a+bx^3\right)^{2/3}+\sqrt[3]{c}\left(-\sqrt{3}\,x+ix\right)\sqrt[3]{a+bx^3}\,\sqrt[3]{bc-ad}-2ix^2(bc-ad)^{2/3}\right)}{108c^{8/3}(bc-ad)^{10/3}}+\frac{8a^3cd^3x+5a^3d^4x^4-18a^2bc^2d^2x-7a^2bcd^3x^4+5a^2bd^4x^7-18ab^2c^2d^2x^4-15ab^2cd^3x^7-18b^3c^4x-36b^3c^3dx^4-18b^3c^2d^2x^7}{18ac^2\sqrt[3]{a+bx^3}\left(c+dx^3\right)^2(ad-bc)^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((a + b*x^3)^(4/3)*(c + d*x^3)^3),x]
```

```
[Out] (-18*b^3*c^4*x - 18*a^2*b*c^2*d^2*x + 8*a^3*c*d^3*x - 36*b^3*c^3*d*x^4 - 18
*a*b^2*c^2*d^2*x^4 - 7*a^2*b*c*d^3*x^4 + 5*a^3*d^4*x^4 - 18*b^3*c^2*d^2*x^7
 - 15*a*b^2*c*d^3*x^7 + 5*a^2*b*d^4*x^7)/(18*a*c^2*(-(b*c) + a*d)^3*(a + b*
x^3)^(1/3)*(c + d*x^3)^2) + ((-81*b^2*c^2*d + (27*I)*Sqrt[3]*b^2*c^2*d + 54
*a*b*c*d^2 - (18*I)*Sqrt[3]*a*b*c*d^2 - 15*a^2*d^3 + (5*I)*Sqrt[3]*a^2*d^3)
*ArcTanh[(I*(b*c - a*d)^(1/3)*x + (-I + Sqrt[3])*c^(1/3)*(a + b*x^3)^(1/3))
/(Sqrt[3]*(b*c - a*d)^(1/3)*x)])/(54*c^(8/3)*(b*c - a*d)^(10/3)) + ((-27*b^
2*c^2*d - (27*I)*Sqrt[3]*b^2*c^2*d + 18*a*b*c*d^2 + (18*I)*Sqrt[3]*a*b*c*d^
2 - 5*a^2*d^3 - (5*I)*Sqrt[3]*a^2*d^3)*Log[2*(b*c - a*d)^(1/3)*x + (1 + I*S
qrt[3])*c^(1/3)*(a + b*x^3)^(1/3)])/(54*c^(8/3)*(b*c - a*d)^(10/3)) + ((27*
b^2*c^2*d + (27*I)*Sqrt[3]*b^2*c^2*d - 18*a*b*c*d^2 - (18*I)*Sqrt[3]*a*b*c*
d^2 + 5*a^2*d^3 + (5*I)*Sqrt[3]*a^2*d^3)*Log[(-2*I)*(b*c - a*d)^(2/3)*x^2 +
 c^(1/3)*(b*c - a*d)^(1/3)*(I*x - Sqrt[3]*x)*(a + b*x^3)^(1/3) + (I + Sqrt[
3])*c^(2/3)*(a + b*x^3)^(2/3)])/(108*c^(8/3)*(b*c - a*d)^(10/3))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^(4/3)/(d*x^3+c)^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^3+a\right)^{\frac{4}{3}}\left(dx^3+c\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^(4/3)/(d*x^3+c)^3,x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^3 + a)^(4/3)*(d*x^3 + c)^3), x)
```

maple [F] time = 0.59, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,x^{3}+a\right)^{\frac{4}{3}}\left(d\,x^{3}+c\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^3+a)^(4/3)/(d*x^3+c)^3,x)
```

```
[Out] int(1/(b*x^3+a)^(4/3)/(d*x^3+c)^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^{3}+a\right)^{\frac{4}{3}}\left(dx^{3}+c\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^(4/3)/(d*x^3+c)^3,x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^3 + a)^(4/3)*(d*x^3 + c)^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(b\,x^{3}+a\right)^{4/3}\,\left(d\,x^{3}+c\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^3)^(4/3)*(c + d*x^3)^3),x)
```

```
[Out] int(1/((a + b*x^3)^(4/3)*(c + d*x^3)^3), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**3+a)**(4/3)/(d*x**3+c)**3,x)
```

```
[Out] Timed out
```

3.81 $\int \frac{1}{\left(a+bx^3\right)^{7/3}\left(c+dx^3\right)^3}\,dx$

Optimal. Leaf size=463

$$\frac{bx\left(-2a^2d^2-42abcd+9b^2c^2\right)}{12a^2c\sqrt[3]{a+bx^3}\,\left(c+dx^3\right)(bc-ad)^3}+\frac{d^2\left(5a^2d^2-24abcd+54b^2c^2\right)\log\left(c+dx^3\right)}{54c^{8/3}(bc-ad)^{13/3}}-\frac{d^2\left(5a^2d^2-24abcd+54b^2c\right.}{18c^{8/3}(b}$$

Rubi [C] time = 8.66, antiderivative size = 1990, normalized size of antiderivative = 4.30, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {430, 429}

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Int[1/((a + b*x^3)^(7/3)*(c + d*x^3)^3),x]
```

```
[Out] -(522756*c^6*(b*c - a*d)^3*x^9*(a + b*x^3)^2 + 1516320*c^5*d*(b*c - a*d)^3*
x^12*(a + b*x^3)^2 + 2198664*c^4*d^2*(b*c - a*d)^3*x^15*(a + b*x^3)^2 + 141
5232*c^3*d^3*(b*c - a*d)^3*x^18*(a + b*x^3)^2 + 341172*c^2*d^4*(b*c - a*d)^
3*x^21*(a + b*x^3)^2 + 28042560*c^7*(b*c - a*d)^2*x^6*(a + b*x^3)^3 + 10760
2560*c^6*d*(b*c - a*d)^2*x^9*(a + b*x^3)^3 + 157697280*c^5*d^2*(b*c - a*d)^
2*x^12*(a + b*x^3)^3 + 101088000*c^4*d^3*(b*c - a*d)^2*x^15*(a + b*x^3)^3 +
 24261120*c^3*d^4*(b*c - a*d)^2*x^18*(a + b*x^3)^3 - 265470660*c^8*(b*c - a
*d)*x^3*(a + b*x^3)^4 - 1019636800*c^7*d*(b*c - a*d)*x^6*(a + b*x^3)^4 - 14
66086440*c^6*d^2*(b*c - a*d)*x^9*(a + b*x^3)^4 - 930252960*c^5*d^3*(b*c - a
*d)*x^12*(a + b*x^3)^4 - 221899860*c^4*d^4*(b*c - a*d)*x^15*(a + b*x^3)^4 +
 335877360*c^9*(a + b*x^3)^5 + 1279532800*c^8*d*x^3*(a + b*x^3)^5 + 1823334
240*c^7*d^2*x^6*(a + b*x^3)^5 + 1151579520*c^6*d^3*x^9*(a + b*x^3)^5 + 2739
39120*c^5*d^4*x^12*(a + b*x^3)^5 - 67420080*c^7*(b*c - a*d)^2*x^6*(a + b*x^
3)^3*Hypergeometric2F1[1/3, 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 25
9692160*c^6*d*(b*c - a*d)^2*x^9*(a + b*x^3)^3*Hypergeometric2F1[1/3, 1, 4/3
, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 377700960*c^5*d^2*(b*c - a*d)^2*x^12
*(a + b*x^3)^3*Hypergeometric2F1[1/3, 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x
^3))] - 241113600*c^4*d^3*(b*c - a*d)^2*x^15*(a + b*x^3)^3*Hypergeometric2F
1[1/3, 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 57723120*c^3*d^4*(b*c -
 a*d)^2*x^18*(a + b*x^3)^3*Hypergeometric2F1[1/3, 1, 4/3, ((b*c - a*d)*x^3)
/(c*(a + b*x^3))] + 349440000*c^8*(b*c - a*d)*x^3*(a + b*x^3)^4*Hypergeomet
ric2F1[1/3, 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 1339520000*c^7*d*(
b*c - a*d)*x^6*(a + b*x^3)^4*Hypergeometric2F1[1/3, 1, 4/3, ((b*c - a*d)*x^
3)/(c*(a + b*x^3))] + 1921920000*c^6*d^2*(b*c - a*d)*x^9*(a + b*x^3)^4*Hype
rgeometric2F1[1/3, 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 1218147840*
c^5*d^3*(b*c - a*d)*x^12*(a + b*x^3)^4*Hypergeometric2F1[1/3, 1, 4/3, ((b*c
 - a*d)*x^3)/(c*(a + b*x^3))] + 290384640*c^4*d^4*(b*c - a*d)*x^15*(a + b*x
^3)^4*Hypergeometric2F1[1/3, 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 3
35877360*c^9*(a + b*x^3)^5*Hypergeometric2F1[1/3, 1, 4/3, ((b*c - a*d)*x^3)
/(c*(a + b*x^3))] - 1279532800*c^8*d*x^3*(a + b*x^3)^5*Hypergeometric2F1[1/
3, 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 1823334240*c^7*d^2*x^6*(a +
 b*x^3)^5*Hypergeometric2F1[1/3, 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))]
 - 1151579520*c^6*d^3*x^9*(a + b*x^3)^5*Hypergeometric2F1[1/3, 1, 4/3, ((b*
c - a*d)*x^3)/(c*(a + b*x^3))] - 273939120*c^5*d^4*x^12*(a + b*x^3)^5*Hyper
geometric2F1[1/3, 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 57834*c^4*(b
*c - a*d)^5*x^15*HypergeometricPFQ[{2, 2, 2, 10/3}, {1, 1, 19/3}, ((b*c - a
*d)*x^3)/(c*(a + b*x^3))] - 224532*c^3*d*(b*c - a*d)^5*x^18*HypergeometricP
FQ[{2, 2, 2, 10/3}, {1, 1, 19/3}, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 3265
92*c^2*d^2*(b*c - a*d)^5*x^21*HypergeometricPFQ[{2, 2, 2, 10/3}, {1, 1, 19/
```

```
3}, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 210924*c*d^3*(b*c - a*d)^5*x^24*Hy
pergeometricPFQ[{2, 2, 2, 10/3}, {1, 1, 19/3}, ((b*c - a*d)*x^3)/(c*(a + b*
x^3))] - 51030*d^4*(b*c - a*d)^5*x^27*HypergeometricPFQ[{2, 2, 2, 10/3}, {1
, 1, 19/3}, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 5103*c^4*(b*c - a*d)^5*x^1
5*HypergeometricPFQ[{2, 2, 2, 2, 10/3}, {1, 1, 1, 19/3}, ((b*c - a*d)*x^3)/
(c*(a + b*x^3))] - 20412*c^3*d*(b*c - a*d)^5*x^18*HypergeometricPFQ[{2, 2,
2, 2, 10/3}, {1, 1, 1, 19/3}, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 30618*c^
2*d^2*(b*c - a*d)^5*x^21*HypergeometricPFQ[{2, 2, 2, 2, 10/3}, {1, 1, 1, 19
/3}, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 20412*c*d^3*(b*c - a*d)^5*x^24*Hy
pergeometricPFQ[{2, 2, 2, 2, 10/3}, {1, 1, 1, 19/3}, ((b*c - a*d)*x^3)/(c*(
a + b*x^3))] - 5103*d^4*(b*c - a*d)^5*x^27*HypergeometricPFQ[{2, 2, 2, 2, 1
0/3}, {1, 1, 1, 19/3}, ((b*c - a*d)*x^3)/(c*(a + b*x^3))])/(524160*c^6*(b*c
 - a*d)^4*x^11*(a + b*x^3)^(10/3)*(c + d*x^3)^2)
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{1}{\left(a+bx^3\right)^{7/3}\left(c+dx^3\right)^3}\,dx = \frac{\sqrt[3]{1+\frac{bx^3}{a}}\int \frac{1}{\left(1+\frac{bx^3}{a}\right)^{7/3}\left(c+dx^3\right)^3}\,dx}{a^2\sqrt[3]{a+bx^3}}$$

$$= -\frac{522756c^6(bc-ad)^3x^9\left(a+bx^3\right)^2 + 1516320c^5d(bc-ad)^3x^{12}\left(a+bx^3\right)^2 + 21986\ldots}{\ldots}$$

Mathematica [A] time = 5.88, size = 337, normalized size = 0.73

$$\frac{1}{36}x\left(a+bx^3\right)^{2/3}\left(\frac{27b^3(bc-5ad)}{a^2\left(a+bx^3\right)(bc-ad)^4}-\frac{9b^3}{a\left(a+bx^3\right)^2(ad-bc)^3}+\frac{2d^3(5ad-21bc)}{c^2\left(c+dx^3\right)(bc-ad)^4}-\frac{6d^3}{c\left(c+dx^3\right)^2(bc-ad)^3}\right)+\frac{d^2\left(5a^2d^2-24abcd+54b^2c^2\right)\left(\log\left(\frac{\sqrt[3]{c}\,x\sqrt[3]{bc-ad}}{\sqrt[3]{ax^3+b}}+\frac{x^2(bc-ad)^{2/3}}{\left(ax^3+b\right)^{2/3}}+c^{2/3}\right)-2\log\left(\sqrt[3]{c}-\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{ax^3+b}}\right)+2\sqrt{3}\tan^{-1}\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\sqrt[3]{ax^3+b}}+1}{\sqrt{3}}\right)\right)}{54c^{8/3}(bc-ad)^{13/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + b*x^3)^(7/3)*(c + d*x^3)^3),x]
```

```
[Out] (x*(a + b*x^3)^(2/3)*((-9*b^3)/(a*(-(b*c) + a*d)^3*(a + b*x^3)^2) + (27*b^3
*(b*c - 5*a*d))/(a^2*(b*c - a*d)^4*(a + b*x^3)) - (6*d^3)/(c*(b*c - a*d)^3*
(c + d*x^3)^2) + (2*d^3*(-21*b*c + 5*a*d))/(c^2*(b*c - a*d)^4*(c + d*x^3)))
)/36 + (d^2*(54*b^2*c^2 - 24*a*b*c*d + 5*a^2*d^2)*(2*Sqrt[3]*ArcTan[(1 + (2
*(b*c - a*d)^(1/3)*x)/(c^(1/3)*(b + a*x^3)^(1/3)))/Sqrt[3]] - 2*Log[c^(1/3)
 - ((b*c - a*d)^(1/3)*x)/(b + a*x^3)^(1/3)] + Log[c^(2/3) + ((b*c - a*d)^(2
/3)*x^2)/(b + a*x^3)^(2/3) + (c^(1/3)*(b*c - a*d)^(1/3)*x)/(b + a*x^3)^(1/3
)]))/(54*c^(8/3)*(b*c - a*d)^(13/3))
```

IntegrateAlgebraic [C] time = 27.99, size = 791, normalized size = 1.71

$$\frac{\left(5i\sqrt{3}a^2d^4+5a^2d^4-24i\sqrt{3}abcd^3-24abcd^3+54i\sqrt{3}b^2c^2d^2+54b^2c^2d^2\right)\log\left(2x\sqrt[3]{bc-ad}+\left(1+i\sqrt{3}\right)\sqrt[3]{c}\sqrt[3]{a+bx^3}\right)}{54c^{8/3}(bc-ad)^{13/3}}+\frac{\left(-5i\sqrt{3}a^2d^4+15a^2d^4+24i\sqrt{3}abcd^3-72abcd^3-54i\sqrt{3}b^2c^2d^2+162b^2c^2d^2\right)\tanh^{-1}\left(\frac{ix\sqrt[3]{bc-ad}+\left(\sqrt{3}-i\right)\sqrt[3]{c}\sqrt[3]{a+bx^3}}{\sqrt{3}x\sqrt[3]{bc-ad}}\right)}{54c^{8/3}(bc-ad)^{13/3}}+\frac{\left(-5i\sqrt{3}a^2d^4-5a^2d^4+24i\sqrt{3}abcd^3+24abcd^3-54i\sqrt{3}b^2c^2d^2-54b^2c^2d^2\right)\log\left(\left(\sqrt{3}+i\right)c^{2/3}\left(a+bx^3\right)^{2/3}+\sqrt[3]{c}\left(-\sqrt{3}x+ix\right)\sqrt[3]{a+bx^3}\sqrt[3]{bc-ad}-2ix^2(bc-ad)^{2/3}\right)}{108c^{8/3}(bc-ad)^{13/3}}+\frac{16a^5cd^4x+10a^5d^5x^4-48a^4bc^2d^3x-10a^4bcd^4x^4+20a^4bd^5x^7-96a^3b^2c^2d^3x^4-68a^3b^2cd^4x^7+10a^3b^2d^5x^{10}-144a^2b^3c^4dx-288a^2b^3c^3d^2x^4-192a^2b^3c^2d^3x^7-42a^2b^3cd^4x^{10}+36ab^4c^5x-63ab^4c^4dx^4-234ab^4c^3d^2x^7-135ab^4c^2d^3x^{10}+27b^5c^5x^4+54b^5c^4dx^7+27b^5c^3d^2x^{10}}{36a^2c^2\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)^2(ad-bc)^4}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((a + b*x^3)^(7/3)*(c + d*x^3)^3),x]
```

```
[Out] (36*a*b^4*c^5*x - 144*a^2*b^3*c^4*d*x - 48*a^4*b*c^2*d^3*x + 16*a^5*c*d^4*x
 + 27*b^5*c^5*x^4 - 63*a*b^4*c^4*d*x^4 - 288*a^2*b^3*c^3*d^2*x^4 - 96*a^3*b
^2*c^2*d^3*x^4 - 10*a^4*b*c*d^4*x^4 + 10*a^5*d^5*x^4 + 54*b^5*c^4*d*x^7 - 2
34*a*b^4*c^3*d^2*x^7 - 192*a^2*b^3*c^2*d^3*x^7 - 68*a^3*b^2*c*d^4*x^7 + 20*
a^4*b*d^5*x^7 + 27*b^5*c^3*d^2*x^10 - 135*a*b^4*c^2*d^3*x^10 - 42*a^2*b^3*c
*d^4*x^10 + 10*a^3*b^2*d^5*x^10)/(36*a^2*c^2*(-(b*c) + a*d)^4*(a + b*x^3)^(
4/3)*(c + d*x^3)^2) + ((162*b^2*c^2*d^2 - (54*I)*Sqrt[3]*b^2*c^2*d^2 - 72*a
*b*c*d^3 + (24*I)*Sqrt[3]*a*b*c*d^3 + 15*a^2*d^4 - (5*I)*Sqrt[3]*a^2*d^4)*A
rcTanh[(I*(b*c - a*d)^(1/3)*x + (-I + Sqrt[3])*c^(1/3)*(a + b*x^3)^(1/3))/(
Sqrt[3]*(b*c - a*d)^(1/3)*x)])/(54*c^(8/3)*(b*c - a*d)^(13/3)) + ((54*b^2*c
^2*d^2 + (54*I)*Sqrt[3]*b^2*c^2*d^2 - 24*a*b*c*d^3 - (24*I)*Sqrt[3]*a*b*c*d
^3 + 5*a^2*d^4 + (5*I)*Sqrt[3]*a^2*d^4)*Log[2*(b*c - a*d)^(1/3)*x + (1 + I*
Sqrt[3])*c^(1/3)*(a + b*x^3)^(1/3)])/(54*c^(8/3)*(b*c - a*d)^(13/3)) + ((-5
4*b^2*c^2*d^2 - (54*I)*Sqrt[3]*b^2*c^2*d^2 + 24*a*b*c*d^3 + (24*I)*Sqrt[3]*
a*b*c*d^3 - 5*a^2*d^4 - (5*I)*Sqrt[3]*a^2*d^4)*Log[(-2*I)*(b*c - a*d)^(2/3)
*x^2 + c^(1/3)*(b*c - a*d)^(1/3)*(I*x - Sqrt[3]*x)*(a + b*x^3)^(1/3) + (I +
 Sqrt[3])*c^(2/3)*(a + b*x^3)^(2/3)])/(108*c^(8/3)*(b*c - a*d)^(13/3))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^(7/3)/(d*x^3+c)^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^3+a\right)^{\frac{7}{3}}\left(dx^3+c\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^(7/3)/(d*x^3+c)^3,x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^3 + a)^(7/3)*(d*x^3 + c)^3), x)
```

maple [F] time = 0.56, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,x^3+a\right)^{\frac{7}{3}}\left(d\,x^3+c\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^3+a)^(7/3)/(d*x^3+c)^3,x)
```

```
[Out] int(1/(b*x^3+a)^(7/3)/(d*x^3+c)^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^3+a\right)^{\frac{7}{3}}\left(dx^3+c\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^(7/3)/(d*x^3+c)^3,x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^3 + a)^(7/3)*(d*x^3 + c)^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(b\,x^3+a\right)^{7/3}\,\left(d\,x^3+c\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^3)^(7/3)*(c + d*x^3)^3),x)
```

```
[Out] int(1/((a + b*x^3)^(7/3)*(c + d*x^3)^3), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**3+a)**(7/3)/(d*x**3+c)**3,x)
```

```
[Out] Timed out
```

3.82 $\int \left(a+bx^3\right)^{-1-\frac{bc}{3bc-3ad}} \left(c+dx^3\right)^{-1+\frac{ad}{3bc-3ad}}\,dx$

Optimal. Leaf size=53

$$\frac{x\left(a+bx^3\right)^{-\frac{bc}{3bc-3ad}}\left(c+dx^3\right)^{\frac{ad}{3bc-3ad}}}{ac}$$

Rubi [A] time = 0.02, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 50, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.020, Rules used = {381}

$$\frac{x\left(a+bx^3\right)^{-\frac{bc}{3bc-3ad}}\left(c+dx^3\right)^{\frac{ad}{3bc-3ad}}}{ac}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^(-1 - (b*c)/(3*b*c - 3*a*d))*(c + d*x^3)^(-1 + (a*d)/(3*b*c
 - 3*a*d)),x]
```

```
[Out] (x*(c + d*x^3)^((a*d)/(3*b*c - 3*a*d)))/(a*c*(a + b*x^3)^((b*c)/(3*b*c - 3*
a*d)))
```

Rule 381

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c), x] /; FreeQ[{a,
b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] &&
EqQ[a*d*(p + 1) + b*c*(q + 1), 0]
```

Rubi steps

$$\int \left(a+bx^3\right)^{-1-\frac{bc}{3bc-3ad}}\left(c+dx^3\right)^{-1+\frac{ad}{3bc-3ad}}\,dx=\frac{x\left(a+bx^3\right)^{-\frac{bc}{3bc-3ad}}\left(c+dx^3\right)^{\frac{ad}{3bc-3ad}}}{ac}$$

Mathematica [A] time = 0.04, size = 52, normalized size = 0.98

$$\frac{x\left(a+bx^3\right)^{\frac{bc}{3ad-3bc}}\left(c+dx^3\right)^{\frac{ad}{3bc-3ad}}}{ac}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^(-1 - (b*c)/(3*b*c - 3*a*d))*(c + d*x^3)^(-1 + (a*d)/
(3*b*c - 3*a*d)),x]
```

```
[Out] (x*(a + b*x^3)^((b*c)/(-3*b*c + 3*a*d))*(c + d*x^3)^((a*d)/(3*b*c - 3*a*d))
)/(a*c)
```

IntegrateAlgebraic [F] time = 0.53, size = 0, normalized size = 0.00

$$\int \left(a+bx^3\right)^{-1-\frac{bc}{3bc-3ad}}\left(c+dx^3\right)^{-1+\frac{ad}{3bc-3ad}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x^3)^(-1 - (b*c)/(3*b*c - 3*a*d))*(c + d*x^3)^(-1
 + (a*d)/(3*b*c - 3*a*d)),x]
```

```
[Out] Defer[IntegrateAlgebraic][(a + b*x^3)^(-1 - (b*c)/(3*b*c - 3*a*d))*(c + d*x
^3)^(-1 + (a*d)/(3*b*c - 3*a*d)), x]
```

fricas [A] time = 1.35, size = 91, normalized size = 1.72

$$\frac{bdx^7 + (bc + ad)x^4 + acx}{\left(bx^3 + a\right)^{\frac{4\,bc-3\,ad}{3\,(bc-ad)}}\left(dx^3 + c\right)^{\frac{3\,bc-4\,ad}{3\,(bc-ad)}}ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(-1-b*c/(-3*a*d+3*b*c))*(d*x^3+c)^(-1+a*d/(-3*a*d+3*b*c
)),x, algorithm="fricas")
```

```
[Out] (b*d*x^7 + (b*c + a*d)*x^4 + a*c*x)/((b*x^3 + a)^(1/3*(4*b*c - 3*a*d)/(b*c
- a*d))*(d*x^3 + c)^(1/3*(3*b*c - 4*a*d)/(b*c - a*d))*a*c)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx^3 + a\right)^{-\frac{bc}{3\,(bc-ad)}-1}\left(dx^3 + c\right)^{\frac{ad}{3\,(bc-ad)}-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(-1-b*c/(-3*a*d+3*b*c))*(d*x^3+c)^(-1+a*d/(-3*a*d+3*b*c
)),x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a)^(-1/3*b*c/(b*c - a*d) - 1)*(d*x^3 + c)^(1/3*a*d/(b*c
- a*d) - 1), x)
```

maple [A] time = 0.04, size = 71, normalized size = 1.34

$$\frac{x\left(b\,x^3 + a\right)^{1-\frac{3ad-4bc}{3(ad-bc)}}\left(d\,x^3 + c\right)^{1-\frac{4ad-3bc}{3(ad-bc)}}}{ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(-1-b*c/(-3*a*d+3*b*c))*(d*x^3+c)^(-1+a*d/(-3*a*d+3*b*c)),x)
```

```
[Out] (b*x^3+a)^(1-1/3*(3*a*d-4*b*c)/(a*d-b*c))*(d*x^3+c)^(1-1/3*(4*a*d-3*b*c)/(a
*d-b*c))/a/c*x
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx^3 + a\right)^{-\frac{bc}{3\,(bc-ad)}-1}\left(dx^3 + c\right)^{\frac{ad}{3\,(bc-ad)}-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(-1-b*c/(-3*a*d+3*b*c))*(d*x^3+c)^(-1+a*d/(-3*a*d+3*b*c
)),x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a)^(-1/3*b*c/(b*c - a*d) - 1)*(d*x^3 + c)^(1/3*a*d/(b*c
- a*d) - 1), x)
```

mupad [B] time = 1.90, size = 131, normalized size = 2.47

$$\frac{x\left(b\,x^3 + a\right)^{\frac{b\,c}{3\,a\,d-3\,b\,c}-1} + \frac{x^4\left(b\,x^3+a\right)^{\frac{b\,c}{3\,a\,d-3\,b\,c}-1}\left(a\,d+b\,c\right)}{a\,c} + \frac{b\,d\,x^7\left(b\,x^3+a\right)^{\frac{b\,c}{3\,a\,d-3\,b\,c}-1}}{a\,c}}{\left(d\,x^3 + c\right)^{\frac{a\,d}{3\,a\,d-3\,b\,c}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^((b*c)/(3*a*d - 3*b*c) - 1)/(c + d*x^3)^((a*d)/(3*a*d - 3*b
*c) + 1),x)
```

```
[Out] (x*(a + b*x^3)^((b*c)/(3*a*d - 3*b*c) - 1) + (x^4*(a + b*x^3)^((b*c)/(3*a*d
 - 3*b*c) - 1)*(a*d + b*c))/(a*c) + (b*d*x^7*(a + b*x^3)^((b*c)/(3*a*d - 3*
b*c) - 1))/(a*c))/(c + d*x^3)^((a*d)/(3*a*d - 3*b*c) + 1)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(-1-b*c/(-3*a*d+3*b*c))*(d*x**3+c)**(-1+a*d/(-3*a*d+3
*b*c)),x)
```

```
[Out] Timed out
```

3.83 $\int \left(a+bx^4\right)\left(c+dx^4\right)^4\,dx$

Optimal. Leaf size=94

$$\frac{1}{5}c^3x^5(4ad+bc)+\frac{2}{9}c^2dx^9(3ad+2bc)+\frac{1}{17}d^3x^{17}(ad+4bc)+\frac{2}{13}cd^2x^{13}(2ad+3bc)+ac^4x+\frac{1}{21}bd^4x^{21}$$

Rubi [A] time = 0.07, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {373}

$$\frac{2}{9}c^2dx^9(3ad+2bc)+\frac{1}{5}c^3x^5(4ad+bc)+\frac{1}{17}d^3x^{17}(ad+4bc)+\frac{2}{13}cd^2x^{13}(2ad+3bc)+ac^4x+\frac{1}{21}bd^4x^{21}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^4)*(c + d*x^4)^4,x]
```

```
[Out] a*c^4*x + (c^3*(b*c + 4*a*d)*x^5)/5 + (2*c^2*d*(2*b*c + 3*a*d)*x^9)/9 + (2*
c*d^2*(3*b*c + 2*a*d)*x^13)/13 + (d^3*(4*b*c + a*d)*x^17)/17 + (b*d^4*x^21)
/21
```

Rule 373

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\int \left(a+bx^4\right)\left(c+dx^4\right)^4\,dx = \int \left(ac^4+c^3(bc+4ad)x^4+2c^2d(2bc+3ad)x^8+2cd^2(3bc+2ad)x^{12}+d^3(4bc+ad)\right.$$
$$= ac^4x+\frac{1}{5}c^3(bc+4ad)x^5+\frac{2}{9}c^2d(2bc+3ad)x^9+\frac{2}{13}cd^2(3bc+2ad)x^{13}+\frac{1}{17}d^3(4bc+$$

Mathematica [A] time = 0.02, size = 94, normalized size = 1.00

$$\frac{1}{5}c^3x^5(4ad+bc)+\frac{2}{9}c^2dx^9(3ad+2bc)+\frac{1}{17}d^3x^{17}(ad+4bc)+\frac{2}{13}cd^2x^{13}(2ad+3bc)+ac^4x+\frac{1}{21}bd^4x^{21}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^4)*(c + d*x^4)^4,x]
```

```
[Out] a*c^4*x + (c^3*(b*c + 4*a*d)*x^5)/5 + (2*c^2*d*(2*b*c + 3*a*d)*x^9)/9 + (2*
c*d^2*(3*b*c + 2*a*d)*x^13)/13 + (d^3*(4*b*c + a*d)*x^17)/17 + (b*d^4*x^21)
/21
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+bx^4\right)\left(c+dx^4\right)^4\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x^4)*(c + d*x^4)^4,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x^4)*(c + d*x^4)^4, x]
```

fricas [A] time = 0.98, size = 98, normalized size = 1.04

$$\frac{1}{21}x^{21}d^4b+\frac{4}{17}x^{17}d^3cb+\frac{1}{17}x^{17}d^4a+\frac{6}{13}x^{13}d^2c^2b+\frac{4}{13}x^{13}d^3ca+\frac{4}{9}x^9dc^3b+\frac{2}{3}x^9d^2c^2a+\frac{1}{5}x^5c^4b+\frac{4}{5}x^5dc^3a+xc^4a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)*(d*x^4+c)^4,x, algorithm="fricas")
```

```
[Out] 1/21*x^21*d^4*b + 4/17*x^17*d^3*c*b + 1/17*x^17*d^4*a + 6/13*x^13*d^2*c^2*b
 + 4/13*x^13*d^3*c*a + 4/9*x^9*d*c^3*b + 2/3*x^9*d^2*c^2*a + 1/5*x^5*c^4*b
+ 4/5*x^5*d*c^3*a + x*c^4*a
```

giac [A] time = 0.16, size = 98, normalized size = 1.04

$$\frac{1}{21}bd^4x^{21}+\frac{4}{17}bcd^3x^{17}+\frac{1}{17}ad^4x^{17}+\frac{6}{13}bc^2d^2x^{13}+\frac{4}{13}acd^3x^{13}+\frac{4}{9}bc^3dx^9+\frac{2}{3}ac^2d^2x^9+\frac{1}{5}bc^4x^5+\frac{4}{5}ac^3dx^5+ac^4x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)*(d*x^4+c)^4,x, algorithm="giac")
```

```
[Out] 1/21*b*d^4*x^21 + 4/17*b*c*d^3*x^17 + 1/17*a*d^4*x^17 + 6/13*b*c^2*d^2*x^13
 + 4/13*a*c*d^3*x^13 + 4/9*b*c^3*d*x^9 + 2/3*a*c^2*d^2*x^9 + 1/5*b*c^4*x^5
+ 4/5*a*c^3*d*x^5 + a*c^4*x
```

maple [A] time = 0.04, size = 97, normalized size = 1.03

$$\frac{b\,d^4x^{21}}{21}+\frac{\left(a\,d^4+4bc\,d^3\right)x^{17}}{17}+\frac{\left(4ac\,d^3+6c^2d^2b\right)x^{13}}{13}+\frac{\left(6a\,c^2d^2+4c^3db\right)x^9}{9}+a\,c^4x+\frac{\left(4a\,c^3d+b\,c^4\right)x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^4+a)*(d*x^4+c)^4,x)
```

```
[Out] 1/21*b*d^4*x^21+1/17*(a*d^4+4*b*c*d^3)*x^17+1/13*(4*a*c*d^3+6*b*c^2*d^2)*x^
13+1/9*(6*a*c^2*d^2+4*b*c^3*d)*x^9+1/5*(4*a*c^3*d+b*c^4)*x^5+a*c^4*x
```

maxima [A] time = 0.65, size = 96, normalized size = 1.02

$$\frac{1}{21}bd^4x^{21}+\frac{1}{17}\left(4\,bcd^3+ad^4\right)x^{17}+\frac{2}{13}\left(3\,bc^2d^2+2\,acd^3\right)x^{13}+\frac{2}{9}\left(2\,bc^3d+3\,ac^2d^2\right)x^9+ac^4x+\frac{1}{5}\left(bc^4+4\,ac^3d\right)x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)*(d*x^4+c)^4,x, algorithm="maxima")
```

```
[Out] 1/21*b*d^4*x^21 + 1/17*(4*b*c*d^3 + a*d^4)*x^17 + 2/13*(3*b*c^2*d^2 + 2*a*c
*d^3)*x^13 + 2/9*(2*b*c^3*d + 3*a*c^2*d^2)*x^9 + a*c^4*x + 1/5*(b*c^4 + 4*a
*c^3*d)*x^5
```

mupad [B] time = 1.30, size = 88, normalized size = 0.94

$$x^5\left(\frac{b\,c^4}{5}+\frac{4\,a\,d\,c^3}{5}\right)+x^{17}\left(\frac{a\,d^4}{17}+\frac{4\,b\,c\,d^3}{17}\right)+\frac{b\,d^4\,x^{21}}{21}+a\,c^4\,x+\frac{2\,c^2\,d\,x^9\,\left(3\,a\,d+2\,b\,c\right)}{9}+\frac{2\,c\,d^2\,x^{13}\,\left(2\,a\,d+3\,b\,c\right)}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^4)*(c + d*x^4)^4,x)
```

```
[Out] x^5*((b*c^4)/5 + (4*a*c^3*d)/5) + x^17*((a*d^4)/17 + (4*b*c*d^3)/17) + (b*d
^4*x^21)/21 + a*c^4*x + (2*c^2*d*x^9*(3*a*d + 2*b*c))/9 + (2*c*d^2*x^13*(2*
a*d + 3*b*c))/13
```

sympy [A] time = 0.09, size = 107, normalized size = 1.14

$$ac^4x + \frac{bd^4x^{21}}{21} + x^{17}\left(\frac{ad^4}{17} + \frac{4bcd^3}{17}\right) + x^{13}\left(\frac{4acd^3}{13} + \frac{6bc^2d^2}{13}\right) + x^9\left(\frac{2ac^2d^2}{3} + \frac{4bc^3d}{9}\right) + x^5\left(\frac{4ac^3d}{5} + \frac{bc^4}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**4+a)*(d*x**4+c)**4,x)
```

```
[Out] a*c**4*x + b*d**4*x**21/21 + x**17*(a*d**4/17 + 4*b*c*d**3/17) + x**13*(4*a
*c*d**3/13 + 6*b*c**2*d**2/13) + x**9*(2*a*c**2*d**2/3 + 4*b*c**3*d/9) + x*
*5*(4*a*c**3*d/5 + b*c**4/5)
```

3.84 $\int \left(a+bx^4\right)\left(c+dx^4\right)^3\,dx$

Optimal. Leaf size=70

$$\frac{1}{5}c^2x^5(3ad+bc)+\frac{1}{13}d^2x^{13}(ad+3bc)+\frac{1}{3}cdx^9(ad+bc)+ac^3x+\frac{1}{17}bd^3x^{17}$$

Rubi [A] time = 0.05, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {373}

$$\frac{1}{5}c^2x^5(3ad+bc)+\frac{1}{13}d^2x^{13}(ad+3bc)+\frac{1}{3}cdx^9(ad+bc)+ac^3x+\frac{1}{17}bd^3x^{17}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^4)*(c + d*x^4)^3,x]
```

```
[Out] a*c^3*x + (c^2*(b*c + 3*a*d)*x^5)/5 + (c*d*(b*c + a*d)*x^9)/3 + (d^2*(3*b*c
 + a*d)*x^13)/13 + (b*d^3*x^17)/17
```

Rule 373

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \left(a+bx^4\right)\left(c+dx^4\right)^3\,dx &= \int \left(ac^3+c^2(bc+3ad)x^4+3cd(bc+ad)x^8+d^2(3bc+ad)x^{12}+bd^3x^{16}\right)dx\\ &= ac^3x+\frac{1}{5}c^2(bc+3ad)x^5+\frac{1}{3}cd(bc+ad)x^9+\frac{1}{13}d^2(3bc+ad)x^{13}+\frac{1}{17}bd^3x^{17}\end{aligned}$$

Mathematica [A] time = 0.02, size = 70, normalized size = 1.00

$$\frac{1}{5}c^2x^5(3ad+bc)+\frac{1}{13}d^2x^{13}(ad+3bc)+\frac{1}{3}cdx^9(ad+bc)+ac^3x+\frac{1}{17}bd^3x^{17}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^4)*(c + d*x^4)^3,x]
```

```
[Out] a*c^3*x + (c^2*(b*c + 3*a*d)*x^5)/5 + (c*d*(b*c + a*d)*x^9)/3 + (d^2*(3*b*c
 + a*d)*x^13)/13 + (b*d^3*x^17)/17
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+bx^4\right)\left(c+dx^4\right)^3\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x^4)*(c + d*x^4)^3,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x^4)*(c + d*x^4)^3, x]
```

fricas [A] time = 0.90, size = 74, normalized size = 1.06

$$\frac{1}{17}x^{17}d^3b+\frac{3}{13}x^{13}d^2cb+\frac{1}{13}x^{13}d^3a+\frac{1}{3}x^9dc^2b+\frac{1}{3}x^9d^2ca+\frac{1}{5}x^5c^3b+\frac{3}{5}x^5dc^2a+xc^3a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)*(d*x^4+c)^3,x, algorithm="fricas")
```

```
[Out] 1/17*x^17*d^3*b + 3/13*x^13*d^2*c*b + 1/13*x^13*d^3*a + 1/3*x^9*d*c^2*b + 1
/3*x^9*d^2*c*a + 1/5*x^5*c^3*b + 3/5*x^5*d*c^2*a + x*c^3*a
```

giac [A] time = 0.15, size = 74, normalized size = 1.06

$$\frac{1}{17}\,bd^3x^{17} + \frac{3}{13}\,bcd^2x^{13} + \frac{1}{13}\,ad^3x^{13} + \frac{1}{3}\,bc^2dx^9 + \frac{1}{3}\,acd^2x^9 + \frac{1}{5}\,bc^3x^5 + \frac{3}{5}\,ac^2dx^5 + ac^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)*(d*x^4+c)^3,x, algorithm="giac")
```

```
[Out] 1/17*b*d^3*x^17 + 3/13*b*c*d^2*x^13 + 1/13*a*d^3*x^13 + 1/3*b*c^2*d*x^9 + 1
/3*a*c*d^2*x^9 + 1/5*b*c^3*x^5 + 3/5*a*c^2*d*x^5 + a*c^3*x
```

maple [A] time = 0.04, size = 73, normalized size = 1.04

$$\frac{b\,d^3x^{17}}{17} + \frac{\left(a\,d^3 + 3bc\,d^2\right)x^{13}}{13} + \frac{\left(3ac\,d^2 + 3b\,c^2d\right)x^9}{9} + a\,c^3x + \frac{\left(3a\,c^2d + b\,c^3\right)x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^4+a)*(d*x^4+c)^3,x)
```

```
[Out] 1/17*b*d^3*x^17+1/13*(a*d^3+3*b*c*d^2)*x^13+1/9*(3*a*c*d^2+3*b*c^2*d)*x^9+1
/5*(3*a*c^2*d+b*c^3)*x^5+a*c^3*x
```

maxima [A] time = 0.65, size = 70, normalized size = 1.00

$$\frac{1}{17}\,bd^3x^{17} + \frac{1}{13}\left(3\,bcd^2 + ad^3\right)x^{13} + \frac{1}{3}\left(bc^2d + acd^2\right)x^9 + \frac{1}{5}\left(bc^3 + 3\,ac^2d\right)x^5 + ac^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)*(d*x^4+c)^3,x, algorithm="maxima")
```

```
[Out] 1/17*b*d^3*x^17 + 1/13*(3*b*c*d^2 + a*d^3)*x^13 + 1/3*(b*c^2*d + a*c*d^2)*x
^9 + 1/5*(b*c^3 + 3*a*c^2*d)*x^5 + a*c^3*x
```

mupad [B] time = 1.24, size = 66, normalized size = 0.94

$$x^5\left(\frac{b\,c^3}{5} + \frac{3\,a\,d\,c^2}{5}\right) + x^{13}\left(\frac{a\,d^3}{13} + \frac{3\,b\,c\,d^2}{13}\right) + \frac{b\,d^3\,x^{17}}{17} + a\,c^3\,x + \frac{c\,d\,x^9\,\left(a\,d + b\,c\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^4)*(c + d*x^4)^3,x)
```

```
[Out] x^5*((b*c^3)/5 + (3*a*c^2*d)/5) + x^13*((a*d^3)/13 + (3*b*c*d^2)/13) + (b*d
^3*x^17)/17 + a*c^3*x + (c*d*x^9*(a*d + b*c))/3
```

sympy [A] time = 0.08, size = 76, normalized size = 1.09

$$ac^3x + \frac{bd^3x^{17}}{17} + x^{13}\left(\frac{ad^3}{13} + \frac{3bcd^2}{13}\right) + x^9\left(\frac{acd^2}{3} + \frac{bc^2d}{3}\right) + x^5\left(\frac{3ac^2d}{5} + \frac{bc^3}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**4+a)*(d*x**4+c)**3,x)
```

```
[Out] a*c**3*x + b*d**3*x**17/17 + x**13*(a*d**3/13 + 3*b*c*d**2/13) + x**9*(a*c*
d**2/3 + b*c**2*d/3) + x**5*(3*a*c**2*d/5 + b*c**3/5)
```

3.85 $\int \left(a+bx^4\right) \left(c+dx^4\right)^2 \, dx$

Optimal. Leaf size=50

$$\frac{1}{9}dx^9(ad+2bc)+\frac{1}{5}cx^5(2ad+bc)+ac^2x+\frac{1}{13}bd^2x^{13}$$

Rubi [A] time = 0.03, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {373}

$$\frac{1}{9}dx^9(ad+2bc)+\frac{1}{5}cx^5(2ad+bc)+ac^2x+\frac{1}{13}bd^2x^{13}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^4)*(c + d*x^4)^2,x]
```

```
[Out] a*c^2*x + (c*(b*c + 2*a*d)*x^5)/5 + (d*(2*b*c + a*d)*x^9)/9 + (b*d^2*x^13)/
13
```

Rule 373

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \left(a+bx^4\right)\left(c+dx^4\right)^2 \, dx &= \int \left(ac^2+c(bc+2ad)x^4+d(2bc+ad)x^8+bd^2x^{12}\right) dx\\ &= ac^2x+\frac{1}{5}c(bc+2ad)x^5+\frac{1}{9}d(2bc+ad)x^9+\frac{1}{13}bd^2x^{13}\end{aligned}$$

Mathematica [A] time = 0.01, size = 50, normalized size = 1.00

$$\frac{1}{9}dx^9(ad+2bc)+\frac{1}{5}cx^5(2ad+bc)+ac^2x+\frac{1}{13}bd^2x^{13}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^4)*(c + d*x^4)^2,x]
```

```
[Out] a*c^2*x + (c*(b*c + 2*a*d)*x^5)/5 + (d*(2*b*c + a*d)*x^9)/9 + (b*d^2*x^13)/
13
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+bx^4\right)\left(c+dx^4\right)^2 \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x^4)*(c + d*x^4)^2,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x^4)*(c + d*x^4)^2, x]
```

fricas [A] time = 0.86, size = 50, normalized size = 1.00

$$\frac{1}{13}x^{13}d^2b+\frac{2}{9}x^9dcb+\frac{1}{9}x^9d^2a+\frac{1}{5}x^5c^2b+\frac{2}{5}x^5dca+xc^2a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)*(d*x^4+c)^2,x, algorithm="fricas")
```

```
[Out] 1/13*x^13*d^2*b + 2/9*x^9*d*c*b + 1/9*x^9*d^2*a + 1/5*x^5*c^2*b + 2/5*x^5*d
*c*a + x*c^2*a
```

giac [A] time = 0.16, size = 50, normalized size = 1.00

$$\frac{1}{13}\,bd^2x^{13}+\frac{2}{9}\,bcdx^9+\frac{1}{9}\,ad^2x^9+\frac{1}{5}\,bc^2x^5+\frac{2}{5}\,acdx^5+ac^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)*(d*x^4+c)^2,x, algorithm="giac")
```

```
[Out] 1/13*b*d^2*x^13 + 2/9*b*c*d*x^9 + 1/9*a*d^2*x^9 + 1/5*b*c^2*x^5 + 2/5*a*c*d
*x^5 + a*c^2*x
```

maple [A] time = 0.04, size = 49, normalized size = 0.98

$$\frac{b\,d^2x^{13}}{13}+\frac{\left(a\,d^2+2bcd\right)x^9}{9}+\frac{\left(2acd+b\,c^2\right)x^5}{5}+a\,c^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^4+a)*(d*x^4+c)^2,x)
```

```
[Out] 1/13*b*d^2*x^13+1/9*(a*d^2+2*b*c*d)*x^9+1/5*(2*a*c*d+b*c^2)*x^5+a*c^2*x
```

maxima [A] time = 0.60, size = 48, normalized size = 0.96

$$\frac{1}{13}\,bd^2x^{13}+\frac{1}{9}\left(2\,bcd+ad^2\right)x^9+\frac{1}{5}\left(bc^2+2\,acd\right)x^5+ac^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)*(d*x^4+c)^2,x, algorithm="maxima")
```

```
[Out] 1/13*b*d^2*x^13 + 1/9*(2*b*c*d + a*d^2)*x^9 + 1/5*(b*c^2 + 2*a*c*d)*x^5 + a
*c^2*x
```

mupad [B] time = 0.05, size = 48, normalized size = 0.96

$$x^5\left(\frac{b\,c^2}{5}+\frac{2\,a\,d\,c}{5}\right)+x^9\left(\frac{a\,d^2}{9}+\frac{2\,b\,c\,d}{9}\right)+\frac{b\,d^2\,x^{13}}{13}+a\,c^2\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^4)*(c + d*x^4)^2,x)
```

```
[Out] x^5*((b*c^2)/5 + (2*a*c*d)/5) + x^9*((a*d^2)/9 + (2*b*c*d)/9) + (b*d^2*x^13
)/13 + a*c^2*x
```

sympy [A] time = 0.08, size = 53, normalized size = 1.06

$$ac^2x+\frac{bd^2x^{13}}{13}+x^9\left(\frac{ad^2}{9}+\frac{2bcd}{9}\right)+x^5\left(\frac{2acd}{5}+\frac{bc^2}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**4+a)*(d*x**4+c)**2,x)
```

```
[Out] a*c**2*x + b*d**2*x**13/13 + x**9*(a*d**2/9 + 2*b*c*d/9) + x**5*(2*a*c*d/5
+ b*c**2/5)
```

3.86 $\int \left(a+bx^4\right)\left(c+dx^4\right)\,dx$

Optimal. Leaf size=28

$$\frac{1}{5}x^5(ad+bc)+acx+\frac{1}{9}bdx^9$$

Rubi [A] time = 0.01, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {373}

$$\frac{1}{5}x^5(ad+bc)+acx+\frac{1}{9}bdx^9$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^4)*(c + d*x^4),x]
```

```
[Out] a*c*x + ((b*c + a*d)*x^5)/5 + (b*d*x^9)/9
```

Rule 373

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \left(a+bx^4\right)\left(c+dx^4\right)\,dx &= \int \left(ac+(bc+ad)x^4+bdx^8\right)\,dx\\ &= acx+\frac{1}{5}(bc+ad)x^5+\frac{1}{9}bdx^9\end{aligned}$$

Mathematica [A] time = 0.01, size = 28, normalized size = 1.00

$$\frac{1}{5}x^5(ad+bc)+acx+\frac{1}{9}bdx^9$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^4)*(c + d*x^4),x]
```

```
[Out] a*c*x + ((b*c + a*d)*x^5)/5 + (b*d*x^9)/9
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+bx^4\right)\left(c+dx^4\right)\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x^4)*(c + d*x^4),x]
```

```
[Out] IntegrateAlgebraic[(a + b*x^4)*(c + d*x^4), x]
```

fricas [A] time = 0.76, size = 26, normalized size = 0.93

$$\frac{1}{9}x^9db+\frac{1}{5}x^5cb+\frac{1}{5}x^5da+xca$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)*(d*x^4+c),x, algorithm="fricas")
[Out] 1/9*x^9*d*b + 1/5*x^5*c*b + 1/5*x^5*d*a + x*c*a
```

giac [A] time = 0.15, size = 26, normalized size = 0.93

$$\frac{1}{9}\,bdx^9+\frac{1}{5}\,bcx^5+\frac{1}{5}\,adx^5+acx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)*(d*x^4+c),x, algorithm="giac")
[Out] 1/9*b*d*x^9 + 1/5*b*c*x^5 + 1/5*a*d*x^5 + a*c*x
```

maple [A] time = 0.04, size = 25, normalized size = 0.89

$$\frac{bd\,x^9}{9}+\frac{(ad+bc)\,x^5}{5}+acx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^4+a)*(d*x^4+c),x)
[Out] a*c*x+1/5*(a*d+b*c)*x^5+1/9*b*d*x^9
```

maxima [A] time = 0.49, size = 24, normalized size = 0.86

$$\frac{1}{9}\,bdx^9+\frac{1}{5}\,(bc+ad)x^5+acx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)*(d*x^4+c),x, algorithm="maxima")
[Out] 1/9*b*d*x^9 + 1/5*(b*c + a*d)*x^5 + a*c*x
```

mupad [B] time = 0.04, size = 25, normalized size = 0.89

$$\frac{b\,d\,x^9}{9}+\left(\frac{a\,d}{5}+\frac{b\,c}{5}\right)\,x^5+a\,c\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^4)*(c + d*x^4),x)
[Out] x^5*((a*d)/5 + (b*c)/5) + a*c*x + (b*d*x^9)/9
```

sympy [A] time = 0.06, size = 26, normalized size = 0.93

$$acx+\frac{bdx^9}{9}+x^5\left(\frac{ad}{5}+\frac{bc}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**4+a)*(d*x**4+c),x)
[Out] a*c*x + b*d*x**9/9 + x**5*(a*d/5 + b*c/5)
```

3.87 $\int \frac{a+bx^4}{c+dx^4}\,dx$

Optimal. Leaf size=223

$$\frac{(bc-ad)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\,x+\sqrt{c}+\sqrt{d}\,x^2\right)}{4\sqrt{2}\,c^{3/4}d^{5/4}}-\frac{(bc-ad)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\,x+\sqrt{c}+\sqrt{d}\,x^2\right)}{4\sqrt{2}\,c^{3/4}d^{5/4}}+\frac{(bc-ad)\tan^{-1}\left(1-\right.}{2\sqrt{2}\,c^{3/4}d^{5/4}}$$

Rubi [A] time = 0.15, antiderivative size = 223, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.412, Rules used = {388, 211, 1165, 628, 1162, 617, 204}

$$\frac{(bc-ad)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\,x+\sqrt{c}+\sqrt{d}\,x^2\right)}{4\sqrt{2}\,c^{3/4}d^{5/4}}-\frac{(bc-ad)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\,x+\sqrt{c}+\sqrt{d}\,x^2\right)}{4\sqrt{2}\,c^{3/4}d^{5/4}}+\frac{(bc-ad)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\,x}{\sqrt[4]{c}}\right)}{2\sqrt{2}\,c^{3/4}d^{5/4}}-\frac{(bc-ad)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}\,x}{\sqrt[4]{c}}+1\right)}{2\sqrt{2}\,c^{3/4}d^{5/4}}+\frac{bx}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^4)/(c + d*x^4),x]
```

```
[Out] (b*x)/d + ((b*c - a*d)*ArcTan[1 - (Sqrt[2]*d^(1/4)*x)/c^(1/4)])/(2*Sqrt[2]*
c^(3/4)*d^(5/4)) - ((b*c - a*d)*ArcTan[1 + (Sqrt[2]*d^(1/4)*x)/c^(1/4)])/(2
*Sqrt[2]*c^(3/4)*d^(5/4)) + ((b*c - a*d)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1
/4)*x + Sqrt[d]*x^2])/(4*Sqrt[2]*c^(3/4)*d^(5/4)) - ((b*c - a*d)*Log[Sqrt[c
] + Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2])/(4*Sqrt[2]*c^(3/4)*d^(5/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{a+bx^4}{c+dx^4}\,dx = \frac{bx}{d} - \frac{(bc-ad)\int \frac{1}{c+dx^4}\,dx}{d}$$

$$= \frac{bx}{d} - \frac{(bc-ad)\int \frac{\sqrt{c}-\sqrt{d}\,x^2}{c+dx^4}\,dx}{2\sqrt{c}\,d} - \frac{(bc-ad)\int \frac{\sqrt{c}+\sqrt{d}\,x^2}{c+dx^4}\,dx}{2\sqrt{c}\,d}$$

$$= \frac{bx}{d} - \frac{(bc-ad)\int \frac{1}{\frac{\sqrt{c}}{\sqrt{d}}-\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{d}}+x^2}\,dx}{4\sqrt{c}\,d^{3/2}} - \frac{(bc-ad)\int \frac{1}{\frac{\sqrt{c}}{\sqrt{d}}+\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{d}}+x^2}\,dx}{4\sqrt{c}\,d^{3/2}} + \frac{(bc-ad)\int \frac{\frac{\sqrt{2}\sqrt[4]{c}}{\sqrt[4]{d}}+2x}{-\frac{\sqrt{c}}{\sqrt{d}}-\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{d}}-x^2}\,dx}{4\sqrt{2}\,c^{3/4}d^{5/4}}$$

$$= \frac{bx}{d} + \frac{(bc-ad)\log\left(\sqrt{c}-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\,x+\sqrt{d}\,x^2\right)}{4\sqrt{2}\,c^{3/4}d^{5/4}} - \frac{(bc-ad)\log\left(\sqrt{c}+\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\,x+\sqrt{d}\,x^2\right)}{4\sqrt{2}\,c^{3/4}d^{5/4}} - \cdots$$

$$= \frac{bx}{d} + \frac{(bc-ad)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\,x}{\sqrt[4]{c}}\right)}{2\sqrt{2}\,c^{3/4}d^{5/4}} - \frac{(bc-ad)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{d}\,x}{\sqrt[4]{c}}\right)}{2\sqrt{2}\,c^{3/4}d^{5/4}} + \frac{(bc-ad)\log\left(\sqrt{c}-\sqrt{2}\sqrt[4]{c}\cdots\right.}{4\sqrt{2}\,c^{3/4}d^{5/4}}$$

Mathematica [A] time = 0.18, size = 196, normalized size = 0.88

$$\frac{\sqrt{2}(bc-ad)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\,x+\sqrt{c}+\sqrt{d}\,x^2\right)-\sqrt{2}(bc-ad)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\,x+\sqrt{c}+\sqrt{d}\,x^2\right)+2\sqrt{2}(bc-ad)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\,x}{\sqrt[4]{c}}\right)-2\sqrt{2}(bc-ad)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}\,x}{\sqrt[4]{c}}+1\right)+8bc^{3/4}\sqrt[4]{d}\,x}{8c^{3/4}d^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^4)/(c + d*x^4),x]
```

```
[Out] (8*b*c^(3/4)*d^(1/4)*x + 2*Sqrt[2]*(b*c - a*d)*ArcTan[1 - (Sqrt[2]*d^(1/4)*
x)/c^(1/4)] - 2*Sqrt[2]*(b*c - a*d)*ArcTan[1 + (Sqrt[2]*d^(1/4)*x)/c^(1/4)]
 + Sqrt[2]*(b*c - a*d)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^
2] - Sqrt[2]*(b*c - a*d)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*
x^2])/(8*c^(3/4)*d^(5/4))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a+bx^4}{c+dx^4}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x^4)/(c + d*x^4),x]
```

```
[Out] IntegrateAlgebraic[(a + b*x^4)/(c + d*x^4), x]
```

fricas [B] time = 1.29, size = 639, normalized size = 2.87

$$\frac{4\,d\left(-\frac{b^4c^4-4\,ab^3c^3d+6\,a^2b^2c^2d^2-4\,a^3bcd^3+a^4d^4}{c^3d^5}\right)^{\frac14}\arctan\left(\frac{c^2d^4x\left(-\frac{b^4c^4-4\,ab^3c^3d+6\,a^2b^2c^2d^2-4\,a^3bcd^3+a^4d^4}{c^3d^5}\right)^{\frac34}-c^2d^4\sqrt{\frac{c^2d^2\sqrt{-\frac{b^4c^4-4\,ab^3c^3d+6\,a^2b^2c^2d^2-4\,a^3bcd^3+a^4d^4}{c^3d^5}}+\left(b^2c^2-2\,abcd+a^2d^2\right)x^2}{b^2c^2-2\,abcd+a^2d^2}}\left(-\frac{b^4c^4-4\,ab^3c^3d+6\,a^2b^2c^2d^2-4\,a^3bcd^3+a^4d^4}{c^3d^5}\right)^{\frac34}}{b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3}\right)+d\left(-\frac{b^4c^4-4\,ab^3c^3d+6\,a^2b^2c^2d^2-4\,a^3bcd^3+a^4d^4}{c^3d^5}\right)^{\frac14}\log\left(cd\left(-\frac{b^4c^4-4\,ab^3c^3d+6\,a^2b^2c^2d^2-4\,a^3bcd^3+a^4d^4}{c^3d^5}\right)^{\frac14}-(bc-ad)x\right)-d\left(-\frac{b^4c^4-4\,ab^3c^3d+6\,a^2b^2c^2d^2-4\,a^3bcd^3+a^4d^4}{c^3d^5}\right)^{\frac14}\log\left(-cd\left(-\frac{b^4c^4-4\,ab^3c^3d+6\,a^2b^2c^2d^2-4\,a^3bcd^3+a^4d^4}{c^3d^5}\right)^{\frac14}-(bc-ad)x\right)+4\,bx}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)/(d*x^4+c),x, algorithm="fricas")
```

```
[Out] 1/4*(4*d*(-(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a
^4*d^4)/(c^3*d^5))^(1/4)*arctan((c^2*d^4*x*(-(b^4*c^4 - 4*a*b^3*c^3*d + 6*a
^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)/(c^3*d^5))^(3/4) - c^2*d^4*sqrt((
c^2*d^2*sqrt(-(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3
+ a^4*d^4)/(c^3*d^5)) + (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*x^2)/(b^2*c^2 - 2*a
*b*c*d + a^2*d^2))*(-(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b
*c*d^3 + a^4*d^4)/(c^3*d^5))^(3/4))/(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^
2 - a^3*d^3)) + d*(-(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*
c*d^3 + a^4*d^4)/(c^3*d^5))^(1/4)*log(c*d*(-(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^
2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)/(c^3*d^5))^(1/4) - (b*c - a*d)*x)
- d*(-(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^
4)/(c^3*d^5))^(1/4)*log(-c*d*(-(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2
 - 4*a^3*b*c*d^3 + a^4*d^4)/(c^3*d^5))^(1/4) - (b*c - a*d)*x) + 4*b*x)/d
```

giac [A] time = 0.16, size = 245, normalized size = 1.10

$$\frac{bx}{d}-\frac{\sqrt{2}\left(\left(cd^3\right)^{\frac14}bc-\left(cd^3\right)^{\frac14}ad\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{c}{d}\right)^{\frac14}\right)}{2\left(\frac{c}{d}\right)^{\frac14}}\right)}{4\,cd^2}-\frac{\sqrt{2}\left(\left(cd^3\right)^{\frac14}bc-\left(cd^3\right)^{\frac14}ad\right)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(\frac{c}{d}\right)^{\frac14}\right)}{2\left(\frac{c}{d}\right)^{\frac14}}\right)}{4\,cd^2}-\frac{\sqrt{2}\left(\left(cd^3\right)^{\frac14}bc-\left(cd^3\right)^{\frac14}ad\right)\log\left(x^2+\sqrt{2}\,x\left(\frac{c}{d}\right)^{\frac14}+\sqrt{\frac{c}{d}}\right)}{8\,cd^2}+\frac{\sqrt{2}\left(\left(cd^3\right)^{\frac14}bc-\left(cd^3\right)^{\frac14}ad\right)\log\left(x^2-\sqrt{2}\,x\left(\frac{c}{d}\right)^{\frac14}+\sqrt{\frac{c}{d}}\right)}{8\,cd^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)/(d*x^4+c),x, algorithm="giac")
```

```
[Out] b*x/d - 1/4*sqrt(2)*((c*d^3)^(1/4)*b*c - (c*d^3)^(1/4)*a*d)*arctan(1/2*sqrt
(2)*(2*x + sqrt(2)*(c/d)^(1/4))/(c/d)^(1/4))/(c*d^2) - 1/4*sqrt(2)*((c*d^3)
^(1/4)*b*c - (c*d^3)^(1/4)*a*d)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(c/d)^(1/
4))/(c/d)^(1/4))/(c*d^2) - 1/8*sqrt(2)*((c*d^3)^(1/4)*b*c - (c*d^3)^(1/4)*a
*d)*log(x^2 + sqrt(2)*x*(c/d)^(1/4) + sqrt(c/d))/(c*d^2) + 1/8*sqrt(2)*((c*
d^3)^(1/4)*b*c - (c*d^3)^(1/4)*a*d)*log(x^2 - sqrt(2)*x*(c/d)^(1/4) + sqrt(
c/d))/(c*d^2)
```

maple [A] time = 0.05, size = 266, normalized size = 1.19

$$\frac{\left(\frac{c}{d}\right)^{\frac14}\sqrt{2}\,a\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{c}{d}\right)^{\frac14}}-1\right)}{4c}+\frac{\left(\frac{c}{d}\right)^{\frac14}\sqrt{2}\,a\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{c}{d}\right)^{\frac14}}+1\right)}{4c}+\frac{\left(\frac{c}{d}\right)^{\frac14}\sqrt{2}\,a\ln\left(\frac{x^2+\left(\frac{c}{d}\right)^{\frac14}\sqrt{2}\,x+\sqrt{\frac{c}{d}}}{x^2-\left(\frac{c}{d}\right)^{\frac14}\sqrt{2}\,x+\sqrt{\frac{c}{d}}}\right)}{8c}+\frac{bx}{d}-\frac{\left(\frac{c}{d}\right)^{\frac14}\sqrt{2}\,b\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{c}{d}\right)^{\frac14}}-1\right)}{4d}-\frac{\left(\frac{c}{d}\right)^{\frac14}\sqrt{2}\,b\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{c}{d}\right)^{\frac14}}+1\right)}{4d}-\frac{\left(\frac{c}{d}\right)^{\frac14}\sqrt{2}\,b\ln\left(\frac{x^2+\left(\frac{c}{d}\right)^{\frac14}\sqrt{2}\,x+\sqrt{\frac{c}{d}}}{x^2-\left(\frac{c}{d}\right)^{\frac14}\sqrt{2}\,x+\sqrt{\frac{c}{d}}}\right)}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^4+a)/(d*x^4+c),x)
```

```
[Out] b/d*x+1/4*(c/d)^(1/4)/c*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x-1)*a-1/4/d*(c/
d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x-1)*b+1/8*(c/d)^(1/4)/c*2^(1/2
)*ln((x^2+(c/d)^(1/4)*x*2^(1/2)+(c/d)^(1/2))/(x^2-(c/d)^(1/4)*x*2^(1/2)+(c/
d)^(1/2)))*a-1/8/d*(c/d)^(1/4)*2^(1/2)*ln((x^2+(c/d)^(1/4)*x*2^(1/2)+(c/d)^
(1/2))/(x^2-(c/d)^(1/4)*x*2^(1/2)+(c/d)^(1/2)))*b+1/4*(c/d)^(1/4)/c*2^(1/2)
*arctan(2^(1/2)/(c/d)^(1/4)*x+1)*a-1/4/d*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)
/(c/d)^(1/4)*x+1)*b
```

maxima [A] time = 1.28, size = 212, normalized size = 0.95

$$\frac{bx}{d}-\frac{\frac{2\sqrt{2}\,(bc-ad)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{d}\,x+\sqrt{2}\,c^{\frac14}d^{\frac14}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{c}\sqrt{\sqrt{c}\sqrt{d}}}+\frac{2\sqrt{2}\,(bc-ad)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{d}\,x-\sqrt{2}\,c^{\frac14}d^{\frac14}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{c}\sqrt{\sqrt{c}\sqrt{d}}}+\frac{\sqrt{2}\,(bc-ad)\log\left(\sqrt{d}\,x^2+\sqrt{2}\,c^{\frac14}d^{\frac14}x+\sqrt{c}\right)}{c^{\frac34}d^{\frac14}}-\frac{\sqrt{2}\,(bc-ad)\log\left(\sqrt{d}\,x^2-\sqrt{2}\,c^{\frac14}d^{\frac14}x+\sqrt{c}\right)}{c^{\frac34}d^{\frac14}}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)/(d*x^4+c),x, algorithm="maxima")
```

```
[Out] b*x/d - 1/8*(2*sqrt(2)*(b*c - a*d)*arctan(1/2*sqrt(2)*(2*sqrt(d)*x + sqrt(2
)*c^(1/4)*d^(1/4))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(c)*sqrt(sqrt(c)*sqrt(d))) +
 2*sqrt(2)*(b*c - a*d)*arctan(1/2*sqrt(2)*(2*sqrt(d)*x - sqrt(2)*c^(1/4)*d^
(1/4))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(c)*sqrt(sqrt(c)*sqrt(d))) + sqrt(2)*(b*
c - a*d)*log(sqrt(d)*x^2 + sqrt(2)*c^(1/4)*d^(1/4)*x + sqrt(c))/(c^(3/4)*d^
(1/4)) - sqrt(2)*(b*c - a*d)*log(sqrt(d)*x^2 - sqrt(2)*c^(1/4)*d^(1/4)*x +
sqrt(c))/(c^(3/4)*d^(1/4)))/d
```

mupad [B] time = 1.48, size = 720, normalized size = 3.23

$$\frac{bx}{d}-\frac{\mathrm{atan}\left(\frac{\frac{\left(x\left(4a^2d^3-8abcd^2+4b^2c^2d\right)-\frac{\left(16bc^2d^2-16acd^3\right)(ad-bc)}{4(-c)^{3/4}d^{5/4}}\right)(ad-bc)\,1\mathrm{i}}{4(-c)^{3/4}d^{5/4}}+\frac{\left(x\left(4a^2d^3-8abcd^2+4b^2c^2d\right)+\frac{\left(16bc^2d^2-16acd^3\right)(ad-bc)}{4(-c)^{3/4}d^{5/4}}\right)(ad-bc)\,1\mathrm{i}}{4(-c)^{3/4}d^{5/4}}}{\frac{\left(x\left(4a^2d^3-8abcd^2+4b^2c^2d\right)-\frac{\left(16bc^2d^2-16acd^3\right)(ad-bc)}{4(-c)^{3/4}d^{5/4}}\right)(ad-bc)}{4(-c)^{3/4}d^{5/4}}-\frac{\left(x\left(4a^2d^3-8abcd^2+4b^2c^2d\right)+\frac{\left(16bc^2d^2-16acd^3\right)(ad-bc)}{4(-c)^{3/4}d^{5/4}}\right)(ad-bc)}{4(-c)^{3/4}d^{5/4}}}\right)(ad-bc)\,1\mathrm{i}}{2(-c)^{3/4}d^{5/4}}-\frac{\mathrm{atan}\left(\frac{\frac{\left(x\left(4a^2d^3-8abcd^2+4b^2c^2d\right)-\frac{\left(16bc^2d^2-16acd^3\right)(ad-bc)\,1\mathrm{i}}{4(-c)^{3/4}d^{5/4}}\right)(ad-bc)}{4(-c)^{3/4}d^{5/4}}+\frac{\left(x\left(4a^2d^3-8abcd^2+4b^2c^2d\right)+\frac{\left(16bc^2d^2-16acd^3\right)(ad-bc)\,1\mathrm{i}}{4(-c)^{3/4}d^{5/4}}\right)(ad-bc)}{4(-c)^{3/4}d^{5/4}}}{\frac{\left(x\left(4a^2d^3-8abcd^2+4b^2c^2d\right)-\frac{\left(16bc^2d^2-16acd^3\right)(ad-bc)\,1\mathrm{i}}{4(-c)^{3/4}d^{5/4}}\right)(ad-bc)\,1\mathrm{i}}{4(-c)^{3/4}d^{5/4}}-\frac{\left(x\left(4a^2d^3-8abcd^2+4b^2c^2d\right)+\frac{\left(16bc^2d^2-16acd^3\right)(ad-bc)\,1\mathrm{i}}{4(-c)^{3/4}d^{5/4}}\right)(ad-bc)\,1\mathrm{i}}{4(-c)^{3/4}d^{5/4}}}\right)(ad-bc)}{2(-c)^{3/4}d^{5/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^4)/(c + d*x^4),x)
```

```
[Out] (b*x)/d - (atan((((x*(4*a^2*d^3 + 4*b^2*c^2*d - 8*a*b*c*d^2) - ((16*b*c^2*d
^2 - 16*a*c*d^3)*(a*d - b*c))/(4*(-c)^(3/4)*d^(5/4)))*(a*d - b*c)*1i)/(4*(-
c)^(3/4)*d^(5/4)) + ((x*(4*a^2*d^3 + 4*b^2*c^2*d - 8*a*b*c*d^2) + ((16*b*c^
2*d^2 - 16*a*c*d^3)*(a*d - b*c))/(4*(-c)^(3/4)*d^(5/4)))*(a*d - b*c)*1i)/(4
*(-c)^(3/4)*d^(5/4)))/(((x*(4*a^2*d^3 + 4*b^2*c^2*d - 8*a*b*c*d^2) - ((16*b
*c^2*d^2 - 16*a*c*d^3)*(a*d - b*c))/(4*(-c)^(3/4)*d^(5/4)))*(a*d - b*c))/(4
*(-c)^(3/4)*d^(5/4)) - ((x*(4*a^2*d^3 + 4*b^2*c^2*d - 8*a*b*c*d^2) + ((16*b
*c^2*d^2 - 16*a*c*d^3)*(a*d - b*c))/(4*(-c)^(3/4)*d^(5/4)))*(a*d - b*c))/(4
*(-c)^(3/4)*d^(5/4))))*(a*d - b*c)*1i)/(2*(-c)^(3/4)*d^(5/4)) - (atan((((x*
(4*a^2*d^3 + 4*b^2*c^2*d - 8*a*b*c*d^2) - ((16*b*c^2*d^2 - 16*a*c*d^3)*(a*d
 - b*c)*1i)/(4*(-c)^(3/4)*d^(5/4)))*(a*d - b*c))/(4*(-c)^(3/4)*d^(5/4)) + (
(x*(4*a^2*d^3 + 4*b^2*c^2*d - 8*a*b*c*d^2) + ((16*b*c^2*d^2 - 16*a*c*d^3)*(
a*d - b*c)*1i)/(4*(-c)^(3/4)*d^(5/4)))*(a*d - b*c))/(4*(-c)^(3/4)*d^(5/4)))
/(((x*(4*a^2*d^3 + 4*b^2*c^2*d - 8*a*b*c*d^2) - ((16*b*c^2*d^2 - 16*a*c*d^3
)*(a*d - b*c)*1i)/(4*(-c)^(3/4)*d^(5/4)))*(a*d - b*c)*1i)/(4*(-c)^(3/4)*d^(
5/4)) - ((x*(4*a^2*d^3 + 4*b^2*c^2*d - 8*a*b*c*d^2) + ((16*b*c^2*d^2 - 16*a
*c*d^3)*(a*d - b*c)*1i)/(4*(-c)^(3/4)*d^(5/4)))*(a*d - b*c)*1i)/(4*(-c)^(3/
4)*d^(5/4))))*(a*d - b*c))/(2*(-c)^(3/4)*d^(5/4))
```

sympy [A] time = 0.66, size = 87, normalized size = 0.39

$$\frac{bx}{d}+\mathrm{RootSum}\left(256t^4c^3d^5+a^4d^4-4a^3bcd^3+6a^2b^2c^2d^2-4ab^3c^3d+b^4c^4,\left(t\mapsto t\log\left(\frac{4tcd}{ad-bc}+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**4+a)/(d*x**4+c),x)
```

```
[Out] b*x/d + RootSum(256*_t**4*c**3*d**5 + a**4*d**4 - 4*a**3*b*c*d**3 + 6*a**2*
b**2*c**2*d**2 - 4*a*b**3*c**3*d + b**4*c**4, Lambda(_t, _t*log(4*_t*c*d/(a
*d - b*c) + x)))
```

3.88 $\int \frac{a+bx^4}{\left(c+dx^4\right)^2} \, dx$

Optimal. Leaf size=245

$$-\frac{(3ad+bc)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\,x+\sqrt{c}+\sqrt{d}\,x^2\right)}{16\sqrt{2}\,c^{7/4}d^{5/4}}+\frac{(3ad+bc)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\,x+\sqrt{c}+\sqrt{d}\,x^2\right)}{16\sqrt{2}\,c^{7/4}d^{5/4}}-\frac{(3ad+bc)\tan^{-1}}{8\sqrt{2}\,c^{7/}}$$

Rubi [A] time = 0.15, antiderivative size = 245, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.412, Rules used = {385, 211, 1165, 628, 1162, 617, 204}

$$-\frac{(3ad+bc)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\,x+\sqrt{c}+\sqrt{d}\,x^2\right)}{16\sqrt{2}\,c^{7/4}d^{5/4}}+\frac{(3ad+bc)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\,x+\sqrt{c}+\sqrt{d}\,x^2\right)}{16\sqrt{2}\,c^{7/4}d^{5/4}}-\frac{(3ad+bc)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\,x}{\sqrt[4]{c}}\right)}{8\sqrt{2}\,c^{7/4}d^{5/4}}+\frac{(3ad+bc)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}\,x}{\sqrt[4]{c}}+1\right)}{8\sqrt{2}\,c^{7/4}d^{5/4}}-\frac{x(bc-ad)}{4cd\left(c+dx^4\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^4)/(c + d*x^4)^2,x]
```

```
[Out] -((b*c - a*d)*x)/(4*c*d*(c + d*x^4)) - ((b*c + 3*a*d)*ArcTan[1 - (Sqrt[2]*d
^(1/4)*x)/c^(1/4)])/(8*Sqrt[2]*c^(7/4)*d^(5/4)) + ((b*c + 3*a*d)*ArcTan[1 +
 (Sqrt[2]*d^(1/4)*x)/c^(1/4)])/(8*Sqrt[2]*c^(7/4)*d^(5/4)) - ((b*c + 3*a*d)
*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2])/(16*Sqrt[2]*c^(7/4
)*d^(5/4)) + ((b*c + 3*a*d)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[
d]*x^2])/(16*Sqrt[2]*c^(7/4)*d^(5/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{a+bx^4}{\left(c+dx^4\right)^2}\,dx = -\frac{(bc-ad)x}{4cd\left(c+dx^4\right)} + \frac{(bc+3ad)\int \frac{1}{c+dx^4}\,dx}{4cd}$$

$$= -\frac{(bc-ad)x}{4cd\left(c+dx^4\right)} + \frac{(bc+3ad)\int \frac{\sqrt{c}-\sqrt{d}x^2}{c+dx^4}\,dx}{8c^{3/2}d} + \frac{(bc+3ad)\int \frac{\sqrt{c}+\sqrt{d}x^2}{c+dx^4}\,dx}{8c^{3/2}d}$$

$$= -\frac{(bc-ad)x}{4cd\left(c+dx^4\right)} + \frac{(bc+3ad)\int \frac{1}{\frac{\sqrt{c}}{\sqrt{d}}-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}}+x^2}\,dx}{16c^{3/2}d^{3/2}} + \frac{(bc+3ad)\int \frac{1}{\frac{\sqrt{c}}{\sqrt{d}}+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}}+x^2}\,dx}{16c^{3/2}d^{3/2}} - \cdots$$

$$= -\frac{(bc-ad)x}{4cd\left(c+dx^4\right)} - \frac{(bc+3ad)\log\left(\sqrt{c}-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{d}x^2\right)}{16\sqrt{2}c^{7/4}d^{5/4}} + \cdots$$

$$= -\frac{(bc-ad)x}{4cd\left(c+dx^4\right)} - \frac{(bc+3ad)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{8\sqrt{2}c^{7/4}d^{5/4}} + \frac{(bc+3ad)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{8\sqrt{2}c^{7/4}d^{5/4}} - \cdots$$

Mathematica [A] time = 0.20, size = 212, normalized size = 0.87

$$\frac{-\frac{8c^{3/4}\sqrt[4]{d}x(bc-ad)}{c+dx^4}-\sqrt{2}(3ad+bc)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{c}+\sqrt{d}x^2\right)+\sqrt{2}(3ad+bc)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{c}+\sqrt{d}x^2\right)-2\sqrt{2}(3ad+bc)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)+2\sqrt{2}(3ad+bc)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}+1\right)}{32c^{7/4}d^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^4)/(c + d*x^4)^2,x]
```

```
[Out] ((-8*c^(3/4)*d^(1/4)*(b*c - a*d)*x)/(c + d*x^4) - 2*Sqrt[2]*(b*c + 3*a*d)*A
rcTan[1 - (Sqrt[2]*d^(1/4)*x)/c^(1/4)] + 2*Sqrt[2]*(b*c + 3*a*d)*ArcTan[1 +
 (Sqrt[2]*d^(1/4)*x)/c^(1/4)] - Sqrt[2]*(b*c + 3*a*d)*Log[Sqrt[c] - Sqrt[2]
*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2] + Sqrt[2]*(b*c + 3*a*d)*Log[Sqrt[c] + Sqr
t[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2])/(32*c^(7/4)*d^(5/4))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a+bx^4}{\left(c+dx^4\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x^4)/(c + d*x^4)^2,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x^4)/(c + d*x^4)^2, x]
```

fricas [B] time = 1.42, size = 711, normalized size = 2.90

$$\frac{4\left(cd^2x^4+c^2d\right)\left(-\frac{b^4c^4+12ab^3c^3d+54a^2b^2c^2d^2+108a^3bcd^3+81a^4d^4}{c^7d^5}\right)^{\frac{1}{4}}\arctan\left(\frac{-c^5d^4x\left(-\frac{b^4c^4+12ab^3c^3d+54a^2b^2c^2d^2+108a^3bcd^3+81a^4d^4}{c^7d^5}\right)^{\frac{3}{4}}-c^5d^4\sqrt{\frac{c^4d^2\sqrt{-\frac{b^4c^4+12ab^3c^3d+54a^2b^2c^2d^2+108a^3bcd^3+81a^4d^4}{c^7d^5}}+\left(b^2c^2+6abcd+9a^2d^2\right)x^2}{b^2c^2+6abcd+9a^2d^2}}\left(-\frac{b^4c^4+12ab^3c^3d+54a^2b^2c^2d^2+108a^3bcd^3+81a^4d^4}{c^7d^5}\right)^{\frac{3}{4}}}{b^3c^3+9ab^2c^2d+27a^2bcd^2+27a^3d^3}\right)+\left(cd^2x^4+c^2d\right)\left(-\frac{b^4c^4+12ab^3c^3d+54a^2b^2c^2d^2+108a^3bcd^3+81a^4d^4}{c^7d^5}\right)^{\frac{1}{4}}\log\left(c^2d\left(-\frac{b^4c^4+12ab^3c^3d+54a^2b^2c^2d^2+108a^3bcd^3+81a^4d^4}{c^7d^5}\right)^{\frac{1}{4}}+(bc+3ad)x\right)-\left(cd^2x^4+c^2d\right)\left(-\frac{b^4c^4+12ab^3c^3d+54a^2b^2c^2d^2+108a^3bcd^3+81a^4d^4}{c^7d^5}\right)^{\frac{1}{4}}\log\left(-c^2d\left(-\frac{b^4c^4+12ab^3c^3d+54a^2b^2c^2d^2+108a^3bcd^3+81a^4d^4}{c^7d^5}\right)^{\frac{1}{4}}+(bc+3ad)x\right)-4(bc-ad)x}{16\left(cd^2x^4+c^2d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)/(d*x^4+c)^2,x, algorithm="fricas")
```

```
[Out] 1/16*(4*(c*d^2*x^4 + c^2*d)*(-(b^4*c^4 + 12*a*b^3*c^3*d + 54*a^2*b^2*c^2*d^
2 + 108*a^3*b*c*d^3 + 81*a^4*d^4)/(c^7*d^5))^(1/4)*arctan(-(c^5*d^4*x*(-(b^
4*c^4 + 12*a*b^3*c^3*d + 54*a^2*b^2*c^2*d^2 + 108*a^3*b*c*d^3 + 81*a^4*d^4)
/(c^7*d^5))^(3/4) - c^5*d^4*sqrt((c^4*d^2*sqrt(-(b^4*c^4 + 12*a*b^3*c^3*d +
 54*a^2*b^2*c^2*d^2 + 108*a^3*b*c*d^3 + 81*a^4*d^4)/(c^7*d^5)) + (b^2*c^2 +
 6*a*b*c*d + 9*a^2*d^2)*x^2)/(b^2*c^2 + 6*a*b*c*d + 9*a^2*d^2))*(-(b^4*c^4
+ 12*a*b^3*c^3*d + 54*a^2*b^2*c^2*d^2 + 108*a^3*b*c*d^3 + 81*a^4*d^4)/(c^7*
d^5))^(3/4))/(b^3*c^3 + 9*a*b^2*c^2*d + 27*a^2*b*c*d^2 + 27*a^3*d^3)) + (c*
d^2*x^4 + c^2*d)*(-(b^4*c^4 + 12*a*b^3*c^3*d + 54*a^2*b^2*c^2*d^2 + 108*a^3
*b*c*d^3 + 81*a^4*d^4)/(c^7*d^5))^(1/4)*log(c^2*d*(-(b^4*c^4 + 12*a*b^3*c^3
*d + 54*a^2*b^2*c^2*d^2 + 108*a^3*b*c*d^3 + 81*a^4*d^4)/(c^7*d^5))^(1/4) +
(b*c + 3*a*d)*x) - (c*d^2*x^4 + c^2*d)*(-(b^4*c^4 + 12*a*b^3*c^3*d + 54*a^2
*b^2*c^2*d^2 + 108*a^3*b*c*d^3 + 81*a^4*d^4)/(c^7*d^5))^(1/4)*log(-c^2*d*(-
(b^4*c^4 + 12*a*b^3*c^3*d + 54*a^2*b^2*c^2*d^2 + 108*a^3*b*c*d^3 + 81*a^4*d
^4)/(c^7*d^5))^(1/4) + (b*c + 3*a*d)*x) - 4*(b*c - a*d)*x)/(c*d^2*x^4 + c^2
*d)
```

giac [A] time = 0.17, size = 266, normalized size = 1.09

$$\frac{\sqrt{2}\left(\left(cd^3\right)^{\frac{1}{4}}bc+3\left(cd^3\right)^{\frac{1}{4}}ad\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{16c^2d^2}+\frac{\sqrt{2}\left(\left(cd^3\right)^{\frac{1}{4}}bc+3\left(cd^3\right)^{\frac{1}{4}}ad\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{16c^2d^2}+\frac{\sqrt{2}\left(\left(cd^3\right)^{\frac{1}{4}}bc+3\left(cd^3\right)^{\frac{1}{4}}ad\right)\log\left(x^2+\sqrt{2}x\left(\frac{c}{d}\right)^{\frac{1}{4}}+\sqrt{\frac{c}{d}}\right)}{32c^2d^2}-\frac{\sqrt{2}\left(\left(cd^3\right)^{\frac{1}{4}}bc+3\left(cd^3\right)^{\frac{1}{4}}ad\right)\log\left(x^2-\sqrt{2}x\left(\frac{c}{d}\right)^{\frac{1}{4}}+\sqrt{\frac{c}{d}}\right)}{32c^2d^2}-\frac{bcx-adx}{4\left(dx^4+c\right)cd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)/(d*x^4+c)^2,x, algorithm="giac")
```

```
[Out] 1/16*sqrt(2)*((c*d^3)^(1/4)*b*c + 3*(c*d^3)^(1/4)*a*d)*arctan(1/2*sqrt(2)*(
2*x + sqrt(2)*(c/d)^(1/4))/(c/d)^(1/4))/(c^2*d^2) + 1/16*sqrt(2)*((c*d^3)^(
1/4)*b*c + 3*(c*d^3)^(1/4)*a*d)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(c/d)^(1/
4))/(c/d)^(1/4))/(c^2*d^2) + 1/32*sqrt(2)*((c*d^3)^(1/4)*b*c + 3*(c*d^3)^(1
/4)*a*d)*log(x^2 + sqrt(2)*x*(c/d)^(1/4) + sqrt(c/d))/(c^2*d^2) - 1/32*sqrt
(2)*((c*d^3)^(1/4)*b*c + 3*(c*d^3)^(1/4)*a*d)*log(x^2 - sqrt(2)*x*(c/d)^(1/
4) + sqrt(c/d))/(c^2*d^2) - 1/4*(b*c*x - a*d*x)/((d*x^4 + c)*c*d)
```

maple [A] time = 0.05, size = 295, normalized size = 1.20

$$\frac{3\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,a\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}-1\right)}{16c^2}+\frac{3\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,a\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}+1\right)}{16c^2}+\frac{3\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,a\ln\left(\frac{x^2+\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{c}{d}}}{x^2-\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{c}{d}}}\right)}{32c^2}+\frac{\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,b\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}-1\right)}{16cd}+\frac{\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,b\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}+1\right)}{16cd}+\frac{\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,b\ln\left(\frac{x^2+\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{c}{d}}}{x^2-\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{c}{d}}}\right)}{32cd}+\frac{(ad-bc)\,x}{4\left(dx^4+c\right)cd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^4+a)/(d*x^4+c)^2,x)
```

```
[Out] 1/4*(a*d-b*c)/d/c*x/(d*x^4+c)+3/16/c^2*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(
c/d)^(1/4)*x+1)*a+1/16/c/d*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x
+1)*b+3/16/c^2*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x-1)*a+1/16/c
/d*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x-1)*b+3/32/c^2*(c/d)^(1/
4)*2^(1/2)*ln((x^2+(c/d)^(1/4)*2^(1/2)*x+(c/d)^(1/2))/(x^2-(c/d)^(1/4)*2^(1
```

```
/2)*x+(c/d)^(1/2)))*a+1/32/c/d*(c/d)^(1/4)*2^(1/2)*ln((x^2+(c/d)^(1/4)*2^(1
/2)*x+(c/d)^(1/2))/(x^2-(c/d)^(1/4)*2^(1/2)*x+(c/d)^(1/2)))*b
```

maxima [A] time = 1.16, size = 236, normalized size = 0.96

$$-\frac{(bc-ad)x}{4\left(cd^2x^4+c^2d\right)}+\frac{\frac{2\sqrt{2}(bc+3ad)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{d}x+\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{c}\sqrt{\sqrt{c}\sqrt{d}}}+\frac{2\sqrt{2}(bc+3ad)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{d}x-\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{c}\sqrt{\sqrt{c}\sqrt{d}}}+\frac{\sqrt{2}(bc+3ad)\log\left(\sqrt{d}x^2+\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}x+\sqrt{c}\right)}{c^{\frac{3}{4}}d^{\frac{1}{4}}}-\frac{\sqrt{2}(bc+3ad)\log\left(\sqrt{d}x^2-\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}x+\sqrt{c}\right)}{c^{\frac{3}{4}}d^{\frac{1}{4}}}}{32cd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)/(d*x^4+c)^2,x, algorithm="maxima")
```

```
[Out] -1/4*(b*c - a*d)*x/(c*d^2*x^4 + c^2*d) + 1/32*(2*sqrt(2)*(b*c + 3*a*d)*arct
an(1/2*sqrt(2)*(2*sqrt(d)*x + sqrt(2)*c^(1/4)*d^(1/4))/sqrt(sqrt(c)*sqrt(d)
))/(sqrt(c)*sqrt(sqrt(c)*sqrt(d))) + 2*sqrt(2)*(b*c + 3*a*d)*arctan(1/2*sqr
t(2)*(2*sqrt(d)*x - sqrt(2)*c^(1/4)*d^(1/4))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(c
)*sqrt(sqrt(c)*sqrt(d))) + sqrt(2)*(b*c + 3*a*d)*log(sqrt(d)*x^2 + sqrt(2)*
c^(1/4)*d^(1/4)*x + sqrt(c))/(c^(3/4)*d^(1/4)) - sqrt(2)*(b*c + 3*a*d)*log(
sqrt(d)*x^2 - sqrt(2)*c^(1/4)*d^(1/4)*x + sqrt(c))/(c^(3/4)*d^(1/4)))/(c*d)
```

mupad [B] time = 1.52, size = 740, normalized size = 3.02

$$\frac{\mathrm{atan}\left(\frac{\frac{\left(\frac{x\left(9a^2d^3+6abcd^2+b^2c^2d\right)}{4c^2}-\frac{(3ad+bc)\left(12ad^3+4bcd^2\right)1i}{16(-c)^{7/4}d^{5/4}}\right)(3ad+bc)}{16(-c)^{7/4}d^{5/4}}+\frac{\left(\frac{x\left(9a^2d^3+6abcd^2+b^2c^2d\right)}{4c^2}+\frac{(3ad+bc)\left(12ad^3+4bcd^2\right)1i}{16(-c)^{7/4}d^{5/4}}\right)(3ad+bc)}{16(-c)^{7/4}d^{5/4}}}{\frac{\left(\frac{x\left(9a^2d^3+6abcd^2+b^2c^2d\right)}{4c^2}-\frac{(3ad+bc)\left(12ad^3+4bcd^2\right)1i}{16(-c)^{7/4}d^{5/4}}\right)(3ad+bc)1i}{16(-c)^{7/4}d^{5/4}}-\frac{\left(\frac{x\left(9a^2d^3+6abcd^2+b^2c^2d\right)}{4c^2}+\frac{(3ad+bc)\left(12ad^3+4bcd^2\right)1i}{16(-c)^{7/4}d^{5/4}}\right)(3ad+bc)1i}{16(-c)^{7/4}d^{5/4}}}\right)(3ad+bc)}{8(-c)^{7/4}d^{5/4}}+\frac{x(ad-bc)}{4cd\left(dx^4+c\right)}+\frac{\mathrm{atan}\left(\frac{\frac{\left(\frac{x\left(9a^2d^3+6abcd^2+b^2c^2d\right)}{4c^2}-\frac{(3ad+bc)\left(12ad^3+4bcd^2\right)}{16(-c)^{7/4}d^{5/4}}\right)(3ad+bc)1i}{16(-c)^{7/4}d^{5/4}}+\frac{\left(\frac{x\left(9a^2d^3+6abcd^2+b^2c^2d\right)}{4c^2}+\frac{(3ad+bc)\left(12ad^3+4bcd^2\right)}{16(-c)^{7/4}d^{5/4}}\right)(3ad+bc)1i}{16(-c)^{7/4}d^{5/4}}}{\frac{\left(\frac{x\left(9a^2d^3+6abcd^2+b^2c^2d\right)}{4c^2}-\frac{(3ad+bc)\left(12ad^3+4bcd^2\right)}{16(-c)^{7/4}d^{5/4}}\right)(3ad+bc)}{16(-c)^{7/4}d^{5/4}}-\frac{\left(\frac{x\left(9a^2d^3+6abcd^2+b^2c^2d\right)}{4c^2}+\frac{(3ad+bc)\left(12ad^3+4bcd^2\right)}{16(-c)^{7/4}d^{5/4}}\right)(3ad+bc)}{16(-c)^{7/4}d^{5/4}}}\right)(3ad+bc)1i}{8(-c)^{7/4}d^{5/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^4)/(c + d*x^4)^2,x)
```

```
[Out] (atan(((((x*(9*a^2*d^3 + b^2*c^2*d + 6*a*b*c*d^2))/(4*c^2) - ((3*a*d + b*c)
*(12*a*d^3 + 4*b*c*d^2))/(16*(-c)^(7/4)*d^(5/4)))*(3*a*d + b*c)*1i)/(16*(-c
)^(7/4)*d^(5/4)) + (((x*(9*a^2*d^3 + b^2*c^2*d + 6*a*b*c*d^2))/(4*c^2) + ((
3*a*d + b*c)*(12*a*d^3 + 4*b*c*d^2))/(16*(-c)^(7/4)*d^(5/4)))*(3*a*d + b*c)
*1i)/(16*(-c)^(7/4)*d^(5/4)))/((((x*(9*a^2*d^3 + b^2*c^2*d + 6*a*b*c*d^2))/
(4*c^2) - ((3*a*d + b*c)*(12*a*d^3 + 4*b*c*d^2))/(16*(-c)^(7/4)*d^(5/4)))*(
3*a*d + b*c))/(16*(-c)^(7/4)*d^(5/4)) - (((x*(9*a^2*d^3 + b^2*c^2*d + 6*a*b
*c*d^2))/(4*c^2) + ((3*a*d + b*c)*(12*a*d^3 + 4*b*c*d^2))/(16*(-c)^(7/4)*d^
(5/4)))*(3*a*d + b*c))/(16*(-c)^(7/4)*d^(5/4))))*(3*a*d + b*c)*1i)/(8*(-c)^
(7/4)*d^(5/4)) + (atan(((((x*(9*a^2*d^3 + b^2*c^2*d + 6*a*b*c*d^2))/(4*c^2)
 - ((3*a*d + b*c)*(12*a*d^3 + 4*b*c*d^2)*1i)/(16*(-c)^(7/4)*d^(5/4)))*(3*a*
d + b*c))/(16*(-c)^(7/4)*d^(5/4)) + (((x*(9*a^2*d^3 + b^2*c^2*d + 6*a*b*c*d
^2))/(4*c^2) + ((3*a*d + b*c)*(12*a*d^3 + 4*b*c*d^2)*1i)/(16*(-c)^(7/4)*d^(
5/4)))*(3*a*d + b*c))/(16*(-c)^(7/4)*d^(5/4)))/((((x*(9*a^2*d^3 + b^2*c^2*d
 + 6*a*b*c*d^2))/(4*c^2) - ((3*a*d + b*c)*(12*a*d^3 + 4*b*c*d^2)*1i)/(16*(-
c)^(7/4)*d^(5/4)))*(3*a*d + b*c)*1i)/(16*(-c)^(7/4)*d^(5/4)) - (((x*(9*a^2*
d^3 + b^2*c^2*d + 6*a*b*c*d^2))/(4*c^2) + ((3*a*d + b*c)*(12*a*d^3 + 4*b*c*
d^2)*1i)/(16*(-c)^(7/4)*d^(5/4)))*(3*a*d + b*c)*1i)/(16*(-c)^(7/4)*d^(5/4))
))*(3*a*d + b*c))/(8*(-c)^(7/4)*d^(5/4)) + (x*(a*d - b*c))/(4*c*d*(c + d*x^
4))
```

sympy [A] time = 0.85, size = 112, normalized size = 0.46

$$\frac{x(ad-bc)}{4c^2d+4cd^2x^4}+\operatorname{RootSum}\left(65536t^4c^7d^5+81a^4d^4+108a^3bcd^3+54a^2b^2c^2d^2+12ab^3c^3d+b^4c^4,\left(t\mapsto t\log\left(\frac{16tc^2d}{3ad+bc}+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**4+a)/(d*x**4+c)**2,x)
```

```
[Out] x*(a*d - b*c)/(4*c**2*d + 4*c*d**2*x**4) + RootSum(65536*_t**4*c**7*d**5 +
81*a**4*d**4 + 108*a**3*b*c*d**3 + 54*a**2*b**2*c**2*d**2 + 12*a*b**3*c**3*
d + b**4*c**4, Lambda(_t, _t*log(16*_t*c**2*d/(3*a*d + b*c) + x)))
```

3.89 $\int \frac{a+bx^4}{(c+dx^4)^3}\,dx$

Optimal. Leaf size=273

$$-\frac{3(7ad+bc)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{c}+\sqrt{d}x^2\right)}{128\sqrt{2}c^{11/4}d^{5/4}}+\frac{3(7ad+bc)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{c}+\sqrt{d}x^2\right)}{128\sqrt{2}c^{11/4}d^{5/4}}-\frac{3(7ad+bc)\,\text{ta}}{64\sqrt{2}}$$

Rubi [A] time = 0.17, antiderivative size = 273, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.471, Rules used = {385, 199, 211, 1165, 628, 1162, 617, 204}

$$-\frac{3(7ad+bc)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{c}+\sqrt{d}x^2\right)}{128\sqrt{2}c^{11/4}d^{5/4}}+\frac{3(7ad+bc)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{c}+\sqrt{d}x^2\right)}{128\sqrt{2}c^{11/4}d^{5/4}}-\frac{3(7ad+bc)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{64\sqrt{2}c^{11/4}d^{5/4}}+\frac{3(7ad+bc)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}+1\right)}{64\sqrt{2}c^{11/4}d^{5/4}}+\frac{x(7ad+bc)}{32c^2d\left(c+dx^4\right)}-\frac{x(bc-ad)}{8cd\left(c+dx^4\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^4)/(c + d*x^4)^3,x]
```

```
[Out] -((b*c - a*d)*x)/(8*c*d*(c + d*x^4)^2) + ((b*c + 7*a*d)*x)/(32*c^2*d*(c + d
*x^4)) - (3*(b*c + 7*a*d)*ArcTan[1 - (Sqrt[2]*d^(1/4)*x)/c^(1/4)])/(64*Sqrt
[2]*c^(11/4)*d^(5/4)) + (3*(b*c + 7*a*d)*ArcTan[1 + (Sqrt[2]*d^(1/4)*x)/c^(
1/4)])/(64*Sqrt[2]*c^(11/4)*d^(5/4)) - (3*(b*c + 7*a*d)*Log[Sqrt[c] - Sqrt[
2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2])/(128*Sqrt[2]*c^(11/4)*d^(5/4)) + (3*(b
*c + 7*a*d)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2])/(128*Sq
rt[2]*c^(11/4)*d^(5/4))
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$
\begin{aligned}
\int \frac{a+bx^4}{\left(c+dx^4\right)^3}\,dx &= -\frac{(bc-ad)x}{8cd\left(c+dx^4\right)^2}+\frac{(bc+7ad)\int \frac{1}{\left(c+dx^4\right)^2}\,dx}{8cd}\\
&= -\frac{(bc-ad)x}{8cd\left(c+dx^4\right)^2}+\frac{(bc+7ad)x}{32c^2d\left(c+dx^4\right)}+\frac{(3(bc+7ad))\int \frac{1}{c+dx^4}\,dx}{32c^2d}\\
&= -\frac{(bc-ad)x}{8cd\left(c+dx^4\right)^2}+\frac{(bc+7ad)x}{32c^2d\left(c+dx^4\right)}+\frac{(3(bc+7ad))\int \frac{\sqrt{c}-\sqrt{d}x^2}{c+dx^4}\,dx}{64c^{5/2}d}+\frac{(3(bc+7ad))\int \frac{\sqrt{c}+\sqrt{d}x^2}{c+dx^4}}{64c^{5/2}d}\\
&= -\frac{(bc-ad)x}{8cd\left(c+dx^4\right)^2}+\frac{(bc+7ad)x}{32c^2d\left(c+dx^4\right)}+\frac{(3(bc+7ad))\int \frac{1}{\frac{\sqrt{c}}{\sqrt{d}}-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}}+x^2}\,dx}{128c^{5/2}d^{3/2}}+\frac{(3(bc+7ad))\int \frac{}{\frac{\sqrt{c}}{\sqrt{d}}+}}{128c^{5/2}d^{3}}\\
&= -\frac{(bc-ad)x}{8cd\left(c+dx^4\right)^2}+\frac{(bc+7ad)x}{32c^2d\left(c+dx^4\right)}-\frac{3(bc+7ad)\log\left(\sqrt{c}-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{d}x^2\right)}{128\sqrt{2}c^{11/4}d^{5/4}}+\frac{3(bc+7}{}\\
&= -\frac{(bc-ad)x}{8cd\left(c+dx^4\right)^2}+\frac{(bc+7ad)x}{32c^2d\left(c+dx^4\right)}-\frac{3(bc+7ad)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{64\sqrt{2}c^{11/4}d^{5/4}}+\frac{3(bc+7ad)\tan^{-1}\left(1\right.}{64\sqrt{2}c^{11/4}d^{5}}
\end{aligned}
$$

Mathematica [A] time = 0.22, size = 243, normalized size = 0.89

$$
\frac{-\frac{32c^{7/4}\sqrt[4]{d}x(bc-ad)}{\left(c+dx^4\right)^2}+\frac{8c^{3/4}\sqrt[4]{d}x(7ad+bc)}{c+dx^4}-3\sqrt{2}(7ad+bc)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{c}+\sqrt{d}x^2\right)+3\sqrt{2}(7ad+bc)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{c}+\sqrt{d}x^2\right)-6\sqrt{2}(7ad+bc)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)+6\sqrt{2}(7ad+bc)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}+1\right)}{256c^{11/4}d^{5/4}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^4)/(c + d*x^4)^3,x]
```

```
[Out] ((-32*c^(7/4)*d^(1/4)*(b*c - a*d)*x)/(c + d*x^4)^2 + (8*c^(3/4)*d^(1/4)*(b*
c + 7*a*d)*x)/(c + d*x^4) - 6*Sqrt[2]*(b*c + 7*a*d)*ArcTan[1 - (Sqrt[2]*d^(
1/4)*x)/c^(1/4)] + 6*Sqrt[2]*(b*c + 7*a*d)*ArcTan[1 + (Sqrt[2]*d^(1/4)*x)/c
^(1/4)] - 3*Sqrt[2]*(b*c + 7*a*d)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*x +
 Sqrt[d]*x^2] + 3*Sqrt[2]*(b*c + 7*a*d)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/
4)*x + Sqrt[d]*x^2])/(256*c^(11/4)*d^(5/4))
```

IntegrateAlgebraic **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + bx^4}{\left(c + dx^4\right)^3} \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x^4)/(c + d*x^4)^3,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x^4)/(c + d*x^4)^3, x]
```

fricas **[B]** time = 1.32, size = 787, normalized size = 2.88

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)/(d*x^4+c)^3,x, algorithm="fricas")
```

```
[Out] 1/128*(4*(b*c*d + 7*a*d^2)*x^5 + 12*(c^2*d^3*x^8 + 2*c^3*d^2*x^4 + c^4*d)*(
-(b^4*c^4 + 28*a*b^3*c^3*d + 294*a^2*b^2*c^2*d^2 + 1372*a^3*b*c*d^3 + 2401*
a^4*d^4)/(c^11*d^5))^(1/4)*arctan(-(c^8*d^4*x*(-(b^4*c^4 + 28*a*b^3*c^3*d +
 294*a^2*b^2*c^2*d^2 + 1372*a^3*b*c*d^3 + 2401*a^4*d^4)/(c^11*d^5))^(3/4) -
 c^8*d^4*sqrt((c^6*d^2*sqrt(-(b^4*c^4 + 28*a*b^3*c^3*d + 294*a^2*b^2*c^2*d^
2 + 1372*a^3*b*c*d^3 + 2401*a^4*d^4)/(c^11*d^5)) + (b^2*c^2 + 14*a*b*c*d +
49*a^2*d^2)*x^2)/(b^2*c^2 + 14*a*b*c*d + 49*a^2*d^2))*(-(b^4*c^4 + 28*a*b^3
*c^3*d + 294*a^2*b^2*c^2*d^2 + 1372*a^3*b*c*d^3 + 2401*a^4*d^4)/(c^11*d^5))
^(3/4))/(b^3*c^3 + 21*a*b^2*c^2*d + 147*a^2*b*c*d^2 + 343*a^3*d^3)) + 3*(c^
2*d^3*x^8 + 2*c^3*d^2*x^4 + c^4*d)*(-(b^4*c^4 + 28*a*b^3*c^3*d + 294*a^2*b^
2*c^2*d^2 + 1372*a^3*b*c*d^3 + 2401*a^4*d^4)/(c^11*d^5))^(1/4)*log(3*c^3*d*
(-(b^4*c^4 + 28*a*b^3*c^3*d + 294*a^2*b^2*c^2*d^2 + 1372*a^3*b*c*d^3 + 2401
*a^4*d^4)/(c^11*d^5))^(1/4) + 3*(b*c + 7*a*d)*x) - 3*(c^2*d^3*x^8 + 2*c^3*d
^2*x^4 + c^4*d)*(-(b^4*c^4 + 28*a*b^3*c^3*d + 294*a^2*b^2*c^2*d^2 + 1372*a^
3*b*c*d^3 + 2401*a^4*d^4)/(c^11*d^5))^(1/4)*log(-3*c^3*d*(-(b^4*c^4 + 28*a*
b^3*c^3*d + 294*a^2*b^2*c^2*d^2 + 1372*a^3*b*c*d^3 + 2401*a^4*d^4)/(c^11*d^
5))^(1/4) + 3*(b*c + 7*a*d)*x) - 4*(3*b*c^2 - 11*a*c*d)*x)/(c^2*d^3*x^8 + 2
*c^3*d^2*x^4 + c^4*d)
```

giac **[A]** time = 0.19, size = 286, normalized size = 1.05

$$\frac{3\sqrt{2}\left(\left(cd^3\right)^{\frac{1}{4}} bc + 7\left(cd^3\right)^{\frac{1}{4}} ad\right)\arctan\left(\frac{\sqrt{2}\left(2x + \sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{128\,c^3d^2} + \frac{3\sqrt{2}\left(\left(cd^3\right)^{\frac{1}{4}} bc + 7\left(cd^3\right)^{\frac{1}{4}} ad\right)\arctan\left(\frac{\sqrt{2}\left(2x - \sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{128\,c^3d^2} + \frac{3\sqrt{2}\left(\left(cd^3\right)^{\frac{1}{4}} bc + 7\left(cd^3\right)^{\frac{1}{4}} ad\right)\log\left(x^2 + \sqrt{2}x\left(\frac{c}{d}\right)^{\frac{1}{4}} + \sqrt{\frac{c}{d}}\right)}{256\,c^3d^2} - \frac{3\sqrt{2}\left(\left(cd^3\right)^{\frac{1}{4}} bc + 7\left(cd^3\right)^{\frac{1}{4}} ad\right)\log\left(x^2 - \sqrt{2}x\left(\frac{c}{d}\right)^{\frac{1}{4}} + \sqrt{\frac{c}{d}}\right)}{256\,c^3d^2} + \frac{bcdx^5 + 7\,ad^2x^5 - 3\,bc^2x + 11\,acdx}{32\left(dx^4 + c\right)^2 c^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)/(d*x^4+c)^3,x, algorithm="giac")
```

```
[Out] 3/128*sqrt(2)*((c*d^3)^(1/4)*b*c + 7*(c*d^3)^(1/4)*a*d)*arctan(1/2*sqrt(2)*
(2*x + sqrt(2)*(c/d)^(1/4))/(c/d)^(1/4))/(c^3*d^2) + 3/128*sqrt(2)*((c*d^3)
^(1/4)*b*c + 7*(c*d^3)^(1/4)*a*d)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(c/d)^(
1/4))/(c/d)^(1/4))/(c^3*d^2) + 3/256*sqrt(2)*((c*d^3)^(1/4)*b*c + 7*(c*d^3)
^(1/4)*a*d)*log(x^2 + sqrt(2)*x*(c/d)^(1/4) + sqrt(c/d))/(c^3*d^2) - 3/256*
sqrt(2)*((c*d^3)^(1/4)*b*c + 7*(c*d^3)^(1/4)*a*d)*log(x^2 - sqrt(2)*x*(c/d)
```

```
^(1/4) + sqrt(c/d))/(c^3*d^2) + 1/32*(b*c*d*x^5 + 7*a*d^2*x^5 - 3*b*c^2*x +
 11*a*c*d*x)/((d*x^4 + c)^2*c^2*d)
```

maple [A] time = 0.05, size = 314, normalized size = 1.15

$$\frac{21\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,a\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}-1\right)}{128c^{3}}+\frac{21\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,a\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}+1\right)}{128c^{3}}+\frac{21\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,a\ln\left(\frac{x^{2}+\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{c}{d}}}{x^{2}-\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{c}{d}}}\right)}{256c^{3}}+\frac{3\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,b\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}-1\right)}{128c^{2}d}+\frac{3\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,b\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}+1\right)}{128c^{2}d}+\frac{3\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,b\ln\left(\frac{x^{2}+\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{c}{d}}}{x^{2}-\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{c}{d}}}\right)}{256c^{2}d}+\frac{\frac{(7ad+bc)x^{5}}{32c^{2}}+\frac{(11ad-3bc)x}{32cd}}{\left(d\,x^{4}+c\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^4+a)/(d*x^4+c)^3,x)
```

```
[Out] (1/32*(7*a*d+b*c)/c^2*x^5+1/32*(11*a*d-3*b*c)/c/d*x)/(d*x^4+c)^2+21/128/c^3
*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x-1)*a+3/128/c^2/d*(c/d)^(1
/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x-1)*b+21/256/c^3*(c/d)^(1/4)*2^(1/2
)*ln((x^2+(c/d)^(1/4)*2^(1/2)*x+(c/d)^(1/2))/(x^2-(c/d)^(1/4)*2^(1/2)*x+(c/
d)^(1/2)))*a+3/256/c^2/d*(c/d)^(1/4)*2^(1/2)*ln((x^2+(c/d)^(1/4)*2^(1/2)*x+
(c/d)^(1/2))/(x^2-(c/d)^(1/4)*2^(1/2)*x+(c/d)^(1/2)))*b+21/128/c^3*(c/d)^(1
/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x+1)*a+3/128/c^2/d*(c/d)^(1/4)*2^(1/
2)*arctan(2^(1/2)/(c/d)^(1/4)*x+1)*b
```

maxima [A] time = 1.21, size = 271, normalized size = 0.99

$$\frac{\left(bcd+7\,ad^{2}\right)x^{5}-\left(3\,bc^{2}-11\,acd\right)x}{32\left(c^{2}d^{3}x^{8}+2\,c^{3}d^{2}x^{4}+c^{4}d\right)}+\frac{3\left(\frac{2\sqrt{2}\,(bc+7\,ad)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{d}\,x+\sqrt{2}\,c^{\frac{1}{4}}d^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{c}\sqrt{\sqrt{c}\sqrt{d}}}+\frac{2\sqrt{2}\,(bc+7\,ad)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{d}\,x-\sqrt{2}\,c^{\frac{1}{4}}d^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{c}\sqrt{\sqrt{c}\sqrt{d}}}+\frac{\sqrt{2}\,(bc+7\,ad)\log\left(\sqrt{d}\,x^{2}+\sqrt{2}\,c^{\frac{1}{4}}d^{\frac{1}{4}}x+\sqrt{c}\right)}{c^{\frac{3}{4}}d^{\frac{1}{4}}}-\frac{\sqrt{2}\,(bc+7\,ad)\log\left(\sqrt{d}\,x^{2}-\sqrt{2}\,c^{\frac{1}{4}}d^{\frac{1}{4}}x+\sqrt{c}\right)}{c^{\frac{3}{4}}d^{\frac{1}{4}}}\right)}{256\,c^{2}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)/(d*x^4+c)^3,x, algorithm="maxima")
```

```
[Out] 1/32*((b*c*d + 7*a*d^2)*x^5 - (3*b*c^2 - 11*a*c*d)*x)/(c^2*d^3*x^8 + 2*c^3*
d^2*x^4 + c^4*d) + 3/256*(2*sqrt(2)*(b*c + 7*a*d)*arctan(1/2*sqrt(2)*(2*sqr
t(d)*x + sqrt(2)*c^(1/4)*d^(1/4))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(c)*sqrt(sqrt
(c)*sqrt(d))) + 2*sqrt(2)*(b*c + 7*a*d)*arctan(1/2*sqrt(2)*(2*sqrt(d)*x - s
qrt(2)*c^(1/4)*d^(1/4))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(c)*sqrt(sqrt(c)*sqrt(d
))) + sqrt(2)*(b*c + 7*a*d)*log(sqrt(d)*x^2 + sqrt(2)*c^(1/4)*d^(1/4)*x + s
qrt(c))/(c^(3/4)*d^(1/4)) - sqrt(2)*(b*c + 7*a*d)*log(sqrt(d)*x^2 - sqrt(2)
*c^(1/4)*d^(1/4)*x + sqrt(c))/(c^(3/4)*d^(1/4)))/(c^2*d)
```

mupad [B] time = 1.58, size = 762, normalized size = 2.79

$$\frac{\frac{x^{5}\,(7\,a\,d+b\,c)}{32\,c^{2}}+\frac{x\,(11\,a\,d-3\,b\,c)}{32\,c\,d}}{c^{2}+2\,c\,d\,x^{4}+d^{2}\,x^{8}}-\frac{\mathrm{atan}\left(\frac{\frac{\left(\frac{9\,x\,\left(49\,a^{2}\,d^{3}+14\,a\,b\,c\,d^{2}+b^{2}\,c^{2}\,d\right)}{256\,c^{4}}-\frac{9\,(7\,a\,d+b\,c)\,\left(7\,a\,d^{3}+b\,c\,d^{2}\right)}{256\,{(-c)}^{15/4}\,d^{5/4}}\right)(7\,a\,d+b\,c)\,3{}\mathrm{i}}{128\,{(-c)}^{11/4}\,d^{5/4}}+\frac{\left(\frac{9\,x\,\left(49\,a^{2}\,d^{3}+14\,a\,b\,c\,d^{2}+b^{2}\,c^{2}\,d\right)}{256\,c^{4}}+\frac{9\,(7\,a\,d+b\,c)\,\left(7\,a\,d^{3}+b\,c\,d^{2}\right)}{256\,{(-c)}^{15/4}\,d^{5/4}}\right)(7\,a\,d+b\,c)\,3{}\mathrm{i}}{128\,{(-c)}^{11/4}\,d^{5/4}}}{\frac{3\left(\frac{9\,x\,\left(49\,a^{2}\,d^{3}+14\,a\,b\,c\,d^{2}+b^{2}\,c^{2}\,d\right)}{256\,c^{4}}-\frac{9\,(7\,a\,d+b\,c)\,\left(7\,a\,d^{3}+b\,c\,d^{2}\right)}{256\,{(-c)}^{15/4}\,d^{5/4}}\right)(7\,a\,d+b\,c)}{128\,{(-c)}^{11/4}\,d^{5/4}}-\frac{3\left(\frac{9\,x\,\left(49\,a^{2}\,d^{3}+14\,a\,b\,c\,d^{2}+b^{2}\,c^{2}\,d\right)}{256\,c^{4}}+\frac{9\,(7\,a\,d+b\,c)\,\left(7\,a\,d^{3}+b\,c\,d^{2}\right)}{256\,{(-c)}^{15/4}\,d^{5/4}}\right)(7\,a\,d+b\,c)}{128\,{(-c)}^{11/4}\,d^{5/4}}}\right)(7\,a\,d+b\,c)\,3{}\mathrm{i}}{64\,{(-c)}^{11/4}\,d^{5/4}}-\frac{3\,\mathrm{atan}\left(\frac{\frac{3\left(\frac{9\,x\,\left(49\,a^{2}\,d^{3}+14\,a\,b\,c\,d^{2}+b^{2}\,c^{2}\,d\right)}{256\,c^{4}}-\frac{(7\,a\,d+b\,c)\,\left(7\,a\,d^{3}+b\,c\,d^{2}\right)9{}\mathrm{i}}{256\,{(-c)}^{15/4}\,d^{5/4}}\right)(7\,a\,d+b\,c)}{128\,{(-c)}^{11/4}\,d^{5/4}}+\frac{3\left(\frac{9\,x\,\left(49\,a^{2}\,d^{3}+14\,a\,b\,c\,d^{2}+b^{2}\,c^{2}\,d\right)}{256\,c^{4}}+\frac{(7\,a\,d+b\,c)\,\left(7\,a\,d^{3}+b\,c\,d^{2}\right)9{}\mathrm{i}}{256\,{(-c)}^{15/4}\,d^{5/4}}\right)(7\,a\,d+b\,c)}{128\,{(-c)}^{11/4}\,d^{5/4}}}{\frac{\left(\frac{9\,x\,\left(49\,a^{2}\,d^{3}+14\,a\,b\,c\,d^{2}+b^{2}\,c^{2}\,d\right)}{256\,c^{4}}-\frac{(7\,a\,d+b\,c)\,\left(7\,a\,d^{3}+b\,c\,d^{2}\right)9{}\mathrm{i}}{256\,{(-c)}^{15/4}\,d^{5/4}}\right)(7\,a\,d+b\,c)\,3{}\mathrm{i}}{128\,{(-c)}^{11/4}\,d^{5/4}}-\frac{\left(\frac{9\,x\,\left(49\,a^{2}\,d^{3}+14\,a\,b\,c\,d^{2}+b^{2}\,c^{2}\,d\right)}{256\,c^{4}}+\frac{(7\,a\,d+b\,c)\,\left(7\,a\,d^{3}+b\,c\,d^{2}\right)9{}\mathrm{i}}{256\,{(-c)}^{15/4}\,d^{5/4}}\right)(7\,a\,d+b\,c)\,3{}\mathrm{i}}{128\,{(-c)}^{11/4}\,d^{5/4}}}\right)(7\,a\,d+b\,c)}{64\,{(-c)}^{11/4}\,d^{5/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^4)/(c + d*x^4)^3,x)
```

```
[Out] ((x^5*(7*a*d + b*c))/(32*c^2) + (x*(11*a*d - 3*b*c))/(32*c*d))/(c^2 + d^2*x
^8 + 2*c*d*x^4) - (atan(((((9*x*(49*a^2*d^3 + b^2*c^2*d + 14*a*b*c*d^2))/(2
56*c^4) - (9*(7*a*d + b*c)*(7*a*d^3 + b*c*d^2))/(256*(-c)^(15/4)*d^(5/4)))*
(7*a*d + b*c)*3i)/(128*(-c)^(11/4)*d^(5/4)) + (((9*x*(49*a^2*d^3 + b^2*c^2*
d + 14*a*b*c*d^2))/(256*c^4) + (9*(7*a*d + b*c)*(7*a*d^3 + b*c*d^2))/(256*(
-c)^(15/4)*d^(5/4)))*(7*a*d + b*c)*3i)/(128*(-c)^(11/4)*d^(5/4)))/((3*((9*x
*(49*a^2*d^3 + b^2*c^2*d + 14*a*b*c*d^2))/(256*c^4) - (9*(7*a*d + b*c)*(7*a
*d^3 + b*c*d^2))/(256*(-c)^(15/4)*d^(5/4)))*(7*a*d + b*c))/(128*(-c)^(11/4)
*d^(5/4)) - (3*((9*x*(49*a^2*d^3 + b^2*c^2*d + 14*a*b*c*d^2))/(256*c^4) + (
9*(7*a*d + b*c)*(7*a*d^3 + b*c*d^2))/(256*(-c)^(15/4)*d^(5/4)))*(7*a*d + b*
c))/(128*(-c)^(11/4)*d^(5/4))))*(7*a*d + b*c)*3i)/(64*(-c)^(11/4)*d^(5/4))
```

```
- (3*atan(((3*((9*x*(49*a^2*d^3 + b^2*c^2*d + 14*a*b*c*d^2))/(256*c^4) - ((
7*a*d + b*c)*(7*a*d^3 + b*c*d^2)*9i)/(256*(-c)^(15/4)*d^(5/4)))*(7*a*d + b*
c))/(128*(-c)^(11/4)*d^(5/4)) + (3*((9*x*(49*a^2*d^3 + b^2*c^2*d + 14*a*b*c
*d^2))/(256*c^4) + ((7*a*d + b*c)*(7*a*d^3 + b*c*d^2)*9i)/(256*(-c)^(15/4)*
d^(5/4)))*(7*a*d + b*c))/(128*(-c)^(11/4)*d^(5/4)))/((((9*x*(49*a^2*d^3 + b
^2*c^2*d + 14*a*b*c*d^2))/(256*c^4) - ((7*a*d + b*c)*(7*a*d^3 + b*c*d^2)*9i
)/(256*(-c)^(15/4)*d^(5/4)))*(7*a*d + b*c)*3i)/(128*(-c)^(11/4)*d^(5/4)) -
(((9*x*(49*a^2*d^3 + b^2*c^2*d + 14*a*b*c*d^2))/(256*c^4) + ((7*a*d + b*c)*
(7*a*d^3 + b*c*d^2)*9i)/(256*(-c)^(15/4)*d^(5/4)))*(7*a*d + b*c)*3i)/(128*(
-c)^(11/4)*d^(5/4))))*(7*a*d + b*c))/(64*(-c)^(11/4)*d^(5/4))
```

sympy [A] time = 1.03, size = 151, normalized size = 0.55

$$\frac{x^5\left(7ad^2+bcd\right)+x\left(11acd-3bc^2\right)}{32c^4d+64c^3d^2x^4+32c^2d^3x^8}+\operatorname{RootSum}\left(268435456t^4c^{11}d^5+194481a^4d^4+111132a^3bcd^3+23814a^2b^2c^2d^2+2268ab^3c^3d+81b^4c^4,\left(t\mapsto t\log\left(\frac{128tc^3d}{21ad+3bc}+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**4+a)/(d*x**4+c)**3,x)
```

```
[Out] (x**5*(7*a*d**2 + b*c*d) + x*(11*a*c*d - 3*b*c**2))/(32*c**4*d + 64*c**3*d*
*2*x**4 + 32*c**2*d**3*x**8) + RootSum(268435456*_t**4*c**11*d**5 + 194481*
a**4*d**4 + 111132*a**3*b*c*d**3 + 23814*a**2*b**2*c**2*d**2 + 2268*a*b**3*
c**3*d + 81*b**4*c**4, Lambda(_t, _t*log(128*_t*c**3*d/(21*a*d + 3*b*c) + x
)))
```

3.90 $\int \left(a+bx^4\right)^2 \left(c+dx^4\right)^4 \, dx$

Optimal. Leaf size=154

$$\frac{1}{17}d^2x^{17}\left(a^2d^2+8abcd+6b^2c^2\right)+\frac{4}{13}cdx^{13}\left(a^2d^2+3abcd+b^2c^2\right)+\frac{1}{9}c^2x^9\left(6a^2d^2+8abcd+b^2c^2\right)+a^2c^4x+\frac{2}{5}ac^3x^5($$

Rubi [A] time = 0.11, antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {373}

$$\frac{1}{17}d^2x^{17}\left(a^2d^2+8abcd+6b^2c^2\right)+\frac{4}{13}cdx^{13}\left(a^2d^2+3abcd+b^2c^2\right)+\frac{1}{9}c^2x^9\left(6a^2d^2+8abcd+b^2c^2\right)+a^2c^4x+\frac{2}{5}ac^3x^5(2ad+bc)+\frac{2}{21}bd^3x^{21}(ad+2bc)+\frac{1}{25}b^2d^4x^{25}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^4)^2*(c + d*x^4)^4,x]
```

```
[Out] a^2*c^4*x + (2*a*c^3*(b*c + 2*a*d)*x^5)/5 + (c^2*(b^2*c^2 + 8*a*b*c*d + 6*a
^2*d^2)*x^9)/9 + (4*c*d*(b^2*c^2 + 3*a*b*c*d + a^2*d^2)*x^13)/13 + (d^2*(6*
b^2*c^2 + 8*a*b*c*d + a^2*d^2)*x^17)/17 + (2*b*d^3*(2*b*c + a*d)*x^21)/21 +
 (b^2*d^4*x^25)/25
```

Rule 373

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\int \left(a+bx^4\right)^2 \left(c+dx^4\right)^4 \, dx = \int \left(a^2c^4+2ac^3(bc+2ad)x^4+c^2\left(b^2c^2+8abcd+6a^2d^2\right)x^8+4cd\left(b^2c^2+3abcd+\right.\right.$$
$$= a^2c^4x+\frac{2}{5}ac^3(bc+2ad)x^5+\frac{1}{9}c^2\left(b^2c^2+8abcd+6a^2d^2\right)x^9+\frac{4}{13}cd\left(b^2c^2+3abcd+\right.$$

Mathematica [A] time = 0.03, size = 154, normalized size = 1.00

$$\frac{1}{17}d^2x^{17}\left(a^2d^2+8abcd+6b^2c^2\right)+\frac{4}{13}cdx^{13}\left(a^2d^2+3abcd+b^2c^2\right)+\frac{1}{9}c^2x^9\left(6a^2d^2+8abcd+b^2c^2\right)+a^2c^4x+\frac{2}{5}ac^3x^5(2ad+bc)+\frac{2}{21}bd^3x^{21}(ad+2bc)+\frac{1}{25}b^2d^4x^{25}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^4)^2*(c + d*x^4)^4,x]
```

```
[Out] a^2*c^4*x + (2*a*c^3*(b*c + 2*a*d)*x^5)/5 + (c^2*(b^2*c^2 + 8*a*b*c*d + 6*a
^2*d^2)*x^9)/9 + (4*c*d*(b^2*c^2 + 3*a*b*c*d + a^2*d^2)*x^13)/13 + (d^2*(6*
b^2*c^2 + 8*a*b*c*d + a^2*d^2)*x^17)/17 + (2*b*d^3*(2*b*c + a*d)*x^21)/21 +
 (b^2*d^4*x^25)/25
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+bx^4\right)^2 \left(c+dx^4\right)^4 \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x^4)^2*(c + d*x^4)^4,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x^4)^2*(c + d*x^4)^4, x]
```

fricas [A] time = 1.05, size = 173, normalized size = 1.12

$$\frac{1}{25}x^{25}d^4b^2+\frac{4}{21}x^{21}d^3cb^2+\frac{2}{21}x^{21}d^4ba+\frac{6}{17}x^{17}d^2c^2b^2+\frac{8}{17}x^{17}d^3cba+\frac{1}{17}x^{17}d^4a^2+\frac{4}{13}x^{13}dc^3b^2+\frac{12}{13}x^{13}d^2c^2ba+\frac{4}{13}x^{13}d^3ca^2+\frac{1}{9}x^9c^4b^2+\frac{8}{9}x^9dc^3ba+\frac{2}{3}x^9d^2c^2a^2+\frac{2}{5}x^5c^4ba+\frac{4}{5}x^5dc^3a^2+xc^4a^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^2*(d*x^4+c)^4,x, algorithm="fricas")
```

```
[Out] 1/25*x^25*d^4*b^2 + 4/21*x^21*d^3*c*b^2 + 2/21*x^21*d^4*b*a + 6/17*x^17*d^2
*c^2*b^2 + 8/17*x^17*d^3*c*b*a + 1/17*x^17*d^4*a^2 + 4/13*x^13*d*c^3*b^2 +
12/13*x^13*d^2*c^2*b*a + 4/13*x^13*d^3*c*a^2 + 1/9*x^9*c^4*b^2 + 8/9*x^9*d*
c^3*b*a + 2/3*x^9*d^2*c^2*a^2 + 2/5*x^5*c^4*b*a + 4/5*x^5*d*c^3*a^2 + x*c^4
*a^2
```

giac [A] time = 0.15, size = 173, normalized size = 1.12

$$\frac{1}{25}b^2d^4x^{25}+\frac{4}{21}b^2cd^3x^{21}+\frac{2}{21}abd^4x^{21}+\frac{6}{17}b^2c^2d^2x^{17}+\frac{8}{17}abcd^3x^{17}+\frac{1}{17}a^2d^4x^{17}+\frac{4}{13}b^2c^3dx^{13}+\frac{12}{13}abc^2d^2x^{13}+\frac{4}{13}a^2cd^3x^{13}+\frac{1}{9}b^2c^4x^9+\frac{8}{9}abc^3dx^9+\frac{2}{3}a^2c^2d^2x^9+\frac{2}{5}abc^4x^5+\frac{4}{5}a^2c^3dx^5+a^2c^4x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^2*(d*x^4+c)^4,x, algorithm="giac")
```

```
[Out] 1/25*b^2*d^4*x^25 + 4/21*b^2*c*d^3*x^21 + 2/21*a*b*d^4*x^21 + 6/17*b^2*c^2*
d^2*x^17 + 8/17*a*b*c*d^3*x^17 + 1/17*a^2*d^4*x^17 + 4/13*b^2*c^3*d*x^13 +
12/13*a*b*c^2*d^2*x^13 + 4/13*a^2*c*d^3*x^13 + 1/9*b^2*c^4*x^9 + 8/9*a*b*c^
3*d*x^9 + 2/3*a^2*c^2*d^2*x^9 + 2/5*a*b*c^4*x^5 + 4/5*a^2*c^3*d*x^5 + a^2*c
^4*x
```

maple [A] time = 0.04, size = 163, normalized size = 1.06

$$\frac{b^2d^4x^{25}}{25}+\frac{\left(2abd^4+4b^2cd^3\right)x^{21}}{21}+\frac{\left(a^2d^4+8abcd^3+6b^2c^2d^2\right)x^{17}}{17}+\frac{\left(4a^2cd^3+12abc^2d^2+4b^2c^3d\right)x^{13}}{13}+\frac{\left(6a^2c^2d^2+8abc^3d+b^2c^4\right)x^9}{9}+a^2c^4x+\frac{\left(4a^2c^3d+2abc^4\right)x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^4+a)^2*(d*x^4+c)^4,x)
```

```
[Out] 1/25*b^2*d^4*x^25+1/21*(2*a*b*d^4+4*b^2*c*d^3)*x^21+1/17*(a^2*d^4+8*a*b*c*d
^3+6*b^2*c^2*d^2)*x^17+1/13*(4*a^2*c*d^3+12*a*b*c^2*d^2+4*b^2*c^3*d)*x^13+1
/9*(6*a^2*c^2*d^2+8*a*b*c^3*d+b^2*c^4)*x^9+1/5*(4*a^2*c^3*d+2*a*b*c^4)*x^5+
a^2*c^4*x
```

maxima [A] time = 0.55, size = 158, normalized size = 1.03

$$\frac{1}{25}b^2d^4x^{25}+\frac{2}{21}\left(2b^2cd^3+abd^4\right)x^{21}+\frac{1}{17}\left(6b^2c^2d^2+8abcd^3+a^2d^4\right)x^{17}+\frac{4}{13}\left(b^2c^3d+3abc^2d^2+a^2cd^3\right)x^{13}+\frac{1}{9}\left(b^2c^4+8abc^3d+6a^2c^2d^2\right)x^9+a^2c^4x+\frac{2}{5}\left(abc^4+2a^2c^3d\right)x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^2*(d*x^4+c)^4,x, algorithm="maxima")
```

```
[Out] 1/25*b^2*d^4*x^25 + 2/21*(2*b^2*c*d^3 + a*b*d^4)*x^21 + 1/17*(6*b^2*c^2*d^2
 + 8*a*b*c*d^3 + a^2*d^4)*x^17 + 4/13*(b^2*c^3*d + 3*a*b*c^2*d^2 + a^2*c*d^
3)*x^13 + 1/9*(b^2*c^4 + 8*a*b*c^3*d + 6*a^2*c^2*d^2)*x^9 + a^2*c^4*x + 2/5
*(a*b*c^4 + 2*a^2*c^3*d)*x^5
```

mupad [B] time = 0.07, size = 146, normalized size = 0.95

$$x^9\left(\frac{2a^2c^2d^2}{3}+\frac{8abc^3d}{9}+\frac{b^2c^4}{9}\right)+x^{17}\left(\frac{a^2d^4}{17}+\frac{8abcd^3}{17}+\frac{6b^2c^2d^2}{17}\right)+a^2c^4x+\frac{b^2d^4x^{25}}{25}+\frac{2ac^3x^5\left(2ad+bc\right)}{5}+\frac{2bd^3x^{21}\left(ad+2bc\right)}{21}+\frac{4cdx^{13}\left(a^2d^2+3abcd+b^2c^2\right)}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^4)^2*(c + d*x^4)^4,x)
```

```
[Out] x^9*((b^2*c^4)/9 + (2*a^2*c^2*d^2)/3 + (8*a*b*c^3*d)/9) + x^17*((a^2*d^4)/1
7 + (6*b^2*c^2*d^2)/17 + (8*a*b*c*d^3)/17) + a^2*c^4*x + (b^2*d^4*x^25)/25
+ (2*a*c^3*x^5*(2*a*d + b*c))/5 + (2*b*d^3*x^21*(a*d + 2*b*c))/21 + (4*c*d*
x^13*(a^2*d^2 + b^2*c^2 + 3*a*b*c*d))/13
```

sympy [A] time = 0.12, size = 185, normalized size = 1.20

$$a^2c^4x + \frac{b^2d^4x^{25}}{25} + x^{21}\left(\frac{2abd^4}{21} + \frac{4b^2cd^3}{21}\right) + x^{17}\left(\frac{a^2d^4}{17} + \frac{8abcd^3}{17} + \frac{6b^2c^2d^2}{17}\right) + x^{13}\left(\frac{4a^2cd^3}{13} + \frac{12abc^2d^2}{13} + \frac{4b^2c^3d}{13}\right) + x^9\left(\frac{2a^2c^2d^2}{3} + \frac{8abc^3d}{9} + \frac{b^2c^4}{9}\right) + x^5\left(\frac{4a^2c^3d}{5} + \frac{2abc^4}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**4+a)**2*(d*x**4+c)**4,x)
```

```
[Out] a**2*c**4*x + b**2*d**4*x**25/25 + x**21*(2*a*b*d**4/21 + 4*b**2*c*d**3/21)
 + x**17*(a**2*d**4/17 + 8*a*b*c*d**3/17 + 6*b**2*c**2*d**2/17) + x**13*(4*
a**2*c*d**3/13 + 12*a*b*c**2*d**2/13 + 4*b**2*c**3*d/13) + x**9*(2*a**2*c**
2*d**2/3 + 8*a*b*c**3*d/9 + b**2*c**4/9) + x**5*(4*a**2*c**3*d/5 + 2*a*b*c*
*4/5)
```

3.91 $\int \left(a+bx^4\right)^2 \left(c+dx^4\right)^3 \, dx$

Optimal. Leaf size=122

$$\frac{1}{13}dx^{13}\left(a^2d^2+6abcd+3b^2c^2\right)+\frac{1}{9}cx^9\left(3a^2d^2+6abcd+b^2c^2\right)+a^2c^3x+\frac{1}{5}ac^2x^5(3ad+2bc)+\frac{1}{17}bd^2x^{17}(2ad+3bc)$$

Rubi [A] time = 0.08, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {373}

$$\frac{1}{13}dx^{13}\left(a^2d^2+6abcd+3b^2c^2\right)+\frac{1}{9}cx^9\left(3a^2d^2+6abcd+b^2c^2\right)+a^2c^3x+\frac{1}{5}ac^2x^5(3ad+2bc)+\frac{1}{17}bd^2x^{17}(2ad+3bc)+\frac{1}{21}b^2d^3x^{21}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^4)^2*(c + d*x^4)^3,x]
```

```
[Out] a^2*c^3*x + (a*c^2*(2*b*c + 3*a*d)*x^5)/5 + (c*(b^2*c^2 + 6*a*b*c*d + 3*a^2
*d^2)*x^9)/9 + (d*(3*b^2*c^2 + 6*a*b*c*d + a^2*d^2)*x^13)/13 + (b*d^2*(3*b*
c + 2*a*d)*x^17)/17 + (b^2*d^3*x^21)/21
```

Rule 373

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\int \left(a+bx^4\right)^2\left(c+dx^4\right)^3\,dx=\int \left(a^2c^3+ac^2(2bc+3ad)x^4+c\left(b^2c^2+6abcd+3a^2d^2\right)x^8+d\left(3b^2c^2+6abcd+\right.\right.$$

$$=a^2c^3x+\frac{1}{5}ac^2(2bc+3ad)x^5+\frac{1}{9}c\left(b^2c^2+6abcd+3a^2d^2\right)x^9+\frac{1}{13}d\left(3b^2c^2+6ab\right.$$

Mathematica [A] time = 0.02, size = 122, normalized size = 1.00

$$\frac{1}{13}dx^{13}\left(a^2d^2+6abcd+3b^2c^2\right)+\frac{1}{9}cx^9\left(3a^2d^2+6abcd+b^2c^2\right)+a^2c^3x+\frac{1}{5}ac^2x^5(3ad+2bc)+\frac{1}{17}bd^2x^{17}(2ad+3bc)+\frac{1}{21}b^2d^3x^{21}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^4)^2*(c + d*x^4)^3,x]
```

```
[Out] a^2*c^3*x + (a*c^2*(2*b*c + 3*a*d)*x^5)/5 + (c*(b^2*c^2 + 6*a*b*c*d + 3*a^2
*d^2)*x^9)/9 + (d*(3*b^2*c^2 + 6*a*b*c*d + a^2*d^2)*x^13)/13 + (b*d^2*(3*b*
c + 2*a*d)*x^17)/17 + (b^2*d^3*x^21)/21
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+bx^4\right)^2\left(c+dx^4\right)^3\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x^4)^2*(c + d*x^4)^3,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x^4)^2*(c + d*x^4)^3, x]
```

fricas [A] time = 1.08, size = 132, normalized size = 1.08

$$\frac{1}{21}x^{21}d^3b^2+\frac{3}{17}x^{17}d^2cb^2+\frac{2}{17}x^{17}d^3ba+\frac{3}{13}x^{13}dc^2b^2+\frac{6}{13}x^{13}d^2cba+\frac{1}{13}x^{13}d^3a^2+\frac{1}{9}x^9c^3b^2+\frac{2}{3}x^9dc^2ba+\frac{1}{3}x^9d^2ca^2+\frac{2}{5}x^5c^3ba+\frac{3}{5}x^5dc^2a^2+xc^3a^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^2*(d*x^4+c)^3,x, algorithm="fricas")
```

```
[Out] 1/21*x^21*d^3*b^2 + 3/17*x^17*d^2*c*b^2 + 2/17*x^17*d^3*b*a + 3/13*x^13*d*c
^2*b^2 + 6/13*x^13*d^2*c*b*a + 1/13*x^13*d^3*a^2 + 1/9*x^9*c^3*b^2 + 2/3*x^
9*d*c^2*b*a + 1/3*x^9*d^2*c*a^2 + 2/5*x^5*c^3*b*a + 3/5*x^5*d*c^2*a^2 + x*c
^3*a^2
```

giac [A] time = 0.15, size = 132, normalized size = 1.08

$$\frac{1}{21}\,b^2d^3x^{21}+\frac{3}{17}\,b^2cd^2x^{17}+\frac{2}{17}\,abd^3x^{17}+\frac{3}{13}\,b^2c^2dx^{13}+\frac{6}{13}\,abcd^2x^{13}+\frac{1}{13}\,a^2d^3x^{13}+\frac{1}{9}\,b^2c^3x^9+\frac{2}{3}\,abc^2dx^9+\frac{1}{3}\,a^2cd^2x^9+\frac{2}{5}\,abc^3x^5+\frac{3}{5}\,a^2c^2dx^5+a^2c^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^2*(d*x^4+c)^3,x, algorithm="giac")
```

```
[Out] 1/21*b^2*d^3*x^21 + 3/17*b^2*c*d^2*x^17 + 2/17*a*b*d^3*x^17 + 3/13*b^2*c^2*
d*x^13 + 6/13*a*b*c*d^2*x^13 + 1/13*a^2*d^3*x^13 + 1/9*b^2*c^3*x^9 + 2/3*a*
b*c^2*d*x^9 + 1/3*a^2*c*d^2*x^9 + 2/5*a*b*c^3*x^5 + 3/5*a^2*c^2*d*x^5 + a^2
*c^3*x
```

maple [A] time = 0.04, size = 125, normalized size = 1.02

$$\frac{b^2d^3x^{21}}{21}+\frac{\left(2ab\,d^3+3b^2c\,d^2\right)x^{17}}{17}+\frac{\left(a^2d^3+6abc\,d^2+3b^2c^2d\right)x^{13}}{13}+\frac{\left(3a^2c\,d^2+6ab\,c^2d+b^2c^3\right)x^9}{9}+a^2c^3x+\frac{\left(3a^2c^2d+2ab\,c^3\right)x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^4+a)^2*(d*x^4+c)^3,x)
```

```
[Out] 1/21*b^2*d^3*x^21+1/17*(2*a*b*d^3+3*b^2*c*d^2)*x^17+1/13*(a^2*d^3+6*a*b*c*d
^2+3*b^2*c^2*d)*x^13+1/9*(3*a^2*c*d^2+6*a*b*c^2*d+b^2*c^3)*x^9+1/5*(3*a^2*c
^2*d+2*a*b*c^3)*x^5+a^2*c^3*x
```

maxima [A] time = 0.54, size = 124, normalized size = 1.02

$$\frac{1}{21}\,b^2d^3x^{21}+\frac{1}{17}\left(3\,b^2cd^2+2\,abd^3\right)x^{17}+\frac{1}{13}\left(3\,b^2c^2d+6\,abcd^2+a^2d^3\right)x^{13}+\frac{1}{9}\left(b^2c^3+6\,abc^2d+3\,a^2cd^2\right)x^9+a^2c^3x+\frac{1}{5}\left(2\,abc^3+3\,a^2c^2d\right)x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^2*(d*x^4+c)^3,x, algorithm="maxima")
```

```
[Out] 1/21*b^2*d^3*x^21 + 1/17*(3*b^2*c*d^2 + 2*a*b*d^3)*x^17 + 1/13*(3*b^2*c^2*d
 + 6*a*b*c*d^2 + a^2*d^3)*x^13 + 1/9*(b^2*c^3 + 6*a*b*c^2*d + 3*a^2*c*d^2)*
x^9 + a^2*c^3*x + 1/5*(2*a*b*c^3 + 3*a^2*c^2*d)*x^5
```

mupad [B] time = 1.30, size = 116, normalized size = 0.95

$$x^9\left(\frac{a^2\,c\,d^2}{3}+\frac{2\,a\,b\,c^2\,d}{3}+\frac{b^2\,c^3}{9}\right)+x^{13}\left(\frac{a^2\,d^3}{13}+\frac{6\,a\,b\,c\,d^2}{13}+\frac{3\,b^2\,c^2\,d}{13}\right)+a^2\,c^3\,x+\frac{b^2\,d^3\,x^{21}}{21}+\frac{a\,c^2\,x^5\,\left(3\,a\,d+2\,b\,c\right)}{5}+\frac{b\,d^2\,x^{17}\,\left(2\,a\,d+3\,b\,c\right)}{17}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^4)^2*(c + d*x^4)^3,x)
```

```
[Out] x^9*((b^2*c^3)/9 + (a^2*c*d^2)/3 + (2*a*b*c^2*d)/3) + x^13*((a^2*d^3)/13 +
(3*b^2*c^2*d)/13 + (6*a*b*c*d^2)/13) + a^2*c^3*x + (b^2*d^3*x^21)/21 + (a*c
^2*x^5*(3*a*d + 2*b*c))/5 + (b*d^2*x^17*(2*a*d + 3*b*c))/17
```

sympy [A] time = 0.10, size = 139, normalized size = 1.14

$$a^2c^3x+\frac{b^2d^3x^{21}}{21}+x^{17}\left(\frac{2abd^3}{17}+\frac{3b^2cd^2}{17}\right)+x^{13}\left(\frac{a^2d^3}{13}+\frac{6abcd^2}{13}+\frac{3b^2c^2d}{13}\right)+x^9\left(\frac{a^2cd^2}{3}+\frac{2abc^2d}{3}+\frac{b^2c^3}{9}\right)+x^5\left(\frac{3a^2c^2d}{5}+\frac{2abc^3}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**4+a)**2*(d*x**4+c)**3,x)
```

```
[Out] a**2*c**3*x + b**2*d**3*x**21/21 + x**17*(2*a*b*d**3/17 + 3*b**2*c*d**2/17)
 + x**13*(a**2*d**3/13 + 6*a*b*c*d**2/13 + 3*b**2*c**2*d/13) + x**9*(a**2*c
*d**2/3 + 2*a*b*c**2*d/3 + b**2*c**3/9) + x**5*(3*a**2*c**2*d/5 + 2*a*b*c**
3/5)
```

3.92 $\int \left(a+b x^4\right)^2 \left(c+d x^4\right)^2 \, dx$

Optimal. Leaf size=82

$$\frac{1}{9} x^9 \left(a^2 d^2+4 a b c d+b^2 c^2\right)+a^2 c^2 x+\frac{2}{13} b d x^{13} (a d+b c)+\frac{2}{5} a c x^5 (a d+b c)+\frac{1}{17} b^2 d^2 x^{17}$$

Rubi [A] time = 0.05, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {373}

$$\frac{1}{9} x^9 \left(a^2 d^2+4 a b c d+b^2 c^2\right)+a^2 c^2 x+\frac{2}{13} b d x^{13} (a d+b c)+\frac{2}{5} a c x^5 (a d+b c)+\frac{1}{17} b^2 d^2 x^{17}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^4)^2*(c + d*x^4)^2,x]
```

```
[Out] a^2*c^2*x + (2*a*c*(b*c + a*d)*x^5)/5 + ((b^2*c^2 + 4*a*b*c*d + a^2*d^2)*x^
9)/9 + (2*b*d*(b*c + a*d)*x^13)/13 + (b^2*d^2*x^17)/17
```

Rule 373

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned} \int \left(a+b x^4\right)^2 \left(c+d x^4\right)^2 \, dx &= \int \left(a^2 c^2+2 a c (b c+a d) x^4+\left(b^2 c^2+4 a b c d+a^2 d^2\right) x^8+2 b d (b c+a d) x^{12}+b^2 d^2 x^{16}\right. \\ &= a^2 c^2 x+\frac{2}{5} a c (b c+a d) x^5+\frac{1}{9} \left(b^2 c^2+4 a b c d+a^2 d^2\right) x^9+\frac{2}{13} b d (b c+a d) x^{13}+\frac{1}{17} b^2 d \end{aligned}$$

Mathematica [A] time = 0.02, size = 82, normalized size = 1.00

$$\frac{1}{9} x^9 \left(a^2 d^2+4 a b c d+b^2 c^2\right)+a^2 c^2 x+\frac{2}{13} b d x^{13} (a d+b c)+\frac{2}{5} a c x^5 (a d+b c)+\frac{1}{17} b^2 d^2 x^{17}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^4)^2*(c + d*x^4)^2,x]
```

```
[Out] a^2*c^2*x + (2*a*c*(b*c + a*d)*x^5)/5 + ((b^2*c^2 + 4*a*b*c*d + a^2*d^2)*x^
9)/9 + (2*b*d*(b*c + a*d)*x^13)/13 + (b^2*d^2*x^17)/17
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+b x^4\right)^2 \left(c+d x^4\right)^2 \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x^4)^2*(c + d*x^4)^2,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x^4)^2*(c + d*x^4)^2, x]
```

fricas [A] time = 0.56, size = 91, normalized size = 1.11

$$\frac{1}{17} x^{17} d^2 b^2+\frac{2}{13} x^{13} d c b^2+\frac{2}{13} x^{13} d^2 b a+\frac{1}{9} x^9 c^2 b^2+\frac{4}{9} x^9 d c b a+\frac{1}{9} x^9 d^2 a^2+\frac{2}{5} x^5 c^2 b a+\frac{2}{5} x^5 d c a^2+x c^2 a^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^2*(d*x^4+c)^2,x, algorithm="fricas")
```

```
[Out] 1/17*x^17*d^2*b^2 + 2/13*x^13*d*c*b^2 + 2/13*x^13*d^2*b*a + 1/9*x^9*c^2*b^2
 + 4/9*x^9*d*c*b*a + 1/9*x^9*d^2*a^2 + 2/5*x^5*c^2*b*a + 2/5*x^5*d*c*a^2 +
x*c^2*a^2
```

giac [A] time = 0.17, size = 91, normalized size = 1.11

$$\frac{1}{17}b^2d^2x^{17} + \frac{2}{13}b^2cdx^{13} + \frac{2}{13}abd^2x^{13} + \frac{1}{9}b^2c^2x^9 + \frac{4}{9}abcdx^9 + \frac{1}{9}a^2d^2x^9 + \frac{2}{5}abc^2x^5 + \frac{2}{5}a^2cdx^5 + a^2c^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^2*(d*x^4+c)^2,x, algorithm="giac")
```

```
[Out] 1/17*b^2*d^2*x^17 + 2/13*b^2*c*d*x^13 + 2/13*a*b*d^2*x^13 + 1/9*b^2*c^2*x^9
 + 4/9*a*b*c*d*x^9 + 1/9*a^2*d^2*x^9 + 2/5*a*b*c^2*x^5 + 2/5*a^2*c*d*x^5 +
a^2*c^2*x
```

maple [A] time = 0.04, size = 87, normalized size = 1.06

$$\frac{b^2d^2x^{17}}{17} + \frac{\left(2ab\,d^2 + 2b^2cd\right)x^{13}}{13} + \frac{\left(a^2d^2 + 4abcd + b^2c^2\right)x^9}{9} + a^2c^2x + \frac{\left(2a^2cd + 2ab\,c^2\right)x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^4+a)^2*(d*x^4+c)^2,x)
```

```
[Out] 1/17*b^2*d^2*x^17+1/13*(2*a*b*d^2+2*b^2*c*d)*x^13+1/9*(a^2*d^2+4*a*b*c*d+b^
2*c^2)*x^9+1/5*(2*a^2*c*d+2*a*b*c^2)*x^5+a^2*c^2*x
```

maxima [A] time = 0.69, size = 82, normalized size = 1.00

$$\frac{1}{17}b^2d^2x^{17} + \frac{2}{13}\left(b^2cd + abd^2\right)x^{13} + \frac{1}{9}\left(b^2c^2 + 4\,abcd + a^2d^2\right)x^9 + \frac{2}{5}\left(abc^2 + a^2cd\right)x^5 + a^2c^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^2*(d*x^4+c)^2,x, algorithm="maxima")
```

```
[Out] 1/17*b^2*d^2*x^17 + 2/13*(b^2*c*d + a*b*d^2)*x^13 + 1/9*(b^2*c^2 + 4*a*b*c*
d + a^2*d^2)*x^9 + 2/5*(a*b*c^2 + a^2*c*d)*x^5 + a^2*c^2*x
```

mupad [B] time = 0.05, size = 75, normalized size = 0.91

$$x^9\left(\frac{a^2\,d^2}{9} + \frac{4\,a\,b\,c\,d}{9} + \frac{b^2\,c^2}{9}\right) + a^2\,c^2\,x + \frac{b^2\,d^2\,x^{17}}{17} + \frac{2\,a\,c\,x^5\,\left(a\,d + b\,c\right)}{5} + \frac{2\,b\,d\,x^{13}\,\left(a\,d + b\,c\right)}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^4)^2*(c + d*x^4)^2,x)
```

```
[Out] x^9*((a^2*d^2)/9 + (b^2*c^2)/9 + (4*a*b*c*d)/9) + a^2*c^2*x + (b^2*d^2*x^17
)/17 + (2*a*c*x^5*(a*d + b*c))/5 + (2*b*d*x^13*(a*d + b*c))/13
```

sympy [A] time = 0.09, size = 97, normalized size = 1.18

$$a^2c^2x + \frac{b^2d^2x^{17}}{17} + x^{13}\left(\frac{2abd^2}{13} + \frac{2b^2cd}{13}\right) + x^9\left(\frac{a^2d^2}{9} + \frac{4abcd}{9} + \frac{b^2c^2}{9}\right) + x^5\left(\frac{2a^2cd}{5} + \frac{2abc^2}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**4+a)**2*(d*x**4+c)**2,x)
```

```
[Out] a**2*c**2*x + b**2*d**2*x**17/17 + x**13*(2*a*b*d**2/13 + 2*b**2*c*d/13) +
x**9*(a**2*d**2/9 + 4*a*b*c*d/9 + b**2*c**2/9) + x**5*(2*a**2*c*d/5 + 2*a*b
*c**2/5)
```

3.93 $\int \left(a+bx^4\right)^2 \left(c+dx^4\right) \, dx$

Optimal. Leaf size=50

$$a^2cx+\frac{1}{9}bx^9(2ad+bc)+\frac{1}{5}ax^5(ad+2bc)+\frac{1}{13}b^2dx^{13}$$

Rubi [A] time = 0.03, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {373}

$$a^2cx+\frac{1}{9}bx^9(2ad+bc)+\frac{1}{5}ax^5(ad+2bc)+\frac{1}{13}b^2dx^{13}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^4)^2*(c + d*x^4),x]
```

```
[Out] a^2*c*x + (a*(2*b*c + a*d)*x^5)/5 + (b*(b*c + 2*a*d)*x^9)/9 + (b^2*d*x^13)/
13
```

Rule 373

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \left(a+bx^4\right)^2\left(c+dx^4\right)\,dx &= \int \left(a^2c+a(2bc+ad)x^4+b(bc+2ad)x^8+b^2dx^{12}\right)dx\\ &= a^2cx+\frac{1}{5}a(2bc+ad)x^5+\frac{1}{9}b(bc+2ad)x^9+\frac{1}{13}b^2dx^{13}\end{aligned}$$

Mathematica [A] time = 0.01, size = 50, normalized size = 1.00

$$a^2cx+\frac{1}{9}bx^9(2ad+bc)+\frac{1}{5}ax^5(ad+2bc)+\frac{1}{13}b^2dx^{13}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^4)^2*(c + d*x^4),x]
```

```
[Out] a^2*c*x + (a*(2*b*c + a*d)*x^5)/5 + (b*(b*c + 2*a*d)*x^9)/9 + (b^2*d*x^13)/
13
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+bx^4\right)^2\left(c+dx^4\right)\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x^4)^2*(c + d*x^4),x]
```

```
[Out] IntegrateAlgebraic[(a + b*x^4)^2*(c + d*x^4), x]
```

fricas [A] time = 0.53, size = 50, normalized size = 1.00

$$\frac{1}{13}x^{13}db^2+\frac{1}{9}x^9cb^2+\frac{2}{9}x^9dba+\frac{2}{5}x^5cba+\frac{1}{5}x^5da^2+xca^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^2*(d*x^4+c),x, algorithm="fricas")
```

```
[Out] 1/13*x^13*d*b^2 + 1/9*x^9*c*b^2 + 2/9*x^9*d*b*a + 2/5*x^5*c*b*a + 1/5*x^5*d
*a^2 + x*c*a^2
```

giac [A] time = 0.15, size = 50, normalized size = 1.00

$$\frac{1}{13}\,b^2dx^{13} + \frac{1}{9}\,b^2cx^9 + \frac{2}{9}\,abdx^9 + \frac{2}{5}\,abcx^5 + \frac{1}{5}\,a^2dx^5 + a^2cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^2*(d*x^4+c),x, algorithm="giac")
```

```
[Out] 1/13*b^2*d*x^13 + 1/9*b^2*c*x^9 + 2/9*a*b*d*x^9 + 2/5*a*b*c*x^5 + 1/5*a^2*d
*x^5 + a^2*c*x
```

maple [A] time = 0.04, size = 49, normalized size = 0.98

$$\frac{b^2d\,x^{13}}{13} + \frac{\left(2abd + b^2c\right)x^9}{9} + \frac{\left(a^2d + 2abc\right)x^5}{5} + a^2cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^4+a)^2*(d*x^4+c),x)
```

```
[Out] 1/13*b^2*d*x^13+1/9*(2*a*b*d+b^2*c)*x^9+1/5*(a^2*d+2*a*b*c)*x^5+a^2*c*x
```

maxima [A] time = 0.48, size = 48, normalized size = 0.96

$$\frac{1}{13}\,b^2dx^{13} + \frac{1}{9}\left(b^2c + 2\,abd\right)x^9 + \frac{1}{5}\left(2\,abc + a^2d\right)x^5 + a^2cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^2*(d*x^4+c),x, algorithm="maxima")
```

```
[Out] 1/13*b^2*d*x^13 + 1/9*(b^2*c + 2*a*b*d)*x^9 + 1/5*(2*a*b*c + a^2*d)*x^5 + a
^2*c*x
```

mupad [B] time = 0.04, size = 48, normalized size = 0.96

$$x^5\left(\frac{d\,a^2}{5} + \frac{2\,b\,c\,a}{5}\right) + x^9\left(\frac{c\,b^2}{9} + \frac{2\,a\,d\,b}{9}\right) + \frac{b^2\,d\,x^{13}}{13} + a^2\,c\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^4)^2*(c + d*x^4),x)
```

```
[Out] x^5*((a^2*d)/5 + (2*a*b*c)/5) + x^9*((b^2*c)/9 + (2*a*b*d)/9) + (b^2*d*x^13
)/13 + a^2*c*x
```

sympy [A] time = 0.08, size = 53, normalized size = 1.06

$$a^2cx + \frac{b^2dx^{13}}{13} + x^9\left(\frac{2abd}{9} + \frac{b^2c}{9}\right) + x^5\left(\frac{a^2d}{5} + \frac{2abc}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**4+a)**2*(d*x**4+c),x)
```

```
[Out] a**2*c*x + b**2*d*x**13/13 + x**9*(2*a*b*d/9 + b**2*c/9) + x**5*(a**2*d/5 +
 2*a*b*c/5)
```

3.94 $\int \frac{(a+bx^4)^2}{c+dx^4}\,dx$

Optimal. Leaf size=253

$$-\frac{(bc-ad)^2\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\,x+\sqrt{c}+\sqrt{d}\,x^2\right)}{4\sqrt{2}\,c^{3/4}d^{9/4}}+\frac{(bc-ad)^2\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\,x+\sqrt{c}+\sqrt{d}\,x^2\right)}{4\sqrt{2}\,c^{3/4}d^{9/4}}-\frac{(bc-ad)^2\tan^{-1}\left(\right.}{2\sqrt{2}\,c^{3/4}}$$

Rubi [A] time = 0.19, antiderivative size = 253, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {390, 211, 1165, 628, 1162, 617, 204}

$$-\frac{(bc-ad)^2\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\,x+\sqrt{c}+\sqrt{d}\,x^2\right)}{4\sqrt{2}\,c^{3/4}d^{9/4}}+\frac{(bc-ad)^2\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\,x+\sqrt{c}+\sqrt{d}\,x^2\right)}{4\sqrt{2}\,c^{3/4}d^{9/4}}-\frac{(bc-ad)^2\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\,x}{\sqrt[4]{c}}\right)}{2\sqrt{2}\,c^{3/4}d^{9/4}}+\frac{(bc-ad)^2\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}\,x}{\sqrt[4]{c}}+1\right)}{2\sqrt{2}\,c^{3/4}d^{9/4}}-\frac{bx(bc-2ad)}{d^2}+\frac{b^2x^5}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^4)^2/(c + d*x^4),x]
```

```
[Out] -((b*(b*c - 2*a*d)*x)/d^2) + (b^2*x^5)/(5*d) - ((b*c - a*d)^2*ArcTan[1 - (S
qrt[2]*d^(1/4)*x)/c^(1/4)])/(2*Sqrt[2]*c^(3/4)*d^(9/4)) + ((b*c - a*d)^2*Ar
cTan[1 + (Sqrt[2]*d^(1/4)*x)/c^(1/4)])/(2*Sqrt[2]*c^(3/4)*d^(9/4)) - ((b*c
- a*d)^2*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2])/(4*Sqrt[2]
*c^(3/4)*d^(9/4)) + ((b*c - a*d)^2*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*x
+ Sqrt[d]*x^2])/(4*Sqrt[2]*c^(3/4)*d^(9/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{\left(a+bx^4\right)^2}{c+dx^4}\,dx=\int \left(-\frac{b(bc-2ad)}{d^2}+\frac{b^2x^4}{d}+\frac{b^2c^2-2abcd+a^2d^2}{d^2\left(c+dx^4\right)}\right)dx$$

$$=-\frac{b(bc-2ad)x}{d^2}+\frac{b^2x^5}{5d}+\frac{(bc-ad)^2\int \frac{1}{c+dx^4}\,dx}{d^2}$$

$$=-\frac{b(bc-2ad)x}{d^2}+\frac{b^2x^5}{5d}+\frac{(bc-ad)^2\int \frac{\sqrt{c}-\sqrt{d}x^2}{c+dx^4}\,dx}{2\sqrt{c}\,d^2}+\frac{(bc-ad)^2\int \frac{\sqrt{c}+\sqrt{d}x^2}{c+dx^4}\,dx}{2\sqrt{c}\,d^2}$$

$$=-\frac{b(bc-2ad)x}{d^2}+\frac{b^2x^5}{5d}+\frac{(bc-ad)^2\int \frac{1}{\frac{\sqrt{c}}{\sqrt{d}}-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}}+x^2}\,dx}{4\sqrt{c}\,d^{5/2}}+\frac{(bc-ad)^2\int \frac{1}{\frac{\sqrt{c}}{\sqrt{d}}+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}}+x^2}\,dx}{4\sqrt{c}\,d^{5/2}}-\frac{(bc \cdots}{\cdots}$$

$$=-\frac{b(bc-2ad)x}{d^2}+\frac{b^2x^5}{5d}-\frac{(bc-ad)^2\log\left(\sqrt{c}-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{d}x^2\right)}{4\sqrt{2}c^{3/4}d^{9/4}}+\frac{(bc-ad)^2\log\left(\sqrt{c}+\cdots\right.}{4\sqrt{2}c^{3}\cdots}$$

$$=-\frac{b(bc-2ad)x}{d^2}+\frac{b^2x^5}{5d}-\frac{(bc-ad)^2\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{2\sqrt{2}c^{3/4}d^{9/4}}+\frac{(bc-ad)^2\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{2\sqrt{2}c^{3/4}d^{9/4}}-\frac{(bc \cdots}{\cdots}$$

Mathematica [A] time = 0.13, size = 231, normalized size = 0.91

$$\frac{-40bc^{3/4}\sqrt[4]{d}x(bc-2ad)-5\sqrt{2}(bc-ad)^2\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{c}+\sqrt{d}x^2\right)+5\sqrt{2}(bc-ad)^2\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{c}+\sqrt{d}x^2\right)-10\sqrt{2}(bc-ad)^2\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)+10\sqrt{2}(bc-ad)^2\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}+1\right)+8b^2c^{3/4}d^{5/4}x^5}{40c^{3/4}d^{9/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^4)^2/(c + d*x^4),x]
```

```
[Out] (-40*b*c^(3/4)*d^(1/4)*(b*c - 2*a*d)*x + 8*b^2*c^(3/4)*d^(5/4)*x^5 - 10*Sqr
t[2]*(b*c - a*d)^2*ArcTan[1 - (Sqrt[2]*d^(1/4)*x)/c^(1/4)] + 10*Sqrt[2]*(b*
c - a*d)^2*ArcTan[1 + (Sqrt[2]*d^(1/4)*x)/c^(1/4)] - 5*Sqrt[2]*(b*c - a*d)^
2*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2] + 5*Sqrt[2]*(b*c -
 a*d)^2*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2])/(40*c^(3/4)
*d^(9/4))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+bx^4\right)^2}{c+dx^4}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x^4)^2/(c + d*x^4),x]
```

```
[Out] IntegrateAlgebraic[(a + b*x^4)^2/(c + d*x^4), x]
```

fricas [B] time = 1.18, size = 1239, normalized size = 4.90

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^2/(d*x^4+c),x, algorithm="fricas")
```

```
[Out] 1/20*(4*b^2*d*x^5 + 20*d^2*(-(b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2
- 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2
*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8)/(c^3*d^9))^(1/4)*arctan(-(c^2*d^7*x*(-(
b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*
b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8
*d^8)/(c^3*d^9))^(3/4) - c^2*d^7*sqrt((c^2*d^4*sqrt(-(b^8*c^8 - 8*a*b^7*c^7
*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*
b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8)/(c^3*d^9)) + (b
^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*x^2)/
(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4))*(-
(b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4
*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^
8*d^8)/(c^3*d^9))^(3/4))/(b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20
*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)) + 5*d^2*(
-(b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^
4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a
^8*d^8)/(c^3*d^9))^(1/4)*log(c*d^2*(-(b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*
c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28
*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8)/(c^3*d^9))^(1/4) + (b^2*c^2 - 2
*a*b*c*d + a^2*d^2)*x) - 5*d^2*(-(b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*
d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6
*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8)/(c^3*d^9))^(1/4)*log(-c*d^2*(-(b^8*
c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*
c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8
)/(c^3*d^9))^(1/4) + (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*x) - 20*(b^2*c - 2*a*b
*d)*x)/d^2
```

giac [A] time = 0.20, size = 353, normalized size = 1.40

$$\frac{\sqrt{2}\left(\left(cd^{3}\right)^{\frac{1}{4}} b^{2}c^{2} - 2\,\left(cd^{3}\right)^{\frac{1}{4}} abcd + \left(cd^{3}\right)^{\frac{1}{4}} a^{2}d^{2}\right)\arctan\left(\frac{\sqrt{2}\left(2\,x + \sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{4\,cd^{3}} + \frac{\sqrt{2}\left(\left(cd^{3}\right)^{\frac{1}{4}} b^{2}c^{2} - 2\,\left(cd^{3}\right)^{\frac{1}{4}} abcd + \left(cd^{3}\right)^{\frac{1}{4}} a^{2}d^{2}\right)\arctan\left(\frac{\sqrt{2}\left(2\,x - \sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{4\,cd^{3}} + \frac{\sqrt{2}\left(\left(cd^{3}\right)^{\frac{1}{4}} b^{2}c^{2} - 2\,\left(cd^{3}\right)^{\frac{1}{4}} abcd + \left(cd^{3}\right)^{\frac{1}{4}} a^{2}d^{2}\right)\log\left(x^{2} + \sqrt{2}\,x\left(\frac{c}{d}\right)^{\frac{1}{4}} + \sqrt{\frac{c}{d}}\right)}{8\,cd^{3}} - \frac{\sqrt{2}\left(\left(cd^{3}\right)^{\frac{1}{4}} b^{2}c^{2} - 2\,\left(cd^{3}\right)^{\frac{1}{4}} abcd + \left(cd^{3}\right)^{\frac{1}{4}} a^{2}d^{2}\right)\log\left(x^{2} - \sqrt{2}\,x\left(\frac{c}{d}\right)^{\frac{1}{4}} + \sqrt{\frac{c}{d}}\right)}{8\,cd^{3}} + \frac{b^{2}d^{4}x^{5} - 5\,b^{2}cd^{3}x + 10\,abd^{4}x}{5\,d^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^2/(d*x^4+c),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(2)*((c*d^3)^(1/4)*b^2*c^2 - 2*(c*d^3)^(1/4)*a*b*c*d + (c*d^3)^(1/4
)*a^2*d^2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(c/d)^(1/4))/(c/d)^(1/4))/(c*d
^3) + 1/4*sqrt(2)*((c*d^3)^(1/4)*b^2*c^2 - 2*(c*d^3)^(1/4)*a*b*c*d + (c*d^3
)^(1/4)*a^2*d^2)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(c/d)^(1/4))/(c/d)^(1/4)
)/(c*d^3) + 1/8*sqrt(2)*((c*d^3)^(1/4)*b^2*c^2 - 2*(c*d^3)^(1/4)*a*b*c*d +
(c*d^3)^(1/4)*a^2*d^2)*log(x^2 + sqrt(2)*x*(c/d)^(1/4) + sqrt(c/d))/(c*d^3)
 - 1/8*sqrt(2)*((c*d^3)^(1/4)*b^2*c^2 - 2*(c*d^3)^(1/4)*a*b*c*d + (c*d^3)^(
1/4)*a^2*d^2)*log(x^2 - sqrt(2)*x*(c/d)^(1/4) + sqrt(c/d))/(c*d^3) + 1/5*(b
^2*d^4*x^5 - 5*b^2*c*d^3*x + 10*a*b*d^4*x)/d^5
```

maple [B] time = 0.05, size = 436, normalized size = 1.72

$$\frac{b^{2}x^{5}}{5d} + \frac{\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,a^{2}\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{c}{d}\right)^{\frac{1}{4}}} - 1\right)}{4c} + \frac{\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,a^{2}\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{c}{d}\right)^{\frac{1}{4}}} + 1\right)}{4c} + \frac{\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,a^{2}\ln\left(\frac{x^{2}+\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{c}{d}}}{x^{2}-\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{c}{d}}}\right)}{8c} + \frac{2abx}{d} - \frac{\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,ab\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{c}{d}\right)^{\frac{1}{4}}} - 1\right)}{2d} - \frac{\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,ab\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{c}{d}\right)^{\frac{1}{4}}} + 1\right)}{2d} - \frac{\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,ab\ln\left(\frac{x^{2}+\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{c}{d}}}{x^{2}-\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{c}{d}}}\right)}{4d} - \frac{b^{2}cx}{d^{2}} + \frac{\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,b^{2}c\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{c}{d}\right)^{\frac{1}{4}}} - 1\right)}{4d^{2}} + \frac{\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,b^{2}c\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{c}{d}\right)^{\frac{1}{4}}} + 1\right)}{4d^{2}} + \frac{\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,b^{2}c\ln\left(\frac{x^{2}+\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{c}{d}}}{x^{2}-\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{c}{d}}}\right)}{8d^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^4+a)^2/(d*x^4+c),x)
```

```
[Out] 1/5*b^2*x^5/d+2*b/d*a*x-b^2/d^2*c*x+1/8*(c/d)^(1/4)/c*2^(1/2)*ln((x^2+(c/d)
^(1/4)*2^(1/2)*x+(c/d)^(1/2))/(x^2-(c/d)^(1/4)*2^(1/2)*x+(c/d)^(1/2)))*a^2-
1/4/d*(c/d)^(1/4)*2^(1/2)*ln((x^2+(c/d)^(1/4)*2^(1/2)*x+(c/d)^(1/2))/(x^2-(
c/d)^(1/4)*2^(1/2)*x+(c/d)^(1/2)))*a*b+1/8/d^2*(c/d)^(1/4)*c*2^(1/2)*ln((x^
2+(c/d)^(1/4)*2^(1/2)*x+(c/d)^(1/2))/(x^2-(c/d)^(1/4)*2^(1/2)*x+(c/d)^(1/2)
))*b^2+1/4*(c/d)^(1/4)/c*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x-1)*a^2-1/2/d*
(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x-1)*a*b+1/4/d^2*(c/d)^(1/4)
*c*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x-1)*b^2+1/4*(c/d)^(1/4)/c*2^(1/2)*ar
ctan(2^(1/2)/(c/d)^(1/4)*x+1)*a^2-1/2/d*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/
(c/d)^(1/4)*x+1)*a*b+1/4/d^2*(c/d)^(1/4)*c*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/
4)*x+1)*b^2
```

maxima [A] time = 1.29, size = 286, normalized size = 1.13

$$\frac{b^{2} d x^{5} - 5 \left(b^{2} c - 2 a b d\right) x}{5 d^{2}} + \frac{\frac{2 \sqrt{2} \left(b^{2} c^{2} - 2 a b c d + a^{2} d^{2}\right) \arctan\left(\frac{\sqrt{2} \left(2 \sqrt{d} x + \sqrt{2} c^{\frac{1}{4}} d^{\frac{1}{4}}\right)}{2 \sqrt{\sqrt{c} \sqrt{d}}}\right)}{\sqrt{c} \sqrt{\sqrt{c} \sqrt{d}}} + \frac{2 \sqrt{2} \left(b^{2} c^{2} - 2 a b c d + a^{2} d^{2}\right) \arctan\left(\frac{\sqrt{2} \left(2 \sqrt{d} x - \sqrt{2} c^{\frac{1}{4}} d^{\frac{1}{4}}\right)}{2 \sqrt{\sqrt{c} \sqrt{d}}}\right)}{\sqrt{c} \sqrt{\sqrt{c} \sqrt{d}}} + \frac{\sqrt{2} \left(b^{2} c^{2} - 2 a b c d + a^{2} d^{2}\right) \log\left(\sqrt{d} x^{2} + \sqrt{2} c^{\frac{1}{4}} d^{\frac{1}{4}} x + \sqrt{c}\right)}{c^{\frac{3}{4}} d^{\frac{1}{4}}} - \frac{\sqrt{2} \left(b^{2} c^{2} - 2 a b c d + a^{2} d^{2}\right) \log\left(\sqrt{d} x^{2} - \sqrt{2} c^{\frac{1}{4}} d^{\frac{1}{4}} x + \sqrt{c}\right)}{c^{\frac{3}{4}} d^{\frac{1}{4}}}}{8 d^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^2/(d*x^4+c),x, algorithm="maxima")
```

```
[Out] 1/5*(b^2*d*x^5 - 5*(b^2*c - 2*a*b*d)*x)/d^2 + 1/8*(2*sqrt(2)*(b^2*c^2 - 2*a
*b*c*d + a^2*d^2)*arctan(1/2*sqrt(2)*(2*sqrt(d)*x + sqrt(2)*c^(1/4)*d^(1/4)
)/sqrt(sqrt(c)*sqrt(d)))/(sqrt(c)*sqrt(sqrt(c)*sqrt(d))) + 2*sqrt(2)*(b^2*c
^2 - 2*a*b*c*d + a^2*d^2)*arctan(1/2*sqrt(2)*(2*sqrt(d)*x - sqrt(2)*c^(1/4)
*d^(1/4))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(c)*sqrt(sqrt(c)*sqrt(d))) + sqrt(2)*
(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*log(sqrt(d)*x^2 + sqrt(2)*c^(1/4)*d^(1/4)*x
 + sqrt(c))/(c^(3/4)*d^(1/4)) - sqrt(2)*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*log
(sqrt(d)*x^2 - sqrt(2)*c^(1/4)*d^(1/4)*x + sqrt(c))/(c^(3/4)*d^(1/4)))/d^2
```

mupad [B] time = 1.48, size = 1081, normalized size = 4.27

$$\frac{b^2 x^5}{5 d} - x \left(\frac{b^2 c}{d^2} - \frac{2 a b}{d}\right) + \frac{\mathrm{atan}\left(\frac{\frac{(a d-b c)^2 \left(\frac{x \left(a^4 d^4-4 a^3 b c d^3+6 a^2 b^2 c^2 d^2-4 a b^3 c^3 d+b^4 c^4\right)}{d}-\frac{(a d-b c)^2 \left(4 a^2 c d^3-8 a b c^2 d^2+4 b^2 c^3 d\right)}{4 (-c)^{3/4} d^{9/4}}\right) 1i}{(-c)^{3/4} d^{9/4}}+\frac{(a d-b c)^2 \left(\frac{x \left(a^4 d^4-4 a^3 b c d^3+6 a^2 b^2 c^2 d^2-4 a b^3 c^3 d+b^4 c^4\right)}{d}+\frac{(a d-b c)^2 \left(4 a^2 c d^3-8 a b c^2 d^2+4 b^2 c^3 d\right)}{4 (-c)^{3/4} d^{9/4}}\right) 1i}{(-c)^{3/4} d^{9/4}}}{\frac{(a d-b c)^2 \left(\frac{x \left(a^4 d^4-4 a^3 b c d^3+6 a^2 b^2 c^2 d^2-4 a b^3 c^3 d+b^4 c^4\right)}{d}-\frac{(a d-b c)^2 \left(4 a^2 c d^3-8 a b c^2 d^2+4 b^2 c^3 d\right)}{4 (-c)^{3/4} d^{9/4}}\right)}{(-c)^{3/4} d^{9/4}}-\frac{(a d-b c)^2 \left(\frac{x \left(a^4 d^4-4 a^3 b c d^3+6 a^2 b^2 c^2 d^2-4 a b^3 c^3 d+b^4 c^4\right)}{d}+\frac{(a d-b c)^2 \left(4 a^2 c d^3-8 a b c^2 d^2+4 b^2 c^3 d\right)}{4 (-c)^{3/4} d^{9/4}}\right)}{(-c)^{3/4} d^{9/4}}}\right) (a d-b c)^2 1i}{2 (-c)^{3/4} d^{9/4}} + \frac{\mathrm{atan}\left(\frac{\frac{(a d-b c)^2 \left(\frac{x \left(a^4 d^4-4 a^3 b c d^3+6 a^2 b^2 c^2 d^2-4 a b^3 c^3 d+b^4 c^4\right)}{d}-\frac{(a d-b c)^2 \left(4 a^2 c d^3-8 a b c^2 d^2+4 b^2 c^3 d\right) 1i}{4 (-c)^{3/4} d^{9/4}}\right)}{(-c)^{3/4} d^{9/4}}+\frac{(a d-b c)^2 \left(\frac{x \left(a^4 d^4-4 a^3 b c d^3+6 a^2 b^2 c^2 d^2-4 a b^3 c^3 d+b^4 c^4\right)}{d}+\frac{(a d-b c)^2 \left(4 a^2 c d^3-8 a b c^2 d^2+4 b^2 c^3 d\right) 1i}{4 (-c)^{3/4} d^{9/4}}\right)}{(-c)^{3/4} d^{9/4}}}{\frac{(a d-b c)^2 \left(\frac{x \left(a^4 d^4-4 a^3 b c d^3+6 a^2 b^2 c^2 d^2-4 a b^3 c^3 d+b^4 c^4\right)}{d}-\frac{(a d-b c)^2 \left(4 a^2 c d^3-8 a b c^2 d^2+4 b^2 c^3 d\right) 1i}{4 (-c)^{3/4} d^{9/4}}\right) 1i}{(-c)^{3/4} d^{9/4}}-\frac{(a d-b c)^2 \left(\frac{x \left(a^4 d^4-4 a^3 b c d^3+6 a^2 b^2 c^2 d^2-4 a b^3 c^3 d+b^4 c^4\right)}{d}+\frac{(a d-b c)^2 \left(4 a^2 c d^3-8 a b c^2 d^2+4 b^2 c^3 d\right) 1i}{4 (-c)^{3/4} d^{9/4}}\right) 1i}{(-c)^{3/4} d^{9/4}}}\right) (a d-b c)^2}{2 (-c)^{3/4} d^{9/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^4)^2/(c + d*x^4),x)
```

```
[Out] (b^2*x^5)/(5*d) - x*((b^2*c)/d^2 - (2*a*b)/d) + (atan((((a*d - b*c)^2*((x*(
a^4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3))/d -
 ((a*d - b*c)^2*(4*a^2*c*d^3 + 4*b^2*c^3*d - 8*a*b*c^2*d^2))/(4*(-c)^(3/4)*
d^(9/4)))*1i)/((-c)^(3/4)*d^(9/4)) + ((a*d - b*c)^2*((x*(a^4*d^4 + b^4*c^4
+ 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3))/d + ((a*d - b*c)^2*(4
*a^2*c*d^3 + 4*b^2*c^3*d - 8*a*b*c^2*d^2))/(4*(-c)^(3/4)*d^(9/4)))*1i)/((-c
)^(3/4)*d^(9/4)))/(((a*d - b*c)^2*((x*(a^4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^
2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3))/d - ((a*d - b*c)^2*(4*a^2*c*d^3 + 4*b^2
*c^3*d - 8*a*b*c^2*d^2))/(4*(-c)^(3/4)*d^(9/4))))/((-c)^(3/4)*d^(9/4)) - ((
a*d - b*c)^2*((x*(a^4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4
*a^3*b*c*d^3))/d + ((a*d - b*c)^2*(4*a^2*c*d^3 + 4*b^2*c^3*d - 8*a*b*c^2*d^
2))/(4*(-c)^(3/4)*d^(9/4))))/((-c)^(3/4)*d^(9/4))))*(a*d - b*c)^2*1i)/(2*(-
c)^(3/4)*d^(9/4)) + (atan((((a*d - b*c)^2*((x*(a^4*d^4 + b^4*c^4 + 6*a^2*b^
2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3))/d - ((a*d - b*c)^2*(4*a^2*c*d^3
 + 4*b^2*c^3*d - 8*a*b*c^2*d^2)*1i)/(4*(-c)^(3/4)*d^(9/4))))/((-c)^(3/4)*d^
(9/4)) + ((a*d - b*c)^2*((x*(a^4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^
3*c^3*d - 4*a^3*b*c*d^3))/d + ((a*d - b*c)^2*(4*a^2*c*d^3 + 4*b^2*c^3*d - 8
*a*b*c^2*d^2)*1i)/(4*(-c)^(3/4)*d^(9/4))))/((-c)^(3/4)*d^(9/4)))/(((a*d - b
```

```
*c)^2*((x*(a^4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*
c*d^3))/d - ((a*d - b*c)^2*(4*a^2*c*d^3 + 4*b^2*c^3*d - 8*a*b*c^2*d^2)*1i)/
(4*(-c)^(3/4)*d^(9/4)))*1i)/((-c)^(3/4)*d^(9/4)) - ((a*d - b*c)^2*((x*(a^4*
d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3))/d + ((a
*d - b*c)^2*(4*a^2*c*d^3 + 4*b^2*c^3*d - 8*a*b*c^2*d^2)*1i)/(4*(-c)^(3/4)*d
^(9/4)))*1i)/((-c)^(3/4)*d^(9/4))))*(a*d - b*c)^2)/(2*(-c)^(3/4)*d^(9/4))
```

sympy [A] time = 1.12, size = 187, normalized size = 0.74

$$\frac{b^2x^5}{5d}+x\left(\frac{2ab}{d}-\frac{b^2c}{d^2}\right)+\operatorname{RootSum}\left(256t^4c^3d^9+a^8d^8-8a^7bcd^7+28a^6b^2c^2d^6-56a^5b^3c^3d^5+70a^4b^4c^4d^4-56a^3b^5c^5d^3+28a^2b^6c^6d^2-8ab^7c^7d+b^8c^8,\left(t\mapsto t\log\left(\frac{4tcd^2}{a^2d^2-2abcd+b^2c^2}+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**4+a)**2/(d*x**4+c),x)
```

```
[Out] b**2*x**5/(5*d) + x*(2*a*b/d - b**2*c/d**2) + RootSum(256*_t**4*c**3*d**9 +
 a**8*d**8 - 8*a**7*b*c*d**7 + 28*a**6*b**2*c**2*d**6 - 56*a**5*b**3*c**3*d
**5 + 70*a**4*b**4*c**4*d**4 - 56*a**3*b**5*c**5*d**3 + 28*a**2*b**6*c**6*d
**2 - 8*a*b**7*c**7*d + b**8*c**8, Lambda(_t, _t*log(4*_t*c*d**2/(a**2*d**2
 - 2*a*b*c*d + b**2*c**2) + x)))
```

3.95 $\int \frac{(a+bx^4)^2}{(c+dx^4)^2} \, dx$

Optimal. Leaf size=291

$$\frac{(bc-ad)(3ad+5bc)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\,x+\sqrt{c}+\sqrt{d}\,x^2\right)}{16\sqrt{2}\,c^{7/4}d^{9/4}}-\frac{(bc-ad)(3ad+5bc)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\,x+\sqrt{c}+\sqrt{d}\,x^2\right)}{16\sqrt{2}\,c^{7/4}d^{9/4}}+$$

Rubi [A] time = 0.37, antiderivative size = 291, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.421, Rules used = {390, 385, 211, 1165, 628, 1162, 617, 204}

$$\frac{(bc-ad)(3ad+5bc)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\,x+\sqrt{c}+\sqrt{d}\,x^2\right)}{16\sqrt{2}\,c^{7/4}d^{9/4}}-\frac{(bc-ad)(3ad+5bc)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\,x+\sqrt{c}+\sqrt{d}\,x^2\right)}{16\sqrt{2}\,c^{7/4}d^{9/4}}+\frac{(bc-ad)(3ad+5bc)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\,x}{\sqrt[4]{c}}\right)}{8\sqrt{2}\,c^{7/4}d^{9/4}}-\frac{(bc-ad)(3ad+5bc)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}\,x}{\sqrt[4]{c}}+1\right)}{8\sqrt{2}\,c^{7/4}d^{9/4}}+\frac{x(bc-ad)^2}{4cd^2\left(c+dx^4\right)}+\frac{b^2x}{d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^4)^2/(c + d*x^4)^2,x]
```

```
[Out] (b^2*x)/d^2 + ((b*c - a*d)^2*x)/(4*c*d^2*(c + d*x^4)) + ((b*c - a*d)*(5*b*c
 + 3*a*d)*ArcTan[1 - (Sqrt[2]*d^(1/4)*x)/c^(1/4)])/(8*Sqrt[2]*c^(7/4)*d^(9/
4)) - ((b*c - a*d)*(5*b*c + 3*a*d)*ArcTan[1 + (Sqrt[2]*d^(1/4)*x)/c^(1/4)])
/(8*Sqrt[2]*c^(7/4)*d^(9/4)) + ((b*c - a*d)*(5*b*c + 3*a*d)*Log[Sqrt[c] - S
qrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2])/(16*Sqrt[2]*c^(7/4)*d^(9/4)) - ((b
*c - a*d)*(5*b*c + 3*a*d)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]
*x^2])/(16*Sqrt[2]*c^(7/4)*d^(9/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{\left(a+bx^4\right)^2}{\left(c+dx^4\right)^2}\,dx = \int \left(\frac{b^2}{d^2}-\frac{b^2c^2-a^2d^2+2bd(bc-ad)x^4}{d^2\left(c+dx^4\right)^2}\right)dx$$

$$= \frac{b^2x}{d^2}-\frac{\int \frac{b^2c^2-a^2d^2+2bd(bc-ad)x^4}{\left(c+dx^4\right)^2}\,dx}{d^2}$$

$$= \frac{b^2x}{d^2}+\frac{(bc-ad)^2x}{4cd^2\left(c+dx^4\right)}-\frac{((bc-ad)(5bc+3ad))\int \frac{1}{c+dx^4}\,dx}{4cd^2}$$

$$= \frac{b^2x}{d^2}+\frac{(bc-ad)^2x}{4cd^2\left(c+dx^4\right)}-\frac{((bc-ad)(5bc+3ad))\int \frac{\sqrt{c}-\sqrt{d}\,x^2}{c+dx^4}\,dx}{8c^{3/2}d^2}-\frac{((bc-ad)(5bc+3ad))\int \frac{\sqrt{}}{}}{8c^{3/2}d^2}$$

$$= \frac{b^2x}{d^2}+\frac{(bc-ad)^2x}{4cd^2\left(c+dx^4\right)}-\frac{((bc-ad)(5bc+3ad))\int \frac{1}{\frac{\sqrt{c}}{\sqrt{d}}-\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{d}}+x^2}\,dx}{16c^{3/2}d^{5/2}}-\frac{((bc-ad)(5bc+3ad))}{16c^{3/2}}$$

$$= \frac{b^2x}{d^2}+\frac{(bc-ad)^2x}{4cd^2\left(c+dx^4\right)}+\frac{(bc-ad)(5bc+3ad)\log\left(\sqrt{c}-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\,x+\sqrt{d}\,x^2\right)}{16\sqrt{2}\,c^{7/4}d^{9/4}}-\frac{(bc-ad)(}{}$$

$$= \frac{b^2x}{d^2}+\frac{(bc-ad)^2x}{4cd^2\left(c+dx^4\right)}+\frac{(bc-ad)(5bc+3ad)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\,x}{\sqrt[4]{c}}\right)}{8\sqrt{2}\,c^{7/4}d^{9/4}}-\frac{(bc-ad)(5bc+3ad)\tan}{8\sqrt{2}\,c^{7/4}}$$

Mathematica [A] time = 0.22, size = 298, normalized size = 1.02

$$\frac{\frac{\sqrt{2}\left(-3a^2d^2-2abcd+5b^2c^2\right)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\,x+\sqrt{c}+\sqrt{d}\,x^2\right)}{c^{7/4}}-\frac{\sqrt{2}\left(-3a^2d^2-2abcd+5b^2c^2\right)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\,x+\sqrt{c}+\sqrt{d}\,x^2\right)}{c^{7/4}}+\frac{2\sqrt{2}\left(-3a^2d^2-2abcd+5b^2c^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\,x}{\sqrt[4]{c}}\right)}{c^{7/4}}-\frac{2\sqrt{2}\left(-3a^2d^2-2abcd+5b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}\,x}{\sqrt[4]{c}}+1\right)}{c^{7/4}}+\frac{8\sqrt[4]{d}\,x(bc-ad)^2}{c\left(c+dx^4\right)}+32b^2\sqrt[4]{d}\,x}{32d^{9/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^4)^2/(c + d*x^4)^2,x]
```

```
[Out] (32*b^2*d^(1/4)*x + (8*d^(1/4)*(b*c - a*d)^2*x)/(c*(c + d*x^4)) + (2*Sqrt[2
]*(5*b^2*c^2 - 2*a*b*c*d - 3*a^2*d^2)*ArcTan[1 - (Sqrt[2]*d^(1/4)*x)/c^(1/4
)])/c^(7/4) - (2*Sqrt[2]*(5*b^2*c^2 - 2*a*b*c*d - 3*a^2*d^2)*ArcTan[1 + (Sq
rt[2]*d^(1/4)*x)/c^(1/4)])/c^(7/4) + (Sqrt[2]*(5*b^2*c^2 - 2*a*b*c*d - 3*a^
2*d^2)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2])/c^(7/4) - (S
qrt[2]*(5*b^2*c^2 - 2*a*b*c*d - 3*a^2*d^2)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^
(1/4)*x + Sqrt[d]*x^2])/c^(7/4))/(32*d^(9/4))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+bx^4\right)^2}{\left(c+dx^4\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x^4)^2/(c + d*x^4)^2,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x^4)^2/(c + d*x^4)^2, x]
```

fricas [B] time = 0.93, size = 1335, normalized size = 4.59

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^2/(d*x^4+c)^2,x, algorithm="fricas")
```

```
[Out] 1/16*(16*b^2*c*d*x^5 + 4*(c*d^3*x^4 + c^2*d^2)*(-(625*b^8*c^8 - 1000*a*b^7*
c^7*d - 900*a^2*b^6*c^6*d^2 + 1640*a^3*b^5*c^5*d^3 + 646*a^4*b^4*c^4*d^4 -
984*a^5*b^3*c^3*d^5 - 324*a^6*b^2*c^2*d^6 + 216*a^7*b*c*d^7 + 81*a^8*d^8)/(
c^7*d^9))^(1/4)*arctan((c^5*d^7*x*(-(625*b^8*c^8 - 1000*a*b^7*c^7*d - 900*a
^2*b^6*c^6*d^2 + 1640*a^3*b^5*c^5*d^3 + 646*a^4*b^4*c^4*d^4 - 984*a^5*b^3*c
^3*d^5 - 324*a^6*b^2*c^2*d^6 + 216*a^7*b*c*d^7 + 81*a^8*d^8)/(c^7*d^9))^(3/
4) - c^5*d^7*sqrt((c^4*d^4*sqrt(-(625*b^8*c^8 - 1000*a*b^7*c^7*d - 900*a^2*
b^6*c^6*d^2 + 1640*a^3*b^5*c^5*d^3 + 646*a^4*b^4*c^4*d^4 - 984*a^5*b^3*c^3*
d^5 - 324*a^6*b^2*c^2*d^6 + 216*a^7*b*c*d^7 + 81*a^8*d^8)/(c^7*d^9)) + (25*
b^4*c^4 - 20*a*b^3*c^3*d - 26*a^2*b^2*c^2*d^2 + 12*a^3*b*c*d^3 + 9*a^4*d^4)
*x^2)/(25*b^4*c^4 - 20*a*b^3*c^3*d - 26*a^2*b^2*c^2*d^2 + 12*a^3*b*c*d^3 +
9*a^4*d^4))*(-(625*b^8*c^8 - 1000*a*b^7*c^7*d - 900*a^2*b^6*c^6*d^2 + 1640*
a^3*b^5*c^5*d^3 + 646*a^4*b^4*c^4*d^4 - 984*a^5*b^3*c^3*d^5 - 324*a^6*b^2*c
^2*d^6 + 216*a^7*b*c*d^7 + 81*a^8*d^8)/(c^7*d^9))^(3/4))/(125*b^6*c^6 - 150
*a*b^5*c^5*d - 165*a^2*b^4*c^4*d^2 + 172*a^3*b^3*c^3*d^3 + 99*a^4*b^2*c^2*d
^4 - 54*a^5*b*c*d^5 - 27*a^6*d^6)) + (c*d^3*x^4 + c^2*d^2)*(-(625*b^8*c^8 -
 1000*a*b^7*c^7*d - 900*a^2*b^6*c^6*d^2 + 1640*a^3*b^5*c^5*d^3 + 646*a^4*b^
4*c^4*d^4 - 984*a^5*b^3*c^3*d^5 - 324*a^6*b^2*c^2*d^6 + 216*a^7*b*c*d^7 + 8
1*a^8*d^8)/(c^7*d^9))^(1/4)*log(c^2*d^2*(-(625*b^8*c^8 - 1000*a*b^7*c^7*d -
 900*a^2*b^6*c^6*d^2 + 1640*a^3*b^5*c^5*d^3 + 646*a^4*b^4*c^4*d^4 - 984*a^5
*b^3*c^3*d^5 - 324*a^6*b^2*c^2*d^6 + 216*a^7*b*c*d^7 + 81*a^8*d^8)/(c^7*d^9
))^(1/4) - (5*b^2*c^2 - 2*a*b*c*d - 3*a^2*d^2)*x) - (c*d^3*x^4 + c^2*d^2)*(
-(625*b^8*c^8 - 1000*a*b^7*c^7*d - 900*a^2*b^6*c^6*d^2 + 1640*a^3*b^5*c^5*d
^3 + 646*a^4*b^4*c^4*d^4 - 984*a^5*b^3*c^3*d^5 - 324*a^6*b^2*c^2*d^6 + 216*
a^7*b*c*d^7 + 81*a^8*d^8)/(c^7*d^9))^(1/4)*log(-c^2*d^2*(-(625*b^8*c^8 - 10
00*a*b^7*c^7*d - 900*a^2*b^6*c^6*d^2 + 1640*a^3*b^5*c^5*d^3 + 646*a^4*b^4*c
^4*d^4 - 984*a^5*b^3*c^3*d^5 - 324*a^6*b^2*c^2*d^6 + 216*a^7*b*c*d^7 + 81*a
^8*d^8)/(c^7*d^9))^(1/4) - (5*b^2*c^2 - 2*a*b*c*d - 3*a^2*d^2)*x) + 4*(5*b^
2*c^2 - 2*a*b*c*d + a^2*d^2)*x)/(c*d^3*x^4 + c^2*d^2)
```

giac [A] time = 0.17, size = 376, normalized size = 1.29

$$\frac{b^2x}{d^2} - \frac{\sqrt{2}\left(5\left(cd^3\right)^{\frac{1}{4}}b^2c^2 - 2\left(cd^3\right)^{\frac{1}{4}}abcd - 3\left(cd^3\right)^{\frac{1}{4}}a^2d^2\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{16\,c^2d^3} - \frac{\sqrt{2}\left(5\left(cd^3\right)^{\frac{1}{4}}b^2c^2 - 2\left(cd^3\right)^{\frac{1}{4}}abcd - 3\left(cd^3\right)^{\frac{1}{4}}a^2d^2\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{16\,c^2d^3} - \frac{\sqrt{2}\left(5\left(cd^3\right)^{\frac{1}{4}}b^2c^2 - 2\left(cd^3\right)^{\frac{1}{4}}abcd - 3\left(cd^3\right)^{\frac{1}{4}}a^2d^2\right)\log\left(x^2+\sqrt{2}x\left(\frac{c}{d}\right)^{\frac{1}{4}}+\sqrt{\frac{c}{d}}\right)}{32\,c^2d^3} + \frac{\sqrt{2}\left(5\left(cd^3\right)^{\frac{1}{4}}b^2c^2 - 2\left(cd^3\right)^{\frac{1}{4}}abcd - 3\left(cd^3\right)^{\frac{1}{4}}a^2d^2\right)\log\left(x^2-\sqrt{2}x\left(\frac{c}{d}\right)^{\frac{1}{4}}+\sqrt{\frac{c}{d}}\right)}{32\,c^2d^3} + \frac{b^2c^2x - 2\,abcdx + a^2d^2x}{4\left(dx^4+c\right)cd^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^2/(d*x^4+c)^2,x, algorithm="giac")
```

```
[Out] b^2*x/d^2 - 1/16*sqrt(2)*(5*(c*d^3)^(1/4)*b^2*c^2 - 2*(c*d^3)^(1/4)*a*b*c*d
 - 3*(c*d^3)^(1/4)*a^2*d^2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(c/d)^(1/4))/
(c/d)^(1/4))/(c^2*d^3) - 1/16*sqrt(2)*(5*(c*d^3)^(1/4)*b^2*c^2 - 2*(c*d^3)^
(1/4)*a*b*c*d - 3*(c*d^3)^(1/4)*a^2*d^2)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*
(c/d)^(1/4))/(c/d)^(1/4))/(c^2*d^3) - 1/32*sqrt(2)*(5*(c*d^3)^(1/4)*b^2*c^2
 - 2*(c*d^3)^(1/4)*a*b*c*d - 3*(c*d^3)^(1/4)*a^2*d^2)*log(x^2 + sqrt(2)*x*(
c/d)^(1/4) + sqrt(c/d))/(c^2*d^3) + 1/32*sqrt(2)*(5*(c*d^3)^(1/4)*b^2*c^2 -
 2*(c*d^3)^(1/4)*a*b*c*d - 3*(c*d^3)^(1/4)*a^2*d^2)*log(x^2 - sqrt(2)*x*(c/
d)^(1/4) + sqrt(c/d))/(c^2*d^3) + 1/4*(b^2*c^2*x - 2*a*b*c*d*x + a^2*d^2*x)
/((d*x^4 + c)*c*d^2)
```

maple [B] time = 0.06, size = 475, normalized size = 1.63

$$\frac{a^2x}{4\left(dx^4+c\right)c} - \frac{abx}{2\left(dx^4+c\right)d} + \frac{b^2cx}{4\left(dx^4+c\right)d^2} + \frac{3\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,a^2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}-1\right)}{16c^2} + \frac{3\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,a^2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}+1\right)}{16c^2} + \frac{3\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,a^2\ln\left(\frac{x^2+\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{c}{d}}}{x^2-\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{c}{d}}}\right)}{32c^2} + \frac{\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,ab\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}-1\right)}{8cd} + \frac{\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,ab\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}+1\right)}{8cd} + \frac{\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,ab\ln\left(\frac{x^2+\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{c}{d}}}{x^2-\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{c}{d}}}\right)}{16cd} + \frac{b^2x}{d^2} - \frac{5\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,b^2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}-1\right)}{16d^2} - \frac{5\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,b^2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}+1\right)}{16d^2} - \frac{5\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,b^2\ln\left(\frac{x^2+\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{c}{d}}}{x^2-\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{c}{d}}}\right)}{32d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^4+a)^2/(d*x^4+c)^2,x)
```

```
[Out] b^2/d^2*x+1/4/c*x/(d*x^4+c)*a^2-1/2/d*x/(d*x^4+c)*a*b+1/4/d^2*c*x/(d*x^4+c)
*b^2+3/16/c^2*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x-1)*a^2+1/8/d
/c*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x-1)*a*b-5/16/d^2*(c/d)^(
1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x-1)*b^2+3/32/c^2*(c/d)^(1/4)*2^(1/
2)*ln((x^2+(c/d)^(1/4)*2^(1/2)*x+(c/d)^(1/2))/(x^2-(c/d)^(1/4)*2^(1/2)*x+(c
/d)^(1/2)))*a^2+1/16/d/c*(c/d)^(1/4)*2^(1/2)*ln((x^2+(c/d)^(1/4)*2^(1/2)*x+
(c/d)^(1/2))/(x^2-(c/d)^(1/4)*2^(1/2)*x+(c/d)^(1/2)))*a*b-5/32/d^2*(c/d)^(1
/4)*2^(1/2)*ln((x^2+(c/d)^(1/4)*2^(1/2)*x+(c/d)^(1/2))/(x^2-(c/d)^(1/4)*2^(
1/2)*x+(c/d)^(1/2)))*b^2+3/16/c^2*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^
(1/4)*x+1)*a^2+1/8/d/c*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x+1)*
a*b-5/16/d^2*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x+1)*b^2
```

maxima [A] time = 1.10, size = 319, normalized size = 1.10

$$\frac{\left(b^2c^2 - 2\,abcd + a^2d^2\right)x}{4\left(cd^3x^4 + c^2d^2\right)} + \frac{b^2x}{d^2} - \frac{\frac{2\sqrt{2}\left(5b^2c^2-2abcd-3a^2d^2\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{d}x+\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{c}\sqrt{\sqrt{c}\sqrt{d}}} + \frac{2\sqrt{2}\left(5b^2c^2-2abcd-3a^2d^2\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{d}x-\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{c}\sqrt{\sqrt{c}\sqrt{d}}} + \frac{\sqrt{2}\left(5b^2c^2-2abcd-3a^2d^2\right)\log\left(\sqrt{d}x^2+\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}x+\sqrt{c}\right)}{c^{\frac{3}{4}}d^{\frac{1}{4}}} - \frac{\sqrt{2}\left(5b^2c^2-2abcd-3a^2d^2\right)\log\left(\sqrt{d}x^2-\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}x+\sqrt{c}\right)}{c^{\frac{3}{4}}d^{\frac{1}{4}}}}{32\,cd^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^2/(d*x^4+c)^2,x, algorithm="maxima")
```

```
[Out] 1/4*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*x/(c*d^3*x^4 + c^2*d^2) + b^2*x/d^2 - 1
/32*(2*sqrt(2)*(5*b^2*c^2 - 2*a*b*c*d - 3*a^2*d^2)*arctan(1/2*sqrt(2)*(2*sq
rt(d)*x + sqrt(2)*c^(1/4)*d^(1/4))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(c)*sqrt(sqr
t(c)*sqrt(d))) + 2*sqrt(2)*(5*b^2*c^2 - 2*a*b*c*d - 3*a^2*d^2)*arctan(1/2*s
qrt(2)*(2*sqrt(d)*x - sqrt(2)*c^(1/4)*d^(1/4))/sqrt(sqrt(c)*sqrt(d)))/(sqrt
(c)*sqrt(sqrt(c)*sqrt(d))) + sqrt(2)*(5*b^2*c^2 - 2*a*b*c*d - 3*a^2*d^2)*lo
g(sqrt(d)*x^2 + sqrt(2)*c^(1/4)*d^(1/4)*x + sqrt(c))/(c^(3/4)*d^(1/4)) - sq
rt(2)*(5*b^2*c^2 - 2*a*b*c*d - 3*a^2*d^2)*log(sqrt(d)*x^2 - sqrt(2)*c^(1/4)
*d^(1/4)*x + sqrt(c))/(c^(3/4)*d^(1/4)))/(c*d^2)
```

mupad [B] time = 1.54, size = 1254, normalized size = 4.31

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^4)^2/(c + d*x^4)^2,x)
```

```
[Out] (b^2*x)/d^2 + (x*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d))/(4*c*(c*d^2 + d^3*x^4)) +
 (atan(((((x*(9*a^4*d^4 + 25*b^4*c^4 - 26*a^2*b^2*c^2*d^2 - 20*a*b^3*c^3*d
+ 12*a^3*b*c*d^3))/(4*c^2*d) - ((a*d - b*c)*(3*a*d + 5*b*c)*(12*a^2*d^3 - 2
0*b^2*c^2*d + 8*a*b*c*d^2))/(16*(-c)^(7/4)*d^(9/4)))*(a*d - b*c)*(3*a*d + 5
*b*c)*1i)/(16*(-c)^(7/4)*d^(9/4)) + (((x*(9*a^4*d^4 + 25*b^4*c^4 - 26*a^2*b
^2*c^2*d^2 - 20*a*b^3*c^3*d + 12*a^3*b*c*d^3))/(4*c^2*d) + ((a*d - b*c)*(3*
a*d + 5*b*c)*(12*a^2*d^3 - 20*b^2*c^2*d + 8*a*b*c*d^2))/(16*(-c)^(7/4)*d^(9
/4)))*(a*d - b*c)*(3*a*d + 5*b*c)*1i)/(16*(-c)^(7/4)*d^(9/4)))/((((x*(9*a^4
*d^4 + 25*b^4*c^4 - 26*a^2*b^2*c^2*d^2 - 20*a*b^3*c^3*d + 12*a^3*b*c*d^3))/
(4*c^2*d) - ((a*d - b*c)*(3*a*d + 5*b*c)*(12*a^2*d^3 - 20*b^2*c^2*d + 8*a*b
*c*d^2))/(16*(-c)^(7/4)*d^(9/4)))*(a*d - b*c)*(3*a*d + 5*b*c))/(16*(-c)^(7/
4)*d^(9/4)) - (((x*(9*a^4*d^4 + 25*b^4*c^4 - 26*a^2*b^2*c^2*d^2 - 20*a*b^3*
c^3*d + 12*a^3*b*c*d^3))/(4*c^2*d) + ((a*d - b*c)*(3*a*d + 5*b*c)*(12*a^2*d
^3 - 20*b^2*c^2*d + 8*a*b*c*d^2))/(16*(-c)^(7/4)*d^(9/4)))*(a*d - b*c)*(3*a
*d + 5*b*c))/(16*(-c)^(7/4)*d^(9/4))))*(a*d - b*c)*(3*a*d + 5*b*c)*1i)/(8*(
-c)^(7/4)*d^(9/4)) + (atan(((((x*(9*a^4*d^4 + 25*b^4*c^4 - 26*a^2*b^2*c^2*d
^2 - 20*a*b^3*c^3*d + 12*a^3*b*c*d^3))/(4*c^2*d) - ((a*d - b*c)*(3*a*d + 5*
b*c)*(12*a^2*d^3 - 20*b^2*c^2*d + 8*a*b*c*d^2)*1i)/(16*(-c)^(7/4)*d^(9/4)))
*(a*d - b*c)*(3*a*d + 5*b*c))/(16*(-c)^(7/4)*d^(9/4)) + (((x*(9*a^4*d^4 + 2
5*b^4*c^4 - 26*a^2*b^2*c^2*d^2 - 20*a*b^3*c^3*d + 12*a^3*b*c*d^3))/(4*c^2*d
) + ((a*d - b*c)*(3*a*d + 5*b*c)*(12*a^2*d^3 - 20*b^2*c^2*d + 8*a*b*c*d^2)*
1i)/(16*(-c)^(7/4)*d^(9/4)))*(a*d - b*c)*(3*a*d + 5*b*c))/(16*(-c)^(7/4)*d^
(9/4)))/((((x*(9*a^4*d^4 + 25*b^4*c^4 - 26*a^2*b^2*c^2*d^2 - 20*a*b^3*c^3*d
 + 12*a^3*b*c*d^3))/(4*c^2*d) - ((a*d - b*c)*(3*a*d + 5*b*c)*(12*a^2*d^3 -
20*b^2*c^2*d + 8*a*b*c*d^2)*1i)/(16*(-c)^(7/4)*d^(9/4)))*(a*d - b*c)*(3*a*d
 + 5*b*c)*1i)/(16*(-c)^(7/4)*d^(9/4)) - (((x*(9*a^4*d^4 + 25*b^4*c^4 - 26*a
^2*b^2*c^2*d^2 - 20*a*b^3*c^3*d + 12*a^3*b*c*d^3))/(4*c^2*d) + ((a*d - b*c)
*(3*a*d + 5*b*c)*(12*a^2*d^3 - 20*b^2*c^2*d + 8*a*b*c*d^2)*1i)/(16*(-c)^(7/
4)*d^(9/4)))*(a*d - b*c)*(3*a*d + 5*b*c)*1i)/(16*(-c)^(7/4)*d^(9/4))))*(a*d
 - b*c)*(3*a*d + 5*b*c))/(8*(-c)^(7/4)*d^(9/4))
```

sympy [A] time = 1.98, size = 219, normalized size = 0.75

$$\frac{b^2x}{d^2}+\frac{x\left(a^2d^2-2abcd+b^2c^2\right)}{4c^2d^2+4cd^3x^4}+\operatorname{RootSum}\left(65536t^4c^7d^9+81a^8d^8+216a^7bcd^7-324a^6b^2c^2d^6-984a^5b^3c^3d^5+646a^4b^4c^4d^4+1640a^3b^5c^5d^3-900a^2b^6c^6d^2-1000ab^7c^7d+625b^8c^8,\left(t\mapsto t\log\left(\frac{16tc^2d^2}{3a^2d^2+2abcd-5b^2c^2}+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**4+a)**2/(d*x**4+c)**2,x)
```

```
[Out] b**2*x/d**2 + x*(a**2*d**2 - 2*a*b*c*d + b**2*c**2)/(4*c**2*d**2 + 4*c*d**3
*x**4) + RootSum(65536*_t**4*c**7*d**9 + 81*a**8*d**8 + 216*a**7*b*c*d**7 -
 324*a**6*b**2*c**2*d**6 - 984*a**5*b**3*c**3*d**5 + 646*a**4*b**4*c**4*d**
4 + 1640*a**3*b**5*c**5*d**3 - 900*a**2*b**6*c**6*d**2 - 1000*a*b**7*c**7*d
 + 625*b**8*c**8, Lambda(_t, _t*log(16*_t*c**2*d**2/(3*a**2*d**2 + 2*a*b*c*
d - 5*b**2*c**2) + x)))
```

3.96 $\int \frac{\left(a+bx^4\right)^2}{\left(c+dx^4\right)^3}\,dx$

Optimal. Leaf size=349

$$-\frac{\left(21a^2d^2+6abcd+5b^2c^2\right)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\,x+\sqrt{c}+\sqrt{d}\,x^2\right)}{128\sqrt{2}\,c^{11/4}d^{9/4}}+\frac{\left(21a^2d^2+6abcd+5b^2c^2\right)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\,x+\cdots\right.}{128\sqrt{2}\,c^{11/4}d^{9/4}}$$

Rubi [A] time = 0.27, antiderivative size = 349, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.421, Rules used = {413, 385, 211, 1165, 628, 1162, 617, 204}

$$-\frac{\left(21a^2d^2+6abcd+5b^2c^2\right)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\,x+\sqrt{c}+\sqrt{d}\,x^2\right)}{128\sqrt{2}\,c^{11/4}d^{9/4}}+\frac{\left(21a^2d^2+6abcd+5b^2c^2\right)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\,x+\sqrt{c}+\sqrt{d}\,x^2\right)}{128\sqrt{2}\,c^{11/4}d^{9/4}}-\frac{\left(21a^2d^2+6abcd+5b^2c^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\,x}{\sqrt[4]{c}}\right)}{64\sqrt{2}\,c^{11/4}d^{9/4}}+\frac{\left(21a^2d^2+6abcd+5b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}\,x}{\sqrt[4]{c}}+1\right)}{64\sqrt{2}\,c^{11/4}d^{9/4}}-\frac{x(bc-ad)(7ad+5bc)}{32c^2d^2\left(c+dx^4\right)}-\frac{x\left(a+bx^4\right)(bc-ad)}{8cd\left(c+dx^4\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^4)^2/(c + d*x^4)^3,x]
```

```
[Out] -((b*c - a*d)*x*(a + b*x^4))/(8*c*d*(c + d*x^4)^2) - ((b*c - a*d)*(5*b*c +
7*a*d)*x)/(32*c^2*d^2*(c + d*x^4)) - ((5*b^2*c^2 + 6*a*b*c*d + 21*a^2*d^2)*
ArcTan[1 - (Sqrt[2]*d^(1/4)*x)/c^(1/4)])/(64*Sqrt[2]*c^(11/4)*d^(9/4)) + ((
5*b^2*c^2 + 6*a*b*c*d + 21*a^2*d^2)*ArcTan[1 + (Sqrt[2]*d^(1/4)*x)/c^(1/4)]
)/(64*Sqrt[2]*c^(11/4)*d^(9/4)) - ((5*b^2*c^2 + 6*a*b*c*d + 21*a^2*d^2)*Log
[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2])/(128*Sqrt[2]*c^(11/4)*
d^(9/4)) + ((5*b^2*c^2 + 6*a*b*c*d + 21*a^2*d^2)*Log[Sqrt[c] + Sqrt[2]*c^(1
/4)*d^(1/4)*x + Sqrt[d]*x^2])/(128*Sqrt[2]*c^(11/4)*d^(9/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 413

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[((a*d - c*b)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*n*(p +
 1)), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{\left(a+bx^4\right)^2}{\left(c+dx^4\right)^3}\,dx = -\frac{(bc-ad)x\left(a+bx^4\right)}{8cd\left(c+dx^4\right)^2}+\frac{\int \frac{a(bc+7ad)+b(5bc+3ad)x^4}{\left(c+dx^4\right)^2}\,dx}{8cd}$$

$$= -\frac{(bc-ad)x\left(a+bx^4\right)}{8cd\left(c+dx^4\right)^2}-\frac{(bc-ad)(5bc+7ad)x}{32c^2d^2\left(c+dx^4\right)}+\frac{\left(5b^2c^2+6abcd+21a^2d^2\right)\int \frac{1}{c+dx^4}\,dx}{32c^2d^2}$$

$$= -\frac{(bc-ad)x\left(a+bx^4\right)}{8cd\left(c+dx^4\right)^2}-\frac{(bc-ad)(5bc+7ad)x}{32c^2d^2\left(c+dx^4\right)}+\frac{\left(5b^2c^2+6abcd+21a^2d^2\right)\int \frac{\sqrt{c}-\sqrt{d}\,x^2}{c+dx^4}\,dx}{64c^{5/2}d^2}+\cdots$$

$$= -\frac{(bc-ad)x\left(a+bx^4\right)}{8cd\left(c+dx^4\right)^2}-\frac{(bc-ad)(5bc+7ad)x}{32c^2d^2\left(c+dx^4\right)}+\frac{\left(5b^2c^2+6abcd+21a^2d^2\right)\int \frac{1}{\frac{\sqrt{c}}{\sqrt{d}}-\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{d}}+x^2}\,d\cdots}{128c^{5/2}d^{5/2}}\cdots$$

$$= -\frac{(bc-ad)x\left(a+bx^4\right)}{8cd\left(c+dx^4\right)^2}-\frac{(bc-ad)(5bc+7ad)x}{32c^2d^2\left(c+dx^4\right)}-\frac{\left(5b^2c^2+6abcd+21a^2d^2\right)\log\left(\sqrt{c}-\sqrt{2}\sqrt[4]{c}\cdots\right.}{128\sqrt{2}\,c^{11/4}d^{9/4}}\cdots$$

$$= -\frac{(bc-ad)x\left(a+bx^4\right)}{8cd\left(c+dx^4\right)^2}-\frac{(bc-ad)(5bc+7ad)x}{32c^2d^2\left(c+dx^4\right)}-\frac{\left(5b^2c^2+6abcd+21a^2d^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\,x\cdots}{\sqrt[4]{c}}\right.}{64\sqrt{2}\,c^{11/4}d^{9/4}}\cdots$$

Mathematica [A] time = 0.23, size = 319, normalized size = 0.91

$$\frac{-\sqrt{2}\left(21a^2d^2+6abcd+5b^2c^2\right)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\,x+\sqrt{c}+\sqrt{d}\,x^2\right)+\sqrt{2}\left(21a^2d^2+6abcd+5b^2c^2\right)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\,x+\sqrt{c}+\sqrt{d}\,x^2\right)-2\sqrt{2}\left(21a^2d^2+6abcd+5b^2c^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\,x}{\sqrt[4]{c}}\right)+2\sqrt{2}\left(21a^2d^2+6abcd+5b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}\,x}{\sqrt[4]{c}}+1\right)-\frac{8c^{3/4}\sqrt[4]{d}\,x\left(-7a^2d^2-2abcd+9b^2c^2\right)}{c+dx^4}+\frac{32c^{7/4}\sqrt[4]{d}\,x(bc-ad)^2}{\left(c+dx^4\right)^2}}{256c^{11/4}d^{9/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^4)^2/(c + d*x^4)^3,x]
```

```
[Out] ((32*c^(7/4)*d^(1/4)*(b*c - a*d)^2*x)/(c + d*x^4)^2 - (8*c^(3/4)*d^(1/4)*(9
*b^2*c^2 - 2*a*b*c*d - 7*a^2*d^2)*x)/(c + d*x^4) - 2*Sqrt[2]*(5*b^2*c^2 + 6
*a*b*c*d + 21*a^2*d^2)*ArcTan[1 - (Sqrt[2]*d^(1/4)*x)/c^(1/4)] + 2*Sqrt[2]*
(5*b^2*c^2 + 6*a*b*c*d + 21*a^2*d^2)*ArcTan[1 + (Sqrt[2]*d^(1/4)*x)/c^(1/4)
] - Sqrt[2]*(5*b^2*c^2 + 6*a*b*c*d + 21*a^2*d^2)*Log[Sqrt[c] - Sqrt[2]*c^(1
/4)*d^(1/4)*x + Sqrt[d]*x^2] + Sqrt[2]*(5*b^2*c^2 + 6*a*b*c*d + 21*a^2*d^2)
*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2])/(256*c^(11/4)*d^(9
/4))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+bx^4\right)^2}{\left(c+dx^4\right)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x^4)^2/(c + d*x^4)^3,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x^4)^2/(c + d*x^4)^3, x]
```

fricas [B] time = 1.20, size = 1411, normalized size = 4.04

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^2/(d*x^4+c)^3,x, algorithm="fricas")
```

```
[Out] -1/128*(4*(9*b^2*c^2*d - 2*a*b*c*d^2 - 7*a^2*d^3)*x^5 - 4*(c^2*d^4*x^8 + 2*
c^3*d^3*x^4 + c^4*d^2)*(-(625*b^8*c^8 + 3000*a*b^7*c^7*d + 15900*a^2*b^6*c^
6*d^2 + 42120*a^3*b^5*c^5*d^3 + 112806*a^4*b^4*c^4*d^4 + 176904*a^5*b^3*c^3
*d^5 + 280476*a^6*b^2*c^2*d^6 + 222264*a^7*b*c*d^7 + 194481*a^8*d^8)/(c^11*
d^9))^(1/4)*arctan(-(c^8*d^7*x*(-(625*b^8*c^8 + 3000*a*b^7*c^7*d + 15900*a^
2*b^6*c^6*d^2 + 42120*a^3*b^5*c^5*d^3 + 112806*a^4*b^4*c^4*d^4 + 176904*a^5
*b^3*c^3*d^5 + 280476*a^6*b^2*c^2*d^6 + 222264*a^7*b*c*d^7 + 194481*a^8*d^8
)/(c^11*d^9))^(3/4) - c^8*d^7*sqrt((c^6*d^4*sqrt(-(625*b^8*c^8 + 3000*a*b^7
*c^7*d + 15900*a^2*b^6*c^6*d^2 + 42120*a^3*b^5*c^5*d^3 + 112806*a^4*b^4*c^4
*d^4 + 176904*a^5*b^3*c^3*d^5 + 280476*a^6*b^2*c^2*d^6 + 222264*a^7*b*c*d^7
 + 194481*a^8*d^8)/(c^11*d^9)) + (25*b^4*c^4 + 60*a*b^3*c^3*d + 246*a^2*b^2
*c^2*d^2 + 252*a^3*b*c*d^3 + 441*a^4*d^4)*x^2)/(25*b^4*c^4 + 60*a*b^3*c^3*d
 + 246*a^2*b^2*c^2*d^2 + 252*a^3*b*c*d^3 + 441*a^4*d^4))*(-(625*b^8*c^8 + 3
000*a*b^7*c^7*d + 15900*a^2*b^6*c^6*d^2 + 42120*a^3*b^5*c^5*d^3 + 112806*a^
4*b^4*c^4*d^4 + 176904*a^5*b^3*c^3*d^5 + 280476*a^6*b^2*c^2*d^6 + 222264*a^
7*b*c*d^7 + 194481*a^8*d^8)/(c^11*d^9))^(3/4))/(125*b^6*c^6 + 450*a*b^5*c^5
*d + 2115*a^2*b^4*c^4*d^2 + 3996*a^3*b^3*c^3*d^3 + 8883*a^4*b^2*c^2*d^4 + 7
938*a^5*b*c*d^5 + 9261*a^6*d^6)) - (c^2*d^4*x^8 + 2*c^3*d^3*x^4 + c^4*d^2)*
(-(625*b^8*c^8 + 3000*a*b^7*c^7*d + 15900*a^2*b^6*c^6*d^2 + 42120*a^3*b^5*c
^5*d^3 + 112806*a^4*b^4*c^4*d^4 + 176904*a^5*b^3*c^3*d^5 + 280476*a^6*b^2*c
^2*d^6 + 222264*a^7*b*c*d^7 + 194481*a^8*d^8)/(c^11*d^9))^(1/4)*log(c^3*d^2
*(-(625*b^8*c^8 + 3000*a*b^7*c^7*d + 15900*a^2*b^6*c^6*d^2 + 42120*a^3*b^5*
c^5*d^3 + 112806*a^4*b^4*c^4*d^4 + 176904*a^5*b^3*c^3*d^5 + 280476*a^6*b^2*
c^2*d^6 + 222264*a^7*b*c*d^7 + 194481*a^8*d^8)/(c^11*d^9))^(1/4) + (5*b^2*c
^2 + 6*a*b*c*d + 21*a^2*d^2)*x) + (c^2*d^4*x^8 + 2*c^3*d^3*x^4 + c^4*d^2)*(
-(625*b^8*c^8 + 3000*a*b^7*c^7*d + 15900*a^2*b^6*c^6*d^2 + 42120*a^3*b^5*c^
5*d^3 + 112806*a^4*b^4*c^4*d^4 + 176904*a^5*b^3*c^3*d^5 + 280476*a^6*b^2*c^
2*d^6 + 222264*a^7*b*c*d^7 + 194481*a^8*d^8)/(c^11*d^9))^(1/4)*log(-c^3*d^2
*(-(625*b^8*c^8 + 3000*a*b^7*c^7*d + 15900*a^2*b^6*c^6*d^2 + 42120*a^3*b^5*
c^5*d^3 + 112806*a^4*b^4*c^4*d^4 + 176904*a^5*b^3*c^3*d^5 + 280476*a^6*b^2*
```

c^2*d^6 + 222264*a^7*b*c*d^7 + 194481*a^8*d^8)/(c^11*d^9))^(1/4) + (5*b^2*c
^2 + 6*a*b*c*d + 21*a^2*d^2)*x) + 4*(5*b^2*c^3 + 6*a*b*c^2*d - 11*a^2*c*d^2
)*x)/(c^2*d^4*x^8 + 2*c^3*d^3*x^4 + c^4*d^2)

giac [A]   time = 0.21, size = 407, normalized size = 1.17

$$\frac{\sqrt{2}\left(5\left(cd^{3}\right)^{\frac{1}{4}}b^{2}c^{2}+6\left(cd^{3}\right)^{\frac{1}{4}}abcd+21\left(cd^{3}\right)^{\frac{1}{4}}a^{2}d^{2}\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{128c^{3}d^{3}}+\frac{\sqrt{2}\left(5\left(cd^{3}\right)^{\frac{1}{4}}b^{2}c^{2}+6\left(cd^{3}\right)^{\frac{1}{4}}abcd+21\left(cd^{3}\right)^{\frac{1}{4}}a^{2}d^{2}\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{128c^{3}d^{3}}+\frac{\sqrt{2}\left(5\left(cd^{3}\right)^{\frac{1}{4}}b^{2}c^{2}+6\left(cd^{3}\right)^{\frac{1}{4}}abcd+21\left(cd^{3}\right)^{\frac{1}{4}}a^{2}d^{2}\right)\log\left(x^{2}+\sqrt{2}x\left(\frac{c}{d}\right)^{\frac{1}{4}}+\sqrt{\frac{c}{d}}\right)}{256c^{3}d^{3}}-\frac{\sqrt{2}\left(5\left(cd^{3}\right)^{\frac{1}{4}}b^{2}c^{2}+6\left(cd^{3}\right)^{\frac{1}{4}}abcd+21\left(cd^{3}\right)^{\frac{1}{4}}a^{2}d^{2}\right)\log\left(x^{2}-\sqrt{2}x\left(\frac{c}{d}\right)^{\frac{1}{4}}+\sqrt{\frac{c}{d}}\right)}{256c^{3}d^{3}}-\frac{9b^{2}c^{2}dx^{5}-2abcd^{2}x^{5}-7a^{2}d^{3}x^{5}+5b^{2}c^{3}x+6abc^{2}dx-11a^{2}cd^{2}x}{32\left(dx^{4}+c\right)^{2}c^{2}d^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^4+a)^2/(d*x^4+c)^3,x, algorithm="giac")

[Out] 1/128*sqrt(2)*(5*(c*d^3)^(1/4)*b^2*c^2 + 6*(c*d^3)^(1/4)*a*b*c*d + 21*(c*d^
3)^(1/4)*a^2*d^2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(c/d)^(1/4))/(c/d)^(1/4
))/(c^3*d^3) + 1/128*sqrt(2)*(5*(c*d^3)^(1/4)*b^2*c^2 + 6*(c*d^3)^(1/4)*a*b
*c*d + 21*(c*d^3)^(1/4)*a^2*d^2)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(c/d)^(1
/4))/(c/d)^(1/4))/(c^3*d^3) + 1/256*sqrt(2)*(5*(c*d^3)^(1/4)*b^2*c^2 + 6*(c
*d^3)^(1/4)*a*b*c*d + 21*(c*d^3)^(1/4)*a^2*d^2)*log(x^2 + sqrt(2)*x*(c/d)^(
1/4) + sqrt(c/d))/(c^3*d^3) - 1/256*sqrt(2)*(5*(c*d^3)^(1/4)*b^2*c^2 + 6*(c
*d^3)^(1/4)*a*b*c*d + 21*(c*d^3)^(1/4)*a^2*d^2)*log(x^2 - sqrt(2)*x*(c/d)^(
1/4) + sqrt(c/d))/(c^3*d^3) - 1/32*(9*b^2*c^2*d*x^5 - 2*a*b*c*d^2*x^5 - 7*a
^2*d^3*x^5 + 5*b^2*c^3*x + 6*a*b*c^2*d*x - 11*a^2*c*d^2*x)/((d*x^4 + c)^2*c
^2*d^2)

maple [A]   time = 0.06, size = 499, normalized size = 1.43

$$\frac{21\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,a^{2}\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}-1\right)}{128c^{3}}+\frac{21\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,a^{2}\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}+1\right)}{128c^{3}}+\frac{21\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,a^{2}\ln\left(\frac{x^{2}+\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{c}{d}}}{x^{2}-\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{c}{d}}}\right)}{256c^{3}}+\frac{3\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,ab\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}-1\right)}{64c^{2}d}+\frac{3\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,ab\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}+1\right)}{64c^{2}d}+\frac{3\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,ab\ln\left(\frac{x^{2}+\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{c}{d}}}{x^{2}-\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{c}{d}}}\right)}{128c^{2}d}+\frac{5\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,b^{2}\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}-1\right)}{128c\,d^{2}}+\frac{5\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,b^{2}\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}+1\right)}{128c\,d^{2}}+\frac{5\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,b^{2}\ln\left(\frac{x^{2}+\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{c}{d}}}{x^{2}-\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{c}{d}}}\right)}{256c\,d^{2}}+\frac{\frac{\left(7a^{2}d^{2}+2abcd-9b^{2}c^{2}\right)x^{5}}{32c^{2}d}+\frac{\left(11a^{2}d^{2}-6abcd-5b^{2}c^{2}\right)x}{32c\,d^{2}}}{\left(d\,x^{4}+c\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x^4+a)^2/(d*x^4+c)^3,x)

[Out] (1/32*(7*a^2*d^2+2*a*b*c*d-9*b^2*c^2)/c^2/d*x^5+1/32*(11*a^2*d^2-6*a*b*c*d-
5*b^2*c^2)/d^2/c*x)/(d*x^4+c)^2+21/128/c^3*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/
2)/(c/d)^(1/4)*x-1)*a^2+3/64/c^2/d*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)
^(1/4)*x-1)*a*b+5/128/c/d^2*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*
x-1)*b^2+21/256/c^3*(c/d)^(1/4)*2^(1/2)*ln((x^2+(c/d)^(1/4)*2^(1/2)*x+(c/d)
^(1/2))/(x^2-(c/d)^(1/4)*2^(1/2)*x+(c/d)^(1/2)))*a^2+3/128/c^2/d*(c/d)^(1/4
)*2^(1/2)*ln((x^2+(c/d)^(1/4)*2^(1/2)*x+(c/d)^(1/2))/(x^2-(c/d)^(1/4)*2^(1/
2)*x+(c/d)^(1/2)))*a*b+5/256/c/d^2*(c/d)^(1/4)*2^(1/2)*ln((x^2+(c/d)^(1/4)*
2^(1/2)*x+(c/d)^(1/2))/(x^2-(c/d)^(1/4)*2^(1/2)*x+(c/d)^(1/2)))*b^2+21/128/
c^3*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x+1)*a^2+3/64/c^2/d*(c/d
)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x+1)*a*b+5/128/c/d^2*(c/d)^(1/4)
*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x+1)*b^2

maxima [A]   time = 1.22, size = 361, normalized size = 1.03

$$-\frac{\left(9\,b^{2}c^{2}d-2\,abcd^{2}-7\,a^{2}d^{3}\right)x^{5}+\left(5\,b^{2}c^{3}+6\,abc^{2}d-11\,a^{2}cd^{2}\right)x}{32\left(c^{2}d^{4}x^{8}+2\,c^{3}d^{3}x^{4}+c^{4}d^{2}\right)}+\frac{\frac{2\sqrt{2}\left(5\,b^{2}c^{2}+6\,abcd+21\,a^{2}d^{2}\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{d}\,x+\sqrt{2}\,c^{\frac{1}{4}}d^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{c}\sqrt{\sqrt{c}\sqrt{d}}}+\frac{2\sqrt{2}\left(5\,b^{2}c^{2}+6\,abcd+21\,a^{2}d^{2}\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{d}\,x-\sqrt{2}\,c^{\frac{1}{4}}d^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{c}\sqrt{\sqrt{c}\sqrt{d}}}+\frac{\sqrt{2}\left(5\,b^{2}c^{2}+6\,abcd+21\,a^{2}d^{2}\right)\log\left(\sqrt{d}\,x^{2}+\sqrt{2}\,c^{\frac{1}{4}}d^{\frac{1}{4}}x+\sqrt{c}\right)}{c^{\frac{3}{4}}d^{\frac{1}{4}}}-\frac{\sqrt{2}\left(5\,b^{2}c^{2}+6\,abcd+21\,a^{2}d^{2}\right)\log\left(\sqrt{d}\,x^{2}-\sqrt{2}\,c^{\frac{1}{4}}d^{\frac{1}{4}}x+\sqrt{c}\right)}{c^{\frac{3}{4}}d^{\frac{1}{4}}}}{256\,c^{2}d^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^4+a)^2/(d*x^4+c)^3,x, algorithm="maxima")

[Out] -1/32*((9*b^2*c^2*d - 2*a*b*c*d^2 - 7*a^2*d^3)*x^5 + (5*b^2*c^3 + 6*a*b*c^2
*d - 11*a^2*c*d^2)*x)/(c^2*d^4*x^8 + 2*c^3*d^3*x^4 + c^4*d^2) + 1/256*(2*sq
rt(2)*(5*b^2*c^2 + 6*a*b*c*d + 21*a^2*d^2)*arctan(1/2*sqrt(2)*(2*sqrt(d)*x
+ sqrt(2)*c^(1/4)*d^(1/4))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(c)*sqrt(sqrt(c)*sqr
t(d))) + 2*sqrt(2)*(5*b^2*c^2 + 6*a*b*c*d + 21*a^2*d^2)*arctan(1/2*sqrt(2)*
(2*sqrt(d)*x - sqrt(2)*c^(1/4)*d^(1/4))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(c)*sqr
t(sqrt(c)*sqrt(d))) + sqrt(2)*(5*b^2*c^2 + 6*a*b*c*d + 21*a^2*d^2)*log(sqrt

```
(d)*x^2 + sqrt(2)*c^(1/4)*d^(1/4)*x + sqrt(c))/(c^(3/4)*d^(1/4)) - sqrt(2)*
(5*b^2*c^2 + 6*a*b*c*d + 21*a^2*d^2)*log(sqrt(d)*x^2 - sqrt(2)*c^(1/4)*d^(1
/4)*x + sqrt(c))/(c^(3/4)*d^(1/4)))/(c^2*d^2)
```

mupad [B] time = 1.66, size = 1401, normalized size = 4.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^4)^2/(c + d*x^4)^3,x)
```

```
[Out] - ((x*(5*b^2*c^2 - 11*a^2*d^2 + 6*a*b*c*d))/(32*c*d^2) - (x^5*(7*a^2*d^2 -
9*b^2*c^2 + 2*a*b*c*d))/(32*c^2*d))/(c^2 + d^2*x^8 + 2*c*d*x^4) - (atan((((
((21*a^2*d^2 + 5*b^2*c^2 + 6*a*b*c*d)*(21*a^2*d^3 + 5*b^2*c^2*d + 6*a*b*c*d
^2))/(256*(-c)^(15/4)*d^(9/4)) - (x*(441*a^4*d^4 + 25*b^4*c^4 + 246*a^2*b^2
*c^2*d^2 + 60*a*b^3*c^3*d + 252*a^3*b*c*d^3))/(256*c^4*d))*(21*a^2*d^2 + 5*
b^2*c^2 + 6*a*b*c*d)*1i)/(128*(-c)^(11/4)*d^(9/4)) - ((((21*a^2*d^2 + 5*b^2
*c^2 + 6*a*b*c*d)*(21*a^2*d^3 + 5*b^2*c^2*d + 6*a*b*c*d^2))/(256*(-c)^(15/4
)*d^(9/4)) + (x*(441*a^4*d^4 + 25*b^4*c^4 + 246*a^2*b^2*c^2*d^2 + 60*a*b^3*
c^3*d + 252*a^3*b*c*d^3))/(256*c^4*d))*(21*a^2*d^2 + 5*b^2*c^2 + 6*a*b*c*d)
*1i)/(128*(-c)^(11/4)*d^(9/4)))/(((((21*a^2*d^2 + 5*b^2*c^2 + 6*a*b*c*d)*(2
1*a^2*d^3 + 5*b^2*c^2*d + 6*a*b*c*d^2))/(256*(-c)^(15/4)*d^(9/4)) - (x*(441
*a^4*d^4 + 25*b^4*c^4 + 246*a^2*b^2*c^2*d^2 + 60*a*b^3*c^3*d + 252*a^3*b*c*
d^3))/(256*c^4*d))*(21*a^2*d^2 + 5*b^2*c^2 + 6*a*b*c*d))/(128*(-c)^(11/4)*d
^(9/4)) + ((((21*a^2*d^2 + 5*b^2*c^2 + 6*a*b*c*d)*(21*a^2*d^3 + 5*b^2*c^2*d
 + 6*a*b*c*d^2))/(256*(-c)^(15/4)*d^(9/4)) + (x*(441*a^4*d^4 + 25*b^4*c^4 +
 246*a^2*b^2*c^2*d^2 + 60*a*b^3*c^3*d + 252*a^3*b*c*d^3))/(256*c^4*d))*(21*
a^2*d^2 + 5*b^2*c^2 + 6*a*b*c*d))/(128*(-c)^(11/4)*d^(9/4))))*(21*a^2*d^2 +
 5*b^2*c^2 + 6*a*b*c*d)*1i)/(64*(-c)^(11/4)*d^(9/4)) - (atan((((((21*a^2*d^
2 + 5*b^2*c^2 + 6*a*b*c*d)*(21*a^2*d^3 + 5*b^2*c^2*d + 6*a*b*c*d^2)*1i)/(25
6*(-c)^(15/4)*d^(9/4)) - (x*(441*a^4*d^4 + 25*b^4*c^4 + 246*a^2*b^2*c^2*d^2
 + 60*a*b^3*c^3*d + 252*a^3*b*c*d^3))/(256*c^4*d))*(21*a^2*d^2 + 5*b^2*c^2
+ 6*a*b*c*d))/(128*(-c)^(11/4)*d^(9/4)) - ((((21*a^2*d^2 + 5*b^2*c^2 + 6*a*
b*c*d)*(21*a^2*d^3 + 5*b^2*c^2*d + 6*a*b*c*d^2)*1i)/(256*(-c)^(15/4)*d^(9/4
)) + (x*(441*a^4*d^4 + 25*b^4*c^4 + 246*a^2*b^2*c^2*d^2 + 60*a*b^3*c^3*d +
252*a^3*b*c*d^3))/(256*c^4*d))*(21*a^2*d^2 + 5*b^2*c^2 + 6*a*b*c*d))/(128*(
-c)^(11/4)*d^(9/4)))/(((((21*a^2*d^2 + 5*b^2*c^2 + 6*a*b*c*d)*(21*a^2*d^3 +
 5*b^2*c^2*d + 6*a*b*c*d^2)*1i)/(256*(-c)^(15/4)*d^(9/4)) - (x*(441*a^4*d^4
 + 25*b^4*c^4 + 246*a^2*b^2*c^2*d^2 + 60*a*b^3*c^3*d + 252*a^3*b*c*d^3))/(2
56*c^4*d))*(21*a^2*d^2 + 5*b^2*c^2 + 6*a*b*c*d)*1i)/(128*(-c)^(11/4)*d^(9/4
)) + ((((21*a^2*d^2 + 5*b^2*c^2 + 6*a*b*c*d)*(21*a^2*d^3 + 5*b^2*c^2*d + 6*
a*b*c*d^2)*1i)/(256*(-c)^(15/4)*d^(9/4)) + (x*(441*a^4*d^4 + 25*b^4*c^4 + 2
46*a^2*b^2*c^2*d^2 + 60*a*b^3*c^3*d + 252*a^3*b*c*d^3))/(256*c^4*d))*(21*a^
2*d^2 + 5*b^2*c^2 + 6*a*b*c*d)*1i)/(128*(-c)^(11/4)*d^(9/4))))*(21*a^2*d^2
+ 5*b^2*c^2 + 6*a*b*c*d))/(64*(-c)^(11/4)*d^(9/4))
```

sympy [A] time = 5.85, size = 264, normalized size = 0.76

$$\frac{x^{5}\left(7a^{2}d^{3}+2abcd^{2}-9b^{2}c^{2}d\right)+x\left(11a^{2}cd^{2}-6abc^{2}d-5b^{2}c^{3}\right)}{32c^{4}d^{2}+64c^{3}d^{3}x^{4}+32c^{2}d^{4}x^{8}}+\operatorname{RootSum}\left(268435456t^{4}c^{11}d^{9}+194481a^{8}d^{8}+222264a^{7}bcd^{7}+280476a^{6}b^{2}c^{2}d^{6}+176904a^{5}b^{3}c^{3}d^{5}+112806a^{4}b^{4}c^{4}d^{4}+42120a^{3}b^{5}c^{5}d^{3}+15900a^{2}b^{6}c^{6}d^{2}+3000ab^{7}c^{7}d+625b^{8}c^{8},\left(t\mapsto t\log\left(\frac{128tc^{3}d^{2}}{21a^{2}d^{2}+6abcd+5b^{2}c^{2}}+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**4+a)**2/(d*x**4+c)**3,x)
```

```
[Out] (x**5*(7*a**2*d**3 + 2*a*b*c*d**2 - 9*b**2*c**2*d) + x*(11*a**2*c*d**2 - 6*
a*b*c**2*d - 5*b**2*c**3))/(32*c**4*d**2 + 64*c**3*d**3*x**4 + 32*c**2*d**4
*x**8) + RootSum(268435456*_t**4*c**11*d**9 + 194481*a**8*d**8 + 222264*a**
7*b*c*d**7 + 280476*a**6*b**2*c**2*d**6 + 176904*a**5*b**3*c**3*d**5 + 1128
06*a**4*b**4*c**4*d**4 + 42120*a**3*b**5*c**5*d**3 + 15900*a**2*b**6*c**6*d
**2 + 3000*a*b**7*c**7*d + 625*b**8*c**8, Lambda(_t, _t*log(128*_t*c**3*d**
2/(21*a**2*d**2 + 6*a*b*c*d + 5*b**2*c**2) + x)))
```

3.97 $\int \frac{\left(c+dx^4\right)^4}{a+bx^4}\,dx$

Optimal. Leaf size=332

$$-\frac{(bc-ad)^4\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)}{4\sqrt{2}\,a^{3/4}b^{17/4}}+\frac{(bc-ad)^4\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)}{4\sqrt{2}\,a^{3/4}b^{17/4}}-\frac{(bc-ad)^4\tan^{-1}\left(1-\right.}{2\sqrt{2}\,a^{3/4}b^{17}}$$

Rubi [A] time = 0.27, antiderivative size = 332, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {390, 211, 1165, 628, 1162, 617, 204}

$$\frac{d^2x^5\left(a^2d^2-4abcd+6b^2c^2\right)}{5b^3}+\frac{dx(2bc-ad)\left(a^2d^2-2abcd+2b^2c^2\right)}{b^4}-\frac{(bc-ad)^4\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)}{4\sqrt{2}\,a^{3/4}b^{17/4}}+\frac{(bc-ad)^4\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)}{4\sqrt{2}\,a^{3/4}b^{17/4}}-\frac{(bc-ad)^4\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,a^{3/4}b^{17/4}}+\frac{(bc-ad)^4\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}+1\right)}{2\sqrt{2}\,a^{3/4}b^{17/4}}+\frac{d^3x^9(4bc-ad)}{9b^2}+\frac{d^4x^{13}}{13b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^4)^4/(a + b*x^4),x]
```

```
[Out] (d*(2*b*c - a*d)*(2*b^2*c^2 - 2*a*b*c*d + a^2*d^2)*x)/b^4 + (d^2*(6*b^2*c^2
 - 4*a*b*c*d + a^2*d^2)*x^5)/(5*b^3) + (d^3*(4*b*c - a*d)*x^9)/(9*b^2) + (d
^4*x^13)/(13*b) - ((b*c - a*d)^4*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(
2*Sqrt[2]*a^(3/4)*b^(17/4)) + ((b*c - a*d)^4*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)
/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*b^(17/4)) - ((b*c - a*d)^4*Log[Sqrt[a] - Sqrt
[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(3/4)*b^(17/4)) + ((b*c
- a*d)^4*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]
*a^(3/4)*b^(17/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{\left(c+dx^4\right)^4}{a+bx^4}\,dx = \int \left(\frac{d(2bc-ad)\left(2b^2c^2-2abcd+a^2d^2\right)}{b^4}+\frac{d^2\left(6b^2c^2-4abcd+a^2d^2\right)x^4}{b^3}+\frac{d^3(4bc-ad)x^8}{b^2}\right.$$

$$= \frac{d(2bc-ad)\left(2b^2c^2-2abcd+a^2d^2\right)x}{b^4}+\frac{d^2\left(6b^2c^2-4abcd+a^2d^2\right)x^5}{5b^3}+\frac{d^3(4bc-ad)x^9}{9b^2}+$$

$$= \frac{d(2bc-ad)\left(2b^2c^2-2abcd+a^2d^2\right)x}{b^4}+\frac{d^2\left(6b^2c^2-4abcd+a^2d^2\right)x^5}{5b^3}+\frac{d^3(4bc-ad)x^9}{9b^2}+$$

$$= \frac{d(2bc-ad)\left(2b^2c^2-2abcd+a^2d^2\right)x}{b^4}+\frac{d^2\left(6b^2c^2-4abcd+a^2d^2\right)x^5}{5b^3}+\frac{d^3(4bc-ad)x^9}{9b^2}+$$

$$= \frac{d(2bc-ad)\left(2b^2c^2-2abcd+a^2d^2\right)x}{b^4}+\frac{d^2\left(6b^2c^2-4abcd+a^2d^2\right)x^5}{5b^3}+\frac{d^3(4bc-ad)x^9}{9b^2}+$$

$$= \frac{d(2bc-ad)\left(2b^2c^2-2abcd+a^2d^2\right)x}{b^4}+\frac{d^2\left(6b^2c^2-4abcd+a^2d^2\right)x^5}{5b^3}+\frac{d^3(4bc-ad)x^9}{9b^2}+$$

Mathematica [A] time = 0.22, size = 322, normalized size = 0.97

$$\frac{-\frac{585\sqrt{2}(bc-ad)^4\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{a^{3/4}}+\frac{585\sqrt{2}(bc-ad)^4\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{a^{3/4}}-\frac{1170\sqrt{2}(bc-ad)^4\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{a^{3/4}}+\frac{1170\sqrt{2}(bc-ad)^4\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{a^{3/4}}+936b^{5/4}d^2x^5\left(a^2d^2-4abcd+6b^2c^2\right)+4680\sqrt[4]{b}dx\left(-a^3d^3+4a^2bcd^2-6ab^2c^2d+4b^3c^3\right)+520b^{9/4}d^3x^9(4bc-ad)+360b^{13/4}d^4x^{13}}{4680b^{17/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^4)^4/(a + b*x^4),x]
```

```
[Out] (4680*b^(1/4)*d*(4*b^3*c^3 - 6*a*b^2*c^2*d + 4*a^2*b*c*d^2 - a^3*d^3)*x + 9
36*b^(5/4)*d^2*(6*b^2*c^2 - 4*a*b*c*d + a^2*d^2)*x^5 + 520*b^(9/4)*d^3*(4*b
*c - a*d)*x^9 + 360*b^(13/4)*d^4*x^13 - (1170*Sqrt[2]*(b*c - a*d)^4*ArcTan[
1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/a^(3/4) + (1170*Sqrt[2]*(b*c - a*d)^4*Arc
Tan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/a^(3/4) - (585*Sqrt[2]*(b*c - a*d)^4*
Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/a^(3/4) + (585*Sqrt
[2]*(b*c - a*d)^4*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/a
^(3/4))/(4680*b^(17/4))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c+dx^4)^4}{a+bx^4}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x^4)^4/(a + b*x^4),x]
```

```
[Out] IntegrateAlgebraic[(c + d*x^4)^4/(a + b*x^4), x]
```

fricas [B] time = 1.22, size = 2477, normalized size = 7.46

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^4+c)^4/(b*x^4+a),x, algorithm="fricas")
```

```
[Out] 1/2340*(180*b^3*d^4*x^13 + 260*(4*b^3*c*d^3 - a*b^2*d^4)*x^9 + 468*(6*b^3*c
^2*d^2 - 4*a*b^2*c*d^3 + a^2*b*d^4)*x^5 + 2340*b^4*(-(b^16*c^16 - 16*a*b^15
*c^15*d + 120*a^2*b^14*c^14*d^2 - 560*a^3*b^13*c^13*d^3 + 1820*a^4*b^12*c^1
2*d^4 - 4368*a^5*b^11*c^11*d^5 + 8008*a^6*b^10*c^10*d^6 - 11440*a^7*b^9*c^9
*d^7 + 12870*a^8*b^8*c^8*d^8 - 11440*a^9*b^7*c^7*d^9 + 8008*a^10*b^6*c^6*d^
10 - 4368*a^11*b^5*c^5*d^11 + 1820*a^12*b^4*c^4*d^12 - 560*a^13*b^3*c^3*d^1
3 + 120*a^14*b^2*c^2*d^14 - 16*a^15*b*c*d^15 + a^16*d^16)/(a^3*b^17))^(1/4)
*arctan(-(a^2*b^13*x*(-(b^16*c^16 - 16*a*b^15*c^15*d + 120*a^2*b^14*c^14*d^
2 - 560*a^3*b^13*c^13*d^3 + 1820*a^4*b^12*c^12*d^4 - 4368*a^5*b^11*c^11*d^5
 + 8008*a^6*b^10*c^10*d^6 - 11440*a^7*b^9*c^9*d^7 + 12870*a^8*b^8*c^8*d^8 -
 11440*a^9*b^7*c^7*d^9 + 8008*a^10*b^6*c^6*d^10 - 4368*a^11*b^5*c^5*d^11 +
1820*a^12*b^4*c^4*d^12 - 560*a^13*b^3*c^3*d^13 + 120*a^14*b^2*c^2*d^14 - 16
*a^15*b*c*d^15 + a^16*d^16)/(a^3*b^17))^(3/4) - a^2*b^13*sqrt((a^2*b^8*sqrt
(-(b^16*c^16 - 16*a*b^15*c^15*d + 120*a^2*b^14*c^14*d^2 - 560*a^3*b^13*c^13
*d^3 + 1820*a^4*b^12*c^12*d^4 - 4368*a^5*b^11*c^11*d^5 + 8008*a^6*b^10*c^10
*d^6 - 11440*a^7*b^9*c^9*d^7 + 12870*a^8*b^8*c^8*d^8 - 11440*a^9*b^7*c^7*d^
9 + 8008*a^10*b^6*c^6*d^10 - 4368*a^11*b^5*c^5*d^11 + 1820*a^12*b^4*c^4*d^1
2 - 560*a^13*b^3*c^3*d^13 + 120*a^14*b^2*c^2*d^14 - 16*a^15*b*c*d^15 + a^16
*d^16)/(a^3*b^17)) + (b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3
*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6
 - 8*a^7*b*c*d^7 + a^8*d^8)*x^2)/(b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*
d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6
*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8))*(-(b^16*c^16 - 16*a*b^15*c^15*d +
120*a^2*b^14*c^14*d^2 - 560*a^3*b^13*c^13*d^3 + 1820*a^4*b^12*c^12*d^4 - 43
68*a^5*b^11*c^11*d^5 + 8008*a^6*b^10*c^10*d^6 - 11440*a^7*b^9*c^9*d^7 + 128
70*a^8*b^8*c^8*d^8 - 11440*a^9*b^7*c^7*d^9 + 8008*a^10*b^6*c^6*d^10 - 4368*
a^11*b^5*c^5*d^11 + 1820*a^12*b^4*c^4*d^12 - 560*a^13*b^3*c^3*d^13 + 120*a^
14*b^2*c^2*d^14 - 16*a^15*b*c*d^15 + a^16*d^16)/(a^3*b^17))^(3/4))/(b^12*c^
12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^
4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5
*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 1
2*a^11*b*c*d^11 + a^12*d^12)) + 585*b^4*(-(b^16*c^16 - 16*a*b^15*c^15*d + 1
20*a^2*b^14*c^14*d^2 - 560*a^3*b^13*c^13*d^3 + 1820*a^4*b^12*c^12*d^4 - 436
8*a^5*b^11*c^11*d^5 + 8008*a^6*b^10*c^10*d^6 - 11440*a^7*b^9*c^9*d^7 + 1287
0*a^8*b^8*c^8*d^8 - 11440*a^9*b^7*c^7*d^9 + 8008*a^10*b^6*c^6*d^10 - 4368*a
^11*b^5*c^5*d^11 + 1820*a^12*b^4*c^4*d^12 - 560*a^13*b^3*c^3*d^13 + 120*a^1
4*b^2*c^2*d^14 - 16*a^15*b*c*d^15 + a^16*d^16)/(a^3*b^17))^(1/4)*log(a*b^4*
(-(b^16*c^16 - 16*a*b^15*c^15*d + 120*a^2*b^14*c^14*d^2 - 560*a^3*b^13*c^13
*d^3 + 1820*a^4*b^12*c^12*d^4 - 4368*a^5*b^11*c^11*d^5 + 8008*a^6*b^10*c^10
*d^6 - 11440*a^7*b^9*c^9*d^7 + 12870*a^8*b^8*c^8*d^8 - 11440*a^9*b^7*c^7*d^
9 + 8008*a^10*b^6*c^6*d^10 - 4368*a^11*b^5*c^5*d^11 + 1820*a^12*b^4*c^4*d^1
2 - 560*a^13*b^3*c^3*d^13 + 120*a^14*b^2*c^2*d^14 - 16*a^15*b*c*d^15 + a^16
*d^16)/(a^3*b^17))^(1/4) + (b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4
```

```
*a^3*b*c*d^3 + a^4*d^4)*x) - 585*b^4*(-(b^16*c^16 - 16*a*b^15*c^15*d + 120*
a^2*b^14*c^14*d^2 - 560*a^3*b^13*c^13*d^3 + 1820*a^4*b^12*c^12*d^4 - 4368*a
^5*b^11*c^11*d^5 + 8008*a^6*b^10*c^10*d^6 - 11440*a^7*b^9*c^9*d^7 + 12870*a
^8*b^8*c^8*d^8 - 11440*a^9*b^7*c^7*d^9 + 8008*a^10*b^6*c^6*d^10 - 4368*a^11
*b^5*c^5*d^11 + 1820*a^12*b^4*c^4*d^12 - 560*a^13*b^3*c^3*d^13 + 120*a^14*b
^2*c^2*d^14 - 16*a^15*b*c*d^15 + a^16*d^16)/(a^3*b^17))^(1/4)*log(-a*b^4*(-
(b^16*c^16 - 16*a*b^15*c^15*d + 120*a^2*b^14*c^14*d^2 - 560*a^3*b^13*c^13*d
^3 + 1820*a^4*b^12*c^12*d^4 - 4368*a^5*b^11*c^11*d^5 + 8008*a^6*b^10*c^10*d
^6 - 11440*a^7*b^9*c^9*d^7 + 12870*a^8*b^8*c^8*d^8 - 11440*a^9*b^7*c^7*d^9
+ 8008*a^10*b^6*c^6*d^10 - 4368*a^11*b^5*c^5*d^11 + 1820*a^12*b^4*c^4*d^12
- 560*a^13*b^3*c^3*d^13 + 120*a^14*b^2*c^2*d^14 - 16*a^15*b*c*d^15 + a^16*d
^16)/(a^3*b^17))^(1/4) + (b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a
^3*b*c*d^3 + a^4*d^4)*x) + 2340*(4*b^3*c^3*d - 6*a*b^2*c^2*d^2 + 4*a^2*b*c*
d^3 - a^3*d^4)*x)/b^4
```

giac [B] time = 0.48, size = 617, normalized size = 1.86

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^4+c)^4/(b*x^4+a),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(2)*((a*b^3)^(1/4)*b^4*c^4 - 4*(a*b^3)^(1/4)*a*b^3*c^3*d + 6*(a*b^3
)^(1/4)*a^2*b^2*c^2*d^2 - 4*(a*b^3)^(1/4)*a^3*b*c*d^3 + (a*b^3)^(1/4)*a^4*d
^4)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a*b^5) + 1
/4*sqrt(2)*((a*b^3)^(1/4)*b^4*c^4 - 4*(a*b^3)^(1/4)*a*b^3*c^3*d + 6*(a*b^3)
^(1/4)*a^2*b^2*c^2*d^2 - 4*(a*b^3)^(1/4)*a^3*b*c*d^3 + (a*b^3)^(1/4)*a^4*d^
4)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a*b^5) + 1/
8*sqrt(2)*((a*b^3)^(1/4)*b^4*c^4 - 4*(a*b^3)^(1/4)*a*b^3*c^3*d + 6*(a*b^3)^
(1/4)*a^2*b^2*c^2*d^2 - 4*(a*b^3)^(1/4)*a^3*b*c*d^3 + (a*b^3)^(1/4)*a^4*d^4
)*log(x^2 + sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a*b^5) - 1/8*sqrt(2)*((a*b^
3)^(1/4)*b^4*c^4 - 4*(a*b^3)^(1/4)*a*b^3*c^3*d + 6*(a*b^3)^(1/4)*a^2*b^2*c^
2*d^2 - 4*(a*b^3)^(1/4)*a^3*b*c*d^3 + (a*b^3)^(1/4)*a^4*d^4)*log(x^2 - sqrt
(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a*b^5) + 1/585*(45*b^12*d^4*x^13 + 260*b^12
*c*d^3*x^9 - 65*a*b^11*d^4*x^9 + 702*b^12*c^2*d^2*x^5 - 468*a*b^11*c*d^3*x^
5 + 117*a^2*b^10*d^4*x^5 + 2340*b^12*c^3*d*x - 3510*a*b^11*c^2*d^2*x + 2340
*a^2*b^10*c*d^3*x - 585*a^3*b^9*d^4*x)/b^13
```

maple [B] time = 0.05, size = 837, normalized size = 2.52

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^4+c)^4/(b*x^4+a),x)
```

```
[Out] -4/5*d^3/b^2*x^5*a*c+4*d^3/b^3*a^2*c*x-6*d^2/b^2*a*c^2*x+1/4*(a/b)^(1/4)/a*
2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x-1)*c^4+1/8*(a/b)^(1/4)/a*2^(1/2)*ln((x
^2+(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2))/(x^2-(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2
)))*c^4+1/4*(a/b)^(1/4)/a*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x+1)*c^4+1/13*
d^4*x^13/b+3/4/b^2*(a/b)^(1/4)*a*2^(1/2)*ln((x^2+(a/b)^(1/4)*x*2^(1/2)+(a/b
)^(1/2))/(x^2-(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2)))*c^2*d^2-1/b^3*(a/b)^(1/4)
*a^2*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x+1)*c*d^3+3/2/b^2*(a/b)^(1/4)*a*2^
(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x+1)*c^2*d^2-1/b^3*(a/b)^(1/4)*a^2*2^(1/2)
*arctan(2^(1/2)/(a/b)^(1/4)*x-1)*c*d^3+3/2/b^2*(a/b)^(1/4)*a*2^(1/2)*arctan
(2^(1/2)/(a/b)^(1/4)*x-1)*c^2*d^2-1/2/b^3*(a/b)^(1/4)*a^2*2^(1/2)*ln((x^2+(
a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2))/(x^2-(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2)))*
c*d^3+6/5*d^2/b*x^5*c^2-d^4/b^4*a^3*x+4*d/b*c^3*x-1/9*d^4/b^2*x^9*a+4/9*d^3
/b*x^9*c+1/5*d^4/b^3*x^5*a^2+1/4/b^4*(a/b)^(1/4)*a^3*2^(1/2)*arctan(2^(1/2)
/(a/b)^(1/4)*x-1)*d^4-1/b*(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x-
1)*c^3*d+1/8/b^4*(a/b)^(1/4)*a^3*2^(1/2)*ln((x^2+(a/b)^(1/4)*x*2^(1/2)+(a/b
```

```
)^(1/2))/(x^2-(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2)))*d^4-1/2/b*(a/b)^(1/4)*2^(
1/2)*ln((x^2+(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2))/(x^2-(a/b)^(1/4)*x*2^(1/2)+
(a/b)^(1/2)))*c^3*d+1/4/b^4*(a/b)^(1/4)*a^3*2^(1/2)*arctan(2^(1/2)/(a/b)^(1
/4)*x+1)*d^4-1/b*(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x+1)*c^3*d
```

maxima [A] time = 1.44, size = 489, normalized size = 1.47

$$\frac{45\,b^3d^4x^{13}+65\left(4\,b^3cd^3-ab^2d^4\right)x^9+117\left(6\,b^3c^2d^2-4\,ab^2cd^3+a^2bd^4\right)x^5+585\left(4\,b^3c^3d-6\,ab^2c^2d^2+4\,a^2bcd^3-a^3d^4\right)x}{585\,b^4}+\frac{\frac{2\sqrt{2}\left(b^4c^4-4\,ab^3c^3d+6\,a^2b^2c^2d^2-4\,a^3bcd^3+a^4d^4\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}+\frac{2\sqrt{2}\left(b^4c^4-4\,ab^3c^3d+6\,a^2b^2c^2d^2-4\,a^3bcd^3+a^4d^4\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}+\frac{\sqrt{2}\left(b^4c^4-4\,ab^3c^3d+6\,a^2b^2c^2d^2-4\,a^3bcd^3+a^4d^4\right)\log\left(\sqrt{b}x^2+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{1}{4}}}-\frac{\sqrt{2}\left(b^4c^4-4\,ab^3c^3d+6\,a^2b^2c^2d^2-4\,a^3bcd^3+a^4d^4\right)\log\left(\sqrt{b}x^2-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{1}{4}}}}{8\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^4+c)^4/(b*x^4+a),x, algorithm="maxima")
```

```
[Out] 1/585*(45*b^3*d^4*x^13 + 65*(4*b^3*c*d^3 - a*b^2*d^4)*x^9 + 117*(6*b^3*c^2*
d^2 - 4*a*b^2*c*d^3 + a^2*b*d^4)*x^5 + 585*(4*b^3*c^3*d - 6*a*b^2*c^2*d^2 +
 4*a^2*b*c*d^3 - a^3*d^4)*x)/b^4 + 1/8*(2*sqrt(2)*(b^4*c^4 - 4*a*b^3*c^3*d
+ 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*arctan(1/2*sqrt(2)*(2*sqrt(b
)*x + sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)
*sqrt(b))) + 2*sqrt(2)*(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3
*b*c*d^3 + a^4*d^4)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x - sqrt(2)*a^(1/4)*b^(1/
4))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + sqrt(2)*(b^4*c
^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*log(sqrt(
b)*x^2 + sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(3/4)*b^(1/4)) - sqrt(2)*(
b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*log(
sqrt(b)*x^2 - sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(3/4)*b^(1/4)))/b^4
```

mupad [B] time = 1.51, size = 1822, normalized size = 5.49

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^4)^4/(a + b*x^4),x)
```

```
[Out] x*((4*c^3*d)/b - (a*((a*((a*d^4)/b^2 - (4*c*d^3)/b))/b + (6*c^2*d^2)/b))/b)
 - x^9*((a*d^4)/(9*b^2) - (4*c*d^3)/(9*b)) + x^5*((a*((a*d^4)/b^2 - (4*c*d^
3)/b))/(5*b) + (6*c^2*d^2)/(5*b)) + (d^4*x^13)/(13*b) + (atan(((((4*x*(a^8*
d^8 + b^8*c^8 + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^
4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a*b^7*c^7*d - 8*a^7*b*c*d^7
))/b^5 - (4*(a*d - b*c)^4*(a^5*d^4 + a*b^4*c^4 - 4*a^2*b^3*c^3*d + 6*a^3*b^
2*c^2*d^2 - 4*a^4*b*c*d^3))/((-a)^(3/4)*b^(21/4)))*(a*d - b*c)^4*1i)/(4*(-a
)^(3/4)*b^(17/4)) + (((4*x*(a^8*d^8 + b^8*c^8 + 28*a^2*b^6*c^6*d^2 - 56*a^3
*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6
 - 8*a*b^7*c^7*d - 8*a^7*b*c*d^7))/b^5 + (4*(a*d - b*c)^4*(a^5*d^4 + a*b^4*
c^4 - 4*a^2*b^3*c^3*d + 6*a^3*b^2*c^2*d^2 - 4*a^4*b*c*d^3))/((-a)^(3/4)*b^(
21/4)))*(a*d - b*c)^4*1i)/(4*(-a)^(3/4)*b^(17/4)))/((((4*x*(a^8*d^8 + b^8*c
^8 + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*
b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a*b^7*c^7*d - 8*a^7*b*c*d^7))/b^5 - (4
*(a*d - b*c)^4*(a^5*d^4 + a*b^4*c^4 - 4*a^2*b^3*c^3*d + 6*a^3*b^2*c^2*d^2 -
 4*a^4*b*c*d^3))/((-a)^(3/4)*b^(21/4)))*(a*d - b*c)^4)/(4*(-a)^(3/4)*b^(17/
4)) - (((4*x*(a^8*d^8 + b^8*c^8 + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 +
 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a*b^7*c^7
*d - 8*a^7*b*c*d^7))/b^5 + (4*(a*d - b*c)^4*(a^5*d^4 + a*b^4*c^4 - 4*a^2*b^
3*c^3*d + 6*a^3*b^2*c^2*d^2 - 4*a^4*b*c*d^3))/((-a)^(3/4)*b^(21/4)))*(a*d -
 b*c)^4)/(4*(-a)^(3/4)*b^(17/4))))*(a*d - b*c)^4*1i)/(2*(-a)^(3/4)*b^(17/4)
) + (atan(((((4*x*(a^8*d^8 + b^8*c^8 + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*
d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a*b^
7*c^7*d - 8*a^7*b*c*d^7))/b^5 - ((a*d - b*c)^4*(a^5*d^4 + a*b^4*c^4 - 4*a^2
*b^3*c^3*d + 6*a^3*b^2*c^2*d^2 - 4*a^4*b*c*d^3)*4i)/((-a)^(3/4)*b^(21/4)))*
(a*d - b*c)^4)/(4*(-a)^(3/4)*b^(17/4)) + (((4*x*(a^8*d^8 + b^8*c^8 + 28*a^2
*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5
 + 28*a^6*b^2*c^2*d^6 - 8*a*b^7*c^7*d - 8*a^7*b*c*d^7))/b^5 + ((a*d - b*c)^
```

```
4*(a^5*d^4 + a*b^4*c^4 - 4*a^2*b^3*c^3*d + 6*a^3*b^2*c^2*d^2 - 4*a^4*b*c*d^
3)*4i)/((-a)^(3/4)*b^(21/4)))*(a*d - b*c)^4)/(4*(-a)^(3/4)*b^(17/4)))/((((4
*x*(a^8*d^8 + b^8*c^8 + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^
4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a*b^7*c^7*d - 8*a^7
*b*c*d^7))/b^5 - ((a*d - b*c)^4*(a^5*d^4 + a*b^4*c^4 - 4*a^2*b^3*c^3*d + 6*
a^3*b^2*c^2*d^2 - 4*a^4*b*c*d^3)*4i)/((-a)^(3/4)*b^(21/4)))*(a*d - b*c)^4*1
i)/(4*(-a)^(3/4)*b^(17/4)) - (((4*x*(a^8*d^8 + b^8*c^8 + 28*a^2*b^6*c^6*d^2
 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^
2*c^2*d^6 - 8*a*b^7*c^7*d - 8*a^7*b*c*d^7))/b^5 + ((a*d - b*c)^4*(a^5*d^4 +
 a*b^4*c^4 - 4*a^2*b^3*c^3*d + 6*a^3*b^2*c^2*d^2 - 4*a^4*b*c*d^3)*4i)/((-a)
^(3/4)*b^(21/4)))*(a*d - b*c)^4*1i)/(4*(-a)^(3/4)*b^(17/4))))*(a*d - b*c)^4
)/(2*(-a)^(3/4)*b^(17/4))
```

sympy [A] time = 3.59, size = 435, normalized size = 1.31

$$x^9\left(-\frac{ad^4}{9b^2}+\frac{4cd^3}{9b}\right)+x^5\left(\frac{a^2d^4}{5b^3}-\frac{4acd^3}{5b^2}+\frac{6c^2d^2}{5b}\right)+x\left(-\frac{a^3d^4}{b^4}+\frac{4a^2cd^3}{b^3}-\frac{6ac^2d^2}{b^2}+\frac{4c^3d}{b}\right)+\operatorname{RootSum}\left(256t^4a^3b^{17}+a^{16}d^{16}-16a^{15}bcd^{15}+120a^{14}b^2c^2d^{14}-560a^{13}b^3c^3d^{13}+1820a^{12}b^4c^4d^{12}-4368a^{11}b^5c^5d^{11}+8008a^{10}b^6c^6d^{10}-11440a^9b^7c^7d^9+12870a^8b^8c^8d^8-11440a^7b^9c^9d^7+8008a^6b^{10}c^{10}d^6-4368a^5b^{11}c^{11}d^5+1820a^4b^{12}c^{12}d^4-560a^3b^{13}c^{13}d^3+120a^2b^{14}c^{14}d^2-16ab^{15}c^{15}d+b^{16}c^{16},\left(t\mapsto t\log\left(\frac{4tab^4}{a^4d^4-4a^3bcd^3+6a^2b^2c^2d^2-4ab^3c^3d+b^4c^4}+x\right)\right)\right)+\frac{d^4x^{13}}{13b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**4+c)**4/(b*x**4+a),x)
```

```
[Out] x**9*(-a*d**4/(9*b**2) + 4*c*d**3/(9*b)) + x**5*(a**2*d**4/(5*b**3) - 4*a*c
*d**3/(5*b**2) + 6*c**2*d**2/(5*b)) + x*(-a**3*d**4/b**4 + 4*a**2*c*d**3/b*
*3 - 6*a*c**2*d**2/b**2 + 4*c**3*d/b) + RootSum(256*_t**4*a**3*b**17 + a**1
6*d**16 - 16*a**15*b*c*d**15 + 120*a**14*b**2*c**2*d**14 - 560*a**13*b**3*c
**3*d**13 + 1820*a**12*b**4*c**4*d**12 - 4368*a**11*b**5*c**5*d**11 + 8008*
a**10*b**6*c**6*d**10 - 11440*a**9*b**7*c**7*d**9 + 12870*a**8*b**8*c**8*d*
*8 - 11440*a**7*b**9*c**9*d**7 + 8008*a**6*b**10*c**10*d**6 - 4368*a**5*b**
11*c**11*d**5 + 1820*a**4*b**12*c**12*d**4 - 560*a**3*b**13*c**13*d**3 + 12
0*a**2*b**14*c**14*d**2 - 16*a*b**15*c**15*d + b**16*c**16, Lambda(_t, _t*l
og(4*_t*a*b**4/(a**4*d**4 - 4*a**3*b*c*d**3 + 6*a**2*b**2*c**2*d**2 - 4*a*b
**3*c**3*d + b**4*c**4) + x))) + d**4*x**13/(13*b)
```

3.98 $\int \frac{(c+dx^4)^3}{a+bx^4}\,dx$

Optimal. Leaf size=288

$$-\frac{(bc-ad)^3\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}b^{13/4}}+\frac{(bc-ad)^3\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}b^{13/4}}-\frac{(bc-ad)^3\tan^{-1}\left(1-\right.}{2\sqrt{2}a^{3/4}b^{13/4}}$$

Rubi [A] time = 0.22, antiderivative size = 288, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {390, 211, 1165, 628, 1162, 617, 204}

$$\frac{dx\left(a^2d^2-3abcd+3b^2c^2\right)}{b^3}-\frac{(bc-ad)^3\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}b^{13/4}}+\frac{(bc-ad)^3\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}b^{13/4}}-\frac{(bc-ad)^3\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}b^{13/4}}+\frac{(bc-ad)^3\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{2\sqrt{2}a^{3/4}b^{13/4}}+\frac{d^2x^5(3bc-ad)}{5b^2}+\frac{d^3x^9}{9b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^4)^3/(a + b*x^4),x]
```

```
[Out] (d*(3*b^2*c^2 - 3*a*b*c*d + a^2*d^2)*x)/b^3 + (d^2*(3*b*c - a*d)*x^5)/(5*b^
2) + (d^3*x^9)/(9*b) - ((b*c - a*d)^3*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4
)])/(2*Sqrt[2]*a^(3/4)*b^(13/4)) + ((b*c - a*d)^3*ArcTan[1 + (Sqrt[2]*b^(1/
4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*b^(13/4)) - ((b*c - a*d)^3*Log[Sqrt[a] -
 Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(3/4)*b^(13/4)) + (
(b*c - a*d)^3*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sq
rt[2]*a^(3/4)*b^(13/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{\left(c+dx^4\right)^3}{a+bx^4}\,dx = \int \left(\frac{d\left(3b^2c^2-3abcd+a^2d^2\right)}{b^3}+\frac{d^2(3bc-ad)x^4}{b^2}+\frac{d^3x^8}{b}+\frac{b^3c^3-3ab^2c^2d+3a^2bcd^2-a^3d^3}{b^3\left(a+bx^4\right)}\right.$$

$$=\frac{d\left(3b^2c^2-3abcd+a^2d^2\right)x}{b^3}+\frac{d^2(3bc-ad)x^5}{5b^2}+\frac{d^3x^9}{9b}+\frac{(bc-ad)^3\int\frac{1}{a+bx^4}\,dx}{b^3}$$

$$=\frac{d\left(3b^2c^2-3abcd+a^2d^2\right)x}{b^3}+\frac{d^2(3bc-ad)x^5}{5b^2}+\frac{d^3x^9}{9b}+\frac{(bc-ad)^3\int\frac{\sqrt{a}-\sqrt{b}\,x^2}{a+bx^4}\,dx}{2\sqrt{a}\,b^3}+\frac{(bc-a}{}$$

$$=\frac{d\left(3b^2c^2-3abcd+a^2d^2\right)x}{b^3}+\frac{d^2(3bc-ad)x^5}{5b^2}+\frac{d^3x^9}{9b}+\frac{(bc-ad)^3\int\frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{b}}+x^2}\,dx}{4\sqrt{a}\,b^{7/2}}+\frac{(b}{}$$

$$=\frac{d\left(3b^2c^2-3abcd+a^2d^2\right)x}{b^3}+\frac{d^2(3bc-ad)x^5}{5b^2}+\frac{d^3x^9}{9b}-\frac{(bc-ad)^3\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\right.}{4\sqrt{2}\,a^{3/4}b^{13/4}}$$

$$=\frac{d\left(3b^2c^2-3abcd+a^2d^2\right)x}{b^3}+\frac{d^2(3bc-ad)x^5}{5b^2}+\frac{d^3x^9}{9b}-\frac{(bc-ad)^3\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,a^{3/4}b^{13/4}}+\frac{(b}{}$$

Mathematica [A] time = 0.17, size = 271, normalized size = 0.94

$$\frac{-72a^{3/4}b^{5/4}d^2x^5(ad-3bc)+40a^{3/4}b^{9/4}d^3x^9+360a^{3/4}\sqrt[4]{b}\,dx\left(a^2d^2-3abcd+3b^2c^2\right)-45\sqrt{2}(bc-ad)^3\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)+45\sqrt{2}(bc-ad)^3\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)-90\sqrt{2}(bc-ad)^3\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)+90\sqrt{2}(bc-ad)^3\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}+1\right)}{360a^{3/4}b^{13/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^4)^3/(a + b*x^4),x]
```

```
[Out] (360*a^(3/4)*b^(1/4)*d*(3*b^2*c^2 - 3*a*b*c*d + a^2*d^2)*x - 72*a^(3/4)*b^(
5/4)*d^2*(-3*b*c + a*d)*x^5 + 40*a^(3/4)*b^(9/4)*d^3*x^9 - 90*Sqrt[2]*(b*c
- a*d)^3*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)] + 90*Sqrt[2]*(b*c - a*d)^3
*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)] - 45*Sqrt[2]*(b*c - a*d)^3*Log[Sqr
t[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2] + 45*Sqrt[2]*(b*c - a*d)^3*
Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(360*a^(3/4)*b^(13/
4))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c+dx^4\right)^3}{a+bx^4}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x^4)^3/(a + b*x^4),x]
```

```
[Out] IntegrateAlgebraic[(c + d*x^4)^3/(a + b*x^4), x]
```

fricas [B] time = 1.48, size = 1855, normalized size = 6.44

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^4+c)^3/(b*x^4+a),x, algorithm="fricas")
```

```
[Out] 1/180*(20*b^2*d^3*x^9 + 36*(3*b^2*c*d^2 - a*b*d^3)*x^5 - 180*b^3*(-(b^12*c^
12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^
4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5
*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 1
2*a^11*b*c*d^11 + a^12*d^12)/(a^3*b^13))^(1/4)*arctan((a^2*b^10*x*(-(b^12*c
^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a
^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^
5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 -
12*a^11*b*c*d^11 + a^12*d^12)/(a^3*b^13))^(3/4) - a^2*b^10*sqrt((a^2*b^6*sq
rt(-(b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*
d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792
*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*
c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(a^3*b^13)) + (b^6*c^6 - 6*a*b^5*c
^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5
*b*c*d^5 + a^6*d^6)*x^2)/(b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20
*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6))*(-(b^12*c
^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a
^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^
5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 -
12*a^11*b*c*d^11 + a^12*d^12)/(a^3*b^13))^(3/4))/(b^9*c^9 - 9*a*b^8*c^8*d +
 36*a^2*b^7*c^7*d^2 - 84*a^3*b^6*c^6*d^3 + 126*a^4*b^5*c^5*d^4 - 126*a^5*b^
4*c^4*d^5 + 84*a^6*b^3*c^3*d^6 - 36*a^7*b^2*c^2*d^7 + 9*a^8*b*c*d^8 - a^9*d
^9)) - 45*b^3*(-(b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*
a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c
^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 +
66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(a^3*b^13))^(1/4)*log(
a*b^3*(-(b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*
c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 -
 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*
b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(a^3*b^13))^(1/4) - (b^3*c^3 -
 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*x) + 45*b^3*(-(b^12*c^12 - 12*a*b
^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d
^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*
a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*
d^11 + a^12*d^12)/(a^3*b^13))^(1/4)*log(-a*b^3*(-(b^12*c^12 - 12*a*b^11*c^1
1*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 79
2*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4
*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 +
a^12*d^12)/(a^3*b^13))^(1/4) - (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a
^3*d^3)*x) + 180*(3*b^2*c^2*d - 3*a*b*c*d^2 + a^2*d^3)*x)/b^3
```

giac [B] time = 0.17, size = 481, normalized size = 1.67

$$\frac{\sqrt{2}\left(\left(ab^3\right)^{\frac{1}{4}}b^3c^3-3\left(ab^3\right)^{\frac{1}{4}}ab^2c^2d+3\left(ab^3\right)^{\frac{1}{4}}a^2bcd^2-\left(ab^3\right)^{\frac{1}{4}}a^3d^3\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4ab^4}+\frac{\sqrt{2}\left(\left(ab^3\right)^{\frac{1}{4}}b^3c^3-3\left(ab^3\right)^{\frac{1}{4}}ab^2c^2d+3\left(ab^3\right)^{\frac{1}{4}}a^2bcd^2-\left(ab^3\right)^{\frac{1}{4}}a^3d^3\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4ab^4}+\frac{\sqrt{2}\left(\left(ab^3\right)^{\frac{1}{4}}b^3c^3-3\left(ab^3\right)^{\frac{1}{4}}ab^2c^2d+3\left(ab^3\right)^{\frac{1}{4}}a^2bcd^2-\left(ab^3\right)^{\frac{1}{4}}a^3d^3\right)\log\left(x^2+\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{8ab^4}-\frac{\sqrt{2}\left(\left(ab^3\right)^{\frac{1}{4}}b^3c^3-3\left(ab^3\right)^{\frac{1}{4}}ab^2c^2d+3\left(ab^3\right)^{\frac{1}{4}}a^2bcd^2-\left(ab^3\right)^{\frac{1}{4}}a^3d^3\right)\log\left(x^2-\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{8ab^4}+\frac{5b^8d^3x^9+27b^8cd^2x^5-9ab^7d^3x^5+135b^8c^2dx-135ab^7cd^2x+45a^2b^6d^3x}{45b^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^4+c)^3/(b*x^4+a),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(2)*((a*b^3)^(1/4)*b^3*c^3 - 3*(a*b^3)^(1/4)*a*b^2*c^2*d + 3*(a*b^3
)^(1/4)*a^2*b*c*d^2 - (a*b^3)^(1/4)*a^3*d^3)*arctan(1/2*sqrt(2)*(2*x + sqrt
(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a*b^4) + 1/4*sqrt(2)*((a*b^3)^(1/4)*b^3*c^3
- 3*(a*b^3)^(1/4)*a*b^2*c^2*d + 3*(a*b^3)^(1/4)*a^2*b*c*d^2 - (a*b^3)^(1/4)
*a^3*d^3)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a*b^
4) + 1/8*sqrt(2)*((a*b^3)^(1/4)*b^3*c^3 - 3*(a*b^3)^(1/4)*a*b^2*c^2*d + 3*(
a*b^3)^(1/4)*a^2*b*c*d^2 - (a*b^3)^(1/4)*a^3*d^3)*log(x^2 + sqrt(2)*x*(a/b)
^(1/4) + sqrt(a/b))/(a*b^4) - 1/8*sqrt(2)*((a*b^3)^(1/4)*b^3*c^3 - 3*(a*b^3
)^(1/4)*a*b^2*c^2*d + 3*(a*b^3)^(1/4)*a^2*b*c*d^2 - (a*b^3)^(1/4)*a^3*d^3)*
log(x^2 - sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a*b^4) + 1/45*(5*b^8*d^3*x^9
+ 27*b^8*c*d^2*x^5 - 9*a*b^7*d^3*x^5 + 135*b^8*c^2*d*x - 135*a*b^7*c*d^2*x
+ 45*a^2*b^6*d^3*x)/b^9
```

maple [B] time = 0.05, size = 627, normalized size = 2.18

$$\frac{d^3x^9}{9b}-\frac{a\,d^3x^5}{5b^2}+\frac{3c\,d^2x^5}{5b}+\frac{a^2d^3x}{b^3}-\frac{\left(\frac{a}{b}\right)^{\frac14}\sqrt{2}\,a^2d^3\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac14}}-1\right)}{4b^3}-\frac{\left(\frac{a}{b}\right)^{\frac14}\sqrt{2}\,a^2d^3\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac14}}+1\right)}{4b^3}-\frac{\left(\frac{a}{b}\right)^{\frac14}\sqrt{2}\,a^2d^3\ln\left(\frac{x^2+\left(\frac{a}{b}\right)^{\frac14}\sqrt{2}\,x+\sqrt{\frac{a}{b}}}{x^2-\left(\frac{a}{b}\right)^{\frac14}\sqrt{2}\,x+\sqrt{\frac{a}{b}}}\right)}{8b^3}-\frac{3ac\,d^2x}{b^2}+\frac{3\left(\frac{a}{b}\right)^{\frac14}\sqrt{2}\,ac\,d^2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac14}}-1\right)}{4b^2}+\frac{3\left(\frac{a}{b}\right)^{\frac14}\sqrt{2}\,ac\,d^2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac14}}+1\right)}{4b^2}+\frac{3\left(\frac{a}{b}\right)^{\frac14}\sqrt{2}\,ac\,d^2\ln\left(\frac{x^2+\left(\frac{a}{b}\right)^{\frac14}\sqrt{2}\,x+\sqrt{\frac{a}{b}}}{x^2-\left(\frac{a}{b}\right)^{\frac14}\sqrt{2}\,x+\sqrt{\frac{a}{b}}}\right)}{8b^2}+\frac{\left(\frac{a}{b}\right)^{\frac14}\sqrt{2}\,c^3\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac14}}-1\right)}{4a}+\frac{\left(\frac{a}{b}\right)^{\frac14}\sqrt{2}\,c^3\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac14}}+1\right)}{4a}+\frac{\left(\frac{a}{b}\right)^{\frac14}\sqrt{2}\,c^3\ln\left(\frac{x^2+\left(\frac{a}{b}\right)^{\frac14}\sqrt{2}\,x+\sqrt{\frac{a}{b}}}{x^2-\left(\frac{a}{b}\right)^{\frac14}\sqrt{2}\,x+\sqrt{\frac{a}{b}}}\right)}{8a}+\frac{3c^2dx}{b}-\frac{3\left(\frac{a}{b}\right)^{\frac14}\sqrt{2}\,c^2d\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac14}}-1\right)}{4b}-\frac{3\left(\frac{a}{b}\right)^{\frac14}\sqrt{2}\,c^2d\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac14}}+1\right)}{4b}-\frac{3\left(\frac{a}{b}\right)^{\frac14}\sqrt{2}\,c^2d\ln\left(\frac{x^2+\left(\frac{a}{b}\right)^{\frac14}\sqrt{2}\,x+\sqrt{\frac{a}{b}}}{x^2-\left(\frac{a}{b}\right)^{\frac14}\sqrt{2}\,x+\sqrt{\frac{a}{b}}}\right)}{8b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^4+c)^3/(b*x^4+a),x)
```

```
[Out] 1/9*d^3*x^9/b-1/5*d^3/b^2*x^5*a+3/5*d^2/b*x^5*c+d^3/b^3*a^2*x-3*d^2/b^2*a*c
*x+3*d/b*c^2*x-1/8/b^3*(a/b)^(1/4)*a^2*2^(1/2)*ln((x^2+(a/b)^(1/4)*2^(1/2)*
x+(a/b)^(1/2))/(x^2-(a/b)^(1/4)*2^(1/2)*x+(a/b)^(1/2)))*d^3+3/8/b^2*(a/b)^(
1/4)*a*2^(1/2)*ln((x^2+(a/b)^(1/4)*2^(1/2)*x+(a/b)^(1/2))/(x^2-(a/b)^(1/4)*
2^(1/2)*x+(a/b)^(1/2)))*c*d^2-3/8/b*(a/b)^(1/4)*2^(1/2)*ln((x^2+(a/b)^(1/4)
*2^(1/2)*x+(a/b)^(1/2))/(x^2-(a/b)^(1/4)*2^(1/2)*x+(a/b)^(1/2)))*c^2*d+1/8*
(a/b)^(1/4)/a*2^(1/2)*ln((x^2+(a/b)^(1/4)*2^(1/2)*x+(a/b)^(1/2))/(x^2-(a/b)
^(1/4)*2^(1/2)*x+(a/b)^(1/2)))*c^3-1/4/b^3*(a/b)^(1/4)*a^2*2^(1/2)*arctan(2
^(1/2)/(a/b)^(1/4)*x-1)*d^3+3/4/b^2*(a/b)^(1/4)*a*2^(1/2)*arctan(2^(1/2)/(a
/b)^(1/4)*x-1)*c*d^2-3/4/b*(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x
-1)*c^2*d+1/4*(a/b)^(1/4)/a*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x-1)*c^3-1/4
/b^3*(a/b)^(1/4)*a^2*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x+1)*d^3+3/4/b^2*(a
/b)^(1/4)*a*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x+1)*c*d^2-3/4/b*(a/b)^(1/4)
*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x+1)*c^2*d+1/4*(a/b)^(1/4)/a*2^(1/2)*ar
ctan(2^(1/2)/(a/b)^(1/4)*x+1)*c^3
```

maxima [A] time = 1.21, size = 385, normalized size = 1.34

$$\frac{5\,b^2d^3x^9+9\left(3\,b^2cd^2-abd^3\right)x^5+45\left(3\,b^2c^2d-3\,abcd^2+a^2d^3\right)x}{45\,b^3}+\frac{\frac{2\sqrt{2}\left(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}\,x+\sqrt{2}\,a^{\frac14}b^{\frac14}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}+\frac{2\sqrt{2}\left(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}\,x-\sqrt{2}\,a^{\frac14}b^{\frac14}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}+\frac{\sqrt{2}\left(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3\right)\log\left(\sqrt{b}\,x^2+\sqrt{2}\,a^{\frac14}b^{\frac14}x+\sqrt{a}\right)}{a^{\frac34}b^{\frac14}}-\frac{\sqrt{2}\left(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3\right)\log\left(\sqrt{b}\,x^2-\sqrt{2}\,a^{\frac14}b^{\frac14}x+\sqrt{a}\right)}{a^{\frac34}b^{\frac14}}}{8\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^4+c)^3/(b*x^4+a),x, algorithm="maxima")
```

```
[Out] 1/45*(5*b^2*d^3*x^9 + 9*(3*b^2*c*d^2 - a*b*d^3)*x^5 + 45*(3*b^2*c^2*d - 3*a
*b*c*d^2 + a^2*d^3)*x)/b^3 + 1/8*(2*sqrt(2)*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^
2*b*c*d^2 - a^3*d^3)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x + sqrt(2)*a^(1/4)*b^(1
/4))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + 2*sqrt(2)*(b^
3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*arctan(1/2*sqrt(2)*(2*sqrt
(b)*x - sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(
a)*sqrt(b))) + sqrt(2)*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*
log(sqrt(b)*x^2 + sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(3/4)*b^(1/4)) -
sqrt(2)*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*log(sqrt(b)*x^2
 - sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(3/4)*b^(1/4)))/b^3
```

mupad [B] time = 1.49, size = 1433, normalized size = 4.98

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^4)^3/(a + b*x^4),x)
```

```
[Out] x*((3*c^2*d)/b + (a*((a*d^3)/b^2 - (3*c*d^2)/b))/b) - x^5*((a*d^3)/(5*b^2)
- (3*c*d^2)/(5*b)) + (d^3*x^9)/(9*b) - (atan(((((x*(a^6*d^6 + b^6*c^6 + 15*
a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a*b^5*c^5*d -
 6*a^5*b*c*d^5))/b^3 - ((a*d - b*c)^3*(4*a^4*d^3 - 4*a*b^3*c^3 + 12*a^2*b^2
*c^2*d - 12*a^3*b*c*d^2))/(4*(-a)^(3/4)*b^(13/4)))*(a*d - b*c)^3*1i)/((-a)^
(3/4)*b^(13/4)) + (((x*(a^6*d^6 + b^6*c^6 + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3
*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a*b^5*c^5*d - 6*a^5*b*c*d^5))/b^3 + ((a*d
 - b*c)^3*(4*a^4*d^3 - 4*a*b^3*c^3 + 12*a^2*b^2*c^2*d - 12*a^3*b*c*d^2))/(4
*(-a)^(3/4)*b^(13/4)))*(a*d - b*c)^3*1i)/((-a)^(3/4)*b^(13/4)))/((((x*(a^6*
d^6 + b^6*c^6 + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^
4 - 6*a*b^5*c^5*d - 6*a^5*b*c*d^5))/b^3 - ((a*d - b*c)^3*(4*a^4*d^3 - 4*a*b
^3*c^3 + 12*a^2*b^2*c^2*d - 12*a^3*b*c*d^2))/(4*(-a)^(3/4)*b^(13/4)))*(a*d
- b*c)^3)/((-a)^(3/4)*b^(13/4)) - (((x*(a^6*d^6 + b^6*c^6 + 15*a^2*b^4*c^4*
d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a*b^5*c^5*d - 6*a^5*b*c*d
^5))/b^3 + ((a*d - b*c)^3*(4*a^4*d^3 - 4*a*b^3*c^3 + 12*a^2*b^2*c^2*d - 12*
a^3*b*c*d^2))/(4*(-a)^(3/4)*b^(13/4)))*(a*d - b*c)^3)/((-a)^(3/4)*b^(13/4))
))*(a*d - b*c)^3*1i)/(2*(-a)^(3/4)*b^(13/4)) - (atan(((((x*(a^6*d^6 + b^6*c
^6 + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a*b^5
*c^5*d - 6*a^5*b*c*d^5))/b^3 - ((a*d - b*c)^3*(4*a^4*d^3 - 4*a*b^3*c^3 + 12
*a^2*b^2*c^2*d - 12*a^3*b*c*d^2)*1i)/(4*(-a)^(3/4)*b^(13/4)))*(a*d - b*c)^3
)/((-a)^(3/4)*b^(13/4)) + (((x*(a^6*d^6 + b^6*c^6 + 15*a^2*b^4*c^4*d^2 - 20
*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a*b^5*c^5*d - 6*a^5*b*c*d^5))/b^3
 + ((a*d - b*c)^3*(4*a^4*d^3 - 4*a*b^3*c^3 + 12*a^2*b^2*c^2*d - 12*a^3*b*c*
d^2)*1i)/(4*(-a)^(3/4)*b^(13/4)))*(a*d - b*c)^3)/((-a)^(3/4)*b^(13/4)))/(((
(x*(a^6*d^6 + b^6*c^6 + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^
2*c^2*d^4 - 6*a*b^5*c^5*d - 6*a^5*b*c*d^5))/b^3 - ((a*d - b*c)^3*(4*a^4*d^3
 - 4*a*b^3*c^3 + 12*a^2*b^2*c^2*d - 12*a^3*b*c*d^2)*1i)/(4*(-a)^(3/4)*b^(13
/4)))*(a*d - b*c)^3*1i)/((-a)^(3/4)*b^(13/4)) - (((x*(a^6*d^6 + b^6*c^6 + 1
5*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a*b^5*c^5*d
 - 6*a^5*b*c*d^5))/b^3 + ((a*d - b*c)^3*(4*a^4*d^3 - 4*a*b^3*c^3 + 12*a^2*b
^2*c^2*d - 12*a^3*b*c*d^2)*1i)/(4*(-a)^(3/4)*b^(13/4)))*(a*d - b*c)^3*1i)/(
(-a)^(3/4)*b^(13/4))))*(a*d - b*c)^3)/(2*(-a)^(3/4)*b^(13/4))
```

sympy [A] time = 1.70, size = 303, normalized size = 1.05

$$x^5\left(-\frac{ad^3}{5b^2}+\frac{3cd^2}{5b}\right)+x\left(\frac{a^2d^3}{b^3}-\frac{3acd^2}{b^2}+\frac{3c^2d}{b}\right)+\operatorname{RootSum}\left(256t^4a^3b^{13}+a^{12}d^{12}-12a^{11}bcd^{11}+66a^{10}b^2c^2d^{10}-220a^9b^3c^3d^9+495a^8b^4c^4d^8-792a^7b^5c^5d^7+924a^6b^6c^6d^6-792a^5b^7c^7d^5+495a^4b^8c^8d^4-220a^3b^9c^9d^3+66a^2b^{10}c^{10}d^2-12ab^{11}c^{11}d+b^{12}c^{12},\left(t\mapsto t\log\left(-\frac{4tab^3}{a^3d^3-3a^2bcd^2+3ab^2c^2d-b^3c^3}+x\right)\right)\right)+\frac{d^3x^9}{9b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**4+c)**3/(b*x**4+a),x)
```

```
[Out] x**5*(-a*d**3/(5*b**2) + 3*c*d**2/(5*b)) + x*(a**2*d**3/b**3 - 3*a*c*d**2/b
**2 + 3*c**2*d/b) + RootSum(256*_t**4*a**3*b**13 + a**12*d**12 - 12*a**11*b
*c*d**11 + 66*a**10*b**2*c**2*d**10 - 220*a**9*b**3*c**3*d**9 + 495*a**8*b*
*4*c**4*d**8 - 792*a**7*b**5*c**5*d**7 + 924*a**6*b**6*c**6*d**6 - 792*a**5
*b**7*c**7*d**5 + 495*a**4*b**8*c**8*d**4 - 220*a**3*b**9*c**9*d**3 + 66*a*
*2*b**10*c**10*d**2 - 12*a*b**11*c**11*d + b**12*c**12, Lambda(_t, _t*log(-
4*_t*a*b**3/(a**3*d**3 - 3*a**2*b*c*d**2 + 3*a*b**2*c**2*d - b**3*c**3) + x
))) + d**3*x**9/(9*b)
```

3.99 $\int \frac{(c+dx^4)^2}{a+bx^4}\,dx$

Optimal. Leaf size=253

$$-\frac{(bc-ad)^2\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}b^{9/4}}+\frac{(bc-ad)^2\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}b^{9/4}}-\frac{(bc-ad)^2\tan^{-1}}{2\sqrt{2}a^{3/4}}$$

Rubi [A] time = 0.19, antiderivative size = 253, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {390, 211, 1165, 628, 1162, 617, 204}

$$-\frac{(bc-ad)^2\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}b^{9/4}}+\frac{(bc-ad)^2\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}b^{9/4}}-\frac{(bc-ad)^2\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}b^{9/4}}+\frac{(bc-ad)^2\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{2\sqrt{2}a^{3/4}b^{9/4}}+\frac{dx(2bc-ad)}{b^2}+\frac{d^2x^5}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^4)^2/(a + b*x^4),x]
```

```
[Out] (d*(2*b*c - a*d)*x)/b^2 + (d^2*x^5)/(5*b) - ((b*c - a*d)^2*ArcTan[1 - (Sqrt
[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*b^(9/4)) + ((b*c - a*d)^2*ArcTa
n[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*b^(9/4)) - ((b*c - a
*d)^2*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^
(3/4)*b^(9/4)) + ((b*c - a*d)^2*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + S
qrt[b]*x^2])/(4*Sqrt[2]*a^(3/4)*b^(9/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{\left(c+dx^4\right)^2}{a+bx^4}\,dx = \int \left(\frac{d(2bc-ad)}{b^2}+\frac{d^2x^4}{b}+\frac{b^2c^2-2abcd+a^2d^2}{b^2\left(a+bx^4\right)}\right)dx$$
$$= \frac{d(2bc-ad)x}{b^2}+\frac{d^2x^5}{5b}+\frac{(bc-ad)^2\int \frac{1}{a+bx^4}\,dx}{b^2}$$
$$= \frac{d(2bc-ad)x}{b^2}+\frac{d^2x^5}{5b}+\frac{(bc-ad)^2\int \frac{\sqrt{a}-\sqrt{b}x^2}{a+bx^4}\,dx}{2\sqrt{a}\,b^2}+\frac{(bc-ad)^2\int \frac{\sqrt{a}+\sqrt{b}x^2}{a+bx^4}\,dx}{2\sqrt{a}\,b^2}$$
$$= \frac{d(2bc-ad)x}{b^2}+\frac{d^2x^5}{5b}+\frac{(bc-ad)^2\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx}{4\sqrt{a}\,b^{5/2}}+\frac{(bc-ad)^2\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx}{4\sqrt{a}\,b^{5/2}}-\frac{(bc-}{}$$
$$= \frac{d(2bc-ad)x}{b^2}+\frac{d^2x^5}{5b}-\frac{(bc-ad)^2\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{4\sqrt{2}\,a^{3/4}b^{9/4}}+\frac{(bc-ad)^2\log\left(\sqrt{a}+\sqrt{}\right.}{4\sqrt{2}\,a^{3/}}$$
$$= \frac{d(2bc-ad)x}{b^2}+\frac{d^2x^5}{5b}-\frac{(bc-ad)^2\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,a^{3/4}b^{9/4}}+\frac{(bc-ad)^2\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,a^{3/4}b^{9/4}}-\frac{(bc-}{}$$

Mathematica [A] time = 0.13, size = 231, normalized size = 0.91

$$\frac{8a^{3/4}b^{5/4}d^2x^5-40a^{3/4}\sqrt[4]{b}\,dx(ad-2bc)-5\sqrt{2}(bc-ad)^2\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)+5\sqrt{2}(bc-ad)^2\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)-10\sqrt{2}(bc-ad)^2\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)+10\sqrt{2}(bc-ad)^2\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{40a^{3/4}b^{9/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^4)^2/(a + b*x^4),x]
```

```
[Out] (-40*a^(3/4)*b^(1/4)*d*(-2*b*c + a*d)*x + 8*a^(3/4)*b^(5/4)*d^2*x^5 - 10*Sq
rt[2]*(b*c - a*d)^2*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)] + 10*Sqrt[2]*(b
*c - a*d)^2*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)] - 5*Sqrt[2]*(b*c - a*d)
^2*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2] + 5*Sqrt[2]*(b*c
- a*d)^2*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(40*a^(3/4
)*b^(9/4))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c+dx^4\right)^2}{a+bx^4}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x^4)^2/(a + b*x^4),x]
```

```
[Out] IntegrateAlgebraic[(c + d*x^4)^2/(a + b*x^4), x]
```

fricas [B] time = 1.43, size = 1240, normalized size = 4.90

$$\frac{4\,bd^2x^5+20\,b^2\left(-\frac{b^8c^8-8\,ab^7c^7d+28\,a^2b^6c^6d^2-56\,a^3b^5c^5d^3+70\,a^4b^4c^4d^4-56\,a^5b^3c^3d^5+28\,a^6b^2c^2d^6-8\,a^7bcd^7+a^8d^8}{a^3b^9}\right)^{\frac{1}{4}}\arctan\left(-\frac{a^2b^7x\left(-\frac{b^8c^8-8\,ab^7c^7d+28\,a^2b^6c^6d^2-56\,a^3b^5c^5d^3+70\,a^4b^4c^4d^4-56\,a^5b^3c^3d^5+28\,a^6b^2c^2d^6-8\,a^7bcd^7+a^8d^8}{a^3b^9}\right)^{\frac{3}{4}}-a^2b^7\sqrt{\frac{a^2b^4\sqrt{-\frac{b^8c^8-8\,ab^7c^7d+28\,a^2b^6c^6d^2-56\,a^3b^5c^5d^3+70\,a^4b^4c^4d^4-56\,a^5b^3c^3d^5+28\,a^6b^2c^2d^6-8\,a^7bcd^7+a^8d^8}{a^3b^9}}+\left(b^4c^4-4\,ab^3c^3d+6\,a^2b^2c^2d^2-4\,a^3bcd^3+a^4d^4\right)x^2}{b^4c^4-4\,ab^3c^3d+6\,a^2b^2c^2d^2-4\,a^3bcd^3+a^4d^4}}\left(-\frac{b^8c^8-8\,ab^7c^7d+28\,a^2b^6c^6d^2-56\,a^3b^5c^5d^3+70\,a^4b^4c^4d^4-56\,a^5b^3c^3d^5+28\,a^6b^2c^2d^6-8\,a^7bcd^7+a^8d^8}{a^3b^9}\right)^{\frac{3}{4}}}{b^6c^6-6\,ab^5c^5d+15\,a^2b^4c^4d^2-20\,a^3b^3c^3d^3+15\,a^4b^2c^2d^4-6\,a^5bcd^5+a^6d^6}\right)+5\,b^2\left(-\frac{b^8c^8-8\,ab^7c^7d+28\,a^2b^6c^6d^2-56\,a^3b^5c^5d^3+70\,a^4b^4c^4d^4-56\,a^5b^3c^3d^5+28\,a^6b^2c^2d^6-8\,a^7bcd^7+a^8d^8}{a^3b^9}\right)^{\frac{1}{4}}\log\left(ab^2\left(-\frac{b^8c^8-8\,ab^7c^7d+28\,a^2b^6c^6d^2-56\,a^3b^5c^5d^3+70\,a^4b^4c^4d^4-56\,a^5b^3c^3d^5+28\,a^6b^2c^2d^6-8\,a^7bcd^7+a^8d^8}{a^3b^9}\right)^{\frac{1}{4}}+\left(b^2c^2-2\,abcd+a^2d^2\right)x\right)-5\,b^2\left(-\frac{b^8c^8-8\,ab^7c^7d+28\,a^2b^6c^6d^2-56\,a^3b^5c^5d^3+70\,a^4b^4c^4d^4-56\,a^5b^3c^3d^5+28\,a^6b^2c^2d^6-8\,a^7bcd^7+a^8d^8}{a^3b^9}\right)^{\frac{1}{4}}\log\left(-ab^2\left(-\frac{b^8c^8-8\,ab^7c^7d+28\,a^2b^6c^6d^2-56\,a^3b^5c^5d^3+70\,a^4b^4c^4d^4-56\,a^5b^3c^3d^5+28\,a^6b^2c^2d^6-8\,a^7bcd^7+a^8d^8}{a^3b^9}\right)^{\frac{1}{4}}+\left(b^2c^2-2\,abcd+a^2d^2\right)x\right)+20\,\left(2\,bcd-ad^2\right)x}{20\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^4+c)^2/(b*x^4+a),x, algorithm="fricas")
```

```
[Out] 1/20*(4*b*d^2*x^5 + 20*b^2*(-(b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2
- 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2
*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8)/(a^3*b^9))^(1/4)*arctan(-(a^2*b^7*x*(-(
b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*
b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8
*d^8)/(a^3*b^9))^(3/4) - a^2*b^7*sqrt((a^2*b^4*sqrt(-(b^8*c^8 - 8*a*b^7*c^7
*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*
b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8)/(a^3*b^9)) + (b
^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*x^2)/
(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4))*(-
(b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4
*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^
8*d^8)/(a^3*b^9))^(3/4))/(b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20
*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)) + 5*b^2*(
-(b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^
4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a
^8*d^8)/(a^3*b^9))^(1/4)*log(a*b^2*(-(b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*
c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28
*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8)/(a^3*b^9))^(1/4) + (b^2*c^2 - 2
*a*b*c*d + a^2*d^2)*x) - 5*b^2*(-(b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*
d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6
*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8)/(a^3*b^9))^(1/4)*log(-a*b^2*(-(b^8*
c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*
c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8
)/(a^3*b^9))^(1/4) + (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*x) + 20*(2*b*c*d - a*d
^2)*x)/b^2
```

giac [A] time = 0.18, size = 353, normalized size = 1.40

$$\frac{\sqrt{2}\left(\left(ab^3\right)^{\frac{1}{4}}b^2c^2-2\left(ab^3\right)^{\frac{1}{4}}abcd+\left(ab^3\right)^{\frac{1}{4}}a^2d^2\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4\,ab^3}+\frac{\sqrt{2}\left(\left(ab^3\right)^{\frac{1}{4}}b^2c^2-2\left(ab^3\right)^{\frac{1}{4}}abcd+\left(ab^3\right)^{\frac{1}{4}}a^2d^2\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4\,ab^3}+\frac{\sqrt{2}\left(\left(ab^3\right)^{\frac{1}{4}}b^2c^2-2\left(ab^3\right)^{\frac{1}{4}}abcd+\left(ab^3\right)^{\frac{1}{4}}a^2d^2\right)\log\left(x^2+\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{8\,ab^3}-\frac{\sqrt{2}\left(\left(ab^3\right)^{\frac{1}{4}}b^2c^2-2\left(ab^3\right)^{\frac{1}{4}}abcd+\left(ab^3\right)^{\frac{1}{4}}a^2d^2\right)\log\left(x^2-\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{8\,ab^3}+\frac{b^4d^2x^5+10\,b^4cdx-5\,ab^3d^2x}{5\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^4+c)^2/(b*x^4+a),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(2)*((a*b^3)^(1/4)*b^2*c^2 - 2*(a*b^3)^(1/4)*a*b*c*d + (a*b^3)^(1/4
)*a^2*d^2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a*b
^3) + 1/4*sqrt(2)*((a*b^3)^(1/4)*b^2*c^2 - 2*(a*b^3)^(1/4)*a*b*c*d + (a*b^3
)^(1/4)*a^2*d^2)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4)
)/(a*b^3) + 1/8*sqrt(2)*((a*b^3)^(1/4)*b^2*c^2 - 2*(a*b^3)^(1/4)*a*b*c*d +
(a*b^3)^(1/4)*a^2*d^2)*log(x^2 + sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a*b^3)
 - 1/8*sqrt(2)*((a*b^3)^(1/4)*b^2*c^2 - 2*(a*b^3)^(1/4)*a*b*c*d + (a*b^3)^(
1/4)*a^2*d^2)*log(x^2 - sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a*b^3) + 1/5*(b
^4*d^2*x^5 + 10*b^4*c*d*x - 5*a*b^3*d^2*x)/b^5
```

maple [B] time = 0.05, size = 436, normalized size = 1.72

$$\frac{d^2x^5}{5b}-\frac{a\,d^2x}{b^2}+\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,a\,d^2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{4b^2}+\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,a\,d^2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{4b^2}+\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,a\,d^2\ln\left(\frac{x^2+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{a}{b}}}{x^2-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{a}{b}}}\right)}{8b^2}+\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,c^2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{4a}+\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,c^2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{4a}+\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,c^2\ln\left(\frac{x^2+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{a}{b}}}{x^2-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{a}{b}}}\right)}{8a}+\frac{2cdx}{b}-\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,cd\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{2b}-\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,cd\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{2b}-\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,cd\ln\left(\frac{x^2+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{a}{b}}}{x^2-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{a}{b}}}\right)}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^4+c)^2/(b*x^4+a),x)
```

```
[Out] 1/5*d^2*x^5/b-d^2/b^2*a*x+2*d/b*c*x+1/8/b^2*(a/b)^(1/4)*a*2^(1/2)*ln((x^2+(
a/b)^(1/4)*2^(1/2)*x+(a/b)^(1/2))/(x^2-(a/b)^(1/4)*2^(1/2)*x+(a/b)^(1/2)))*
d^2-1/4/b*(a/b)^(1/4)*2^(1/2)*ln((x^2+(a/b)^(1/4)*2^(1/2)*x+(a/b)^(1/2))/(x
^2-(a/b)^(1/4)*2^(1/2)*x+(a/b)^(1/2)))*c*d+1/8*(a/b)^(1/4)/a*2^(1/2)*ln((x^
2+(a/b)^(1/4)*2^(1/2)*x+(a/b)^(1/2))/(x^2-(a/b)^(1/4)*2^(1/2)*x+(a/b)^(1/2)
))*c^2+1/4/b^2*(a/b)^(1/4)*a*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x-1)*d^2-1/
2/b*(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x-1)*c*d+1/4*(a/b)^(1/4)
/a*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x-1)*c^2+1/4/b^2*(a/b)^(1/4)*a*2^(1/2
)*arctan(2^(1/2)/(a/b)^(1/4)*x+1)*d^2-1/2/b*(a/b)^(1/4)*2^(1/2)*arctan(2^(1
/2)/(a/b)^(1/4)*x+1)*c*d+1/4*(a/b)^(1/4)/a*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/
4)*x+1)*c^2
```

maxima [A] time = 1.09, size = 287, normalized size = 1.13

$$\frac{bd^2x^5+5\left(2\,bcd-ad^2\right)x}{5\,b^2}+\frac{\frac{2\,\sqrt{2}\left(b^2c^2-2\,abcd+a^2d^2\right)\arctan\left(\frac{\sqrt{2}\left(2\,\sqrt{b}x+\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\,\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{a}\,\sqrt{\sqrt{a}\,\sqrt{b}}}+\frac{2\,\sqrt{2}\left(b^2c^2-2\,abcd+a^2d^2\right)\arctan\left(\frac{\sqrt{2}\left(2\,\sqrt{b}x-\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\,\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{a}\,\sqrt{\sqrt{a}\,\sqrt{b}}}+\frac{\sqrt{2}\left(b^2c^2-2\,abcd+a^2d^2\right)\log\left(\sqrt{b}x^2+\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{1}{4}}}-\frac{\sqrt{2}\left(b^2c^2-2\,abcd+a^2d^2\right)\log\left(\sqrt{b}x^2-\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{1}{4}}}}{8\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^4+c)^2/(b*x^4+a),x, algorithm="maxima")
```

```
[Out] 1/5*(b*d^2*x^5 + 5*(2*b*c*d - a*d^2)*x)/b^2 + 1/8*(2*sqrt(2)*(b^2*c^2 - 2*a
*b*c*d + a^2*d^2)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x + sqrt(2)*a^(1/4)*b^(1/4)
)/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + 2*sqrt(2)*(b^2*c
^2 - 2*a*b*c*d + a^2*d^2)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x - sqrt(2)*a^(1/4)
*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + sqrt(2)*
(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*log(sqrt(b)*x^2 + sqrt(2)*a^(1/4)*b^(1/4)*x
 + sqrt(a))/(a^(3/4)*b^(1/4)) - sqrt(2)*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*log
(sqrt(b)*x^2 - sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(3/4)*b^(1/4)))/b^2
```

mupad [B] time = 1.47, size = 1081, normalized size = 4.27

$$\frac{d^2x^5}{5\,b}-x\left(\frac{a\,d^2}{b^2}-\frac{2\,c\,d}{b}\right)+\frac{\mathrm{atan}\left(\frac{\frac{(a\,d-b\,c)^2\left(\frac{x\left(a^4d^4-4\,a^3b\,c\,d^3+6\,a^2b^2c^2d^2-4\,a\,b^3c^3d+b^4c^4\right)}{b}-\frac{(a\,d-b\,c)^2\left(4\,a^3b\,d^2-8\,a^2b^2c\,d+4\,a\,b^3c^2\right)}{4\,(-a)^{3/4}\,b^{9/4}}\right)1i}{(-a)^{3/4}\,b^{9/4}}+\frac{(a\,d-b\,c)^2\left(\frac{x\left(a^4d^4-4\,a^3b\,c\,d^3+6\,a^2b^2c^2d^2-4\,a\,b^3c^3d+b^4c^4\right)}{b}+\frac{(a\,d-b\,c)^2\left(4\,a^3b\,d^2-8\,a^2b^2c\,d+4\,a\,b^3c^2\right)}{4\,(-a)^{3/4}\,b^{9/4}}\right)1i}{(-a)^{3/4}\,b^{9/4}}}{\frac{(a\,d-b\,c)^2\left(\frac{x\left(a^4d^4-4\,a^3b\,c\,d^3+6\,a^2b^2c^2d^2-4\,a\,b^3c^3d+b^4c^4\right)}{b}-\frac{(a\,d-b\,c)^2\left(4\,a^3b\,d^2-8\,a^2b^2c\,d+4\,a\,b^3c^2\right)}{4\,(-a)^{3/4}\,b^{9/4}}\right)}{(-a)^{3/4}\,b^{9/4}}-\frac{(a\,d-b\,c)^2\left(\frac{x\left(a^4d^4-4\,a^3b\,c\,d^3+6\,a^2b^2c^2d^2-4\,a\,b^3c^3d+b^4c^4\right)}{b}+\frac{(a\,d-b\,c)^2\left(4\,a^3b\,d^2-8\,a^2b^2c\,d+4\,a\,b^3c^2\right)}{4\,(-a)^{3/4}\,b^{9/4}}\right)}{(-a)^{3/4}\,b^{9/4}}}\right)(a\,d-b\,c)^2\,1i}{2\,(-a)^{3/4}\,b^{9/4}}+\frac{\mathrm{atan}\left(\frac{\frac{(a\,d-b\,c)^2\left(\frac{x\left(a^4d^4-4\,a^3b\,c\,d^3+6\,a^2b^2c^2d^2-4\,a\,b^3c^3d+b^4c^4\right)}{b}-\frac{(a\,d-b\,c)^2\left(4\,a^3b\,d^2-8\,a^2b^2c\,d+4\,a\,b^3c^2\right)1i}{4\,(-a)^{3/4}\,b^{9/4}}\right)}{(-a)^{3/4}\,b^{9/4}}+\frac{(a\,d-b\,c)^2\left(\frac{x\left(a^4d^4-4\,a^3b\,c\,d^3+6\,a^2b^2c^2d^2-4\,a\,b^3c^3d+b^4c^4\right)}{b}+\frac{(a\,d-b\,c)^2\left(4\,a^3b\,d^2-8\,a^2b^2c\,d+4\,a\,b^3c^2\right)1i}{4\,(-a)^{3/4}\,b^{9/4}}\right)}{(-a)^{3/4}\,b^{9/4}}}{\frac{(a\,d-b\,c)^2\left(\frac{x\left(a^4d^4-4\,a^3b\,c\,d^3+6\,a^2b^2c^2d^2-4\,a\,b^3c^3d+b^4c^4\right)}{b}-\frac{(a\,d-b\,c)^2\left(4\,a^3b\,d^2-8\,a^2b^2c\,d+4\,a\,b^3c^2\right)1i}{4\,(-a)^{3/4}\,b^{9/4}}\right)1i}{(-a)^{3/4}\,b^{9/4}}-\frac{(a\,d-b\,c)^2\left(\frac{x\left(a^4d^4-4\,a^3b\,c\,d^3+6\,a^2b^2c^2d^2-4\,a\,b^3c^3d+b^4c^4\right)}{b}+\frac{(a\,d-b\,c)^2\left(4\,a^3b\,d^2-8\,a^2b^2c\,d+4\,a\,b^3c^2\right)1i}{4\,(-a)^{3/4}\,b^{9/4}}\right)1i}{(-a)^{3/4}\,b^{9/4}}}\right)(a\,d-b\,c)^2}{2\,(-a)^{3/4}\,b^{9/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^4)^2/(a + b*x^4),x)
```

```
[Out] (d^2*x^5)/(5*b) - x*((a*d^2)/b^2 - (2*c*d)/b) + (atan((((a*d - b*c)^2*((x*(
a^4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3))/b -
 ((a*d - b*c)^2*(4*a*b^3*c^2 + 4*a^3*b*d^2 - 8*a^2*b^2*c*d))/(4*(-a)^(3/4)*
b^(9/4)))*1i)/((-a)^(3/4)*b^(9/4)) + ((a*d - b*c)^2*((x*(a^4*d^4 + b^4*c^4
+ 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3))/b + ((a*d - b*c)^2*(4
*a*b^3*c^2 + 4*a^3*b*d^2 - 8*a^2*b^2*c*d))/(4*(-a)^(3/4)*b^(9/4)))*1i)/((-a
)^(3/4)*b^(9/4)))/(((a*d - b*c)^2*((x*(a^4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^
2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3))/b - ((a*d - b*c)^2*(4*a*b^3*c^2 + 4*a^3
*b*d^2 - 8*a^2*b^2*c*d))/(4*(-a)^(3/4)*b^(9/4))))/((-a)^(3/4)*b^(9/4)) - ((
a*d - b*c)^2*((x*(a^4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4
*a^3*b*c*d^3))/b + ((a*d - b*c)^2*(4*a*b^3*c^2 + 4*a^3*b*d^2 - 8*a^2*b^2*c*
d))/(4*(-a)^(3/4)*b^(9/4))))/((-a)^(3/4)*b^(9/4))))*(a*d - b*c)^2*1i)/(2*(-
a)^(3/4)*b^(9/4)) + (atan((((a*d - b*c)^2*((x*(a^4*d^4 + b^4*c^4 + 6*a^2*b^
2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3))/b - ((a*d - b*c)^2*(4*a*b^3*c^2
 + 4*a^3*b*d^2 - 8*a^2*b^2*c*d)*1i)/(4*(-a)^(3/4)*b^(9/4))))/((-a)^(3/4)*b^
(9/4)) + ((a*d - b*c)^2*((x*(a^4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^
3*c^3*d - 4*a^3*b*c*d^3))/b + ((a*d - b*c)^2*(4*a*b^3*c^2 + 4*a^3*b*d^2 - 8
*a^2*b^2*c*d)*1i)/(4*(-a)^(3/4)*b^(9/4))))/((-a)^(3/4)*b^(9/4)))/(((a*d - b
```

```
*c)^2*((x*(a^4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*
c*d^3))/b - ((a*d - b*c)^2*(4*a*b^3*c^2 + 4*a^3*b*d^2 - 8*a^2*b^2*c*d)*1i)/
(4*(-a)^(3/4)*b^(9/4)))*1i)/((-a)^(3/4)*b^(9/4)) - ((a*d - b*c)^2*((x*(a^4*
d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3))/b + ((a
*d - b*c)^2*(4*a*b^3*c^2 + 4*a^3*b*d^2 - 8*a^2*b^2*c*d)*1i)/(4*(-a)^(3/4)*b
^(9/4)))*1i)/((-a)^(3/4)*b^(9/4))))*(a*d - b*c)^2)/(2*(-a)^(3/4)*b^(9/4))
```

sympy [A] time = 1.08, size = 187, normalized size = 0.74

$$x\left(-\frac{ad^2}{b^2}+\frac{2cd}{b}\right)+\operatorname{RootSum}\left(256t^4a^3b^9+a^8d^8-8a^7bcd^7+28a^6b^2c^2d^6-56a^5b^3c^3d^5+70a^4b^4c^4d^4-56a^3b^5c^5d^3+28a^2b^6c^6d^2-8ab^7c^7d+b^8c^8,\left(t\mapsto t\log\left(\frac{4tab^2}{a^2d^2-2abcd+b^2c^2}+x\right)\right)\right)+\frac{d^2x^5}{5b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**4+c)**2/(b*x**4+a),x)
```

```
[Out] x*(-a*d**2/b**2 + 2*c*d/b) + RootSum(256*_t**4*a**3*b**9 + a**8*d**8 - 8*a*
*7*b*c*d**7 + 28*a**6*b**2*c**2*d**6 - 56*a**5*b**3*c**3*d**5 + 70*a**4*b**
4*c**4*d**4 - 56*a**3*b**5*c**5*d**3 + 28*a**2*b**6*c**6*d**2 - 8*a*b**7*c*
*7*d + b**8*c**8, Lambda(_t, _t*log(4*_t*a*b**2/(a**2*d**2 - 2*a*b*c*d + b*
*2*c**2) + x))) + d**2*x**5/(5*b)
```

3.100 $\int \frac{c+dx^4}{a+bx^4}\,dx$

Optimal. Leaf size=223

$$-\frac{(bc-ad)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}b^{5/4}}+\frac{(bc-ad)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}b^{5/4}}-\frac{(bc-ad)\tan^{-1}\left(1-\right.}{2\sqrt{2}a^{3/4}b^{5/4}}$$

Rubi [A] time = 0.14, antiderivative size = 223, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.412$, Rules used = {388, 211, 1165, 628, 1162, 617, 204}

$$-\frac{(bc-ad)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}b^{5/4}}+\frac{(bc-ad)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}b^{5/4}}-\frac{(bc-ad)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}b^{5/4}}+\frac{(bc-ad)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{2\sqrt{2}a^{3/4}b^{5/4}}+\frac{dx}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^4)/(a + b*x^4),x]
```

```
[Out] (d*x)/b - ((b*c - a*d)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*
a^(3/4)*b^(5/4)) + ((b*c - a*d)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2
*Sqrt[2]*a^(3/4)*b^(5/4)) - ((b*c - a*d)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1
/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(3/4)*b^(5/4)) + ((b*c - a*d)*Log[Sqrt[a
] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(3/4)*b^(5/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{c+dx^4}{a+bx^4}\,dx = \frac{dx}{b} - \frac{(-bc+ad)\int \frac{1}{a+bx^4}\,dx}{b}$$

$$= \frac{dx}{b} + \frac{(bc-ad)\int \frac{\sqrt{a}-\sqrt{b}x^2}{a+bx^4}\,dx}{2\sqrt{a}\,b} + \frac{(bc-ad)\int \frac{\sqrt{a}+\sqrt{b}x^2}{a+bx^4}\,dx}{2\sqrt{a}\,b}$$

$$= \frac{dx}{b} + \frac{(bc-ad)\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx}{4\sqrt{a}\,b^{3/2}} + \frac{(bc-ad)\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx}{4\sqrt{a}\,b^{3/2}} - \frac{(bc-ad)\int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{b}}+2x}{-\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}-x^2}}{4\sqrt{2}\,a^{3/4}b^{5/4}}$$

$$= \frac{dx}{b} - \frac{(bc-ad)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{4\sqrt{2}\,a^{3/4}b^{5/4}} + \frac{(bc-ad)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{4\sqrt{2}\,a^{3/4}b^{5/4}}$$

$$= \frac{dx}{b} - \frac{(bc-ad)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,a^{3/4}b^{5/4}} + \frac{(bc-ad)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,a^{3/4}b^{5/4}} - \frac{(bc-ad)\log\left(\sqrt{a}-\sqrt{2}\right.}{4\sqrt{2}\,a^{3/4}b}$$

Mathematica [A] time = 0.15, size = 196, normalized size = 0.88

$$\frac{8a^{3/4}\sqrt[4]{b}\,dx-\sqrt{2}(bc-ad)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)+\sqrt{2}(bc-ad)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)-2\sqrt{2}(bc-ad)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)+2\sqrt{2}(bc-ad)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{8a^{3/4}b^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^4)/(a + b*x^4),x]
```

```
[Out] (8*a^(3/4)*b^(1/4)*d*x - 2*Sqrt[2]*(b*c - a*d)*ArcTan[1 - (Sqrt[2]*b^(1/4)*
x)/a^(1/4)] + 2*Sqrt[2]*(b*c - a*d)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)]
 - Sqrt[2]*(b*c - a*d)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^
2] + Sqrt[2]*(b*c - a*d)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*
x^2])/(8*a^(3/4)*b^(5/4))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{c+dx^4}{a+bx^4}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x^4)/(a + b*x^4),x]
```

[Out] IntegrateAlgebraic[(c + d*x^4)/(a + b*x^4), x]

fricas [B] time = 0.74, size = 639, normalized size = 2.87

$$-\frac{4\,b\left(-\frac{b^4c^4-4\,ab^3c^3d+6\,a^2b^2c^2d^2-4\,a^3bcd^3+a^4d^4}{a^3b^5}\right)^{\frac{1}{4}}\arctan\left(\frac{a^2b^4x\left(-\frac{b^4c^4-4\,ab^3c^3d+6\,a^2b^2c^2d^2-4\,a^3bcd^3+a^4d^4}{a^3b^5}\right)^{\frac{3}{4}}-a^2b^4\sqrt{\frac{a^2b^2\sqrt{-\frac{b^4c^4-4\,ab^3c^3d+6\,a^2b^2c^2d^2-4\,a^3bcd^3+a^4d^4}{a^3b^5}}+\left(b^2c^2-2\,abcd+a^2d^2\right)x^2}{b^2c^2-2\,abcd+a^2d^2}}\left(-\frac{b^4c^4-4\,ab^3c^3d+6\,a^2b^2c^2d^2-4\,a^3bcd^3+a^4d^4}{a^3b^5}\right)^{\frac{3}{4}}}{b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3}\right)+b\left(-\frac{b^4c^4-4\,ab^3c^3d+6\,a^2b^2c^2d^2-4\,a^3bcd^3+a^4d^4}{a^3b^5}\right)^{\frac{1}{4}}\log\left(ab\left(-\frac{b^4c^4-4\,ab^3c^3d+6\,a^2b^2c^2d^2-4\,a^3bcd^3+a^4d^4}{a^3b^5}\right)^{\frac{1}{4}}-(bc-ad)x\right)-b\left(-\frac{b^4c^4-4\,ab^3c^3d+6\,a^2b^2c^2d^2-4\,a^3bcd^3+a^4d^4}{a^3b^5}\right)^{\frac{1}{4}}\log\left(-ab\left(-\frac{b^4c^4-4\,ab^3c^3d+6\,a^2b^2c^2d^2-4\,a^3bcd^3+a^4d^4}{a^3b^5}\right)^{\frac{1}{4}}-(bc-ad)x\right)-4\,dx}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^4+c)/(b*x^4+a),x, algorithm="fricas")
```

```
[Out] -1/4*(4*b*(-(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 +
a^4*d^4)/(a^3*b^5))^(1/4)*arctan((a^2*b^4*x*(-(b^4*c^4 - 4*a*b^3*c^3*d + 6*
a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)/(a^3*b^5))^(3/4) - a^2*b^4*sqrt(
(a^2*b^2*sqrt(-(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3
 + a^4*d^4)/(a^3*b^5)) + (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*x^2)/(b^2*c^2 - 2*
a*b*c*d + a^2*d^2))*(-(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*
b*c*d^3 + a^4*d^4)/(a^3*b^5))^(3/4))/(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d
^2 - a^3*d^3)) + b*(-(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b
*c*d^3 + a^4*d^4)/(a^3*b^5))^(1/4)*log(a*b*(-(b^4*c^4 - 4*a*b^3*c^3*d + 6*a
^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)/(a^3*b^5))^(1/4) - (b*c - a*d)*x)
 - b*(-(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d
^4)/(a^3*b^5))^(1/4)*log(-a*b*(-(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^
2 - 4*a^3*b*c*d^3 + a^4*d^4)/(a^3*b^5))^(1/4) - (b*c - a*d)*x) - 4*d*x)/b
```

giac [A] time = 0.17, size = 245, normalized size = 1.10

$$\frac{dx}{b}+\frac{\sqrt{2}\left(\left(ab^3\right)^{\frac{1}{4}}bc-\left(ab^3\right)^{\frac{1}{4}}ad\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4\,ab^2}+\frac{\sqrt{2}\left(\left(ab^3\right)^{\frac{1}{4}}bc-\left(ab^3\right)^{\frac{1}{4}}ad\right)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4\,ab^2}+\frac{\sqrt{2}\left(\left(ab^3\right)^{\frac{1}{4}}bc-\left(ab^3\right)^{\frac{1}{4}}ad\right)\log\left(x^2+\sqrt{2}\,x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{8\,ab^2}-\frac{\sqrt{2}\left(\left(ab^3\right)^{\frac{1}{4}}bc-\left(ab^3\right)^{\frac{1}{4}}ad\right)\log\left(x^2-\sqrt{2}\,x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{8\,ab^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^4+c)/(b*x^4+a),x, algorithm="giac")
```

```
[Out] d*x/b + 1/4*sqrt(2)*((a*b^3)^(1/4)*b*c - (a*b^3)^(1/4)*a*d)*arctan(1/2*sqrt
(2)*(2*x + sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a*b^2) + 1/4*sqrt(2)*((a*b^3)
^(1/4)*b*c - (a*b^3)^(1/4)*a*d)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/b)^(1/
4))/(a/b)^(1/4))/(a*b^2) + 1/8*sqrt(2)*((a*b^3)^(1/4)*b*c - (a*b^3)^(1/4)*a
*d)*log(x^2 + sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a*b^2) - 1/8*sqrt(2)*((a*
b^3)^(1/4)*b*c - (a*b^3)^(1/4)*a*d)*log(x^2 - sqrt(2)*x*(a/b)^(1/4) + sqrt(
a/b))/(a*b^2)
```

maple [A] time = 0.05, size = 266, normalized size = 1.19

$$\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,c\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{4a}+\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,c\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{4a}+\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,c\ln\left(\frac{x^2+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{a}{b}}}{x^2-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{a}{b}}}\right)}{8a}+\frac{dx}{b}-\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,d\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{4b}-\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,d\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{4b}-\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,d\ln\left(\frac{x^2+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{a}{b}}}{x^2-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{a}{b}}}\right)}{8b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^4+c)/(b*x^4+a),x)
```

```
[Out] 1/b*d*x-1/4/b*(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x-1)*d+1/4*(a/
b)^(1/4)/a*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x-1)*c-1/8/b*(a/b)^(1/4)*2^(1
/2)*ln((x^2+(a/b)^(1/4)*2^(1/2)*x+(a/b)^(1/2))/(x^2-(a/b)^(1/4)*2^(1/2)*x+(
a/b)^(1/2)))*d+1/8*(a/b)^(1/4)/a*2^(1/2)*ln((x^2+(a/b)^(1/4)*2^(1/2)*x+(a/b
)^(1/2))/(x^2-(a/b)^(1/4)*2^(1/2)*x+(a/b)^(1/2)))*c-1/4/b*(a/b)^(1/4)*2^(1/
2)*arctan(2^(1/2)/(a/b)^(1/4)*x+1)*d+1/4*(a/b)^(1/4)/a*2^(1/2)*arctan(2^(1/
2)/(a/b)^(1/4)*x+1)*c
```

maxima [A] time = 1.13, size = 212, normalized size = 0.95

$$\frac{dx}{b}+\frac{\frac{2\sqrt{2}\,(bc-ad)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}\,x+\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}+\frac{2\sqrt{2}\,(bc-ad)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}\,x-\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}+\frac{\sqrt{2}\,(bc-ad)\log\left(\sqrt{b}\,x^2+\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{1}{4}}}-\frac{\sqrt{2}\,(bc-ad)\log\left(\sqrt{b}\,x^2-\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{1}{4}}}}{8\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^4+c)/(b*x^4+a),x, algorithm="maxima")
```

```
[Out] d*x/b + 1/8*(2*sqrt(2)*(b*c - a*d)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x + sqrt(2
)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) +
 2*sqrt(2)*(b*c - a*d)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x - sqrt(2)*a^(1/4)*b^
(1/4))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + sqrt(2)*(b*
c - a*d)*log(sqrt(b)*x^2 + sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(3/4)*b^
(1/4)) - sqrt(2)*(b*c - a*d)*log(sqrt(b)*x^2 - sqrt(2)*a^(1/4)*b^(1/4)*x +
sqrt(a))/(a^(3/4)*b^(1/4)))/b
```

mupad [B] time = 0.22, size = 720, normalized size = 3.23

$$\frac{dx}{b}-\frac{\mathrm{atan}\left(\frac{\frac{\left(x\left(4a^2bd^2-8ab^2cd+4b^3c^2\right)-\frac{\left(16a^2b^2d-16ab^3c\right)(ad-bc)}{4(-a)^{3/4}b^{5/4}}\right)(ad-bc)\,1i}{4(-a)^{3/4}b^{5/4}}+\frac{\left(x\left(4a^2bd^2-8ab^2cd+4b^3c^2\right)+\frac{\left(16a^2b^2d-16ab^3c\right)(ad-bc)}{4(-a)^{3/4}b^{5/4}}\right)(ad-bc)\,1i}{4(-a)^{3/4}b^{5/4}}}{\frac{\left(x\left(4a^2bd^2-8ab^2cd+4b^3c^2\right)-\frac{\left(16a^2b^2d-16ab^3c\right)(ad-bc)}{4(-a)^{3/4}b^{5/4}}\right)(ad-bc)}{4(-a)^{3/4}b^{5/4}}-\frac{\left(x\left(4a^2bd^2-8ab^2cd+4b^3c^2\right)+\frac{\left(16a^2b^2d-16ab^3c\right)(ad-bc)}{4(-a)^{3/4}b^{5/4}}\right)(ad-bc)}{4(-a)^{3/4}b^{5/4}}}\right)(ad-bc)\,1i}{2(-a)^{3/4}b^{5/4}}-\frac{\mathrm{atan}\left(\frac{\frac{\left(x\left(4a^2bd^2-8ab^2cd+4b^3c^2\right)-\frac{\left(16a^2b^2d-16ab^3c\right)(ad-bc)\,1i}{4(-a)^{3/4}b^{5/4}}\right)(ad-bc)}{4(-a)^{3/4}b^{5/4}}+\frac{\left(x\left(4a^2bd^2-8ab^2cd+4b^3c^2\right)+\frac{\left(16a^2b^2d-16ab^3c\right)(ad-bc)\,1i}{4(-a)^{3/4}b^{5/4}}\right)(ad-bc)}{4(-a)^{3/4}b^{5/4}}}{\frac{\left(x\left(4a^2bd^2-8ab^2cd+4b^3c^2\right)-\frac{\left(16a^2b^2d-16ab^3c\right)(ad-bc)\,1i}{4(-a)^{3/4}b^{5/4}}\right)(ad-bc)\,1i}{4(-a)^{3/4}b^{5/4}}-\frac{\left(x\left(4a^2bd^2-8ab^2cd+4b^3c^2\right)+\frac{\left(16a^2b^2d-16ab^3c\right)(ad-bc)\,1i}{4(-a)^{3/4}b^{5/4}}\right)(ad-bc)\,1i}{4(-a)^{3/4}b^{5/4}}}\right)(ad-bc)}{2(-a)^{3/4}b^{5/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^4)/(a + b*x^4),x)
```

```
[Out] (d*x)/b - (atan((((x*(4*b^3*c^2 + 4*a^2*b*d^2 - 8*a*b^2*c*d) - ((16*a^2*b^2
*d - 16*a*b^3*c)*(a*d - b*c))/(4*(-a)^(3/4)*b^(5/4)))*(a*d - b*c)*1i)/(4*(-
a)^(3/4)*b^(5/4)) + ((x*(4*b^3*c^2 + 4*a^2*b*d^2 - 8*a*b^2*c*d) + ((16*a^2*
b^2*d - 16*a*b^3*c)*(a*d - b*c))/(4*(-a)^(3/4)*b^(5/4)))*(a*d - b*c)*1i)/(4
*(-a)^(3/4)*b^(5/4)))/(((x*(4*b^3*c^2 + 4*a^2*b*d^2 - 8*a*b^2*c*d) - ((16*a
^2*b^2*d - 16*a*b^3*c)*(a*d - b*c))/(4*(-a)^(3/4)*b^(5/4)))*(a*d - b*c))/(4
*(-a)^(3/4)*b^(5/4)) - ((x*(4*b^3*c^2 + 4*a^2*b*d^2 - 8*a*b^2*c*d) + ((16*a
^2*b^2*d - 16*a*b^3*c)*(a*d - b*c))/(4*(-a)^(3/4)*b^(5/4)))*(a*d - b*c))/(4
*(-a)^(3/4)*b^(5/4))))*(a*d - b*c)*1i)/(2*(-a)^(3/4)*b^(5/4)) - (atan((((x*
(4*b^3*c^2 + 4*a^2*b*d^2 - 8*a*b^2*c*d) - ((16*a^2*b^2*d - 16*a*b^3*c)*(a*d
 - b*c)*1i)/(4*(-a)^(3/4)*b^(5/4)))*(a*d - b*c))/(4*(-a)^(3/4)*b^(5/4)) + (
(x*(4*b^3*c^2 + 4*a^2*b*d^2 - 8*a*b^2*c*d) + ((16*a^2*b^2*d - 16*a*b^3*c)*(
a*d - b*c)*1i)/(4*(-a)^(3/4)*b^(5/4)))*(a*d - b*c))/(4*(-a)^(3/4)*b^(5/4)))
/(((x*(4*b^3*c^2 + 4*a^2*b*d^2 - 8*a*b^2*c*d) - ((16*a^2*b^2*d - 16*a*b^3*c
)*(a*d - b*c)*1i)/(4*(-a)^(3/4)*b^(5/4)))*(a*d - b*c)*1i)/(4*(-a)^(3/4)*b^(
5/4)) - ((x*(4*b^3*c^2 + 4*a^2*b*d^2 - 8*a*b^2*c*d) + ((16*a^2*b^2*d - 16*a
*b^3*c)*(a*d - b*c)*1i)/(4*(-a)^(3/4)*b^(5/4)))*(a*d - b*c)*1i)/(4*(-a)^(3/
4)*b^(5/4))))*(a*d - b*c))/(2*(-a)^(3/4)*b^(5/4))
```

sympy [A] time = 0.61, size = 87, normalized size = 0.39

$$\operatorname{RootSum}\left(256t^4a^3b^5+a^4d^4-4a^3bcd^3+6a^2b^2c^2d^2-4ab^3c^3d+b^4c^4,\left(t\mapsto t\log\left(-\frac{4tab}{ad-bc}+x\right)\right)\right)+\frac{dx}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**4+c)/(b*x**4+a),x)
```

```
[Out] RootSum(256*_t**4*a**3*b**5 + a**4*d**4 - 4*a**3*b*c*d**3 + 6*a**2*b**2*c**
2*d**2 - 4*a*b**3*c**3*d + b**4*c**4, Lambda(_t, _t*log(-4*_t*a*b/(a*d - b*
c) + x))) + d*x/b
```

3.101 $\int \frac{1}{(a+bx^4)(c+dx^4)}\,dx$

Optimal. Leaf size=449

$$-\frac{b^{3/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}(bc-ad)}+\frac{b^{3/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}(bc-ad)}-\frac{b^{3/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}(bc-ad)}+\frac{b^{3/4}\tan}{2\sqrt{2}}$$

Rubi [A] time = 0.27, antiderivative size = 449, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {391, 211, 1165, 628, 1162, 617, 204}

$$-\frac{b^{3/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}(bc-ad)}+\frac{b^{3/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}(bc-ad)}-\frac{b^{3/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}(bc-ad)}+\frac{b^{3/4}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{2\sqrt{2}a^{3/4}(bc-ad)}+\frac{d^{3/4}\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{c}+\sqrt{d}x^2\right)}{4\sqrt{2}c^{3/4}(bc-ad)}-\frac{d^{3/4}\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{c}+\sqrt{d}x^2\right)}{4\sqrt{2}c^{3/4}(bc-ad)}+\frac{d^{3/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{2\sqrt{2}c^{3/4}(bc-ad)}-\frac{d^{3/4}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}+1\right)}{2\sqrt{2}c^{3/4}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^4)*(c + d*x^4)),x]
```

```
[Out] -(b^(3/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*(b*c
- a*d)) + (b^(3/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3
/4)*(b*c - a*d)) + (d^(3/4)*ArcTan[1 - (Sqrt[2]*d^(1/4)*x)/c^(1/4)])/(2*Sqr
t[2]*c^(3/4)*(b*c - a*d)) - (d^(3/4)*ArcTan[1 + (Sqrt[2]*d^(1/4)*x)/c^(1/4)
])/(2*Sqrt[2]*c^(3/4)*(b*c - a*d)) - (b^(3/4)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)
*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(3/4)*(b*c - a*d)) + (b^(3/4)*Log[S
qrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(3/4)*(b*c
- a*d)) + (d^(3/4)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2])/
(4*Sqrt[2]*c^(3/4)*(b*c - a*d)) - (d^(3/4)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^
(1/4)*x + Sqrt[d]*x^2])/(4*Sqrt[2]*c^(3/4)*(b*c - a*d))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 391

```
Int[1/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))), x_Symbol] :> Dis
t[b/(b*c - a*d), Int[1/(a + b*x^n), x], x] - Dist[d/(b*c - a*d), Int[1/(c +
 d*x^n), x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{1}{\left(a+bx^4\right)\left(c+dx^4\right)}\,dx = \frac{b\int \frac{1}{a+bx^4}\,dx}{bc-ad} - \frac{d\int \frac{1}{c+dx^4}\,dx}{bc-ad}$$

$$= \frac{b\int \frac{\sqrt{a}-\sqrt{b}x^2}{a+bx^4}\,dx}{2\sqrt{a}(bc-ad)} + \frac{b\int \frac{\sqrt{a}+\sqrt{b}x^2}{a+bx^4}\,dx}{2\sqrt{a}(bc-ad)} - \frac{d\int \frac{\sqrt{c}-\sqrt{d}x^2}{c+dx^4}\,dx}{2\sqrt{c}(bc-ad)} - \frac{d\int \frac{\sqrt{c}+\sqrt{d}x^2}{c+dx^4}\,dx}{2\sqrt{c}(bc-ad)}$$

$$= \frac{\sqrt{b}\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx}{4\sqrt{a}(bc-ad)} + \frac{\sqrt{b}\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx}{4\sqrt{a}(bc-ad)} - \frac{b^{3/4}\int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{b}}+2x}{-\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}-x^2}\,dx}{4\sqrt{2}a^{3/4}(bc-ad)} - \frac{b^{3/4}\ldots}{\ldots}$$

$$= -\frac{b^{3/4}\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}(bc-ad)} + \frac{b^{3/4}\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}(bc-ad)} + \ldots$$

$$= -\frac{b^{3/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}(bc-ad)} + \frac{b^{3/4}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}(bc-ad)} + \frac{d^{3/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{2\sqrt{2}c^{3/4}(bc-ad)} - \ldots$$

Mathematica [A] time = 0.14, size = 340, normalized size = 0.76

$$\frac{a^{3/4}d^{3/4}\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{c}+\sqrt{d}x^2\right)-a^{3/4}d^{3/4}\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{c}+\sqrt{d}x^2\right)+2a^{3/4}d^{3/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)-2a^{3/4}d^{3/4}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}+1\right)-b^{3/4}c^{3/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)+b^{3/4}c^{3/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)-2b^{3/4}c^{3/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)+2b^{3/4}c^{3/4}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{4\sqrt{2}a^{3/4}c^{3/4}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x^4)*(c + d*x^4)),x]
```

```
[Out] (-2*b^(3/4)*c^(3/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)] + 2*b^(3/4)*c^(
3/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)] + 2*a^(3/4)*d^(3/4)*ArcTan[1 -
 (Sqrt[2]*d^(1/4)*x)/c^(1/4)] - 2*a^(3/4)*d^(3/4)*ArcTan[1 + (Sqrt[2]*d^(1/
4)*x)/c^(1/4)] - b^(3/4)*c^(3/4)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x +
Sqrt[b]*x^2] + b^(3/4)*c^(3/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sq
rt[b]*x^2] + a^(3/4)*d^(3/4)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt
[d]*x^2] - a^(3/4)*d^(3/4)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d
]*x^2])/(4*Sqrt[2]*a^(3/4)*c^(3/4)*(b*c - a*d))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+bx^4\right)\left(c+dx^4\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/((a + b*x^4)*(c + d*x^4)),x]
```

```
[Out] IntegrateAlgebraic[1/((a + b*x^4)*(c + d*x^4)), x]
```

fricas [B] time = 1.74, size = 1356, normalized size = 3.02

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a)/(d*x^4+c),x, algorithm="fricas")
```

```
[Out] -(-b^3/(a^3*b^4*c^4 - 4*a^4*b^3*c^3*d + 6*a^5*b^2*c^2*d^2 - 4*a^6*b*c*d^3 +
 a^7*d^4))^(1/4)*arctan(((a^2*b^3*c^3 - 3*a^3*b^2*c^2*d + 3*a^4*b*c*d^2 - a
^5*d^3)*(-b^3/(a^3*b^4*c^4 - 4*a^4*b^3*c^3*d + 6*a^5*b^2*c^2*d^2 - 4*a^6*b*
c*d^3 + a^7*d^4))^(3/4)*x - (a^2*b^3*c^3 - 3*a^3*b^2*c^2*d + 3*a^4*b*c*d^2
- a^5*d^3)*(-b^3/(a^3*b^4*c^4 - 4*a^4*b^3*c^3*d + 6*a^5*b^2*c^2*d^2 - 4*a^6
*b*c*d^3 + a^7*d^4))^(3/4)*sqrt((b^2*x^2 + (a^2*b^2*c^2 - 2*a^3*b*c*d + a^4
*d^2)*sqrt(-b^3/(a^3*b^4*c^4 - 4*a^4*b^3*c^3*d + 6*a^5*b^2*c^2*d^2 - 4*a^6*
b*c*d^3 + a^7*d^4)))/b^2))/b^2) + (-d^3/(b^4*c^7 - 4*a*b^3*c^6*d + 6*a^2*b^
2*c^5*d^2 - 4*a^3*b*c^4*d^3 + a^4*c^3*d^4))^(1/4)*arctan(((b^3*c^5 - 3*a*b^
2*c^4*d + 3*a^2*b*c^3*d^2 - a^3*c^2*d^3)*(-d^3/(b^4*c^7 - 4*a*b^3*c^6*d + 6
*a^2*b^2*c^5*d^2 - 4*a^3*b*c^4*d^3 + a^4*c^3*d^4))^(3/4)*x - (b^3*c^5 - 3*a
*b^2*c^4*d + 3*a^2*b*c^3*d^2 - a^3*c^2*d^3)*(-d^3/(b^4*c^7 - 4*a*b^3*c^6*d
+ 6*a^2*b^2*c^5*d^2 - 4*a^3*b*c^4*d^3 + a^4*c^3*d^4))^(3/4)*sqrt((d^2*x^2 +
 (b^2*c^4 - 2*a*b*c^3*d + a^2*c^2*d^2)*sqrt(-d^3/(b^4*c^7 - 4*a*b^3*c^6*d +
 6*a^2*b^2*c^5*d^2 - 4*a^3*b*c^4*d^3 + a^4*c^3*d^4)))/d^2))/d^2) + 1/4*(-b^
3/(a^3*b^4*c^4 - 4*a^4*b^3*c^3*d + 6*a^5*b^2*c^2*d^2 - 4*a^6*b*c*d^3 + a^7*
d^4))^(1/4)*log(b*x + (a*b*c - a^2*d)*(-b^3/(a^3*b^4*c^4 - 4*a^4*b^3*c^3*d
+ 6*a^5*b^2*c^2*d^2 - 4*a^6*b*c*d^3 + a^7*d^4))^(1/4)) - 1/4*(-b^3/(a^3*b^4
*c^4 - 4*a^4*b^3*c^3*d + 6*a^5*b^2*c^2*d^2 - 4*a^6*b*c*d^3 + a^7*d^4))^(1/4
)*log(b*x - (a*b*c - a^2*d)*(-b^3/(a^3*b^4*c^4 - 4*a^4*b^3*c^3*d + 6*a^5*b^
2*c^2*d^2 - 4*a^6*b*c*d^3 + a^7*d^4))^(1/4)) - 1/4*(-d^3/(b^4*c^7 - 4*a*b^3
*c^6*d + 6*a^2*b^2*c^5*d^2 - 4*a^3*b*c^4*d^3 + a^4*c^3*d^4))^(1/4)*log(d*x
+ (b*c^2 - a*c*d)*(-d^3/(b^4*c^7 - 4*a*b^3*c^6*d + 6*a^2*b^2*c^5*d^2 - 4*a^
3*b*c^4*d^3 + a^4*c^3*d^4))^(1/4)) + 1/4*(-d^3/(b^4*c^7 - 4*a*b^3*c^6*d + 6
*a^2*b^2*c^5*d^2 - 4*a^3*b*c^4*d^3 + a^4*c^3*d^4))^(1/4)*log(d*x - (b*c^2 -
 a*c*d)*(-d^3/(b^4*c^7 - 4*a*b^3*c^6*d + 6*a^2*b^2*c^5*d^2 - 4*a^3*b*c^4*d^
3 + a^4*c^3*d^4))^(1/4))
```

giac [A] time = 0.21, size = 437, normalized size = 0.97

$$\frac{\left(ab^3\right)^{\frac{1}{4}} \arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}abc-\sqrt{2}a^2d\right)}+\frac{\left(ab^3\right)^{\frac{1}{4}} \arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}abc-\sqrt{2}a^2d\right)}-\frac{\left(cd^3\right)^{\frac{1}{4}} \arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}bc^2-\sqrt{2}acd\right)}-\frac{\left(cd^3\right)^{\frac{1}{4}} \arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}bc^2-\sqrt{2}acd\right)}+\frac{\left(ab^3\right)^{\frac{1}{4}}\log\left(x^2+\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{4\left(\sqrt{2}abc-\sqrt{2}a^2d\right)}-\frac{\left(ab^3\right)^{\frac{1}{4}}\log\left(x^2-\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{4\left(\sqrt{2}abc-\sqrt{2}a^2d\right)}-\frac{\left(cd^3\right)^{\frac{1}{4}}\log\left(x^2+\sqrt{2}x\left(\frac{c}{d}\right)^{\frac{1}{4}}+\sqrt{\frac{c}{d}}\right)}{4\left(\sqrt{2}bc^2-\sqrt{2}acd\right)}+\frac{\left(cd^3\right)^{\frac{1}{4}}\log\left(x^2-\sqrt{2}x\left(\frac{c}{d}\right)^{\frac{1}{4}}+\sqrt{\frac{c}{d}}\right)}{4\left(\sqrt{2}bc^2-\sqrt{2}acd\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a)/(d*x^4+c),x, algorithm="giac")
```

```
[Out] 1/2*(a*b^3)^(1/4)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4
))/(sqrt(2)*a*b*c - sqrt(2)*a^2*d) + 1/2*(a*b^3)^(1/4)*arctan(1/2*sqrt(2)*(
2*x - sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(sqrt(2)*a*b*c - sqrt(2)*a^2*d) - 1
/2*(c*d^3)^(1/4)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(c/d)^(1/4))/(c/d)^(1/4)
)/(sqrt(2)*b*c^2 - sqrt(2)*a*c*d) - 1/2*(c*d^3)^(1/4)*arctan(1/2*sqrt(2)*(2
*x - sqrt(2)*(c/d)^(1/4))/(c/d)^(1/4))/(sqrt(2)*b*c^2 - sqrt(2)*a*c*d) + 1/
4*(a*b^3)^(1/4)*log(x^2 + sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(sqrt(2)*a*b*c
 - sqrt(2)*a^2*d) - 1/4*(a*b^3)^(1/4)*log(x^2 - sqrt(2)*x*(a/b)^(1/4) + sqr
t(a/b))/(sqrt(2)*a*b*c - sqrt(2)*a^2*d) - 1/4*(c*d^3)^(1/4)*log(x^2 + sqrt(
2)*x*(c/d)^(1/4) + sqrt(c/d))/(sqrt(2)*b*c^2 - sqrt(2)*a*c*d) + 1/4*(c*d^3)
^(1/4)*log(x^2 - sqrt(2)*x*(c/d)^(1/4) + sqrt(c/d))/(sqrt(2)*b*c^2 - sqrt(2
)*a*c*d)
```

maple [A]   time = 0.06, size = 320, normalized size = 0.71

$$-\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\, b \arctan\left(\frac{\sqrt{2}\, x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{4\left(ad-bc\right)a}-\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\, b \arctan\left(\frac{\sqrt{2}\, x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{4\left(ad-bc\right)a}-\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\, b \ln\left(\frac{x^{2}+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\, x+\sqrt{\frac{a}{b}}}{x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\, x+\sqrt{\frac{a}{b}}}\right)}{8\left(ad-bc\right)a}+\frac{\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\, d \arctan\left(\frac{\sqrt{2}\, x}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}-1\right)}{4\left(ad-bc\right)c}+\frac{\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\, d \arctan\left(\frac{\sqrt{2}\, x}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}+1\right)}{4\left(ad-bc\right)c}+\frac{\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\, d \ln\left(\frac{x^{2}+\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\, x+\sqrt{\frac{c}{d}}}{x^{2}-\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\, x+\sqrt{\frac{c}{d}}}\right)}{8\left(ad-bc\right)c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^4+a)/(d*x^4+c),x)
```

```
[Out] 1/8*d/(a*d-b*c)*(c/d)^(1/4)/c*2^(1/2)*ln((x^2+(c/d)^(1/4)*2^(1/2)*x+(c/d)^(
1/2))/(x^2-(c/d)^(1/4)*2^(1/2)*x+(c/d)^(1/2)))+1/4*d/(a*d-b*c)*(c/d)^(1/4)/
c*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x+1)+1/4*d/(a*d-b*c)*(c/d)^(1/4)/c*2^(
1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x-1)-1/8*b/(a*d-b*c)*(a/b)^(1/4)/a*2^(1/2)*
ln((x^2+(a/b)^(1/4)*2^(1/2)*x+(a/b)^(1/2))/(x^2-(a/b)^(1/4)*2^(1/2)*x+(a/b)
^(1/2)))-1/4*b/(a*d-b*c)*(a/b)^(1/4)/a*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x
+1)-1/4*b/(a*d-b*c)*(a/b)^(1/4)/a*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x-1)
```

maxima [A]   time = 1.43, size = 365, normalized size = 0.81

$$\frac{\frac{2\sqrt{2}\, b \arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}+\frac{2\sqrt{2}\, b \arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}+\frac{\sqrt{2}b^{\frac{3}{4}}\log\left(\sqrt{b}x^{2}+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}}-\frac{\sqrt{2}b^{\frac{3}{4}}\log\left(\sqrt{b}x^{2}-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}}}{8\left(bc-ad\right)}-\frac{\frac{2\sqrt{2}\, d \arctan\left(\frac{\sqrt{2}\left(2\sqrt{d}x+\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{c}\sqrt{\sqrt{c}\sqrt{d}}}+\frac{2\sqrt{2}\, d \arctan\left(\frac{\sqrt{2}\left(2\sqrt{d}x-\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{c}\sqrt{\sqrt{c}\sqrt{d}}}+\frac{\sqrt{2}d^{\frac{3}{4}}\log\left(\sqrt{d}x^{2}+\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}x+\sqrt{c}\right)}{c^{\frac{3}{4}}}-\frac{\sqrt{2}d^{\frac{3}{4}}\log\left(\sqrt{d}x^{2}-\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}x+\sqrt{c}\right)}{c^{\frac{3}{4}}}}{8\left(bc-ad\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a)/(d*x^4+c),x, algorithm="maxima")
```

```
[Out] 1/8*(2*sqrt(2)*b*arctan(1/2*sqrt(2)*(2*sqrt(b)*x + sqrt(2)*a^(1/4)*b^(1/4))
/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + 2*sqrt(2)*b*arcta
n(1/2*sqrt(2)*(2*sqrt(b)*x - sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b))
)/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + sqrt(2)*b^(3/4)*log(sqrt(b)*x^2 + sqrt(
2)*a^(1/4)*b^(1/4)*x + sqrt(a))/a^(3/4) - sqrt(2)*b^(3/4)*log(sqrt(b)*x^2 -
 sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/a^(3/4))/(b*c - a*d) - 1/8*(2*sqrt(2)
*d*arctan(1/2*sqrt(2)*(2*sqrt(d)*x + sqrt(2)*c^(1/4)*d^(1/4))/sqrt(sqrt(c)*
sqrt(d)))/(sqrt(c)*sqrt(sqrt(c)*sqrt(d))) + 2*sqrt(2)*d*arctan(1/2*sqrt(2)*
(2*sqrt(d)*x - sqrt(2)*c^(1/4)*d^(1/4))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(c)*sqr
t(sqrt(c)*sqrt(d))) + sqrt(2)*d^(3/4)*log(sqrt(d)*x^2 + sqrt(2)*c^(1/4)*d^(
1/4)*x + sqrt(c))/c^(3/4) - sqrt(2)*d^(3/4)*log(sqrt(d)*x^2 - sqrt(2)*c^(1/
4)*d^(1/4)*x + sqrt(c))/c^(3/4))/(b*c - a*d)
```

mupad [B]   time = 2.76, size = 6153, normalized size = 13.70

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^4)*(c + d*x^4)),x)
```

```
[Out] - atan((((-d^3/(256*b^4*c^7 + 256*a^4*c^3*d^4 - 1024*a^3*b*c^4*d^3 + 1536*a
^2*b^2*c^5*d^2 - 1024*a*b^3*c^6*d))^(1/4)*((-d^3/(256*b^4*c^7 + 256*a^4*c^3
*d^4 - 1024*a^3*b*c^4*d^3 + 1536*a^2*b^2*c^5*d^2 - 1024*a*b^3*c^6*d))^(3/4)
*((-d^3/(256*b^4*c^7 + 256*a^4*c^3*d^4 - 1024*a^3*b*c^4*d^3 + 1536*a^2*b^2*
c^5*d^2 - 1024*a*b^3*c^6*d))^(1/4)*(4096*a*b^11*c^8*d^4 + 4096*a^8*b^4*c*d^
11 - 20480*a^2*b^10*c^7*d^5 + 36864*a^3*b^9*c^6*d^6 - 20480*a^4*b^8*c^5*d^7
 - 20480*a^5*b^7*c^4*d^8 + 36864*a^6*b^6*c^3*d^9 - 20480*a^7*b^5*c^2*d^10)
+ x*(1024*a^7*b^4*d^11 + 1024*b^11*c^7*d^4 - 4096*a*b^10*c^6*d^5 - 4096*a^6
*b^5*c*d^10 + 6144*a^2*b^9*c^5*d^6 - 3072*a^3*b^8*c^4*d^7 - 3072*a^4*b^7*c^
3*d^8 + 6144*a^5*b^6*c^2*d^9)) - 16*a^2*b^6*d^8 - 16*b^8*c^2*d^6 + 32*a*b^7
*c*d^7) + 8*b^7*d^7*x)*(-d^3/(256*b^4*c^7 + 256*a^4*c^3*d^4 - 1024*a^3*b*c^
4*d^3 + 1536*a^2*b^2*c^5*d^2 - 1024*a*b^3*c^6*d))^(1/4)*1i - ((-d^3/(256*b^
4*c^7 + 256*a^4*c^3*d^4 - 1024*a^3*b*c^4*d^3 + 1536*a^2*b^2*c^5*d^2 - 1024*
a*b^3*c^6*d))^(1/4)*((-d^3/(256*b^4*c^7 + 256*a^4*c^3*d^4 - 1024*a^3*b*c^4*
d^3 + 1536*a^2*b^2*c^5*d^2 - 1024*a*b^3*c^6*d))^(3/4)*((-d^3/(256*b^4*c^7 +
```

256*a^4*c^3*d^4 - 1024*a^3*b*c^4*d^3 + 1536*a^2*b^2*c^5*d^2 - 1024*a*b^3*c
^6*d))^(1/4)*(4096*a*b^11*c^8*d^4 + 4096*a^8*b^4*c*d^11 - 20480*a^2*b^10*c^
7*d^5 + 36864*a^3*b^9*c^6*d^6 - 20480*a^4*b^8*c^5*d^7 - 20480*a^5*b^7*c^4*d
^8 + 36864*a^6*b^6*c^3*d^9 - 20480*a^7*b^5*c^2*d^10) - x*(1024*a^7*b^4*d^11
 + 1024*b^11*c^7*d^4 - 4096*a*b^10*c^6*d^5 - 4096*a^6*b^5*c*d^10 + 6144*a^2
*b^9*c^5*d^6 - 3072*a^3*b^8*c^4*d^7 - 3072*a^4*b^7*c^3*d^8 + 6144*a^5*b^6*c
^2*d^9)) - 16*a^2*b^6*d^8 - 16*b^8*c^2*d^6 + 32*a*b^7*c*d^7) - 8*b^7*d^7*x)
*(-d^3/(256*b^4*c^7 + 256*a^4*c^3*d^4 - 1024*a^3*b*c^4*d^3 + 1536*a^2*b^2*c
^5*d^2 - 1024*a*b^3*c^6*d))^(1/4)*1i)/(((-d^3/(256*b^4*c^7 + 256*a^4*c^3*d^
4 - 1024*a^3*b*c^4*d^3 + 1536*a^2*b^2*c^5*d^2 - 1024*a*b^3*c^6*d))^(1/4)*((
-d^3/(256*b^4*c^7 + 256*a^4*c^3*d^4 - 1024*a^3*b*c^4*d^3 + 1536*a^2*b^2*c^5
*d^2 - 1024*a*b^3*c^6*d))^(3/4)*((-d^3/(256*b^4*c^7 + 256*a^4*c^3*d^4 - 102
4*a^3*b*c^4*d^3 + 1536*a^2*b^2*c^5*d^2 - 1024*a*b^3*c^6*d))^(1/4)*(4096*a*b
^11*c^8*d^4 + 4096*a^8*b^4*c*d^11 - 20480*a^2*b^10*c^7*d^5 + 36864*a^3*b^9*
c^6*d^6 - 20480*a^4*b^8*c^5*d^7 - 20480*a^5*b^7*c^4*d^8 + 36864*a^6*b^6*c^3
*d^9 - 20480*a^7*b^5*c^2*d^10) + x*(1024*a^7*b^4*d^11 + 1024*b^11*c^7*d^4 -
 4096*a*b^10*c^6*d^5 - 4096*a^6*b^5*c*d^10 + 6144*a^2*b^9*c^5*d^6 - 3072*a^
3*b^8*c^4*d^7 - 3072*a^4*b^7*c^3*d^8 + 6144*a^5*b^6*c^2*d^9)) - 16*a^2*b^6*
d^8 - 16*b^8*c^2*d^6 + 32*a*b^7*c*d^7) + 8*b^7*d^7*x)*(-d^3/(256*b^4*c^7 +
256*a^4*c^3*d^4 - 1024*a^3*b*c^4*d^3 + 1536*a^2*b^2*c^5*d^2 - 1024*a*b^3*c^
6*d))^(1/4) + ((-d^3/(256*b^4*c^7 + 256*a^4*c^3*d^4 - 1024*a^3*b*c^4*d^3 +
1536*a^2*b^2*c^5*d^2 - 1024*a*b^3*c^6*d))^(1/4)*((-d^3/(256*b^4*c^7 + 256*a
^4*c^3*d^4 - 1024*a^3*b*c^4*d^3 + 1536*a^2*b^2*c^5*d^2 - 1024*a*b^3*c^6*d))
^(3/4)*((-d^3/(256*b^4*c^7 + 256*a^4*c^3*d^4 - 1024*a^3*b*c^4*d^3 + 1536*a^
2*b^2*c^5*d^2 - 1024*a*b^3*c^6*d))^(1/4)*(4096*a*b^11*c^8*d^4 + 4096*a^8*b^
4*c*d^11 - 20480*a^2*b^10*c^7*d^5 + 36864*a^3*b^9*c^6*d^6 - 20480*a^4*b^8*c
^5*d^7 - 20480*a^5*b^7*c^4*d^8 + 36864*a^6*b^6*c^3*d^9 - 20480*a^7*b^5*c^2*
d^10) - x*(1024*a^7*b^4*d^11 + 1024*b^11*c^7*d^4 - 4096*a*b^10*c^6*d^5 - 40
96*a^6*b^5*c*d^10 + 6144*a^2*b^9*c^5*d^6 - 3072*a^3*b^8*c^4*d^7 - 3072*a^4*
b^7*c^3*d^8 + 6144*a^5*b^6*c^2*d^9)) - 16*a^2*b^6*d^8 - 16*b^8*c^2*d^6 + 32
*a*b^7*c*d^7) - 8*b^7*d^7*x)*(-d^3/(256*b^4*c^7 + 256*a^4*c^3*d^4 - 1024*a^
3*b*c^4*d^3 + 1536*a^2*b^2*c^5*d^2 - 1024*a*b^3*c^6*d))^(1/4)))*(-d^3/(256*
b^4*c^7 + 256*a^4*c^3*d^4 - 1024*a^3*b*c^4*d^3 + 1536*a^2*b^2*c^5*d^2 - 102
4*a*b^3*c^6*d))^(1/4)*2i - atan((((-b^3/(256*a^7*d^4 + 256*a^3*b^4*c^4 - 10
24*a^4*b^3*c^3*d + 1536*a^5*b^2*c^2*d^2 - 1024*a^6*b*c*d^3))^(1/4)*((-b^3/(
256*a^7*d^4 + 256*a^3*b^4*c^4 - 1024*a^4*b^3*c^3*d + 1536*a^5*b^2*c^2*d^2 -
 1024*a^6*b*c*d^3))^(3/4)*((-b^3/(256*a^7*d^4 + 256*a^3*b^4*c^4 - 1024*a^4*
b^3*c^3*d + 1536*a^5*b^2*c^2*d^2 - 1024*a^6*b*c*d^3))^(1/4)*(4096*a*b^11*c^
8*d^4 + 4096*a^8*b^4*c*d^11 - 20480*a^2*b^10*c^7*d^5 + 36864*a^3*b^9*c^6*d^
6 - 20480*a^4*b^8*c^5*d^7 - 20480*a^5*b^7*c^4*d^8 + 36864*a^6*b^6*c^3*d^9 -
 20480*a^7*b^5*c^2*d^10) + x*(1024*a^7*b^4*d^11 + 1024*b^11*c^7*d^4 - 4096*
a*b^10*c^6*d^5 - 4096*a^6*b^5*c*d^10 + 6144*a^2*b^9*c^5*d^6 - 3072*a^3*b^8*
c^4*d^7 - 3072*a^4*b^7*c^3*d^8 + 6144*a^5*b^6*c^2*d^9)) - 16*a^2*b^6*d^8 -
16*b^8*c^2*d^6 + 32*a*b^7*c*d^7) + 8*b^7*d^7*x)*(-b^3/(256*a^7*d^4 + 256*a^
3*b^4*c^4 - 1024*a^4*b^3*c^3*d + 1536*a^5*b^2*c^2*d^2 - 1024*a^6*b*c*d^3))^
(1/4)*1i - ((-b^3/(256*a^7*d^4 + 256*a^3*b^4*c^4 - 1024*a^4*b^3*c^3*d + 153
6*a^5*b^2*c^2*d^2 - 1024*a^6*b*c*d^3))^(1/4)*((-b^3/(256*a^7*d^4 + 256*a^3*
b^4*c^4 - 1024*a^4*b^3*c^3*d + 1536*a^5*b^2*c^2*d^2 - 1024*a^6*b*c*d^3))^(3
/4)*((-b^3/(256*a^7*d^4 + 256*a^3*b^4*c^4 - 1024*a^4*b^3*c^3*d + 1536*a^5*b
^2*c^2*d^2 - 1024*a^6*b*c*d^3))^(1/4)*(4096*a*b^11*c^8*d^4 + 4096*a^8*b^4*c
*d^11 - 20480*a^2*b^10*c^7*d^5 + 36864*a^3*b^9*c^6*d^6 - 20480*a^4*b^8*c^5*
d^7 - 20480*a^5*b^7*c^4*d^8 + 36864*a^6*b^6*c^3*d^9 - 20480*a^7*b^5*c^2*d^1
0) - x*(1024*a^7*b^4*d^11 + 1024*b^11*c^7*d^4 - 4096*a*b^10*c^6*d^5 - 4096*
a^6*b^5*c*d^10 + 6144*a^2*b^9*c^5*d^6 - 3072*a^3*b^8*c^4*d^7 - 3072*a^4*b^7
*c^3*d^8 + 6144*a^5*b^6*c^2*d^9)) - 16*a^2*b^6*d^8 - 16*b^8*c^2*d^6 + 32*a*
b^7*c*d^7) - 8*b^7*d^7*x)*(-b^3/(256*a^7*d^4 + 256*a^3*b^4*c^4 - 1024*a^4*b
^3*c^3*d + 1536*a^5*b^2*c^2*d^2 - 1024*a^6*b*c*d^3))^(1/4)*1i)/(((-b^3/(256
*a^7*d^4 + 256*a^3*b^4*c^4 - 1024*a^4*b^3*c^3*d + 1536*a^5*b^2*c^2*d^2 - 10
24*a^6*b*c*d^3))^(1/4)*((-b^3/(256*a^7*d^4 + 256*a^3*b^4*c^4 - 1024*a^4*b^3

```
*c^3*d + 1536*a^5*b^2*c^2*d^2 - 1024*a^6*b*c*d^3))^(3/4)*((-b^3/(256*a^7*d^
4 + 256*a^3*b^4*c^4 - 1024*a^4*b^3*c^3*d + 1536*a^5*b^2*c^2*d^2 - 1024*a^6*
b*c*d^3))^(1/4)*(4096*a*b^11*c^8*d^4 + 4096*a^8*b^4*c*d^11 - 20480*a^2*b^10
*c^7*d^5 + 36864*a^3*b^9*c^6*d^6 - 20480*a^4*b^8*c^5*d^7 - 20480*a^5*b^7*c^
4*d^8 + 36864*a^6*b^6*c^3*d^9 - 20480*a^7*b^5*c^2*d^10) + x*(1024*a^7*b^4*d
^11 + 1024*b^11*c^7*d^4 - 4096*a*b^10*c^6*d^5 - 4096*a^6*b^5*c*d^10 + 6144*
a^2*b^9*c^5*d^6 - 3072*a^3*b^8*c^4*d^7 - 3072*a^4*b^7*c^3*d^8 + 6144*a^5*b^
6*c^2*d^9)) - 16*a^2*b^6*d^8 - 16*b^8*c^2*d^6 + 32*a*b^7*c*d^7) + 8*b^7*d^7
*x)*(-b^3/(256*a^7*d^4 + 256*a^3*b^4*c^4 - 1024*a^4*b^3*c^3*d + 1536*a^5*b^
2*c^2*d^2 - 1024*a^6*b*c*d^3))^(1/4) + ((-b^3/(256*a^7*d^4 + 256*a^3*b^4*c^
4 - 1024*a^4*b^3*c^3*d + 1536*a^5*b^2*c^2*d^2 - 1024*a^6*b*c*d^3))^(1/4)*((
-b^3/(256*a^7*d^4 + 256*a^3*b^4*c^4 - 1024*a^4*b^3*c^3*d + 1536*a^5*b^2*c^2
*d^2 - 1024*a^6*b*c*d^3))^(3/4)*((-b^3/(256*a^7*d^4 + 256*a^3*b^4*c^4 - 102
4*a^4*b^3*c^3*d + 1536*a^5*b^2*c^2*d^2 - 1024*a^6*b*c*d^3))^(1/4)*(4096*a*b
^11*c^8*d^4 + 4096*a^8*b^4*c*d^11 - 20480*a^2*b^10*c^7*d^5 + 36864*a^3*b^9*
c^6*d^6 - 20480*a^4*b^8*c^5*d^7 - 20480*a^5*b^7*c^4*d^8 + 36864*a^6*b^6*c^3
*d^9 - 20480*a^7*b^5*c^2*d^10) - x*(1024*a^7*b^4*d^11 + 1024*b^11*c^7*d^4 -
 4096*a*b^10*c^6*d^5 - 4096*a^6*b^5*c*d^10 + 6144*a^2*b^9*c^5*d^6 - 3072*a^
3*b^8*c^4*d^7 - 3072*a^4*b^7*c^3*d^8 + 6144*a^5*b^6*c^2*d^9)) - 16*a^2*b^6*
d^8 - 16*b^8*c^2*d^6 + 32*a*b^7*c*d^7) - 8*b^7*d^7*x)*(-b^3/(256*a^7*d^4 +
256*a^3*b^4*c^4 - 1024*a^4*b^3*c^3*d + 1536*a^5*b^2*c^2*d^2 - 1024*a^6*b*c*
d^3))^(1/4)))*(-b^3/(256*a^7*d^4 + 256*a^3*b^4*c^4 - 1024*a^4*b^3*c^3*d + 1
536*a^5*b^2*c^2*d^2 - 1024*a^6*b*c*d^3))^(1/4)*2i - 2*atan((b^3*d^3*x - (12
8*b^10*c^7*x)/(256*a^7*d^4 + 256*a^3*b^4*c^4 - 1024*a^4*b^3*c^3*d + 1536*a^
5*b^2*c^2*d^2 - 1024*a^6*b*c*d^3) - (128*a^7*b^3*d^7*x)/(256*a^7*d^4 + 256*
a^3*b^4*c^4 - 1024*a^4*b^3*c^3*d + 1536*a^5*b^2*c^2*d^2 - 1024*a^6*b*c*d^3)
 - (768*a^2*b^8*c^5*d^2*x)/(256*a^7*d^4 + 256*a^3*b^4*c^4 - 1024*a^4*b^3*c^
3*d + 1536*a^5*b^2*c^2*d^2 - 1024*a^6*b*c*d^3) + (384*a^3*b^7*c^4*d^3*x)/(2
56*a^7*d^4 + 256*a^3*b^4*c^4 - 1024*a^4*b^3*c^3*d + 1536*a^5*b^2*c^2*d^2 -
1024*a^6*b*c*d^3) + (384*a^4*b^6*c^3*d^4*x)/(256*a^7*d^4 + 256*a^3*b^4*c^4
- 1024*a^4*b^3*c^3*d + 1536*a^5*b^2*c^2*d^2 - 1024*a^6*b*c*d^3) - (768*a^5*
b^5*c^2*d^5*x)/(256*a^7*d^4 + 256*a^3*b^4*c^4 - 1024*a^4*b^3*c^3*d + 1536*a
^5*b^2*c^2*d^2 - 1024*a^6*b*c*d^3) + (512*a*b^9*c^6*d*x)/(256*a^7*d^4 + 256
*a^3*b^4*c^4 - 1024*a^4*b^3*c^3*d + 1536*a^5*b^2*c^2*d^2 - 1024*a^6*b*c*d^3
) + (512*a^6*b^4*c*d^6*x)/(256*a^7*d^4 + 256*a^3*b^4*c^4 - 1024*a^4*b^3*c^3
*d + 1536*a^5*b^2*c^2*d^2 - 1024*a^6*b*c*d^3))/((-b^3/(256*a^7*d^4 + 256*a^
3*b^4*c^4 - 1024*a^4*b^3*c^3*d + 1536*a^5*b^2*c^2*d^2 - 1024*a^6*b*c*d^3))^
(1/4)*((b^3*(512*a*b^7*c^8 + 512*a^8*c*d^7 - 2560*a^2*b^6*c^7*d - 2560*a^7*
b*c^2*d^6 + 4608*a^3*b^5*c^6*d^2 - 2560*a^4*b^4*c^5*d^3 - 2560*a^5*b^3*c^4*
d^4 + 4608*a^6*b^2*c^3*d^5))/(256*a^7*d^4 + 256*a^3*b^4*c^4 - 1024*a^4*b^3*
c^3*d + 1536*a^5*b^2*c^2*d^2 - 1024*a^6*b*c*d^3) + 2*a^2*b^2*d^4 + 2*b^4*c^
2*d^2 - 4*a*b^3*c*d^3)))*(-b^3/(256*a^7*d^4 + 256*a^3*b^4*c^4 - 1024*a^4*b^
3*c^3*d + 1536*a^5*b^2*c^2*d^2 - 1024*a^6*b*c*d^3))^(1/4) - 2*atan((b^3*d^3
*x - (128*a^7*d^10*x)/(256*b^4*c^7 + 256*a^4*c^3*d^4 - 1024*a^3*b*c^4*d^3 +
 1536*a^2*b^2*c^5*d^2 - 1024*a*b^3*c^6*d) - (128*b^7*c^7*d^3*x)/(256*b^4*c^
7 + 256*a^4*c^3*d^4 - 1024*a^3*b*c^4*d^3 + 1536*a^2*b^2*c^5*d^2 - 1024*a*b^
3*c^6*d) - (768*a^2*b^5*c^5*d^5*x)/(256*b^4*c^7 + 256*a^4*c^3*d^4 - 1024*a^
3*b*c^4*d^3 + 1536*a^2*b^2*c^5*d^2 - 1024*a*b^3*c^6*d) + (384*a^3*b^4*c^4*d
^6*x)/(256*b^4*c^7 + 256*a^4*c^3*d^4 - 1024*a^3*b*c^4*d^3 + 1536*a^2*b^2*c^
5*d^2 - 1024*a*b^3*c^6*d) + (384*a^4*b^3*c^3*d^7*x)/(256*b^4*c^7 + 256*a^4*
c^3*d^4 - 1024*a^3*b*c^4*d^3 + 1536*a^2*b^2*c^5*d^2 - 1024*a*b^3*c^6*d) - (
768*a^5*b^2*c^2*d^8*x)/(256*b^4*c^7 + 256*a^4*c^3*d^4 - 1024*a^3*b*c^4*d^3
+ 1536*a^2*b^2*c^5*d^2 - 1024*a*b^3*c^6*d) + (512*a^6*b*c*d^9*x)/(256*b^4*c
^7 + 256*a^4*c^3*d^4 - 1024*a^3*b*c^4*d^3 + 1536*a^2*b^2*c^5*d^2 - 1024*a*b
^3*c^6*d) + (512*a*b^6*c^6*d^4*x)/(256*b^4*c^7 + 256*a^4*c^3*d^4 - 1024*a^3
*b*c^4*d^3 + 1536*a^2*b^2*c^5*d^2 - 1024*a*b^3*c^6*d))/((-d^3/(256*b^4*c^7
+ 256*a^4*c^3*d^4 - 1024*a^3*b*c^4*d^3 + 1536*a^2*b^2*c^5*d^2 - 1024*a*b^3*
c^6*d))^(1/4)*((d^3*(512*a*b^7*c^8 + 512*a^8*c*d^7 - 2560*a^2*b^6*c^7*d - 2
560*a^7*b*c^2*d^6 + 4608*a^3*b^5*c^6*d^2 - 2560*a^4*b^4*c^5*d^3 - 2560*a^5*
```

```
b^3*c^4*d^4 + 4608*a^6*b^2*c^3*d^5))/(256*b^4*c^7 + 256*a^4*c^3*d^4 - 1024*
a^3*b*c^4*d^3 + 1536*a^2*b^2*c^5*d^2 - 1024*a*b^3*c^6*d) + 2*a^2*b^2*d^4 +
2*b^4*c^2*d^2 - 4*a*b^3*c*d^3)))*(-d^3/(256*b^4*c^7 + 256*a^4*c^3*d^4 - 102
4*a^3*b*c^4*d^3 + 1536*a^2*b^2*c^5*d^2 - 1024*a*b^3*c^6*d))^(1/4)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**4+a)/(d*x**4+c),x)
[Out] Timed out
```

3.102 $\int \frac{1}{\left(a+bx^4\right)\left(c+dx^4\right)^2}\,dx$

Optimal. Leaf size=513

$$-\frac{b^{7/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)}{4\sqrt{2}\,a^{3/4}(bc-ad)^2}+\frac{b^{7/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)}{4\sqrt{2}\,a^{3/4}(bc-ad)^2}-\frac{b^{7/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,a^{3/4}(bc-ad)^2}+\frac{b^{7/4}\ldots}{2\ldots}$$

Rubi [A] time = 0.42, antiderivative size = 513, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.421, Rules used = {414, 522, 211, 1165, 628, 1162, 617, 204}

$$-\frac{b^{7/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)}{4\sqrt{2}\,a^{3/4}(bc-ad)^2}+\frac{b^{7/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)}{4\sqrt{2}\,a^{3/4}(bc-ad)^2}-\frac{b^{7/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,a^{3/4}(bc-ad)^2}+\frac{b^{7/4}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}+1\right)}{2\sqrt{2}\,a^{3/4}(bc-ad)^2}+\frac{d^{3/4}(7bc-3ad)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\,x+\sqrt{c}+\sqrt{d}\,x^2\right)}{16\sqrt{2}\,c^{7/4}(bc-ad)^2}-\frac{d^{3/4}(7bc-3ad)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\,x+\sqrt{c}+\sqrt{d}\,x^2\right)}{16\sqrt{2}\,c^{7/4}(bc-ad)^2}+\frac{d^{3/4}(7bc-3ad)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\,x}{\sqrt[4]{c}}\right)}{8\sqrt{2}\,c^{7/4}(bc-ad)^2}-\frac{d^{3/4}(7bc-3ad)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}\,x}{\sqrt[4]{c}}+1\right)}{8\sqrt{2}\,c^{7/4}(bc-ad)^2}-\frac{dx}{4c\left(c+dx^4\right)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^4)*(c + d*x^4)^2),x]
```

```
[Out] -(d*x)/(4*c*(b*c - a*d)*(c + d*x^4)) - (b^(7/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)
*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*(b*c - a*d)^2) + (b^(7/4)*ArcTan[1 + (Sqrt
[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*(b*c - a*d)^2) + (d^(3/4)*(7*b*
c - 3*a*d)*ArcTan[1 - (Sqrt[2]*d^(1/4)*x)/c^(1/4)])/(8*Sqrt[2]*c^(7/4)*(b*c
 - a*d)^2) - (d^(3/4)*(7*b*c - 3*a*d)*ArcTan[1 + (Sqrt[2]*d^(1/4)*x)/c^(1/4
)])/(8*Sqrt[2]*c^(7/4)*(b*c - a*d)^2) - (b^(7/4)*Log[Sqrt[a] - Sqrt[2]*a^(1
/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(3/4)*(b*c - a*d)^2) + (b^(7/4)*
Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(3/4)*
(b*c - a*d)^2) + (d^(3/4)*(7*b*c - 3*a*d)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(
1/4)*x + Sqrt[d]*x^2])/(16*Sqrt[2]*c^(7/4)*(b*c - a*d)^2) - (d^(3/4)*(7*b*c
 - 3*a*d)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2])/(16*Sqrt[
2]*c^(7/4)*(b*c - a*d)^2)
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
```

```
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{1}{\left(a+bx^4\right)\left(c+dx^4\right)^2}\,dx = -\frac{dx}{4c(bc-ad)\left(c+dx^4\right)}+\frac{\int \frac{4bc-3ad-3bdx^4}{\left(a+bx^4\right)\left(c+dx^4\right)}\,dx}{4c(bc-ad)}$$

$$= -\frac{dx}{4c(bc-ad)\left(c+dx^4\right)}+\frac{b^2\int \frac{1}{a+bx^4}\,dx}{(bc-ad)^2}-\frac{(d(7bc-3ad))\int \frac{1}{c+dx^4}\,dx}{4c(bc-ad)^2}$$

$$= -\frac{dx}{4c(bc-ad)\left(c+dx^4\right)}+\frac{b^2\int \frac{\sqrt{a}-\sqrt{b}\,x^2}{a+bx^4}\,dx}{2\sqrt{a}\,(bc-ad)^2}+\frac{b^2\int \frac{\sqrt{a}+\sqrt{b}\,x^2}{a+bx^4}\,dx}{2\sqrt{a}\,(bc-ad)^2}-\frac{(d(7bc-3ad))\int \sqrt{\cdots}}{8c^{3/2}(bc-a\cdots}$$

$$= -\frac{dx}{4c(bc-ad)\left(c+dx^4\right)}+\frac{b^{3/2}\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{b}}+x^2}\,dx}{4\sqrt{a}\,(bc-ad)^2}+\frac{b^{3/2}\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{b}}+x^2}\,dx}{4\sqrt{a}\,(bc-ad)^2}-\frac{b^{7/4}\int\cdots}{4\sqrt{\cdots}}$$

$$= -\frac{dx}{4c(bc-ad)\left(c+dx^4\right)}-\frac{b^{7/4}\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{b}\,x^2\right)}{4\sqrt{2}\,a^{3/4}(bc-ad)^2}+\frac{b^{7/4}\log\left(\sqrt{a}+\sqrt{2}\cdots\right.}{4\sqrt{2}\,a^{3/4}(\cdots}$$

$$= -\frac{dx}{4c(bc-ad)\left(c+dx^4\right)}-\frac{b^{7/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,a^{3/4}(bc-ad)^2}+\frac{b^{7/4}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,a^{3/4}(bc-ad)^2}+\frac{d^{3/4}(7\cdots}{\cdots}$$

Mathematica [A] time = 0.33, size = 498, normalized size = 0.97

$$\frac{8a^{3/4}c^{3/4}dx(ad-bc)-2\sqrt{2}a^{3/4}d^{3/4}\left(c+dx^4\right)(3ad-7bc)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)+2\sqrt{2}a^{3/4}d^{3/4}\left(c+dx^4\right)(3ad-7bc)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}+1\right)+\sqrt{2}a^{3/4}d^{3/4}\left(c+dx^4\right)(7bc-3ad)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{c}+\sqrt{d}x^2\right)+\sqrt{2}a^{3/4}d^{3/4}\left(c+dx^4\right)(3ad-7bc)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{c}+\sqrt{d}x^2\right)-8\sqrt{2}b^{7/4}c^{7/4}\left(c+dx^4\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)+8\sqrt{2}b^{7/4}c^{7/4}\left(c+dx^4\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)-4\sqrt{2}b^{7/4}c^{7/4}\left(c+dx^4\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)+4\sqrt{2}b^{7/4}c^{7/4}\left(c+dx^4\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{32a^{3/4}c^{7/4}\left(c+dx^4\right)(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x^4)*(c + d*x^4)^2),x]
```

```
[Out] (8*a^(3/4)*c^(3/4)*d*(-(b*c) + a*d)*x - 8*Sqrt[2]*b^(7/4)*c^(7/4)*(c + d*x^
4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)] + 8*Sqrt[2]*b^(7/4)*c^(7/4)*(c +
 d*x^4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)] - 2*Sqrt[2]*a^(3/4)*d^(3/4)
*(-7*b*c + 3*a*d)*(c + d*x^4)*ArcTan[1 - (Sqrt[2]*d^(1/4)*x)/c^(1/4)] + 2*S
qrt[2]*a^(3/4)*d^(3/4)*(-7*b*c + 3*a*d)*(c + d*x^4)*ArcTan[1 + (Sqrt[2]*d^(
1/4)*x)/c^(1/4)] - 4*Sqrt[2]*b^(7/4)*c^(7/4)*(c + d*x^4)*Log[Sqrt[a] - Sqrt
[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2] + 4*Sqrt[2]*b^(7/4)*c^(7/4)*(c + d*x^4
)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2] + Sqrt[2]*a^(3/4)*
d^(3/4)*(7*b*c - 3*a*d)*(c + d*x^4)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*x
 + Sqrt[d]*x^2] + Sqrt[2]*a^(3/4)*d^(3/4)*(-7*b*c + 3*a*d)*(c + d*x^4)*Log[
Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2])/(32*a^(3/4)*c^(7/4)*(b*
c - a*d)^2*(c + d*x^4))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+bx^4\right)\left(c+dx^4\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/((a + b*x^4)*(c + d*x^4)^2),x]
```

```
[Out] IntegrateAlgebraic[1/((a + b*x^4)*(c + d*x^4)^2), x]
```

fricas [B] time = 58.70, size = 3299, normalized size = 6.43

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a)/(d*x^4+c)^2,x, algorithm="fricas")
```

```
[Out] 1/16*(4*((b*c^2*d - a*c*d^2)*x^4 + b*c^3 - a*c^2*d)*(-(2401*b^4*c^4*d^3 - 4
116*a*b^3*c^3*d^4 + 2646*a^2*b^2*c^2*d^5 - 756*a^3*b*c*d^6 + 81*a^4*d^7)/(b
^8*c^15 - 8*a*b^7*c^14*d + 28*a^2*b^6*c^13*d^2 - 56*a^3*b^5*c^12*d^3 + 70*a
^4*b^4*c^11*d^4 - 56*a^5*b^3*c^10*d^5 + 28*a^6*b^2*c^9*d^6 - 8*a^7*b*c^8*d^
7 + a^8*c^7*d^8))^(1/4)*arctan(((b^6*c^11 - 6*a*b^5*c^10*d + 15*a^2*b^4*c^9
*d^2 - 20*a^3*b^3*c^8*d^3 + 15*a^4*b^2*c^7*d^4 - 6*a^5*b*c^6*d^5 + a^6*c^5*
d^6)*x*(-(2401*b^4*c^4*d^3 - 4116*a*b^3*c^3*d^4 + 2646*a^2*b^2*c^2*d^5 - 75
6*a^3*b*c*d^6 + 81*a^4*d^7)/(b^8*c^15 - 8*a*b^7*c^14*d + 28*a^2*b^6*c^13*d^
2 - 56*a^3*b^5*c^12*d^3 + 70*a^4*b^4*c^11*d^4 - 56*a^5*b^3*c^10*d^5 + 28*a^
6*b^2*c^9*d^6 - 8*a^7*b*c^8*d^7 + a^8*c^7*d^8))^(3/4) - (b^6*c^11 - 6*a*b^5
*c^10*d + 15*a^2*b^4*c^9*d^2 - 20*a^3*b^3*c^8*d^3 + 15*a^4*b^2*c^7*d^4 - 6*
a^5*b*c^6*d^5 + a^6*c^5*d^6)*(-(2401*b^4*c^4*d^3 - 4116*a*b^3*c^3*d^4 + 264
6*a^2*b^2*c^2*d^5 - 756*a^3*b*c*d^6 + 81*a^4*d^7)/(b^8*c^15 - 8*a*b^7*c^14*
d + 28*a^2*b^6*c^13*d^2 - 56*a^3*b^5*c^12*d^3 + 70*a^4*b^4*c^11*d^4 - 56*a^
5*b^3*c^10*d^5 + 28*a^6*b^2*c^9*d^6 - 8*a^7*b*c^8*d^7 + a^8*c^7*d^8))^(3/4)
*sqrt(((49*b^2*c^2*d^2 - 42*a*b*c*d^3 + 9*a^2*d^4)*x^2 + (b^4*c^8 - 4*a*b^3
*c^7*d + 6*a^2*b^2*c^6*d^2 - 4*a^3*b*c^5*d^3 + a^4*c^4*d^4)*sqrt(-(2401*b^4
*c^4*d^3 - 4116*a*b^3*c^3*d^4 + 2646*a^2*b^2*c^2*d^5 - 756*a^3*b*c*d^6 + 81
*a^4*d^7)/(b^8*c^15 - 8*a*b^7*c^14*d + 28*a^2*b^6*c^13*d^2 - 56*a^3*b^5*c^1
2*d^3 + 70*a^4*b^4*c^11*d^4 - 56*a^5*b^3*c^10*d^5 + 28*a^6*b^2*c^9*d^6 - 8*
a^7*b*c^8*d^7 + a^8*c^7*d^8)))/(49*b^2*c^2*d^2 - 42*a*b*c*d^3 + 9*a^2*d^4))
)/(343*b^3*c^3*d^2 - 441*a*b^2*c^2*d^3 + 189*a^2*b*c*d^4 - 27*a^3*d^5)) + 1
```

```
6*(-b^7/(a^3*b^8*c^8 - 8*a^4*b^7*c^7*d + 28*a^5*b^6*c^6*d^2 - 56*a^6*b^5*c^
5*d^3 + 70*a^7*b^4*c^4*d^4 - 56*a^8*b^3*c^3*d^5 + 28*a^9*b^2*c^2*d^6 - 8*a^
10*b*c*d^7 + a^11*d^8))^(1/4)*((b*c^2*d - a*c*d^2)*x^4 + b*c^3 - a*c^2*d)*a
rctan(-((a^2*b^6*c^6 - 6*a^3*b^5*c^5*d + 15*a^4*b^4*c^4*d^2 - 20*a^5*b^3*c^
3*d^3 + 15*a^6*b^2*c^2*d^4 - 6*a^7*b*c*d^5 + a^8*d^6)*(-b^7/(a^3*b^8*c^8 -
8*a^4*b^7*c^7*d + 28*a^5*b^6*c^6*d^2 - 56*a^6*b^5*c^5*d^3 + 70*a^7*b^4*c^4*
d^4 - 56*a^8*b^3*c^3*d^5 + 28*a^9*b^2*c^2*d^6 - 8*a^10*b*c*d^7 + a^11*d^8))
^(3/4)*x - (a^2*b^6*c^6 - 6*a^3*b^5*c^5*d + 15*a^4*b^4*c^4*d^2 - 20*a^5*b^3
*c^3*d^3 + 15*a^6*b^2*c^2*d^4 - 6*a^7*b*c*d^5 + a^8*d^6)*(-b^7/(a^3*b^8*c^8
 - 8*a^4*b^7*c^7*d + 28*a^5*b^6*c^6*d^2 - 56*a^6*b^5*c^5*d^3 + 70*a^7*b^4*c
^4*d^4 - 56*a^8*b^3*c^3*d^5 + 28*a^9*b^2*c^2*d^6 - 8*a^10*b*c*d^7 + a^11*d^
8))^(3/4)*sqrt((b^4*x^2 + (a^2*b^4*c^4 - 4*a^3*b^3*c^3*d + 6*a^4*b^2*c^2*d^
2 - 4*a^5*b*c*d^3 + a^6*d^4)*sqrt(-b^7/(a^3*b^8*c^8 - 8*a^4*b^7*c^7*d + 28*
a^5*b^6*c^6*d^2 - 56*a^6*b^5*c^5*d^3 + 70*a^7*b^4*c^4*d^4 - 56*a^8*b^3*c^3*
d^5 + 28*a^9*b^2*c^2*d^6 - 8*a^10*b*c*d^7 + a^11*d^8)))/b^4))/b^5) + 4*(-b^
7/(a^3*b^8*c^8 - 8*a^4*b^7*c^7*d + 28*a^5*b^6*c^6*d^2 - 56*a^6*b^5*c^5*d^3
+ 70*a^7*b^4*c^4*d^4 - 56*a^8*b^3*c^3*d^5 + 28*a^9*b^2*c^2*d^6 - 8*a^10*b*c
*d^7 + a^11*d^8))^(1/4)*((b*c^2*d - a*c*d^2)*x^4 + b*c^3 - a*c^2*d)*log(b^2
*x + (-b^7/(a^3*b^8*c^8 - 8*a^4*b^7*c^7*d + 28*a^5*b^6*c^6*d^2 - 56*a^6*b^5
*c^5*d^3 + 70*a^7*b^4*c^4*d^4 - 56*a^8*b^3*c^3*d^5 + 28*a^9*b^2*c^2*d^6 - 8
*a^10*b*c*d^7 + a^11*d^8))^(1/4)*(a*b^2*c^2 - 2*a^2*b*c*d + a^3*d^2)) - 4*(
-b^7/(a^3*b^8*c^8 - 8*a^4*b^7*c^7*d + 28*a^5*b^6*c^6*d^2 - 56*a^6*b^5*c^5*d
^3 + 70*a^7*b^4*c^4*d^4 - 56*a^8*b^3*c^3*d^5 + 28*a^9*b^2*c^2*d^6 - 8*a^10*
b*c*d^7 + a^11*d^8))^(1/4)*((b*c^2*d - a*c*d^2)*x^4 + b*c^3 - a*c^2*d)*log(
b^2*x - (-b^7/(a^3*b^8*c^8 - 8*a^4*b^7*c^7*d + 28*a^5*b^6*c^6*d^2 - 56*a^6*
b^5*c^5*d^3 + 70*a^7*b^4*c^4*d^4 - 56*a^8*b^3*c^3*d^5 + 28*a^9*b^2*c^2*d^6
- 8*a^10*b*c*d^7 + a^11*d^8))^(1/4)*(a*b^2*c^2 - 2*a^2*b*c*d + a^3*d^2)) +
((b*c^2*d - a*c*d^2)*x^4 + b*c^3 - a*c^2*d)*(-(2401*b^4*c^4*d^3 - 4116*a*b^
3*c^3*d^4 + 2646*a^2*b^2*c^2*d^5 - 756*a^3*b*c*d^6 + 81*a^4*d^7)/(b^8*c^15
- 8*a*b^7*c^14*d + 28*a^2*b^6*c^13*d^2 - 56*a^3*b^5*c^12*d^3 + 70*a^4*b^4*c
^11*d^4 - 56*a^5*b^3*c^10*d^5 + 28*a^6*b^2*c^9*d^6 - 8*a^7*b*c^8*d^7 + a^8*
c^7*d^8))^(1/4)*log(-(7*b*c*d - 3*a*d^2)*x + (b^2*c^4 - 2*a*b*c^3*d + a^2*c
^2*d^2)*(-(2401*b^4*c^4*d^3 - 4116*a*b^3*c^3*d^4 + 2646*a^2*b^2*c^2*d^5 - 7
56*a^3*b*c*d^6 + 81*a^4*d^7)/(b^8*c^15 - 8*a*b^7*c^14*d + 28*a^2*b^6*c^13*d
^2 - 56*a^3*b^5*c^12*d^3 + 70*a^4*b^4*c^11*d^4 - 56*a^5*b^3*c^10*d^5 + 28*a
^6*b^2*c^9*d^6 - 8*a^7*b*c^8*d^7 + a^8*c^7*d^8))^(1/4)) - ((b*c^2*d - a*c*d
^2)*x^4 + b*c^3 - a*c^2*d)*(-(2401*b^4*c^4*d^3 - 4116*a*b^3*c^3*d^4 + 2646*
a^2*b^2*c^2*d^5 - 756*a^3*b*c*d^6 + 81*a^4*d^7)/(b^8*c^15 - 8*a*b^7*c^14*d
+ 28*a^2*b^6*c^13*d^2 - 56*a^3*b^5*c^12*d^3 + 70*a^4*b^4*c^11*d^4 - 56*a^5*
b^3*c^10*d^5 + 28*a^6*b^2*c^9*d^6 - 8*a^7*b*c^8*d^7 + a^8*c^7*d^8))^(1/4)*l
og(-(7*b*c*d - 3*a*d^2)*x - (b^2*c^4 - 2*a*b*c^3*d + a^2*c^2*d^2)*(-(2401*b
^4*c^4*d^3 - 4116*a*b^3*c^3*d^4 + 2646*a^2*b^2*c^2*d^5 - 756*a^3*b*c*d^6 +
81*a^4*d^7)/(b^8*c^15 - 8*a*b^7*c^14*d + 28*a^2*b^6*c^13*d^2 - 56*a^3*b^5*c
^12*d^3 + 70*a^4*b^4*c^11*d^4 - 56*a^5*b^3*c^10*d^5 + 28*a^6*b^2*c^9*d^6 -
8*a^7*b*c^8*d^7 + a^8*c^7*d^8))^(1/4)) - 4*d*x)/((b*c^2*d - a*c*d^2)*x^4 +
b*c^3 - a*c^2*d)
```

giac [A] time = 0.19, size = 667, normalized size = 1.30

$$\frac{\left(ab^{3}\right)^{\frac{1}{4}} b \arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}ab^{2}c^{2}-2\sqrt{2}a^{2}bcd+\sqrt{2}a^{3}d^{2}\right)}+\frac{\left(ab^{3}\right)^{\frac{1}{4}} b \arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}ab^{2}c^{2}-2\sqrt{2}a^{2}bcd+\sqrt{2}a^{3}d^{2}\right)}+\frac{\left(ab^{3}\right)^{\frac{1}{4}} b \log\left(x^{2}+\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{4\left(\sqrt{2}ab^{2}c^{2}-2\sqrt{2}a^{2}bcd+\sqrt{2}a^{3}d^{2}\right)}-\frac{\left(ab^{3}\right)^{\frac{1}{4}} b \log\left(x^{2}-\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{4\left(\sqrt{2}ab^{2}c^{2}-2\sqrt{2}a^{2}bcd+\sqrt{2}a^{3}d^{2}\right)}-\frac{\left(7\left(cd^{3}\right)^{\frac{1}{4}}bc-3\left(cd^{3}\right)^{\frac{1}{4}}ad\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{8\left(\sqrt{2}b^{2}c^{4}-2\sqrt{2}abc^{3}d+\sqrt{2}a^{2}c^{2}d^{2}\right)}-\frac{\left(7\left(cd^{3}\right)^{\frac{1}{4}}bc-3\left(cd^{3}\right)^{\frac{1}{4}}ad\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{8\left(\sqrt{2}b^{2}c^{4}-2\sqrt{2}abc^{3}d+\sqrt{2}a^{2}c^{2}d^{2}\right)}-\frac{\left(7\left(cd^{3}\right)^{\frac{1}{4}}bc-3\left(cd^{3}\right)^{\frac{1}{4}}ad\right)\log\left(x^{2}+\sqrt{2}x\left(\frac{c}{d}\right)^{\frac{1}{4}}+\sqrt{\frac{c}{d}}\right)}{16\left(\sqrt{2}b^{2}c^{4}-2\sqrt{2}abc^{3}d+\sqrt{2}a^{2}c^{2}d^{2}\right)}+\frac{\left(7\left(cd^{3}\right)^{\frac{1}{4}}bc-3\left(cd^{3}\right)^{\frac{1}{4}}ad\right)\log\left(x^{2}-\sqrt{2}x\left(\frac{c}{d}\right)^{\frac{1}{4}}+\sqrt{\frac{c}{d}}\right)}{16\left(\sqrt{2}b^{2}c^{4}-2\sqrt{2}abc^{3}d+\sqrt{2}a^{2}c^{2}d^{2}\right)}-\frac{dx}{4\left(dx^{4}+c\right)\left(bc^{2}-acd\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a)/(d*x^4+c)^2,x, algorithm="giac")
```

```
[Out] 1/2*(a*b^3)^(1/4)*b*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/b)^(1/4))/(a/b)^(1
/4))/(sqrt(2)*a*b^2*c^2 - 2*sqrt(2)*a^2*b*c*d + sqrt(2)*a^3*d^2) + 1/2*(a*b
^3)^(1/4)*b*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(sq
rt(2)*a*b^2*c^2 - 2*sqrt(2)*a^2*b*c*d + sqrt(2)*a^3*d^2) + 1/4*(a*b^3)^(1/4
)*b*log(x^2 + sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(sqrt(2)*a*b^2*c^2 - 2*sqr
```

```
t(2)*a^2*b*c*d + sqrt(2)*a^3*d^2) - 1/4*(a*b^3)^(1/4)*b*log(x^2 - sqrt(2)*x
*(a/b)^(1/4) + sqrt(a/b))/(sqrt(2)*a*b^2*c^2 - 2*sqrt(2)*a^2*b*c*d + sqrt(2
)*a^3*d^2) - 1/8*(7*(c*d^3)^(1/4)*b*c - 3*(c*d^3)^(1/4)*a*d)*arctan(1/2*sqr
t(2)*(2*x + sqrt(2)*(c/d)^(1/4))/(c/d)^(1/4))/(sqrt(2)*b^2*c^4 - 2*sqrt(2)*
a*b*c^3*d + sqrt(2)*a^2*c^2*d^2) - 1/8*(7*(c*d^3)^(1/4)*b*c - 3*(c*d^3)^(1/
4)*a*d)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(c/d)^(1/4))/(c/d)^(1/4))/(sqrt(2
)*b^2*c^4 - 2*sqrt(2)*a*b*c^3*d + sqrt(2)*a^2*c^2*d^2) - 1/16*(7*(c*d^3)^(1
/4)*b*c - 3*(c*d^3)^(1/4)*a*d)*log(x^2 + sqrt(2)*x*(c/d)^(1/4) + sqrt(c/d))
/(sqrt(2)*b^2*c^4 - 2*sqrt(2)*a*b*c^3*d + sqrt(2)*a^2*c^2*d^2) + 1/16*(7*(c
*d^3)^(1/4)*b*c - 3*(c*d^3)^(1/4)*a*d)*log(x^2 - sqrt(2)*x*(c/d)^(1/4) + sq
rt(c/d))/(sqrt(2)*b^2*c^4 - 2*sqrt(2)*a*b*c^3*d + sqrt(2)*a^2*c^2*d^2) - 1/
4*d*x/((d*x^4 + c)*(b*c^2 - a*c*d))
```

maple [A]   time = 0.06, size = 550, normalized size = 1.07

$$\frac{a\,d^2x}{4\,(ad-bc)^2\left(d\,x^4+c\right)c}-\frac{bdx}{4\,(ad-bc)^2\left(d\,x^4+c\right)}+\frac{3\left(\frac{c}{d}\right)^{\frac14}\sqrt{2}\,a\,d^2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{c}{d}\right)^{\frac14}}-1\right)}{16\,(ad-bc)^2\,c^2}+\frac{3\left(\frac{c}{d}\right)^{\frac14}\sqrt{2}\,a\,d^2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{c}{d}\right)^{\frac14}}+1\right)}{16\,(ad-bc)^2\,c^2}+\frac{3\left(\frac{c}{d}\right)^{\frac14}\sqrt{2}\,a\,d^2\ln\left(\frac{x^2+\left(\frac{c}{d}\right)^{\frac14}\sqrt{2}\,x+\sqrt{\frac{c}{d}}}{x^2-\left(\frac{c}{d}\right)^{\frac14}\sqrt{2}\,x+\sqrt{\frac{c}{d}}}\right)}{32\,(ad-bc)^2\,c^2}+\frac{\left(\frac{a}{b}\right)^{\frac14}\sqrt{2}\,b^2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac14}}-1\right)}{4\,(ad-bc)^2\,a}+\frac{\left(\frac{a}{b}\right)^{\frac14}\sqrt{2}\,b^2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac14}}+1\right)}{4\,(ad-bc)^2\,a}+\frac{\left(\frac{a}{b}\right)^{\frac14}\sqrt{2}\,b^2\ln\left(\frac{x^2+\left(\frac{a}{b}\right)^{\frac14}\sqrt{2}\,x+\sqrt{\frac{a}{b}}}{x^2-\left(\frac{a}{b}\right)^{\frac14}\sqrt{2}\,x+\sqrt{\frac{a}{b}}}\right)}{8\,(ad-bc)^2\,a}-\frac{7\left(\frac{c}{d}\right)^{\frac14}\sqrt{2}\,bd\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{c}{d}\right)^{\frac14}}-1\right)}{16\,(ad-bc)^2\,c}-\frac{7\left(\frac{c}{d}\right)^{\frac14}\sqrt{2}\,bd\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{c}{d}\right)^{\frac14}}+1\right)}{16\,(ad-bc)^2\,c}-\frac{7\left(\frac{c}{d}\right)^{\frac14}\sqrt{2}\,bd\ln\left(\frac{x^2+\left(\frac{c}{d}\right)^{\frac14}\sqrt{2}\,x+\sqrt{\frac{c}{d}}}{x^2-\left(\frac{c}{d}\right)^{\frac14}\sqrt{2}\,x+\sqrt{\frac{c}{d}}}\right)}{32\,(ad-bc)^2\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^4+a)/(d*x^4+c)^2,x)
```

```
[Out] 1/4*d^2/(a*d-b*c)^2/c*x/(d*x^4+c)*a-1/4*d/(a*d-b*c)^2*x/(d*x^4+c)*b+3/16*d^
2/(a*d-b*c)^2/c^2*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x-1)*a-7/1
6*d/(a*d-b*c)^2/c*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x-1)*b+3/3
2*d^2/(a*d-b*c)^2/c^2*(c/d)^(1/4)*2^(1/2)*ln((x^2+(c/d)^(1/4)*2^(1/2)*x+(c/
d)^(1/2))/(x^2-(c/d)^(1/4)*2^(1/2)*x+(c/d)^(1/2)))*a-7/32*d/(a*d-b*c)^2/c*(
c/d)^(1/4)*2^(1/2)*ln((x^2+(c/d)^(1/4)*2^(1/2)*x+(c/d)^(1/2))/(x^2-(c/d)^(1
/4)*2^(1/2)*x+(c/d)^(1/2)))*b+3/16*d^2/(a*d-b*c)^2/c^2*(c/d)^(1/4)*2^(1/2)*
arctan(2^(1/2)/(c/d)^(1/4)*x+1)*a-7/16*d/(a*d-b*c)^2/c*(c/d)^(1/4)*2^(1/2)*
arctan(2^(1/2)/(c/d)^(1/4)*x+1)*b+1/8*b^2/(a*d-b*c)^2*(a/b)^(1/4)/a*2^(1/2)
*ln((x^2+(a/b)^(1/4)*2^(1/2)*x+(a/b)^(1/2))/(x^2-(a/b)^(1/4)*2^(1/2)*x+(a/b
)^(1/2)))+1/4*b^2/(a*d-b*c)^2*(a/b)^(1/4)/a*2^(1/2)*arctan(2^(1/2)/(a/b)^(1
/4)*x+1)+1/4*b^2/(a*d-b*c)^2*(a/b)^(1/4)/a*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/
4)*x-1)
```

maxima [A]   time = 1.57, size = 481, normalized size = 0.94

$$-\frac{dx}{4\left(\left(bc^2d-acd^2\right)x^4+bc^3-ac^2d\right)}+\frac{\frac{2\sqrt{2}\,b^2\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}\,x+\sqrt{2}\,a^{\frac14}b^{\frac14}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}+\frac{2\sqrt{2}\,b^2\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}\,x-\sqrt{2}\,a^{\frac14}b^{\frac14}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}+\frac{\sqrt{2}\,b^{\frac74}\log\left(\sqrt{b}\,x^2+\sqrt{2}\,a^{\frac14}b^{\frac14}x+\sqrt{a}\right)}{a^{\frac34}}-\frac{\sqrt{2}\,b^{\frac74}\log\left(\sqrt{b}\,x^2-\sqrt{2}\,a^{\frac14}b^{\frac14}x+\sqrt{a}\right)}{a^{\frac34}}}{8\left(b^2c^2-2\,abcd+a^2d^2\right)}-\frac{\frac{2\sqrt{2}\left(7\,bcd-3\,ad^2\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{d}\,x+\sqrt{2}\,c^{\frac14}d^{\frac14}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{c}\sqrt{\sqrt{c}\sqrt{d}}}+\frac{2\sqrt{2}\left(7\,bcd-3\,ad^2\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{d}\,x-\sqrt{2}\,c^{\frac14}d^{\frac14}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{c}\sqrt{\sqrt{c}\sqrt{d}}}+\frac{\sqrt{2}\left(7\,bcd-3\,ad^2\right)\log\left(\sqrt{d}\,x^2+\sqrt{2}\,c^{\frac14}d^{\frac14}x+\sqrt{c}\right)}{c^{\frac34}d^{\frac14}}-\frac{\sqrt{2}\left(7\,bcd-3\,ad^2\right)\log\left(\sqrt{d}\,x^2-\sqrt{2}\,c^{\frac14}d^{\frac14}x+\sqrt{c}\right)}{c^{\frac34}d^{\frac14}}}{32\left(b^2c^3-2\,abc^2d+a^2cd^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a)/(d*x^4+c)^2,x, algorithm="maxima")
```

```
[Out] -1/4*d*x/((b*c^2*d - a*c*d^2)*x^4 + b*c^3 - a*c^2*d) + 1/8*(2*sqrt(2)*b^2*a
rctan(1/2*sqrt(2)*(2*sqrt(b)*x + sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt
(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + 2*sqrt(2)*b^2*arctan(1/2*sqrt(2)*(2
*sqrt(b)*x - sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(
sqrt(a)*sqrt(b))) + sqrt(2)*b^(7/4)*log(sqrt(b)*x^2 + sqrt(2)*a^(1/4)*b^(1/
4)*x + sqrt(a))/a^(3/4) - sqrt(2)*b^(7/4)*log(sqrt(b)*x^2 - sqrt(2)*a^(1/4)
*b^(1/4)*x + sqrt(a))/a^(3/4))/(b^2*c^2 - 2*a*b*c*d + a^2*d^2) - 1/32*(2*sq
rt(2)*(7*b*c*d - 3*a*d^2)*arctan(1/2*sqrt(2)*(2*sqrt(d)*x + sqrt(2)*c^(1/4)
*d^(1/4))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(c)*sqrt(sqrt(c)*sqrt(d))) + 2*sqrt(2
)*(7*b*c*d - 3*a*d^2)*arctan(1/2*sqrt(2)*(2*sqrt(d)*x - sqrt(2)*c^(1/4)*d^(
1/4))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(c)*sqrt(sqrt(c)*sqrt(d))) + sqrt(2)*(7*b
*c*d - 3*a*d^2)*log(sqrt(d)*x^2 + sqrt(2)*c^(1/4)*d^(1/4)*x + sqrt(c))/(c^(
3/4)*d^(1/4)) - sqrt(2)*(7*b*c*d - 3*a*d^2)*log(sqrt(d)*x^2 - sqrt(2)*c^(1/
4)*d^(1/4)*x + sqrt(c))/(c^(3/4)*d^(1/4)))/(b^2*c^3 - 2*a*b*c^2*d + a^2*c*d
^2)
```

mupad [B]   time = 4.00, size = 21975, normalized size = 42.84

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^4)*(c + d*x^4)^2),x)
```

```
[Out] 2*atan(((-(81*a^4*d^7 + 2401*b^4*c^4*d^3 - 4116*a*b^3*c^3*d^4 + 2646*a^2*b^
2*c^2*d^5 - 756*a^3*b*c*d^6)/(65536*b^8*c^15 + 65536*a^8*c^7*d^8 - 524288*a
^7*b*c^8*d^7 + 1835008*a^2*b^6*c^13*d^2 - 3670016*a^3*b^5*c^12*d^3 + 458752
0*a^4*b^4*c^11*d^4 - 3670016*a^5*b^3*c^10*d^5 + 1835008*a^6*b^2*c^9*d^6 - 5
24288*a*b^7*c^14*d))^(1/4)*((-(81*a^4*d^7 + 2401*b^4*c^4*d^3 - 4116*a*b^3*c
^3*d^4 + 2646*a^2*b^2*c^2*d^5 - 756*a^3*b*c*d^6)/(65536*b^8*c^15 + 65536*a^
8*c^7*d^8 - 524288*a^7*b*c^8*d^7 + 1835008*a^2*b^6*c^13*d^2 - 3670016*a^3*b
^5*c^12*d^3 + 4587520*a^4*b^4*c^11*d^4 - 3670016*a^5*b^3*c^10*d^5 + 1835008
*a^6*b^2*c^9*d^6 - 524288*a*b^7*c^14*d))^(1/4)*((((81*a^4*b^7*d^10)/16 + 28
*b^11*c^4*d^6 - (2145*a*b^10*c^3*d^7)/16 - (675*a^3*b^8*c*d^9)/16 + (1971*a
^2*b^9*c^2*d^8)/16)*1i)/(b^3*c^7 - a^3*c^4*d^3 + 3*a^2*b*c^5*d^2 - 3*a*b^2*
c^6*d) + (-(81*a^4*d^7 + 2401*b^4*c^4*d^3 - 4116*a*b^3*c^3*d^4 + 2646*a^2*b
^2*c^2*d^5 - 756*a^3*b*c*d^6)/(65536*b^8*c^15 + 65536*a^8*c^7*d^8 - 524288*
a^7*b*c^8*d^7 + 1835008*a^2*b^6*c^13*d^2 - 3670016*a^3*b^5*c^12*d^3 + 45875
20*a^4*b^4*c^11*d^4 - 3670016*a^5*b^3*c^10*d^5 + 1835008*a^6*b^2*c^9*d^6 -
524288*a*b^7*c^14*d))^(3/4)*(((-(81*a^4*d^7 + 2401*b^4*c^4*d^3 - 4116*a*b^3
*c^3*d^4 + 2646*a^2*b^2*c^2*d^5 - 756*a^3*b*c*d^6)/(65536*b^8*c^15 + 65536*
a^8*c^7*d^8 - 524288*a^7*b*c^8*d^7 + 1835008*a^2*b^6*c^13*d^2 - 3670016*a^3
*b^5*c^12*d^3 + 4587520*a^4*b^4*c^11*d^4 - 3670016*a^5*b^3*c^10*d^5 + 18350
08*a^6*b^2*c^9*d^6 - 524288*a*b^7*c^14*d))^(1/4)*(28672*a^2*b^13*c^13*d^5 -
 4096*a*b^14*c^14*d^4 - 78848*a^3*b^12*c^12*d^6 + 90112*a^4*b^11*c^11*d^7 +
 28672*a^5*b^10*c^10*d^8 - 229376*a^6*b^9*c^9*d^9 + 329728*a^7*b^8*c^8*d^10
 - 253952*a^8*b^7*c^7*d^11 + 114688*a^9*b^6*c^6*d^12 - 28672*a^10*b^5*c^5*d
^13 + 3072*a^11*b^4*c^4*d^14))/(b^3*c^7 - a^3*c^4*d^3 + 3*a^2*b*c^5*d^2 - 3
*a*b^2*c^6*d) - (x*(65536*b^17*c^15*d^4 - 524288*a*b^16*c^14*d^5 + 1835008*
a^2*b^15*c^13*d^6 - 3469312*a^3*b^14*c^12*d^7 + 2809856*a^4*b^13*c^11*d^8 +
 3362816*a^5*b^12*c^10*d^9 - 14516224*a^6*b^11*c^9*d^10 + 24190976*a^7*b^10
*c^8*d^11 - 25280512*a^8*b^9*c^7*d^12 + 17833984*a^9*b^8*c^6*d^13 - 8486912
*a^10*b^7*c^5*d^14 + 2609152*a^11*b^6*c^4*d^15 - 466944*a^12*b^5*c^3*d^16 +
 36864*a^13*b^4*c^2*d^17)*1i)/(64*(b^6*c^10 + a^6*c^4*d^6 - 6*a^5*b*c^5*d^5
 + 15*a^2*b^4*c^8*d^2 - 20*a^3*b^3*c^7*d^3 + 15*a^4*b^2*c^6*d^4 - 6*a*b^5*c
^9*d)))*1i) + (x*(81*a^4*b^9*d^11 + 3185*b^13*c^4*d^7 - 4788*a*b^12*c^3*d^8
 - 756*a^3*b^10*c*d^10 + 2790*a^2*b^11*c^2*d^9))/(64*(b^6*c^10 + a^6*c^4*d^
6 - 6*a^5*b*c^5*d^5 + 15*a^2*b^4*c^8*d^2 - 20*a^3*b^3*c^7*d^3 + 15*a^4*b^2*
c^6*d^4 - 6*a*b^5*c^9*d))) - (-(81*a^4*d^7 + 2401*b^4*c^4*d^3 - 4116*a*b^3*
c^3*d^4 + 2646*a^2*b^2*c^2*d^5 - 756*a^3*b*c*d^6)/(65536*b^8*c^15 + 65536*a
^8*c^7*d^8 - 524288*a^7*b*c^8*d^7 + 1835008*a^2*b^6*c^13*d^2 - 3670016*a^3*
b^5*c^12*d^3 + 4587520*a^4*b^4*c^11*d^4 - 3670016*a^5*b^3*c^10*d^5 + 183500
8*a^6*b^2*c^9*d^6 - 524288*a*b^7*c^14*d))^(1/4)*((-(81*a^4*d^7 + 2401*b^4*c
^4*d^3 - 4116*a*b^3*c^3*d^4 + 2646*a^2*b^2*c^2*d^5 - 756*a^3*b*c*d^6)/(6553
6*b^8*c^15 + 65536*a^8*c^7*d^8 - 524288*a^7*b*c^8*d^7 + 1835008*a^2*b^6*c^1
3*d^2 - 3670016*a^3*b^5*c^12*d^3 + 4587520*a^4*b^4*c^11*d^4 - 3670016*a^5*b
^3*c^10*d^5 + 1835008*a^6*b^2*c^9*d^6 - 524288*a*b^7*c^14*d))^(1/4)*((((81*
a^4*b^7*d^10)/16 + 28*b^11*c^4*d^6 - (2145*a*b^10*c^3*d^7)/16 - (675*a^3*b^
8*c*d^9)/16 + (1971*a^2*b^9*c^2*d^8)/16)*1i)/(b^3*c^7 - a^3*c^4*d^3 + 3*a^2
*b*c^5*d^2 - 3*a*b^2*c^6*d) + (-(81*a^4*d^7 + 2401*b^4*c^4*d^3 - 4116*a*b^3
*c^3*d^4 + 2646*a^2*b^2*c^2*d^5 - 756*a^3*b*c*d^6)/(65536*b^8*c^15 + 65536*
a^8*c^7*d^8 - 524288*a^7*b*c^8*d^7 + 1835008*a^2*b^6*c^13*d^2 - 3670016*a^3
*b^5*c^12*d^3 + 4587520*a^4*b^4*c^11*d^4 - 3670016*a^5*b^3*c^10*d^5 + 18350
08*a^6*b^2*c^9*d^6 - 524288*a*b^7*c^14*d))^(3/4)*(((-(81*a^4*d^7 + 2401*b^4
*c^4*d^3 - 4116*a*b^3*c^3*d^4 + 2646*a^2*b^2*c^2*d^5 - 756*a^3*b*c*d^6)/(65
536*b^8*c^15 + 65536*a^8*c^7*d^8 - 524288*a^7*b*c^8*d^7 + 1835008*a^2*b^6*c
^13*d^2 - 3670016*a^3*b^5*c^12*d^3 + 4587520*a^4*b^4*c^11*d^4 - 3670016*a^5
*b^3*c^10*d^5 + 1835008*a^6*b^2*c^9*d^6 - 524288*a*b^7*c^14*d))^(1/4)*(2867
```

2*a^2*b^13*c^13*d^5 - 4096*a*b^14*c^14*d^4 - 78848*a^3*b^12*c^12*d^6 + 9011
2*a^4*b^11*c^11*d^7 + 28672*a^5*b^10*c^10*d^8 - 229376*a^6*b^9*c^9*d^9 + 32
9728*a^7*b^8*c^8*d^10 - 253952*a^8*b^7*c^7*d^11 + 114688*a^9*b^6*c^6*d^12 -
28672*a^10*b^5*c^5*d^13 + 3072*a^11*b^4*c^4*d^14))/(b^3*c^7 - a^3*c^4*d^3
+ 3*a^2*b*c^5*d^2 - 3*a*b^2*c^6*d) + (x*(65536*b^17*c^15*d^4 - 524288*a*b^1
6*c^14*d^5 + 1835008*a^2*b^15*c^13*d^6 - 3469312*a^3*b^14*c^12*d^7 + 280985
6*a^4*b^13*c^11*d^8 + 3362816*a^5*b^12*c^10*d^9 - 14516224*a^6*b^11*c^9*d^1
0 + 24190976*a^7*b^10*c^8*d^11 - 25280512*a^8*b^9*c^7*d^12 + 17833984*a^9*b
^8*c^6*d^13 - 8486912*a^10*b^7*c^5*d^14 + 2609152*a^11*b^6*c^4*d^15 - 46694
4*a^12*b^5*c^3*d^16 + 36864*a^13*b^4*c^2*d^17)*1i)/(64*(b^6*c^10 + a^6*c^4*
d^6 - 6*a^5*b*c^5*d^5 + 15*a^2*b^4*c^8*d^2 - 20*a^3*b^3*c^7*d^3 + 15*a^4*b^
2*c^6*d^4 - 6*a*b^5*c^9*d)))*1i) - (x*(81*a^4*b^9*d^11 + 3185*b^13*c^4*d^7
- 4788*a*b^12*c^3*d^8 - 756*a^3*b^10*c*d^10 + 2790*a^2*b^11*c^2*d^9))/(64*(
b^6*c^10 + a^6*c^4*d^6 - 6*a^5*b*c^5*d^5 + 15*a^2*b^4*c^8*d^2 - 20*a^3*b^3*
c^7*d^3 + 15*a^4*b^2*c^6*d^4 - 6*a*b^5*c^9*d))))/((-(81*a^4*d^7 + 2401*b^4*
c^4*d^3 - 4116*a*b^3*c^3*d^4 + 2646*a^2*b^2*c^2*d^5 - 756*a^3*b*c*d^6)/(655
36*b^8*c^15 + 65536*a^8*c^7*d^8 - 524288*a^7*b*c^8*d^7 + 1835008*a^2*b^6*c^
13*d^2 - 3670016*a^3*b^5*c^12*d^3 + 4587520*a^4*b^4*c^11*d^4 - 3670016*a^5*
b^3*c^10*d^5 + 1835008*a^6*b^2*c^9*d^6 - 524288*a*b^7*c^14*d))^(1/4)*((-(81
*a^4*d^7 + 2401*b^4*c^4*d^3 - 4116*a*b^3*c^3*d^4 + 2646*a^2*b^2*c^2*d^5 - 7
56*a^3*b*c*d^6)/(65536*b^8*c^15 + 65536*a^8*c^7*d^8 - 524288*a^7*b*c^8*d^7
+ 1835008*a^2*b^6*c^13*d^2 - 3670016*a^3*b^5*c^12*d^3 + 4587520*a^4*b^4*c^1
1*d^4 - 3670016*a^5*b^3*c^10*d^5 + 1835008*a^6*b^2*c^9*d^6 - 524288*a*b^7*c
^14*d))^(1/4)*((((81*a^4*b^7*d^10)/16 + 28*b^11*c^4*d^6 - (2145*a*b^10*c^3*
d^7)/16 - (675*a^3*b^8*c*d^9)/16 + (1971*a^2*b^9*c^2*d^8)/16)*1i)/(b^3*c^7
- a^3*c^4*d^3 + 3*a^2*b*c^5*d^2 - 3*a*b^2*c^6*d) + (-(81*a^4*d^7 + 2401*b^4
*c^4*d^3 - 4116*a*b^3*c^3*d^4 + 2646*a^2*b^2*c^2*d^5 - 756*a^3*b*c*d^6)/(65
536*b^8*c^15 + 65536*a^8*c^7*d^8 - 524288*a^7*b*c^8*d^7 + 1835008*a^2*b^6*c
^13*d^2 - 3670016*a^3*b^5*c^12*d^3 + 4587520*a^4*b^4*c^11*d^4 - 3670016*a^5
*b^3*c^10*d^5 + 1835008*a^6*b^2*c^9*d^6 - 524288*a*b^7*c^14*d))^(3/4)*(((-(
81*a^4*d^7 + 2401*b^4*c^4*d^3 - 4116*a*b^3*c^3*d^4 + 2646*a^2*b^2*c^2*d^5 -
756*a^3*b*c*d^6)/(65536*b^8*c^15 + 65536*a^8*c^7*d^8 - 524288*a^7*b*c^8*d^
7 + 1835008*a^2*b^6*c^13*d^2 - 3670016*a^3*b^5*c^12*d^3 + 4587520*a^4*b^4*c
^11*d^4 - 3670016*a^5*b^3*c^10*d^5 + 1835008*a^6*b^2*c^9*d^6 - 524288*a*b^7
*c^14*d))^(1/4)*(28672*a^2*b^13*c^13*d^5 - 4096*a*b^14*c^14*d^4 - 78848*a^3
*b^12*c^12*d^6 + 90112*a^4*b^11*c^11*d^7 + 28672*a^5*b^10*c^10*d^8 - 229376
*a^6*b^9*c^9*d^9 + 329728*a^7*b^8*c^8*d^10 - 253952*a^8*b^7*c^7*d^11 + 1146
88*a^9*b^6*c^6*d^12 - 28672*a^10*b^5*c^5*d^13 + 3072*a^11*b^4*c^4*d^14))/(b
^3*c^7 - a^3*c^4*d^3 + 3*a^2*b*c^5*d^2 - 3*a*b^2*c^6*d) - (x*(65536*b^17*c^
15*d^4 - 524288*a*b^16*c^14*d^5 + 1835008*a^2*b^15*c^13*d^6 - 3469312*a^3*b
^14*c^12*d^7 + 2809856*a^4*b^13*c^11*d^8 + 3362816*a^5*b^12*c^10*d^9 - 1451
6224*a^6*b^11*c^9*d^10 + 24190976*a^7*b^10*c^8*d^11 - 25280512*a^8*b^9*c^7*
d^12 + 17833984*a^9*b^8*c^6*d^13 - 8486912*a^10*b^7*c^5*d^14 + 2609152*a^11
*b^6*c^4*d^15 - 466944*a^12*b^5*c^3*d^16 + 36864*a^13*b^4*c^2*d^17)*1i)/(64
*(b^6*c^10 + a^6*c^4*d^6 - 6*a^5*b*c^5*d^5 + 15*a^2*b^4*c^8*d^2 - 20*a^3*b^
3*c^7*d^3 + 15*a^4*b^2*c^6*d^4 - 6*a*b^5*c^9*d)))*1i)*1i + (x*(81*a^4*b^9*d
^11 + 3185*b^13*c^4*d^7 - 4788*a*b^12*c^3*d^8 - 756*a^3*b^10*c*d^10 + 2790*
a^2*b^11*c^2*d^9)*1i)/(64*(b^6*c^10 + a^6*c^4*d^6 - 6*a^5*b*c^5*d^5 + 15*a^
2*b^4*c^8*d^2 - 20*a^3*b^3*c^7*d^3 + 15*a^4*b^2*c^6*d^4 - 6*a*b^5*c^9*d)))
+ (-(81*a^4*d^7 + 2401*b^4*c^4*d^3 - 4116*a*b^3*c^3*d^4 + 2646*a^2*b^2*c^2*
d^5 - 756*a^3*b*c*d^6)/(65536*b^8*c^15 + 65536*a^8*c^7*d^8 - 524288*a^7*b*c
^8*d^7 + 1835008*a^2*b^6*c^13*d^2 - 3670016*a^3*b^5*c^12*d^3 + 4587520*a^4*
b^4*c^11*d^4 - 3670016*a^5*b^3*c^10*d^5 + 1835008*a^6*b^2*c^9*d^6 - 524288*
a*b^7*c^14*d))^(1/4)*((-(81*a^4*d^7 + 2401*b^4*c^4*d^3 - 4116*a*b^3*c^3*d^4
+ 2646*a^2*b^2*c^2*d^5 - 756*a^3*b*c*d^6)/(65536*b^8*c^15 + 65536*a^8*c^7*
d^8 - 524288*a^7*b*c^8*d^7 + 1835008*a^2*b^6*c^13*d^2 - 3670016*a^3*b^5*c^1
2*d^3 + 4587520*a^4*b^4*c^11*d^4 - 3670016*a^5*b^3*c^10*d^5 + 1835008*a^6*b
^2*c^9*d^6 - 524288*a*b^7*c^14*d))^(1/4)*((((81*a^4*b^7*d^10)/16 + 28*b^11*
c^4*d^6 - (2145*a*b^10*c^3*d^7)/16 - (675*a^3*b^8*c*d^9)/16 + (1971*a^2*b^9

```
*c^2*d^8)/16)*1i)/(b^3*c^7 - a^3*c^4*d^3 + 3*a^2*b*c^5*d^2 - 3*a*b^2*c^6*d)
 + (-(81*a^4*d^7 + 2401*b^4*c^4*d^3 - 4116*a*b^3*c^3*d^4 + 2646*a^2*b^2*c^2
*d^5 - 756*a^3*b*c*d^6)/(65536*b^8*c^15 + 65536*a^8*c^7*d^8 - 524288*a^7*b*
c^8*d^7 + 1835008*a^2*b^6*c^13*d^2 - 3670016*a^3*b^5*c^12*d^3 + 4587520*a^4
*b^4*c^11*d^4 - 3670016*a^5*b^3*c^10*d^5 + 1835008*a^6*b^2*c^9*d^6 - 524288
*a*b^7*c^14*d))^(3/4)*(((-(81*a^4*d^7 + 2401*b^4*c^4*d^3 - 4116*a*b^3*c^3*d
^4 + 2646*a^2*b^2*c^2*d^5 - 756*a^3*b*c*d^6)/(65536*b^8*c^15 + 65536*a^8*c^
7*d^8 - 524288*a^7*b*c^8*d^7 + 1835008*a^2*b^6*c^13*d^2 - 3670016*a^3*b^5*c
^12*d^3 + 4587520*a^4*b^4*c^11*d^4 - 3670016*a^5*b^3*c^10*d^5 + 1835008*a^6
*b^2*c^9*d^6 - 524288*a*b^7*c^14*d))^(1/4)*(28672*a^2*b^13*c^13*d^5 - 4096*
a*b^14*c^14*d^4 - 78848*a^3*b^12*c^12*d^6 + 90112*a^4*b^11*c^11*d^7 + 28672
*a^5*b^10*c^10*d^8 - 229376*a^6*b^9*c^9*d^9 + 329728*a^7*b^8*c^8*d^10 - 253
952*a^8*b^7*c^7*d^11 + 114688*a^9*b^6*c^6*d^12 - 28672*a^10*b^5*c^5*d^13 +
3072*a^11*b^4*c^4*d^14))/(b^3*c^7 - a^3*c^4*d^3 + 3*a^2*b*c^5*d^2 - 3*a*b^2
*c^6*d) + (x*(65536*b^17*c^15*d^4 - 524288*a*b^16*c^14*d^5 + 1835008*a^2*b^
15*c^13*d^6 - 3469312*a^3*b^14*c^12*d^7 + 2809856*a^4*b^13*c^11*d^8 + 33628
16*a^5*b^12*c^10*d^9 - 14516224*a^6*b^11*c^9*d^10 + 24190976*a^7*b^10*c^8*d
^11 - 25280512*a^8*b^9*c^7*d^12 + 17833984*a^9*b^8*c^6*d^13 - 8486912*a^10*
b^7*c^5*d^14 + 2609152*a^11*b^6*c^4*d^15 - 466944*a^12*b^5*c^3*d^16 + 36864
*a^13*b^4*c^2*d^17)*1i)/(64*(b^6*c^10 + a^6*c^4*d^6 - 6*a^5*b*c^5*d^5 + 15*
a^2*b^4*c^8*d^2 - 20*a^3*b^3*c^7*d^3 + 15*a^4*b^2*c^6*d^4 - 6*a*b^5*c^9*d))
)*1i)*1i - (x*(81*a^4*b^9*d^11 + 3185*b^13*c^4*d^7 - 4788*a*b^12*c^3*d^8 -
756*a^3*b^10*c*d^10 + 2790*a^2*b^11*c^2*d^9)*1i)/(64*(b^6*c^10 + a^6*c^4*d^
6 - 6*a^5*b*c^5*d^5 + 15*a^2*b^4*c^8*d^2 - 20*a^3*b^3*c^7*d^3 + 15*a^4*b^2*
c^6*d^4 - 6*a*b^5*c^9*d)))))*(-(81*a^4*d^7 + 2401*b^4*c^4*d^3 - 4116*a*b^3*
c^3*d^4 + 2646*a^2*b^2*c^2*d^5 - 756*a^3*b*c*d^6)/(65536*b^8*c^15 + 65536*a
^8*c^7*d^8 - 524288*a^7*b*c^8*d^7 + 1835008*a^2*b^6*c^13*d^2 - 3670016*a^3*
b^5*c^12*d^3 + 4587520*a^4*b^4*c^11*d^4 - 3670016*a^5*b^3*c^10*d^5 + 183500
8*a^6*b^2*c^9*d^6 - 524288*a*b^7*c^14*d))^(1/4) - atan(((-(81*a^4*d^7 + 240
1*b^4*c^4*d^3 - 4116*a*b^3*c^3*d^4 + 2646*a^2*b^2*c^2*d^5 - 756*a^3*b*c*d^6
)/(65536*b^8*c^15 + 65536*a^8*c^7*d^8 - 524288*a^7*b*c^8*d^7 + 1835008*a^2*
b^6*c^13*d^2 - 3670016*a^3*b^5*c^12*d^3 + 4587520*a^4*b^4*c^11*d^4 - 367001
6*a^5*b^3*c^10*d^5 + 1835008*a^6*b^2*c^9*d^6 - 524288*a*b^7*c^14*d))^(1/4)*
((-(81*a^4*d^7 + 2401*b^4*c^4*d^3 - 4116*a*b^3*c^3*d^4 + 2646*a^2*b^2*c^2*d
^5 - 756*a^3*b*c*d^6)/(65536*b^8*c^15 + 65536*a^8*c^7*d^8 - 524288*a^7*b*c^
8*d^7 + 1835008*a^2*b^6*c^13*d^2 - 3670016*a^3*b^5*c^12*d^3 + 4587520*a^4*b
^4*c^11*d^4 - 3670016*a^5*b^3*c^10*d^5 + 1835008*a^6*b^2*c^9*d^6 - 524288*a
*b^7*c^14*d))^(1/4)*(((81*a^4*b^7*d^10)/16 + 28*b^11*c^4*d^6 - (2145*a*b^10
*c^3*d^7)/16 - (675*a^3*b^8*c*d^9)/16 + (1971*a^2*b^9*c^2*d^8)/16)/(b^3*c^7
 - a^3*c^4*d^3 + 3*a^2*b*c^5*d^2 - 3*a*b^2*c^6*d) + (-(81*a^4*d^7 + 2401*b^
4*c^4*d^3 - 4116*a*b^3*c^3*d^4 + 2646*a^2*b^2*c^2*d^5 - 756*a^3*b*c*d^6)/(6
5536*b^8*c^15 + 65536*a^8*c^7*d^8 - 524288*a^7*b*c^8*d^7 + 1835008*a^2*b^6*
c^13*d^2 - 3670016*a^3*b^5*c^12*d^3 + 4587520*a^4*b^4*c^11*d^4 - 3670016*a^
5*b^3*c^10*d^5 + 1835008*a^6*b^2*c^9*d^6 - 524288*a*b^7*c^14*d))^(3/4)*(((-
(81*a^4*d^7 + 2401*b^4*c^4*d^3 - 4116*a*b^3*c^3*d^4 + 2646*a^2*b^2*c^2*d^5
- 756*a^3*b*c*d^6)/(65536*b^8*c^15 + 65536*a^8*c^7*d^8 - 524288*a^7*b*c^8*d
^7 + 1835008*a^2*b^6*c^13*d^2 - 3670016*a^3*b^5*c^12*d^3 + 4587520*a^4*b^4*
c^11*d^4 - 3670016*a^5*b^3*c^10*d^5 + 1835008*a^6*b^2*c^9*d^6 - 524288*a*b^
7*c^14*d))^(1/4)*(28672*a^2*b^13*c^13*d^5 - 4096*a*b^14*c^14*d^4 - 78848*a^
3*b^12*c^12*d^6 + 90112*a^4*b^11*c^11*d^7 + 28672*a^5*b^10*c^10*d^8 - 22937
6*a^6*b^9*c^9*d^9 + 329728*a^7*b^8*c^8*d^10 - 253952*a^8*b^7*c^7*d^11 + 114
688*a^9*b^6*c^6*d^12 - 28672*a^10*b^5*c^5*d^13 + 3072*a^11*b^4*c^4*d^14))/(
b^3*c^7 - a^3*c^4*d^3 + 3*a^2*b*c^5*d^2 - 3*a*b^2*c^6*d) - (x*(65536*b^17*c
^15*d^4 - 524288*a*b^16*c^14*d^5 + 1835008*a^2*b^15*c^13*d^6 - 3469312*a^3*
b^14*c^12*d^7 + 2809856*a^4*b^13*c^11*d^8 + 3362816*a^5*b^12*c^10*d^9 - 145
16224*a^6*b^11*c^9*d^10 + 24190976*a^7*b^10*c^8*d^11 - 25280512*a^8*b^9*c^7
*d^12 + 17833984*a^9*b^8*c^6*d^13 - 8486912*a^10*b^7*c^5*d^14 + 2609152*a^1
1*b^6*c^4*d^15 - 466944*a^12*b^5*c^3*d^16 + 36864*a^13*b^4*c^2*d^17))/(64*(
b^6*c^10 + a^6*c^4*d^6 - 6*a^5*b*c^5*d^5 + 15*a^2*b^4*c^8*d^2 - 20*a^3*b^3*
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c^7*d^3 + 15*a^4*b^2*c^6*d^4 - 6*a*b^5*c^9*d))))*1i - (x*(81*a^4*b^9*d^11 + 3185*b^13*c^4*d^7 - 4788*a*b^12*c^3*d^8 - 756*a^3*b^10*c*d^10 + 2790*a^2*b^11*c^2*d^9)*1i)/(64*(b^6*c^10 + a^6*c^4*d^6 - 6*a^5*b*c^5*d^5 + 15*a^2*b^4*c^8*d^2 - 20*a^3*b^3*c^7*d^3 + 15*a^4*b^2*c^6*d^4 - 6*a*b^5*c^9*d))) - (-(81*a^4*d^7 + 2401*b^4*c^4*d^3 - 4116*a*b^3*c^3*d^4 + 2646*a^2*b^2*c^2*d^5 - 756*a^3*b*c*d^6)/(65536*b^8*c^15 + 65536*a^8*c^7*d^8 - 524288*a^7*b*c^8*d^7 + 1835008*a^2*b^6*c^13*d^2 - 3670016*a^3*b^5*c^12*d^3 + 4587520*a^4*b^4*c^11*d^4 - 3670016*a^5*b^3*c^10*d^5 + 1835008*a^6*b^2*c^9*d^6 - 524288*a*b^7*c^14*d))^(1/4)*((-(81*a^4*d^7 + 2401*b^4*c^4*d^3 - 4116*a*b^3*c^3*d^4 + 2646*a^2*b^2*c^2*d^5 - 756*a^3*b*c*d^6)/(65536*b^8*c^15 + 65536*a^8*c^7*d^8 - 524288*a^7*b*c^8*d^7 + 1835008*a^2*b^6*c^13*d^2 - 3670016*a^3*b^5*c^12*d^3 + 4587520*a^4*b^4*c^11*d^4 - 3670016*a^5*b^3*c^10*d^5 + 1835008*a^6*b^2*c^9*d^6 - 524288*a*b^7*c^14*d))^(1/4)*(((81*a^4*b^7*d^10)/16 + 28*b^11*c^4*d^6 - (2145*a*b^10*c^3*d^7)/16 - (675*a^3*b^8*c*d^9)/16 + (1971*a^2*b^9*c^2*d^8)/16)/(b^3*c^7 - a^3*c^4*d^3 + 3*a^2*b*c^5*d^2 - 3*a*b^2*c^6*d) + (-(81*a^4*d^7 + 2401*b^4*c^4*d^3 - 4116*a*b^3*c^3*d^4 + 2646*a^2*b^2*c^2*d^5 - 756*a^3*b*c*d^6)/(65536*b^8*c^15 + 65536*a^8*c^7*d^8 - 524288*a^7*b*c^8*d^7 + 1835008*a^2*b^6*c^13*d^2 - 3670016*a^3*b^5*c^12*d^3 + 4587520*a^4*b^4*c^11*d^4 - 3670016*a^5*b^3*c^10*d^5 + 1835008*a^6*b^2*c^9*d^6 - 524288*a*b^7*c^14*d))^(3/4)*(((-(81*a^4*d^7 + 2401*b^4*c^4*d^3 - 4116*a*b^3*c^3*d^4 + 2646*a^2*b^2*c^2*d^5 - 756*a^3*b*c*d^6)/(65536*b^8*c^15 + 65536*a^8*c^7*d^8 - 524288*a^7*b*c^8*d^7 + 1835008*a^2*b^6*c^13*d^2 - 3670016*a^3*b^5*c^12*d^3 + 4587520*a^4*b^4*c^11*d^4 - 3670016*a^5*b^3*c^10*d^5 + 1835008*a^6*b^2*c^9*d^6 - 524288*a*b^7*c^14*d))^(1/4)*(28672*a^2*b^13*c^13*d^5 - 4096*a*b^14*c^14*d^4 - 78848*a^3*b^12*c^12*d^6 + 90112*a^4*b^11*c^11*d^7 + 28672*a^5*b^10*c^10*d^8 - 229376*a^6*b^9*c^9*d^9 + 329728*a^7*b^8*c^8*d^10 - 253952*a^8*b^7*c^7*d^11 + 114688*a^9*b^6*c^6*d^12 - 28672*a^10*b^5*c^5*d^13 + 3072*a^11*b^4*c^4*d^14))/(b^3*c^7 - a^3*c^4*d^3 + 3*a^2*b*c^5*d^2 - 3*a*b^2*c^6*d) + (x*(65536*b^17*c^15*d^4 - 524288*a*b^16*c^14*d^5 + 1835008*a^2*b^15*c^13*d^6 - 3469312*a^3*b^14*c^12*d^7 + 2809856*a^4*b^13*c^11*d^8 + 3362816*a^5*b^12*c^10*d^9 - 14516224*a^6*b^11*c^9*d^10 + 24190976*a^7*b^10*c^8*d^11 - 25280512*a^8*b^9*c^7*d^12 + 17833984*a^9*b^8*c^6*d^13 - 8486912*a^10*b^7*c^5*d^14 + 2609152*a^11*b^6*c^4*d^15 - 466944*a^12*b^5*c^3*d^16 + 36864*a^13*b^4*c^2*d^17))/(64*(b^6*c^10 + a^6*c^4*d^6 - 6*a^5*b*c^5*d^5 + 15*a^2*b^4*c^8*d^2 - 20*a^3*b^3*c^7*d^3 + 15*a^4*b^2*c^6*d^4 - 6*a*b^5*c^9*d))))*1i + (x*(81*a^4*b^9*d^11 + 3185*b^13*c^4*d^7 - 4788*a*b^12*c^3*d^8 - 756*a^3*b^10*c*d^10 + 2790*a^2*b^11*c^2*d^9)*1i)/(64*(b^6*c^10 + a^6*c^4*d^6 - 6*a^5*b*c^5*d^5 + 15*a^2*b^4*c^8*d^2 - 20*a^3*b^3*c^7*d^3 + 15*a^4*b^2*c^6*d^4 - 6*a*b^5*c^9*d))))/((-(81*a^4*d^7 + 2401*b^4*c^4*d^3 - 4116*a*b^3*c^3*d^4 + 2646*a^2*b^2*c^2*d^5 - 756*a^3*b*c*d^6)/(65536*b^8*c^15 + 65536*a^8*c^7*d^8 - 524288*a^7*b*c^8*d^7 + 1835008*a^2*b^6*c^13*d^2 - 3670016*a^3*b^5*c^12*d^3 + 4587520*a^4*b^4*c^11*d^4 - 3670016*a^5*b^3*c^10*d^5 + 1835008*a^6*b^2*c^9*d^6 - 524288*a*b^7*c^14*d))^(1/4)*((-(81*a^4*d^7 + 2401*b^4*c^4*d^3 - 4116*a*b^3*c^3*d^4 + 2646*a^2*b^2*c^2*d^5 - 756*a^3*b*c*d^6)/(65536*b^8*c^15 + 65536*a^8*c^7*d^8 - 524288*a^7*b*c^8*d^7 + 1835008*a^2*b^6*c^13*d^2 - 3670016*a^3*b^5*c^12*d^3 + 4587520*a^4*b^4*c^11*d^4 - 3670016*a^5*b^3*c^10*d^5 + 1835008*a^6*b^2*c^9*d^6 - 524288*a*b^7*c^14*d))^(1/4)*(((81*a^4*b^7*d^10)/16 + 28*b^11*c^4*d^6 - (2145*a*b^10*c^3*d^7)/16 - (675*a^3*b^8*c*d^9)/16 + (1971*a^2*b^9*c^2*d^8)/16)/(b^3*c^7 - a^3*c^4*d^3 + 3*a^2*b*c^5*d^2 - 3*a*b^2*c^6*d) + (-(81*a^4*d^7 + 2401*b^4*c^4*d^3 - 4116*a*b^3*c^3*d^4 + 2646*a^2*b^2*c^2*d^5 - 756*a^3*b*c*d^6)/(65536*b^8*c^15 + 65536*a^8*c^7*d^8 - 524288*a^7*b*c^8*d^7 + 1835008*a^2*b^6*c^13*d^2 - 3670016*a^3*b^5*c^12*d^3 + 4587520*a^4*b^4*c^11*d^4 - 3670016*a^5*b^3*c^10*d^5 + 1835008*a^6*b^2*c^9*d^6 - 524288*a*b^7*c^14*d))^(3/4)*(((-(81*a^4*d^7 + 2401*b^4*c^4*d^3 - 4116*a*b^3*c^3*d^4 + 2646*a^2*b^2*c^2*d^5 - 756*a^3*b*c*d^6)/(65536*b^8*c^15 + 65536*a^8*c^7*d^8 - 524288*a^7*b*c^8*d^7 + 1835008*a^2*b^6*c^13*d^2 - 3670016*a^3*b^5*c^12*d^3 + 4587520*a^4*b^4*c^11*d^4 - 3670016*a^5*b^3*c^10*d^5 + 1835008*a^6*b^2*c^9*d^6 - 524288*a*b^7*c^14*d))^(1/4)*(28672*a^2*b^13*c^13*d^5 - 4096*a*b^14*c^14*d^4 - 78848*a^3*b^12*c^12*d^6 + 90112*a^4*b^11*c^11*d^7 +

28672*a^5*b^10*c^10*d^8 - 229376*a^6*b^9*c^9*d^9 + 329728*a^7*b^8*c^8*d^10 - 253952*a^8*b^7*c^7*d^11 + 114688*a^9*b^6*c^6*d^12 - 28672*a^10*b^5*c^5*d^13 + 3072*a^11*b^4*c^4*d^14))/(b^3*c^7 - a^3*c^4*d^3 + 3*a^2*b*c^5*d^2 - 3*a*b^2*c^6*d) - (x*(65536*b^17*c^15*d^4 - 524288*a*b^16*c^14*d^5 + 1835008*a^2*b^15*c^13*d^6 - 3469312*a^3*b^14*c^12*d^7 + 2809856*a^4*b^13*c^11*d^8 + 3362816*a^5*b^12*c^10*d^9 - 14516224*a^6*b^11*c^9*d^10 + 24190976*a^7*b^10*c^8*d^11 - 25280512*a^8*b^9*c^7*d^12 + 17833984*a^9*b^8*c^6*d^13 - 8486912*a^10*b^7*c^5*d^14 + 2609152*a^11*b^6*c^4*d^15 - 466944*a^12*b^5*c^3*d^16 + 36864*a^13*b^4*c^2*d^17))/(64*(b^6*c^10 + a^6*c^4*d^6 - 6*a^5*b*c^5*d^5 + 15*a^2*b^4*c^8*d^2 - 20*a^3*b^3*c^7*d^3 + 15*a^4*b^2*c^6*d^4 - 6*a*b^5*c^9*d)))) - (x*(81*a^4*b^9*d^11 + 3185*b^13*c^4*d^7 - 4788*a*b^12*c^3*d^8 - 756*a^3*b^10*c*d^10 + 2790*a^2*b^11*c^2*d^9))/(64*(b^6*c^10 + a^6*c^4*d^6 - 6*a^5*b*c^5*d^5 + 15*a^2*b^4*c^8*d^2 - 20*a^3*b^3*c^7*d^3 + 15*a^4*b^2*c^6*d^4 - 6*a*b^5*c^9*d))) + (-(81*a^4*d^7 + 2401*b^4*c^4*d^3 - 4116*a*b^3*c^3*d^4 + 2646*a^2*b^2*c^2*d^5 - 756*a^3*b*c*d^6)/(65536*b^8*c^15 + 65536*a^8*c^7*d^8 - 524288*a^7*b*c^8*d^7 + 1835008*a^2*b^6*c^13*d^2 - 3670016*a^3*b^5*c^12*d^3 + 4587520*a^4*b^4*c^11*d^4 - 3670016*a^5*b^3*c^10*d^5 + 1835008*a^6*b^2*c^9*d^6 - 524288*a*b^7*c^14*d))^(1/4)*((-(81*a^4*d^7 + 2401*b^4*c^4*d^3 - 4116*a*b^3*c^3*d^4 + 2646*a^2*b^2*c^2*d^5 - 756*a^3*b*c*d^6)/(65536*b^8*c^15 + 65536*a^8*c^7*d^8 - 524288*a^7*b*c^8*d^7 + 1835008*a^2*b^6*c^13*d^2 - 3670016*a^3*b^5*c^12*d^3 + 4587520*a^4*b^4*c^11*d^4 - 3670016*a^5*b^3*c^10*d^5 + 1835008*a^6*b^2*c^9*d^6 - 524288*a*b^7*c^14*d))^(1/4)*((((81*a^4*b^7*d^10)/16 + 28*b^11*c^4*d^6 - (2145*a*b^10*c^3*d^7)/16 - (675*a^3*b^8*c*d^9)/16 + (1971*a^2*b^9*c^2*d^8)/16)/(b^3*c^7 - a^3*c^4*d^3 + 3*a^2*b*c^5*d^2 - 3*a*b^2*c^6*d) + (-(81*a^4*d^7 + 2401*b^4*c^4*d^3 - 4116*a*b^3*c^3*d^4 + 2646*a^2*b^2*c^2*d^5 - 756*a^3*b*c*d^6)/(65536*b^8*c^15 + 65536*a^8*c^7*d^8 - 524288*a^7*b*c^8*d^7 + 1835008*a^2*b^6*c^13*d^2 - 3670016*a^3*b^5*c^12*d^3 + 4587520*a^4*b^4*c^11*d^4 - 3670016*a^5*b^3*c^10*d^5 + 1835008*a^6*b^2*c^9*d^6 - 524288*a*b^7*c^14*d))^(3/4)*(((-(81*a^4*d^7 + 2401*b^4*c^4*d^3 - 4116*a*b^3*c^3*d^4 + 2646*a^2*b^2*c^2*d^5 - 756*a^3*b*c*d^6)/(65536*b^8*c^15 + 65536*a^8*c^7*d^8 - 524288*a^7*b*c^8*d^7 + 1835008*a^2*b^6*c^13*d^2 - 3670016*a^3*b^5*c^12*d^3 + 4587520*a^4*b^4*c^11*d^4 - 3670016*a^5*b^3*c^10*d^5 + 1835008*a^6*b^2*c^9*d^6 - 524288*a*b^7*c^14*d))^(1/4)*(28672*a^2*b^13*c^13*d^5 - 4096*a*b^14*c^14*d^4 - 78848*a^3*b^12*c^12*d^6 + 90112*a^4*b^11*c^11*d^7 + 28672*a^5*b^10*c^10*d^8 - 229376*a^6*b^9*c^9*d^9 + 329728*a^7*b^8*c^8*d^10 - 253952*a^8*b^7*c^7*d^11 + 114688*a^9*b^6*c^6*d^12 - 28672*a^10*b^5*c^5*d^13 + 3072*a^11*b^4*c^4*d^14))/(b^3*c^7 - a^3*c^4*d^3 + 3*a^2*b*c^5*d^2 - 3*a*b^2*c^6*d) + (x*(65536*b^17*c^15*d^4 - 524288*a*b^16*c^14*d^5 + 1835008*a^2*b^15*c^13*d^6 - 3469312*a^3*b^14*c^12*d^7 + 2809856*a^4*b^13*c^11*d^8 + 3362816*a^5*b^12*c^10*d^9 - 14516224*a^6*b^11*c^9*d^10 + 24190976*a^7*b^10*c^8*d^11 - 25280512*a^8*b^9*c^7*d^12 + 17833984*a^9*b^8*c^6*d^13 - 8486912*a^10*b^7*c^5*d^14 + 2609152*a^11*b^6*c^4*d^15 - 466944*a^12*b^5*c^3*d^16 + 36864*a^13*b^4*c^2*d^17))/(64*(b^6*c^10 + a^6*c^4*d^6 - 6*a^5*b*c^5*d^5 + 15*a^2*b^4*c^8*d^2 - 20*a^3*b^3*c^7*d^3 + 15*a^4*b^2*c^6*d^4 - 6*a*b^5*c^9*d)))) + (x*(81*a^4*b^9*d^11 + 3185*b^13*c^4*d^7 - 4788*a*b^12*c^3*d^8 - 756*a^3*b^10*c*d^10 + 2790*a^2*b^11*c^2*d^9))/(64*(b^6*c^10 + a^6*c^4*d^6 - 6*a^5*b*c^5*d^5 + 15*a^2*b^4*c^8*d^2 - 20*a^3*b^3*c^7*d^3 + 15*a^4*b^2*c^6*d^4 - 6*a*b^5*c^9*d)))))*(-(81*a^4*d^7 + 2401*b^4*c^4*d^3 - 4116*a*b^3*c^3*d^4 + 2646*a^2*b^2*c^2*d^5 - 756*a^3*b*c*d^6)/(65536*b^8*c^15 + 65536*a^8*c^7*d^8 - 524288*a^7*b*c^8*d^7 + 1835008*a^2*b^6*c^13*d^2 - 3670016*a^3*b^5*c^12*d^3 + 4587520*a^4*b^4*c^11*d^4 - 3670016*a^5*b^3*c^10*d^5 + 1835008*a^6*b^2*c^9*d^6 - 524288*a*b^7*c^14*d))^(1/4)*2i - atan(((-b^7/(256*a^11*d^8 + 256*a^3*b^8*c^8 - 2048*a^4*b^7*c^7*d + 7168*a^5*b^6*c^6*d^2 - 14336*a^6*b^5*c^5*d^3 + 17920*a^7*b^4*c^4*d^4 - 14336*a^8*b^3*c^3*d^5 + 7168*a^9*b^2*c^2*d^6 - 2048*a^10*b*c*d^7))^(1/4)*((-b^7/(256*a^11*d^8 + 256*a^3*b^8*c^8 - 2048*a^4*b^7*c^7*d + 7168*a^5*b^6*c^6*d^2 - 14336*a^6*b^5*c^5*d^3 + 17920*a^7*b^4*c^4*d^4 - 14336*a^8*b^3*c^3*d^5 + 7168*a^9*b^2*c^2*d^6 - 2048*a^10*b*c*d^7))^(1/4)*((-b^7/(256*a^11*d^8 + 256*a^3*b^8*c^8 - 2048*a^4*b^7*c^7*d + 7168*a^5*b^6*c^6*d^2 - 14336*a^6*b^5*c^5*d^3 + 17920*a^7*b^4*c^4

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*d^4 - 14336*a^8*b^3*c^3*d^5 + 7168*a^9*b^2*c^2*d^6 - 2048*a^10*b*c*d^7))^(
3/4)*(((-b^7/(256*a^11*d^8 + 256*a^3*b^8*c^8 - 2048*a^4*b^7*c^7*d + 7168*a^
5*b^6*c^6*d^2 - 14336*a^6*b^5*c^5*d^3 + 17920*a^7*b^4*c^4*d^4 - 14336*a^8*b
^3*c^3*d^5 + 7168*a^9*b^2*c^2*d^6 - 2048*a^10*b*c*d^7))^(1/4)*(28672*a^2*b^
13*c^13*d^5 - 4096*a*b^14*c^14*d^4 - 78848*a^3*b^12*c^12*d^6 + 90112*a^4*b^
11*c^11*d^7 + 28672*a^5*b^10*c^10*d^8 - 229376*a^6*b^9*c^9*d^9 + 329728*a^7
*b^8*c^8*d^10 - 253952*a^8*b^7*c^7*d^11 + 114688*a^9*b^6*c^6*d^12 - 28672*a
^10*b^5*c^5*d^13 + 3072*a^11*b^4*c^4*d^14))/(b^3*c^7 - a^3*c^4*d^3 + 3*a^2*
b*c^5*d^2 - 3*a*b^2*c^6*d) - (x*(65536*b^17*c^15*d^4 - 524288*a*b^16*c^14*d
^5 + 1835008*a^2*b^15*c^13*d^6 - 3469312*a^3*b^14*c^12*d^7 + 2809856*a^4*b^
13*c^11*d^8 + 3362816*a^5*b^12*c^10*d^9 - 14516224*a^6*b^11*c^9*d^10 + 2419
0976*a^7*b^10*c^8*d^11 - 25280512*a^8*b^9*c^7*d^12 + 17833984*a^9*b^8*c^6*d
^13 - 8486912*a^10*b^7*c^5*d^14 + 2609152*a^11*b^6*c^4*d^15 - 466944*a^12*b
^5*c^3*d^16 + 36864*a^13*b^4*c^2*d^17))/(64*(b^6*c^10 + a^6*c^4*d^6 - 6*a^5
*b*c^5*d^5 + 15*a^2*b^4*c^8*d^2 - 20*a^3*b^3*c^7*d^3 + 15*a^4*b^2*c^6*d^4 -
 6*a*b^5*c^9*d))) + ((81*a^4*b^7*d^10)/16 + 28*b^11*c^4*d^6 - (2145*a*b^10*
c^3*d^7)/16 - (675*a^3*b^8*c*d^9)/16 + (1971*a^2*b^9*c^2*d^8)/16)/(b^3*c^7
- a^3*c^4*d^3 + 3*a^2*b*c^5*d^2 - 3*a*b^2*c^6*d))*1i - (x*(81*a^4*b^9*d^11
+ 3185*b^13*c^4*d^7 - 4788*a*b^12*c^3*d^8 - 756*a^3*b^10*c*d^10 + 2790*a^2*
b^11*c^2*d^9)*1i)/(64*(b^6*c^10 + a^6*c^4*d^6 - 6*a^5*b*c^5*d^5 + 15*a^2*b^
4*c^8*d^2 - 20*a^3*b^3*c^7*d^3 + 15*a^4*b^2*c^6*d^4 - 6*a*b^5*c^9*d))) - (-
b^7/(256*a^11*d^8 + 256*a^3*b^8*c^8 - 2048*a^4*b^7*c^7*d + 7168*a^5*b^6*c^6
*d^2 - 14336*a^6*b^5*c^5*d^3 + 17920*a^7*b^4*c^4*d^4 - 14336*a^8*b^3*c^3*d^
5 + 7168*a^9*b^2*c^2*d^6 - 2048*a^10*b*c*d^7))^(1/4)*((-b^7/(256*a^11*d^8 +
 256*a^3*b^8*c^8 - 2048*a^4*b^7*c^7*d + 7168*a^5*b^6*c^6*d^2 - 14336*a^6*b^
5*c^5*d^3 + 17920*a^7*b^4*c^4*d^4 - 14336*a^8*b^3*c^3*d^5 + 7168*a^9*b^2*c^
2*d^6 - 2048*a^10*b*c*d^7))^(1/4)*((-b^7/(256*a^11*d^8 + 256*a^3*b^8*c^8 -
2048*a^4*b^7*c^7*d + 7168*a^5*b^6*c^6*d^2 - 14336*a^6*b^5*c^5*d^3 + 17920*a
^7*b^4*c^4*d^4 - 14336*a^8*b^3*c^3*d^5 + 7168*a^9*b^2*c^2*d^6 - 2048*a^10*b
*c*d^7))^(3/4)*(((-b^7/(256*a^11*d^8 + 256*a^3*b^8*c^8 - 2048*a^4*b^7*c^7*d
 + 7168*a^5*b^6*c^6*d^2 - 14336*a^6*b^5*c^5*d^3 + 17920*a^7*b^4*c^4*d^4 - 1
4336*a^8*b^3*c^3*d^5 + 7168*a^9*b^2*c^2*d^6 - 2048*a^10*b*c*d^7))^(1/4)*(28
672*a^2*b^13*c^13*d^5 - 4096*a*b^14*c^14*d^4 - 78848*a^3*b^12*c^12*d^6 + 90
112*a^4*b^11*c^11*d^7 + 28672*a^5*b^10*c^10*d^8 - 229376*a^6*b^9*c^9*d^9 +
329728*a^7*b^8*c^8*d^10 - 253952*a^8*b^7*c^7*d^11 + 114688*a^9*b^6*c^6*d^12
 - 28672*a^10*b^5*c^5*d^13 + 3072*a^11*b^4*c^4*d^14))/(b^3*c^7 - a^3*c^4*d^
3 + 3*a^2*b*c^5*d^2 - 3*a*b^2*c^6*d) + (x*(65536*b^17*c^15*d^4 - 524288*a*b
^16*c^14*d^5 + 1835008*a^2*b^15*c^13*d^6 - 3469312*a^3*b^14*c^12*d^7 + 2809
856*a^4*b^13*c^11*d^8 + 3362816*a^5*b^12*c^10*d^9 - 14516224*a^6*b^11*c^9*d
^10 + 24190976*a^7*b^10*c^8*d^11 - 25280512*a^8*b^9*c^7*d^12 + 17833984*a^9
*b^8*c^6*d^13 - 8486912*a^10*b^7*c^5*d^14 + 2609152*a^11*b^6*c^4*d^15 - 466
944*a^12*b^5*c^3*d^16 + 36864*a^13*b^4*c^2*d^17))/(64*(b^6*c^10 + a^6*c^4*d
^6 - 6*a^5*b*c^5*d^5 + 15*a^2*b^4*c^8*d^2 - 20*a^3*b^3*c^7*d^3 + 15*a^4*b^2
*c^6*d^4 - 6*a*b^5*c^9*d))) + ((81*a^4*b^7*d^10)/16 + 28*b^11*c^4*d^6 - (21
45*a*b^10*c^3*d^7)/16 - (675*a^3*b^8*c*d^9)/16 + (1971*a^2*b^9*c^2*d^8)/16)
/(b^3*c^7 - a^3*c^4*d^3 + 3*a^2*b*c^5*d^2 - 3*a*b^2*c^6*d))*1i + (x*(81*a^4
*b^9*d^11 + 3185*b^13*c^4*d^7 - 4788*a*b^12*c^3*d^8 - 756*a^3*b^10*c*d^10 +
 2790*a^2*b^11*c^2*d^9)*1i)/(64*(b^6*c^10 + a^6*c^4*d^6 - 6*a^5*b*c^5*d^5 +
 15*a^2*b^4*c^8*d^2 - 20*a^3*b^3*c^7*d^3 + 15*a^4*b^2*c^6*d^4 - 6*a*b^5*c^9
*d))))/((-b^7/(256*a^11*d^8 + 256*a^3*b^8*c^8 - 2048*a^4*b^7*c^7*d + 7168*a
^5*b^6*c^6*d^2 - 14336*a^6*b^5*c^5*d^3 + 17920*a^7*b^4*c^4*d^4 - 14336*a^8*
b^3*c^3*d^5 + 7168*a^9*b^2*c^2*d^6 - 2048*a^10*b*c*d^7))^(1/4)*((-b^7/(256*
a^11*d^8 + 256*a^3*b^8*c^8 - 2048*a^4*b^7*c^7*d + 7168*a^5*b^6*c^6*d^2 - 14
336*a^6*b^5*c^5*d^3 + 17920*a^7*b^4*c^4*d^4 - 14336*a^8*b^3*c^3*d^5 + 7168*
a^9*b^2*c^2*d^6 - 2048*a^10*b*c*d^7))^(1/4)*((-b^7/(256*a^11*d^8 + 256*a^3*
b^8*c^8 - 2048*a^4*b^7*c^7*d + 7168*a^5*b^6*c^6*d^2 - 14336*a^6*b^5*c^5*d^3
 + 17920*a^7*b^4*c^4*d^4 - 14336*a^8*b^3*c^3*d^5 + 7168*a^9*b^2*c^2*d^6 - 2
048*a^10*b*c*d^7))^(3/4)*(((-b^7/(256*a^11*d^8 + 256*a^3*b^8*c^8 - 2048*a^4
*b^7*c^7*d + 7168*a^5*b^6*c^6*d^2 - 14336*a^6*b^5*c^5*d^3 + 17920*a^7*b^4*c
```

^4*d^4 - 14336*a^8*b^3*c^3*d^5 + 7168*a^9*b^2*c^2*d^6 - 2048*a^10*b*c*d^7))
^(1/4)*(28672*a^2*b^13*c^13*d^5 - 4096*a*b^14*c^14*d^4 - 78848*a^3*b^12*c^1
2*d^6 + 90112*a^4*b^11*c^11*d^7 + 28672*a^5*b^10*c^10*d^8 - 229376*a^6*b^9*
c^9*d^9 + 329728*a^7*b^8*c^8*d^10 - 253952*a^8*b^7*c^7*d^11 + 114688*a^9*b^
6*c^6*d^12 - 28672*a^10*b^5*c^5*d^13 + 3072*a^11*b^4*c^4*d^14))/(b^3*c^7 -
a^3*c^4*d^3 + 3*a^2*b*c^5*d^2 - 3*a*b^2*c^6*d) - (x*(65536*b^17*c^15*d^4 -
524288*a*b^16*c^14*d^5 + 1835008*a^2*b^15*c^13*d^6 - 3469312*a^3*b^14*c^12*
d^7 + 2809856*a^4*b^13*c^11*d^8 + 3362816*a^5*b^12*c^10*d^9 - 14516224*a^6*
b^11*c^9*d^10 + 24190976*a^7*b^10*c^8*d^11 - 25280512*a^8*b^9*c^7*d^12 + 17
833984*a^9*b^8*c^6*d^13 - 8486912*a^10*b^7*c^5*d^14 + 2609152*a^11*b^6*c^4*
d^15 - 466944*a^12*b^5*c^3*d^16 + 36864*a^13*b^4*c^2*d^17))/(64*(b^6*c^10 +
a^6*c^4*d^6 - 6*a^5*b*c^5*d^5 + 15*a^2*b^4*c^8*d^2 - 20*a^3*b^3*c^7*d^3 +
15*a^4*b^2*c^6*d^4 - 6*a*b^5*c^9*d))) + ((81*a^4*b^7*d^10)/16 + 28*b^11*c^4
*d^6 - (2145*a*b^10*c^3*d^7)/16 - (675*a^3*b^8*c*d^9)/16 + (1971*a^2*b^9*c^
2*d^8)/16)/(b^3*c^7 - a^3*c^4*d^3 + 3*a^2*b*c^5*d^2 - 3*a*b^2*c^6*d)) - (x*
(81*a^4*b^9*d^11 + 3185*b^13*c^4*d^7 - 4788*a*b^12*c^3*d^8 - 756*a^3*b^10*c
*d^10 + 2790*a^2*b^11*c^2*d^9))/(64*(b^6*c^10 + a^6*c^4*d^6 - 6*a^5*b*c^5*d
^5 + 15*a^2*b^4*c^8*d^2 - 20*a^3*b^3*c^7*d^3 + 15*a^4*b^2*c^6*d^4 - 6*a*b^5
*c^9*d))) + (-b^7/(256*a^11*d^8 + 256*a^3*b^8*c^8 - 2048*a^4*b^7*c^7*d + 71
68*a^5*b^6*c^6*d^2 - 14336*a^6*b^5*c^5*d^3 + 17920*a^7*b^4*c^4*d^4 - 14336*
a^8*b^3*c^3*d^5 + 7168*a^9*b^2*c^2*d^6 - 2048*a^10*b*c*d^7))^(1/4)*((-b^7/(
256*a^11*d^8 + 256*a^3*b^8*c^8 - 2048*a^4*b^7*c^7*d + 7168*a^5*b^6*c^6*d^2
- 14336*a^6*b^5*c^5*d^3 + 17920*a^7*b^4*c^4*d^4 - 14336*a^8*b^3*c^3*d^5 + 7
168*a^9*b^2*c^2*d^6 - 2048*a^10*b*c*d^7))^(1/4)*((-b^7/(256*a^11*d^8 + 256*
a^3*b^8*c^8 - 2048*a^4*b^7*c^7*d + 7168*a^5*b^6*c^6*d^2 - 14336*a^6*b^5*c^5
*d^3 + 17920*a^7*b^4*c^4*d^4 - 14336*a^8*b^3*c^3*d^5 + 7168*a^9*b^2*c^2*d^6
- 2048*a^10*b*c*d^7))^(3/4)*(((-b^7/(256*a^11*d^8 + 256*a^3*b^8*c^8 - 2048
*a^4*b^7*c^7*d + 7168*a^5*b^6*c^6*d^2 - 14336*a^6*b^5*c^5*d^3 + 17920*a^7*b
^4*c^4*d^4 - 14336*a^8*b^3*c^3*d^5 + 7168*a^9*b^2*c^2*d^6 - 2048*a^10*b*c*d
^7))^(1/4)*(28672*a^2*b^13*c^13*d^5 - 4096*a*b^14*c^14*d^4 - 78848*a^3*b^12
*c^12*d^6 + 90112*a^4*b^11*c^11*d^7 + 28672*a^5*b^10*c^10*d^8 - 229376*a^6*
b^9*c^9*d^9 + 329728*a^7*b^8*c^8*d^10 - 253952*a^8*b^7*c^7*d^11 + 114688*a^
9*b^6*c^6*d^12 - 28672*a^10*b^5*c^5*d^13 + 3072*a^11*b^4*c^4*d^14))/(b^3*c^
7 - a^3*c^4*d^3 + 3*a^2*b*c^5*d^2 - 3*a*b^2*c^6*d) + (x*(65536*b^17*c^15*d^
4 - 524288*a*b^16*c^14*d^5 + 1835008*a^2*b^15*c^13*d^6 - 3469312*a^3*b^14*c
^12*d^7 + 2809856*a^4*b^13*c^11*d^8 + 3362816*a^5*b^12*c^10*d^9 - 14516224*
a^6*b^11*c^9*d^10 + 24190976*a^7*b^10*c^8*d^11 - 25280512*a^8*b^9*c^7*d^12
+ 17833984*a^9*b^8*c^6*d^13 - 8486912*a^10*b^7*c^5*d^14 + 2609152*a^11*b^6*
c^4*d^15 - 466944*a^12*b^5*c^3*d^16 + 36864*a^13*b^4*c^2*d^17))/(64*(b^6*c^
10 + a^6*c^4*d^6 - 6*a^5*b*c^5*d^5 + 15*a^2*b^4*c^8*d^2 - 20*a^3*b^3*c^7*d^
3 + 15*a^4*b^2*c^6*d^4 - 6*a*b^5*c^9*d))) + ((81*a^4*b^7*d^10)/16 + 28*b^11
*c^4*d^6 - (2145*a*b^10*c^3*d^7)/16 - (675*a^3*b^8*c*d^9)/16 + (1971*a^2*b^
9*c^2*d^8)/16)/(b^3*c^7 - a^3*c^4*d^3 + 3*a^2*b*c^5*d^2 - 3*a*b^2*c^6*d)) +
(x*(81*a^4*b^9*d^11 + 3185*b^13*c^4*d^7 - 4788*a*b^12*c^3*d^8 - 756*a^3*b^
10*c*d^10 + 2790*a^2*b^11*c^2*d^9))/(64*(b^6*c^10 + a^6*c^4*d^6 - 6*a^5*b*c
^5*d^5 + 15*a^2*b^4*c^8*d^2 - 20*a^3*b^3*c^7*d^3 + 15*a^4*b^2*c^6*d^4 - 6*a
*b^5*c^9*d)))))*(-b^7/(256*a^11*d^8 + 256*a^3*b^8*c^8 - 2048*a^4*b^7*c^7*d
+ 7168*a^5*b^6*c^6*d^2 - 14336*a^6*b^5*c^5*d^3 + 17920*a^7*b^4*c^4*d^4 - 14
336*a^8*b^3*c^3*d^5 + 7168*a^9*b^2*c^2*d^6 - 2048*a^10*b*c*d^7))^(1/4)*2i +
2*atan(((-b^7/(256*a^11*d^8 + 256*a^3*b^8*c^8 - 2048*a^4*b^7*c^7*d + 7168*
a^5*b^6*c^6*d^2 - 14336*a^6*b^5*c^5*d^3 + 17920*a^7*b^4*c^4*d^4 - 14336*a^8
*b^3*c^3*d^5 + 7168*a^9*b^2*c^2*d^6 - 2048*a^10*b*c*d^7))^(1/4)*((-b^7/(256
*a^11*d^8 + 256*a^3*b^8*c^8 - 2048*a^4*b^7*c^7*d + 7168*a^5*b^6*c^6*d^2 - 1
4336*a^6*b^5*c^5*d^3 + 17920*a^7*b^4*c^4*d^4 - 14336*a^8*b^3*c^3*d^5 + 7168
*a^9*b^2*c^2*d^6 - 2048*a^10*b*c*d^7))^(1/4)*((-b^7/(256*a^11*d^8 + 256*a^3
*b^8*c^8 - 2048*a^4*b^7*c^7*d + 7168*a^5*b^6*c^6*d^2 - 14336*a^6*b^5*c^5*d^
3 + 17920*a^7*b^4*c^4*d^4 - 14336*a^8*b^3*c^3*d^5 + 7168*a^9*b^2*c^2*d^6 -
2048*a^10*b*c*d^7))^(3/4)*(((-b^7/(256*a^11*d^8 + 256*a^3*b^8*c^8 - 2048*a^
4*b^7*c^7*d + 7168*a^5*b^6*c^6*d^2 - 14336*a^6*b^5*c^5*d^3 + 17920*a^7*b^4*

c^4*d^4 - 14336*a^8*b^3*c^3*d^5 + 7168*a^9*b^2*c^2*d^6 - 2048*a^10*b*c*d^7)
)^(1/4)*(28672*a^2*b^13*c^13*d^5 - 4096*a*b^14*c^14*d^4 - 78848*a^3*b^12*c^
12*d^6 + 90112*a^4*b^11*c^11*d^7 + 28672*a^5*b^10*c^10*d^8 - 229376*a^6*b^9
*c^9*d^9 + 329728*a^7*b^8*c^8*d^10 - 253952*a^8*b^7*c^7*d^11 + 114688*a^9*b
^6*c^6*d^12 - 28672*a^10*b^5*c^5*d^13 + 3072*a^11*b^4*c^4*d^14))/(b^3*c^7 -
a^3*c^4*d^3 + 3*a^2*b*c^5*d^2 - 3*a*b^2*c^6*d) - (x*(65536*b^17*c^15*d^4 -
524288*a*b^16*c^14*d^5 + 1835008*a^2*b^15*c^13*d^6 - 3469312*a^3*b^14*c^12
*d^7 + 2809856*a^4*b^13*c^11*d^8 + 3362816*a^5*b^12*c^10*d^9 - 14516224*a^6
*b^11*c^9*d^10 + 24190976*a^7*b^10*c^8*d^11 - 25280512*a^8*b^9*c^7*d^12 + 1
7833984*a^9*b^8*c^6*d^13 - 8486912*a^10*b^7*c^5*d^14 + 2609152*a^11*b^6*c^4
*d^15 - 466944*a^12*b^5*c^3*d^16 + 36864*a^13*b^4*c^2*d^17)*1i)/(64*(b^6*c^
10 + a^6*c^4*d^6 - 6*a^5*b*c^5*d^5 + 15*a^2*b^4*c^8*d^2 - 20*a^3*b^3*c^7*d^
3 + 15*a^4*b^2*c^6*d^4 - 6*a*b^5*c^9*d)))*1i + (((81*a^4*b^7*d^10)/16 + 28*
b^11*c^4*d^6 - (2145*a*b^10*c^3*d^7)/16 - (675*a^3*b^8*c*d^9)/16 + (1971*a^
2*b^9*c^2*d^8)/16)*1i)/(b^3*c^7 - a^3*c^4*d^3 + 3*a^2*b*c^5*d^2 - 3*a*b^2*c
^6*d)) + (x*(81*a^4*b^9*d^11 + 3185*b^13*c^4*d^7 - 4788*a*b^12*c^3*d^8 - 75
6*a^3*b^10*c*d^10 + 2790*a^2*b^11*c^2*d^9))/(64*(b^6*c^10 + a^6*c^4*d^6 - 6
*a^5*b*c^5*d^5 + 15*a^2*b^4*c^8*d^2 - 20*a^3*b^3*c^7*d^3 + 15*a^4*b^2*c^6*d
^4 - 6*a*b^5*c^9*d))) - (-b^7/(256*a^11*d^8 + 256*a^3*b^8*c^8 - 2048*a^4*b^
7*c^7*d + 7168*a^5*b^6*c^6*d^2 - 14336*a^6*b^5*c^5*d^3 + 17920*a^7*b^4*c^4*
d^4 - 14336*a^8*b^3*c^3*d^5 + 7168*a^9*b^2*c^2*d^6 - 2048*a^10*b*c*d^7))^(1
/4)*((-b^7/(256*a^11*d^8 + 256*a^3*b^8*c^8 - 2048*a^4*b^7*c^7*d + 7168*a^5*
b^6*c^6*d^2 - 14336*a^6*b^5*c^5*d^3 + 17920*a^7*b^4*c^4*d^4 - 14336*a^8*b^3
*c^3*d^5 + 7168*a^9*b^2*c^2*d^6 - 2048*a^10*b*c*d^7))^(1/4)*((-b^7/(256*a^1
1*d^8 + 256*a^3*b^8*c^8 - 2048*a^4*b^7*c^7*d + 7168*a^5*b^6*c^6*d^2 - 14336
*a^6*b^5*c^5*d^3 + 17920*a^7*b^4*c^4*d^4 - 14336*a^8*b^3*c^3*d^5 + 7168*a^9
*b^2*c^2*d^6 - 2048*a^10*b*c*d^7))^(3/4)*(((-b^7/(256*a^11*d^8 + 256*a^3*b^
8*c^8 - 2048*a^4*b^7*c^7*d + 7168*a^5*b^6*c^6*d^2 - 14336*a^6*b^5*c^5*d^3 +
17920*a^7*b^4*c^4*d^4 - 14336*a^8*b^3*c^3*d^5 + 7168*a^9*b^2*c^2*d^6 - 204
8*a^10*b*c*d^7))^(1/4)*(28672*a^2*b^13*c^13*d^5 - 4096*a*b^14*c^14*d^4 - 78
848*a^3*b^12*c^12*d^6 + 90112*a^4*b^11*c^11*d^7 + 28672*a^5*b^10*c^10*d^8 -
229376*a^6*b^9*c^9*d^9 + 329728*a^7*b^8*c^8*d^10 - 253952*a^8*b^7*c^7*d^11
+ 114688*a^9*b^6*c^6*d^12 - 28672*a^10*b^5*c^5*d^13 + 3072*a^11*b^4*c^4*d^
14))/(b^3*c^7 - a^3*c^4*d^3 + 3*a^2*b*c^5*d^2 - 3*a*b^2*c^6*d) + (x*(65536*
b^17*c^15*d^4 - 524288*a*b^16*c^14*d^5 + 1835008*a^2*b^15*c^13*d^6 - 346931
2*a^3*b^14*c^12*d^7 + 2809856*a^4*b^13*c^11*d^8 + 3362816*a^5*b^12*c^10*d^9
- 14516224*a^6*b^11*c^9*d^10 + 24190976*a^7*b^10*c^8*d^11 - 25280512*a^8*b
^9*c^7*d^12 + 17833984*a^9*b^8*c^6*d^13 - 8486912*a^10*b^7*c^5*d^14 + 26091
52*a^11*b^6*c^4*d^15 - 466944*a^12*b^5*c^3*d^16 + 36864*a^13*b^4*c^2*d^17)*
1i)/(64*(b^6*c^10 + a^6*c^4*d^6 - 6*a^5*b*c^5*d^5 + 15*a^2*b^4*c^8*d^2 - 20
*a^3*b^3*c^7*d^3 + 15*a^4*b^2*c^6*d^4 - 6*a*b^5*c^9*d)))*1i + (((81*a^4*b^7
*d^10)/16 + 28*b^11*c^4*d^6 - (2145*a*b^10*c^3*d^7)/16 - (675*a^3*b^8*c*d^9
)/16 + (1971*a^2*b^9*c^2*d^8)/16)*1i)/(b^3*c^7 - a^3*c^4*d^3 + 3*a^2*b*c^5*
d^2 - 3*a*b^2*c^6*d)) - (x*(81*a^4*b^9*d^11 + 3185*b^13*c^4*d^7 - 4788*a*b^
12*c^3*d^8 - 756*a^3*b^10*c*d^10 + 2790*a^2*b^11*c^2*d^9))/(64*(b^6*c^10 +
a^6*c^4*d^6 - 6*a^5*b*c^5*d^5 + 15*a^2*b^4*c^8*d^2 - 20*a^3*b^3*c^7*d^3 + 1
5*a^4*b^2*c^6*d^4 - 6*a*b^5*c^9*d))))/((-b^7/(256*a^11*d^8 + 256*a^3*b^8*c^
8 - 2048*a^4*b^7*c^7*d + 7168*a^5*b^6*c^6*d^2 - 14336*a^6*b^5*c^5*d^3 + 179
20*a^7*b^4*c^4*d^4 - 14336*a^8*b^3*c^3*d^5 + 7168*a^9*b^2*c^2*d^6 - 2048*a^
10*b*c*d^7))^(1/4)*((-b^7/(256*a^11*d^8 + 256*a^3*b^8*c^8 - 2048*a^4*b^7*c^
7*d + 7168*a^5*b^6*c^6*d^2 - 14336*a^6*b^5*c^5*d^3 + 17920*a^7*b^4*c^4*d^4
- 14336*a^8*b^3*c^3*d^5 + 7168*a^9*b^2*c^2*d^6 - 2048*a^10*b*c*d^7))^(1/4)*
((-b^7/(256*a^11*d^8 + 256*a^3*b^8*c^8 - 2048*a^4*b^7*c^7*d + 7168*a^5*b^6*
c^6*d^2 - 14336*a^6*b^5*c^5*d^3 + 17920*a^7*b^4*c^4*d^4 - 14336*a^8*b^3*c^3
*d^5 + 7168*a^9*b^2*c^2*d^6 - 2048*a^10*b*c*d^7))^(3/4)*(((-b^7/(256*a^11*d
^8 + 256*a^3*b^8*c^8 - 2048*a^4*b^7*c^7*d + 7168*a^5*b^6*c^6*d^2 - 14336*a^
6*b^5*c^5*d^3 + 17920*a^7*b^4*c^4*d^4 - 14336*a^8*b^3*c^3*d^5 + 7168*a^9*b^
2*c^2*d^6 - 2048*a^10*b*c*d^7))^(1/4)*(28672*a^2*b^13*c^13*d^5 - 4096*a*b^1
4*c^14*d^4 - 78848*a^3*b^12*c^12*d^6 + 90112*a^4*b^11*c^11*d^7 + 28672*a^5*

```
b^10*c^10*d^8 - 229376*a^6*b^9*c^9*d^9 + 329728*a^7*b^8*c^8*d^10 - 253952*a
^8*b^7*c^7*d^11 + 114688*a^9*b^6*c^6*d^12 - 28672*a^10*b^5*c^5*d^13 + 3072*
a^11*b^4*c^4*d^14))/(b^3*c^7 - a^3*c^4*d^3 + 3*a^2*b*c^5*d^2 - 3*a*b^2*c^6*
d) - (x*(65536*b^17*c^15*d^4 - 524288*a*b^16*c^14*d^5 + 1835008*a^2*b^15*c^
13*d^6 - 3469312*a^3*b^14*c^12*d^7 + 2809856*a^4*b^13*c^11*d^8 + 3362816*a^
5*b^12*c^10*d^9 - 14516224*a^6*b^11*c^9*d^10 + 24190976*a^7*b^10*c^8*d^11 -
 25280512*a^8*b^9*c^7*d^12 + 17833984*a^9*b^8*c^6*d^13 - 8486912*a^10*b^7*c
^5*d^14 + 2609152*a^11*b^6*c^4*d^15 - 466944*a^12*b^5*c^3*d^16 + 36864*a^13
*b^4*c^2*d^17)*1i)/(64*(b^6*c^10 + a^6*c^4*d^6 - 6*a^5*b*c^5*d^5 + 15*a^2*b
^4*c^8*d^2 - 20*a^3*b^3*c^7*d^3 + 15*a^4*b^2*c^6*d^4 - 6*a*b^5*c^9*d)))*1i
+ (((81*a^4*b^7*d^10)/16 + 28*b^11*c^4*d^6 - (2145*a*b^10*c^3*d^7)/16 - (67
5*a^3*b^8*c*d^9)/16 + (1971*a^2*b^9*c^2*d^8)/16)*1i)/(b^3*c^7 - a^3*c^4*d^3
 + 3*a^2*b*c^5*d^2 - 3*a*b^2*c^6*d))*1i + (x*(81*a^4*b^9*d^11 + 3185*b^13*c
^4*d^7 - 4788*a*b^12*c^3*d^8 - 756*a^3*b^10*c*d^10 + 2790*a^2*b^11*c^2*d^9)
*1i)/(64*(b^6*c^10 + a^6*c^4*d^6 - 6*a^5*b*c^5*d^5 + 15*a^2*b^4*c^8*d^2 - 2
0*a^3*b^3*c^7*d^3 + 15*a^4*b^2*c^6*d^4 - 6*a*b^5*c^9*d))) + (-b^7/(256*a^11
*d^8 + 256*a^3*b^8*c^8 - 2048*a^4*b^7*c^7*d + 7168*a^5*b^6*c^6*d^2 - 14336*
a^6*b^5*c^5*d^3 + 17920*a^7*b^4*c^4*d^4 - 14336*a^8*b^3*c^3*d^5 + 7168*a^9*
b^2*c^2*d^6 - 2048*a^10*b*c*d^7))^(1/4)*((-b^7/(256*a^11*d^8 + 256*a^3*b^8*
c^8 - 2048*a^4*b^7*c^7*d + 7168*a^5*b^6*c^6*d^2 - 14336*a^6*b^5*c^5*d^3 + 1
7920*a^7*b^4*c^4*d^4 - 14336*a^8*b^3*c^3*d^5 + 7168*a^9*b^2*c^2*d^6 - 2048*
a^10*b*c*d^7))^(1/4)*((-b^7/(256*a^11*d^8 + 256*a^3*b^8*c^8 - 2048*a^4*b^7*
c^7*d + 7168*a^5*b^6*c^6*d^2 - 14336*a^6*b^5*c^5*d^3 + 17920*a^7*b^4*c^4*d^
4 - 14336*a^8*b^3*c^3*d^5 + 7168*a^9*b^2*c^2*d^6 - 2048*a^10*b*c*d^7))^(3/4
)*(((-b^7/(256*a^11*d^8 + 256*a^3*b^8*c^8 - 2048*a^4*b^7*c^7*d + 7168*a^5*b
^6*c^6*d^2 - 14336*a^6*b^5*c^5*d^3 + 17920*a^7*b^4*c^4*d^4 - 14336*a^8*b^3*
c^3*d^5 + 7168*a^9*b^2*c^2*d^6 - 2048*a^10*b*c*d^7))^(1/4)*(28672*a^2*b^13*
c^13*d^5 - 4096*a*b^14*c^14*d^4 - 78848*a^3*b^12*c^12*d^6 + 90112*a^4*b^11*
c^11*d^7 + 28672*a^5*b^10*c^10*d^8 - 229376*a^6*b^9*c^9*d^9 + 329728*a^7*b^
8*c^8*d^10 - 253952*a^8*b^7*c^7*d^11 + 114688*a^9*b^6*c^6*d^12 - 28672*a^10
*b^5*c^5*d^13 + 3072*a^11*b^4*c^4*d^14))/(b^3*c^7 - a^3*c^4*d^3 + 3*a^2*b*c
^5*d^2 - 3*a*b^2*c^6*d) + (x*(65536*b^17*c^15*d^4 - 524288*a*b^16*c^14*d^5
+ 1835008*a^2*b^15*c^13*d^6 - 3469312*a^3*b^14*c^12*d^7 + 2809856*a^4*b^13*
c^11*d^8 + 3362816*a^5*b^12*c^10*d^9 - 14516224*a^6*b^11*c^9*d^10 + 2419097
6*a^7*b^10*c^8*d^11 - 25280512*a^8*b^9*c^7*d^12 + 17833984*a^9*b^8*c^6*d^13
 - 8486912*a^10*b^7*c^5*d^14 + 2609152*a^11*b^6*c^4*d^15 - 466944*a^12*b^5*
c^3*d^16 + 36864*a^13*b^4*c^2*d^17)*1i)/(64*(b^6*c^10 + a^6*c^4*d^6 - 6*a^5
*b*c^5*d^5 + 15*a^2*b^4*c^8*d^2 - 20*a^3*b^3*c^7*d^3 + 15*a^4*b^2*c^6*d^4 -
 6*a*b^5*c^9*d)))*1i + (((81*a^4*b^7*d^10)/16 + 28*b^11*c^4*d^6 - (2145*a*b
^10*c^3*d^7)/16 - (675*a^3*b^8*c*d^9)/16 + (1971*a^2*b^9*c^2*d^8)/16)*1i)/(
b^3*c^7 - a^3*c^4*d^3 + 3*a^2*b*c^5*d^2 - 3*a*b^2*c^6*d))*1i - (x*(81*a^4*b
^9*d^11 + 3185*b^13*c^4*d^7 - 4788*a*b^12*c^3*d^8 - 756*a^3*b^10*c*d^10 + 2
790*a^2*b^11*c^2*d^9)*1i)/(64*(b^6*c^10 + a^6*c^4*d^6 - 6*a^5*b*c^5*d^5 + 1
5*a^2*b^4*c^8*d^2 - 20*a^3*b^3*c^7*d^3 + 15*a^4*b^2*c^6*d^4 - 6*a*b^5*c^9*d
)))))*(-b^7/(256*a^11*d^8 + 256*a^3*b^8*c^8 - 2048*a^4*b^7*c^7*d + 7168*a^5
*b^6*c^6*d^2 - 14336*a^6*b^5*c^5*d^3 + 17920*a^7*b^4*c^4*d^4 - 14336*a^8*b^
3*c^3*d^5 + 7168*a^9*b^2*c^2*d^6 - 2048*a^10*b*c*d^7))^(1/4) + (d*x)/(4*c*(
c + d*x^4)*(a*d - b*c))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**4+a)/(d*x**4+c)**2,x)
```

```
[Out] Timed out
```

3.103 $\int \frac{(c+dx^4)^5}{(a+bx^4)^2}\,dx$

Optimal. Leaf size=407

$$-\frac{(bc-ad)^4(17ad+3bc)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)}{16\sqrt{2}\,a^{7/4}b^{21/4}}+\frac{(bc-ad)^4(17ad+3bc)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{a}+\right.}{16\sqrt{2}\,a^{7/4}b^{21/4}}$$

Rubi [A] time = 0.40, antiderivative size = 407, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$, Rules used = {390, 385, 211, 1165, 628, 1162, 617, 204}

$$\frac{d^3x^5\left(3a^2d^2-10abcd+10b^2c^2\right)}{5b^4}+\frac{d^2x\left(15a^2bcd^2-4a^3d^3-20ab^2c^2d+10b^3c^3\right)}{b^5}-\frac{(bc-ad)^4(17ad+3bc)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)}{16\sqrt{2}\,a^{7/4}b^{21/4}}+\frac{(bc-ad)^4(17ad+3bc)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)}{16\sqrt{2}\,a^{7/4}b^{21/4}}-\frac{(bc-ad)^4(17ad+3bc)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{8\sqrt{2}\,a^{7/4}b^{21/4}}+\frac{(bc-ad)^4(17ad+3bc)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}+1\right)}{8\sqrt{2}\,a^{7/4}b^{21/4}}+\frac{d^4x^9(5bc-2ad)}{9b^3}+\frac{x(bc-ad)^5}{4ab^5\left(a+bx^4\right)}+\frac{d^5x^{13}}{13b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^4)^5/(a + b*x^4)^2,x]
```

```
[Out] (d^2*(10*b^3*c^3 - 20*a*b^2*c^2*d + 15*a^2*b*c*d^2 - 4*a^3*d^3)*x)/b^5 + (d
^3*(10*b^2*c^2 - 10*a*b*c*d + 3*a^2*d^2)*x^5)/(5*b^4) + (d^4*(5*b*c - 2*a*d
)*x^9)/(9*b^3) + (d^5*x^13)/(13*b^2) + ((b*c - a*d)^5*x)/(4*a*b^5*(a + b*x^
4)) - ((b*c - a*d)^4*(3*b*c + 17*a*d)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4
)])/(8*Sqrt[2]*a^(7/4)*b^(21/4)) + ((b*c - a*d)^4*(3*b*c + 17*a*d)*ArcTan[1
 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*b^(21/4)) - ((b*c - a*d
)^4*(3*b*c + 17*a*d)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2]
)/(16*Sqrt[2]*a^(7/4)*b^(21/4)) + ((b*c - a*d)^4*(3*b*c + 17*a*d)*Log[Sqrt[
a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(16*Sqrt[2]*a^(7/4)*b^(21/4)
)
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{\left(c+dx^4\right)^5}{\left(a+bx^4\right)^2}\,dx = \int \left(\frac{d^2\left(10b^3c^3-20ab^2c^2d+15a^2bcd^2-4a^3d^3\right)}{b^5}+\frac{d^3\left(10b^2c^2-10abcd+3a^2d^2\right)x^4}{b^4}+\frac{d^4(5bc}{b}\right.$$

$$=\frac{d^2\left(10b^3c^3-20ab^2c^2d+15a^2bcd^2-4a^3d^3\right)x}{b^5}+\frac{d^3\left(10b^2c^2-10abcd+3a^2d^2\right)x^5}{5b^4}+\frac{d^4(5bc-}{9b^3}$$

$$=\frac{d^2\left(10b^3c^3-20ab^2c^2d+15a^2bcd^2-4a^3d^3\right)x}{b^5}+\frac{d^3\left(10b^2c^2-10abcd+3a^2d^2\right)x^5}{5b^4}+\frac{d^4(5bc-}{9b^3}$$

$$=\frac{d^2\left(10b^3c^3-20ab^2c^2d+15a^2bcd^2-4a^3d^3\right)x}{b^5}+\frac{d^3\left(10b^2c^2-10abcd+3a^2d^2\right)x^5}{5b^4}+\frac{d^4(5bc-}{9b^3}$$

$$=\frac{d^2\left(10b^3c^3-20ab^2c^2d+15a^2bcd^2-4a^3d^3\right)x}{b^5}+\frac{d^3\left(10b^2c^2-10abcd+3a^2d^2\right)x^5}{5b^4}+\frac{d^4(5bc-}{9b^3}$$

$$=\frac{d^2\left(10b^3c^3-20ab^2c^2d+15a^2bcd^2-4a^3d^3\right)x}{b^5}+\frac{d^3\left(10b^2c^2-10abcd+3a^2d^2\right)x^5}{5b^4}+\frac{d^4(5bc-}{9b^3}$$

$$=\frac{d^2\left(10b^3c^3-20ab^2c^2d+15a^2bcd^2-4a^3d^3\right)x}{b^5}+\frac{d^3\left(10b^2c^2-10abcd+3a^2d^2\right)x^5}{5b^4}+\frac{d^4(5bc-}{9b^3}$$

Mathematica [A] time = 0.45, size = 391, normalized size = 0.96

$$\frac{-\frac{585\sqrt{2}(bc-ad)^4(17ad+3bc)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{a^{7/4}}+\frac{585\sqrt{2}(bc-ad)^4(17ad+3bc)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{a^{7/4}}-\frac{1170\sqrt{2}(bc-ad)^4(17ad+3bc)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{a^{7/4}}+\frac{1170\sqrt{2}(bc-ad)^4(17ad+3bc)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{a^{7/4}}+3744b^{5/4}d^3x^5\left(3a^2d^2-10abcd+10b^2c^2\right)+18720\sqrt[4]{b}d^2x\left(-4a^3d^3+15a^2bcd^2-20ab^2c^2d+10b^3c^3\right)+2080b^{9/4}d^4x^9(5bc-2ad)+\frac{4680\sqrt[4]{b}x(bc-ad)^5}{a\left(a+bx^4\right)}+1440b^{13/4}d^5x^{13}}{18720b^{21/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^4)^5/(a + b*x^4)^2,x]
```

```
[Out] (18720*b^(1/4)*d^2*(10*b^3*c^3 - 20*a*b^2*c^2*d + 15*a^2*b*c*d^2 - 4*a^3*d^
3)*x + 3744*b^(5/4)*d^3*(10*b^2*c^2 - 10*a*b*c*d + 3*a^2*d^2)*x^5 + 2080*b^
(9/4)*d^4*(5*b*c - 2*a*d)*x^9 + 1440*b^(13/4)*d^5*x^13 + (4680*b^(1/4)*(b*c
 - a*d)^5*x)/(a*(a + b*x^4)) - (1170*Sqrt[2]*(b*c - a*d)^4*(3*b*c + 17*a*d)
*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/a^(7/4) + (1170*Sqrt[2]*(b*c - a*
d)^4*(3*b*c + 17*a*d)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/a^(7/4) - (5
85*Sqrt[2]*(b*c - a*d)^4*(3*b*c + 17*a*d)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(
1/4)*x + Sqrt[b]*x^2])/a^(7/4) + (585*Sqrt[2]*(b*c - a*d)^4*(3*b*c + 17*a*d
)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/a^(7/4))/(18720*b
^(21/4))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c+dx^4\right)^5}{\left(a+bx^4\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x^4)^5/(a + b*x^4)^2,x]
```

```
[Out] IntegrateAlgebraic[(c + d*x^4)^5/(a + b*x^4)^2, x]
```

fricas [B] time = 1.49, size = 3222, normalized size = 7.92

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^4+c)^5/(b*x^4+a)^2,x, algorithm="fricas")
```

```
[Out] 1/9360*(720*a*b^4*d^5*x^17 + 80*(65*a*b^4*c*d^4 - 17*a^2*b^3*d^5)*x^13 + 20
8*(90*a*b^4*c^2*d^3 - 65*a^2*b^3*c*d^4 + 17*a^3*b^2*d^5)*x^9 + 1872*(50*a*b
^4*c^3*d^2 - 90*a^2*b^3*c^2*d^3 + 65*a^3*b^2*c*d^4 - 17*a^4*b*d^5)*x^5 + 23
40*(a*b^6*x^4 + a^2*b^5)*(-(81*b^20*c^20 + 540*a*b^19*c^19*d - 4050*a^2*b^1
8*c^18*d^2 - 15780*a^3*b^17*c^17*d^3 + 132205*a^4*b^16*c^16*d^4 - 13264*a^5
*b^15*c^15*d^5 - 1960920*a^6*b^14*c^14*d^6 + 6137200*a^7*b^13*c^13*d^7 - 50
0110*a^8*b^12*c^12*d^8 - 48530040*a^9*b^11*c^11*d^9 + 174873556*a^10*b^10*c
^10*d^10 - 360900280*a^11*b^9*c^9*d^11 + 517559250*a^12*b^8*c^8*d^12 - 5482
31440*a^13*b^7*c^7*d^13 + 438700840*a^14*b^6*c^6*d^14 - 266040144*a^15*b^5*
c^5*d^15 + 120836285*a^16*b^4*c^4*d^16 - 39944900*a^17*b^3*c^3*d^17 + 90948
30*a^18*b^2*c^2*d^18 - 1277380*a^19*b*c*d^19 + 83521*a^20*d^20)/(a^7*b^21))
^(1/4)*arctan(-(a^5*b^16*x*(-(81*b^20*c^20 + 540*a*b^19*c^19*d - 4050*a^2*b
^18*c^18*d^2 - 15780*a^3*b^17*c^17*d^3 + 132205*a^4*b^16*c^16*d^4 - 13264*a
^5*b^15*c^15*d^5 - 1960920*a^6*b^14*c^14*d^6 + 6137200*a^7*b^13*c^13*d^7 -
500110*a^8*b^12*c^12*d^8 - 48530040*a^9*b^11*c^11*d^9 + 174873556*a^10*b^10
*c^10*d^10 - 360900280*a^11*b^9*c^9*d^11 + 517559250*a^12*b^8*c^8*d^12 - 54
8231440*a^13*b^7*c^7*d^13 + 438700840*a^14*b^6*c^6*d^14 - 266040144*a^15*b^
5*c^5*d^15 + 120836285*a^16*b^4*c^4*d^16 - 39944900*a^17*b^3*c^3*d^17 + 909
4830*a^18*b^2*c^2*d^18 - 1277380*a^19*b*c*d^19 + 83521*a^20*d^20)/(a^7*b^21
))^(3/4) - a^5*b^16*sqrt((a^4*b^10*sqrt(-(81*b^20*c^20 + 540*a*b^19*c^19*d
- 4050*a^2*b^18*c^18*d^2 - 15780*a^3*b^17*c^17*d^3 + 132205*a^4*b^16*c^16*d
^4 - 13264*a^5*b^15*c^15*d^5 - 1960920*a^6*b^14*c^14*d^6 + 6137200*a^7*b^13
*c^13*d^7 - 500110*a^8*b^12*c^12*d^8 - 48530040*a^9*b^11*c^11*d^9 + 1748735
56*a^10*b^10*c^10*d^10 - 360900280*a^11*b^9*c^9*d^11 + 517559250*a^12*b^8*c
```

```
^8*d^12 - 548231440*a^13*b^7*c^7*d^13 + 438700840*a^14*b^6*c^6*d^14 - 26604
0144*a^15*b^5*c^5*d^15 + 120836285*a^16*b^4*c^4*d^16 - 39944900*a^17*b^3*c^
3*d^17 + 9094830*a^18*b^2*c^2*d^18 - 1277380*a^19*b*c*d^19 + 83521*a^20*d^2
0)/(a^7*b^21)) + (9*b^10*c^10 + 30*a*b^9*c^9*d - 275*a^2*b^8*c^8*d^2 + 40*a
^3*b^7*c^7*d^3 + 3010*a^4*b^6*c^6*d^4 - 9548*a^5*b^5*c^5*d^5 + 14770*a^6*b^
4*c^4*d^6 - 13400*a^7*b^3*c^3*d^7 + 7285*a^8*b^2*c^2*d^8 - 2210*a^9*b*c*d^9
 + 289*a^10*d^10)*x^2)/(9*b^10*c^10 + 30*a*b^9*c^9*d - 275*a^2*b^8*c^8*d^2
+ 40*a^3*b^7*c^7*d^3 + 3010*a^4*b^6*c^6*d^4 - 9548*a^5*b^5*c^5*d^5 + 14770*
a^6*b^4*c^4*d^6 - 13400*a^7*b^3*c^3*d^7 + 7285*a^8*b^2*c^2*d^8 - 2210*a^9*b
*c*d^9 + 289*a^10*d^10))*(-(81*b^20*c^20 + 540*a*b^19*c^19*d - 4050*a^2*b^1
8*c^18*d^2 - 15780*a^3*b^17*c^17*d^3 + 132205*a^4*b^16*c^16*d^4 - 13264*a^5
*b^15*c^15*d^5 - 1960920*a^6*b^14*c^14*d^6 + 6137200*a^7*b^13*c^13*d^7 - 50
0110*a^8*b^12*c^12*d^8 - 48530040*a^9*b^11*c^11*d^9 + 174873556*a^10*b^10*c
^10*d^10 - 360900280*a^11*b^9*c^9*d^11 + 517559250*a^12*b^8*c^8*d^12 - 5482
31440*a^13*b^7*c^7*d^13 + 438700840*a^14*b^6*c^6*d^14 - 266040144*a^15*b^5*
c^5*d^15 + 120836285*a^16*b^4*c^4*d^16 - 39944900*a^17*b^3*c^3*d^17 + 90948
30*a^18*b^2*c^2*d^18 - 1277380*a^19*b*c*d^19 + 83521*a^20*d^20)/(a^7*b^21))
^(3/4))/(27*b^15*c^15 + 135*a*b^14*c^14*d - 1125*a^2*b^13*c^13*d^2 - 1945*a
^3*b^12*c^12*d^3 + 25095*a^4*b^11*c^11*d^4 - 42141*a^5*b^10*c^10*d^5 - 1319
45*a^6*b^9*c^9*d^6 + 774675*a^7*b^8*c^8*d^7 - 1837935*a^8*b^7*c^7*d^8 + 270
0885*a^9*b^6*c^6*d^9 - 2702799*a^10*b^5*c^5*d^10 + 1889685*a^11*b^4*c^4*d^1
1 - 914675*a^12*b^3*c^3*d^12 + 293505*a^13*b^2*c^2*d^13 - 56355*a^14*b*c*d^
14 + 4913*a^15*d^15)) + 585*(a*b^6*x^4 + a^2*b^5)*(-(81*b^20*c^20 + 540*a*b
^19*c^19*d - 4050*a^2*b^18*c^18*d^2 - 15780*a^3*b^17*c^17*d^3 + 132205*a^4*
b^16*c^16*d^4 - 13264*a^5*b^15*c^15*d^5 - 1960920*a^6*b^14*c^14*d^6 + 61372
00*a^7*b^13*c^13*d^7 - 500110*a^8*b^12*c^12*d^8 - 48530040*a^9*b^11*c^11*d^
9 + 174873556*a^10*b^10*c^10*d^10 - 360900280*a^11*b^9*c^9*d^11 + 517559250
*a^12*b^8*c^8*d^12 - 548231440*a^13*b^7*c^7*d^13 + 438700840*a^14*b^6*c^6*d
^14 - 266040144*a^15*b^5*c^5*d^15 + 120836285*a^16*b^4*c^4*d^16 - 39944900*
a^17*b^3*c^3*d^17 + 9094830*a^18*b^2*c^2*d^18 - 1277380*a^19*b*c*d^19 + 835
21*a^20*d^20)/(a^7*b^21))^(1/4)*log(a^2*b^5*(-(81*b^20*c^20 + 540*a*b^19*c^
19*d - 4050*a^2*b^18*c^18*d^2 - 15780*a^3*b^17*c^17*d^3 + 132205*a^4*b^16*c
^16*d^4 - 13264*a^5*b^15*c^15*d^5 - 1960920*a^6*b^14*c^14*d^6 + 6137200*a^7
*b^13*c^13*d^7 - 500110*a^8*b^12*c^12*d^8 - 48530040*a^9*b^11*c^11*d^9 + 17
4873556*a^10*b^10*c^10*d^10 - 360900280*a^11*b^9*c^9*d^11 + 517559250*a^12*
b^8*c^8*d^12 - 548231440*a^13*b^7*c^7*d^13 + 438700840*a^14*b^6*c^6*d^14 -
266040144*a^15*b^5*c^5*d^15 + 120836285*a^16*b^4*c^4*d^16 - 39944900*a^17*b
^3*c^3*d^17 + 9094830*a^18*b^2*c^2*d^18 - 1277380*a^19*b*c*d^19 + 83521*a^2
0*d^20)/(a^7*b^21))^(1/4) + (3*b^5*c^5 + 5*a*b^4*c^4*d - 50*a^2*b^3*c^3*d^2
 + 90*a^3*b^2*c^2*d^3 - 65*a^4*b*c*d^4 + 17*a^5*d^5)*x) - 585*(a*b^6*x^4 +
a^2*b^5)*(-(81*b^20*c^20 + 540*a*b^19*c^19*d - 4050*a^2*b^18*c^18*d^2 - 157
80*a^3*b^17*c^17*d^3 + 132205*a^4*b^16*c^16*d^4 - 13264*a^5*b^15*c^15*d^5 -
 1960920*a^6*b^14*c^14*d^6 + 6137200*a^7*b^13*c^13*d^7 - 500110*a^8*b^12*c^
12*d^8 - 48530040*a^9*b^11*c^11*d^9 + 174873556*a^10*b^10*c^10*d^10 - 36090
0280*a^11*b^9*c^9*d^11 + 517559250*a^12*b^8*c^8*d^12 - 548231440*a^13*b^7*c
^7*d^13 + 438700840*a^14*b^6*c^6*d^14 - 266040144*a^15*b^5*c^5*d^15 + 12083
6285*a^16*b^4*c^4*d^16 - 39944900*a^17*b^3*c^3*d^17 + 9094830*a^18*b^2*c^2*
d^18 - 1277380*a^19*b*c*d^19 + 83521*a^20*d^20)/(a^7*b^21))^(1/4)*log(-a^2*
b^5*(-(81*b^20*c^20 + 540*a*b^19*c^19*d - 4050*a^2*b^18*c^18*d^2 - 15780*a^
3*b^17*c^17*d^3 + 132205*a^4*b^16*c^16*d^4 - 13264*a^5*b^15*c^15*d^5 - 1960
920*a^6*b^14*c^14*d^6 + 6137200*a^7*b^13*c^13*d^7 - 500110*a^8*b^12*c^12*d^
8 - 48530040*a^9*b^11*c^11*d^9 + 174873556*a^10*b^10*c^10*d^10 - 360900280*
a^11*b^9*c^9*d^11 + 517559250*a^12*b^8*c^8*d^12 - 548231440*a^13*b^7*c^7*d^
13 + 438700840*a^14*b^6*c^6*d^14 - 266040144*a^15*b^5*c^5*d^15 + 120836285*
a^16*b^4*c^4*d^16 - 39944900*a^17*b^3*c^3*d^17 + 9094830*a^18*b^2*c^2*d^18
- 1277380*a^19*b*c*d^19 + 83521*a^20*d^20)/(a^7*b^21))^(1/4) + (3*b^5*c^5 +
 5*a*b^4*c^4*d - 50*a^2*b^3*c^3*d^2 + 90*a^3*b^2*c^2*d^3 - 65*a^4*b*c*d^4 +
 17*a^5*d^5)*x) + 2340*(b^5*c^5 - 5*a*b^4*c^4*d + 50*a^2*b^3*c^3*d^2 - 90*a
^3*b^2*c^2*d^3 + 65*a^4*b*c*d^4 - 17*a^5*d^5)*x)/(a*b^6*x^4 + a^2*b^5)
```

giac [B] time = 0.18, size = 798, normalized size = 1.96

$$\frac{\sqrt{2}\left(3\left(ab^{3}\right)^{\frac{1}{4}}b^{5}c^{5}+5\left(ab^{3}\right)^{\frac{1}{4}}ab^{4}c^{4}d-50\left(ab^{3}\right)^{\frac{1}{4}}a^{2}b^{3}c^{3}d^{2}+90\left(ab^{3}\right)^{\frac{1}{4}}a^{3}b^{2}c^{2}d^{3}-65\left(ab^{3}\right)^{\frac{1}{4}}a^{4}bcd^{4}+17\left(ab^{3}\right)^{\frac{1}{4}}a^{5}d^{5}\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{16a^{2}b^{6}}+\frac{\sqrt{2}\left(3\left(ab^{3}\right)^{\frac{1}{4}}b^{5}c^{5}+5\left(ab^{3}\right)^{\frac{1}{4}}ab^{4}c^{4}d-50\left(ab^{3}\right)^{\frac{1}{4}}a^{2}b^{3}c^{3}d^{2}+90\left(ab^{3}\right)^{\frac{1}{4}}a^{3}b^{2}c^{2}d^{3}-65\left(ab^{3}\right)^{\frac{1}{4}}a^{4}bcd^{4}+17\left(ab^{3}\right)^{\frac{1}{4}}a^{5}d^{5}\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{16a^{2}b^{6}}+\frac{\sqrt{2}\left(3\left(ab^{3}\right)^{\frac{1}{4}}b^{5}c^{5}+5\left(ab^{3}\right)^{\frac{1}{4}}ab^{4}c^{4}d-50\left(ab^{3}\right)^{\frac{1}{4}}a^{2}b^{3}c^{3}d^{2}+90\left(ab^{3}\right)^{\frac{1}{4}}a^{3}b^{2}c^{2}d^{3}-65\left(ab^{3}\right)^{\frac{1}{4}}a^{4}bcd^{4}+17\left(ab^{3}\right)^{\frac{1}{4}}a^{5}d^{5}\right)\log\left(x^{2}+\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{32a^{2}b^{6}}-\frac{\sqrt{2}\left(3\left(ab^{3}\right)^{\frac{1}{4}}b^{5}c^{5}+5\left(ab^{3}\right)^{\frac{1}{4}}ab^{4}c^{4}d-50\left(ab^{3}\right)^{\frac{1}{4}}a^{2}b^{3}c^{3}d^{2}+90\left(ab^{3}\right)^{\frac{1}{4}}a^{3}b^{2}c^{2}d^{3}-65\left(ab^{3}\right)^{\frac{1}{4}}a^{4}bcd^{4}+17\left(ab^{3}\right)^{\frac{1}{4}}a^{5}d^{5}\right)\log\left(x^{2}-\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{32a^{2}b^{6}}+\frac{b^{5}c^{5}x-5ab^{4}c^{4}dx+10a^{2}b^{3}c^{3}d^{2}x-10a^{3}b^{2}c^{2}d^{3}x+5a^{4}bcd^{4}x-a^{5}d^{5}x}{4\left(bx^{4}+a\right)ab^{5}}+\frac{45b^{24}d^{5}x^{13}+325b^{24}cd^{4}x^{9}-130ab^{23}d^{5}x^{9}+1170b^{24}c^{2}d^{3}x^{5}-1170ab^{23}cd^{4}x^{5}+351a^{2}b^{22}d^{5}x^{5}+5850b^{24}c^{3}d^{2}x-11700ab^{23}c^{2}d^{3}x+8775a^{2}b^{22}cd^{4}x-2340a^{3}b^{21}d^{5}x}{585b^{26}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^4+c)^5/(b*x^4+a)^2,x, algorithm="giac")
```

```
[Out] 1/16*sqrt(2)*(3*(a*b^3)^(1/4)*b^5*c^5 + 5*(a*b^3)^(1/4)*a*b^4*c^4*d - 50*(a
*b^3)^(1/4)*a^2*b^3*c^3*d^2 + 90*(a*b^3)^(1/4)*a^3*b^2*c^2*d^3 - 65*(a*b^3)
^(1/4)*a^4*b*c*d^4 + 17*(a*b^3)^(1/4)*a^5*d^5)*arctan(1/2*sqrt(2)*(2*x + sq
rt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a^2*b^6) + 1/16*sqrt(2)*(3*(a*b^3)^(1/4)*b
^5*c^5 + 5*(a*b^3)^(1/4)*a*b^4*c^4*d - 50*(a*b^3)^(1/4)*a^2*b^3*c^3*d^2 + 9
0*(a*b^3)^(1/4)*a^3*b^2*c^2*d^3 - 65*(a*b^3)^(1/4)*a^4*b*c*d^4 + 17*(a*b^3)
^(1/4)*a^5*d^5)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))
/(a^2*b^6) + 1/32*sqrt(2)*(3*(a*b^3)^(1/4)*b^5*c^5 + 5*(a*b^3)^(1/4)*a*b^4*
c^4*d - 50*(a*b^3)^(1/4)*a^2*b^3*c^3*d^2 + 90*(a*b^3)^(1/4)*a^3*b^2*c^2*d^3
 - 65*(a*b^3)^(1/4)*a^4*b*c*d^4 + 17*(a*b^3)^(1/4)*a^5*d^5)*log(x^2 + sqrt(
2)*x*(a/b)^(1/4) + sqrt(a/b))/(a^2*b^6) - 1/32*sqrt(2)*(3*(a*b^3)^(1/4)*b^5
*c^5 + 5*(a*b^3)^(1/4)*a*b^4*c^4*d - 50*(a*b^3)^(1/4)*a^2*b^3*c^3*d^2 + 90*
(a*b^3)^(1/4)*a^3*b^2*c^2*d^3 - 65*(a*b^3)^(1/4)*a^4*b*c*d^4 + 17*(a*b^3)^(
1/4)*a^5*d^5)*log(x^2 - sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a^2*b^6) + 1/4*
(b^5*c^5*x - 5*a*b^4*c^4*d*x + 10*a^2*b^3*c^3*d^2*x - 10*a^3*b^2*c^2*d^3*x
+ 5*a^4*b*c*d^4*x - a^5*d^5*x)/((b*x^4 + a)*a*b^5) + 1/585*(45*b^24*d^5*x^1
3 + 325*b^24*c*d^4*x^9 - 130*a*b^23*d^5*x^9 + 1170*b^24*c^2*d^3*x^5 - 1170*
a*b^23*c*d^4*x^5 + 351*a^2*b^22*d^5*x^5 + 5850*b^24*c^3*d^2*x - 11700*a*b^2
3*c^2*d^3*x + 8775*a^2*b^22*c*d^4*x - 2340*a^3*b^21*d^5*x)/b^26
```

maple [B] time = 0.06, size = 1118, normalized size = 2.75

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^4+c)^5/(b*x^4+a)^2,x)
```

```
[Out] 5/32/b/a*(a/b)^(1/4)*2^(1/2)*ln((x^2+(a/b)^(1/4)*2^(1/2)*x+(a/b)^(1/2))/(x^
2-(a/b)^(1/4)*2^(1/2)*x+(a/b)^(1/2)))*c^4*d-65/16/b^4*a^2*(a/b)^(1/4)*2^(1/
2)*arctan(2^(1/2)/(a/b)^(1/4)*x+1)*c*d^4+45/8/b^3*a*(a/b)^(1/4)*2^(1/2)*arc
tan(2^(1/2)/(a/b)^(1/4)*x+1)*c^2*d^3+5/16/b/a*(a/b)^(1/4)*2^(1/2)*arctan(2^
(1/2)/(a/b)^(1/4)*x+1)*c^4*d-65/16/b^4*a^2*(a/b)^(1/4)*2^(1/2)*arctan(2^(1/
2)/(a/b)^(1/4)*x-1)*c*d^4+45/8/b^3*a*(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/
b)^(1/4)*x-1)*c^2*d^3+5/16/b/a*(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/
4)*x-1)*c^4*d-65/32/b^4*a^2*(a/b)^(1/4)*2^(1/2)*ln((x^2+(a/b)^(1/4)*2^(1/2)
*x+(a/b)^(1/2))/(x^2-(a/b)^(1/4)*2^(1/2)*x+(a/b)^(1/2)))*c*d^4+45/16/b^3*a*
(a/b)^(1/4)*2^(1/2)*ln((x^2+(a/b)^(1/4)*2^(1/2)*x+(a/b)^(1/2))/(x^2-(a/b)^(
1/4)*2^(1/2)*x+(a/b)^(1/2)))*c^2*d^3+1/13*d^5*x^13/b^2+2*d^3/b^2*x^5*c^2-4*
d^5/b^5*a^3*x+10*d^2/b^2*c^3*x+1/4/a*x/(b*x^4+a)*c^5-2/9*d^5/b^3*x^9*a+5/9*
d^4/b^2*x^9*c+3/5*d^5/b^4*x^5*a^2-5/2/b^3*a^2*x/(b*x^4+a)*c^2*d^3+5/2/b^2*a
*x/(b*x^4+a)*c^3*d^2+17/16/b^5*a^3*(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)
^(1/4)*x-1)*d^5-25/8/b^2*(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x-1
)*c^3*d^2+17/32/b^5*a^3*(a/b)^(1/4)*2^(1/2)*ln((x^2+(a/b)^(1/4)*2^(1/2)*x+(
a/b)^(1/2))/(x^2-(a/b)^(1/4)*2^(1/2)*x+(a/b)^(1/2)))*d^5-25/16/b^2*(a/b)^(1
/4)*2^(1/2)*ln((x^2+(a/b)^(1/4)*2^(1/2)*x+(a/b)^(1/2))/(x^2-(a/b)^(1/4)*2^(
1/2)*x+(a/b)^(1/2)))*c^3*d^2+17/16/b^5*a^3*(a/b)^(1/4)*2^(1/2)*arctan(2^(1/
2)/(a/b)^(1/4)*x+1)*d^5-25/8/b^2*(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(
1/4)*x+1)*c^3*d^2+5/4/b^4*a^3*x/(b*x^4+a)*c*d^4-1/4/b^5*a^4*x/(b*x^4+a)*d^5
-5/4/b*x/(b*x^4+a)*c^4*d+3/16/a^2*(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^
(1/4)*x-1)*c^5+3/32/a^2*(a/b)^(1/4)*2^(1/2)*ln((x^2+(a/b)^(1/4)*2^(1/2)*x+(
a/b)^(1/2))/(x^2-(a/b)^(1/4)*2^(1/2)*x+(a/b)^(1/2)))*c^5+3/16/a^2*(a/b)^(1/
4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x+1)*c^5-2*d^4/b^3*x^5*a*c+15*d^4/b^4
*a^2*c*x-20*a/b^3*c^2*d^3*x
```

maxima [A] time = 1.45, size = 644, normalized size = 1.58

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^4+c)^5/(b*x^4+a)^2,x, algorithm="maxima")
```

```
[Out] 1/4*(b^5*c^5 - 5*a*b^4*c^4*d + 10*a^2*b^3*c^3*d^2 - 10*a^3*b^2*c^2*d^3 + 5*
a^4*b*c*d^4 - a^5*d^5)*x/(a*b^6*x^4 + a^2*b^5) + 1/585*(45*b^3*d^5*x^13 + 6
5*(5*b^3*c*d^4 - 2*a*b^2*d^5)*x^9 + 117*(10*b^3*c^2*d^3 - 10*a*b^2*c*d^4 +
3*a^2*b*d^5)*x^5 + 585*(10*b^3*c^3*d^2 - 20*a*b^2*c^2*d^3 + 15*a^2*b*c*d^4
- 4*a^3*d^5)*x)/b^5 + 1/32*(2*sqrt(2)*(3*b^5*c^5 + 5*a*b^4*c^4*d - 50*a^2*b
^3*c^3*d^2 + 90*a^3*b^2*c^2*d^3 - 65*a^4*b*c*d^4 + 17*a^5*d^5)*arctan(1/2*s
qrt(2)*(2*sqrt(b)*x + sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(sqrt
(a)*sqrt(sqrt(a)*sqrt(b))) + 2*sqrt(2)*(3*b^5*c^5 + 5*a*b^4*c^4*d - 50*a^2*
b^3*c^3*d^2 + 90*a^3*b^2*c^2*d^3 - 65*a^4*b*c*d^4 + 17*a^5*d^5)*arctan(1/2*
sqrt(2)*(2*sqrt(b)*x - sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(sqr
t(a)*sqrt(sqrt(a)*sqrt(b))) + sqrt(2)*(3*b^5*c^5 + 5*a*b^4*c^4*d - 50*a^2*b
^3*c^3*d^2 + 90*a^3*b^2*c^2*d^3 - 65*a^4*b*c*d^4 + 17*a^5*d^5)*log(sqrt(b)*
x^2 + sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(3/4)*b^(1/4)) - sqrt(2)*(3*b
^5*c^5 + 5*a*b^4*c^4*d - 50*a^2*b^3*c^3*d^2 + 90*a^3*b^2*c^2*d^3 - 65*a^4*b
*c*d^4 + 17*a^5*d^5)*log(sqrt(b)*x^2 - sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))
/(a^(3/4)*b^(1/4)))/(a*b^5)
```

mupad [B] time = 1.71, size = 2490, normalized size = 6.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^4)^5/(a + b*x^4)^2,x)
```

```
[Out] x*((10*c^3*d^2)/b^2 - (2*a*((2*a*((2*a*d^5)/b^3 - (5*c*d^4)/b^2))/b - (a^2*
d^5)/b^4 + (10*c^2*d^3)/b^2))/b + (a^2*((2*a*d^5)/b^3 - (5*c*d^4)/b^2))/b^2
) - x^9*((2*a*d^5)/(9*b^3) - (5*c*d^4)/(9*b^2)) + x^5*((2*a*((2*a*d^5)/b^3
- (5*c*d^4)/b^2))/(5*b) - (a^2*d^5)/(5*b^4) + (2*c^2*d^3)/b^2) + (d^5*x^13)
/(13*b^2) - (x*(a^5*d^5 - b^5*c^5 - 10*a^2*b^3*c^3*d^2 + 10*a^3*b^2*c^2*d^3
 + 5*a*b^4*c^4*d - 5*a^4*b*c*d^4))/(4*a*(a*b^5 + b^6*x^4)) + (atan(((((x*(2
89*a^10*d^10 + 9*b^10*c^10 - 275*a^2*b^8*c^8*d^2 + 40*a^3*b^7*c^7*d^3 + 301
0*a^4*b^6*c^6*d^4 - 9548*a^5*b^5*c^5*d^5 + 14770*a^6*b^4*c^4*d^6 - 13400*a^
7*b^3*c^3*d^7 + 7285*a^8*b^2*c^2*d^8 + 30*a*b^9*c^9*d - 2210*a^9*b*c*d^9))/
(4*a^2*b^7) - ((a*d - b*c)^4*(17*a*d + 3*b*c)*(17*a^5*d^5 + 3*b^5*c^5 - 50*
a^2*b^3*c^3*d^2 + 90*a^3*b^2*c^2*d^3 + 5*a*b^4*c^4*d - 65*a^4*b*c*d^4))/(4*
(-a)^(7/4)*b^(29/4)))*(a*d - b*c)^4*(17*a*d + 3*b*c)*1i)/(16*(-a)^(7/4)*b^(
21/4)) + (((x*(289*a^10*d^10 + 9*b^10*c^10 - 275*a^2*b^8*c^8*d^2 + 40*a^3*b
^7*c^7*d^3 + 3010*a^4*b^6*c^6*d^4 - 9548*a^5*b^5*c^5*d^5 + 14770*a^6*b^4*c^
4*d^6 - 13400*a^7*b^3*c^3*d^7 + 7285*a^8*b^2*c^2*d^8 + 30*a*b^9*c^9*d - 221
0*a^9*b*c*d^9))/(4*a^2*b^7) + ((a*d - b*c)^4*(17*a*d + 3*b*c)*(17*a^5*d^5 +
 3*b^5*c^5 - 50*a^2*b^3*c^3*d^2 + 90*a^3*b^2*c^2*d^3 + 5*a*b^4*c^4*d - 65*a
^4*b*c*d^4))/(4*(-a)^(7/4)*b^(29/4)))*(a*d - b*c)^4*(17*a*d + 3*b*c)*1i)/(1
6*(-a)^(7/4)*b^(21/4)))/((((x*(289*a^10*d^10 + 9*b^10*c^10 - 275*a^2*b^8*c^
8*d^2 + 40*a^3*b^7*c^7*d^3 + 3010*a^4*b^6*c^6*d^4 - 9548*a^5*b^5*c^5*d^5 +
14770*a^6*b^4*c^4*d^6 - 13400*a^7*b^3*c^3*d^7 + 7285*a^8*b^2*c^2*d^8 + 30*a
*b^9*c^9*d - 2210*a^9*b*c*d^9))/(4*a^2*b^7) - ((a*d - b*c)^4*(17*a*d + 3*b*
c)*(17*a^5*d^5 + 3*b^5*c^5 - 50*a^2*b^3*c^3*d^2 + 90*a^3*b^2*c^2*d^3 + 5*a*
b^4*c^4*d - 65*a^4*b*c*d^4))/(4*(-a)^(7/4)*b^(29/4)))*(a*d - b*c)^4*(17*a*d
 + 3*b*c))/(16*(-a)^(7/4)*b^(21/4)) - (((x*(289*a^10*d^10 + 9*b^10*c^10 - 2
75*a^2*b^8*c^8*d^2 + 40*a^3*b^7*c^7*d^3 + 3010*a^4*b^6*c^6*d^4 - 9548*a^5*b
^5*c^5*d^5 + 14770*a^6*b^4*c^4*d^6 - 13400*a^7*b^3*c^3*d^7 + 7285*a^8*b^2*c
^2*d^8 + 30*a*b^9*c^9*d - 2210*a^9*b*c*d^9))/(4*a^2*b^7) + ((a*d - b*c)^4*(
17*a*d + 3*b*c)*(17*a^5*d^5 + 3*b^5*c^5 - 50*a^2*b^3*c^3*d^2 + 90*a^3*b^2*c
```

```
^2*d^3 + 5*a*b^4*c^4*d - 65*a^4*b*c*d^4))/(4*(-a)^(7/4)*b^(29/4)))*(a*d - b
*c)^4*(17*a*d + 3*b*c))/(16*(-a)^(7/4)*b^(21/4))))*(a*d - b*c)^4*(17*a*d +
3*b*c)*1i)/(8*(-a)^(7/4)*b^(21/4)) + (atan(((((x*(289*a^10*d^10 + 9*b^10*c^
10 - 275*a^2*b^8*c^8*d^2 + 40*a^3*b^7*c^7*d^3 + 3010*a^4*b^6*c^6*d^4 - 9548
*a^5*b^5*c^5*d^5 + 14770*a^6*b^4*c^4*d^6 - 13400*a^7*b^3*c^3*d^7 + 7285*a^8
*b^2*c^2*d^8 + 30*a*b^9*c^9*d - 2210*a^9*b*c*d^9))/(4*a^2*b^7) - ((a*d - b*
c)^4*(17*a*d + 3*b*c)*(17*a^5*d^5 + 3*b^5*c^5 - 50*a^2*b^3*c^3*d^2 + 90*a^3
*b^2*c^2*d^3 + 5*a*b^4*c^4*d - 65*a^4*b*c*d^4)*1i)/(4*(-a)^(7/4)*b^(29/4)))
*(a*d - b*c)^4*(17*a*d + 3*b*c))/(16*(-a)^(7/4)*b^(21/4)) + (((x*(289*a^10*
d^10 + 9*b^10*c^10 - 275*a^2*b^8*c^8*d^2 + 40*a^3*b^7*c^7*d^3 + 3010*a^4*b^
6*c^6*d^4 - 9548*a^5*b^5*c^5*d^5 + 14770*a^6*b^4*c^4*d^6 - 13400*a^7*b^3*c^
3*d^7 + 7285*a^8*b^2*c^2*d^8 + 30*a*b^9*c^9*d - 2210*a^9*b*c*d^9))/(4*a^2*b
^7) + ((a*d - b*c)^4*(17*a*d + 3*b*c)*(17*a^5*d^5 + 3*b^5*c^5 - 50*a^2*b^3*
c^3*d^2 + 90*a^3*b^2*c^2*d^3 + 5*a*b^4*c^4*d - 65*a^4*b*c*d^4)*1i)/(4*(-a)^
(7/4)*b^(29/4)))*(a*d - b*c)^4*(17*a*d + 3*b*c))/(16*(-a)^(7/4)*b^(21/4)))/
((((x*(289*a^10*d^10 + 9*b^10*c^10 - 275*a^2*b^8*c^8*d^2 + 40*a^3*b^7*c^7*d
^3 + 3010*a^4*b^6*c^6*d^4 - 9548*a^5*b^5*c^5*d^5 + 14770*a^6*b^4*c^4*d^6 -
13400*a^7*b^3*c^3*d^7 + 7285*a^8*b^2*c^2*d^8 + 30*a*b^9*c^9*d - 2210*a^9*b*
c*d^9))/(4*a^2*b^7) - ((a*d - b*c)^4*(17*a*d + 3*b*c)*(17*a^5*d^5 + 3*b^5*c
^5 - 50*a^2*b^3*c^3*d^2 + 90*a^3*b^2*c^2*d^3 + 5*a*b^4*c^4*d - 65*a^4*b*c*d
^4)*1i)/(4*(-a)^(7/4)*b^(29/4)))*(a*d - b*c)^4*(17*a*d + 3*b*c)*1i)/(16*(-a
)^(7/4)*b^(21/4)) - (((x*(289*a^10*d^10 + 9*b^10*c^10 - 275*a^2*b^8*c^8*d^2
 + 40*a^3*b^7*c^7*d^3 + 3010*a^4*b^6*c^6*d^4 - 9548*a^5*b^5*c^5*d^5 + 14770
*a^6*b^4*c^4*d^6 - 13400*a^7*b^3*c^3*d^7 + 7285*a^8*b^2*c^2*d^8 + 30*a*b^9*
c^9*d - 2210*a^9*b*c*d^9))/(4*a^2*b^7) + ((a*d - b*c)^4*(17*a*d + 3*b*c)*(1
7*a^5*d^5 + 3*b^5*c^5 - 50*a^2*b^3*c^3*d^2 + 90*a^3*b^2*c^2*d^3 + 5*a*b^4*c
^4*d - 65*a^4*b*c*d^4)*1i)/(4*(-a)^(7/4)*b^(29/4)))*(a*d - b*c)^4*(17*a*d +
 3*b*c)*1i)/(16*(-a)^(7/4)*b^(21/4))))*(a*d - b*c)^4*(17*a*d + 3*b*c))/(8*(
-a)^(7/4)*b^(21/4))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**4+c)**5/(b*x**4+a)**2,x)
```

```
[Out] Timed out
```

3.104 $\int \frac{(c+dx^4)^4}{(a+bx^4)^2}\,dx$

Optimal. Leaf size=357

$$-\frac{(bc-ad)^3(13ad+3bc)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{16\sqrt{2}a^{7/4}b^{17/4}}+\frac{(bc-ad)^3(13ad+3bc)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}\right.}{16\sqrt{2}a^{7/4}b^{17/4}}$$

Rubi [A] time = 0.37, antiderivative size = 357, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.421, Rules used = {390, 385, 211, 1165, 628, 1162, 617, 204}

$$\frac{d^2x\left(3a^2d^2-8abcd+6b^2c^2\right)}{b^4}-\frac{(bc-ad)^3(13ad+3bc)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{16\sqrt{2}a^{7/4}b^{17/4}}+\frac{(bc-ad)^3(13ad+3bc)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{16\sqrt{2}a^{7/4}b^{17/4}}-\frac{(bc-ad)^3(13ad+3bc)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}b^{17/4}}+\frac{(bc-ad)^3(13ad+3bc)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{8\sqrt{2}a^{7/4}b^{17/4}}+\frac{2d^3x^5(2bc-ad)}{5b^3}+\frac{x(bc-ad)^4}{4ab^4\left(a+bx^4\right)}+\frac{d^4x^9}{9b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^4)^4/(a + b*x^4)^2,x]
```

```
[Out] (d^2*(6*b^2*c^2 - 8*a*b*c*d + 3*a^2*d^2)*x)/b^4 + (2*d^3*(2*b*c - a*d)*x^5)
/(5*b^3) + (d^4*x^9)/(9*b^2) + ((b*c - a*d)^4*x)/(4*a*b^4*(a + b*x^4)) - ((
b*c - a*d)^3*(3*b*c + 13*a*d)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(8*S
qrt[2]*a^(7/4)*b^(17/4)) + ((b*c - a*d)^3*(3*b*c + 13*a*d)*ArcTan[1 + (Sqrt
[2]*b^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*b^(17/4)) - ((b*c - a*d)^3*(3*b
*c + 13*a*d)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(16*Sq
rt[2]*a^(7/4)*b^(17/4)) + ((b*c - a*d)^3*(3*b*c + 13*a*d)*Log[Sqrt[a] + Sqr
t[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(16*Sqrt[2]*a^(7/4)*b^(17/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{\left(c+dx^4\right)^4}{\left(a+bx^4\right)^2}\,dx = \int \left(\frac{d^2\left(6b^2c^2-8abcd+3a^2d^2\right)}{b^4}+\frac{2d^3(2bc-ad)x^4}{b^3}+\frac{d^4x^8}{b^2}+\frac{(bc-ad)^3(bc+3ad)+4bd(b\cdots}{b^4\left(a+bx^4\right)^2}\cdots\right.$$

$$= \frac{d^2\left(6b^2c^2-8abcd+3a^2d^2\right)x}{b^4}+\frac{2d^3(2bc-ad)x^5}{5b^3}+\frac{d^4x^9}{9b^2}+\frac{\int \frac{(bc-ad)^3(bc+3ad)+4bd(bc-ad)^3x^4}{\left(a+bx^4\right)^2}\,dx}{b^4}\cdots$$

$$= \frac{d^2\left(6b^2c^2-8abcd+3a^2d^2\right)x}{b^4}+\frac{2d^3(2bc-ad)x^5}{5b^3}+\frac{d^4x^9}{9b^2}+\frac{(bc-ad)^4x}{4ab^4\left(a+bx^4\right)}+\frac{\left((bc-ad)^3(3\cdots\right.}{\cdots}$$

$$= \frac{d^2\left(6b^2c^2-8abcd+3a^2d^2\right)x}{b^4}+\frac{2d^3(2bc-ad)x^5}{5b^3}+\frac{d^4x^9}{9b^2}+\frac{(bc-ad)^4x}{4ab^4\left(a+bx^4\right)}+\frac{\left((bc-ad)^3(3\cdots\right.}{\cdots}$$

$$= \frac{d^2\left(6b^2c^2-8abcd+3a^2d^2\right)x}{b^4}+\frac{2d^3(2bc-ad)x^5}{5b^3}+\frac{d^4x^9}{9b^2}+\frac{(bc-ad)^4x}{4ab^4\left(a+bx^4\right)}+\frac{\left((bc-ad)^3(3\cdots\right.}{\cdots}$$

$$= \frac{d^2\left(6b^2c^2-8abcd+3a^2d^2\right)x}{b^4}+\frac{2d^3(2bc-ad)x^5}{5b^3}+\frac{d^4x^9}{9b^2}+\frac{(bc-ad)^4x}{4ab^4\left(a+bx^4\right)}-\frac{(bc-ad)^3(3b\cdots}{\cdots}$$

$$= \frac{d^2\left(6b^2c^2-8abcd+3a^2d^2\right)x}{b^4}+\frac{2d^3(2bc-ad)x^5}{5b^3}+\frac{d^4x^9}{9b^2}+\frac{(bc-ad)^4x}{4ab^4\left(a+bx^4\right)}-\frac{(bc-ad)^3(3b\cdots}{\cdots}$$

Mathematica [A] time = 0.35, size = 341, normalized size = 0.96

$$\frac{\frac{45\sqrt{2}(ad-bc)^3(13ad+3bc)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{a^{7/4}}+\frac{45\sqrt{2}(bc-ad)^3(13ad+3bc)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{a^{7/4}}+\frac{90\sqrt{2}(ad-bc)^3(13ad+3bc)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{a^{7/4}}+\frac{90\sqrt{2}(bc-ad)^3(13ad+3bc)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{a^{7/4}}+1440\sqrt[4]{b}d^2x\left(3a^2d^2-8abcd+6b^2c^2\right)+576b^{5/4}d^3x^5(2bc-ad)+\frac{360\sqrt[4]{b}x(bc-ad)^4}{a\left(a+bx^4\right)}+160b^{9/4}d^4x^9}{1440b^{17/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^4)^4/(a + b*x^4)^2,x]
```

```
[Out] (1440*b^(1/4)*d^2*(6*b^2*c^2 - 8*a*b*c*d + 3*a^2*d^2)*x + 576*b^(5/4)*d^3*(
2*b*c - a*d)*x^5 + 160*b^(9/4)*d^4*x^9 + (360*b^(1/4)*(b*c - a*d)^4*x)/(a*(
a + b*x^4)) + (90*Sqrt[2]*(-(b*c) + a*d)^3*(3*b*c + 13*a*d)*ArcTan[1 - (Sqr
t[2]*b^(1/4)*x)/a^(1/4)])/a^(7/4) + (90*Sqrt[2]*(b*c - a*d)^3*(3*b*c + 13*a
*d)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/a^(7/4) + (45*Sqrt[2]*(-(b*c)
+ a*d)^3*(3*b*c + 13*a*d)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]
*x^2])/a^(7/4) + (45*Sqrt[2]*(b*c - a*d)^3*(3*b*c + 13*a*d)*Log[Sqrt[a] + S
qrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/a^(7/4))/(1440*b^(17/4))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c+dx^4\right)^4}{\left(a+bx^4\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x^4)^4/(a + b*x^4)^2,x]
```

```
[Out] IntegrateAlgebraic[(c + d*x^4)^4/(a + b*x^4)^2, x]
```

fricas [B] time = 1.37, size = 2580, normalized size = 7.23

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^4+c)^4/(b*x^4+a)^2,x, algorithm="fricas")
```

```
[Out] 1/720*(80*a*b^3*d^4*x^13 + 16*(36*a*b^3*c*d^3 - 13*a^2*b^2*d^4)*x^9 + 144*(
30*a*b^3*c^2*d^2 - 36*a^2*b^2*c*d^3 + 13*a^3*b*d^4)*x^5 - 180*(a*b^5*x^4 +
a^2*b^4)*(-(81*b^16*c^16 + 432*a*b^15*c^15*d - 2376*a^2*b^14*c^14*d^2 - 830
4*a^3*b^13*c^13*d^3 + 45724*a^4*b^12*c^12*d^4 + 20400*a^5*b^11*c^11*d^5 - 4
34808*a^6*b^10*c^10*d^6 + 772112*a^7*b^9*c^9*d^7 + 617958*a^8*b^8*c^8*d^8 -
 4810608*a^9*b^7*c^7*d^9 + 9723912*a^10*b^6*c^6*d^10 - 11486160*a^11*b^5*c^
5*d^11 + 8923164*a^12*b^4*c^4*d^12 - 4651504*a^13*b^3*c^3*d^13 + 1577784*a^
14*b^2*c^2*d^14 - 316368*a^15*b*c*d^15 + 28561*a^16*d^16)/(a^7*b^17))^(1/4)
*arctan((a^5*b^13*x*(-(81*b^16*c^16 + 432*a*b^15*c^15*d - 2376*a^2*b^14*c^1
4*d^2 - 8304*a^3*b^13*c^13*d^3 + 45724*a^4*b^12*c^12*d^4 + 20400*a^5*b^11*c
^11*d^5 - 434808*a^6*b^10*c^10*d^6 + 772112*a^7*b^9*c^9*d^7 + 617958*a^8*b^
8*c^8*d^8 - 4810608*a^9*b^7*c^7*d^9 + 9723912*a^10*b^6*c^6*d^10 - 11486160*
a^11*b^5*c^5*d^11 + 8923164*a^12*b^4*c^4*d^12 - 4651504*a^13*b^3*c^3*d^13 +
 1577784*a^14*b^2*c^2*d^14 - 316368*a^15*b*c*d^15 + 28561*a^16*d^16)/(a^7*b
^17))^(3/4) - a^5*b^13*sqrt((a^4*b^8*sqrt(-(81*b^16*c^16 + 432*a*b^15*c^15*
d - 2376*a^2*b^14*c^14*d^2 - 8304*a^3*b^13*c^13*d^3 + 45724*a^4*b^12*c^12*d
^4 + 20400*a^5*b^11*c^11*d^5 - 434808*a^6*b^10*c^10*d^6 + 772112*a^7*b^9*c^
9*d^7 + 617958*a^8*b^8*c^8*d^8 - 4810608*a^9*b^7*c^7*d^9 + 9723912*a^10*b^6
*c^6*d^10 - 11486160*a^11*b^5*c^5*d^11 + 8923164*a^12*b^4*c^4*d^12 - 465150
4*a^13*b^3*c^3*d^13 + 1577784*a^14*b^2*c^2*d^14 - 316368*a^15*b*c*d^15 + 28
561*a^16*d^16)/(a^7*b^17)) + (9*b^8*c^8 + 24*a*b^7*c^7*d - 164*a^2*b^6*c^6*
d^2 - 24*a^3*b^5*c^5*d^3 + 1110*a^4*b^4*c^4*d^4 - 2264*a^5*b^3*c^3*d^5 + 20
76*a^6*b^2*c^2*d^6 - 936*a^7*b*c*d^7 + 169*a^8*d^8)*x^2)/(9*b^8*c^8 + 24*a*
b^7*c^7*d - 164*a^2*b^6*c^6*d^2 - 24*a^3*b^5*c^5*d^3 + 1110*a^4*b^4*c^4*d^4
 - 2264*a^5*b^3*c^3*d^5 + 2076*a^6*b^2*c^2*d^6 - 936*a^7*b*c*d^7 + 169*a^8*
d^8))*(-(81*b^16*c^16 + 432*a*b^15*c^15*d - 2376*a^2*b^14*c^14*d^2 - 8304*a
^3*b^13*c^13*d^3 + 45724*a^4*b^12*c^12*d^4 + 20400*a^5*b^11*c^11*d^5 - 4348
08*a^6*b^10*c^10*d^6 + 772112*a^7*b^9*c^9*d^7 + 617958*a^8*b^8*c^8*d^8 - 48
10608*a^9*b^7*c^7*d^9 + 9723912*a^10*b^6*c^6*d^10 - 11486160*a^11*b^5*c^5*d
^11 + 8923164*a^12*b^4*c^4*d^12 - 4651504*a^13*b^3*c^3*d^13 + 1577784*a^14*
```

```
b^2*c^2*d^14 - 316368*a^15*b*c*d^15 + 28561*a^16*d^16)/(a^7*b^17))^(3/4))/(
27*b^12*c^12 + 108*a*b^11*c^11*d - 666*a^2*b^10*c^10*d^2 - 1124*a^3*b^9*c^9
*d^3 + 8901*a^4*b^8*c^8*d^4 - 7848*a^5*b^7*c^7*d^5 - 34860*a^6*b^6*c^6*d^6
+ 113688*a^7*b^5*c^5*d^7 - 161451*a^8*b^4*c^4*d^8 + 132924*a^9*b^3*c^3*d^9
- 65754*a^10*b^2*c^2*d^10 + 18252*a^11*b*c*d^11 - 2197*a^12*d^12)) - 45*(a*
b^5*x^4 + a^2*b^4)*(-(81*b^16*c^16 + 432*a*b^15*c^15*d - 2376*a^2*b^14*c^14
*d^2 - 8304*a^3*b^13*c^13*d^3 + 45724*a^4*b^12*c^12*d^4 + 20400*a^5*b^11*c^
11*d^5 - 434808*a^6*b^10*c^10*d^6 + 772112*a^7*b^9*c^9*d^7 + 617958*a^8*b^8
*c^8*d^8 - 4810608*a^9*b^7*c^7*d^9 + 9723912*a^10*b^6*c^6*d^10 - 11486160*a
^11*b^5*c^5*d^11 + 8923164*a^12*b^4*c^4*d^12 - 4651504*a^13*b^3*c^3*d^13 +
1577784*a^14*b^2*c^2*d^14 - 316368*a^15*b*c*d^15 + 28561*a^16*d^16)/(a^7*b^
17))^(1/4)*log(a^2*b^4*(-(81*b^16*c^16 + 432*a*b^15*c^15*d - 2376*a^2*b^14*
c^14*d^2 - 8304*a^3*b^13*c^13*d^3 + 45724*a^4*b^12*c^12*d^4 + 20400*a^5*b^1
1*c^11*d^5 - 434808*a^6*b^10*c^10*d^6 + 772112*a^7*b^9*c^9*d^7 + 617958*a^8
*b^8*c^8*d^8 - 4810608*a^9*b^7*c^7*d^9 + 9723912*a^10*b^6*c^6*d^10 - 114861
60*a^11*b^5*c^5*d^11 + 8923164*a^12*b^4*c^4*d^12 - 4651504*a^13*b^3*c^3*d^1
3 + 1577784*a^14*b^2*c^2*d^14 - 316368*a^15*b*c*d^15 + 28561*a^16*d^16)/(a^
7*b^17))^(1/4) - (3*b^4*c^4 + 4*a*b^3*c^3*d - 30*a^2*b^2*c^2*d^2 + 36*a^3*b
*c*d^3 - 13*a^4*d^4)*x) + 45*(a*b^5*x^4 + a^2*b^4)*(-(81*b^16*c^16 + 432*a*
b^15*c^15*d - 2376*a^2*b^14*c^14*d^2 - 8304*a^3*b^13*c^13*d^3 + 45724*a^4*b
^12*c^12*d^4 + 20400*a^5*b^11*c^11*d^5 - 434808*a^6*b^10*c^10*d^6 + 772112*
a^7*b^9*c^9*d^7 + 617958*a^8*b^8*c^8*d^8 - 4810608*a^9*b^7*c^7*d^9 + 972391
2*a^10*b^6*c^6*d^10 - 11486160*a^11*b^5*c^5*d^11 + 8923164*a^12*b^4*c^4*d^1
2 - 4651504*a^13*b^3*c^3*d^13 + 1577784*a^14*b^2*c^2*d^14 - 316368*a^15*b*c
*d^15 + 28561*a^16*d^16)/(a^7*b^17))^(1/4)*log(-a^2*b^4*(-(81*b^16*c^16 + 4
32*a*b^15*c^15*d - 2376*a^2*b^14*c^14*d^2 - 8304*a^3*b^13*c^13*d^3 + 45724*
a^4*b^12*c^12*d^4 + 20400*a^5*b^11*c^11*d^5 - 434808*a^6*b^10*c^10*d^6 + 77
2112*a^7*b^9*c^9*d^7 + 617958*a^8*b^8*c^8*d^8 - 4810608*a^9*b^7*c^7*d^9 + 9
723912*a^10*b^6*c^6*d^10 - 11486160*a^11*b^5*c^5*d^11 + 8923164*a^12*b^4*c^
4*d^12 - 4651504*a^13*b^3*c^3*d^13 + 1577784*a^14*b^2*c^2*d^14 - 316368*a^1
5*b*c*d^15 + 28561*a^16*d^16)/(a^7*b^17))^(1/4) - (3*b^4*c^4 + 4*a*b^3*c^3*
d - 30*a^2*b^2*c^2*d^2 + 36*a^3*b*c*d^3 - 13*a^4*d^4)*x) + 180*(b^4*c^4 - 4
*a*b^3*c^3*d + 30*a^2*b^2*c^2*d^2 - 36*a^3*b*c*d^3 + 13*a^4*d^4)*x)/(a*b^5*
x^4 + a^2*b^4)
```

giac [B] time = 0.17, size = 642, normalized size = 1.80

$$\frac{\sqrt{2}\left(3\left(ab^3\right)^{\frac14}b^4c^4+4\left(ab^3\right)^{\frac14}ab^3c^3d-30\left(ab^3\right)^{\frac14}a^2b^2c^2d^2+36\left(ab^3\right)^{\frac14}a^3bcd^3-13\left(ab^3\right)^{\frac14}a^4d^4\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac14}\right)}{2\left(\frac{a}{b}\right)^{\frac14}}\right)}{16a^2b^5}+\frac{\sqrt{2}\left(3\left(ab^3\right)^{\frac14}b^4c^4+4\left(ab^3\right)^{\frac14}ab^3c^3d-30\left(ab^3\right)^{\frac14}a^2b^2c^2d^2+36\left(ab^3\right)^{\frac14}a^3bcd^3-13\left(ab^3\right)^{\frac14}a^4d^4\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{b}\right)^{\frac14}\right)}{2\left(\frac{a}{b}\right)^{\frac14}}\right)}{16a^2b^5}+\frac{\sqrt{2}\left(3\left(ab^3\right)^{\frac14}b^4c^4+4\left(ab^3\right)^{\frac14}ab^3c^3d-30\left(ab^3\right)^{\frac14}a^2b^2c^2d^2+36\left(ab^3\right)^{\frac14}a^3bcd^3-13\left(ab^3\right)^{\frac14}a^4d^4\right)\log\left(x^2+\sqrt{2}x\left(\frac{a}{b}\right)^{\frac14}+\sqrt{\frac{a}{b}}\right)}{32a^2b^5}-\frac{\sqrt{2}\left(3\left(ab^3\right)^{\frac14}b^4c^4+4\left(ab^3\right)^{\frac14}ab^3c^3d-30\left(ab^3\right)^{\frac14}a^2b^2c^2d^2+36\left(ab^3\right)^{\frac14}a^3bcd^3-13\left(ab^3\right)^{\frac14}a^4d^4\right)\log\left(x^2-\sqrt{2}x\left(\frac{a}{b}\right)^{\frac14}+\sqrt{\frac{a}{b}}\right)}{32a^2b^5}+\frac{b^4c^4x-4ab^3c^3dx+6a^2b^2c^2d^2x-4a^3bcd^3x+a^4d^4x}{4\left(bx^4+a\right)ab^4}+\frac{5b^{16}d^4x^9+36b^{16}cd^3x^5-18ab^{15}d^4x^5+270b^{16}c^2d^2x-360ab^{15}cd^3x+135a^2b^{14}d^4x}{45b^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^4+c)^4/(b*x^4+a)^2,x, algorithm="giac")
```

```
[Out] 1/16*sqrt(2)*(3*(a*b^3)^(1/4)*b^4*c^4 + 4*(a*b^3)^(1/4)*a*b^3*c^3*d - 30*(a
*b^3)^(1/4)*a^2*b^2*c^2*d^2 + 36*(a*b^3)^(1/4)*a^3*b*c*d^3 - 13*(a*b^3)^(1/
4)*a^4*d^4)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a^
2*b^5) + 1/16*sqrt(2)*(3*(a*b^3)^(1/4)*b^4*c^4 + 4*(a*b^3)^(1/4)*a*b^3*c^3*
d - 30*(a*b^3)^(1/4)*a^2*b^2*c^2*d^2 + 36*(a*b^3)^(1/4)*a^3*b*c*d^3 - 13*(a
*b^3)^(1/4)*a^4*d^4)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/b)^(1/4))/(a/b)^(
1/4))/(a^2*b^5) + 1/32*sqrt(2)*(3*(a*b^3)^(1/4)*b^4*c^4 + 4*(a*b^3)^(1/4)*a
*b^3*c^3*d - 30*(a*b^3)^(1/4)*a^2*b^2*c^2*d^2 + 36*(a*b^3)^(1/4)*a^3*b*c*d^
3 - 13*(a*b^3)^(1/4)*a^4*d^4)*log(x^2 + sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/
(a^2*b^5) - 1/32*sqrt(2)*(3*(a*b^3)^(1/4)*b^4*c^4 + 4*(a*b^3)^(1/4)*a*b^3*c
^3*d - 30*(a*b^3)^(1/4)*a^2*b^2*c^2*d^2 + 36*(a*b^3)^(1/4)*a^3*b*c*d^3 - 13
*(a*b^3)^(1/4)*a^4*d^4)*log(x^2 - sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a^2*b
^5) + 1/4*(b^4*c^4*x - 4*a*b^3*c^3*d*x + 6*a^2*b^2*c^2*d^2*x - 4*a^3*b*c*d^
3*x + a^4*d^4*x)/((b*x^4 + a)*a*b^4) + 1/45*(5*b^16*d^4*x^9 + 36*b^16*c*d^3
*x^5 - 18*a*b^15*d^4*x^5 + 270*b^16*c^2*d^2*x - 360*a*b^15*c*d^3*x + 135*a^
2*b^14*d^4*x)/b^18
```

maple [B] time = 0.06, size = 885, normalized size = 2.48

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^4+c)^4/(b*x^4+a)^2,x)
```

```
[Out] 1/4/b/a*(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x-1)*c^3*d+9/8/b^3*a
*(a/b)^(1/4)*2^(1/2)*ln((x^2+(a/b)^(1/4)*2^(1/2)*x+(a/b)^(1/2))/(x^2-(a/b)^
(1/4)*2^(1/2)*x+(a/b)^(1/2)))*c*d^3+1/8/b/a*(a/b)^(1/4)*2^(1/2)*ln((x^2+(a/
b)^(1/4)*2^(1/2)*x+(a/b)^(1/2))/(x^2-(a/b)^(1/4)*2^(1/2)*x+(a/b)^(1/2)))*c^
3*d+9/4/b^3*a*(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x+1)*c*d^3+1/4
/b/a*(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x+1)*c^3*d+9/4/b^3*a*(a
/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x-1)*c*d^3+1/9*d^4*x^9/b^2+1/4
/a*x/(b*x^4+a)*c^4-2/5*d^4/b^3*x^5*a+4/5*d^3/b^2*x^5*c+6/b^2*c^2*d^2*x+3*a^
2/b^4*d^4*x-13/16/b^4*a^2*(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x-
1)*d^4-15/8/b^2*(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x-1)*c^2*d^2
-1/b^3*a^2*x/(b*x^4+a)*c*d^3+3/2/b^2*a*x/(b*x^4+a)*c^2*d^2-13/16/b^4*a^2*(a
/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x+1)*d^4-15/8/b^2*(a/b)^(1/4)*
2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x+1)*c^2*d^2-13/32/b^4*a^2*(a/b)^(1/4)*2
^(1/2)*ln((x^2+(a/b)^(1/4)*2^(1/2)*x+(a/b)^(1/2))/(x^2-(a/b)^(1/4)*2^(1/2)*
x+(a/b)^(1/2)))*d^4-15/16/b^2*(a/b)^(1/4)*2^(1/2)*ln((x^2+(a/b)^(1/4)*2^(1/
2)*x+(a/b)^(1/2))/(x^2-(a/b)^(1/4)*2^(1/2)*x+(a/b)^(1/2)))*c^2*d^2+1/4/b^4*
a^3*x/(b*x^4+a)*d^4-1/b*x/(b*x^4+a)*c^3*d+3/16/a^2*(a/b)^(1/4)*2^(1/2)*arct
an(2^(1/2)/(a/b)^(1/4)*x-1)*c^4+3/32/a^2*(a/b)^(1/4)*2^(1/2)*ln((x^2+(a/b)^
(1/4)*2^(1/2)*x+(a/b)^(1/2))/(x^2-(a/b)^(1/4)*2^(1/2)*x+(a/b)^(1/2)))*c^4-8
*d^3/b^3*a*c*x+3/16/a^2*(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x+1)
*c^4
```

maxima [A] time = 1.45, size = 521, normalized size = 1.46

$$\frac{(b^4c^4 - 4ab^3c^3d + 6a^2b^2c^2d^2 - 4a^3bcd^3 + a^4d^4)x}{4(ab^5x^4 + a^2b^4)} + \frac{5b^2d^4x^9 + 18(2b^2cd^3 - abd^4)x^5 + 45(6b^2c^2d^2 - 8abcd^3 + 3a^2d^4)x}{45b^4} + \frac{\frac{2\sqrt{2}(3b^4c^4+4ab^3c^3d-30a^2b^2c^2d^2+36a^3bcd^3-13a^4d^4)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}} + \frac{2\sqrt{2}(3b^4c^4+4ab^3c^3d-30a^2b^2c^2d^2+36a^3bcd^3-13a^4d^4)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}} + \frac{\sqrt{2}(3b^4c^4+4ab^3c^3d-30a^2b^2c^2d^2+36a^3bcd^3-13a^4d^4)\log\left(\sqrt{b}x^2+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{1}{4}}} - \frac{\sqrt{2}(3b^4c^4+4ab^3c^3d-30a^2b^2c^2d^2+36a^3bcd^3-13a^4d^4)\log\left(\sqrt{b}x^2-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{1}{4}}}}{32ab^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^4+c)^4/(b*x^4+a)^2,x, algorithm="maxima")
```

```
[Out] 1/4*(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)
*x/(a*b^5*x^4 + a^2*b^4) + 1/45*(5*b^2*d^4*x^9 + 18*(2*b^2*c*d^3 - a*b*d^4)
*x^5 + 45*(6*b^2*c^2*d^2 - 8*a*b*c*d^3 + 3*a^2*d^4)*x)/b^4 + 1/32*(2*sqrt(2
)*(3*b^4*c^4 + 4*a*b^3*c^3*d - 30*a^2*b^2*c^2*d^2 + 36*a^3*b*c*d^3 - 13*a^4
*d^4)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x + sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(
a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + 2*sqrt(2)*(3*b^4*c^4 + 4*a*b
^3*c^3*d - 30*a^2*b^2*c^2*d^2 + 36*a^3*b*c*d^3 - 13*a^4*d^4)*arctan(1/2*sqr
t(2)*(2*sqrt(b)*x - sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a
)*sqrt(sqrt(a)*sqrt(b))) + sqrt(2)*(3*b^4*c^4 + 4*a*b^3*c^3*d - 30*a^2*b^2*
c^2*d^2 + 36*a^3*b*c*d^3 - 13*a^4*d^4)*log(sqrt(b)*x^2 + sqrt(2)*a^(1/4)*b^
(1/4)*x + sqrt(a))/(a^(3/4)*b^(1/4)) - sqrt(2)*(3*b^4*c^4 + 4*a*b^3*c^3*d -
 30*a^2*b^2*c^2*d^2 + 36*a^3*b*c*d^3 - 13*a^4*d^4)*log(sqrt(b)*x^2 - sqrt(2
)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(3/4)*b^(1/4)))/(a*b^4)
```

mupad [B] time = 0.30, size = 2043, normalized size = 5.72

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^4)^4/(a + b*x^4)^2,x)
```

```
[Out] x*((2*a*((2*a*d^4)/b^3 - (4*c*d^3)/b^2))/b - (a^2*d^4)/b^4 + (6*c^2*d^2)/b^
2) - x^5*((2*a*d^4)/(5*b^3) - (4*c*d^3)/(5*b^2)) + (d^4*x^9)/(9*b^2) + (x*(
a^4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3))/(4*
a*(a*b^4 + b^5*x^4)) + (atan(((((x*(169*a^8*d^8 + 9*b^8*c^8 - 164*a^2*b^6*c
^6*d^2 - 24*a^3*b^5*c^5*d^3 + 1110*a^4*b^4*c^4*d^4 - 2264*a^5*b^3*c^3*d^5 +
```

```
 2076*a^6*b^2*c^2*d^6 + 24*a*b^7*c^7*d - 936*a^7*b*c*d^7))/(4*a^2*b^5) - ((
a*d - b*c)^3*(13*a*d + 3*b*c)*(3*b^4*c^4 - 13*a^4*d^4 - 30*a^2*b^2*c^2*d^2
+ 4*a*b^3*c^3*d + 36*a^3*b*c*d^3))/(4*(-a)^(7/4)*b^(21/4)))*(a*d - b*c)^3*(
13*a*d + 3*b*c)*1i)/(16*(-a)^(7/4)*b^(17/4)) + (((x*(169*a^8*d^8 + 9*b^8*c^
8 - 164*a^2*b^6*c^6*d^2 - 24*a^3*b^5*c^5*d^3 + 1110*a^4*b^4*c^4*d^4 - 2264*
a^5*b^3*c^3*d^5 + 2076*a^6*b^2*c^2*d^6 + 24*a*b^7*c^7*d - 936*a^7*b*c*d^7))
/(4*a^2*b^5) + ((a*d - b*c)^3*(13*a*d + 3*b*c)*(3*b^4*c^4 - 13*a^4*d^4 - 30
*a^2*b^2*c^2*d^2 + 4*a*b^3*c^3*d + 36*a^3*b*c*d^3))/(4*(-a)^(7/4)*b^(21/4))
)*(a*d - b*c)^3*(13*a*d + 3*b*c)*1i)/(16*(-a)^(7/4)*b^(17/4)))/((((x*(169*a
^8*d^8 + 9*b^8*c^8 - 164*a^2*b^6*c^6*d^2 - 24*a^3*b^5*c^5*d^3 + 1110*a^4*b^
4*c^4*d^4 - 2264*a^5*b^3*c^3*d^5 + 2076*a^6*b^2*c^2*d^6 + 24*a*b^7*c^7*d -
936*a^7*b*c*d^7))/(4*a^2*b^5) - ((a*d - b*c)^3*(13*a*d + 3*b*c)*(3*b^4*c^4
- 13*a^4*d^4 - 30*a^2*b^2*c^2*d^2 + 4*a*b^3*c^3*d + 36*a^3*b*c*d^3))/(4*(-a
)^(7/4)*b^(21/4)))*(a*d - b*c)^3*(13*a*d + 3*b*c))/(16*(-a)^(7/4)*b^(17/4))
 - (((x*(169*a^8*d^8 + 9*b^8*c^8 - 164*a^2*b^6*c^6*d^2 - 24*a^3*b^5*c^5*d^3
 + 1110*a^4*b^4*c^4*d^4 - 2264*a^5*b^3*c^3*d^5 + 2076*a^6*b^2*c^2*d^6 + 24*
a*b^7*c^7*d - 936*a^7*b*c*d^7))/(4*a^2*b^5) + ((a*d - b*c)^3*(13*a*d + 3*b*
c)*(3*b^4*c^4 - 13*a^4*d^4 - 30*a^2*b^2*c^2*d^2 + 4*a*b^3*c^3*d + 36*a^3*b*
c*d^3))/(4*(-a)^(7/4)*b^(21/4)))*(a*d - b*c)^3*(13*a*d + 3*b*c))/(16*(-a)^(
7/4)*b^(17/4))))*(a*d - b*c)^3*(13*a*d + 3*b*c)*1i)/(8*(-a)^(7/4)*b^(17/4))
 + (atan(((((x*(169*a^8*d^8 + 9*b^8*c^8 - 164*a^2*b^6*c^6*d^2 - 24*a^3*b^5*
c^5*d^3 + 1110*a^4*b^4*c^4*d^4 - 2264*a^5*b^3*c^3*d^5 + 2076*a^6*b^2*c^2*d^
6 + 24*a*b^7*c^7*d - 936*a^7*b*c*d^7))/(4*a^2*b^5) - ((a*d - b*c)^3*(13*a*d
 + 3*b*c)*(3*b^4*c^4 - 13*a^4*d^4 - 30*a^2*b^2*c^2*d^2 + 4*a*b^3*c^3*d + 36
*a^3*b*c*d^3)*1i)/(4*(-a)^(7/4)*b^(21/4)))*(a*d - b*c)^3*(13*a*d + 3*b*c))/
(16*(-a)^(7/4)*b^(17/4)) + (((x*(169*a^8*d^8 + 9*b^8*c^8 - 164*a^2*b^6*c^6*
d^2 - 24*a^3*b^5*c^5*d^3 + 1110*a^4*b^4*c^4*d^4 - 2264*a^5*b^3*c^3*d^5 + 20
76*a^6*b^2*c^2*d^6 + 24*a*b^7*c^7*d - 936*a^7*b*c*d^7))/(4*a^2*b^5) + ((a*d
 - b*c)^3*(13*a*d + 3*b*c)*(3*b^4*c^4 - 13*a^4*d^4 - 30*a^2*b^2*c^2*d^2 + 4
*a*b^3*c^3*d + 36*a^3*b*c*d^3)*1i)/(4*(-a)^(7/4)*b^(21/4)))*(a*d - b*c)^3*(
13*a*d + 3*b*c))/(16*(-a)^(7/4)*b^(17/4)))/((((x*(169*a^8*d^8 + 9*b^8*c^8 -
 164*a^2*b^6*c^6*d^2 - 24*a^3*b^5*c^5*d^3 + 1110*a^4*b^4*c^4*d^4 - 2264*a^5
*b^3*c^3*d^5 + 2076*a^6*b^2*c^2*d^6 + 24*a*b^7*c^7*d - 936*a^7*b*c*d^7))/(4
*a^2*b^5) - ((a*d - b*c)^3*(13*a*d + 3*b*c)*(3*b^4*c^4 - 13*a^4*d^4 - 30*a^
2*b^2*c^2*d^2 + 4*a*b^3*c^3*d + 36*a^3*b*c*d^3)*1i)/(4*(-a)^(7/4)*b^(21/4))
)*(a*d - b*c)^3*(13*a*d + 3*b*c)*1i)/(16*(-a)^(7/4)*b^(17/4)) - (((x*(169*a
^8*d^8 + 9*b^8*c^8 - 164*a^2*b^6*c^6*d^2 - 24*a^3*b^5*c^5*d^3 + 1110*a^4*b^
4*c^4*d^4 - 2264*a^5*b^3*c^3*d^5 + 2076*a^6*b^2*c^2*d^6 + 24*a*b^7*c^7*d -
936*a^7*b*c*d^7))/(4*a^2*b^5) + ((a*d - b*c)^3*(13*a*d + 3*b*c)*(3*b^4*c^4
- 13*a^4*d^4 - 30*a^2*b^2*c^2*d^2 + 4*a*b^3*c^3*d + 36*a^3*b*c*d^3)*1i)/(4*
(-a)^(7/4)*b^(21/4)))*(a*d - b*c)^3*(13*a*d + 3*b*c)*1i)/(16*(-a)^(7/4)*b^(
17/4))))*(a*d - b*c)^3*(13*a*d + 3*b*c))/(8*(-a)^(7/4)*b^(17/4))
```

sympy [A] time = 47.54, size = 471, normalized size = 1.32

$$x^5\left(-\frac{2ad^4}{5b^3}+\frac{4cd^3}{5b^2}\right)+x\left(\frac{3a^2d^4}{b^4}-\frac{8acd^3}{b^3}+\frac{6c^2d^2}{b^2}\right)+\frac{x\left(a^4d^4-4a^3bcd^3+6a^2b^2c^2d^2-4ab^3c^3d+b^4c^4\right)}{4a^2b^4+4ab^5x^4}+\operatorname{RootSum}\left(65536t^4a^7b^{17}+28561a^{16}d^{16}-316368a^{15}bcd^{15}+1577784a^{14}b^2c^2d^{14}-4651504a^{13}b^3c^3d^{13}+8923164a^{12}b^4c^4d^{12}-11486160a^{11}b^5c^5d^{11}+9723912a^{10}b^6c^6d^{10}-4810608a^9b^7c^7d^9+617958a^8b^8c^8d^8+772112a^7b^9c^9d^7-434808a^6b^{10}c^{10}d^6+20400a^5b^{11}c^{11}d^5+45724a^4b^{12}c^{12}d^4-8304a^3b^{13}c^{13}d^3-2376a^2b^{14}c^{14}d^2+432ab^{15}c^{15}d+81b^{16}c^{16},\left(t\mapsto t\log\left(-\frac{16ta^2b^4}{13a^4d^4-36a^3bcd^3+30a^2b^2c^2d^2-4ab^3c^3d-3b^4c^4}+x\right)\right)\right)+\frac{d^4x^9}{9b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**4+c)**4/(b*x**4+a)**2,x)
```

```
[Out] x**5*(-2*a*d**4/(5*b**3) + 4*c*d**3/(5*b**2)) + x*(3*a**2*d**4/b**4 - 8*a*c
*d**3/b**3 + 6*c**2*d**2/b**2) + x*(a**4*d**4 - 4*a**3*b*c*d**3 + 6*a**2*b*
*2*c**2*d**2 - 4*a*b**3*c**3*d + b**4*c**4)/(4*a**2*b**4 + 4*a*b**5*x**4) +
 RootSum(65536*_t**4*a**7*b**17 + 28561*a**16*d**16 - 316368*a**15*b*c*d**1
5 + 1577784*a**14*b**2*c**2*d**14 - 4651504*a**13*b**3*c**3*d**13 + 8923164
*a**12*b**4*c**4*d**12 - 11486160*a**11*b**5*c**5*d**11 + 9723912*a**10*b**
6*c**6*d**10 - 4810608*a**9*b**7*c**7*d**9 + 617958*a**8*b**8*c**8*d**8 + 7
72112*a**7*b**9*c**9*d**7 - 434808*a**6*b**10*c**10*d**6 + 20400*a**5*b**11
*c**11*d**5 + 45724*a**4*b**12*c**12*d**4 - 8304*a**3*b**13*c**13*d**3 - 23
76*a**2*b**14*c**14*d**2 + 432*a*b**15*c**15*d + 81*b**16*c**16, Lambda(_t,
```

```
 _t*log(-16*_t*a**2*b**4/(13*a**4*d**4 - 36*a**3*b*c*d**3 + 30*a**2*b**2*c*
*2*d**2 - 4*a*b**3*c**3*d - 3*b**4*c**4) + x))) + d**4*x**9/(9*b**2)
```

3.105 $\int \frac{(c+dx^4)^3}{(a+bx^4)^2}\,dx$

Optimal. Leaf size=317

$$-\frac{3(bc-ad)^2(3ad+bc)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{16\sqrt{2}a^{7/4}b^{13/4}}+\frac{3(bc-ad)^2(3ad+bc)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}\right.}{16\sqrt{2}a^{7/4}b^{13/4}}$$

Rubi [A] time = 0.32, antiderivative size = 317, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.421, Rules used = {390, 385, 211, 1165, 628, 1162, 617, 204}

$$-\frac{3(bc-ad)^2(3ad+bc)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{16\sqrt{2}a^{7/4}b^{13/4}}+\frac{3(bc-ad)^2(3ad+bc)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{16\sqrt{2}a^{7/4}b^{13/4}}-\frac{3(bc-ad)^2(3ad+bc)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}b^{13/4}}+\frac{3(bc-ad)^2(3ad+bc)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{8\sqrt{2}a^{7/4}b^{13/4}}+\frac{d^2x(3bc-2ad)}{b^3}+\frac{x(bc-ad)^3}{4ab^3\left(a+bx^4\right)}+\frac{d^3x^5}{5b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^4)^3/(a + b*x^4)^2,x]
```

```
[Out] (d^2*(3*b*c - 2*a*d)*x)/b^3 + (d^3*x^5)/(5*b^2) + ((b*c - a*d)^3*x)/(4*a*b^
3*(a + b*x^4)) - (3*(b*c - a*d)^2*(b*c + 3*a*d)*ArcTan[1 - (Sqrt[2]*b^(1/4)
*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*b^(13/4)) + (3*(b*c - a*d)^2*(b*c + 3*a*d)
*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*b^(13/4)) - (3
*(b*c - a*d)^2*(b*c + 3*a*d)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt
[b]*x^2])/(16*Sqrt[2]*a^(7/4)*b^(13/4)) + (3*(b*c - a*d)^2*(b*c + 3*a*d)*Lo
g[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(16*Sqrt[2]*a^(7/4)*b
^(13/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{\left(c+dx^4\right)^3}{\left(a+bx^4\right)^2}\,dx = \int \left(\frac{d^2(3bc-2ad)}{b^3}+\frac{d^3x^4}{b^2}+\frac{(bc-ad)^2(bc+2ad)+3bd(bc-ad)^2x^4}{b^3\left(a+bx^4\right)^2}\right)dx$$

$$= \frac{d^2(3bc-2ad)x}{b^3}+\frac{d^3x^5}{5b^2}+\frac{\int \frac{(bc-ad)^2(bc+2ad)+3bd(bc-ad)^2x^4}{\left(a+bx^4\right)^2}\,dx}{b^3}$$

$$= \frac{d^2(3bc-2ad)x}{b^3}+\frac{d^3x^5}{5b^2}+\frac{(bc-ad)^3x}{4ab^3\left(a+bx^4\right)}+\frac{\left(3(bc-ad)^2(bc+3ad)\right)\int \frac{1}{a+bx^4}\,dx}{4ab^3}$$

$$= \frac{d^2(3bc-2ad)x}{b^3}+\frac{d^3x^5}{5b^2}+\frac{(bc-ad)^3x}{4ab^3\left(a+bx^4\right)}+\frac{\left(3(bc-ad)^2(bc+3ad)\right)\int \frac{\sqrt{a}-\sqrt{b}x^2}{a+bx^4}\,dx}{8a^{3/2}b^3}+\frac{\left(3(bc-\right.}{}$$

$$= \frac{d^2(3bc-2ad)x}{b^3}+\frac{d^3x^5}{5b^2}+\frac{(bc-ad)^3x}{4ab^3\left(a+bx^4\right)}+\frac{\left(3(bc-ad)^2(bc+3ad)\right)\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx}{16a^{3/2}b^{7/2}}+\frac{(3(}{}$$

$$= \frac{d^2(3bc-2ad)x}{b^3}+\frac{d^3x^5}{5b^2}+\frac{(bc-ad)^3x}{4ab^3\left(a+bx^4\right)}-\frac{3(bc-ad)^2(bc+3ad)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{\right.}}{16\sqrt{2}a^{7/4}b^{13/4}}$$

$$= \frac{d^2(3bc-2ad)x}{b^3}+\frac{d^3x^5}{5b^2}+\frac{(bc-ad)^3x}{4ab^3\left(a+bx^4\right)}-\frac{3(bc-ad)^2(bc+3ad)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}b^{13/4}}+\frac{3(bc-}{}$$

Mathematica [A] time = 0.25, size = 301, normalized size = 0.95

$$\frac{-\frac{15\sqrt{2}(bc-ad)^2(3ad+bc)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{a^{7/4}}+\frac{15\sqrt{2}(bc-ad)^2(3ad+bc)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{a^{7/4}}-\frac{30\sqrt{2}(bc-ad)^2(3ad+bc)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{a^{7/4}}+\frac{30\sqrt{2}(bc-ad)^2(3ad+bc)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{a^{7/4}}+160\sqrt[4]{b}d^2x(3bc-2ad)+\frac{40\sqrt[4]{b}x(bc-ad)^3}{a\left(a+bx^4\right)}+32b^{5/4}d^3x^5}{160b^{13/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^4)^3/(a + b*x^4)^2,x]
```

```
[Out] (160*b^(1/4)*d^2*(3*b*c - 2*a*d)*x + 32*b^(5/4)*d^3*x^5 + (40*b^(1/4)*(b*c
- a*d)^3*x)/(a*(a + b*x^4)) - (30*Sqrt[2]*(b*c - a*d)^2*(b*c + 3*a*d)*ArcTa
n[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/a^(7/4) + (30*Sqrt[2]*(b*c - a*d)^2*(b*
c + 3*a*d)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/a^(7/4) - (15*Sqrt[2]*(
b*c - a*d)^2*(b*c + 3*a*d)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b
]*x^2])/a^(7/4) + (15*Sqrt[2]*(b*c - a*d)^2*(b*c + 3*a*d)*Log[Sqrt[a] + Sqr
t[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/a^(7/4))/(160*b^(13/4))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c+dx^4\right)^3}{\left(a+bx^4\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x^4)^3/(a + b*x^4)^2,x]
```

```
[Out] IntegrateAlgebraic[(c + d*x^4)^3/(a + b*x^4)^2, x]
```

fricas [B] time = 1.11, size = 1938, normalized size = 6.11

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^4+c)^3/(b*x^4+a)^2,x, algorithm="fricas")
```

```
[Out] 1/80*(16*a*b^2*d^3*x^9 + 48*(5*a*b^2*c*d^2 - 3*a^2*b*d^3)*x^5 + 60*(a*b^4*x
^4 + a^2*b^3)*(-(b^12*c^12 + 4*a*b^11*c^11*d - 14*a^2*b^10*c^10*d^2 - 44*a^
3*b^9*c^9*d^3 + 127*a^4*b^8*c^8*d^4 + 136*a^5*b^7*c^7*d^5 - 644*a^6*b^6*c^6
*d^6 + 328*a^7*b^5*c^5*d^7 + 1039*a^8*b^4*c^4*d^8 - 1932*a^9*b^3*c^3*d^9 +
1458*a^10*b^2*c^2*d^10 - 540*a^11*b*c*d^11 + 81*a^12*d^12)/(a^7*b^13))^(1/4
)*arctan(-(a^5*b^10*x*(-(b^12*c^12 + 4*a*b^11*c^11*d - 14*a^2*b^10*c^10*d^2
 - 44*a^3*b^9*c^9*d^3 + 127*a^4*b^8*c^8*d^4 + 136*a^5*b^7*c^7*d^5 - 644*a^6
*b^6*c^6*d^6 + 328*a^7*b^5*c^5*d^7 + 1039*a^8*b^4*c^4*d^8 - 1932*a^9*b^3*c^
3*d^9 + 1458*a^10*b^2*c^2*d^10 - 540*a^11*b*c*d^11 + 81*a^12*d^12)/(a^7*b^1
3))^(3/4) - a^5*b^10*sqrt((a^4*b^6*sqrt(-(b^12*c^12 + 4*a*b^11*c^11*d - 14*
a^2*b^10*c^10*d^2 - 44*a^3*b^9*c^9*d^3 + 127*a^4*b^8*c^8*d^4 + 136*a^5*b^7*
c^7*d^5 - 644*a^6*b^6*c^6*d^6 + 328*a^7*b^5*c^5*d^7 + 1039*a^8*b^4*c^4*d^8
- 1932*a^9*b^3*c^3*d^9 + 1458*a^10*b^2*c^2*d^10 - 540*a^11*b*c*d^11 + 81*a^
12*d^12)/(a^7*b^13)) + (b^6*c^6 + 2*a*b^5*c^5*d - 9*a^2*b^4*c^4*d^2 - 4*a^3
*b^3*c^3*d^3 + 31*a^4*b^2*c^2*d^4 - 30*a^5*b*c*d^5 + 9*a^6*d^6)*x^2)/(b^6*c
^6 + 2*a*b^5*c^5*d - 9*a^2*b^4*c^4*d^2 - 4*a^3*b^3*c^3*d^3 + 31*a^4*b^2*c^2
*d^4 - 30*a^5*b*c*d^5 + 9*a^6*d^6))*(-(b^12*c^12 + 4*a*b^11*c^11*d - 14*a^2
*b^10*c^10*d^2 - 44*a^3*b^9*c^9*d^3 + 127*a^4*b^8*c^8*d^4 + 136*a^5*b^7*c^7
*d^5 - 644*a^6*b^6*c^6*d^6 + 328*a^7*b^5*c^5*d^7 + 1039*a^8*b^4*c^4*d^8 - 1
932*a^9*b^3*c^3*d^9 + 1458*a^10*b^2*c^2*d^10 - 540*a^11*b*c*d^11 + 81*a^12*
d^12)/(a^7*b^13))^(3/4))/(b^9*c^9 + 3*a*b^8*c^8*d - 12*a^2*b^7*c^7*d^2 - 20
*a^3*b^6*c^6*d^3 + 78*a^4*b^5*c^5*d^4 - 6*a^5*b^4*c^4*d^5 - 188*a^6*b^3*c^3
*d^6 + 252*a^7*b^2*c^2*d^7 - 135*a^8*b*c*d^8 + 27*a^9*d^9)) + 15*(a*b^4*x^4
 + a^2*b^3)*(-(b^12*c^12 + 4*a*b^11*c^11*d - 14*a^2*b^10*c^10*d^2 - 44*a^3*
b^9*c^9*d^3 + 127*a^4*b^8*c^8*d^4 + 136*a^5*b^7*c^7*d^5 - 644*a^6*b^6*c^6*d
^6 + 328*a^7*b^5*c^5*d^7 + 1039*a^8*b^4*c^4*d^8 - 1932*a^9*b^3*c^3*d^9 + 14
58*a^10*b^2*c^2*d^10 - 540*a^11*b*c*d^11 + 81*a^12*d^12)/(a^7*b^13))^(1/4)*
log(3*a^2*b^3*(-(b^12*c^12 + 4*a*b^11*c^11*d - 14*a^2*b^10*c^10*d^2 - 44*a^
3*b^9*c^9*d^3 + 127*a^4*b^8*c^8*d^4 + 136*a^5*b^7*c^7*d^5 - 644*a^6*b^6*c^6
*d^6 + 328*a^7*b^5*c^5*d^7 + 1039*a^8*b^4*c^4*d^8 - 1932*a^9*b^3*c^3*d^9 +
1458*a^10*b^2*c^2*d^10 - 540*a^11*b*c*d^11 + 81*a^12*d^12)/(a^7*b^13))^(1/4
```

```
) + 3*(b^3*c^3 + a*b^2*c^2*d - 5*a^2*b*c*d^2 + 3*a^3*d^3)*x) - 15*(a*b^4*x^
4 + a^2*b^3)*(-(b^12*c^12 + 4*a*b^11*c^11*d - 14*a^2*b^10*c^10*d^2 - 44*a^3
*b^9*c^9*d^3 + 127*a^4*b^8*c^8*d^4 + 136*a^5*b^7*c^7*d^5 - 644*a^6*b^6*c^6*
d^6 + 328*a^7*b^5*c^5*d^7 + 1039*a^8*b^4*c^4*d^8 - 1932*a^9*b^3*c^3*d^9 + 1
458*a^10*b^2*c^2*d^10 - 540*a^11*b*c*d^11 + 81*a^12*d^12)/(a^7*b^13))^(1/4)
*log(-3*a^2*b^3*(-(b^12*c^12 + 4*a*b^11*c^11*d - 14*a^2*b^10*c^10*d^2 - 44*
a^3*b^9*c^9*d^3 + 127*a^4*b^8*c^8*d^4 + 136*a^5*b^7*c^7*d^5 - 644*a^6*b^6*c
^6*d^6 + 328*a^7*b^5*c^5*d^7 + 1039*a^8*b^4*c^4*d^8 - 1932*a^9*b^3*c^3*d^9
+ 1458*a^10*b^2*c^2*d^10 - 540*a^11*b*c*d^11 + 81*a^12*d^12)/(a^7*b^13))^(1
/4) + 3*(b^3*c^3 + a*b^2*c^2*d - 5*a^2*b*c*d^2 + 3*a^3*d^3)*x) + 20*(b^3*c^
3 - 3*a*b^2*c^2*d + 15*a^2*b*c*d^2 - 9*a^3*d^3)*x)/(a*b^4*x^4 + a^2*b^3)
```

giac [A] time = 0.18, size = 496, normalized size = 1.56

$$\frac{3\sqrt{2}\left(\left(ab^{3}\right)^{\frac{1}{4}}b^{3}c^{3}+\left(ab^{3}\right)^{\frac{1}{4}}ab^{2}c^{2}d-5\left(ab^{3}\right)^{\frac{1}{4}}a^{2}bcd^{2}+3\left(ab^{3}\right)^{\frac{1}{4}}a^{3}d^{3}\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{16a^{2}b^{4}}+\frac{3\sqrt{2}\left(\left(ab^{3}\right)^{\frac{1}{4}}b^{3}c^{3}+\left(ab^{3}\right)^{\frac{1}{4}}ab^{2}c^{2}d-5\left(ab^{3}\right)^{\frac{1}{4}}a^{2}bcd^{2}+3\left(ab^{3}\right)^{\frac{1}{4}}a^{3}d^{3}\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{16a^{2}b^{4}}+\frac{3\sqrt{2}\left(\left(ab^{3}\right)^{\frac{1}{4}}b^{3}c^{3}+\left(ab^{3}\right)^{\frac{1}{4}}ab^{2}c^{2}d-5\left(ab^{3}\right)^{\frac{1}{4}}a^{2}bcd^{2}+3\left(ab^{3}\right)^{\frac{1}{4}}a^{3}d^{3}\right)\log\left(x^{2}+\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{32a^{2}b^{4}}-\frac{3\sqrt{2}\left(\left(ab^{3}\right)^{\frac{1}{4}}b^{3}c^{3}+\left(ab^{3}\right)^{\frac{1}{4}}ab^{2}c^{2}d-5\left(ab^{3}\right)^{\frac{1}{4}}a^{2}bcd^{2}+3\left(ab^{3}\right)^{\frac{1}{4}}a^{3}d^{3}\right)\log\left(x^{2}-\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{32a^{2}b^{4}}+\frac{b^{3}c^{3}x-3ab^{2}c^{2}dx+3a^{2}bcd^{2}x-a^{3}d^{3}x}{4\left(bx^{4}+a\right)ab^{3}}+\frac{b^{8}d^{3}x^{5}+15b^{8}cd^{2}x-10ab^{7}d^{3}x}{5b^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^4+c)^3/(b*x^4+a)^2,x, algorithm="giac")
```

```
[Out] 3/16*sqrt(2)*((a*b^3)^(1/4)*b^3*c^3 + (a*b^3)^(1/4)*a*b^2*c^2*d - 5*(a*b^3)
^(1/4)*a^2*b*c*d^2 + 3*(a*b^3)^(1/4)*a^3*d^3)*arctan(1/2*sqrt(2)*(2*x + sqr
t(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a^2*b^4) + 3/16*sqrt(2)*((a*b^3)^(1/4)*b^3*
c^3 + (a*b^3)^(1/4)*a*b^2*c^2*d - 5*(a*b^3)^(1/4)*a^2*b*c*d^2 + 3*(a*b^3)^(
1/4)*a^3*d^3)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(
a^2*b^4) + 3/32*sqrt(2)*((a*b^3)^(1/4)*b^3*c^3 + (a*b^3)^(1/4)*a*b^2*c^2*d
- 5*(a*b^3)^(1/4)*a^2*b*c*d^2 + 3*(a*b^3)^(1/4)*a^3*d^3)*log(x^2 + sqrt(2)*
x*(a/b)^(1/4) + sqrt(a/b))/(a^2*b^4) - 3/32*sqrt(2)*((a*b^3)^(1/4)*b^3*c^3
+ (a*b^3)^(1/4)*a*b^2*c^2*d - 5*(a*b^3)^(1/4)*a^2*b*c*d^2 + 3*(a*b^3)^(1/4)
*a^3*d^3)*log(x^2 - sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a^2*b^4) + 1/4*(b^3
*c^3*x - 3*a*b^2*c^2*d*x + 3*a^2*b*c*d^2*x - a^3*d^3*x)/((b*x^4 + a)*a*b^3)
 + 1/5*(b^8*d^3*x^5 + 15*b^8*c*d^2*x - 10*a*b^7*d^3*x)/b^10
```

maple [B] time = 0.06, size = 669, normalized size = 2.11

$$\frac{d^{3}x^{5}}{5b^{2}}-\frac{a^{2}d^{3}x}{4\left(bx^{4}+a\right)b^{3}}+\frac{3acd^{2}x}{4\left(bx^{4}+a\right)b^{2}}+\frac{c^{3}x}{4\left(bx^{4}+a\right)a}-\frac{3c^{2}dx}{4\left(bx^{4}+a\right)b}-\frac{2ad^{3}x}{b^{3}}+\frac{9\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,ad^{3}\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{16b^{3}}+\frac{9\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,ad^{3}\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{16b^{3}}+\frac{9\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,ad^{3}\ln\left(\frac{x^{2}+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{a}{b}}}{x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{a}{b}}}\right)}{32b^{3}}+\frac{3\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,c^{2}d\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{16ab}+\frac{3\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,c^{2}d\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{16ab}+\frac{3\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,c^{2}d\ln\left(\frac{x^{2}+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{a}{b}}}{x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{a}{b}}}\right)}{32ab}+\frac{3\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,c^{3}\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{16a^{2}}+\frac{3\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,c^{3}\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{16a^{2}}+\frac{3\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,c^{3}\ln\left(\frac{x^{2}+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{a}{b}}}{x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{a}{b}}}\right)}{32a^{2}}+\frac{3cd^{2}x}{b^{2}}-\frac{15\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,cd^{2}\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{16b^{2}}-\frac{15\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,cd^{2}\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{16b^{2}}-\frac{15\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,cd^{2}\ln\left(\frac{x^{2}+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{a}{b}}}{x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{a}{b}}}\right)}{32b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^4+c)^3/(b*x^4+a)^2,x)
```

```
[Out] 1/5*d^3*x^5/b^2-2*a/b^3*d^3*x+3/b^2*c*d^2*x-1/4/b^3*a^2*x/(b*x^4+a)*d^3+3/4
/b^2*a*x/(b*x^4+a)*c*d^2-3/4/b*x/(b*x^4+a)*c^2*d+1/4/a*x/(b*x^4+a)*c^3+9/16
/b^3*a*(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x-1)*d^3-15/16/b^2*(a
/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x-1)*c*d^2+3/16/b/a*(a/b)^(1/4
)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x-1)*c^2*d+3/16/a^2*(a/b)^(1/4)*2^(1/2
)*arctan(2^(1/2)/(a/b)^(1/4)*x-1)*c^3+9/32/b^3*a*(a/b)^(1/4)*2^(1/2)*ln((x^
2+(a/b)^(1/4)*2^(1/2)*x+(a/b)^(1/2))/(x^2-(a/b)^(1/4)*2^(1/2)*x+(a/b)^(1/2)
))*d^3-15/32/b^2*(a/b)^(1/4)*2^(1/2)*ln((x^2+(a/b)^(1/4)*2^(1/2)*x+(a/b)^(1
/2))/(x^2-(a/b)^(1/4)*2^(1/2)*x+(a/b)^(1/2)))*c*d^2+3/32/b/a*(a/b)^(1/4)*2^
(1/2)*ln((x^2+(a/b)^(1/4)*2^(1/2)*x+(a/b)^(1/2))/(x^2-(a/b)^(1/4)*2^(1/2)*x
+(a/b)^(1/2)))*c^2*d+3/32/a^2*(a/b)^(1/4)*2^(1/2)*ln((x^2+(a/b)^(1/4)*2^(1/
2)*x+(a/b)^(1/2))/(x^2-(a/b)^(1/4)*2^(1/2)*x+(a/b)^(1/2)))*c^3+9/16/b^3*a*(
a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x+1)*d^3-15/16/b^2*(a/b)^(1/4
)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x+1)*c*d^2+3/16/b/a*(a/b)^(1/4)*2^(1/2
)*arctan(2^(1/2)/(a/b)^(1/4)*x+1)*c^2*d+3/16/a^2*(a/b)^(1/4)*2^(1/2)*arctan
(2^(1/2)/(a/b)^(1/4)*x+1)*c^3
```

maxima [A] time = 1.31, size = 405, normalized size = 1.28

$$\frac{\left(b^{3}c^{3}-3ab^{2}c^{2}d+3a^{2}bcd^{2}-a^{3}d^{3}\right)x}{4\left(ab^{4}x^{4}+a^{2}b^{3}\right)}+\frac{bd^{3}x^{5}+5\left(3bcd^{2}-2ad^{3}\right)x}{5b^{3}}+\frac{3\left(\frac{2\sqrt{2}\left(b^{3}c^{3}+ab^{2}c^{2}d-5a^{2}bcd^{2}+3a^{3}d^{3}\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}+\frac{2\sqrt{2}\left(b^{3}c^{3}+ab^{2}c^{2}d-5a^{2}bcd^{2}+3a^{3}d^{3}\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}+\frac{\sqrt{2}\left(b^{3}c^{3}+ab^{2}c^{2}d-5a^{2}bcd^{2}+3a^{3}d^{3}\right)\log\left(\sqrt{b}x^{2}+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{1}{4}}}-\frac{\sqrt{2}\left(b^{3}c^{3}+ab^{2}c^{2}d-5a^{2}bcd^{2}+3a^{3}d^{3}\right)\log\left(\sqrt{b}x^{2}-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{1}{4}}}\right)}{32ab^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^4+c)^3/(b*x^4+a)^2,x, algorithm="maxima")
```

```
[Out] 1/4*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*x/(a*b^4*x^4 + a^2*
b^3) + 1/5*(b*d^3*x^5 + 5*(3*b*c*d^2 - 2*a*d^3)*x)/b^3 + 3/32*(2*sqrt(2)*(b
^3*c^3 + a*b^2*c^2*d - 5*a^2*b*c*d^2 + 3*a^3*d^3)*arctan(1/2*sqrt(2)*(2*sqr
t(b)*x + sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt
(a)*sqrt(b))) + 2*sqrt(2)*(b^3*c^3 + a*b^2*c^2*d - 5*a^2*b*c*d^2 + 3*a^3*d^
3)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x - sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*
sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + sqrt(2)*(b^3*c^3 + a*b^2*c^2*d
- 5*a^2*b*c*d^2 + 3*a^3*d^3)*log(sqrt(b)*x^2 + sqrt(2)*a^(1/4)*b^(1/4)*x +
sqrt(a))/(a^(3/4)*b^(1/4)) - sqrt(2)*(b^3*c^3 + a*b^2*c^2*d - 5*a^2*b*c*d^2
 + 3*a^3*d^3)*log(sqrt(b)*x^2 - sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(3/
4)*b^(1/4)))/(a*b^3)
```

mupad [B] time = 1.53, size = 1616, normalized size = 5.10

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^4)^3/(a + b*x^4)^2,x)
```

```
[Out] (d^3*x^5)/(5*b^2) - x*((2*a*d^3)/b^3 - (3*c*d^2)/b^2) - (x*(a^3*d^3 - b^3*c
^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2))/(4*a*(a*b^3 + b^4*x^4)) + (atan((((a*d
 - b*c)^2*(3*a*d + b*c)*((9*x*(9*a^6*d^6 + b^6*c^6 - 9*a^2*b^4*c^4*d^2 - 4*
a^3*b^3*c^3*d^3 + 31*a^4*b^2*c^2*d^4 + 2*a*b^5*c^5*d - 30*a^5*b*c*d^5))/(4*
a^2*b^3) - (3*(a*d - b*c)^2*(3*a*d + b*c)*(36*a^3*d^3 + 12*b^3*c^3 + 12*a*b
^2*c^2*d - 60*a^2*b*c*d^2))/(16*(-a)^(7/4)*b^(13/4)))*3i)/(16*(-a)^(7/4)*b^
(13/4)) + ((a*d - b*c)^2*(3*a*d + b*c)*((9*x*(9*a^6*d^6 + b^6*c^6 - 9*a^2*b
^4*c^4*d^2 - 4*a^3*b^3*c^3*d^3 + 31*a^4*b^2*c^2*d^4 + 2*a*b^5*c^5*d - 30*a^
5*b*c*d^5))/(4*a^2*b^3) + (3*(a*d - b*c)^2*(3*a*d + b*c)*(36*a^3*d^3 + 12*b
^3*c^3 + 12*a*b^2*c^2*d - 60*a^2*b*c*d^2))/(16*(-a)^(7/4)*b^(13/4)))*3i)/(1
6*(-a)^(7/4)*b^(13/4)))/((3*(a*d - b*c)^2*(3*a*d + b*c)*((9*x*(9*a^6*d^6 +
b^6*c^6 - 9*a^2*b^4*c^4*d^2 - 4*a^3*b^3*c^3*d^3 + 31*a^4*b^2*c^2*d^4 + 2*a*
b^5*c^5*d - 30*a^5*b*c*d^5))/(4*a^2*b^3) - (3*(a*d - b*c)^2*(3*a*d + b*c)*(
36*a^3*d^3 + 12*b^3*c^3 + 12*a*b^2*c^2*d - 60*a^2*b*c*d^2))/(16*(-a)^(7/4)*
b^(13/4))))/(16*(-a)^(7/4)*b^(13/4)) - (3*(a*d - b*c)^2*(3*a*d + b*c)*((9*x
*(9*a^6*d^6 + b^6*c^6 - 9*a^2*b^4*c^4*d^2 - 4*a^3*b^3*c^3*d^3 + 31*a^4*b^2*
c^2*d^4 + 2*a*b^5*c^5*d - 30*a^5*b*c*d^5))/(4*a^2*b^3) + (3*(a*d - b*c)^2*(
3*a*d + b*c)*(36*a^3*d^3 + 12*b^3*c^3 + 12*a*b^2*c^2*d - 60*a^2*b*c*d^2))/(
16*(-a)^(7/4)*b^(13/4))))/(16*(-a)^(7/4)*b^(13/4))))*(a*d - b*c)^2*(3*a*d +
 b*c)*3i)/(8*(-a)^(7/4)*b^(13/4)) + (3*atan(((3*(a*d - b*c)^2*(3*a*d + b*c)
*((9*x*(9*a^6*d^6 + b^6*c^6 - 9*a^2*b^4*c^4*d^2 - 4*a^3*b^3*c^3*d^3 + 31*a^
4*b^2*c^2*d^4 + 2*a*b^5*c^5*d - 30*a^5*b*c*d^5))/(4*a^2*b^3) - ((a*d - b*c)
^2*(3*a*d + b*c)*(36*a^3*d^3 + 12*b^3*c^3 + 12*a*b^2*c^2*d - 60*a^2*b*c*d^2
)*3i)/(16*(-a)^(7/4)*b^(13/4))))/(16*(-a)^(7/4)*b^(13/4)) + (3*(a*d - b*c)^
2*(3*a*d + b*c)*((9*x*(9*a^6*d^6 + b^6*c^6 - 9*a^2*b^4*c^4*d^2 - 4*a^3*b^3*
c^3*d^3 + 31*a^4*b^2*c^2*d^4 + 2*a*b^5*c^5*d - 30*a^5*b*c*d^5))/(4*a^2*b^3)
 + ((a*d - b*c)^2*(3*a*d + b*c)*(36*a^3*d^3 + 12*b^3*c^3 + 12*a*b^2*c^2*d -
 60*a^2*b*c*d^2)*3i)/(16*(-a)^(7/4)*b^(13/4))))/(16*(-a)^(7/4)*b^(13/4)))/(
((a*d - b*c)^2*(3*a*d + b*c)*((9*x*(9*a^6*d^6 + b^6*c^6 - 9*a^2*b^4*c^4*d^2
 - 4*a^3*b^3*c^3*d^3 + 31*a^4*b^2*c^2*d^4 + 2*a*b^5*c^5*d - 30*a^5*b*c*d^5)
)/(4*a^2*b^3) - ((a*d - b*c)^2*(3*a*d + b*c)*(36*a^3*d^3 + 12*b^3*c^3 + 12*
a*b^2*c^2*d - 60*a^2*b*c*d^2)*3i)/(16*(-a)^(7/4)*b^(13/4)))*3i)/(16*(-a)^(7
/4)*b^(13/4)) - ((a*d - b*c)^2*(3*a*d + b*c)*((9*x*(9*a^6*d^6 + b^6*c^6 - 9
*a^2*b^4*c^4*d^2 - 4*a^3*b^3*c^3*d^3 + 31*a^4*b^2*c^2*d^4 + 2*a*b^5*c^5*d -
 30*a^5*b*c*d^5))/(4*a^2*b^3) + ((a*d - b*c)^2*(3*a*d + b*c)*(36*a^3*d^3 +
12*b^3*c^3 + 12*a*b^2*c^2*d - 60*a^2*b*c*d^2)*3i)/(16*(-a)^(7/4)*b^(13/4)))
*3i)/(16*(-a)^(7/4)*b^(13/4))))*(a*d - b*c)^2*(3*a*d + b*c))/(8*(-a)^(7/4)*
b^(13/4))
```

sympy [**A**] time = 6.99, size = 337, normalized size = 1.06

$$x\left(-\frac{2ad^3}{b^3}+\frac{3cd^2}{b^2}\right)+\frac{x\left(-a^3d^3+3a^2bcd^2-3ab^2c^2d+b^3c^3\right)}{4a^2b^3+4ab^4x^4}+\operatorname{RootSum}\left(65536t^4a^7b^{13}+6561a^{12}d^{12}-43740a^{11}bcd^{11}+118098a^{10}b^2c^2d^{10}-156492a^9b^3c^3d^9+84159a^8b^4c^4d^8+26568a^7b^5c^5d^7-52164a^6b^6c^6d^6+11016a^5b^7c^7d^5+10287a^4b^8c^8d^4-3564a^3b^9c^9d^3-1134a^2b^{10}c^{10}d^2+324ab^{11}c^{11}d+81b^{12}c^{12},\left(t\mapsto t\log\left(\frac{16ta^2b^3}{9a^3d^3-15a^2bcd^2+3ab^2c^2d+3b^3c^3}+x\right)\right)\right)+\frac{d^3x^5}{5b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**4+c)**3/(b*x**4+a)**2,x)
```

```
[Out] x*(-2*a*d**3/b**3 + 3*c*d**2/b**2) + x*(-a**3*d**3 + 3*a**2*b*c*d**2 - 3*a*
b**2*c**2*d + b**3*c**3)/(4*a**2*b**3 + 4*a*b**4*x**4) + RootSum(65536*_t**
4*a**7*b**13 + 6561*a**12*d**12 - 43740*a**11*b*c*d**11 + 118098*a**10*b**2
*c**2*d**10 - 156492*a**9*b**3*c**3*d**9 + 84159*a**8*b**4*c**4*d**8 + 2656
8*a**7*b**5*c**5*d**7 - 52164*a**6*b**6*c**6*d**6 + 11016*a**5*b**7*c**7*d*
*5 + 10287*a**4*b**8*c**8*d**4 - 3564*a**3*b**9*c**9*d**3 - 1134*a**2*b**10
*c**10*d**2 + 324*a*b**11*c**11*d + 81*b**12*c**12, Lambda(_t, _t*log(16*_t
*a**2*b**3/(9*a**3*d**3 - 15*a**2*b*c*d**2 + 3*a*b**2*c**2*d + 3*b**3*c**3)
 + x))) + d**3*x**5/(5*b**2)
```

3.106 $\int \frac{(c+dx^4)^2}{(a+bx^4)^2}\,dx$

Optimal. Leaf size=291

$$-\frac{(bc-ad)(5ad+3bc)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)}{16\sqrt{2}\,a^{7/4}b^{9/4}}+\frac{(bc-ad)(5ad+3bc)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x\right.}{16\sqrt{2}\,a^{7/4}b^{9/4}}$$

Rubi [A] time = 0.38, antiderivative size = 291, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.421, Rules used = {390, 385, 211, 1165, 628, 1162, 617, 204}

$$-\frac{(bc-ad)(5ad+3bc)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)}{16\sqrt{2}\,a^{7/4}b^{9/4}}+\frac{(bc-ad)(5ad+3bc)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)}{16\sqrt{2}\,a^{7/4}b^{9/4}}-\frac{(bc-ad)(5ad+3bc)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{8\sqrt{2}\,a^{7/4}b^{9/4}}+\frac{(bc-ad)(5ad+3bc)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}+1\right)}{8\sqrt{2}\,a^{7/4}b^{9/4}}+\frac{x(bc-ad)^2}{4ab^2\left(a+bx^4\right)}+\frac{d^2x}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^4)^2/(a + b*x^4)^2,x]
```

```
[Out] (d^2*x)/b^2 + ((b*c - a*d)^2*x)/(4*a*b^2*(a + b*x^4)) - ((b*c - a*d)*(3*b*c
 + 5*a*d)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*b^(9/
4)) + ((b*c - a*d)*(3*b*c + 5*a*d)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])
/(8*Sqrt[2]*a^(7/4)*b^(9/4)) - ((b*c - a*d)*(3*b*c + 5*a*d)*Log[Sqrt[a] - S
qrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(16*Sqrt[2]*a^(7/4)*b^(9/4)) + ((b
*c - a*d)*(3*b*c + 5*a*d)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]
*x^2])/(16*Sqrt[2]*a^(7/4)*b^(9/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{\left(c+dx^4\right)^2}{\left(a+bx^4\right)^2}\,dx = \int \left(\frac{d^2}{b^2}+\frac{b^2c^2-a^2d^2+2bd(bc-ad)x^4}{b^2\left(a+bx^4\right)^2}\right)dx$$

$$= \frac{d^2x}{b^2}+\frac{\int \frac{b^2c^2-a^2d^2+2bd(bc-ad)x^4}{\left(a+bx^4\right)^2}\,dx}{b^2}$$

$$= \frac{d^2x}{b^2}+\frac{(bc-ad)^2x}{4ab^2\left(a+bx^4\right)}+\frac{((bc-ad)(3bc+5ad))\int \frac{1}{a+bx^4}\,dx}{4ab^2}$$

$$= \frac{d^2x}{b^2}+\frac{(bc-ad)^2x}{4ab^2\left(a+bx^4\right)}+\frac{((bc-ad)(3bc+5ad))\int \frac{\sqrt{a}-\sqrt{b}\,x^2}{a+bx^4}\,dx}{8a^{3/2}b^2}+\frac{((bc-ad)(3bc+5ad))\int \frac{\sqrt{a}+\cdots}{a+\cdots}}{8a^{3/2}b^2}$$

$$= \frac{d^2x}{b^2}+\frac{(bc-ad)^2x}{4ab^2\left(a+bx^4\right)}+\frac{((bc-ad)(3bc+5ad))\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{b}}+x^2}\,dx}{16a^{3/2}b^{5/2}}+\frac{((bc-ad)(3bc+5ad))\int \cdots}{16a^{3/2}b^{5/\cdots}}$$

$$= \frac{d^2x}{b^2}+\frac{(bc-ad)^2x}{4ab^2\left(a+bx^4\right)}-\frac{(bc-ad)(3bc+5ad)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{b}\,x^2\right)}{16\sqrt{2}\,a^{7/4}b^{9/4}}+\frac{(bc-ad)(3b\cdots}{\cdots}$$

$$= \frac{d^2x}{b^2}+\frac{(bc-ad)^2x}{4ab^2\left(a+bx^4\right)}-\frac{(bc-ad)(3bc+5ad)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{8\sqrt{2}\,a^{7/4}b^{9/4}}+\frac{(bc-ad)(3bc+5ad)\tan^{-\cdots}}{8\sqrt{2}\,a^{7/4}b^{9/4}}$$

Mathematica [A]   time = 0.23, size = 297, normalized size = 1.02

$$\frac{\frac{\sqrt{2}\left(5a^2d^2-2abcd-3b^2c^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)}{a^{7/4}}+\frac{\sqrt{2}\left(-5a^2d^2+2abcd+3b^2c^2\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)}{a^{7/4}}+\frac{2\sqrt{2}\left(5a^2d^2-2abcd-3b^2c^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{a^{7/4}}+\frac{2\sqrt{2}\left(-5a^2d^2+2abcd+3b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}+1\right)}{a^{7/4}}+\frac{8\sqrt[4]{b}\,x(bc-ad)^2}{a\left(a+bx^4\right)}+32\sqrt[4]{b}\,d^2x}{32b^{9/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^4)^2/(a + b*x^4)^2,x]
```

```
[Out] (32*b^(1/4)*d^2*x + (8*b^(1/4)*(b*c - a*d)^2*x)/(a*(a + b*x^4)) + (2*Sqrt[2
]*(-3*b^2*c^2 - 2*a*b*c*d + 5*a^2*d^2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/
4)])/a^(7/4) + (2*Sqrt[2]*(3*b^2*c^2 + 2*a*b*c*d - 5*a^2*d^2)*ArcTan[1 + (S
qrt[2]*b^(1/4)*x)/a^(1/4)])/a^(7/4) + (Sqrt[2]*(-3*b^2*c^2 - 2*a*b*c*d + 5*
a^2*d^2)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/a^(7/4) +
(Sqrt[2]*(3*b^2*c^2 + 2*a*b*c*d - 5*a^2*d^2)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*
b^(1/4)*x + Sqrt[b]*x^2])/a^(7/4))/(32*b^(9/4))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c+dx^4\right)^2}{\left(a+bx^4\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x^4)^2/(a + b*x^4)^2,x]
```

```
[Out] IntegrateAlgebraic[(c + d*x^4)^2/(a + b*x^4)^2, x]
```

fricas [B] time = 1.38, size = 1335, normalized size = 4.59

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^4+c)^2/(b*x^4+a)^2,x, algorithm="fricas")
```

```
[Out] 1/16*(16*a*b*d^2*x^5 - 4*(a*b^3*x^4 + a^2*b^2)*(-(81*b^8*c^8 + 216*a*b^7*c^
7*d - 324*a^2*b^6*c^6*d^2 - 984*a^3*b^5*c^5*d^3 + 646*a^4*b^4*c^4*d^4 + 164
0*a^5*b^3*c^3*d^5 - 900*a^6*b^2*c^2*d^6 - 1000*a^7*b*c*d^7 + 625*a^8*d^8)/(
a^7*b^9))^(1/4)*arctan((a^5*b^7*x*(-(81*b^8*c^8 + 216*a*b^7*c^7*d - 324*a^2
*b^6*c^6*d^2 - 984*a^3*b^5*c^5*d^3 + 646*a^4*b^4*c^4*d^4 + 1640*a^5*b^3*c^3
*d^5 - 900*a^6*b^2*c^2*d^6 - 1000*a^7*b*c*d^7 + 625*a^8*d^8)/(a^7*b^9))^(3/
4) - a^5*b^7*sqrt((a^4*b^4*sqrt(-(81*b^8*c^8 + 216*a*b^7*c^7*d - 324*a^2*b^
6*c^6*d^2 - 984*a^3*b^5*c^5*d^3 + 646*a^4*b^4*c^4*d^4 + 1640*a^5*b^3*c^3*d^
5 - 900*a^6*b^2*c^2*d^6 - 1000*a^7*b*c*d^7 + 625*a^8*d^8)/(a^7*b^9)) + (9*b
^4*c^4 + 12*a*b^3*c^3*d - 26*a^2*b^2*c^2*d^2 - 20*a^3*b*c*d^3 + 25*a^4*d^4)
*x^2)/(9*b^4*c^4 + 12*a*b^3*c^3*d - 26*a^2*b^2*c^2*d^2 - 20*a^3*b*c*d^3 + 2
5*a^4*d^4))*(-(81*b^8*c^8 + 216*a*b^7*c^7*d - 324*a^2*b^6*c^6*d^2 - 984*a^3
*b^5*c^5*d^3 + 646*a^4*b^4*c^4*d^4 + 1640*a^5*b^3*c^3*d^5 - 900*a^6*b^2*c^2
*d^6 - 1000*a^7*b*c*d^7 + 625*a^8*d^8)/(a^7*b^9))^(3/4))/(27*b^6*c^6 + 54*a
*b^5*c^5*d - 99*a^2*b^4*c^4*d^2 - 172*a^3*b^3*c^3*d^3 + 165*a^4*b^2*c^2*d^4
 + 150*a^5*b*c*d^5 - 125*a^6*d^6)) - (a*b^3*x^4 + a^2*b^2)*(-(81*b^8*c^8 +
216*a*b^7*c^7*d - 324*a^2*b^6*c^6*d^2 - 984*a^3*b^5*c^5*d^3 + 646*a^4*b^4*c
^4*d^4 + 1640*a^5*b^3*c^3*d^5 - 900*a^6*b^2*c^2*d^6 - 1000*a^7*b*c*d^7 + 62
5*a^8*d^8)/(a^7*b^9))^(1/4)*log(a^2*b^2*(-(81*b^8*c^8 + 216*a*b^7*c^7*d - 3
24*a^2*b^6*c^6*d^2 - 984*a^3*b^5*c^5*d^3 + 646*a^4*b^4*c^4*d^4 + 1640*a^5*b
^3*c^3*d^5 - 900*a^6*b^2*c^2*d^6 - 1000*a^7*b*c*d^7 + 625*a^8*d^8)/(a^7*b^9
))^(1/4) - (3*b^2*c^2 + 2*a*b*c*d - 5*a^2*d^2)*x) + (a*b^3*x^4 + a^2*b^2)*(
-(81*b^8*c^8 + 216*a*b^7*c^7*d - 324*a^2*b^6*c^6*d^2 - 984*a^3*b^5*c^5*d^3
+ 646*a^4*b^4*c^4*d^4 + 1640*a^5*b^3*c^3*d^5 - 900*a^6*b^2*c^2*d^6 - 1000*a
^7*b*c*d^7 + 625*a^8*d^8)/(a^7*b^9))^(1/4)*log(-a^2*b^2*(-(81*b^8*c^8 + 216
*a*b^7*c^7*d - 324*a^2*b^6*c^6*d^2 - 984*a^3*b^5*c^5*d^3 + 646*a^4*b^4*c^4*
d^4 + 1640*a^5*b^3*c^3*d^5 - 900*a^6*b^2*c^2*d^6 - 1000*a^7*b*c*d^7 + 625*a
^8*d^8)/(a^7*b^9))^(1/4) - (3*b^2*c^2 + 2*a*b*c*d - 5*a^2*d^2)*x) + 4*(b^2*
c^2 - 2*a*b*c*d + 5*a^2*d^2)*x)/(a*b^3*x^4 + a^2*b^2)
```

giac [A] time = 0.17, size = 376, normalized size = 1.29

$$\frac{d^2x}{b^2}+\frac{\sqrt{2}\left(3\left(ab^3\right)^{\frac{1}{4}}b^2c^2+2\left(ab^3\right)^{\frac{1}{4}}abcd-5\left(ab^3\right)^{\frac{1}{4}}a^2d^2\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{16a^2b^3}+\frac{\sqrt{2}\left(3\left(ab^3\right)^{\frac{1}{4}}b^2c^2+2\left(ab^3\right)^{\frac{1}{4}}abcd-5\left(ab^3\right)^{\frac{1}{4}}a^2d^2\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{16a^2b^3}+\frac{\sqrt{2}\left(3\left(ab^3\right)^{\frac{1}{4}}b^2c^2+2\left(ab^3\right)^{\frac{1}{4}}abcd-5\left(ab^3\right)^{\frac{1}{4}}a^2d^2\right)\log\left(x^2+\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{32a^2b^3}-\frac{\sqrt{2}\left(3\left(ab^3\right)^{\frac{1}{4}}b^2c^2+2\left(ab^3\right)^{\frac{1}{4}}abcd-5\left(ab^3\right)^{\frac{1}{4}}a^2d^2\right)\log\left(x^2-\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{32a^2b^3}+\frac{b^2c^2x-2abcdx+a^2d^2x}{4\left(bx^4+a\right)ab^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^4+c)^2/(b*x^4+a)^2,x, algorithm="giac")
```

```
[Out] d^2*x/b^2 + 1/16*sqrt(2)*(3*(a*b^3)^(1/4)*b^2*c^2 + 2*(a*b^3)^(1/4)*a*b*c*d
 - 5*(a*b^3)^(1/4)*a^2*d^2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/b)^(1/4))/
(a/b)^(1/4))/(a^2*b^3) + 1/16*sqrt(2)*(3*(a*b^3)^(1/4)*b^2*c^2 + 2*(a*b^3)^
(1/4)*a*b*c*d - 5*(a*b^3)^(1/4)*a^2*d^2)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*
(a/b)^(1/4))/(a/b)^(1/4))/(a^2*b^3) + 1/32*sqrt(2)*(3*(a*b^3)^(1/4)*b^2*c^2
 + 2*(a*b^3)^(1/4)*a*b*c*d - 5*(a*b^3)^(1/4)*a^2*d^2)*log(x^2 + sqrt(2)*x*(
a/b)^(1/4) + sqrt(a/b))/(a^2*b^3) - 1/32*sqrt(2)*(3*(a*b^3)^(1/4)*b^2*c^2 +
 2*(a*b^3)^(1/4)*a*b*c*d - 5*(a*b^3)^(1/4)*a^2*d^2)*log(x^2 - sqrt(2)*x*(a/
b)^(1/4) + sqrt(a/b))/(a^2*b^3) + 1/4*(b^2*c^2*x - 2*a*b*c*d*x + a^2*d^2*x)
/((b*x^4 + a)*a*b^2)
```

maple [B] time = 0.06, size = 475, normalized size = 1.63

$$\frac{ad^2x}{4\left(bx^4+a\right)b^2}+\frac{c^2x}{4\left(bx^4+a\right)a}-\frac{cdx}{2\left(bx^4+a\right)b}+\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,cd\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{8ab}+\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,cd\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{8ab}+\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,cd\ln\left(\frac{x^2+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{a}{b}}}{x^2-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{a}{b}}}\right)}{16ab}+\frac{3\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,c^2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{16a^2}+\frac{3\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,c^2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{16a^2}+\frac{3\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,c^2\ln\left(\frac{x^2+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{a}{b}}}{x^2-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{a}{b}}}\right)}{32a^2}+\frac{d^2x}{b^2}-\frac{5\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,d^2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{16b^2}-\frac{5\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,d^2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{16b^2}-\frac{5\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,d^2\ln\left(\frac{x^2+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{a}{b}}}{x^2-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{a}{b}}}\right)}{32b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^4+c)^2/(b*x^4+a)^2,x)
```

```
[Out] 1/b^2*d^2*x+1/4/b^2*a*x/(b*x^4+a)*d^2-1/2/b*x/(b*x^4+a)*c*d+1/4/a*x/(b*x^4+
a)*c^2-5/16/b^2*(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x-1)*d^2+1/8
/b/a*(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x-1)*c*d+3/16/a^2*(a/b)
^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x-1)*c^2-5/32/b^2*(a/b)^(1/4)*2^(
1/2)*ln((x^2+(a/b)^(1/4)*2^(1/2)*x+(a/b)^(1/2))/(x^2-(a/b)^(1/4)*2^(1/2)*x+
(a/b)^(1/2)))*d^2+1/16/b/a*(a/b)^(1/4)*2^(1/2)*ln((x^2+(a/b)^(1/4)*2^(1/2)*
x+(a/b)^(1/2))/(x^2-(a/b)^(1/4)*2^(1/2)*x+(a/b)^(1/2)))*c*d+3/32/a^2*(a/b)^
(1/4)*2^(1/2)*ln((x^2+(a/b)^(1/4)*2^(1/2)*x+(a/b)^(1/2))/(x^2-(a/b)^(1/4)*2
^(1/2)*x+(a/b)^(1/2)))*c^2-5/16/b^2*(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b
)^(1/4)*x+1)*d^2+1/8/b/a*(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x+1
)*c*d+3/16/a^2*(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x+1)*c^2
```

maxima [A] time = 1.26, size = 319, normalized size = 1.10

$$\frac{\left(b^2c^2-2abcd+a^2d^2\right)x}{4\left(ab^3x^4+a^2b^2\right)}+\frac{d^2x}{b^2}+\frac{\frac{2\sqrt{2}\left(3b^2c^2+2abcd-5a^2d^2\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}+\frac{2\sqrt{2}\left(3b^2c^2+2abcd-5a^2d^2\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}+\frac{\sqrt{2}\left(3b^2c^2+2abcd-5a^2d^2\right)\log\left(\sqrt{b}x^2+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{1}{4}}}-\frac{\sqrt{2}\left(3b^2c^2+2abcd-5a^2d^2\right)\log\left(\sqrt{b}x^2-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{1}{4}}}}{32ab^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^4+c)^2/(b*x^4+a)^2,x, algorithm="maxima")
```

```
[Out] 1/4*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*x/(a*b^3*x^4 + a^2*b^2) + d^2*x/b^2 + 1
/32*(2*sqrt(2)*(3*b^2*c^2 + 2*a*b*c*d - 5*a^2*d^2)*arctan(1/2*sqrt(2)*(2*sq
rt(b)*x + sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqr
t(a)*sqrt(b))) + 2*sqrt(2)*(3*b^2*c^2 + 2*a*b*c*d - 5*a^2*d^2)*arctan(1/2*s
qrt(2)*(2*sqrt(b)*x - sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(sqrt
(a)*sqrt(sqrt(a)*sqrt(b))) + sqrt(2)*(3*b^2*c^2 + 2*a*b*c*d - 5*a^2*d^2)*lo
g(sqrt(b)*x^2 + sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(3/4)*b^(1/4)) - sq
rt(2)*(3*b^2*c^2 + 2*a*b*c*d - 5*a^2*d^2)*log(sqrt(b)*x^2 - sqrt(2)*a^(1/4)
*b^(1/4)*x + sqrt(a))/(a^(3/4)*b^(1/4)))/(a*b^2)
```

mupad [B] time = 0.30, size = 1254, normalized size = 4.31

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^4)^2/(a + b*x^4)^2,x)
```

```
[Out] (d^2*x)/b^2 + (x*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d))/(4*a*(a*b^2 + b^3*x^4)) +
 (atan((((a*d - b*c)*(5*a*d + 3*b*c)*((x*(25*a^4*d^4 + 9*b^4*c^4 - 26*a^2*b
^2*c^2*d^2 + 12*a*b^3*c^3*d - 20*a^3*b*c*d^3))/(4*a^2*b) - ((a*d - b*c)*(5*
a*d + 3*b*c)*(12*b^3*c^2 - 20*a^2*b*d^2 + 8*a*b^2*c*d))/(16*(-a)^(7/4)*b^(9
/4)))*1i)/(16*(-a)^(7/4)*b^(9/4)) + ((a*d - b*c)*(5*a*d + 3*b*c)*((x*(25*a^
4*d^4 + 9*b^4*c^4 - 26*a^2*b^2*c^2*d^2 + 12*a*b^3*c^3*d - 20*a^3*b*c*d^3))/
(4*a^2*b) + ((a*d - b*c)*(5*a*d + 3*b*c)*(12*b^3*c^2 - 20*a^2*b*d^2 + 8*a*b
^2*c*d))/(16*(-a)^(7/4)*b^(9/4)))*1i)/(16*(-a)^(7/4)*b^(9/4)))/(((a*d - b*c
)*(5*a*d + 3*b*c)*((x*(25*a^4*d^4 + 9*b^4*c^4 - 26*a^2*b^2*c^2*d^2 + 12*a*b
^3*c^3*d - 20*a^3*b*c*d^3))/(4*a^2*b) - ((a*d - b*c)*(5*a*d + 3*b*c)*(12*b^
3*c^2 - 20*a^2*b*d^2 + 8*a*b^2*c*d))/(16*(-a)^(7/4)*b^(9/4))))/(16*(-a)^(7/
4)*b^(9/4)) - ((a*d - b*c)*(5*a*d + 3*b*c)*((x*(25*a^4*d^4 + 9*b^4*c^4 - 26
*a^2*b^2*c^2*d^2 + 12*a*b^3*c^3*d - 20*a^3*b*c*d^3))/(4*a^2*b) + ((a*d - b*
c)*(5*a*d + 3*b*c)*(12*b^3*c^2 - 20*a^2*b*d^2 + 8*a*b^2*c*d))/(16*(-a)^(7/4
)*b^(9/4))))/(16*(-a)^(7/4)*b^(9/4))))*(a*d - b*c)*(5*a*d + 3*b*c)*1i)/(8*(
-a)^(7/4)*b^(9/4)) + (atan((((a*d - b*c)*(5*a*d + 3*b*c)*((x*(25*a^4*d^4 +
9*b^4*c^4 - 26*a^2*b^2*c^2*d^2 + 12*a*b^3*c^3*d - 20*a^3*b*c*d^3))/(4*a^2*b
) - ((a*d - b*c)*(5*a*d + 3*b*c)*(12*b^3*c^2 - 20*a^2*b*d^2 + 8*a*b^2*c*d)*
1i)/(16*(-a)^(7/4)*b^(9/4))))/(16*(-a)^(7/4)*b^(9/4)) + ((a*d - b*c)*(5*a*d
 + 3*b*c)*((x*(25*a^4*d^4 + 9*b^4*c^4 - 26*a^2*b^2*c^2*d^2 + 12*a*b^3*c^3*d
 - 20*a^3*b*c*d^3))/(4*a^2*b) + ((a*d - b*c)*(5*a*d + 3*b*c)*(12*b^3*c^2 -
20*a^2*b*d^2 + 8*a*b^2*c*d)*1i)/(16*(-a)^(7/4)*b^(9/4))))/(16*(-a)^(7/4)*b^
(9/4)))/(((a*d - b*c)*(5*a*d + 3*b*c)*((x*(25*a^4*d^4 + 9*b^4*c^4 - 26*a^2*
b^2*c^2*d^2 + 12*a*b^3*c^3*d - 20*a^3*b*c*d^3))/(4*a^2*b) - ((a*d - b*c)*(5
*a*d + 3*b*c)*(12*b^3*c^2 - 20*a^2*b*d^2 + 8*a*b^2*c*d)*1i)/(16*(-a)^(7/4)*
b^(9/4)))*1i)/(16*(-a)^(7/4)*b^(9/4)) - ((a*d - b*c)*(5*a*d + 3*b*c)*((x*(2
5*a^4*d^4 + 9*b^4*c^4 - 26*a^2*b^2*c^2*d^2 + 12*a*b^3*c^3*d - 20*a^3*b*c*d^
3))/(4*a^2*b) + ((a*d - b*c)*(5*a*d + 3*b*c)*(12*b^3*c^2 - 20*a^2*b*d^2 + 8
*a*b^2*c*d)*1i)/(16*(-a)^(7/4)*b^(9/4)))*1i)/(16*(-a)^(7/4)*b^(9/4))))*(a*d
 - b*c)*(5*a*d + 3*b*c))/(8*(-a)^(7/4)*b^(9/4))
```

sympy [A] time = 2.17, size = 219, normalized size = 0.75

$$\frac{x\left(a^2d^2 - 2abcd + b^2c^2\right)}{4a^2b^2 + 4ab^3x^4} + \operatorname{RootSum}\left(65536t^4a^7b^9 + 625a^8d^8 - 1000a^7bcd^7 - 900a^6b^2c^2d^6 + 1640a^5b^3c^3d^5 + 646a^4b^4c^4d^4 - 984a^3b^5c^5d^3 - 324a^2b^6c^6d^2 + 216ab^7c^7d + 81b^8c^8, \left(t \mapsto t\log\left(-\frac{16ta^2b^2}{5a^2d^2 - 2abcd - 3b^2c^2} + x\right)\right)\right) + \frac{d^2x}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**4+c)**2/(b*x**4+a)**2,x)
```

```
[Out] x*(a**2*d**2 - 2*a*b*c*d + b**2*c**2)/(4*a**2*b**2 + 4*a*b**3*x**4) + RootS
um(65536*_t**4*a**7*b**9 + 625*a**8*d**8 - 1000*a**7*b*c*d**7 - 900*a**6*b*
*2*c**2*d**6 + 1640*a**5*b**3*c**3*d**5 + 646*a**4*b**4*c**4*d**4 - 984*a**
3*b**5*c**5*d**3 - 324*a**2*b**6*c**6*d**2 + 216*a*b**7*c**7*d + 81*b**8*c*
*8, Lambda(_t, _t*log(-16*_t*a**2*b**2/(5*a**2*d**2 - 2*a*b*c*d - 3*b**2*c*
*2) + x))) + d**2*x/b**2
```

3.107 $\int \frac{c+dx^4}{\left(a+bx^4\right)^2} \, dx$

Optimal. Leaf size=245

$$-\frac{(ad+3bc)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)}{16\sqrt{2}\,a^{7/4}b^{5/4}}+\frac{(ad+3bc)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)}{16\sqrt{2}\,a^{7/4}b^{5/4}}-\frac{(ad+3bc)\tan^{-1}\left(1\right.}{8\sqrt{2}\,a^{7/4}b^{5}}$$

Rubi [A] time = 0.15, antiderivative size = 245, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.412, Rules used = {385, 211, 1165, 628, 1162, 617, 204}

$$-\frac{(ad+3bc)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)}{16\sqrt{2}\,a^{7/4}b^{5/4}}+\frac{(ad+3bc)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)}{16\sqrt{2}\,a^{7/4}b^{5/4}}-\frac{(ad+3bc)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{8\sqrt{2}\,a^{7/4}b^{5/4}}+\frac{(ad+3bc)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}+1\right)}{8\sqrt{2}\,a^{7/4}b^{5/4}}+\frac{x(bc-ad)}{4ab\left(a+bx^4\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^4)/(a + b*x^4)^2,x]
```

```
[Out] ((b*c - a*d)*x)/(4*a*b*(a + b*x^4)) - ((3*b*c + a*d)*ArcTan[1 - (Sqrt[2]*b^
(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*b^(5/4)) + ((3*b*c + a*d)*ArcTan[1 +
(Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*b^(5/4)) - ((3*b*c + a*d)*
Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(16*Sqrt[2]*a^(7/4)
*b^(5/4)) + ((3*b*c + a*d)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b
]*x^2])/(16*Sqrt[2]*a^(7/4)*b^(5/4))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{c+dx^4}{\left(a+bx^4\right)^2}\,dx = \frac{(bc-ad)x}{4ab\left(a+bx^4\right)} + \frac{(3bc+ad)\int \frac{1}{a+bx^4}\,dx}{4ab}$$

$$= \frac{(bc-ad)x}{4ab\left(a+bx^4\right)} + \frac{(3bc+ad)\int \frac{\sqrt{a}-\sqrt{b}x^2}{a+bx^4}\,dx}{8a^{3/2}b} + \frac{(3bc+ad)\int \frac{\sqrt{a}+\sqrt{b}x^2}{a+bx^4}\,dx}{8a^{3/2}b}$$

$$= \frac{(bc-ad)x}{4ab\left(a+bx^4\right)} + \frac{(3bc+ad)\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx}{16a^{3/2}b^{3/2}} + \frac{(3bc+ad)\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx}{16a^{3/2}b^{3/2}} - \frac{(3bc+a}{}$$

$$= \frac{(bc-ad)x}{4ab\left(a+bx^4\right)} - \frac{(3bc+ad)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{16\sqrt{2}a^{7/4}b^{5/4}} + \frac{(3bc+ad)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{}\right.}{16\sqrt{2}a^{7/4}b^{5/}}$$

$$= \frac{(bc-ad)x}{4ab\left(a+bx^4\right)} - \frac{(3bc+ad)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}b^{5/4}} + \frac{(3bc+ad)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}b^{5/4}} - \frac{(3bc+a}{}$$

Mathematica [A] time = 0.17, size = 212, normalized size = 0.87

$$\frac{-\frac{8a^{3/4}\sqrt[4]{b}x(ad-bc)}{a+bx^4}-\sqrt{2}(ad+3bc)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)+\sqrt{2}(ad+3bc)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)-2\sqrt{2}(ad+3bc)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)+2\sqrt{2}(ad+3bc)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{32a^{7/4}b^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^4)/(a + b*x^4)^2,x]
```

```
[Out] ((-8*a^(3/4)*b^(1/4)*(-(b*c) + a*d)*x)/(a + b*x^4) - 2*Sqrt[2]*(3*b*c + a*d
)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)] + 2*Sqrt[2]*(3*b*c + a*d)*ArcTan[
1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)] - Sqrt[2]*(3*b*c + a*d)*Log[Sqrt[a] - Sqrt
[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2] + Sqrt[2]*(3*b*c + a*d)*Log[Sqrt[a] +
Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(32*a^(7/4)*b^(5/4))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{c+dx^4}{\left(a+bx^4\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x^4)/(a + b*x^4)^2,x]
```

```
[Out] IntegrateAlgebraic[(c + d*x^4)/(a + b*x^4)^2, x]
```

fricas [B] time = 0.83, size = 711, normalized size = 2.90

$$\frac{4\left(ab^{2}x^{4}+a^{2}b\right)\left(-\frac{81b^{4}c^{4}+108ab^{3}c^{3}d+54a^{2}b^{2}c^{2}d^{2}+12a^{3}bcd^{3}+a^{4}d^{4}}{a^{7}b^{5}}\right)^{\frac{1}{4}}\arctan\left(-\frac{a^{5}b^{4}x\left(-\frac{81b^{4}c^{4}+108ab^{3}c^{3}d+54a^{2}b^{2}c^{2}d^{2}+12a^{3}bcd^{3}+a^{4}d^{4}}{a^{7}b^{5}}\right)^{\frac{3}{4}}-a^{5}b^{4}\sqrt{\frac{a^{4}b^{2}\sqrt{-\frac{81b^{4}c^{4}+108ab^{3}c^{3}d+54a^{2}b^{2}c^{2}d^{2}+12a^{3}bcd^{3}+a^{4}d^{4}}{a^{7}b^{5}}}+\left(9b^{2}c^{2}+6abcd+a^{2}d^{2}\right)x^{2}}{9b^{2}c^{2}+6abcd+a^{2}d^{2}}}\left(-\frac{81b^{4}c^{4}+108ab^{3}c^{3}d+54a^{2}b^{2}c^{2}d^{2}+12a^{3}bcd^{3}+a^{4}d^{4}}{a^{7}b^{5}}\right)^{\frac{3}{4}}}{27b^{3}c^{3}+27ab^{2}c^{2}d+9a^{2}bcd^{2}+a^{3}d^{3}}\right)+\left(ab^{2}x^{4}+a^{2}b\right)\left(-\frac{81b^{4}c^{4}+108ab^{3}c^{3}d+54a^{2}b^{2}c^{2}d^{2}+12a^{3}bcd^{3}+a^{4}d^{4}}{a^{7}b^{5}}\right)^{\frac{1}{4}}\log\left(a^{2}b\left(-\frac{81b^{4}c^{4}+108ab^{3}c^{3}d+54a^{2}b^{2}c^{2}d^{2}+12a^{3}bcd^{3}+a^{4}d^{4}}{a^{7}b^{5}}\right)^{\frac{1}{4}}+\left(3bc+ad\right)x\right)-\left(ab^{2}x^{4}+a^{2}b\right)\left(-\frac{81b^{4}c^{4}+108ab^{3}c^{3}d+54a^{2}b^{2}c^{2}d^{2}+12a^{3}bcd^{3}+a^{4}d^{4}}{a^{7}b^{5}}\right)^{\frac{1}{4}}\log\left(-a^{2}b\left(-\frac{81b^{4}c^{4}+108ab^{3}c^{3}d+54a^{2}b^{2}c^{2}d^{2}+12a^{3}bcd^{3}+a^{4}d^{4}}{a^{7}b^{5}}\right)^{\frac{1}{4}}+\left(3bc+ad\right)x\right)+4\left(bc-ad\right)x}{16\left(ab^{2}x^{4}+a^{2}b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^4+c)/(b*x^4+a)^2,x, algorithm="fricas")
```

```
[Out] 1/16*(4*(a*b^2*x^4 + a^2*b)*(-(81*b^4*c^4 + 108*a*b^3*c^3*d + 54*a^2*b^2*c^
2*d^2 + 12*a^3*b*c*d^3 + a^4*d^4)/(a^7*b^5))^(1/4)*arctan(-(a^5*b^4*x*(-(81
*b^4*c^4 + 108*a*b^3*c^3*d + 54*a^2*b^2*c^2*d^2 + 12*a^3*b*c*d^3 + a^4*d^4)
/(a^7*b^5))^(3/4) - a^5*b^4*sqrt((a^4*b^2*sqrt(-(81*b^4*c^4 + 108*a*b^3*c^3
*d + 54*a^2*b^2*c^2*d^2 + 12*a^3*b*c*d^3 + a^4*d^4)/(a^7*b^5)) + (9*b^2*c^2
 + 6*a*b*c*d + a^2*d^2)*x^2)/(9*b^2*c^2 + 6*a*b*c*d + a^2*d^2))*(-(81*b^4*c
^4 + 108*a*b^3*c^3*d + 54*a^2*b^2*c^2*d^2 + 12*a^3*b*c*d^3 + a^4*d^4)/(a^7*
b^5))^(3/4))/(27*b^3*c^3 + 27*a*b^2*c^2*d + 9*a^2*b*c*d^2 + a^3*d^3)) + (a*
b^2*x^4 + a^2*b)*(-(81*b^4*c^4 + 108*a*b^3*c^3*d + 54*a^2*b^2*c^2*d^2 + 12*
a^3*b*c*d^3 + a^4*d^4)/(a^7*b^5))^(1/4)*log(a^2*b*(-(81*b^4*c^4 + 108*a*b^3
*c^3*d + 54*a^2*b^2*c^2*d^2 + 12*a^3*b*c*d^3 + a^4*d^4)/(a^7*b^5))^(1/4) +
(3*b*c + a*d)*x) - (a*b^2*x^4 + a^2*b)*(-(81*b^4*c^4 + 108*a*b^3*c^3*d + 54
*a^2*b^2*c^2*d^2 + 12*a^3*b*c*d^3 + a^4*d^4)/(a^7*b^5))^(1/4)*log(-a^2*b*(-
(81*b^4*c^4 + 108*a*b^3*c^3*d + 54*a^2*b^2*c^2*d^2 + 12*a^3*b*c*d^3 + a^4*d
^4)/(a^7*b^5))^(1/4) + (3*b*c + a*d)*x) + 4*(b*c - a*d)*x)/(a*b^2*x^4 + a^2
*b)
```

giac [A] time = 0.17, size = 266, normalized size = 1.09

$$\frac{\sqrt{2}\left(3\left(ab^{3}\right)^{\frac{1}{4}}bc+\left(ab^{3}\right)^{\frac{1}{4}}ad\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{16a^{2}b^{2}}+\frac{\sqrt{2}\left(3\left(ab^{3}\right)^{\frac{1}{4}}bc+\left(ab^{3}\right)^{\frac{1}{4}}ad\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{16a^{2}b^{2}}+\frac{\sqrt{2}\left(3\left(ab^{3}\right)^{\frac{1}{4}}bc+\left(ab^{3}\right)^{\frac{1}{4}}ad\right)\log\left(x^{2}+\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{32a^{2}b^{2}}-\frac{\sqrt{2}\left(3\left(ab^{3}\right)^{\frac{1}{4}}bc+\left(ab^{3}\right)^{\frac{1}{4}}ad\right)\log\left(x^{2}-\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{32a^{2}b^{2}}+\frac{bcx-adx}{4\left(bx^{4}+a\right)ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^4+c)/(b*x^4+a)^2,x, algorithm="giac")
```

```
[Out] 1/16*sqrt(2)*(3*(a*b^3)^(1/4)*b*c + (a*b^3)^(1/4)*a*d)*arctan(1/2*sqrt(2)*(
2*x + sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a^2*b^2) + 1/16*sqrt(2)*(3*(a*b^3)
^(1/4)*b*c + (a*b^3)^(1/4)*a*d)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/b)^(1/
4))/(a/b)^(1/4))/(a^2*b^2) + 1/32*sqrt(2)*(3*(a*b^3)^(1/4)*b*c + (a*b^3)^(1
/4)*a*d)*log(x^2 + sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a^2*b^2) - 1/32*sqrt
(2)*(3*(a*b^3)^(1/4)*b*c + (a*b^3)^(1/4)*a*d)*log(x^2 - sqrt(2)*x*(a/b)^(1/
4) + sqrt(a/b))/(a^2*b^2) + 1/4*(b*c*x - a*d*x)/((b*x^4 + a)*a*b)
```

maple [A] time = 0.05, size = 295, normalized size = 1.20

$$\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,d\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{16ab}+\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,d\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{16ab}+\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,d\ln\left(\frac{x^{2}+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{a}{b}}}{x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{a}{b}}}\right)}{32ab}+\frac{3\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,c\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{16a^{2}}+\frac{3\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,c\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{16a^{2}}+\frac{3\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,c\ln\left(\frac{x^{2}+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{a}{b}}}{x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{a}{b}}}\right)}{32a^{2}}-\frac{\left(ad-bc\right)x}{4\left(bx^{4}+a\right)ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^4+c)/(b*x^4+a)^2,x)
```

```
[Out] -1/4*(a*d-b*c)/a/b*x/(b*x^4+a)+1/16/a/b*(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/
(a/b)^(1/4)*x-1)*d+3/16/a^2*(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*
x-1)*c+1/32/a/b*(a/b)^(1/4)*2^(1/2)*ln((x^2+(a/b)^(1/4)*2^(1/2)*x+(a/b)^(1/
2))/(x^2-(a/b)^(1/4)*2^(1/2)*x+(a/b)^(1/2)))*d+3/32/a^2*(a/b)^(1/4)*2^(1/2)
*ln((x^2+(a/b)^(1/4)*2^(1/2)*x+(a/b)^(1/2))/(x^2-(a/b)^(1/4)*2^(1/2)*x+(a/b
```

```
)^(1/2)))*c+1/16/a/b*(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x+1)*d+
3/16/a^2*(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x+1)*c
```

maxima [A] time = 1.35, size = 236, normalized size = 0.96

$$\frac{(bc-ad)x}{4\left(ab^2x^4+a^2b\right)}+\frac{\frac{2\sqrt{2}(3bc+ad)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}+\frac{2\sqrt{2}(3bc+ad)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}+\frac{\sqrt{2}(3bc+ad)\log\left(\sqrt{b}x^2+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{1}{4}}}-\frac{\sqrt{2}(3bc+ad)\log\left(\sqrt{b}x^2-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{1}{4}}}}{32ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^4+c)/(b*x^4+a)^2,x, algorithm="maxima")
```

```
[Out] 1/4*(b*c - a*d)*x/(a*b^2*x^4 + a^2*b) + 1/32*(2*sqrt(2)*(3*b*c + a*d)*arcta
n(1/2*sqrt(2)*(2*sqrt(b)*x + sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b))
)/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + 2*sqrt(2)*(3*b*c + a*d)*arctan(1/2*sqrt
(2)*(2*sqrt(b)*x - sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)
*sqrt(sqrt(a)*sqrt(b))) + sqrt(2)*(3*b*c + a*d)*log(sqrt(b)*x^2 + sqrt(2)*a
^(1/4)*b^(1/4)*x + sqrt(a))/(a^(3/4)*b^(1/4)) - sqrt(2)*(3*b*c + a*d)*log(s
qrt(b)*x^2 - sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(3/4)*b^(1/4)))/(a*b)
```

mupad [B] time = 1.53, size = 740, normalized size = 3.02

$$\frac{\mathrm{atan}\left(\frac{\frac{\left(\frac{x\left(a^2bd^2+6ab^2cd+9b^3c^2\right)}{4a^2}-\frac{(ad+3bc)\left(12cb^3+4adb^2\right)}{16(-a)^{7/4}b^{5/4}}\right)(ad+3bc)\,1i}{16(-a)^{7/4}b^{5/4}}+\frac{\left(\frac{x\left(a^2bd^2+6ab^2cd+9b^3c^2\right)}{4a^2}+\frac{(ad+3bc)\left(12cb^3+4adb^2\right)}{16(-a)^{7/4}b^{5/4}}\right)(ad+3bc)\,1i}{16(-a)^{7/4}b^{5/4}}}{\frac{\left(\frac{x\left(a^2bd^2+6ab^2cd+9b^3c^2\right)}{4a^2}-\frac{(ad+3bc)\left(12cb^3+4adb^2\right)}{16(-a)^{7/4}b^{5/4}}\right)(ad+3bc)}{16(-a)^{7/4}b^{5/4}}-\frac{\left(\frac{x\left(a^2bd^2+6ab^2cd+9b^3c^2\right)}{4a^2}+\frac{(ad+3bc)\left(12cb^3+4adb^2\right)}{16(-a)^{7/4}b^{5/4}}\right)(ad+3bc)}{16(-a)^{7/4}b^{5/4}}}\right)(ad+3bc)}{8(-a)^{7/4}b^{5/4}}-\frac{x(ad-bc)}{4ab\left(bx^4+a\right)}+\frac{\mathrm{atan}\left(\frac{\frac{\left(\frac{x\left(a^2bd^2+6ab^2cd+9b^3c^2\right)}{4a^2}-\frac{(ad+3bc)\left(12cb^3+4adb^2\right)1i}{16(-a)^{7/4}b^{5/4}}\right)(ad+3bc)}{16(-a)^{7/4}b^{5/4}}+\frac{\left(\frac{x\left(a^2bd^2+6ab^2cd+9b^3c^2\right)}{4a^2}+\frac{(ad+3bc)\left(12cb^3+4adb^2\right)1i}{16(-a)^{7/4}b^{5/4}}\right)(ad+3bc)}{16(-a)^{7/4}b^{5/4}}}{\frac{\left(\frac{x\left(a^2bd^2+6ab^2cd+9b^3c^2\right)}{4a^2}-\frac{(ad+3bc)\left(12cb^3+4adb^2\right)1i}{16(-a)^{7/4}b^{5/4}}\right)(ad+3bc)\,1i}{16(-a)^{7/4}b^{5/4}}-\frac{\left(\frac{x\left(a^2bd^2+6ab^2cd+9b^3c^2\right)}{4a^2}+\frac{(ad+3bc)\left(12cb^3+4adb^2\right)1i}{16(-a)^{7/4}b^{5/4}}\right)(ad+3bc)\,1i}{16(-a)^{7/4}b^{5/4}}}\right)(ad+3bc)\,1i}{8(-a)^{7/4}b^{5/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^4)/(a + b*x^4)^2,x)
```

```
[Out] (atan(((((x*(9*b^3*c^2 + a^2*b*d^2 + 6*a*b^2*c*d))/(4*a^2) - ((a*d + 3*b*c)
*(12*b^3*c + 4*a*b^2*d))/(16*(-a)^(7/4)*b^(5/4)))*(a*d + 3*b*c)*1i)/(16*(-a
)^(7/4)*b^(5/4)) + (((x*(9*b^3*c^2 + a^2*b*d^2 + 6*a*b^2*c*d))/(4*a^2) + ((
a*d + 3*b*c)*(12*b^3*c + 4*a*b^2*d))/(16*(-a)^(7/4)*b^(5/4)))*(a*d + 3*b*c)
*1i)/(16*(-a)^(7/4)*b^(5/4)))/((((x*(9*b^3*c^2 + a^2*b*d^2 + 6*a*b^2*c*d))/
(4*a^2) - ((a*d + 3*b*c)*(12*b^3*c + 4*a*b^2*d))/(16*(-a)^(7/4)*b^(5/4)))*(
a*d + 3*b*c))/(16*(-a)^(7/4)*b^(5/4)) - (((x*(9*b^3*c^2 + a^2*b*d^2 + 6*a*b
^2*c*d))/(4*a^2) + ((a*d + 3*b*c)*(12*b^3*c + 4*a*b^2*d))/(16*(-a)^(7/4)*b^
(5/4)))*(a*d + 3*b*c))/(16*(-a)^(7/4)*b^(5/4))))*(a*d + 3*b*c)*1i)/(8*(-a)^
(7/4)*b^(5/4)) + (atan(((((x*(9*b^3*c^2 + a^2*b*d^2 + 6*a*b^2*c*d))/(4*a^2)
 - ((a*d + 3*b*c)*(12*b^3*c + 4*a*b^2*d)*1i)/(16*(-a)^(7/4)*b^(5/4)))*(a*d
+ 3*b*c))/(16*(-a)^(7/4)*b^(5/4)) + (((x*(9*b^3*c^2 + a^2*b*d^2 + 6*a*b^2*c
*d))/(4*a^2) + ((a*d + 3*b*c)*(12*b^3*c + 4*a*b^2*d)*1i)/(16*(-a)^(7/4)*b^(
5/4)))*(a*d + 3*b*c))/(16*(-a)^(7/4)*b^(5/4)))/((((x*(9*b^3*c^2 + a^2*b*d^2
 + 6*a*b^2*c*d))/(4*a^2) - ((a*d + 3*b*c)*(12*b^3*c + 4*a*b^2*d)*1i)/(16*(-
a)^(7/4)*b^(5/4)))*(a*d + 3*b*c)*1i)/(16*(-a)^(7/4)*b^(5/4)) - (((x*(9*b^3*
c^2 + a^2*b*d^2 + 6*a*b^2*c*d))/(4*a^2) + ((a*d + 3*b*c)*(12*b^3*c + 4*a*b^
2*d)*1i)/(16*(-a)^(7/4)*b^(5/4)))*(a*d + 3*b*c)*1i)/(16*(-a)^(7/4)*b^(5/4))
))*(a*d + 3*b*c))/(8*(-a)^(7/4)*b^(5/4)) - (x*(a*d - b*c))/(4*a*b*(a + b*x^
4))
```

sympy [A] time = 0.97, size = 112, normalized size = 0.46

$$\frac{x(-ad+bc)}{4a^2b+4ab^2x^4}+\operatorname{RootSum}\left(65536t^4a^7b^5+a^4d^4+12a^3bcd^3+54a^2b^2c^2d^2+108ab^3c^3d+81b^4c^4,\left(t\mapsto t\log\left(\frac{16ta^2b}{ad+3bc}+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**4+c)/(b*x**4+a)**2,x)
```

```
[Out] x*(-a*d + b*c)/(4*a**2*b + 4*a*b**2*x**4) + RootSum(65536*_t**4*a**7*b**5 +
 a**4*d**4 + 12*a**3*b*c*d**3 + 54*a**2*b**2*c**2*d**2 + 108*a*b**3*c**3*d
+ 81*b**4*c**4, Lambda(_t, _t*log(16*_t*a**2*b/(a*d + 3*b*c) + x)))
```

3.108 $\int \frac{1}{(a+bx^4)^2 (c+dx^4)} \, dx$

Optimal. Leaf size=513

$$-\frac{b^{3/4}(3bc-7ad)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{16\sqrt{2}a^{7/4}(bc-ad)^2}+\frac{b^{3/4}(3bc-7ad)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{16\sqrt{2}a^{7/4}(bc-ad)^2}-\frac{b^{3/4}(3bc-}{8}$$

Rubi [A] time = 0.43, antiderivative size = 513, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.421, Rules used = {414, 522, 211, 1165, 628, 1162, 617, 204}

$$-\frac{b^{3/4}(3bc-7ad)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{16\sqrt{2}a^{7/4}(bc-ad)^2}+\frac{b^{3/4}(3bc-7ad)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{16\sqrt{2}a^{7/4}(bc-ad)^2}-\frac{b^{3/4}(3bc-7ad)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}(bc-ad)^2}+\frac{b^{3/4}(3bc-7ad)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{8\sqrt{2}a^{7/4}(bc-ad)^2}-\frac{d^{7/4}\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{c}+\sqrt{d}x^2\right)}{4\sqrt{2}c^{3/4}(bc-ad)^2}+\frac{d^{7/4}\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{c}+\sqrt{d}x^2\right)}{4\sqrt{2}c^{3/4}(bc-ad)^2}-\frac{d^{7/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{2\sqrt{2}c^{3/4}(bc-ad)^2}+\frac{d^{7/4}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}+1\right)}{2\sqrt{2}c^{3/4}(bc-ad)^2}+\frac{bx}{4a\left(a+bx^4\right)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^4)^2*(c + d*x^4)),x]
```

```
[Out] (b*x)/(4*a*(b*c - a*d)*(a + b*x^4)) - (b^(3/4)*(3*b*c - 7*a*d)*ArcTan[1 - (
Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*(b*c - a*d)^2) + (b^(3/4)*(
3*b*c - 7*a*d)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*
(b*c - a*d)^2) - (d^(7/4)*ArcTan[1 - (Sqrt[2]*d^(1/4)*x)/c^(1/4)])/(2*Sqrt[
2]*c^(3/4)*(b*c - a*d)^2) + (d^(7/4)*ArcTan[1 + (Sqrt[2]*d^(1/4)*x)/c^(1/4)
])/(2*Sqrt[2]*c^(3/4)*(b*c - a*d)^2) - (b^(3/4)*(3*b*c - 7*a*d)*Log[Sqrt[a]
 - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(16*Sqrt[2]*a^(7/4)*(b*c - a*d
)^2) + (b^(3/4)*(3*b*c - 7*a*d)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + S
qrt[b]*x^2])/(16*Sqrt[2]*a^(7/4)*(b*c - a*d)^2) - (d^(7/4)*Log[Sqrt[c] - Sq
rt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2])/(4*Sqrt[2]*c^(3/4)*(b*c - a*d)^2) +
 (d^(7/4)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2])/(4*Sqrt[2
]*c^(3/4)*(b*c - a*d)^2)
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
```

```
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{1}{\left(a+bx^4\right)^2\left(c+dx^4\right)}\,dx = \frac{bx}{4a(bc-ad)\left(a+bx^4\right)} - \frac{\int \frac{-3bc+4ad-3bdx^4}{\left(a+bx^4\right)\left(c+dx^4\right)}\,dx}{4a(bc-ad)}$$

$$= \frac{bx}{4a(bc-ad)\left(a+bx^4\right)} + \frac{d^2\int \frac{1}{c+dx^4}\,dx}{(bc-ad)^2} + \frac{(b(3bc-7ad))\int \frac{1}{a+bx^4}\,dx}{4a(bc-ad)^2}$$

$$= \frac{bx}{4a(bc-ad)\left(a+bx^4\right)} + \frac{d^2\int \frac{\sqrt{c}-\sqrt{d}\,x^2}{c+dx^4}\,dx}{2\sqrt{c}\,(bc-ad)^2} + \frac{d^2\int \frac{\sqrt{c}+\sqrt{d}\,x^2}{c+dx^4}\,dx}{2\sqrt{c}\,(bc-ad)^2} + \frac{(b(3bc-7ad))\int}{8a^{3/2}(bc-}$$

$$= \frac{bx}{4a(bc-ad)\left(a+bx^4\right)} + \frac{d^{3/2}\int \frac{1}{\frac{\sqrt{c}}{\sqrt{d}}-\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{d}}+x^2}\,dx}{4\sqrt{c}\,(bc-ad)^2} + \frac{d^{3/2}\int \frac{1}{\frac{\sqrt{c}}{\sqrt{d}}+\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{d}}+x^2}\,dx}{4\sqrt{c}\,(bc-ad)^2} - \frac{d^{7/4}}{4}$$

$$= \frac{bx}{4a(bc-ad)\left(a+bx^4\right)} - \frac{b^{3/4}(3bc-7ad)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{b}\,x^2\right)}{16\sqrt{2}\,a^{7/4}(bc-ad)^2} + \frac{b^{3/4}(}{}$$

$$= \frac{bx}{4a(bc-ad)\left(a+bx^4\right)} - \frac{b^{3/4}(3bc-7ad)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{8\sqrt{2}\,a^{7/4}(bc-ad)^2} + \frac{b^{3/4}(3bc-7ad)\tan}{8\sqrt{2}\,a^{7/4}(b}$$

Mathematica [A] time = 0.35, size = 499, normalized size = 0.97

$$\frac{8a^{3/4}bc^{3/4}x(bc-ad)-8\sqrt{2}a^{7/4}d^{7/4}\left(a+bx^4\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)+8\sqrt{2}a^{7/4}d^{7/4}\left(a+bx^4\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}+1\right)-4\sqrt{2}a^{7/4}d^{7/4}\left(a+bx^4\right)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{c}+\sqrt{d}x^2\right)+4\sqrt{2}a^{7/4}d^{7/4}\left(a+bx^4\right)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{c}+\sqrt{d}x^2\right)-2\sqrt{2}b^{3/4}c^{3/4}\left(a+bx^4\right)(3bc-7ad)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)+2\sqrt{2}b^{3/4}c^{3/4}\left(a+bx^4\right)(3bc-7ad)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)-\sqrt{2}b^{3/4}c^{3/4}\left(a+bx^4\right)(3bc-7ad)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)+\sqrt{2}b^{3/4}c^{3/4}\left(a+bx^4\right)(3bc-7ad)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{32a^{7/4}c^{3/4}\left(a+bx^4\right)(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x^4)^2*(c + d*x^4)),x]
```

```
[Out] (8*a^(3/4)*b*c^(3/4)*(b*c - a*d)*x - 2*Sqrt[2]*b^(3/4)*c^(3/4)*(3*b*c - 7*a
*d)*(a + b*x^4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)] + 2*Sqrt[2]*b^(3/4)
*c^(3/4)*(3*b*c - 7*a*d)*(a + b*x^4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)
] - 8*Sqrt[2]*a^(7/4)*d^(7/4)*(a + b*x^4)*ArcTan[1 - (Sqrt[2]*d^(1/4)*x)/c^
(1/4)] + 8*Sqrt[2]*a^(7/4)*d^(7/4)*(a + b*x^4)*ArcTan[1 + (Sqrt[2]*d^(1/4)*
x)/c^(1/4)] - Sqrt[2]*b^(3/4)*c^(3/4)*(3*b*c - 7*a*d)*(a + b*x^4)*Log[Sqrt[
a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2] + Sqrt[2]*b^(3/4)*c^(3/4)*(3*
b*c - 7*a*d)*(a + b*x^4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*
x^2] - 4*Sqrt[2]*a^(7/4)*d^(7/4)*(a + b*x^4)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*
d^(1/4)*x + Sqrt[d]*x^2] + 4*Sqrt[2]*a^(7/4)*d^(7/4)*(a + b*x^4)*Log[Sqrt[c
] + Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2])/(32*a^(7/4)*c^(3/4)*(b*c - a*
d)^2*(a + b*x^4))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+bx^4\right)^2\left(c+dx^4\right)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/((a + b*x^4)^2*(c + d*x^4)),x]
```

```
[Out] IntegrateAlgebraic[1/((a + b*x^4)^2*(c + d*x^4)), x]
```

fricas [B] time = 59.89, size = 3299, normalized size = 6.43

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a)^2/(d*x^4+c),x, algorithm="fricas")
```

```
[Out] -1/16*(4*((a*b^2*c - a^2*b*d)*x^4 + a^2*b*c - a^3*d)*(-(81*b^7*c^4 - 756*a*
b^6*c^3*d + 2646*a^2*b^5*c^2*d^2 - 4116*a^3*b^4*c*d^3 + 2401*a^4*b^3*d^4)/(
a^7*b^8*c^8 - 8*a^8*b^7*c^7*d + 28*a^9*b^6*c^6*d^2 - 56*a^10*b^5*c^5*d^3 +
70*a^11*b^4*c^4*d^4 - 56*a^12*b^3*c^3*d^5 + 28*a^13*b^2*c^2*d^6 - 8*a^14*b*
c*d^7 + a^15*d^8))^(1/4)*arctan(((a^5*b^6*c^6 - 6*a^6*b^5*c^5*d + 15*a^7*b^
4*c^4*d^2 - 20*a^8*b^3*c^3*d^3 + 15*a^9*b^2*c^2*d^4 - 6*a^10*b*c*d^5 + a^11
*d^6)*x*(-(81*b^7*c^4 - 756*a*b^6*c^3*d + 2646*a^2*b^5*c^2*d^2 - 4116*a^3*b
^4*c*d^3 + 2401*a^4*b^3*d^4)/(a^7*b^8*c^8 - 8*a^8*b^7*c^7*d + 28*a^9*b^6*c^
6*d^2 - 56*a^10*b^5*c^5*d^3 + 70*a^11*b^4*c^4*d^4 - 56*a^12*b^3*c^3*d^5 + 2
8*a^13*b^2*c^2*d^6 - 8*a^14*b*c*d^7 + a^15*d^8))^(3/4) - (a^5*b^6*c^6 - 6*a
^6*b^5*c^5*d + 15*a^7*b^4*c^4*d^2 - 20*a^8*b^3*c^3*d^3 + 15*a^9*b^2*c^2*d^4
 - 6*a^10*b*c*d^5 + a^11*d^6)*(-(81*b^7*c^4 - 756*a*b^6*c^3*d + 2646*a^2*b^
5*c^2*d^2 - 4116*a^3*b^4*c*d^3 + 2401*a^4*b^3*d^4)/(a^7*b^8*c^8 - 8*a^8*b^7
*c^7*d + 28*a^9*b^6*c^6*d^2 - 56*a^10*b^5*c^5*d^3 + 70*a^11*b^4*c^4*d^4 - 5
6*a^12*b^3*c^3*d^5 + 28*a^13*b^2*c^2*d^6 - 8*a^14*b*c*d^7 + a^15*d^8))^(3/4
)*sqrt(((9*b^4*c^2 - 42*a*b^3*c*d + 49*a^2*b^2*d^2)*x^2 + (a^4*b^4*c^4 - 4*
a^5*b^3*c^3*d + 6*a^6*b^2*c^2*d^2 - 4*a^7*b*c*d^3 + a^8*d^4)*sqrt(-(81*b^7*
c^4 - 756*a*b^6*c^3*d + 2646*a^2*b^5*c^2*d^2 - 4116*a^3*b^4*c*d^3 + 2401*a^
4*b^3*d^4)/(a^7*b^8*c^8 - 8*a^8*b^7*c^7*d + 28*a^9*b^6*c^6*d^2 - 56*a^10*b^
5*c^5*d^3 + 70*a^11*b^4*c^4*d^4 - 56*a^12*b^3*c^3*d^5 + 28*a^13*b^2*c^2*d^6
 - 8*a^14*b*c*d^7 + a^15*d^8)))/(9*b^4*c^2 - 42*a*b^3*c*d + 49*a^2*b^2*d^2)
))/(27*b^5*c^3 - 189*a*b^4*c^2*d + 441*a^2*b^3*c*d^2 - 343*a^3*b^2*d^3)) -
```

```
16*(-d^7/(b^8*c^11 - 8*a*b^7*c^10*d + 28*a^2*b^6*c^9*d^2 - 56*a^3*b^5*c^8*d
^3 + 70*a^4*b^4*c^7*d^4 - 56*a^5*b^3*c^6*d^5 + 28*a^6*b^2*c^5*d^6 - 8*a^7*b
*c^4*d^7 + a^8*c^3*d^8))^(1/4)*((a*b^2*c - a^2*b*d)*x^4 + a^2*b*c - a^3*d)*
arctan(-((b^6*c^8 - 6*a*b^5*c^7*d + 15*a^2*b^4*c^6*d^2 - 20*a^3*b^3*c^5*d^3
 + 15*a^4*b^2*c^4*d^4 - 6*a^5*b*c^3*d^5 + a^6*c^2*d^6)*(-d^7/(b^8*c^11 - 8*
a*b^7*c^10*d + 28*a^2*b^6*c^9*d^2 - 56*a^3*b^5*c^8*d^3 + 70*a^4*b^4*c^7*d^4
 - 56*a^5*b^3*c^6*d^5 + 28*a^6*b^2*c^5*d^6 - 8*a^7*b*c^4*d^7 + a^8*c^3*d^8)
)^(3/4)*x - (b^6*c^8 - 6*a*b^5*c^7*d + 15*a^2*b^4*c^6*d^2 - 20*a^3*b^3*c^5*
d^3 + 15*a^4*b^2*c^4*d^4 - 6*a^5*b*c^3*d^5 + a^6*c^2*d^6)*(-d^7/(b^8*c^11 -
 8*a*b^7*c^10*d + 28*a^2*b^6*c^9*d^2 - 56*a^3*b^5*c^8*d^3 + 70*a^4*b^4*c^7*
d^4 - 56*a^5*b^3*c^6*d^5 + 28*a^6*b^2*c^5*d^6 - 8*a^7*b*c^4*d^7 + a^8*c^3*d
^8))^(3/4)*sqrt((d^4*x^2 + (b^4*c^6 - 4*a*b^3*c^5*d + 6*a^2*b^2*c^4*d^2 - 4
*a^3*b*c^3*d^3 + a^4*c^2*d^4)*sqrt(-d^7/(b^8*c^11 - 8*a*b^7*c^10*d + 28*a^2
*b^6*c^9*d^2 - 56*a^3*b^5*c^8*d^3 + 70*a^4*b^4*c^7*d^4 - 56*a^5*b^3*c^6*d^5
 + 28*a^6*b^2*c^5*d^6 - 8*a^7*b*c^4*d^7 + a^8*c^3*d^8)))/d^4))/d^5) - 4*(-d
^7/(b^8*c^11 - 8*a*b^7*c^10*d + 28*a^2*b^6*c^9*d^2 - 56*a^3*b^5*c^8*d^3 + 7
0*a^4*b^4*c^7*d^4 - 56*a^5*b^3*c^6*d^5 + 28*a^6*b^2*c^5*d^6 - 8*a^7*b*c^4*d
^7 + a^8*c^3*d^8))^(1/4)*((a*b^2*c - a^2*b*d)*x^4 + a^2*b*c - a^3*d)*log(d^
2*x + (-d^7/(b^8*c^11 - 8*a*b^7*c^10*d + 28*a^2*b^6*c^9*d^2 - 56*a^3*b^5*c^
8*d^3 + 70*a^4*b^4*c^7*d^4 - 56*a^5*b^3*c^6*d^5 + 28*a^6*b^2*c^5*d^6 - 8*a^
7*b*c^4*d^7 + a^8*c^3*d^8))^(1/4)*(b^2*c^3 - 2*a*b*c^2*d + a^2*c*d^2)) + 4*
(-d^7/(b^8*c^11 - 8*a*b^7*c^10*d + 28*a^2*b^6*c^9*d^2 - 56*a^3*b^5*c^8*d^3
+ 70*a^4*b^4*c^7*d^4 - 56*a^5*b^3*c^6*d^5 + 28*a^6*b^2*c^5*d^6 - 8*a^7*b*c^
4*d^7 + a^8*c^3*d^8))^(1/4)*((a*b^2*c - a^2*b*d)*x^4 + a^2*b*c - a^3*d)*log
(d^2*x - (-d^7/(b^8*c^11 - 8*a*b^7*c^10*d + 28*a^2*b^6*c^9*d^2 - 56*a^3*b^5
*c^8*d^3 + 70*a^4*b^4*c^7*d^4 - 56*a^5*b^3*c^6*d^5 + 28*a^6*b^2*c^5*d^6 - 8
*a^7*b*c^4*d^7 + a^8*c^3*d^8))^(1/4)*(b^2*c^3 - 2*a*b*c^2*d + a^2*c*d^2)) +
 ((a*b^2*c - a^2*b*d)*x^4 + a^2*b*c - a^3*d)*(-(81*b^7*c^4 - 756*a*b^6*c^3*
d + 2646*a^2*b^5*c^2*d^2 - 4116*a^3*b^4*c*d^3 + 2401*a^4*b^3*d^4)/(a^7*b^8*
c^8 - 8*a^8*b^7*c^7*d + 28*a^9*b^6*c^6*d^2 - 56*a^10*b^5*c^5*d^3 + 70*a^11*
b^4*c^4*d^4 - 56*a^12*b^3*c^3*d^5 + 28*a^13*b^2*c^2*d^6 - 8*a^14*b*c*d^7 +
a^15*d^8))^(1/4)*log(-(3*b^2*c - 7*a*b*d)*x + (a^2*b^2*c^2 - 2*a^3*b*c*d +
a^4*d^2)*(-(81*b^7*c^4 - 756*a*b^6*c^3*d + 2646*a^2*b^5*c^2*d^2 - 4116*a^3*
b^4*c*d^3 + 2401*a^4*b^3*d^4)/(a^7*b^8*c^8 - 8*a^8*b^7*c^7*d + 28*a^9*b^6*c
^6*d^2 - 56*a^10*b^5*c^5*d^3 + 70*a^11*b^4*c^4*d^4 - 56*a^12*b^3*c^3*d^5 +
28*a^13*b^2*c^2*d^6 - 8*a^14*b*c*d^7 + a^15*d^8))^(1/4)) - ((a*b^2*c - a^2*
b*d)*x^4 + a^2*b*c - a^3*d)*(-(81*b^7*c^4 - 756*a*b^6*c^3*d + 2646*a^2*b^5*
c^2*d^2 - 4116*a^3*b^4*c*d^3 + 2401*a^4*b^3*d^4)/(a^7*b^8*c^8 - 8*a^8*b^7*c
^7*d + 28*a^9*b^6*c^6*d^2 - 56*a^10*b^5*c^5*d^3 + 70*a^11*b^4*c^4*d^4 - 56*
a^12*b^3*c^3*d^5 + 28*a^13*b^2*c^2*d^6 - 8*a^14*b*c*d^7 + a^15*d^8))^(1/4)*
log(-(3*b^2*c - 7*a*b*d)*x - (a^2*b^2*c^2 - 2*a^3*b*c*d + a^4*d^2)*(-(81*b^
7*c^4 - 756*a*b^6*c^3*d + 2646*a^2*b^5*c^2*d^2 - 4116*a^3*b^4*c*d^3 + 2401*
a^4*b^3*d^4)/(a^7*b^8*c^8 - 8*a^8*b^7*c^7*d + 28*a^9*b^6*c^6*d^2 - 56*a^10*
b^5*c^5*d^3 + 70*a^11*b^4*c^4*d^4 - 56*a^12*b^3*c^3*d^5 + 28*a^13*b^2*c^2*d
^6 - 8*a^14*b*c*d^7 + a^15*d^8))^(1/4)) - 4*b*x)/((a*b^2*c - a^2*b*d)*x^4 +
 a^2*b*c - a^3*d)
```

giac [A] time = 0.22, size = 667, normalized size = 1.30

$$\frac{\left(cd^{3}\right)^{\frac{1}{4}} d \arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}b^{2}c^{3}-2\sqrt{2}abc^{2}d+\sqrt{2}a^{2}cd^{2}\right)}+\frac{\left(cd^{3}\right)^{\frac{1}{4}} d \arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}b^{2}c^{3}-2\sqrt{2}abc^{2}d+\sqrt{2}a^{2}cd^{2}\right)}+\frac{\left(cd^{3}\right)^{\frac{1}{4}} d \log\left(x^{2}+\sqrt{2}x\left(\frac{c}{d}\right)^{\frac{1}{4}}+\sqrt{\frac{c}{d}}\right)}{4\left(\sqrt{2}b^{2}c^{3}-2\sqrt{2}abc^{2}d+\sqrt{2}a^{2}cd^{2}\right)}-\frac{\left(cd^{3}\right)^{\frac{1}{4}} d \log\left(x^{2}-\sqrt{2}x\left(\frac{c}{d}\right)^{\frac{1}{4}}+\sqrt{\frac{c}{d}}\right)}{4\left(\sqrt{2}b^{2}c^{3}-2\sqrt{2}abc^{2}d+\sqrt{2}a^{2}cd^{2}\right)}+\frac{\left(3\left(ab^{3}\right)^{\frac{1}{4}}bc-7\left(ab^{3}\right)^{\frac{1}{4}}ad\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{8\left(\sqrt{2}a^{2}b^{2}c^{2}-2\sqrt{2}a^{3}bcd+\sqrt{2}a^{4}d^{2}\right)}+\frac{\left(3\left(ab^{3}\right)^{\frac{1}{4}}bc-7\left(ab^{3}\right)^{\frac{1}{4}}ad\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{8\left(\sqrt{2}a^{2}b^{2}c^{2}-2\sqrt{2}a^{3}bcd+\sqrt{2}a^{4}d^{2}\right)}+\frac{\left(3\left(ab^{3}\right)^{\frac{1}{4}}bc-7\left(ab^{3}\right)^{\frac{1}{4}}ad\right)\log\left(x^{2}+\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{16\left(\sqrt{2}a^{2}b^{2}c^{2}-2\sqrt{2}a^{3}bcd+\sqrt{2}a^{4}d^{2}\right)}-\frac{\left(3\left(ab^{3}\right)^{\frac{1}{4}}bc-7\left(ab^{3}\right)^{\frac{1}{4}}ad\right)\log\left(x^{2}-\sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{16\left(\sqrt{2}a^{2}b^{2}c^{2}-2\sqrt{2}a^{3}bcd+\sqrt{2}a^{4}d^{2}\right)}+\frac{bx}{4\left(bx^{4}+a\right)\left(abc-a^{2}d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a)^2/(d*x^4+c),x, algorithm="giac")
```

```
[Out] 1/2*(c*d^3)^(1/4)*d*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(c/d)^(1/4))/(c/d)^(1
/4))/(sqrt(2)*b^2*c^3 - 2*sqrt(2)*a*b*c^2*d + sqrt(2)*a^2*c*d^2) + 1/2*(c*d
^3)^(1/4)*d*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(c/d)^(1/4))/(c/d)^(1/4))/(sq
rt(2)*b^2*c^3 - 2*sqrt(2)*a*b*c^2*d + sqrt(2)*a^2*c*d^2) + 1/4*(c*d^3)^(1/4
)*d*log(x^2 + sqrt(2)*x*(c/d)^(1/4) + sqrt(c/d))/(sqrt(2)*b^2*c^3 - 2*sqrt(
```

```
2)*a*b*c^2*d + sqrt(2)*a^2*c*d^2) - 1/4*(c*d^3)^(1/4)*d*log(x^2 - sqrt(2)*x
*(c/d)^(1/4) + sqrt(c/d))/(sqrt(2)*b^2*c^3 - 2*sqrt(2)*a*b*c^2*d + sqrt(2)*
a^2*c*d^2) + 1/8*(3*(a*b^3)^(1/4)*b*c - 7*(a*b^3)^(1/4)*a*d)*arctan(1/2*sqr
t(2)*(2*x + sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(sqrt(2)*a^2*b^2*c^2 - 2*sqrt
(2)*a^3*b*c*d + sqrt(2)*a^4*d^2) + 1/8*(3*(a*b^3)^(1/4)*b*c - 7*(a*b^3)^(1/
4)*a*d)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(sqrt(2
)*a^2*b^2*c^2 - 2*sqrt(2)*a^3*b*c*d + sqrt(2)*a^4*d^2) + 1/16*(3*(a*b^3)^(1
/4)*b*c - 7*(a*b^3)^(1/4)*a*d)*log(x^2 + sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))
/(sqrt(2)*a^2*b^2*c^2 - 2*sqrt(2)*a^3*b*c*d + sqrt(2)*a^4*d^2) - 1/16*(3*(a
*b^3)^(1/4)*b*c - 7*(a*b^3)^(1/4)*a*d)*log(x^2 - sqrt(2)*x*(a/b)^(1/4) + sq
rt(a/b))/(sqrt(2)*a^2*b^2*c^2 - 2*sqrt(2)*a^3*b*c*d + sqrt(2)*a^4*d^2) + 1/
4*b*x/((b*x^4 + a)*(a*b*c - a^2*d))
```

maple [A] time = 0.06, size = 550, normalized size = 1.07

$$\frac{b^2cx}{4(ad-bc)^2(bx^4+a)a}-\frac{bdx}{4(ad-bc)^2(bx^4+a)}-\frac{7\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,bd\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{16(ad-bc)^2a}-\frac{7\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,bd\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{16(ad-bc)^2a}-\frac{7\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,bd\ln\left(\frac{x^2+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{a}{b}}}{x^2-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{a}{b}}}\right)}{32(ad-bc)^2a}+\frac{3\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,b^2c\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{16(ad-bc)^2a^2}+\frac{3\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,b^2c\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{16(ad-bc)^2a^2}+\frac{3\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,b^2c\ln\left(\frac{x^2+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{a}{b}}}{x^2-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{a}{b}}}\right)}{32(ad-bc)^2a^2}+\frac{\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,d^2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}-1\right)}{4(ad-bc)^2c}+\frac{\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,d^2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}+1\right)}{4(ad-bc)^2c}+\frac{\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,d^2\ln\left(\frac{x^2+\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{c}{d}}}{x^2-\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{c}{d}}}\right)}{8(ad-bc)^2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^4+a)^2/(d*x^4+c),x)
```

```
[Out] 1/8*d^2/(a*d-b*c)^2*(c/d)^(1/4)/c*2^(1/2)*ln((x^2+(c/d)^(1/4)*2^(1/2)*x+(c/
d)^(1/2))/(x^2-(c/d)^(1/4)*2^(1/2)*x+(c/d)^(1/2)))+1/4*d^2/(a*d-b*c)^2*(c/d
)^(1/4)/c*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x+1)+1/4*d^2/(a*d-b*c)^2*(c/d)
^(1/4)/c*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x-1)-1/4*b/(a*d-b*c)^2*x/(b*x^4
+a)*d+1/4*b^2/(a*d-b*c)^2/a*x/(b*x^4+a)*c-7/16*b/(a*d-b*c)^2/a*(a/b)^(1/4)*
2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x+1)*d+3/16*b^2/(a*d-b*c)^2/a^2*(a/b)^(1
/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x+1)*c-7/16*b/(a*d-b*c)^2/a*(a/b)^(1
/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x-1)*d+3/16*b^2/(a*d-b*c)^2/a^2*(a/b
)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x-1)*c-7/32*b/(a*d-b*c)^2/a*(a/b
)^(1/4)*2^(1/2)*ln((x^2+(a/b)^(1/4)*2^(1/2)*x+(a/b)^(1/2))/(x^2-(a/b)^(1/4)
*2^(1/2)*x+(a/b)^(1/2)))*d+3/32*b^2/(a*d-b*c)^2/a^2*(a/b)^(1/4)*2^(1/2)*ln(
(x^2+(a/b)^(1/4)*2^(1/2)*x+(a/b)^(1/2))/(x^2-(a/b)^(1/4)*2^(1/2)*x+(a/b)^(1
/2)))*c
```

maxima [A] time = 1.46, size = 470, normalized size = 0.92

$$\frac{\left(\frac{2\sqrt{2}(3bc-7ad)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}+\frac{2\sqrt{2}(3bc-7ad)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}+\frac{\sqrt{2}(3bc-7ad)\log\left(\sqrt{b}x^2+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{1}{4}}}-\frac{\sqrt{2}(3bc-7ad)\log\left(\sqrt{b}x^2-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{1}{4}}}\right)b}{32\left(ab^2c^2-2a^2bcd+a^3d^2\right)}+\frac{bx}{4\left(\left(ab^2c-a^2bd\right)x^4+a^2bc-a^3d\right)}+\frac{\frac{2\sqrt{2}d^2\arctan\left(\frac{\sqrt{2}\left(2\sqrt{d}x+\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{c}\sqrt{\sqrt{c}\sqrt{d}}}+\frac{2\sqrt{2}d^2\arctan\left(\frac{\sqrt{2}\left(2\sqrt{d}x-\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{c}\sqrt{\sqrt{c}\sqrt{d}}}+\frac{\sqrt{2}d^{\frac{7}{4}}\log\left(\sqrt{d}x^2+\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}x+\sqrt{c}\right)}{c^{\frac{3}{4}}}-\frac{\sqrt{2}d^{\frac{7}{4}}\log\left(\sqrt{d}x^2-\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}x+\sqrt{c}\right)}{c^{\frac{3}{4}}}}{8\left(b^2c^2-2abcd+a^2d^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a)^2/(d*x^4+c),x, algorithm="maxima")
```

```
[Out] 1/32*(2*sqrt(2)*(3*b*c - 7*a*d)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x + sqrt(2)*a
^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + 2*
sqrt(2)*(3*b*c - 7*a*d)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x - sqrt(2)*a^(1/4)*b
^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + sqrt(2)*(3
*b*c - 7*a*d)*log(sqrt(b)*x^2 + sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(3/
4)*b^(1/4)) - sqrt(2)*(3*b*c - 7*a*d)*log(sqrt(b)*x^2 - sqrt(2)*a^(1/4)*b^(
1/4)*x + sqrt(a))/(a^(3/4)*b^(1/4)))*b/(a*b^2*c^2 - 2*a^2*b*c*d + a^3*d^2)
+ 1/4*b*x/((a*b^2*c - a^2*b*d)*x^4 + a^2*b*c - a^3*d) + 1/8*(2*sqrt(2)*d^2*
arctan(1/2*sqrt(2)*(2*sqrt(d)*x + sqrt(2)*c^(1/4)*d^(1/4))/sqrt(sqrt(c)*sqr
t(d)))/(sqrt(c)*sqrt(sqrt(c)*sqrt(d))) + 2*sqrt(2)*d^2*arctan(1/2*sqrt(2)*(
2*sqrt(d)*x - sqrt(2)*c^(1/4)*d^(1/4))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(c)*sqrt
(sqrt(c)*sqrt(d))) + sqrt(2)*d^(7/4)*log(sqrt(d)*x^2 + sqrt(2)*c^(1/4)*d^(1
/4)*x + sqrt(c))/c^(3/4) - sqrt(2)*d^(7/4)*log(sqrt(d)*x^2 - sqrt(2)*c^(1/4
)*d^(1/4)*x + sqrt(c))/c^(3/4))/(b^2*c^2 - 2*a*b*c*d + a^2*d^2)
```

mupad [B] time = 3.82, size = 21975, normalized size = 42.84

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^4)^2*(c + d*x^4)),x)
```

```
[Out] 2*atan((((-(81*b^7*c^4 + 2401*a^4*b^3*d^4 - 4116*a^3*b^4*c*d^3 + 2646*a^2*b
^5*c^2*d^2 - 756*a*b^6*c^3*d)/(65536*a^15*d^8 + 65536*a^7*b^8*c^8 - 524288*
a^8*b^7*c^7*d + 1835008*a^9*b^6*c^6*d^2 - 3670016*a^10*b^5*c^5*d^3 + 458752
0*a^11*b^4*c^4*d^4 - 3670016*a^12*b^3*c^3*d^5 + 1835008*a^13*b^2*c^2*d^6 -
524288*a^14*b*c*d^7))^(1/4)*(((28*a^4*b^6*d^11 + (81*b^10*c^4*d^7)/16 - (67
5*a*b^9*c^3*d^8)/16 - (2145*a^3*b^7*c*d^10)/16 + (1971*a^2*b^8*c^2*d^9)/16)
*1i)/(a^7*d^3 - a^4*b^3*c^3 + 3*a^5*b^2*c^2*d - 3*a^6*b*c*d^2) + (-(81*b^7*
c^4 + 2401*a^4*b^3*d^4 - 4116*a^3*b^4*c*d^3 + 2646*a^2*b^5*c^2*d^2 - 756*a*
b^6*c^3*d)/(65536*a^15*d^8 + 65536*a^7*b^8*c^8 - 524288*a^8*b^7*c^7*d + 183
5008*a^9*b^6*c^6*d^2 - 3670016*a^10*b^5*c^5*d^3 + 4587520*a^11*b^4*c^4*d^4
- 3670016*a^12*b^3*c^3*d^5 + 1835008*a^13*b^2*c^2*d^6 - 524288*a^14*b*c*d^7
))^(3/4)*(((-(81*b^7*c^4 + 2401*a^4*b^3*d^4 - 4116*a^3*b^4*c*d^3 + 2646*a^2
*b^5*c^2*d^2 - 756*a*b^6*c^3*d)/(65536*a^15*d^8 + 65536*a^7*b^8*c^8 - 52428
8*a^8*b^7*c^7*d + 1835008*a^9*b^6*c^6*d^2 - 3670016*a^10*b^5*c^5*d^3 + 4587
520*a^11*b^4*c^4*d^4 - 3670016*a^12*b^3*c^3*d^5 + 1835008*a^13*b^2*c^2*d^6
- 524288*a^14*b*c*d^7))^(1/4)*(3072*a^4*b^14*c^11*d^4 - 4096*a^14*b^4*c*d^1
4 - 28672*a^5*b^13*c^10*d^5 + 114688*a^6*b^12*c^9*d^6 - 253952*a^7*b^11*c^8
*d^7 + 329728*a^8*b^10*c^7*d^8 - 229376*a^9*b^9*c^6*d^9 + 28672*a^10*b^8*c^
5*d^10 + 90112*a^11*b^7*c^4*d^11 - 78848*a^12*b^6*c^3*d^12 + 28672*a^13*b^5
*c^2*d^13))/(a^7*d^3 - a^4*b^3*c^3 + 3*a^5*b^2*c^2*d - 3*a^6*b*c*d^2) - (x*
(65536*a^15*b^4*d^17 - 524288*a^14*b^5*c*d^16 + 36864*a^2*b^17*c^13*d^4 - 4
66944*a^3*b^16*c^12*d^5 + 2609152*a^4*b^15*c^11*d^6 - 8486912*a^5*b^14*c^10
*d^7 + 17833984*a^6*b^13*c^9*d^8 - 25280512*a^7*b^12*c^8*d^9 + 24190976*a^8
*b^11*c^7*d^10 - 14516224*a^9*b^10*c^6*d^11 + 3362816*a^10*b^9*c^5*d^12 + 2
809856*a^11*b^8*c^4*d^13 - 3469312*a^12*b^7*c^3*d^14 + 1835008*a^13*b^6*c^2
*d^15)*1i)/(64*(a^10*d^6 + a^4*b^6*c^6 - 6*a^5*b^5*c^5*d + 15*a^6*b^4*c^4*d
^2 - 20*a^7*b^3*c^3*d^3 + 15*a^8*b^2*c^2*d^4 - 6*a^9*b*c*d^5)))*1i) + (x*(3
185*a^4*b^7*d^13 + 81*b^11*c^4*d^9 - 756*a*b^10*c^3*d^10 - 4788*a^3*b^8*c*d
^12 + 2790*a^2*b^9*c^2*d^11))/(64*(a^10*d^6 + a^4*b^6*c^6 - 6*a^5*b^5*c^5*d
 + 15*a^6*b^4*c^4*d^2 - 20*a^7*b^3*c^3*d^3 + 15*a^8*b^2*c^2*d^4 - 6*a^9*b*c
*d^5)))*(-(81*b^7*c^4 + 2401*a^4*b^3*d^4 - 4116*a^3*b^4*c*d^3 + 2646*a^2*b^
5*c^2*d^2 - 756*a*b^6*c^3*d)/(65536*a^15*d^8 + 65536*a^7*b^8*c^8 - 524288*a
^8*b^7*c^7*d + 1835008*a^9*b^6*c^6*d^2 - 3670016*a^10*b^5*c^5*d^3 + 4587520
*a^11*b^4*c^4*d^4 - 3670016*a^12*b^3*c^3*d^5 + 1835008*a^13*b^2*c^2*d^6 - 5
24288*a^14*b*c*d^7))^(1/4) - ((-(81*b^7*c^4 + 2401*a^4*b^3*d^4 - 4116*a^3*b
^4*c*d^3 + 2646*a^2*b^5*c^2*d^2 - 756*a*b^6*c^3*d)/(65536*a^15*d^8 + 65536*
a^7*b^8*c^8 - 524288*a^8*b^7*c^7*d + 1835008*a^9*b^6*c^6*d^2 - 3670016*a^10
*b^5*c^5*d^3 + 4587520*a^11*b^4*c^4*d^4 - 3670016*a^12*b^3*c^3*d^5 + 183500
8*a^13*b^2*c^2*d^6 - 524288*a^14*b*c*d^7))^(1/4)*(((28*a^4*b^6*d^11 + (81*b
^10*c^4*d^7)/16 - (675*a*b^9*c^3*d^8)/16 - (2145*a^3*b^7*c*d^10)/16 + (1971
*a^2*b^8*c^2*d^9)/16)*1i)/(a^7*d^3 - a^4*b^3*c^3 + 3*a^5*b^2*c^2*d - 3*a^6*
b*c*d^2) + (-(81*b^7*c^4 + 2401*a^4*b^3*d^4 - 4116*a^3*b^4*c*d^3 + 2646*a^2
*b^5*c^2*d^2 - 756*a*b^6*c^3*d)/(65536*a^15*d^8 + 65536*a^7*b^8*c^8 - 52428
8*a^8*b^7*c^7*d + 1835008*a^9*b^6*c^6*d^2 - 3670016*a^10*b^5*c^5*d^3 + 4587
520*a^11*b^4*c^4*d^4 - 3670016*a^12*b^3*c^3*d^5 + 1835008*a^13*b^2*c^2*d^6
- 524288*a^14*b*c*d^7))^(3/4)*(((-(81*b^7*c^4 + 2401*a^4*b^3*d^4 - 4116*a^3
*b^4*c*d^3 + 2646*a^2*b^5*c^2*d^2 - 756*a*b^6*c^3*d)/(65536*a^15*d^8 + 6553
6*a^7*b^8*c^8 - 524288*a^8*b^7*c^7*d + 1835008*a^9*b^6*c^6*d^2 - 3670016*a^
10*b^5*c^5*d^3 + 4587520*a^11*b^4*c^4*d^4 - 3670016*a^12*b^3*c^3*d^5 + 1835
008*a^13*b^2*c^2*d^6 - 524288*a^14*b*c*d^7))^(1/4)*(3072*a^4*b^14*c^11*d^4
- 4096*a^14*b^4*c*d^14 - 28672*a^5*b^13*c^10*d^5 + 114688*a^6*b^12*c^9*d^6
- 253952*a^7*b^11*c^8*d^7 + 329728*a^8*b^10*c^7*d^8 - 229376*a^9*b^9*c^6*d^
9 + 28672*a^10*b^8*c^5*d^10 + 90112*a^11*b^7*c^4*d^11 - 78848*a^12*b^6*c^3*
d^12 + 28672*a^13*b^5*c^2*d^13))/(a^7*d^3 - a^4*b^3*c^3 + 3*a^5*b^2*c^2*d -
 3*a^6*b*c*d^2) + (x*(65536*a^15*b^4*d^17 - 524288*a^14*b^5*c*d^16 + 36864*
a^2*b^17*c^13*d^4 - 466944*a^3*b^16*c^12*d^5 + 2609152*a^4*b^15*c^11*d^6 -
```

```
8486912*a^5*b^14*c^10*d^7 + 17833984*a^6*b^13*c^9*d^8 - 25280512*a^7*b^12*c
^8*d^9 + 24190976*a^8*b^11*c^7*d^10 - 14516224*a^9*b^10*c^6*d^11 + 3362816*
a^10*b^9*c^5*d^12 + 2809856*a^11*b^8*c^4*d^13 - 3469312*a^12*b^7*c^3*d^14 +
 1835008*a^13*b^6*c^2*d^15)*1i)/(64*(a^10*d^6 + a^4*b^6*c^6 - 6*a^5*b^5*c^5
*d + 15*a^6*b^4*c^4*d^2 - 20*a^7*b^3*c^3*d^3 + 15*a^8*b^2*c^2*d^4 - 6*a^9*b
*c*d^5)))*1i) - (x*(3185*a^4*b^7*d^13 + 81*b^11*c^4*d^9 - 756*a*b^10*c^3*d^
10 - 4788*a^3*b^8*c*d^12 + 2790*a^2*b^9*c^2*d^11))/(64*(a^10*d^6 + a^4*b^6*
c^6 - 6*a^5*b^5*c^5*d + 15*a^6*b^4*c^4*d^2 - 20*a^7*b^3*c^3*d^3 + 15*a^8*b^
2*c^2*d^4 - 6*a^9*b*c*d^5)))*(-(81*b^7*c^4 + 2401*a^4*b^3*d^4 - 4116*a^3*b^
4*c*d^3 + 2646*a^2*b^5*c^2*d^2 - 756*a*b^6*c^3*d)/(65536*a^15*d^8 + 65536*a
^7*b^8*c^8 - 524288*a^8*b^7*c^7*d + 1835008*a^9*b^6*c^6*d^2 - 3670016*a^10*
b^5*c^5*d^3 + 4587520*a^11*b^4*c^4*d^4 - 3670016*a^12*b^3*c^3*d^5 + 1835008
*a^13*b^2*c^2*d^6 - 524288*a^14*b*c*d^7))^(1/4))/(((-(81*b^7*c^4 + 2401*a^4
*b^3*d^4 - 4116*a^3*b^4*c*d^3 + 2646*a^2*b^5*c^2*d^2 - 756*a*b^6*c^3*d)/(65
536*a^15*d^8 + 65536*a^7*b^8*c^8 - 524288*a^8*b^7*c^7*d + 1835008*a^9*b^6*c
^6*d^2 - 3670016*a^10*b^5*c^5*d^3 + 4587520*a^11*b^4*c^4*d^4 - 3670016*a^12
*b^3*c^3*d^5 + 1835008*a^13*b^2*c^2*d^6 - 524288*a^14*b*c*d^7))^(1/4)*(((28
*a^4*b^6*d^11 + (81*b^10*c^4*d^7)/16 - (675*a*b^9*c^3*d^8)/16 - (2145*a^3*b
^7*c*d^10)/16 + (1971*a^2*b^8*c^2*d^9)/16)*1i)/(a^7*d^3 - a^4*b^3*c^3 + 3*a
^5*b^2*c^2*d - 3*a^6*b*c*d^2) + (-(81*b^7*c^4 + 2401*a^4*b^3*d^4 - 4116*a^3
*b^4*c*d^3 + 2646*a^2*b^5*c^2*d^2 - 756*a*b^6*c^3*d)/(65536*a^15*d^8 + 6553
6*a^7*b^8*c^8 - 524288*a^8*b^7*c^7*d + 1835008*a^9*b^6*c^6*d^2 - 3670016*a^
10*b^5*c^5*d^3 + 4587520*a^11*b^4*c^4*d^4 - 3670016*a^12*b^3*c^3*d^5 + 1835
008*a^13*b^2*c^2*d^6 - 524288*a^14*b*c*d^7))^(3/4)*(((-(81*b^7*c^4 + 2401*a
^4*b^3*d^4 - 4116*a^3*b^4*c*d^3 + 2646*a^2*b^5*c^2*d^2 - 756*a*b^6*c^3*d)/(
65536*a^15*d^8 + 65536*a^7*b^8*c^8 - 524288*a^8*b^7*c^7*d + 1835008*a^9*b^6
*c^6*d^2 - 3670016*a^10*b^5*c^5*d^3 + 4587520*a^11*b^4*c^4*d^4 - 3670016*a^
12*b^3*c^3*d^5 + 1835008*a^13*b^2*c^2*d^6 - 524288*a^14*b*c*d^7))^(1/4)*(30
72*a^4*b^14*c^11*d^4 - 4096*a^14*b^4*c*d^14 - 28672*a^5*b^13*c^10*d^5 + 114
688*a^6*b^12*c^9*d^6 - 253952*a^7*b^11*c^8*d^7 + 329728*a^8*b^10*c^7*d^8 -
229376*a^9*b^9*c^6*d^9 + 28672*a^10*b^8*c^5*d^10 + 90112*a^11*b^7*c^4*d^11
- 78848*a^12*b^6*c^3*d^12 + 28672*a^13*b^5*c^2*d^13))/(a^7*d^3 - a^4*b^3*c^
3 + 3*a^5*b^2*c^2*d - 3*a^6*b*c*d^2) - (x*(65536*a^15*b^4*d^17 - 524288*a^1
4*b^5*c*d^16 + 36864*a^2*b^17*c^13*d^4 - 466944*a^3*b^16*c^12*d^5 + 2609152
*a^4*b^15*c^11*d^6 - 8486912*a^5*b^14*c^10*d^7 + 17833984*a^6*b^13*c^9*d^8
- 25280512*a^7*b^12*c^8*d^9 + 24190976*a^8*b^11*c^7*d^10 - 14516224*a^9*b^1
0*c^6*d^11 + 3362816*a^10*b^9*c^5*d^12 + 2809856*a^11*b^8*c^4*d^13 - 346931
2*a^12*b^7*c^3*d^14 + 1835008*a^13*b^6*c^2*d^15)*1i)/(64*(a^10*d^6 + a^4*b^
6*c^6 - 6*a^5*b^5*c^5*d + 15*a^6*b^4*c^4*d^2 - 20*a^7*b^3*c^3*d^3 + 15*a^8*
b^2*c^2*d^4 - 6*a^9*b*c*d^5)))*1i)*1i + (x*(3185*a^4*b^7*d^13 + 81*b^11*c^4
*d^9 - 756*a*b^10*c^3*d^10 - 4788*a^3*b^8*c*d^12 + 2790*a^2*b^9*c^2*d^11)*1
i)/(64*(a^10*d^6 + a^4*b^6*c^6 - 6*a^5*b^5*c^5*d + 15*a^6*b^4*c^4*d^2 - 20*
a^7*b^3*c^3*d^3 + 15*a^8*b^2*c^2*d^4 - 6*a^9*b*c*d^5)))*(-(81*b^7*c^4 + 240
1*a^4*b^3*d^4 - 4116*a^3*b^4*c*d^3 + 2646*a^2*b^5*c^2*d^2 - 756*a*b^6*c^3*d
)/(65536*a^15*d^8 + 65536*a^7*b^8*c^8 - 524288*a^8*b^7*c^7*d + 1835008*a^9*
b^6*c^6*d^2 - 3670016*a^10*b^5*c^5*d^3 + 4587520*a^11*b^4*c^4*d^4 - 3670016
*a^12*b^3*c^3*d^5 + 1835008*a^13*b^2*c^2*d^6 - 524288*a^14*b*c*d^7))^(1/4)
+ ((-(81*b^7*c^4 + 2401*a^4*b^3*d^4 - 4116*a^3*b^4*c*d^3 + 2646*a^2*b^5*c^2
*d^2 - 756*a*b^6*c^3*d)/(65536*a^15*d^8 + 65536*a^7*b^8*c^8 - 524288*a^8*b^
7*c^7*d + 1835008*a^9*b^6*c^6*d^2 - 3670016*a^10*b^5*c^5*d^3 + 4587520*a^11
*b^4*c^4*d^4 - 3670016*a^12*b^3*c^3*d^5 + 1835008*a^13*b^2*c^2*d^6 - 524288
*a^14*b*c*d^7))^(1/4)*(((28*a^4*b^6*d^11 + (81*b^10*c^4*d^7)/16 - (675*a*b^
9*c^3*d^8)/16 - (2145*a^3*b^7*c*d^10)/16 + (1971*a^2*b^8*c^2*d^9)/16)*1i)/(
a^7*d^3 - a^4*b^3*c^3 + 3*a^5*b^2*c^2*d - 3*a^6*b*c*d^2) + (-(81*b^7*c^4 +
2401*a^4*b^3*d^4 - 4116*a^3*b^4*c*d^3 + 2646*a^2*b^5*c^2*d^2 - 756*a*b^6*c^
3*d)/(65536*a^15*d^8 + 65536*a^7*b^8*c^8 - 524288*a^8*b^7*c^7*d + 1835008*a
^9*b^6*c^6*d^2 - 3670016*a^10*b^5*c^5*d^3 + 4587520*a^11*b^4*c^4*d^4 - 3670
016*a^12*b^3*c^3*d^5 + 1835008*a^13*b^2*c^2*d^6 - 524288*a^14*b*c*d^7))^(3/
4)*(((-(81*b^7*c^4 + 2401*a^4*b^3*d^4 - 4116*a^3*b^4*c*d^3 + 2646*a^2*b^5*c
```

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^2*d^2 - 756*a*b^6*c^3*d)/(65536*a^15*d^8 + 65536*a^7*b^8*c^8 - 524288*a^8*
b^7*c^7*d + 1835008*a^9*b^6*c^6*d^2 - 3670016*a^10*b^5*c^5*d^3 + 4587520*a^
11*b^4*c^4*d^4 - 3670016*a^12*b^3*c^3*d^5 + 1835008*a^13*b^2*c^2*d^6 - 5242
88*a^14*b*c*d^7))^(1/4)*(3072*a^4*b^14*c^11*d^4 - 4096*a^14*b^4*c*d^14 - 28
672*a^5*b^13*c^10*d^5 + 114688*a^6*b^12*c^9*d^6 - 253952*a^7*b^11*c^8*d^7 +
 329728*a^8*b^10*c^7*d^8 - 229376*a^9*b^9*c^6*d^9 + 28672*a^10*b^8*c^5*d^10
 + 90112*a^11*b^7*c^4*d^11 - 78848*a^12*b^6*c^3*d^12 + 28672*a^13*b^5*c^2*d
^13))/(a^7*d^3 - a^4*b^3*c^3 + 3*a^5*b^2*c^2*d - 3*a^6*b*c*d^2) + (x*(65536
*a^15*b^4*d^17 - 524288*a^14*b^5*c*d^16 + 36864*a^2*b^17*c^13*d^4 - 466944*
a^3*b^16*c^12*d^5 + 2609152*a^4*b^15*c^11*d^6 - 8486912*a^5*b^14*c^10*d^7 +
 17833984*a^6*b^13*c^9*d^8 - 25280512*a^7*b^12*c^8*d^9 + 24190976*a^8*b^11*
c^7*d^10 - 14516224*a^9*b^10*c^6*d^11 + 3362816*a^10*b^9*c^5*d^12 + 2809856
*a^11*b^8*c^4*d^13 - 3469312*a^12*b^7*c^3*d^14 + 1835008*a^13*b^6*c^2*d^15)
*1i)/(64*(a^10*d^6 + a^4*b^6*c^6 - 6*a^5*b^5*c^5*d + 15*a^6*b^4*c^4*d^2 - 2
0*a^7*b^3*c^3*d^3 + 15*a^8*b^2*c^2*d^4 - 6*a^9*b*c*d^5)))*1i)*1i - (x*(3185
*a^4*b^7*d^13 + 81*b^11*c^4*d^9 - 756*a*b^10*c^3*d^10 - 4788*a^3*b^8*c*d^12
 + 2790*a^2*b^9*c^2*d^11)*1i)/(64*(a^10*d^6 + a^4*b^6*c^6 - 6*a^5*b^5*c^5*d
 + 15*a^6*b^4*c^4*d^2 - 20*a^7*b^3*c^3*d^3 + 15*a^8*b^2*c^2*d^4 - 6*a^9*b*c
*d^5)))*(-(81*b^7*c^4 + 2401*a^4*b^3*d^4 - 4116*a^3*b^4*c*d^3 + 2646*a^2*b^
5*c^2*d^2 - 756*a*b^6*c^3*d)/(65536*a^15*d^8 + 65536*a^7*b^8*c^8 - 524288*a
^8*b^7*c^7*d + 1835008*a^9*b^6*c^6*d^2 - 3670016*a^10*b^5*c^5*d^3 + 4587520
*a^11*b^4*c^4*d^4 - 3670016*a^12*b^3*c^3*d^5 + 1835008*a^13*b^2*c^2*d^6 - 5
24288*a^14*b*c*d^7))^(1/4)))*(-(81*b^7*c^4 + 2401*a^4*b^3*d^4 - 4116*a^3*b^
4*c*d^3 + 2646*a^2*b^5*c^2*d^2 - 756*a*b^6*c^3*d)/(65536*a^15*d^8 + 65536*a
^7*b^8*c^8 - 524288*a^8*b^7*c^7*d + 1835008*a^9*b^6*c^6*d^2 - 3670016*a^10*
b^5*c^5*d^3 + 4587520*a^11*b^4*c^4*d^4 - 3670016*a^12*b^3*c^3*d^5 + 1835008
*a^13*b^2*c^2*d^6 - 524288*a^14*b*c*d^7))^(1/4) - atan((((-(81*b^7*c^4 + 24
01*a^4*b^3*d^4 - 4116*a^3*b^4*c*d^3 + 2646*a^2*b^5*c^2*d^2 - 756*a*b^6*c^3*
d)/(65536*a^15*d^8 + 65536*a^7*b^8*c^8 - 524288*a^8*b^7*c^7*d + 1835008*a^9
*b^6*c^6*d^2 - 3670016*a^10*b^5*c^5*d^3 + 4587520*a^11*b^4*c^4*d^4 - 367001
6*a^12*b^3*c^3*d^5 + 1835008*a^13*b^2*c^2*d^6 - 524288*a^14*b*c*d^7))^(1/4)
*((28*a^4*b^6*d^11 + (81*b^10*c^4*d^7)/16 - (675*a*b^9*c^3*d^8)/16 - (2145*
a^3*b^7*c*d^10)/16 + (1971*a^2*b^8*c^2*d^9)/16)/(a^7*d^3 - a^4*b^3*c^3 + 3*
a^5*b^2*c^2*d - 3*a^6*b*c*d^2) + (-(81*b^7*c^4 + 2401*a^4*b^3*d^4 - 4116*a^
3*b^4*c*d^3 + 2646*a^2*b^5*c^2*d^2 - 756*a*b^6*c^3*d)/(65536*a^15*d^8 + 655
36*a^7*b^8*c^8 - 524288*a^8*b^7*c^7*d + 1835008*a^9*b^6*c^6*d^2 - 3670016*a
^10*b^5*c^5*d^3 + 4587520*a^11*b^4*c^4*d^4 - 3670016*a^12*b^3*c^3*d^5 + 183
5008*a^13*b^2*c^2*d^6 - 524288*a^14*b*c*d^7))^(3/4)*(((-(81*b^7*c^4 + 2401*
a^4*b^3*d^4 - 4116*a^3*b^4*c*d^3 + 2646*a^2*b^5*c^2*d^2 - 756*a*b^6*c^3*d)/
(65536*a^15*d^8 + 65536*a^7*b^8*c^8 - 524288*a^8*b^7*c^7*d + 1835008*a^9*b^
6*c^6*d^2 - 3670016*a^10*b^5*c^5*d^3 + 4587520*a^11*b^4*c^4*d^4 - 3670016*a
^12*b^3*c^3*d^5 + 1835008*a^13*b^2*c^2*d^6 - 524288*a^14*b*c*d^7))^(1/4)*(3
072*a^4*b^14*c^11*d^4 - 4096*a^14*b^4*c*d^14 - 28672*a^5*b^13*c^10*d^5 + 11
4688*a^6*b^12*c^9*d^6 - 253952*a^7*b^11*c^8*d^7 + 329728*a^8*b^10*c^7*d^8 -
 229376*a^9*b^9*c^6*d^9 + 28672*a^10*b^8*c^5*d^10 + 90112*a^11*b^7*c^4*d^11
 - 78848*a^12*b^6*c^3*d^12 + 28672*a^13*b^5*c^2*d^13))/(a^7*d^3 - a^4*b^3*c
^3 + 3*a^5*b^2*c^2*d - 3*a^6*b*c*d^2) - (x*(65536*a^15*b^4*d^17 - 524288*a^
14*b^5*c*d^16 + 36864*a^2*b^17*c^13*d^4 - 466944*a^3*b^16*c^12*d^5 + 260915
2*a^4*b^15*c^11*d^6 - 8486912*a^5*b^14*c^10*d^7 + 17833984*a^6*b^13*c^9*d^8
 - 25280512*a^7*b^12*c^8*d^9 + 24190976*a^8*b^11*c^7*d^10 - 14516224*a^9*b^
10*c^6*d^11 + 3362816*a^10*b^9*c^5*d^12 + 2809856*a^11*b^8*c^4*d^13 - 34693
12*a^12*b^7*c^3*d^14 + 1835008*a^13*b^6*c^2*d^15))/(64*(a^10*d^6 + a^4*b^6*
c^6 - 6*a^5*b^5*c^5*d + 15*a^6*b^4*c^4*d^2 - 20*a^7*b^3*c^3*d^3 + 15*a^8*b^
2*c^2*d^4 - 6*a^9*b*c*d^5))))*1i - (x*(3185*a^4*b^7*d^13 + 81*b^11*c^4*d^9
- 756*a*b^10*c^3*d^10 - 4788*a^3*b^8*c*d^12 + 2790*a^2*b^9*c^2*d^11)*1i)/(6
4*(a^10*d^6 + a^4*b^6*c^6 - 6*a^5*b^5*c^5*d + 15*a^6*b^4*c^4*d^2 - 20*a^7*b
^3*c^3*d^3 + 15*a^8*b^2*c^2*d^4 - 6*a^9*b*c*d^5)))*(-(81*b^7*c^4 + 2401*a^4
*b^3*d^4 - 4116*a^3*b^4*c*d^3 + 2646*a^2*b^5*c^2*d^2 - 756*a*b^6*c^3*d)/(65
536*a^15*d^8 + 65536*a^7*b^8*c^8 - 524288*a^8*b^7*c^7*d + 1835008*a^9*b^6*c
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^6*d^2 - 3670016*a^10*b^5*c^5*d^3 + 4587520*a^11*b^4*c^4*d^4 - 3670016*a^12*b^3*c^3*d^5 + 1835008*a^13*b^2*c^2*d^6 - 524288*a^14*b*c*d^7))^(1/4) - ((-(81*b^7*c^4 + 2401*a^4*b^3*d^4 - 4116*a^3*b^4*c*d^3 + 2646*a^2*b^5*c^2*d^2 - 756*a*b^6*c^3*d)/(65536*a^15*d^8 + 65536*a^7*b^8*c^8 - 524288*a^8*b^7*c^7*d + 1835008*a^9*b^6*c^6*d^2 - 3670016*a^10*b^5*c^5*d^3 + 4587520*a^11*b^4*c^4*d^4 - 3670016*a^12*b^3*c^3*d^5 + 1835008*a^13*b^2*c^2*d^6 - 524288*a^14*b*c*d^7))^(1/4)*((28*a^4*b^6*d^11 + (81*b^10*c^4*d^7)/16 - (675*a*b^9*c^3*d^8)/16 - (2145*a^3*b^7*c*d^10)/16 + (1971*a^2*b^8*c^2*d^9)/16)/(a^7*d^3 - a^4*b^3*c^3 + 3*a^5*b^2*c^2*d - 3*a^6*b*c*d^2) + (-(81*b^7*c^4 + 2401*a^4*b^3*d^4 - 4116*a^3*b^4*c*d^3 + 2646*a^2*b^5*c^2*d^2 - 756*a*b^6*c^3*d)/(65536*a^15*d^8 + 65536*a^7*b^8*c^8 - 524288*a^8*b^7*c^7*d + 1835008*a^9*b^6*c^6*d^2 - 3670016*a^10*b^5*c^5*d^3 + 4587520*a^11*b^4*c^4*d^4 - 3670016*a^12*b^3*c^3*d^5 + 1835008*a^13*b^2*c^2*d^6 - 524288*a^14*b*c*d^7))^(3/4)*(((-(81*b^7*c^4 + 2401*a^4*b^3*d^4 - 4116*a^3*b^4*c*d^3 + 2646*a^2*b^5*c^2*d^2 - 756*a*b^6*c^3*d)/(65536*a^15*d^8 + 65536*a^7*b^8*c^8 - 524288*a^8*b^7*c^7*d + 1835008*a^9*b^6*c^6*d^2 - 3670016*a^10*b^5*c^5*d^3 + 4587520*a^11*b^4*c^4*d^4 - 3670016*a^12*b^3*c^3*d^5 + 1835008*a^13*b^2*c^2*d^6 - 524288*a^14*b*c*d^7))^(1/4)*(3072*a^4*b^14*c^11*d^4 - 4096*a^14*b^4*c*d^14 - 28672*a^5*b^13*c^10*d^5 + 114688*a^6*b^12*c^9*d^6 - 253952*a^7*b^11*c^8*d^7 + 329728*a^8*b^10*c^7*d^8 - 229376*a^9*b^9*c^6*d^9 + 28672*a^10*b^8*c^5*d^10 + 90112*a^11*b^7*c^4*d^11 - 78848*a^12*b^6*c^3*d^12 + 28672*a^13*b^5*c^2*d^13))/(a^7*d^3 - a^4*b^3*c^3 + 3*a^5*b^2*c^2*d - 3*a^6*b*c*d^2) + (x*(65536*a^15*b^4*d^17 - 524288*a^14*b^5*c*d^16 + 36864*a^2*b^17*c^13*d^4 - 466944*a^3*b^16*c^12*d^5 + 2609152*a^4*b^15*c^11*d^6 - 8486912*a^5*b^14*c^10*d^7 + 17833984*a^6*b^13*c^9*d^8 - 25280512*a^7*b^12*c^8*d^9 + 24190976*a^8*b^11*c^7*d^10 - 14516224*a^9*b^10*c^6*d^11 + 3362816*a^10*b^9*c^5*d^12 + 2809856*a^11*b^8*c^4*d^13 - 3469312*a^12*b^7*c^3*d^14 + 1835008*a^13*b^6*c^2*d^15))/(64*(a^10*d^6 + a^4*b^6*c^6 - 6*a^5*b^5*c^5*d + 15*a^6*b^4*c^4*d^2 - 20*a^7*b^3*c^3*d^3 + 15*a^8*b^2*c^2*d^4 - 6*a^9*b*c*d^5))))*1i + (x*(3185*a^4*b^7*d^13 + 81*b^11*c^4*d^9 - 756*a*b^10*c^3*d^10 - 4788*a^3*b^8*c*d^12 + 2790*a^2*b^9*c^2*d^11)*1i)/(64*(a^10*d^6 + a^4*b^6*c^6 - 6*a^5*b^5*c^5*d + 15*a^6*b^4*c^4*d^2 - 20*a^7*b^3*c^3*d^3 + 15*a^8*b^2*c^2*d^4 - 6*a^9*b*c*d^5)))*(-(81*b^7*c^4 + 2401*a^4*b^3*d^4 - 4116*a^3*b^4*c*d^3 + 2646*a^2*b^5*c^2*d^2 - 756*a*b^6*c^3*d)/(65536*a^15*d^8 + 65536*a^7*b^8*c^8 - 524288*a^8*b^7*c^7*d + 1835008*a^9*b^6*c^6*d^2 - 3670016*a^10*b^5*c^5*d^3 + 4587520*a^11*b^4*c^4*d^4 - 3670016*a^12*b^3*c^3*d^5 + 1835008*a^13*b^2*c^2*d^6 - 524288*a^14*b*c*d^7))^(1/4))/(((-(81*b^7*c^4 + 2401*a^4*b^3*d^4 - 4116*a^3*b^4*c*d^3 + 2646*a^2*b^5*c^2*d^2 - 756*a*b^6*c^3*d)/(65536*a^15*d^8 + 65536*a^7*b^8*c^8 - 524288*a^8*b^7*c^7*d + 1835008*a^9*b^6*c^6*d^2 - 3670016*a^10*b^5*c^5*d^3 + 4587520*a^11*b^4*c^4*d^4 - 3670016*a^12*b^3*c^3*d^5 + 1835008*a^13*b^2*c^2*d^6 - 524288*a^14*b*c*d^7))^(1/4)*((28*a^4*b^6*d^11 + (81*b^10*c^4*d^7)/16 - (675*a*b^9*c^3*d^8)/16 - (2145*a^3*b^7*c*d^10)/16 + (1971*a^2*b^8*c^2*d^9)/16)/(a^7*d^3 - a^4*b^3*c^3 + 3*a^5*b^2*c^2*d - 3*a^6*b*c*d^2) + (-(81*b^7*c^4 + 2401*a^4*b^3*d^4 - 4116*a^3*b^4*c*d^3 + 2646*a^2*b^5*c^2*d^2 - 756*a*b^6*c^3*d)/(65536*a^15*d^8 + 65536*a^7*b^8*c^8 - 524288*a^8*b^7*c^7*d + 1835008*a^9*b^6*c^6*d^2 - 3670016*a^10*b^5*c^5*d^3 + 4587520*a^11*b^4*c^4*d^4 - 3670016*a^12*b^3*c^3*d^5 + 1835008*a^13*b^2*c^2*d^6 - 524288*a^14*b*c*d^7))^(3/4)*(((-(81*b^7*c^4 + 2401*a^4*b^3*d^4 - 4116*a^3*b^4*c*d^3 + 2646*a^2*b^5*c^2*d^2 - 756*a*b^6*c^3*d)/(65536*a^15*d^8 + 65536*a^7*b^8*c^8 - 524288*a^8*b^7*c^7*d + 1835008*a^9*b^6*c^6*d^2 - 3670016*a^10*b^5*c^5*d^3 + 4587520*a^11*b^4*c^4*d^4 - 3670016*a^12*b^3*c^3*d^5 + 1835008*a^13*b^2*c^2*d^6 - 524288*a^14*b*c*d^7))^(1/4)*(3072*a^4*b^14*c^11*d^4 - 4096*a^14*b^4*c*d^14 - 28672*a^5*b^13*c^10*d^5 + 114688*a^6*b^12*c^9*d^6 - 253952*a^7*b^11*c^8*d^7 + 329728*a^8*b^10*c^7*d^8 - 229376*a^9*b^9*c^6*d^9 + 28672*a^10*b^8*c^5*d^10 + 90112*a^11*b^7*c^4*d^11 - 78848*a^12*b^6*c^3*d^12 + 28672*a^13*b^5*c^2*d^13))/(a^7*d^3 - a^4*b^3*c^3 + 3*a^5*b^2*c^2*d - 3*a^6*b*c*d^2) - (x*(65536*a^15*b^4*d^17 - 524288*a^14*b^5*c*d^16 + 36864*a^2*b^17*c^13*d^4 - 466944*a^3*b^16*c^12*d^5 + 2609152*a^4*b^15*c^11*d^6 - 8486912*a^5*b^14*c^10*d^7 + 17833984*a^6*b^13*c^9*d^8 - 25280512*a^7*b^12*c^8*d^9 + 24190976*a^8

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*b^11*c^7*d^10 - 14516224*a^9*b^10*c^6*d^11 + 3362816*a^10*b^9*c^5*d^12 + 2
809856*a^11*b^8*c^4*d^13 - 3469312*a^12*b^7*c^3*d^14 + 1835008*a^13*b^6*c^2
*d^15))/(64*(a^10*d^6 + a^4*b^6*c^6 - 6*a^5*b^5*c^5*d + 15*a^6*b^4*c^4*d^2
- 20*a^7*b^3*c^3*d^3 + 15*a^8*b^2*c^2*d^4 - 6*a^9*b*c*d^5)))) - (x*(3185*a^
4*b^7*d^13 + 81*b^11*c^4*d^9 - 756*a*b^10*c^3*d^10 - 4788*a^3*b^8*c*d^12 +
2790*a^2*b^9*c^2*d^11))/(64*(a^10*d^6 + a^4*b^6*c^6 - 6*a^5*b^5*c^5*d + 15*
a^6*b^4*c^4*d^2 - 20*a^7*b^3*c^3*d^3 + 15*a^8*b^2*c^2*d^4 - 6*a^9*b*c*d^5))
)*(-(81*b^7*c^4 + 2401*a^4*b^3*d^4 - 4116*a^3*b^4*c*d^3 + 2646*a^2*b^5*c^2*
d^2 - 756*a*b^6*c^3*d)/(65536*a^15*d^8 + 65536*a^7*b^8*c^8 - 524288*a^8*b^7
*c^7*d + 1835008*a^9*b^6*c^6*d^2 - 3670016*a^10*b^5*c^5*d^3 + 4587520*a^11*
b^4*c^4*d^4 - 3670016*a^12*b^3*c^3*d^5 + 1835008*a^13*b^2*c^2*d^6 - 524288*
a^14*b*c*d^7))^(1/4) + ((-(81*b^7*c^4 + 2401*a^4*b^3*d^4 - 4116*a^3*b^4*c*d
^3 + 2646*a^2*b^5*c^2*d^2 - 756*a*b^6*c^3*d)/(65536*a^15*d^8 + 65536*a^7*b^
8*c^8 - 524288*a^8*b^7*c^7*d + 1835008*a^9*b^6*c^6*d^2 - 3670016*a^10*b^5*c
^5*d^3 + 4587520*a^11*b^4*c^4*d^4 - 3670016*a^12*b^3*c^3*d^5 + 1835008*a^13
*b^2*c^2*d^6 - 524288*a^14*b*c*d^7))^(1/4)*((28*a^4*b^6*d^11 + (81*b^10*c^4
*d^7)/16 - (675*a*b^9*c^3*d^8)/16 - (2145*a^3*b^7*c*d^10)/16 + (1971*a^2*b^
8*c^2*d^9)/16)/(a^7*d^3 - a^4*b^3*c^3 + 3*a^5*b^2*c^2*d - 3*a^6*b*c*d^2) +
(-(81*b^7*c^4 + 2401*a^4*b^3*d^4 - 4116*a^3*b^4*c*d^3 + 2646*a^2*b^5*c^2*d^
2 - 756*a*b^6*c^3*d)/(65536*a^15*d^8 + 65536*a^7*b^8*c^8 - 524288*a^8*b^7*c
^7*d + 1835008*a^9*b^6*c^6*d^2 - 3670016*a^10*b^5*c^5*d^3 + 4587520*a^11*b^
4*c^4*d^4 - 3670016*a^12*b^3*c^3*d^5 + 1835008*a^13*b^2*c^2*d^6 - 524288*a^
14*b*c*d^7))^(3/4)*(((-(81*b^7*c^4 + 2401*a^4*b^3*d^4 - 4116*a^3*b^4*c*d^3
+ 2646*a^2*b^5*c^2*d^2 - 756*a*b^6*c^3*d)/(65536*a^15*d^8 + 65536*a^7*b^8*c
^8 - 524288*a^8*b^7*c^7*d + 1835008*a^9*b^6*c^6*d^2 - 3670016*a^10*b^5*c^5*
d^3 + 4587520*a^11*b^4*c^4*d^4 - 3670016*a^12*b^3*c^3*d^5 + 1835008*a^13*b^
2*c^2*d^6 - 524288*a^14*b*c*d^7))^(1/4)*(3072*a^4*b^14*c^11*d^4 - 4096*a^14
*b^4*c*d^14 - 28672*a^5*b^13*c^10*d^5 + 114688*a^6*b^12*c^9*d^6 - 253952*a^
7*b^11*c^8*d^7 + 329728*a^8*b^10*c^7*d^8 - 229376*a^9*b^9*c^6*d^9 + 28672*a
^10*b^8*c^5*d^10 + 90112*a^11*b^7*c^4*d^11 - 78848*a^12*b^6*c^3*d^12 + 2867
2*a^13*b^5*c^2*d^13))/(a^7*d^3 - a^4*b^3*c^3 + 3*a^5*b^2*c^2*d - 3*a^6*b*c*
d^2) + (x*(65536*a^15*b^4*d^17 - 524288*a^14*b^5*c*d^16 + 36864*a^2*b^17*c^
13*d^4 - 466944*a^3*b^16*c^12*d^5 + 2609152*a^4*b^15*c^11*d^6 - 8486912*a^5
*b^14*c^10*d^7 + 17833984*a^6*b^13*c^9*d^8 - 25280512*a^7*b^12*c^8*d^9 + 24
190976*a^8*b^11*c^7*d^10 - 14516224*a^9*b^10*c^6*d^11 + 3362816*a^10*b^9*c^
5*d^12 + 2809856*a^11*b^8*c^4*d^13 - 3469312*a^12*b^7*c^3*d^14 + 1835008*a^
13*b^6*c^2*d^15))/(64*(a^10*d^6 + a^4*b^6*c^6 - 6*a^5*b^5*c^5*d + 15*a^6*b^
4*c^4*d^2 - 20*a^7*b^3*c^3*d^3 + 15*a^8*b^2*c^2*d^4 - 6*a^9*b*c*d^5)))) + (
x*(3185*a^4*b^7*d^13 + 81*b^11*c^4*d^9 - 756*a*b^10*c^3*d^10 - 4788*a^3*b^8
*c*d^12 + 2790*a^2*b^9*c^2*d^11))/(64*(a^10*d^6 + a^4*b^6*c^6 - 6*a^5*b^5*c
^5*d + 15*a^6*b^4*c^4*d^2 - 20*a^7*b^3*c^3*d^3 + 15*a^8*b^2*c^2*d^4 - 6*a^9
*b*c*d^5)))*(-(81*b^7*c^4 + 2401*a^4*b^3*d^4 - 4116*a^3*b^4*c*d^3 + 2646*a^
2*b^5*c^2*d^2 - 756*a*b^6*c^3*d)/(65536*a^15*d^8 + 65536*a^7*b^8*c^8 - 5242
88*a^8*b^7*c^7*d + 1835008*a^9*b^6*c^6*d^2 - 3670016*a^10*b^5*c^5*d^3 + 458
7520*a^11*b^4*c^4*d^4 - 3670016*a^12*b^3*c^3*d^5 + 1835008*a^13*b^2*c^2*d^6
 - 524288*a^14*b*c*d^7))^(1/4)))*(-(81*b^7*c^4 + 2401*a^4*b^3*d^4 - 4116*a^
3*b^4*c*d^3 + 2646*a^2*b^5*c^2*d^2 - 756*a*b^6*c^3*d)/(65536*a^15*d^8 + 655
36*a^7*b^8*c^8 - 524288*a^8*b^7*c^7*d + 1835008*a^9*b^6*c^6*d^2 - 3670016*a
^10*b^5*c^5*d^3 + 4587520*a^11*b^4*c^4*d^4 - 3670016*a^12*b^3*c^3*d^5 + 183
5008*a^13*b^2*c^2*d^6 - 524288*a^14*b*c*d^7))^(1/4)*2i - atan(((-d^7/(256*b
^8*c^11 + 256*a^8*c^3*d^8 - 2048*a^7*b*c^4*d^7 + 7168*a^2*b^6*c^9*d^2 - 143
36*a^3*b^5*c^8*d^3 + 17920*a^4*b^4*c^7*d^4 - 14336*a^5*b^3*c^6*d^5 + 7168*a
^6*b^2*c^5*d^6 - 2048*a*b^7*c^10*d))^(1/4)*((-d^7/(256*b^8*c^11 + 256*a^8*c
^3*d^8 - 2048*a^7*b*c^4*d^7 + 7168*a^2*b^6*c^9*d^2 - 14336*a^3*b^5*c^8*d^3
+ 17920*a^4*b^4*c^7*d^4 - 14336*a^5*b^3*c^6*d^5 + 7168*a^6*b^2*c^5*d^6 - 20
48*a*b^7*c^10*d))^(1/4)*((-d^7/(256*b^8*c^11 + 256*a^8*c^3*d^8 - 2048*a^7*b
*c^4*d^7 + 7168*a^2*b^6*c^9*d^2 - 14336*a^3*b^5*c^8*d^3 + 17920*a^4*b^4*c^7
*d^4 - 14336*a^5*b^3*c^6*d^5 + 7168*a^6*b^2*c^5*d^6 - 2048*a*b^7*c^10*d))^(
3/4)*(((-d^7/(256*b^8*c^11 + 256*a^8*c^3*d^8 - 2048*a^7*b*c^4*d^7 + 7168*a^
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2*b^6*c^9*d^2 - 14336*a^3*b^5*c^8*d^3 + 17920*a^4*b^4*c^7*d^4 - 14336*a^5*b
^3*c^6*d^5 + 7168*a^6*b^2*c^5*d^6 - 2048*a*b^7*c^10*d))^(1/4)*(3072*a^4*b^1
4*c^11*d^4 - 4096*a^14*b^4*c*d^14 - 28672*a^5*b^13*c^10*d^5 + 114688*a^6*b^
12*c^9*d^6 - 253952*a^7*b^11*c^8*d^7 + 329728*a^8*b^10*c^7*d^8 - 229376*a^9
*b^9*c^6*d^9 + 28672*a^10*b^8*c^5*d^10 + 90112*a^11*b^7*c^4*d^11 - 78848*a^
12*b^6*c^3*d^12 + 28672*a^13*b^5*c^2*d^13))/(a^7*d^3 - a^4*b^3*c^3 + 3*a^5*
b^2*c^2*d - 3*a^6*b*c*d^2) - (x*(65536*a^15*b^4*d^17 - 524288*a^14*b^5*c*d^
16 + 36864*a^2*b^17*c^13*d^4 - 466944*a^3*b^16*c^12*d^5 + 2609152*a^4*b^15*
c^11*d^6 - 8486912*a^5*b^14*c^10*d^7 + 17833984*a^6*b^13*c^9*d^8 - 25280512
*a^7*b^12*c^8*d^9 + 24190976*a^8*b^11*c^7*d^10 - 14516224*a^9*b^10*c^6*d^11
 + 3362816*a^10*b^9*c^5*d^12 + 2809856*a^11*b^8*c^4*d^13 - 3469312*a^12*b^7
*c^3*d^14 + 1835008*a^13*b^6*c^2*d^15))/(64*(a^10*d^6 + a^4*b^6*c^6 - 6*a^5
*b^5*c^5*d + 15*a^6*b^4*c^4*d^2 - 20*a^7*b^3*c^3*d^3 + 15*a^8*b^2*c^2*d^4 -
 6*a^9*b*c*d^5))) + (28*a^4*b^6*d^11 + (81*b^10*c^4*d^7)/16 - (675*a*b^9*c^
3*d^8)/16 - (2145*a^3*b^7*c*d^10)/16 + (1971*a^2*b^8*c^2*d^9)/16)/(a^7*d^3
- a^4*b^3*c^3 + 3*a^5*b^2*c^2*d - 3*a^6*b*c*d^2))*1i - (x*(3185*a^4*b^7*d^1
3 + 81*b^11*c^4*d^9 - 756*a*b^10*c^3*d^10 - 4788*a^3*b^8*c*d^12 + 2790*a^2*
b^9*c^2*d^11)*1i)/(64*(a^10*d^6 + a^4*b^6*c^6 - 6*a^5*b^5*c^5*d + 15*a^6*b^
4*c^4*d^2 - 20*a^7*b^3*c^3*d^3 + 15*a^8*b^2*c^2*d^4 - 6*a^9*b*c*d^5))) - (-
d^7/(256*b^8*c^11 + 256*a^8*c^3*d^8 - 2048*a^7*b*c^4*d^7 + 7168*a^2*b^6*c^9
*d^2 - 14336*a^3*b^5*c^8*d^3 + 17920*a^4*b^4*c^7*d^4 - 14336*a^5*b^3*c^6*d^
5 + 7168*a^6*b^2*c^5*d^6 - 2048*a*b^7*c^10*d))^(1/4)*((-d^7/(256*b^8*c^11 +
 256*a^8*c^3*d^8 - 2048*a^7*b*c^4*d^7 + 7168*a^2*b^6*c^9*d^2 - 14336*a^3*b^
5*c^8*d^3 + 17920*a^4*b^4*c^7*d^4 - 14336*a^5*b^3*c^6*d^5 + 7168*a^6*b^2*c^
5*d^6 - 2048*a*b^7*c^10*d))^(1/4)*((-d^7/(256*b^8*c^11 + 256*a^8*c^3*d^8 -
2048*a^7*b*c^4*d^7 + 7168*a^2*b^6*c^9*d^2 - 14336*a^3*b^5*c^8*d^3 + 17920*a
^4*b^4*c^7*d^4 - 14336*a^5*b^3*c^6*d^5 + 7168*a^6*b^2*c^5*d^6 - 2048*a*b^7*
c^10*d))^(3/4)*(((-d^7/(256*b^8*c^11 + 256*a^8*c^3*d^8 - 2048*a^7*b*c^4*d^7
 + 7168*a^2*b^6*c^9*d^2 - 14336*a^3*b^5*c^8*d^3 + 17920*a^4*b^4*c^7*d^4 - 1
4336*a^5*b^3*c^6*d^5 + 7168*a^6*b^2*c^5*d^6 - 2048*a*b^7*c^10*d))^(1/4)*(30
72*a^4*b^14*c^11*d^4 - 4096*a^14*b^4*c*d^14 - 28672*a^5*b^13*c^10*d^5 + 114
688*a^6*b^12*c^9*d^6 - 253952*a^7*b^11*c^8*d^7 + 329728*a^8*b^10*c^7*d^8 -
229376*a^9*b^9*c^6*d^9 + 28672*a^10*b^8*c^5*d^10 + 90112*a^11*b^7*c^4*d^11
- 78848*a^12*b^6*c^3*d^12 + 28672*a^13*b^5*c^2*d^13))/(a^7*d^3 - a^4*b^3*c^
3 + 3*a^5*b^2*c^2*d - 3*a^6*b*c*d^2) + (x*(65536*a^15*b^4*d^17 - 524288*a^1
4*b^5*c*d^16 + 36864*a^2*b^17*c^13*d^4 - 466944*a^3*b^16*c^12*d^5 + 2609152
*a^4*b^15*c^11*d^6 - 8486912*a^5*b^14*c^10*d^7 + 17833984*a^6*b^13*c^9*d^8
- 25280512*a^7*b^12*c^8*d^9 + 24190976*a^8*b^11*c^7*d^10 - 14516224*a^9*b^1
0*c^6*d^11 + 3362816*a^10*b^9*c^5*d^12 + 2809856*a^11*b^8*c^4*d^13 - 346931
2*a^12*b^7*c^3*d^14 + 1835008*a^13*b^6*c^2*d^15))/(64*(a^10*d^6 + a^4*b^6*c
^6 - 6*a^5*b^5*c^5*d + 15*a^6*b^4*c^4*d^2 - 20*a^7*b^3*c^3*d^3 + 15*a^8*b^2
*c^2*d^4 - 6*a^9*b*c*d^5))) + (28*a^4*b^6*d^11 + (81*b^10*c^4*d^7)/16 - (67
5*a*b^9*c^3*d^8)/16 - (2145*a^3*b^7*c*d^10)/16 + (1971*a^2*b^8*c^2*d^9)/16)
/(a^7*d^3 - a^4*b^3*c^3 + 3*a^5*b^2*c^2*d - 3*a^6*b*c*d^2))*1i + (x*(3185*a
^4*b^7*d^13 + 81*b^11*c^4*d^9 - 756*a*b^10*c^3*d^10 - 4788*a^3*b^8*c*d^12 +
 2790*a^2*b^9*c^2*d^11)*1i)/(64*(a^10*d^6 + a^4*b^6*c^6 - 6*a^5*b^5*c^5*d +
 15*a^6*b^4*c^4*d^2 - 20*a^7*b^3*c^3*d^3 + 15*a^8*b^2*c^2*d^4 - 6*a^9*b*c*d
^5))))/((-d^7/(256*b^8*c^11 + 256*a^8*c^3*d^8 - 2048*a^7*b*c^4*d^7 + 7168*a
^2*b^6*c^9*d^2 - 14336*a^3*b^5*c^8*d^3 + 17920*a^4*b^4*c^7*d^4 - 14336*a^5*
b^3*c^6*d^5 + 7168*a^6*b^2*c^5*d^6 - 2048*a*b^7*c^10*d))^(1/4)*((-d^7/(256*
b^8*c^11 + 256*a^8*c^3*d^8 - 2048*a^7*b*c^4*d^7 + 7168*a^2*b^6*c^9*d^2 - 14
336*a^3*b^5*c^8*d^3 + 17920*a^4*b^4*c^7*d^4 - 14336*a^5*b^3*c^6*d^5 + 7168*
a^6*b^2*c^5*d^6 - 2048*a*b^7*c^10*d))^(1/4)*((-d^7/(256*b^8*c^11 + 256*a^8*
c^3*d^8 - 2048*a^7*b*c^4*d^7 + 7168*a^2*b^6*c^9*d^2 - 14336*a^3*b^5*c^8*d^3
 + 17920*a^4*b^4*c^7*d^4 - 14336*a^5*b^3*c^6*d^5 + 7168*a^6*b^2*c^5*d^6 - 2
048*a*b^7*c^10*d))^(3/4)*(((-d^7/(256*b^8*c^11 + 256*a^8*c^3*d^8 - 2048*a^7
*b*c^4*d^7 + 7168*a^2*b^6*c^9*d^2 - 14336*a^3*b^5*c^8*d^3 + 17920*a^4*b^4*c
^7*d^4 - 14336*a^5*b^3*c^6*d^5 + 7168*a^6*b^2*c^5*d^6 - 2048*a*b^7*c^10*d))
^(1/4)*(3072*a^4*b^14*c^11*d^4 - 4096*a^14*b^4*c*d^14 - 28672*a^5*b^13*c^10

*d^5 + 114688*a^6*b^12*c^9*d^6 - 253952*a^7*b^11*c^8*d^7 + 329728*a^8*b^10*
c^7*d^8 - 229376*a^9*b^9*c^6*d^9 + 28672*a^10*b^8*c^5*d^10 + 90112*a^11*b^7
*c^4*d^11 - 78848*a^12*b^6*c^3*d^12 + 28672*a^13*b^5*c^2*d^13))/(a^7*d^3 -
a^4*b^3*c^3 + 3*a^5*b^2*c^2*d - 3*a^6*b*c*d^2) - (x*(65536*a^15*b^4*d^17 -
524288*a^14*b^5*c*d^16 + 36864*a^2*b^17*c^13*d^4 - 466944*a^3*b^16*c^12*d^5
 + 2609152*a^4*b^15*c^11*d^6 - 8486912*a^5*b^14*c^10*d^7 + 17833984*a^6*b^1
3*c^9*d^8 - 25280512*a^7*b^12*c^8*d^9 + 24190976*a^8*b^11*c^7*d^10 - 145162
24*a^9*b^10*c^6*d^11 + 3362816*a^10*b^9*c^5*d^12 + 2809856*a^11*b^8*c^4*d^1
3 - 3469312*a^12*b^7*c^3*d^14 + 1835008*a^13*b^6*c^2*d^15))/(64*(a^10*d^6 +
 a^4*b^6*c^6 - 6*a^5*b^5*c^5*d + 15*a^6*b^4*c^4*d^2 - 20*a^7*b^3*c^3*d^3 +
15*a^8*b^2*c^2*d^4 - 6*a^9*b*c*d^5))) + (28*a^4*b^6*d^11 + (81*b^10*c^4*d^7
)/16 - (675*a*b^9*c^3*d^8)/16 - (2145*a^3*b^7*c*d^10)/16 + (1971*a^2*b^8*c^
2*d^9)/16)/(a^7*d^3 - a^4*b^3*c^3 + 3*a^5*b^2*c^2*d - 3*a^6*b*c*d^2)) - (x*
(3185*a^4*b^7*d^13 + 81*b^11*c^4*d^9 - 756*a*b^10*c^3*d^10 - 4788*a^3*b^8*c
*d^12 + 2790*a^2*b^9*c^2*d^11))/(64*(a^10*d^6 + a^4*b^6*c^6 - 6*a^5*b^5*c^5
*d + 15*a^6*b^4*c^4*d^2 - 20*a^7*b^3*c^3*d^3 + 15*a^8*b^2*c^2*d^4 - 6*a^9*b
*c*d^5))) + (-d^7/(256*b^8*c^11 + 256*a^8*c^3*d^8 - 2048*a^7*b*c^4*d^7 + 71
68*a^2*b^6*c^9*d^2 - 14336*a^3*b^5*c^8*d^3 + 17920*a^4*b^4*c^7*d^4 - 14336*
a^5*b^3*c^6*d^5 + 7168*a^6*b^2*c^5*d^6 - 2048*a*b^7*c^10*d))^(1/4)*((-d^7/(
256*b^8*c^11 + 256*a^8*c^3*d^8 - 2048*a^7*b*c^4*d^7 + 7168*a^2*b^6*c^9*d^2
- 14336*a^3*b^5*c^8*d^3 + 17920*a^4*b^4*c^7*d^4 - 14336*a^5*b^3*c^6*d^5 + 7
168*a^6*b^2*c^5*d^6 - 2048*a*b^7*c^10*d))^(1/4)*((-d^7/(256*b^8*c^11 + 256*
a^8*c^3*d^8 - 2048*a^7*b*c^4*d^7 + 7168*a^2*b^6*c^9*d^2 - 14336*a^3*b^5*c^8
*d^3 + 17920*a^4*b^4*c^7*d^4 - 14336*a^5*b^3*c^6*d^5 + 7168*a^6*b^2*c^5*d^6
 - 2048*a*b^7*c^10*d))^(3/4)*(((-d^7/(256*b^8*c^11 + 256*a^8*c^3*d^8 - 2048
*a^7*b*c^4*d^7 + 7168*a^2*b^6*c^9*d^2 - 14336*a^3*b^5*c^8*d^3 + 17920*a^4*b
^4*c^7*d^4 - 14336*a^5*b^3*c^6*d^5 + 7168*a^6*b^2*c^5*d^6 - 2048*a*b^7*c^10
d))^(1/4)(3072*a^4*b^14*c^11*d^4 - 4096*a^14*b^4*c*d^14 - 28672*a^5*b^13*
c^10*d^5 + 114688*a^6*b^12*c^9*d^6 - 253952*a^7*b^11*c^8*d^7 + 329728*a^8*b
^10*c^7*d^8 - 229376*a^9*b^9*c^6*d^9 + 28672*a^10*b^8*c^5*d^10 + 90112*a^11
*b^7*c^4*d^11 - 78848*a^12*b^6*c^3*d^12 + 28672*a^13*b^5*c^2*d^13))/(a^7*d^
3 - a^4*b^3*c^3 + 3*a^5*b^2*c^2*d - 3*a^6*b*c*d^2) + (x*(65536*a^15*b^4*d^1
7 - 524288*a^14*b^5*c*d^16 + 36864*a^2*b^17*c^13*d^4 - 466944*a^3*b^16*c^12
*d^5 + 2609152*a^4*b^15*c^11*d^6 - 8486912*a^5*b^14*c^10*d^7 + 17833984*a^6
*b^13*c^9*d^8 - 25280512*a^7*b^12*c^8*d^9 + 24190976*a^8*b^11*c^7*d^10 - 14
516224*a^9*b^10*c^6*d^11 + 3362816*a^10*b^9*c^5*d^12 + 2809856*a^11*b^8*c^4
*d^13 - 3469312*a^12*b^7*c^3*d^14 + 1835008*a^13*b^6*c^2*d^15))/(64*(a^10*d
^6 + a^4*b^6*c^6 - 6*a^5*b^5*c^5*d + 15*a^6*b^4*c^4*d^2 - 20*a^7*b^3*c^3*d^
3 + 15*a^8*b^2*c^2*d^4 - 6*a^9*b*c*d^5))) + (28*a^4*b^6*d^11 + (81*b^10*c^4
*d^7)/16 - (675*a*b^9*c^3*d^8)/16 - (2145*a^3*b^7*c*d^10)/16 + (1971*a^2*b^
8*c^2*d^9)/16)/(a^7*d^3 - a^4*b^3*c^3 + 3*a^5*b^2*c^2*d - 3*a^6*b*c*d^2)) +
 (x*(3185*a^4*b^7*d^13 + 81*b^11*c^4*d^9 - 756*a*b^10*c^3*d^10 - 4788*a^3*b
^8*c*d^12 + 2790*a^2*b^9*c^2*d^11))/(64*(a^10*d^6 + a^4*b^6*c^6 - 6*a^5*b^5
*c^5*d + 15*a^6*b^4*c^4*d^2 - 20*a^7*b^3*c^3*d^3 + 15*a^8*b^2*c^2*d^4 - 6*a
^9*b*c*d^5)))))*(-d^7/(256*b^8*c^11 + 256*a^8*c^3*d^8 - 2048*a^7*b*c^4*d^7
+ 7168*a^2*b^6*c^9*d^2 - 14336*a^3*b^5*c^8*d^3 + 17920*a^4*b^4*c^7*d^4 - 14
336*a^5*b^3*c^6*d^5 + 7168*a^6*b^2*c^5*d^6 - 2048*a*b^7*c^10*d))^(1/4)*2i +
 2*atan(((-d^7/(256*b^8*c^11 + 256*a^8*c^3*d^8 - 2048*a^7*b*c^4*d^7 + 7168*
a^2*b^6*c^9*d^2 - 14336*a^3*b^5*c^8*d^3 + 17920*a^4*b^4*c^7*d^4 - 14336*a^5
*b^3*c^6*d^5 + 7168*a^6*b^2*c^5*d^6 - 2048*a*b^7*c^10*d))^(1/4)*((-d^7/(256
*b^8*c^11 + 256*a^8*c^3*d^8 - 2048*a^7*b*c^4*d^7 + 7168*a^2*b^6*c^9*d^2 - 1
4336*a^3*b^5*c^8*d^3 + 17920*a^4*b^4*c^7*d^4 - 14336*a^5*b^3*c^6*d^5 + 7168
*a^6*b^2*c^5*d^6 - 2048*a*b^7*c^10*d))^(1/4)*((-d^7/(256*b^8*c^11 + 256*a^8
*c^3*d^8 - 2048*a^7*b*c^4*d^7 + 7168*a^2*b^6*c^9*d^2 - 14336*a^3*b^5*c^8*d^
3 + 17920*a^4*b^4*c^7*d^4 - 14336*a^5*b^3*c^6*d^5 + 7168*a^6*b^2*c^5*d^6 -
2048*a*b^7*c^10*d))^(3/4)*(((-d^7/(256*b^8*c^11 + 256*a^8*c^3*d^8 - 2048*a^
7*b*c^4*d^7 + 7168*a^2*b^6*c^9*d^2 - 14336*a^3*b^5*c^8*d^3 + 17920*a^4*b^4*
c^7*d^4 - 14336*a^5*b^3*c^6*d^5 + 7168*a^6*b^2*c^5*d^6 - 2048*a*b^7*c^10*d)
)^(1/4)*(3072*a^4*b^14*c^11*d^4 - 4096*a^14*b^4*c*d^14 - 28672*a^5*b^13*c^1

0*d^5 + 114688*a^6*b^12*c^9*d^6 - 253952*a^7*b^11*c^8*d^7 + 329728*a^8*b^10
*c^7*d^8 - 229376*a^9*b^9*c^6*d^9 + 28672*a^10*b^8*c^5*d^10 + 90112*a^11*b^
7*c^4*d^11 - 78848*a^12*b^6*c^3*d^12 + 28672*a^13*b^5*c^2*d^13))/(a^7*d^3 -
 a^4*b^3*c^3 + 3*a^5*b^2*c^2*d - 3*a^6*b*c*d^2) - (x*(65536*a^15*b^4*d^17 -
 524288*a^14*b^5*c*d^16 + 36864*a^2*b^17*c^13*d^4 - 466944*a^3*b^16*c^12*d^
5 + 2609152*a^4*b^15*c^11*d^6 - 8486912*a^5*b^14*c^10*d^7 + 17833984*a^6*b^
13*c^9*d^8 - 25280512*a^7*b^12*c^8*d^9 + 24190976*a^8*b^11*c^7*d^10 - 14516
224*a^9*b^10*c^6*d^11 + 3362816*a^10*b^9*c^5*d^12 + 2809856*a^11*b^8*c^4*d^
13 - 3469312*a^12*b^7*c^3*d^14 + 1835008*a^13*b^6*c^2*d^15)*1i)/(64*(a^10*d
^6 + a^4*b^6*c^6 - 6*a^5*b^5*c^5*d + 15*a^6*b^4*c^4*d^2 - 20*a^7*b^3*c^3*d^
3 + 15*a^8*b^2*c^2*d^4 - 6*a^9*b*c*d^5)))*1i + ((28*a^4*b^6*d^11 + (81*b^10
*c^4*d^7)/16 - (675*a*b^9*c^3*d^8)/16 - (2145*a^3*b^7*c*d^10)/16 + (1971*a^
2*b^8*c^2*d^9)/16)*1i)/(a^7*d^3 - a^4*b^3*c^3 + 3*a^5*b^2*c^2*d - 3*a^6*b*c
d^2)) + (x(3185*a^4*b^7*d^13 + 81*b^11*c^4*d^9 - 756*a*b^10*c^3*d^10 - 47
88*a^3*b^8*c*d^12 + 2790*a^2*b^9*c^2*d^11))/(64*(a^10*d^6 + a^4*b^6*c^6 - 6
*a^5*b^5*c^5*d + 15*a^6*b^4*c^4*d^2 - 20*a^7*b^3*c^3*d^3 + 15*a^8*b^2*c^2*d
^4 - 6*a^9*b*c*d^5))) - (-d^7/(256*b^8*c^11 + 256*a^8*c^3*d^8 - 2048*a^7*b*
c^4*d^7 + 7168*a^2*b^6*c^9*d^2 - 14336*a^3*b^5*c^8*d^3 + 17920*a^4*b^4*c^7*
d^4 - 14336*a^5*b^3*c^6*d^5 + 7168*a^6*b^2*c^5*d^6 - 2048*a*b^7*c^10*d))^(1
/4)*((-d^7/(256*b^8*c^11 + 256*a^8*c^3*d^8 - 2048*a^7*b*c^4*d^7 + 7168*a^2*
b^6*c^9*d^2 - 14336*a^3*b^5*c^8*d^3 + 17920*a^4*b^4*c^7*d^4 - 14336*a^5*b^3
*c^6*d^5 + 7168*a^6*b^2*c^5*d^6 - 2048*a*b^7*c^10*d))^(1/4)*((-d^7/(256*b^8
*c^11 + 256*a^8*c^3*d^8 - 2048*a^7*b*c^4*d^7 + 7168*a^2*b^6*c^9*d^2 - 14336
*a^3*b^5*c^8*d^3 + 17920*a^4*b^4*c^7*d^4 - 14336*a^5*b^3*c^6*d^5 + 7168*a^6
*b^2*c^5*d^6 - 2048*a*b^7*c^10*d))^(3/4)*(((-d^7/(256*b^8*c^11 + 256*a^8*c^
3*d^8 - 2048*a^7*b*c^4*d^7 + 7168*a^2*b^6*c^9*d^2 - 14336*a^3*b^5*c^8*d^3 +
 17920*a^4*b^4*c^7*d^4 - 14336*a^5*b^3*c^6*d^5 + 7168*a^6*b^2*c^5*d^6 - 204
8*a*b^7*c^10*d))^(1/4)*(3072*a^4*b^14*c^11*d^4 - 4096*a^14*b^4*c*d^14 - 286
72*a^5*b^13*c^10*d^5 + 114688*a^6*b^12*c^9*d^6 - 253952*a^7*b^11*c^8*d^7 +
329728*a^8*b^10*c^7*d^8 - 229376*a^9*b^9*c^6*d^9 + 28672*a^10*b^8*c^5*d^10
+ 90112*a^11*b^7*c^4*d^11 - 78848*a^12*b^6*c^3*d^12 + 28672*a^13*b^5*c^2*d^
13))/(a^7*d^3 - a^4*b^3*c^3 + 3*a^5*b^2*c^2*d - 3*a^6*b*c*d^2) + (x*(65536*
a^15*b^4*d^17 - 524288*a^14*b^5*c*d^16 + 36864*a^2*b^17*c^13*d^4 - 466944*a
^3*b^16*c^12*d^5 + 2609152*a^4*b^15*c^11*d^6 - 8486912*a^5*b^14*c^10*d^7 +
17833984*a^6*b^13*c^9*d^8 - 25280512*a^7*b^12*c^8*d^9 + 24190976*a^8*b^11*c
^7*d^10 - 14516224*a^9*b^10*c^6*d^11 + 3362816*a^10*b^9*c^5*d^12 + 2809856*
a^11*b^8*c^4*d^13 - 3469312*a^12*b^7*c^3*d^14 + 1835008*a^13*b^6*c^2*d^15)*
1i)/(64*(a^10*d^6 + a^4*b^6*c^6 - 6*a^5*b^5*c^5*d + 15*a^6*b^4*c^4*d^2 - 20
*a^7*b^3*c^3*d^3 + 15*a^8*b^2*c^2*d^4 - 6*a^9*b*c*d^5)))*1i + ((28*a^4*b^6*
d^11 + (81*b^10*c^4*d^7)/16 - (675*a*b^9*c^3*d^8)/16 - (2145*a^3*b^7*c*d^10
)/16 + (1971*a^2*b^8*c^2*d^9)/16)*1i)/(a^7*d^3 - a^4*b^3*c^3 + 3*a^5*b^2*c^
2*d - 3*a^6*b*c*d^2)) - (x*(3185*a^4*b^7*d^13 + 81*b^11*c^4*d^9 - 756*a*b^1
0*c^3*d^10 - 4788*a^3*b^8*c*d^12 + 2790*a^2*b^9*c^2*d^11))/(64*(a^10*d^6 +
a^4*b^6*c^6 - 6*a^5*b^5*c^5*d + 15*a^6*b^4*c^4*d^2 - 20*a^7*b^3*c^3*d^3 + 1
5*a^8*b^2*c^2*d^4 - 6*a^9*b*c*d^5))))/((-d^7/(256*b^8*c^11 + 256*a^8*c^3*d^
8 - 2048*a^7*b*c^4*d^7 + 7168*a^2*b^6*c^9*d^2 - 14336*a^3*b^5*c^8*d^3 + 179
20*a^4*b^4*c^7*d^4 - 14336*a^5*b^3*c^6*d^5 + 7168*a^6*b^2*c^5*d^6 - 2048*a*
b^7*c^10*d))^(1/4)*((-d^7/(256*b^8*c^11 + 256*a^8*c^3*d^8 - 2048*a^7*b*c^4*
d^7 + 7168*a^2*b^6*c^9*d^2 - 14336*a^3*b^5*c^8*d^3 + 17920*a^4*b^4*c^7*d^4
- 14336*a^5*b^3*c^6*d^5 + 7168*a^6*b^2*c^5*d^6 - 2048*a*b^7*c^10*d))^(1/4)*
((-d^7/(256*b^8*c^11 + 256*a^8*c^3*d^8 - 2048*a^7*b*c^4*d^7 + 7168*a^2*b^6*
c^9*d^2 - 14336*a^3*b^5*c^8*d^3 + 17920*a^4*b^4*c^7*d^4 - 14336*a^5*b^3*c^6
*d^5 + 7168*a^6*b^2*c^5*d^6 - 2048*a*b^7*c^10*d))^(3/4)*(((-d^7/(256*b^8*c^
11 + 256*a^8*c^3*d^8 - 2048*a^7*b*c^4*d^7 + 7168*a^2*b^6*c^9*d^2 - 14336*a^
3*b^5*c^8*d^3 + 17920*a^4*b^4*c^7*d^4 - 14336*a^5*b^3*c^6*d^5 + 7168*a^6*b^
2*c^5*d^6 - 2048*a*b^7*c^10*d))^(1/4)*(3072*a^4*b^14*c^11*d^4 - 4096*a^14*b
^4*c*d^14 - 28672*a^5*b^13*c^10*d^5 + 114688*a^6*b^12*c^9*d^6 - 253952*a^7*
b^11*c^8*d^7 + 329728*a^8*b^10*c^7*d^8 - 229376*a^9*b^9*c^6*d^9 + 28672*a^1
0*b^8*c^5*d^10 + 90112*a^11*b^7*c^4*d^11 - 78848*a^12*b^6*c^3*d^12 + 28672*

```
a^13*b^5*c^2*d^13))/(a^7*d^3 - a^4*b^3*c^3 + 3*a^5*b^2*c^2*d - 3*a^6*b*c*d^
2) - (x*(65536*a^15*b^4*d^17 - 524288*a^14*b^5*c*d^16 + 36864*a^2*b^17*c^13
*d^4 - 466944*a^3*b^16*c^12*d^5 + 2609152*a^4*b^15*c^11*d^6 - 8486912*a^5*b
^14*c^10*d^7 + 17833984*a^6*b^13*c^9*d^8 - 25280512*a^7*b^12*c^8*d^9 + 2419
0976*a^8*b^11*c^7*d^10 - 14516224*a^9*b^10*c^6*d^11 + 3362816*a^10*b^9*c^5*
d^12 + 2809856*a^11*b^8*c^4*d^13 - 3469312*a^12*b^7*c^3*d^14 + 1835008*a^13
*b^6*c^2*d^15)*1i)/(64*(a^10*d^6 + a^4*b^6*c^6 - 6*a^5*b^5*c^5*d + 15*a^6*b
^4*c^4*d^2 - 20*a^7*b^3*c^3*d^3 + 15*a^8*b^2*c^2*d^4 - 6*a^9*b*c*d^5)))*1i
+ ((28*a^4*b^6*d^11 + (81*b^10*c^4*d^7)/16 - (675*a*b^9*c^3*d^8)/16 - (2145
*a^3*b^7*c*d^10)/16 + (1971*a^2*b^8*c^2*d^9)/16)*1i)/(a^7*d^3 - a^4*b^3*c^3
 + 3*a^5*b^2*c^2*d - 3*a^6*b*c*d^2))*1i + (x*(3185*a^4*b^7*d^13 + 81*b^11*c
^4*d^9 - 756*a*b^10*c^3*d^10 - 4788*a^3*b^8*c*d^12 + 2790*a^2*b^9*c^2*d^11)
*1i)/(64*(a^10*d^6 + a^4*b^6*c^6 - 6*a^5*b^5*c^5*d + 15*a^6*b^4*c^4*d^2 - 2
0*a^7*b^3*c^3*d^3 + 15*a^8*b^2*c^2*d^4 - 6*a^9*b*c*d^5))) + (-d^7/(256*b^8*
c^11 + 256*a^8*c^3*d^8 - 2048*a^7*b*c^4*d^7 + 7168*a^2*b^6*c^9*d^2 - 14336*
a^3*b^5*c^8*d^3 + 17920*a^4*b^4*c^7*d^4 - 14336*a^5*b^3*c^6*d^5 + 7168*a^6*
b^2*c^5*d^6 - 2048*a*b^7*c^10*d))^(1/4)*((-d^7/(256*b^8*c^11 + 256*a^8*c^3*
d^8 - 2048*a^7*b*c^4*d^7 + 7168*a^2*b^6*c^9*d^2 - 14336*a^3*b^5*c^8*d^3 + 1
7920*a^4*b^4*c^7*d^4 - 14336*a^5*b^3*c^6*d^5 + 7168*a^6*b^2*c^5*d^6 - 2048*
a*b^7*c^10*d))^(1/4)*((-d^7/(256*b^8*c^11 + 256*a^8*c^3*d^8 - 2048*a^7*b*c^
4*d^7 + 7168*a^2*b^6*c^9*d^2 - 14336*a^3*b^5*c^8*d^3 + 17920*a^4*b^4*c^7*d^
4 - 14336*a^5*b^3*c^6*d^5 + 7168*a^6*b^2*c^5*d^6 - 2048*a*b^7*c^10*d))^(3/4
)*(((-d^7/(256*b^8*c^11 + 256*a^8*c^3*d^8 - 2048*a^7*b*c^4*d^7 + 7168*a^2*b
^6*c^9*d^2 - 14336*a^3*b^5*c^8*d^3 + 17920*a^4*b^4*c^7*d^4 - 14336*a^5*b^3*
c^6*d^5 + 7168*a^6*b^2*c^5*d^6 - 2048*a*b^7*c^10*d))^(1/4)*(3072*a^4*b^14*c
^11*d^4 - 4096*a^14*b^4*c*d^14 - 28672*a^5*b^13*c^10*d^5 + 114688*a^6*b^12*
c^9*d^6 - 253952*a^7*b^11*c^8*d^7 + 329728*a^8*b^10*c^7*d^8 - 229376*a^9*b^
9*c^6*d^9 + 28672*a^10*b^8*c^5*d^10 + 90112*a^11*b^7*c^4*d^11 - 78848*a^12*
b^6*c^3*d^12 + 28672*a^13*b^5*c^2*d^13))/(a^7*d^3 - a^4*b^3*c^3 + 3*a^5*b^2
*c^2*d - 3*a^6*b*c*d^2) + (x*(65536*a^15*b^4*d^17 - 524288*a^14*b^5*c*d^16
+ 36864*a^2*b^17*c^13*d^4 - 466944*a^3*b^16*c^12*d^5 + 2609152*a^4*b^15*c^1
1*d^6 - 8486912*a^5*b^14*c^10*d^7 + 17833984*a^6*b^13*c^9*d^8 - 25280512*a^
7*b^12*c^8*d^9 + 24190976*a^8*b^11*c^7*d^10 - 14516224*a^9*b^10*c^6*d^11 +
3362816*a^10*b^9*c^5*d^12 + 2809856*a^11*b^8*c^4*d^13 - 3469312*a^12*b^7*c^
3*d^14 + 1835008*a^13*b^6*c^2*d^15)*1i)/(64*(a^10*d^6 + a^4*b^6*c^6 - 6*a^5
*b^5*c^5*d + 15*a^6*b^4*c^4*d^2 - 20*a^7*b^3*c^3*d^3 + 15*a^8*b^2*c^2*d^4 -
 6*a^9*b*c*d^5)))*1i + ((28*a^4*b^6*d^11 + (81*b^10*c^4*d^7)/16 - (675*a*b^
9*c^3*d^8)/16 - (2145*a^3*b^7*c*d^10)/16 + (1971*a^2*b^8*c^2*d^9)/16)*1i)/(
a^7*d^3 - a^4*b^3*c^3 + 3*a^5*b^2*c^2*d - 3*a^6*b*c*d^2))*1i - (x*(3185*a^4
*b^7*d^13 + 81*b^11*c^4*d^9 - 756*a*b^10*c^3*d^10 - 4788*a^3*b^8*c*d^12 + 2
790*a^2*b^9*c^2*d^11)*1i)/(64*(a^10*d^6 + a^4*b^6*c^6 - 6*a^5*b^5*c^5*d + 1
5*a^6*b^4*c^4*d^2 - 20*a^7*b^3*c^3*d^3 + 15*a^8*b^2*c^2*d^4 - 6*a^9*b*c*d^5
)))))*(-d^7/(256*b^8*c^11 + 256*a^8*c^3*d^8 - 2048*a^7*b*c^4*d^7 + 7168*a^2
*b^6*c^9*d^2 - 14336*a^3*b^5*c^8*d^3 + 17920*a^4*b^4*c^7*d^4 - 14336*a^5*b^
3*c^6*d^5 + 7168*a^6*b^2*c^5*d^6 - 2048*a*b^7*c^10*d))^(1/4) - (b*x)/(4*a*(
a + b*x^4)*(a*d - b*c))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**4+a)**2/(d*x**4+c),x)
```

```
[Out] Timed out
```

3.109 $\int \frac{1}{(a+bx^4)^2(c+dx^4)^2}\,dx$

Optimal. Leaf size=596

$$-\frac{b^{7/4}(3bc-11ad)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{16\sqrt{2}a^{7/4}(bc-ad)^3}+\frac{b^{7/4}(3bc-11ad)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{16\sqrt{2}a^{7/4}(bc-ad)^3}-\frac{b^{7/4}(3bc}{}$$

Rubi [A] time = 0.74, antiderivative size = 596, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 9, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.474, Rules used = {414, 527, 522, 211, 1165, 628, 1162, 617, 204}

$$-\frac{b^{7/4}(3bc-11ad)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{16\sqrt{2}a^{7/4}(bc-ad)^3}+\frac{b^{7/4}(3bc-11ad)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{16\sqrt{2}a^{7/4}(bc-ad)^3}-\frac{b^{7/4}(3bc-11ad)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}(bc-ad)^3}+\frac{b^{7/4}(3bc-11ad)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{8\sqrt{2}a^{7/4}(bc-ad)^3}-\frac{d^{7/4}(11bc-3ad)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{c}+\sqrt{d}x^2\right)}{16\sqrt{2}c^{7/4}(bc-ad)^3}+\frac{d^{7/4}(11bc-3ad)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{c}+\sqrt{d}x^2\right)}{16\sqrt{2}c^{7/4}(bc-ad)^3}-\frac{d^{7/4}(11bc-3ad)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{8\sqrt{2}c^{7/4}(bc-ad)^3}+\frac{d^{7/4}(11bc-3ad)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}+1\right)}{8\sqrt{2}c^{7/4}(bc-ad)^3}+\frac{bx}{4a\left(a+bx^4\right)\left(c+dx^4\right)(bc-ad)}+\frac{dx(ad+bc)}{4ac\left(c+dx^4\right)(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^4)^2*(c + d*x^4)^2),x]
```

```
[Out] (d*(b*c + a*d)*x)/(4*a*c*(b*c - a*d)^2*(c + d*x^4)) + (b*x)/(4*a*(b*c - a*d
)*(a + b*x^4)*(c + d*x^4)) - (b^(7/4)*(3*b*c - 11*a*d)*ArcTan[1 - (Sqrt[2]*
b^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*(b*c - a*d)^3) + (b^(7/4)*(3*b*c -
11*a*d)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*(b*c -
a*d)^3) - (d^(7/4)*(11*b*c - 3*a*d)*ArcTan[1 - (Sqrt[2]*d^(1/4)*x)/c^(1/4)]
)/(8*Sqrt[2]*c^(7/4)*(b*c - a*d)^3) + (d^(7/4)*(11*b*c - 3*a*d)*ArcTan[1 +
(Sqrt[2]*d^(1/4)*x)/c^(1/4)])/(8*Sqrt[2]*c^(7/4)*(b*c - a*d)^3) - (b^(7/4)*
(3*b*c - 11*a*d)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(1
6*Sqrt[2]*a^(7/4)*(b*c - a*d)^3) + (b^(7/4)*(3*b*c - 11*a*d)*Log[Sqrt[a] +
Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(16*Sqrt[2]*a^(7/4)*(b*c - a*d)^3
) - (d^(7/4)*(11*b*c - 3*a*d)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqr
t[d]*x^2])/(16*Sqrt[2]*c^(7/4)*(b*c - a*d)^3) + (d^(7/4)*(11*b*c - 3*a*d)*L
og[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2])/(16*Sqrt[2]*c^(7/4)*
(b*c - a*d)^3)
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{1}{\left(a+bx^4\right)^2\left(c+dx^4\right)^2}\,dx = \frac{bx}{4a(bc-ad)\left(a+bx^4\right)\left(c+dx^4\right)} - \frac{\int \frac{-3bc+4ad-7bdx^4}{\left(a+bx^4\right)\left(c+dx^4\right)^2}\,dx}{4a(bc-ad)}$$

$$= \frac{d(bc+ad)x}{4ac(bc-ad)^2\left(c+dx^4\right)} + \frac{bx}{4a(bc-ad)\left(a+bx^4\right)\left(c+dx^4\right)} - \frac{\int \frac{-4\left(3b^2c^2-8abcd+3a^2d^2\right)-\cdots}{\left(a+bx^4\right)\left(c+d\cdots\right)}}{16ac(bc-\cdots)}$$

$$= \frac{d(bc+ad)x}{4ac(bc-ad)^2\left(c+dx^4\right)} + \frac{bx}{4a(bc-ad)\left(a+bx^4\right)\left(c+dx^4\right)} + \frac{\left(b^2(3bc-11ad)\right)\int \frac{1}{a+\cdots}}{4a(bc-ad)^3}\cdots$$

$$= \frac{d(bc+ad)x}{4ac(bc-ad)^2\left(c+dx^4\right)} + \frac{bx}{4a(bc-ad)\left(a+bx^4\right)\left(c+dx^4\right)} + \frac{\left(b^2(3bc-11ad)\right)\int \frac{\sqrt{a}\cdots}{a\cdots}}{8a^{3/2}(bc-ad)\cdots}$$

$$= \frac{d(bc+ad)x}{4ac(bc-ad)^2\left(c+dx^4\right)} + \frac{bx}{4a(bc-ad)\left(a+bx^4\right)\left(c+dx^4\right)} + \frac{\left(b^{3/2}(3bc-11ad)\right)\int \cdots}{16a^{3/2}(bc-\cdots)}$$

$$= \frac{d(bc+ad)x}{4ac(bc-ad)^2\left(c+dx^4\right)} + \frac{bx}{4a(bc-ad)\left(a+bx^4\right)\left(c+dx^4\right)} - \frac{b^{7/4}(3bc-11ad)\log\left(\cdots\right.}{16\sqrt{2}\,a\cdots}$$

$$= \frac{d(bc+ad)x}{4ac(bc-ad)^2\left(c+dx^4\right)} + \frac{bx}{4a(bc-ad)\left(a+bx^4\right)\left(c+dx^4\right)} - \frac{b^{7/4}(3bc-11ad)\tan^{-1}\cdots}{8\sqrt{2}\,a^{7/4}(bc-\cdots)}$$

Mathematica [A] time = 6.19, size = 629, normalized size = 1.06

$$\frac{b^{7/4}(11ad-3bc)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{16\sqrt{2}a^{7/4}(bc-ad)^3} - \frac{b^{7/4}(11ad-3bc)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{16\sqrt{2}a^{7/4}(bc-ad)^3} - \frac{b^{7/4}(11ad-3bc)\tan^{-1}\left(\frac{2\sqrt[4]{b}x-\sqrt{2}\sqrt[4]{a}}{\sqrt{2}\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}(bc-ad)^3} - \frac{b^{7/4}(11ad-3bc)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}+2\sqrt[4]{b}x}{\sqrt{2}\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}(bc-ad)^3} + \frac{b^2x}{4a\left(a+bx^4\right)(ad-bc)^2} + \frac{d^{7/4}(11bc-3ad)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{c}+\sqrt{d}x^2\right)}{16\sqrt{2}c^{7/4}(ad-bc)^3} - \frac{d^{7/4}(11bc-3ad)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{c}+\sqrt{d}x^2\right)}{16\sqrt{2}c^{7/4}(ad-bc)^3} - \frac{d^{7/4}(11bc-3ad)\tan^{-1}\left(\frac{2\sqrt[4]{d}x-\sqrt{2}\sqrt[4]{c}}{\sqrt{2}\sqrt[4]{c}}\right)}{8\sqrt{2}c^{7/4}(ad-bc)^3} - \frac{d^{7/4}(11bc-3ad)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}+2\sqrt[4]{d}x}{\sqrt{2}\sqrt[4]{c}}\right)}{8\sqrt{2}c^{7/4}(ad-bc)^3} + \frac{d^2x}{4c\left(c+dx^4\right)(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x^4)^2*(c + d*x^4)^2),x]
```

```
[Out] (b^2*x)/(4*a*(-(b*c) + a*d)^2*(a + b*x^4)) + (d^2*x)/(4*c*(b*c - a*d)^2*(c
+ d*x^4)) - (b^(7/4)*(-3*b*c + 11*a*d)*ArcTan[(-(Sqrt[2]*a^(1/4)) + 2*b^(1/
4)*x)/(Sqrt[2]*a^(1/4))])/(8*Sqrt[2]*a^(7/4)*(b*c - a*d)^3) - (b^(7/4)*(-3*
b*c + 11*a*d)*ArcTan[(Sqrt[2]*a^(1/4) + 2*b^(1/4)*x)/(Sqrt[2]*a^(1/4))])/(8
*Sqrt[2]*a^(7/4)*(b*c - a*d)^3) - (d^(7/4)*(11*b*c - 3*a*d)*ArcTan[(-(Sqrt[
2]*c^(1/4)) + 2*d^(1/4)*x)/(Sqrt[2]*c^(1/4))])/(8*Sqrt[2]*c^(7/4)*(-(b*c) +
 a*d)^3) - (d^(7/4)*(11*b*c - 3*a*d)*ArcTan[(Sqrt[2]*c^(1/4) + 2*d^(1/4)*x)
/(Sqrt[2]*c^(1/4))])/(8*Sqrt[2]*c^(7/4)*(-(b*c) + a*d)^3) + (b^(7/4)*(-3*b*
c + 11*a*d)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(16*Sqr
t[2]*a^(7/4)*(b*c - a*d)^3) - (b^(7/4)*(-3*b*c + 11*a*d)*Log[Sqrt[a] + Sqrt
[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(16*Sqrt[2]*a^(7/4)*(b*c - a*d)^3) +
(d^(7/4)*(11*b*c - 3*a*d)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]
*x^2])/(16*Sqrt[2]*c^(7/4)*(-(b*c) + a*d)^3) - (d^(7/4)*(11*b*c - 3*a*d)*Lo
g[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2])/(16*Sqrt[2]*c^(7/4)*(
-(b*c) + a*d)^3)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+bx^4\right)^2\left(c+dx^4\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/((a + b*x^4)^2*(c + d*x^4)^2),x]
```

```
[Out] IntegrateAlgebraic[1/((a + b*x^4)^2*(c + d*x^4)^2), x]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a)^2/(d*x^4+c)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [B] time = 0.22, size = 967, normalized size = 1.62

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a)^2/(d*x^4+c)^2,x, algorithm="giac")
```

```
[Out] 1/8*(3*(a*b^3)^(1/4)*b^2*c - 11*(a*b^3)^(1/4)*a*b*d)*arctan(1/2*sqrt(2)*(2*
x + sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(sqrt(2)*a^2*b^3*c^3 - 3*sqrt(2)*a^3*
b^2*c^2*d + 3*sqrt(2)*a^4*b*c*d^2 - sqrt(2)*a^5*d^3) + 1/8*(3*(a*b^3)^(1/4)
*b^2*c - 11*(a*b^3)^(1/4)*a*b*d)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/b)^(1
/4))/(a/b)^(1/4))/(sqrt(2)*a^2*b^3*c^3 - 3*sqrt(2)*a^3*b^2*c^2*d + 3*sqrt(2
)*a^4*b*c*d^2 - sqrt(2)*a^5*d^3) + 1/8*(11*(c*d^3)^(1/4)*b*c*d - 3*(c*d^3)^
(1/4)*a*d^2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(c/d)^(1/4))/(c/d)^(1/4))/(s
qrt(2)*b^3*c^5 - 3*sqrt(2)*a*b^2*c^4*d + 3*sqrt(2)*a^2*b*c^3*d^2 - sqrt(2)*
a^3*c^2*d^3) + 1/8*(11*(c*d^3)^(1/4)*b*c*d - 3*(c*d^3)^(1/4)*a*d^2)*arctan(
1/2*sqrt(2)*(2*x - sqrt(2)*(c/d)^(1/4))/(c/d)^(1/4))/(sqrt(2)*b^3*c^5 - 3*s
qrt(2)*a*b^2*c^4*d + 3*sqrt(2)*a^2*b*c^3*d^2 - sqrt(2)*a^3*c^2*d^3) + 1/16*
(3*(a*b^3)^(1/4)*b^2*c - 11*(a*b^3)^(1/4)*a*b*d)*log(x^2 + sqrt(2)*x*(a/b)^
(1/4) + sqrt(a/b))/(sqrt(2)*a^2*b^3*c^3 - 3*sqrt(2)*a^3*b^2*c^2*d + 3*sqrt(
2)*a^4*b*c*d^2 - sqrt(2)*a^5*d^3) - 1/16*(3*(a*b^3)^(1/4)*b^2*c - 11*(a*b^3
)^(1/4)*a*b*d)*log(x^2 - sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(sqrt(2)*a^2*b^
3*c^3 - 3*sqrt(2)*a^3*b^2*c^2*d + 3*sqrt(2)*a^4*b*c*d^2 - sqrt(2)*a^5*d^3)
+ 1/16*(11*(c*d^3)^(1/4)*b*c*d - 3*(c*d^3)^(1/4)*a*d^2)*log(x^2 + sqrt(2)*x
*(c/d)^(1/4) + sqrt(c/d))/(sqrt(2)*b^3*c^5 - 3*sqrt(2)*a*b^2*c^4*d + 3*sqrt
(2)*a^2*b*c^3*d^2 - sqrt(2)*a^3*c^2*d^3) - 1/16*(11*(c*d^3)^(1/4)*b*c*d - 3
*(c*d^3)^(1/4)*a*d^2)*log(x^2 - sqrt(2)*x*(c/d)^(1/4) + sqrt(c/d))/(sqrt(2)
*b^3*c^5 - 3*sqrt(2)*a*b^2*c^4*d + 3*sqrt(2)*a^2*b*c^3*d^2 - sqrt(2)*a^3*c^
2*d^3) + 1/4*(b^2*c*d*x^5 + a*b*d^2*x^5 + b^2*c^2*x + a^2*d^2*x)/((b*d*x^8
+ b*c*x^4 + a*d*x^4 + a*c)*(a*b^2*c^3 - 2*a^2*b*c^2*d + a^3*c*d^2))
```

maple [A] time = 0.07, size = 784, normalized size = 1.32

$$\frac{a d^3 x}{4 (ad-bc)^3 (d x^4+c) c}-\frac{b^3 c x}{4 (ad-bc)^3 (b x^4+a) a}+\frac{b^2 d x}{4 (ad-bc)^3 (b x^4+a)}-\frac{b d^2 x}{4 (ad-bc)^3 (d x^4+c)}+\frac{3 \left(\frac{c}{d}\right)^{\frac{1}{4}} \sqrt{2}\, a d^3 \arctan\left(\frac{\sqrt{2}\, x}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}-1\right)}{16 (ad-bc)^3 c^2}+\frac{3 \left(\frac{c}{d}\right)^{\frac{1}{4}} \sqrt{2}\, a d^3 \arctan\left(\frac{\sqrt{2}\, x}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}+1\right)}{16 (ad-bc)^3 c^2}+\frac{3 \left(\frac{c}{d}\right)^{\frac{1}{4}} \sqrt{2}\, a d^3 \ln\left(\frac{x^2+\left(\frac{c}{d}\right)^{\frac{1}{4}} \sqrt{2}\, x+\sqrt{\frac{c}{d}}}{x^2-\left(\frac{c}{d}\right)^{\frac{1}{4}} \sqrt{2}\, x+\sqrt{\frac{c}{d}}}\right)}{32 (ad-bc)^3 c^2}+\frac{11 \left(\frac{a}{b}\right)^{\frac{1}{4}} \sqrt{2}\, b^2 d \arctan\left(\frac{\sqrt{2}\, x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{16 (ad-bc)^3 a}+\frac{11 \left(\frac{a}{b}\right)^{\frac{1}{4}} \sqrt{2}\, b^2 d \arctan\left(\frac{\sqrt{2}\, x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{16 (ad-bc)^3 a}+\frac{11 \left(\frac{a}{b}\right)^{\frac{1}{4}} \sqrt{2}\, b^2 d \ln\left(\frac{x^2+\left(\frac{a}{b}\right)^{\frac{1}{4}} \sqrt{2}\, x+\sqrt{\frac{a}{b}}}{x^2-\left(\frac{a}{b}\right)^{\frac{1}{4}} \sqrt{2}\, x+\sqrt{\frac{a}{b}}}\right)}{32 (ad-bc)^3 a}-\frac{3 \left(\frac{a}{b}\right)^{\frac{1}{4}} \sqrt{2}\, b^3 c \arctan\left(\frac{\sqrt{2}\, x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{16 (ad-bc)^3 a^2}-\frac{3 \left(\frac{a}{b}\right)^{\frac{1}{4}} \sqrt{2}\, b^3 c \arctan\left(\frac{\sqrt{2}\, x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{16 (ad-bc)^3 a^2}-\frac{3 \left(\frac{a}{b}\right)^{\frac{1}{4}} \sqrt{2}\, b^3 c \ln\left(\frac{x^2+\left(\frac{a}{b}\right)^{\frac{1}{4}} \sqrt{2}\, x+\sqrt{\frac{a}{b}}}{x^2-\left(\frac{a}{b}\right)^{\frac{1}{4}} \sqrt{2}\, x+\sqrt{\frac{a}{b}}}\right)}{32 (ad-bc)^3 a^2}-\frac{11 \left(\frac{c}{d}\right)^{\frac{1}{4}} \sqrt{2}\, b d^2 \arctan\left(\frac{\sqrt{2}\, x}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}-1\right)}{16 (ad-bc)^3 c}-\frac{11 \left(\frac{c}{d}\right)^{\frac{1}{4}} \sqrt{2}\, b d^2 \arctan\left(\frac{\sqrt{2}\, x}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}+1\right)}{16 (ad-bc)^3 c}-\frac{11 \left(\frac{c}{d}\right)^{\frac{1}{4}} \sqrt{2}\, b d^2 \ln\left(\frac{x^2+\left(\frac{c}{d}\right)^{\frac{1}{4}} \sqrt{2}\, x+\sqrt{\frac{c}{d}}}{x^2-\left(\frac{c}{d}\right)^{\frac{1}{4}} \sqrt{2}\, x+\sqrt{\frac{c}{d}}}\right)}{32 (ad-bc)^3 c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^4+a)^2/(d*x^4+c)^2,x)
```

```
[Out] 1/4*d^3/(a*d-b*c)^3/c*x/(d*x^4+c)*a-1/4*d^2/(a*d-b*c)^3*x/(d*x^4+c)*b+3/16*
d^3/(a*d-b*c)^3/c^2*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x-1)*a-1
1/16*d^2/(a*d-b*c)^3/c*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x-1)*
b+3/32*d^3/(a*d-b*c)^3/c^2*(c/d)^(1/4)*2^(1/2)*ln((x^2+(c/d)^(1/4)*2^(1/2)*
x+(c/d)^(1/2))/(x^2-(c/d)^(1/4)*2^(1/2)*x+(c/d)^(1/2)))*a-11/32*d^2/(a*d-b*
c)^3/c*(c/d)^(1/4)*2^(1/2)*ln((x^2+(c/d)^(1/4)*2^(1/2)*x+(c/d)^(1/2))/(x^2-
(c/d)^(1/4)*2^(1/2)*x+(c/d)^(1/2)))*b+3/16*d^3/(a*d-b*c)^3/c^2*(c/d)^(1/4)*
2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x+1)*a-11/16*d^2/(a*d-b*c)^3/c*(c/d)^(1/
4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x+1)*b+1/4*b^2/(a*d-b*c)^3*x/(b*x^4+a
)*d-1/4*b^3/(a*d-b*c)^3/a*x/(b*x^4+a)*c+11/32*b^2/(a*d-b*c)^3/a*(a/b)^(1/4)
```

*2^(1/2)*ln((x^2+(a/b)^(1/4)*2^(1/2)*x+(a/b)^(1/2))/(x^2-(a/b)^(1/4)*2^(1/2)*x+(a/b)^(1/2)))*d-3/32*b^3/(a*d-b*c)^3/a^2*(a/b)^(1/4)*2^(1/2)*ln((x^2+(a/b)^(1/4)*2^(1/2)*x+(a/b)^(1/2))/(x^2-(a/b)^(1/4)*2^(1/2)*x+(a/b)^(1/2)))*c+11/16*b^2/(a*d-b*c)^3/a*(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x-1)*d-3/16*b^3/(a*d-b*c)^3/a^2*(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x-1)*c+11/16*b^2/(a*d-b*c)^3/a*(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x+1)*d-3/16*b^3/(a*d-b*c)^3/a^2*(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x+1)*c

maxima [A] time = 1.29, size = 670, normalized size = 1.12

$$\frac{\left(\frac{2\sqrt{2}(3bc-11ad)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}+\frac{2\sqrt{2}(3bc-11ad)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}+\frac{\sqrt{2}(3bc-11ad)\log\left(\sqrt{b}x^2+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{1}{4}}}-\frac{\sqrt{2}(3bc-11ad)\log\left(\sqrt{b}x^2-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{1}{4}}}\right)b^2}{32\left(ab^3c^3-3a^2b^2c^2d+3a^3bcd^2-a^4d^3\right)}+\frac{\left(b^2cd+abd^2\right)x^5+\left(b^2c^2+a^2d^2\right)x}{4\left(\left(ab^3c^3d-2a^2b^2c^2d^2+a^3bcd^3\right)x^8+a^2b^2c^4-2a^3bc^3d+a^4c^2d^2+\left(ab^3c^4-a^2b^2c^3d-a^3bc^2d^2+a^4cd^3\right)x^4\right)}+\frac{\frac{2\sqrt{2}\left(11bcd^2-3ad^3\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{d}x+\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{c}\sqrt{\sqrt{c}\sqrt{d}}}+\frac{2\sqrt{2}\left(11bcd^2-3ad^3\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{d}x-\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{c}\sqrt{\sqrt{c}\sqrt{d}}}+\frac{\sqrt{2}\left(11bcd^2-3ad^3\right)\log\left(\sqrt{d}x^2+\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}x+\sqrt{c}\right)}{c^{\frac{3}{4}}d^{\frac{1}{4}}}-\frac{\sqrt{2}\left(11bcd^2-3ad^3\right)\log\left(\sqrt{d}x^2-\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}x+\sqrt{c}\right)}{c^{\frac{3}{4}}d^{\frac{1}{4}}}}{32\left(b^3c^4-3ab^2c^3d+3a^2bc^2d^2-a^3cd^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a)^2/(d*x^4+c)^2,x, algorithm="maxima")
```

```
[Out] 1/32*(2*sqrt(2)*(3*b*c - 11*a*d)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x + sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + 2*sqrt(2)*(3*b*c - 11*a*d)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x - sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + sqrt(2)*(3*b*c - 11*a*d)*log(sqrt(b)*x^2 + sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(3/4)*b^(1/4)) - sqrt(2)*(3*b*c - 11*a*d)*log(sqrt(b)*x^2 - sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(3/4)*b^(1/4)))*b^2/(a*b^3*c^3 - 3*a^2*b^2*c^2*d + 3*a^3*b*c*d^2 - a^4*d^3) + 1/4*((b^2*c*d + a*b*d^2)*x^5 + (b^2*c^2 + a^2*d^2)*x)/((a*b^3*c^3*d - 2*a^2*b^2*c^2*d^2 + a^3*b*c*d^3)*x^8 + a^2*b^2*c^4 - 2*a^3*b*c^3*d + a^4*c^2*d^2 + (a*b^3*c^4 - a^2*b^2*c^3*d - a^3*b*c^2*d^2 + a^4*c*d^3)*x^4) + 1/32*(2*sqrt(2)*(11*b*c*d^2 - 3*a*d^3)*arctan(1/2*sqrt(2)*(2*sqrt(d)*x + sqrt(2)*c^(1/4)*d^(1/4))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(c)*sqrt(sqrt(c)*sqrt(d))) + 2*sqrt(2)*(11*b*c*d^2 - 3*a*d^3)*arctan(1/2*sqrt(2)*(2*sqrt(d)*x - sqrt(2)*c^(1/4)*d^(1/4))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(c)*sqrt(sqrt(c)*sqrt(d))) + sqrt(2)*(11*b*c*d^2 - 3*a*d^3)*log(sqrt(d)*x^2 + sqrt(2)*c^(1/4)*d^(1/4)*x + sqrt(c))/(c^(3/4)*d^(1/4)) - sqrt(2)*(11*b*c*d^2 - 3*a*d^3)*log(sqrt(d)*x^2 - sqrt(2)*c^(1/4)*d^(1/4)*x + sqrt(c))/(c^(3/4)*d^(1/4)))/(b^3*c^4 - 3*a*b^2*c^3*d + 3*a^2*b*c^2*d^2 - a^3*c*d^3)
```

mupad [B] time = 5.62, size = 37266, normalized size = 62.53

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^4)^2*(c + d*x^4)^2),x)
```

```
[Out] ((x*(a^2*d^2 + b^2*c^2))/(4*a*c*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)) + (b*d*x^5*(a*d + b*c))/(4*a*c*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)))/(a*c + x^4*(a*d + b*c) + b*d*x^8) - atan(((-(81*a^4*d^11 + 14641*b^4*c^4*d^7 - 15972*a*b^3*c^3*d^8 + 6534*a^2*b^2*c^2*d^9 - 1188*a^3*b*c*d^10)/(65536*b^12*c^19 + 65536*a^12*c^7*d^12 - 786432*a^11*b*c^8*d^11 + 4325376*a^2*b^10*c^17*d^2 - 14417920*a^3*b^9*c^16*d^3 + 32440320*a^4*b^8*c^15*d^4 - 51904512*a^5*b^7*c^14*d^5 + 60555264*a^6*b^6*c^13*d^6 - 51904512*a^7*b^5*c^12*d^7 + 32440320*a^8*b^4*c^11*d^8 - 14417920*a^9*b^3*c^10*d^9 + 4325376*a^10*b^2*c^9*d^10 - 786432*a*b^11*c^18*d))^(1/4)*((-(81*a^4*d^11 + 14641*b^4*c^4*d^7 - 15972*a*b^3*c^3*d^8 + 6534*a^2*b^2*c^2*d^9 - 1188*a^3*b*c*d^10)/(65536*b^12*c^19 + 65536*a^12*c^7*d^12 - 786432*a^11*b*c^8*d^11 + 4325376*a^2*b^10*c^17*d^2 - 14417920*a^3*b^9*c^16*d^3 + 32440320*a^4*b^8*c^15*d^4 - 51904512*a^5*b^7*c^14*d^5 + 60555264*a^6*b^6*c^13*d^6 - 51904512*a^7*b^5*c^12*d^7 + 32440320*a^8*b^4*c^11*d^8 - 14417920*a^9*b^3*c^10*d^9 + 4325376*a^10*b^2*c^9*d^10 - 786432*a*b^11*c^18*d))^(1/4)*(((891*a^8*b^7*d^15)/64 + (891*b^15*c^8*d^7)/64 - (3105*a*b^14*c^7*d^8)/16 - (3105*a^7*b^8*c*d^14)/16 + (31509*a^2*b^13*c^6*d^9)/32 - (33069*a^3*b^12*c^5*d^10)/16 + (60307*a^4*b^11*c^4*d^11)/32 - (33069*a^5*b^10*c^3*d^12)/16 + (31509*a^6*b^9*c^2*d^13)/32)/(a^4*b^8*c^12 + a^12*c^4*
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d^8 - 8*a^5*b^7*c^11*d - 8*a^11*b*c^5*d^7 + 28*a^6*b^6*c^10*d^2 - 56*a^7*b^
5*c^9*d^3 + 70*a^8*b^4*c^8*d^4 - 56*a^9*b^3*c^7*d^5 + 28*a^10*b^2*c^6*d^6)
+ (-(81*a^4*d^11 + 14641*b^4*c^4*d^7 - 15972*a*b^3*c^3*d^8 + 6534*a^2*b^2*c
^2*d^9 - 1188*a^3*b*c*d^10)/(65536*b^12*c^19 + 65536*a^12*c^7*d^12 - 786432
*a^11*b*c^8*d^11 + 4325376*a^2*b^10*c^17*d^2 - 14417920*a^3*b^9*c^16*d^3 +
32440320*a^4*b^8*c^15*d^4 - 51904512*a^5*b^7*c^14*d^5 + 60555264*a^6*b^6*c^
13*d^6 - 51904512*a^7*b^5*c^12*d^7 + 32440320*a^8*b^4*c^11*d^8 - 14417920*a
^9*b^3*c^10*d^9 + 4325376*a^10*b^2*c^9*d^10 - 786432*a*b^11*c^18*d))^(3/4)*
((x*(589824*a^2*b^23*c^21*d^4 - 11403264*a^3*b^22*c^20*d^5 + 98762752*a^4*b
^21*c^19*d^6 - 510394368*a^5*b^20*c^18*d^7 + 1766916096*a^6*b^19*c^17*d^8 -
 4344840192*a^7*b^18*c^16*d^9 + 7796490240*a^8*b^17*c^15*d^10 - 10168369152
*a^9*b^16*c^14*d^11 + 9007726592*a^10*b^15*c^13*d^12 - 3635478528*a^11*b^14
*c^12*d^13 - 3635478528*a^12*b^13*c^11*d^14 + 9007726592*a^13*b^12*c^10*d^1
5 - 10168369152*a^14*b^11*c^9*d^16 + 7796490240*a^15*b^10*c^8*d^17 - 434484
0192*a^16*b^9*c^7*d^18 + 1766916096*a^17*b^8*c^6*d^19 - 510394368*a^18*b^7*
c^5*d^20 + 98762752*a^19*b^6*c^4*d^21 - 11403264*a^20*b^5*c^3*d^22 + 589824
*a^21*b^4*c^2*d^23))/(1024*(a^4*b^12*c^16 + a^16*c^4*d^12 - 12*a^5*b^11*c^1
5*d - 12*a^15*b*c^5*d^11 + 66*a^6*b^10*c^14*d^2 - 220*a^7*b^9*c^13*d^3 + 49
5*a^8*b^8*c^12*d^4 - 792*a^9*b^7*c^11*d^5 + 924*a^10*b^6*c^10*d^6 - 792*a^1
1*b^5*c^9*d^7 + 495*a^12*b^4*c^8*d^8 - 220*a^13*b^3*c^7*d^9 + 66*a^14*b^2*c
^6*d^10)) + ((-(81*a^4*d^11 + 14641*b^4*c^4*d^7 - 15972*a*b^3*c^3*d^8 + 653
4*a^2*b^2*c^2*d^9 - 1188*a^3*b*c*d^10)/(65536*b^12*c^19 + 65536*a^12*c^7*d^
12 - 786432*a^11*b*c^8*d^11 + 4325376*a^2*b^10*c^17*d^2 - 14417920*a^3*b^9*
c^16*d^3 + 32440320*a^4*b^8*c^15*d^4 - 51904512*a^5*b^7*c^14*d^5 + 60555264
*a^6*b^6*c^13*d^6 - 51904512*a^7*b^5*c^12*d^7 + 32440320*a^8*b^4*c^11*d^8 -
 14417920*a^9*b^3*c^10*d^9 + 4325376*a^10*b^2*c^9*d^10 - 786432*a*b^11*c^18
*d))^(1/4)*(3072*a^4*b^19*c^19*d^4 - 45056*a^5*b^18*c^18*d^5 + 292864*a^6*b
^17*c^17*d^6 - 1115136*a^7*b^16*c^16*d^7 + 2745344*a^8*b^15*c^15*d^8 - 4483
072*a^9*b^14*c^14*d^9 + 4595712*a^10*b^13*c^13*d^10 - 1993728*a^11*b^12*c^1
2*d^11 - 1993728*a^12*b^11*c^11*d^12 + 4595712*a^13*b^10*c^10*d^13 - 448307
2*a^14*b^9*c^9*d^14 + 2745344*a^15*b^8*c^8*d^15 - 1115136*a^16*b^7*c^7*d^16
 + 292864*a^17*b^6*c^6*d^17 - 45056*a^18*b^5*c^5*d^18 + 3072*a^19*b^4*c^4*d
^19))/(a^4*b^8*c^12 + a^12*c^4*d^8 - 8*a^5*b^7*c^11*d - 8*a^11*b*c^5*d^7 +
28*a^6*b^6*c^10*d^2 - 56*a^7*b^5*c^9*d^3 + 70*a^8*b^4*c^8*d^4 - 56*a^9*b^3*
c^7*d^5 + 28*a^10*b^2*c^6*d^6)))*1i + (x*(9801*a^8*b^9*d^17 + 9801*b^17*c^8
*d^9 - 149094*a*b^16*c^7*d^10 - 149094*a^7*b^10*c*d^16 + 1001520*a^2*b^15*c
^6*d^11 - 3484602*a^3*b^14*c^5*d^12 + 5769038*a^4*b^13*c^4*d^13 - 3484602*a
^5*b^12*c^3*d^14 + 1001520*a^6*b^11*c^2*d^15)*1i)/(1024*(a^4*b^12*c^16 + a^
16*c^4*d^12 - 12*a^5*b^11*c^15*d - 12*a^15*b*c^5*d^11 + 66*a^6*b^10*c^14*d^
2 - 220*a^7*b^9*c^13*d^3 + 495*a^8*b^8*c^12*d^4 - 792*a^9*b^7*c^11*d^5 + 92
4*a^10*b^6*c^10*d^6 - 792*a^11*b^5*c^9*d^7 + 495*a^12*b^4*c^8*d^8 - 220*a^1
3*b^3*c^7*d^9 + 66*a^14*b^2*c^6*d^10))) - (-(81*a^4*d^11 + 14641*b^4*c^4*d^
7 - 15972*a*b^3*c^3*d^8 + 6534*a^2*b^2*c^2*d^9 - 1188*a^3*b*c*d^10)/(65536*
b^12*c^19 + 65536*a^12*c^7*d^12 - 786432*a^11*b*c^8*d^11 + 4325376*a^2*b^10
*c^17*d^2 - 14417920*a^3*b^9*c^16*d^3 + 32440320*a^4*b^8*c^15*d^4 - 5190451
2*a^5*b^7*c^14*d^5 + 60555264*a^6*b^6*c^13*d^6 - 51904512*a^7*b^5*c^12*d^7
 + 32440320*a^8*b^4*c^11*d^8 - 14417920*a^9*b^3*c^10*d^9 + 4325376*a^10*b^2*
c^9*d^10 - 786432*a*b^11*c^18*d))^(1/4)*((-(81*a^4*d^11 + 14641*b^4*c^4*d^7
 - 15972*a*b^3*c^3*d^8 + 6534*a^2*b^2*c^2*d^9 - 1188*a^3*b*c*d^10)/(65536*b
^12*c^19 + 65536*a^12*c^7*d^12 - 786432*a^11*b*c^8*d^11 + 4325376*a^2*b^10*
c^17*d^2 - 14417920*a^3*b^9*c^16*d^3 + 32440320*a^4*b^8*c^15*d^4 - 51904512
*a^5*b^7*c^14*d^5 + 60555264*a^6*b^6*c^13*d^6 - 51904512*a^7*b^5*c^12*d^7 +
 32440320*a^8*b^4*c^11*d^8 - 14417920*a^9*b^3*c^10*d^9 + 4325376*a^10*b^2*c
^9*d^10 - 786432*a*b^11*c^18*d))^(1/4)*(((891*a^8*b^7*d^15)/64 + (891*b^15*
c^8*d^7)/64 - (3105*a*b^14*c^7*d^8)/16 - (3105*a^7*b^8*c*d^14)/16 + (31509*
a^2*b^13*c^6*d^9)/32 - (33069*a^3*b^12*c^5*d^10)/16 + (60307*a^4*b^11*c^4*d
^11)/32 - (33069*a^5*b^10*c^3*d^12)/16 + (31509*a^6*b^9*c^2*d^13)/32)/(a^4*
b^8*c^12 + a^12*c^4*d^8 - 8*a^5*b^7*c^11*d - 8*a^11*b*c^5*d^7 + 28*a^6*b^6*
c^10*d^2 - 56*a^7*b^5*c^9*d^3 + 70*a^8*b^4*c^8*d^4 - 56*a^9*b^3*c^7*d^5 + 2
```

8*a^10*b^2*c^6*d^6) - (-(81*a^4*d^11 + 14641*b^4*c^4*d^7 - 15972*a*b^3*c^3*
d^8 + 6534*a^2*b^2*c^2*d^9 - 1188*a^3*b*c*d^10)/(65536*b^12*c^19 + 65536*a^
12*c^7*d^12 - 786432*a^11*b*c^8*d^11 + 4325376*a^2*b^10*c^17*d^2 - 14417920
*a^3*b^9*c^16*d^3 + 32440320*a^4*b^8*c^15*d^4 - 51904512*a^5*b^7*c^14*d^5 +
60555264*a^6*b^6*c^13*d^6 - 51904512*a^7*b^5*c^12*d^7 + 32440320*a^8*b^4*c
^11*d^8 - 14417920*a^9*b^3*c^10*d^9 + 4325376*a^10*b^2*c^9*d^10 - 786432*a*
b^11*c^18*d))^(3/4)*((x*(589824*a^2*b^23*c^21*d^4 - 11403264*a^3*b^22*c^20*
d^5 + 98762752*a^4*b^21*c^19*d^6 - 510394368*a^5*b^20*c^18*d^7 + 1766916096
*a^6*b^19*c^17*d^8 - 4344840192*a^7*b^18*c^16*d^9 + 7796490240*a^8*b^17*c^1
5*d^10 - 10168369152*a^9*b^16*c^14*d^11 + 9007726592*a^10*b^15*c^13*d^12 -
3635478528*a^11*b^14*c^12*d^13 - 3635478528*a^12*b^13*c^11*d^14 + 900772659
2*a^13*b^12*c^10*d^15 - 10168369152*a^14*b^11*c^9*d^16 + 7796490240*a^15*b^
10*c^8*d^17 - 4344840192*a^16*b^9*c^7*d^18 + 1766916096*a^17*b^8*c^6*d^19 -
510394368*a^18*b^7*c^5*d^20 + 98762752*a^19*b^6*c^4*d^21 - 11403264*a^20*b
^5*c^3*d^22 + 589824*a^21*b^4*c^2*d^23))/(1024*(a^4*b^12*c^16 + a^16*c^4*d^
12 - 12*a^5*b^11*c^15*d - 12*a^15*b*c^5*d^11 + 66*a^6*b^10*c^14*d^2 - 220*a
^7*b^9*c^13*d^3 + 495*a^8*b^8*c^12*d^4 - 792*a^9*b^7*c^11*d^5 + 924*a^10*b^
6*c^10*d^6 - 792*a^11*b^5*c^9*d^7 + 495*a^12*b^4*c^8*d^8 - 220*a^13*b^3*c^7
*d^9 + 66*a^14*b^2*c^6*d^10)) - ((-(81*a^4*d^11 + 14641*b^4*c^4*d^7 - 15972
*a*b^3*c^3*d^8 + 6534*a^2*b^2*c^2*d^9 - 1188*a^3*b*c*d^10)/(65536*b^12*c^19
+ 65536*a^12*c^7*d^12 - 786432*a^11*b*c^8*d^11 + 4325376*a^2*b^10*c^17*d^2
- 14417920*a^3*b^9*c^16*d^3 + 32440320*a^4*b^8*c^15*d^4 - 51904512*a^5*b^7
*c^14*d^5 + 60555264*a^6*b^6*c^13*d^6 - 51904512*a^7*b^5*c^12*d^7 + 3244032
0*a^8*b^4*c^11*d^8 - 14417920*a^9*b^3*c^10*d^9 + 4325376*a^10*b^2*c^9*d^10
- 786432*a*b^11*c^18*d))^(1/4)*(3072*a^4*b^19*c^19*d^4 - 45056*a^5*b^18*c^1
8*d^5 + 292864*a^6*b^17*c^17*d^6 - 1115136*a^7*b^16*c^16*d^7 + 2745344*a^8*
b^15*c^15*d^8 - 4483072*a^9*b^14*c^14*d^9 + 4595712*a^10*b^13*c^13*d^10 - 1
993728*a^11*b^12*c^12*d^11 - 1993728*a^12*b^11*c^11*d^12 + 4595712*a^13*b^1
0*c^10*d^13 - 4483072*a^14*b^9*c^9*d^14 + 2745344*a^15*b^8*c^8*d^15 - 11151
36*a^16*b^7*c^7*d^16 + 292864*a^17*b^6*c^6*d^17 - 45056*a^18*b^5*c^5*d^18 +
3072*a^19*b^4*c^4*d^19))/(a^4*b^8*c^12 + a^12*c^4*d^8 - 8*a^5*b^7*c^11*d -
8*a^11*b*c^5*d^7 + 28*a^6*b^6*c^10*d^2 - 56*a^7*b^5*c^9*d^3 + 70*a^8*b^4*c
^8*d^4 - 56*a^9*b^3*c^7*d^5 + 28*a^10*b^2*c^6*d^6)))*1i - (x*(9801*a^8*b^9*
d^17 + 9801*b^17*c^8*d^9 - 149094*a*b^16*c^7*d^10 - 149094*a^7*b^10*c*d^16
+ 1001520*a^2*b^15*c^6*d^11 - 3484602*a^3*b^14*c^5*d^12 + 5769038*a^4*b^13*
c^4*d^13 - 3484602*a^5*b^12*c^3*d^14 + 1001520*a^6*b^11*c^2*d^15)*1i)/(1024
*(a^4*b^12*c^16 + a^16*c^4*d^12 - 12*a^5*b^11*c^15*d - 12*a^15*b*c^5*d^11 +
66*a^6*b^10*c^14*d^2 - 220*a^7*b^9*c^13*d^3 + 495*a^8*b^8*c^12*d^4 - 792*a
^9*b^7*c^11*d^5 + 924*a^10*b^6*c^10*d^6 - 792*a^11*b^5*c^9*d^7 + 495*a^12*b
^4*c^8*d^8 - 220*a^13*b^3*c^7*d^9 + 66*a^14*b^2*c^6*d^10))))/((-(81*a^4*d^1
1 + 14641*b^4*c^4*d^7 - 15972*a*b^3*c^3*d^8 + 6534*a^2*b^2*c^2*d^9 - 1188*a
^3*b*c*d^10)/(65536*b^12*c^19 + 65536*a^12*c^7*d^12 - 786432*a^11*b*c^8*d^1
1 + 4325376*a^2*b^10*c^17*d^2 - 14417920*a^3*b^9*c^16*d^3 + 32440320*a^4*b^
8*c^15*d^4 - 51904512*a^5*b^7*c^14*d^5 + 60555264*a^6*b^6*c^13*d^6 - 519045
12*a^7*b^5*c^12*d^7 + 32440320*a^8*b^4*c^11*d^8 - 14417920*a^9*b^3*c^10*d^9
+ 4325376*a^10*b^2*c^9*d^10 - 786432*a*b^11*c^18*d))^(1/4)*((-(81*a^4*d^11
+ 14641*b^4*c^4*d^7 - 15972*a*b^3*c^3*d^8 + 6534*a^2*b^2*c^2*d^9 - 1188*a^
3*b*c*d^10)/(65536*b^12*c^19 + 65536*a^12*c^7*d^12 - 786432*a^11*b*c^8*d^11
+ 4325376*a^2*b^10*c^17*d^2 - 14417920*a^3*b^9*c^16*d^3 + 32440320*a^4*b^8
*c^15*d^4 - 51904512*a^5*b^7*c^14*d^5 + 60555264*a^6*b^6*c^13*d^6 - 5190451
2*a^7*b^5*c^12*d^7 + 32440320*a^8*b^4*c^11*d^8 - 14417920*a^9*b^3*c^10*d^9
+ 4325376*a^10*b^2*c^9*d^10 - 786432*a*b^11*c^18*d))^(1/4)*(((891*a^8*b^7*d
^15)/64 + (891*b^15*c^8*d^7)/64 - (3105*a*b^14*c^7*d^8)/16 - (3105*a^7*b^8*
c*d^14)/16 + (31509*a^2*b^13*c^6*d^9)/32 - (33069*a^3*b^12*c^5*d^10)/16 + (
60307*a^4*b^11*c^4*d^11)/32 - (33069*a^5*b^10*c^3*d^12)/16 + (31509*a^6*b^9
*c^2*d^13)/32)/(a^4*b^8*c^12 + a^12*c^4*d^8 - 8*a^5*b^7*c^11*d - 8*a^11*b*c
^5*d^7 + 28*a^6*b^6*c^10*d^2 - 56*a^7*b^5*c^9*d^3 + 70*a^8*b^4*c^8*d^4 - 56
*a^9*b^3*c^7*d^5 + 28*a^10*b^2*c^6*d^6) + (-(81*a^4*d^11 + 14641*b^4*c^4*d^
7 - 15972*a*b^3*c^3*d^8 + 6534*a^2*b^2*c^2*d^9 - 1188*a^3*b*c*d^10)/(65536*

b^12*c^19 + 65536*a^12*c^7*d^12 - 786432*a^11*b*c^8*d^11 + 4325376*a^2*b^10
*c^17*d^2 - 14417920*a^3*b^9*c^16*d^3 + 32440320*a^4*b^8*c^15*d^4 - 5190451
2*a^5*b^7*c^14*d^5 + 60555264*a^6*b^6*c^13*d^6 - 51904512*a^7*b^5*c^12*d^7
+ 32440320*a^8*b^4*c^11*d^8 - 14417920*a^9*b^3*c^10*d^9 + 4325376*a^10*b^2*
c^9*d^10 - 786432*a*b^11*c^18*d))^(3/4)*((x*(589824*a^2*b^23*c^21*d^4 - 114
03264*a^3*b^22*c^20*d^5 + 98762752*a^4*b^21*c^19*d^6 - 510394368*a^5*b^20*c
^18*d^7 + 1766916096*a^6*b^19*c^17*d^8 - 4344840192*a^7*b^18*c^16*d^9 + 779
6490240*a^8*b^17*c^15*d^10 - 10168369152*a^9*b^16*c^14*d^11 + 9007726592*a^
10*b^15*c^13*d^12 - 3635478528*a^11*b^14*c^12*d^13 - 3635478528*a^12*b^13*c
^11*d^14 + 9007726592*a^13*b^12*c^10*d^15 - 10168369152*a^14*b^11*c^9*d^16
+ 7796490240*a^15*b^10*c^8*d^17 - 4344840192*a^16*b^9*c^7*d^18 + 1766916096
*a^17*b^8*c^6*d^19 - 510394368*a^18*b^7*c^5*d^20 + 98762752*a^19*b^6*c^4*d^
21 - 11403264*a^20*b^5*c^3*d^22 + 589824*a^21*b^4*c^2*d^23))/(1024*(a^4*b^1
2*c^16 + a^16*c^4*d^12 - 12*a^5*b^11*c^15*d - 12*a^15*b*c^5*d^11 + 66*a^6*b
^10*c^14*d^2 - 220*a^7*b^9*c^13*d^3 + 495*a^8*b^8*c^12*d^4 - 792*a^9*b^7*c^
11*d^5 + 924*a^10*b^6*c^10*d^6 - 792*a^11*b^5*c^9*d^7 + 495*a^12*b^4*c^8*d^
8 - 220*a^13*b^3*c^7*d^9 + 66*a^14*b^2*c^6*d^10)) + ((-(81*a^4*d^11 + 14641
*b^4*c^4*d^7 - 15972*a*b^3*c^3*d^8 + 6534*a^2*b^2*c^2*d^9 - 1188*a^3*b*c*d^
10)/(65536*b^12*c^19 + 65536*a^12*c^7*d^12 - 786432*a^11*b*c^8*d^11 + 43253
76*a^2*b^10*c^17*d^2 - 14417920*a^3*b^9*c^16*d^3 + 32440320*a^4*b^8*c^15*d^
4 - 51904512*a^5*b^7*c^14*d^5 + 60555264*a^6*b^6*c^13*d^6 - 51904512*a^7*b^
5*c^12*d^7 + 32440320*a^8*b^4*c^11*d^8 - 14417920*a^9*b^3*c^10*d^9 + 432537
6*a^10*b^2*c^9*d^10 - 786432*a*b^11*c^18*d))^(1/4)*(3072*a^4*b^19*c^19*d^4
- 45056*a^5*b^18*c^18*d^5 + 292864*a^6*b^17*c^17*d^6 - 1115136*a^7*b^16*c^1
6*d^7 + 2745344*a^8*b^15*c^15*d^8 - 4483072*a^9*b^14*c^14*d^9 + 4595712*a^1
0*b^13*c^13*d^10 - 1993728*a^11*b^12*c^12*d^11 - 1993728*a^12*b^11*c^11*d^1
2 + 4595712*a^13*b^10*c^10*d^13 - 4483072*a^14*b^9*c^9*d^14 + 2745344*a^15*
b^8*c^8*d^15 - 1115136*a^16*b^7*c^7*d^16 + 292864*a^17*b^6*c^6*d^17 - 45056
*a^18*b^5*c^5*d^18 + 3072*a^19*b^4*c^4*d^19))/(a^4*b^8*c^12 + a^12*c^4*d^8
- 8*a^5*b^7*c^11*d - 8*a^11*b*c^5*d^7 + 28*a^6*b^6*c^10*d^2 - 56*a^7*b^5*c^
9*d^3 + 70*a^8*b^4*c^8*d^4 - 56*a^9*b^3*c^7*d^5 + 28*a^10*b^2*c^6*d^6))) +
(x*(9801*a^8*b^9*d^17 + 9801*b^17*c^8*d^9 - 149094*a*b^16*c^7*d^10 - 149094
*a^7*b^10*c*d^16 + 1001520*a^2*b^15*c^6*d^11 - 3484602*a^3*b^14*c^5*d^12 +
5769038*a^4*b^13*c^4*d^13 - 3484602*a^5*b^12*c^3*d^14 + 1001520*a^6*b^11*c^
2*d^15))/(1024*(a^4*b^12*c^16 + a^16*c^4*d^12 - 12*a^5*b^11*c^15*d - 12*a^1
5*b*c^5*d^11 + 66*a^6*b^10*c^14*d^2 - 220*a^7*b^9*c^13*d^3 + 495*a^8*b^8*c^
12*d^4 - 792*a^9*b^7*c^11*d^5 + 924*a^10*b^6*c^10*d^6 - 792*a^11*b^5*c^9*d^
7 + 495*a^12*b^4*c^8*d^8 - 220*a^13*b^3*c^7*d^9 + 66*a^14*b^2*c^6*d^10))) +
(-(81*a^4*d^11 + 14641*b^4*c^4*d^7 - 15972*a*b^3*c^3*d^8 + 6534*a^2*b^2*c^
2*d^9 - 1188*a^3*b*c*d^10)/(65536*b^12*c^19 + 65536*a^12*c^7*d^12 - 786432*
a^11*b*c^8*d^11 + 4325376*a^2*b^10*c^17*d^2 - 14417920*a^3*b^9*c^16*d^3 + 3
2440320*a^4*b^8*c^15*d^4 - 51904512*a^5*b^7*c^14*d^5 + 60555264*a^6*b^6*c^1
3*d^6 - 51904512*a^7*b^5*c^12*d^7 + 32440320*a^8*b^4*c^11*d^8 - 14417920*a^
9*b^3*c^10*d^9 + 4325376*a^10*b^2*c^9*d^10 - 786432*a*b^11*c^18*d))^(1/4)*(
(-(81*a^4*d^11 + 14641*b^4*c^4*d^7 - 15972*a*b^3*c^3*d^8 + 6534*a^2*b^2*c^2
*d^9 - 1188*a^3*b*c*d^10)/(65536*b^12*c^19 + 65536*a^12*c^7*d^12 - 786432*a
^11*b*c^8*d^11 + 4325376*a^2*b^10*c^17*d^2 - 14417920*a^3*b^9*c^16*d^3 + 32
440320*a^4*b^8*c^15*d^4 - 51904512*a^5*b^7*c^14*d^5 + 60555264*a^6*b^6*c^13
*d^6 - 51904512*a^7*b^5*c^12*d^7 + 32440320*a^8*b^4*c^11*d^8 - 14417920*a^9
*b^3*c^10*d^9 + 4325376*a^10*b^2*c^9*d^10 - 786432*a*b^11*c^18*d))^(1/4)*((
(891*a^8*b^7*d^15)/64 + (891*b^15*c^8*d^7)/64 - (3105*a*b^14*c^7*d^8)/16 -
(3105*a^7*b^8*c*d^14)/16 + (31509*a^2*b^13*c^6*d^9)/32 - (33069*a^3*b^12*c^
5*d^10)/16 + (60307*a^4*b^11*c^4*d^11)/32 - (33069*a^5*b^10*c^3*d^12)/16 +
(31509*a^6*b^9*c^2*d^13)/32)/(a^4*b^8*c^12 + a^12*c^4*d^8 - 8*a^5*b^7*c^11*
d - 8*a^11*b*c^5*d^7 + 28*a^6*b^6*c^10*d^2 - 56*a^7*b^5*c^9*d^3 + 70*a^8*b^
4*c^8*d^4 - 56*a^9*b^3*c^7*d^5 + 28*a^10*b^2*c^6*d^6) - (-(81*a^4*d^11 + 14
641*b^4*c^4*d^7 - 15972*a*b^3*c^3*d^8 + 6534*a^2*b^2*c^2*d^9 - 1188*a^3*b*c
*d^10)/(65536*b^12*c^19 + 65536*a^12*c^7*d^12 - 786432*a^11*b*c^8*d^11 + 43
25376*a^2*b^10*c^17*d^2 - 14417920*a^3*b^9*c^16*d^3 + 32440320*a^4*b^8*c^15

*d^4 - 51904512*a^5*b^7*c^14*d^5 + 60555264*a^6*b^6*c^13*d^6 - 51904512*a^7
*b^5*c^12*d^7 + 32440320*a^8*b^4*c^11*d^8 - 14417920*a^9*b^3*c^10*d^9 + 432
5376*a^10*b^2*c^9*d^10 - 786432*a*b^11*c^18*d))^(3/4)*((x*(589824*a^2*b^23*
c^21*d^4 - 11403264*a^3*b^22*c^20*d^5 + 98762752*a^4*b^21*c^19*d^6 - 510394
368*a^5*b^20*c^18*d^7 + 1766916096*a^6*b^19*c^17*d^8 - 4344840192*a^7*b^18*
c^16*d^9 + 7796490240*a^8*b^17*c^15*d^10 - 10168369152*a^9*b^16*c^14*d^11 +
 9007726592*a^10*b^15*c^13*d^12 - 3635478528*a^11*b^14*c^12*d^13 - 36354785
28*a^12*b^13*c^11*d^14 + 9007726592*a^13*b^12*c^10*d^15 - 10168369152*a^14*
b^11*c^9*d^16 + 7796490240*a^15*b^10*c^8*d^17 - 4344840192*a^16*b^9*c^7*d^1
8 + 1766916096*a^17*b^8*c^6*d^19 - 510394368*a^18*b^7*c^5*d^20 + 98762752*a
^19*b^6*c^4*d^21 - 11403264*a^20*b^5*c^3*d^22 + 589824*a^21*b^4*c^2*d^23))/
(1024*(a^4*b^12*c^16 + a^16*c^4*d^12 - 12*a^5*b^11*c^15*d - 12*a^15*b*c^5*d
^11 + 66*a^6*b^10*c^14*d^2 - 220*a^7*b^9*c^13*d^3 + 495*a^8*b^8*c^12*d^4 -
792*a^9*b^7*c^11*d^5 + 924*a^10*b^6*c^10*d^6 - 792*a^11*b^5*c^9*d^7 + 495*a
^12*b^4*c^8*d^8 - 220*a^13*b^3*c^7*d^9 + 66*a^14*b^2*c^6*d^10)) - ((-(81*a^
4*d^11 + 14641*b^4*c^4*d^7 - 15972*a*b^3*c^3*d^8 + 6534*a^2*b^2*c^2*d^9 - 1
188*a^3*b*c*d^10)/(65536*b^12*c^19 + 65536*a^12*c^7*d^12 - 786432*a^11*b*c^
8*d^11 + 4325376*a^2*b^10*c^17*d^2 - 14417920*a^3*b^9*c^16*d^3 + 32440320*a
^4*b^8*c^15*d^4 - 51904512*a^5*b^7*c^14*d^5 + 60555264*a^6*b^6*c^13*d^6 - 5
1904512*a^7*b^5*c^12*d^7 + 32440320*a^8*b^4*c^11*d^8 - 14417920*a^9*b^3*c^1
0*d^9 + 4325376*a^10*b^2*c^9*d^10 - 786432*a*b^11*c^18*d))^(1/4)*(3072*a^4*
b^19*c^19*d^4 - 45056*a^5*b^18*c^18*d^5 + 292864*a^6*b^17*c^17*d^6 - 111513
6*a^7*b^16*c^16*d^7 + 2745344*a^8*b^15*c^15*d^8 - 4483072*a^9*b^14*c^14*d^9
 + 4595712*a^10*b^13*c^13*d^10 - 1993728*a^11*b^12*c^12*d^11 - 1993728*a^12
*b^11*c^11*d^12 + 4595712*a^13*b^10*c^10*d^13 - 4483072*a^14*b^9*c^9*d^14 +
 2745344*a^15*b^8*c^8*d^15 - 1115136*a^16*b^7*c^7*d^16 + 292864*a^17*b^6*c^
6*d^17 - 45056*a^18*b^5*c^5*d^18 + 3072*a^19*b^4*c^4*d^19))/(a^4*b^8*c^12 +
 a^12*c^4*d^8 - 8*a^5*b^7*c^11*d - 8*a^11*b*c^5*d^7 + 28*a^6*b^6*c^10*d^2 -
 56*a^7*b^5*c^9*d^3 + 70*a^8*b^4*c^8*d^4 - 56*a^9*b^3*c^7*d^5 + 28*a^10*b^2
*c^6*d^6))) - (x*(9801*a^8*b^9*d^17 + 9801*b^17*c^8*d^9 - 149094*a*b^16*c^7
*d^10 - 149094*a^7*b^10*c*d^16 + 1001520*a^2*b^15*c^6*d^11 - 3484602*a^3*b^
14*c^5*d^12 + 5769038*a^4*b^13*c^4*d^13 - 3484602*a^5*b^12*c^3*d^14 + 10015
20*a^6*b^11*c^2*d^15))/(1024*(a^4*b^12*c^16 + a^16*c^4*d^12 - 12*a^5*b^11*c
^15*d - 12*a^15*b*c^5*d^11 + 66*a^6*b^10*c^14*d^2 - 220*a^7*b^9*c^13*d^3 +
495*a^8*b^8*c^12*d^4 - 792*a^9*b^7*c^11*d^5 + 924*a^10*b^6*c^10*d^6 - 792*a
^11*b^5*c^9*d^7 + 495*a^12*b^4*c^8*d^8 - 220*a^13*b^3*c^7*d^9 + 66*a^14*b^2
*c^6*d^10)))))*(-(81*a^4*d^11 + 14641*b^4*c^4*d^7 - 15972*a*b^3*c^3*d^8 + 6
534*a^2*b^2*c^2*d^9 - 1188*a^3*b*c*d^10)/(65536*b^12*c^19 + 65536*a^12*c^7*
d^12 - 786432*a^11*b*c^8*d^11 + 4325376*a^2*b^10*c^17*d^2 - 14417920*a^3*b^
9*c^16*d^3 + 32440320*a^4*b^8*c^15*d^4 - 51904512*a^5*b^7*c^14*d^5 + 605552
64*a^6*b^6*c^13*d^6 - 51904512*a^7*b^5*c^12*d^7 + 32440320*a^8*b^4*c^11*d^8
 - 14417920*a^9*b^3*c^10*d^9 + 4325376*a^10*b^2*c^9*d^10 - 786432*a*b^11*c^
18*d))^(1/4)*2i + 2*atan(((-(81*a^4*d^11 + 14641*b^4*c^4*d^7 - 15972*a*b^3*
c^3*d^8 + 6534*a^2*b^2*c^2*d^9 - 1188*a^3*b*c*d^10)/(65536*b^12*c^19 + 6553
6*a^12*c^7*d^12 - 786432*a^11*b*c^8*d^11 + 4325376*a^2*b^10*c^17*d^2 - 1441
7920*a^3*b^9*c^16*d^3 + 32440320*a^4*b^8*c^15*d^4 - 51904512*a^5*b^7*c^14*d
^5 + 60555264*a^6*b^6*c^13*d^6 - 51904512*a^7*b^5*c^12*d^7 + 32440320*a^8*b
^4*c^11*d^8 - 14417920*a^9*b^3*c^10*d^9 + 4325376*a^10*b^2*c^9*d^10 - 78643
2*a*b^11*c^18*d))^(1/4)*((-(81*a^4*d^11 + 14641*b^4*c^4*d^7 - 15972*a*b^3*c
^3*d^8 + 6534*a^2*b^2*c^2*d^9 - 1188*a^3*b*c*d^10)/(65536*b^12*c^19 + 65536
*a^12*c^7*d^12 - 786432*a^11*b*c^8*d^11 + 4325376*a^2*b^10*c^17*d^2 - 14417
920*a^3*b^9*c^16*d^3 + 32440320*a^4*b^8*c^15*d^4 - 51904512*a^5*b^7*c^14*d^
5 + 60555264*a^6*b^6*c^13*d^6 - 51904512*a^7*b^5*c^12*d^7 + 32440320*a^8*b^
4*c^11*d^8 - 14417920*a^9*b^3*c^10*d^9 + 4325376*a^10*b^2*c^9*d^10 - 786432
*a*b^11*c^18*d))^(1/4)*((((891*a^8*b^7*d^15)/64 + (891*b^15*c^8*d^7)/64 - (
3105*a*b^14*c^7*d^8)/16 - (3105*a^7*b^8*c*d^14)/16 + (31509*a^2*b^13*c^6*d^
9)/32 - (33069*a^3*b^12*c^5*d^10)/16 + (60307*a^4*b^11*c^4*d^11)/32 - (3306
9*a^5*b^10*c^3*d^12)/16 + (31509*a^6*b^9*c^2*d^13)/32)*1i)/(a^4*b^8*c^12 +
a^12*c^4*d^8 - 8*a^5*b^7*c^11*d - 8*a^11*b*c^5*d^7 + 28*a^6*b^6*c^10*d^2 -

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56*a^7*b^5*c^9*d^3 + 70*a^8*b^4*c^8*d^4 - 56*a^9*b^3*c^7*d^5 + 28*a^10*b^2*
c^6*d^6) + (-(81*a^4*d^11 + 14641*b^4*c^4*d^7 - 15972*a*b^3*c^3*d^8 + 6534*
a^2*b^2*c^2*d^9 - 1188*a^3*b*c*d^10)/(65536*b^12*c^19 + 65536*a^12*c^7*d^12
 - 786432*a^11*b*c^8*d^11 + 4325376*a^2*b^10*c^17*d^2 - 14417920*a^3*b^9*c^
16*d^3 + 32440320*a^4*b^8*c^15*d^4 - 51904512*a^5*b^7*c^14*d^5 + 60555264*a
^6*b^6*c^13*d^6 - 51904512*a^7*b^5*c^12*d^7 + 32440320*a^8*b^4*c^11*d^8 - 1
4417920*a^9*b^3*c^10*d^9 + 4325376*a^10*b^2*c^9*d^10 - 786432*a*b^11*c^18*d
))^(3/4)*((x*(589824*a^2*b^23*c^21*d^4 - 11403264*a^3*b^22*c^20*d^5 + 98762
752*a^4*b^21*c^19*d^6 - 510394368*a^5*b^20*c^18*d^7 + 1766916096*a^6*b^19*c
^17*d^8 - 4344840192*a^7*b^18*c^16*d^9 + 7796490240*a^8*b^17*c^15*d^10 - 10
168369152*a^9*b^16*c^14*d^11 + 9007726592*a^10*b^15*c^13*d^12 - 3635478528*
a^11*b^14*c^12*d^13 - 3635478528*a^12*b^13*c^11*d^14 + 9007726592*a^13*b^12
*c^10*d^15 - 10168369152*a^14*b^11*c^9*d^16 + 7796490240*a^15*b^10*c^8*d^17
 - 4344840192*a^16*b^9*c^7*d^18 + 1766916096*a^17*b^8*c^6*d^19 - 510394368*
a^18*b^7*c^5*d^20 + 98762752*a^19*b^6*c^4*d^21 - 11403264*a^20*b^5*c^3*d^22
 + 589824*a^21*b^4*c^2*d^23)*1i)/(1024*(a^4*b^12*c^16 + a^16*c^4*d^12 - 12*
a^5*b^11*c^15*d - 12*a^15*b*c^5*d^11 + 66*a^6*b^10*c^14*d^2 - 220*a^7*b^9*c
^13*d^3 + 495*a^8*b^8*c^12*d^4 - 792*a^9*b^7*c^11*d^5 + 924*a^10*b^6*c^10*d
^6 - 792*a^11*b^5*c^9*d^7 + 495*a^12*b^4*c^8*d^8 - 220*a^13*b^3*c^7*d^9 + 6
6*a^14*b^2*c^6*d^10)) + ((-(81*a^4*d^11 + 14641*b^4*c^4*d^7 - 15972*a*b^3*c
^3*d^8 + 6534*a^2*b^2*c^2*d^9 - 1188*a^3*b*c*d^10)/(65536*b^12*c^19 + 65536
*a^12*c^7*d^12 - 786432*a^11*b*c^8*d^11 + 4325376*a^2*b^10*c^17*d^2 - 14417
920*a^3*b^9*c^16*d^3 + 32440320*a^4*b^8*c^15*d^4 - 51904512*a^5*b^7*c^14*d^
5 + 60555264*a^6*b^6*c^13*d^6 - 51904512*a^7*b^5*c^12*d^7 + 32440320*a^8*b^
4*c^11*d^8 - 14417920*a^9*b^3*c^10*d^9 + 4325376*a^10*b^2*c^9*d^10 - 786432
*a*b^11*c^18*d))^(1/4)*(3072*a^4*b^19*c^19*d^4 - 45056*a^5*b^18*c^18*d^5 +
292864*a^6*b^17*c^17*d^6 - 1115136*a^7*b^16*c^16*d^7 + 2745344*a^8*b^15*c^1
5*d^8 - 4483072*a^9*b^14*c^14*d^9 + 4595712*a^10*b^13*c^13*d^10 - 1993728*a
^11*b^12*c^12*d^11 - 1993728*a^12*b^11*c^11*d^12 + 4595712*a^13*b^10*c^10*d
^13 - 4483072*a^14*b^9*c^9*d^14 + 2745344*a^15*b^8*c^8*d^15 - 1115136*a^16*
b^7*c^7*d^16 + 292864*a^17*b^6*c^6*d^17 - 45056*a^18*b^5*c^5*d^18 + 3072*a^
19*b^4*c^4*d^19))/(a^4*b^8*c^12 + a^12*c^4*d^8 - 8*a^5*b^7*c^11*d - 8*a^11*
b*c^5*d^7 + 28*a^6*b^6*c^10*d^2 - 56*a^7*b^5*c^9*d^3 + 70*a^8*b^4*c^8*d^4 -
 56*a^9*b^3*c^7*d^5 + 28*a^10*b^2*c^6*d^6))*1i) - (x*(9801*a^8*b^9*d^17 + 9
801*b^17*c^8*d^9 - 149094*a*b^16*c^7*d^10 - 149094*a^7*b^10*c*d^16 + 100152
0*a^2*b^15*c^6*d^11 - 3484602*a^3*b^14*c^5*d^12 + 5769038*a^4*b^13*c^4*d^13
 - 3484602*a^5*b^12*c^3*d^14 + 1001520*a^6*b^11*c^2*d^15))/(1024*(a^4*b^12*
c^16 + a^16*c^4*d^12 - 12*a^5*b^11*c^15*d - 12*a^15*b*c^5*d^11 + 66*a^6*b^1
0*c^14*d^2 - 220*a^7*b^9*c^13*d^3 + 495*a^8*b^8*c^12*d^4 - 792*a^9*b^7*c^11
*d^5 + 924*a^10*b^6*c^10*d^6 - 792*a^11*b^5*c^9*d^7 + 495*a^12*b^4*c^8*d^8
- 220*a^13*b^3*c^7*d^9 + 66*a^14*b^2*c^6*d^10))) - (-(81*a^4*d^11 + 14641*b
^4*c^4*d^7 - 15972*a*b^3*c^3*d^8 + 6534*a^2*b^2*c^2*d^9 - 1188*a^3*b*c*d^10
)/(65536*b^12*c^19 + 65536*a^12*c^7*d^12 - 786432*a^11*b*c^8*d^11 + 4325376
*a^2*b^10*c^17*d^2 - 14417920*a^3*b^9*c^16*d^3 + 32440320*a^4*b^8*c^15*d^4
- 51904512*a^5*b^7*c^14*d^5 + 60555264*a^6*b^6*c^13*d^6 - 51904512*a^7*b^5*
c^12*d^7 + 32440320*a^8*b^4*c^11*d^8 - 14417920*a^9*b^3*c^10*d^9 + 4325376*
a^10*b^2*c^9*d^10 - 786432*a*b^11*c^18*d))^(1/4)*((-(81*a^4*d^11 + 14641*b^
4*c^4*d^7 - 15972*a*b^3*c^3*d^8 + 6534*a^2*b^2*c^2*d^9 - 1188*a^3*b*c*d^10)
/(65536*b^12*c^19 + 65536*a^12*c^7*d^12 - 786432*a^11*b*c^8*d^11 + 4325376*
a^2*b^10*c^17*d^2 - 14417920*a^3*b^9*c^16*d^3 + 32440320*a^4*b^8*c^15*d^4 -
 51904512*a^5*b^7*c^14*d^5 + 60555264*a^6*b^6*c^13*d^6 - 51904512*a^7*b^5*c
^12*d^7 + 32440320*a^8*b^4*c^11*d^8 - 14417920*a^9*b^3*c^10*d^9 + 4325376*a
^10*b^2*c^9*d^10 - 786432*a*b^11*c^18*d))^(1/4)*((((891*a^8*b^7*d^15)/64 +
(891*b^15*c^8*d^7)/64 - (3105*a*b^14*c^7*d^8)/16 - (3105*a^7*b^8*c*d^14)/16
 + (31509*a^2*b^13*c^6*d^9)/32 - (33069*a^3*b^12*c^5*d^10)/16 + (60307*a^4*
b^11*c^4*d^11)/32 - (33069*a^5*b^10*c^3*d^12)/16 + (31509*a^6*b^9*c^2*d^13)
/32)*1i)/(a^4*b^8*c^12 + a^12*c^4*d^8 - 8*a^5*b^7*c^11*d - 8*a^11*b*c^5*d^7
 + 28*a^6*b^6*c^10*d^2 - 56*a^7*b^5*c^9*d^3 + 70*a^8*b^4*c^8*d^4 - 56*a^9*b
^3*c^7*d^5 + 28*a^10*b^2*c^6*d^6) - (-(81*a^4*d^11 + 14641*b^4*c^4*d^7 - 15
```

972*a*b^3*c^3*d^8 + 6534*a^2*b^2*c^2*d^9 - 1188*a^3*b*c*d^10)/(65536*b^12*c
^19 + 65536*a^12*c^7*d^12 - 786432*a^11*b*c^8*d^11 + 4325376*a^2*b^10*c^17*
d^2 - 14417920*a^3*b^9*c^16*d^3 + 32440320*a^4*b^8*c^15*d^4 - 51904512*a^5*
b^7*c^14*d^5 + 60555264*a^6*b^6*c^13*d^6 - 51904512*a^7*b^5*c^12*d^7 + 3244
0320*a^8*b^4*c^11*d^8 - 14417920*a^9*b^3*c^10*d^9 + 4325376*a^10*b^2*c^9*d^
10 - 786432*a*b^11*c^18*d))^(3/4)*((x*(589824*a^2*b^23*c^21*d^4 - 11403264*
a^3*b^22*c^20*d^5 + 98762752*a^4*b^21*c^19*d^6 - 510394368*a^5*b^20*c^18*d^
7 + 1766916096*a^6*b^19*c^17*d^8 - 4344840192*a^7*b^18*c^16*d^9 + 779649024
0*a^8*b^17*c^15*d^10 - 10168369152*a^9*b^16*c^14*d^11 + 9007726592*a^10*b^1
5*c^13*d^12 - 3635478528*a^11*b^14*c^12*d^13 - 3635478528*a^12*b^13*c^11*d^
14 + 9007726592*a^13*b^12*c^10*d^15 - 10168369152*a^14*b^11*c^9*d^16 + 7796
490240*a^15*b^10*c^8*d^17 - 4344840192*a^16*b^9*c^7*d^18 + 1766916096*a^17*
b^8*c^6*d^19 - 510394368*a^18*b^7*c^5*d^20 + 98762752*a^19*b^6*c^4*d^21 - 1
1403264*a^20*b^5*c^3*d^22 + 589824*a^21*b^4*c^2*d^23)*1i)/(1024*(a^4*b^12*c
^16 + a^16*c^4*d^12 - 12*a^5*b^11*c^15*d - 12*a^15*b*c^5*d^11 + 66*a^6*b^10
*c^14*d^2 - 220*a^7*b^9*c^13*d^3 + 495*a^8*b^8*c^12*d^4 - 792*a^9*b^7*c^11*
d^5 + 924*a^10*b^6*c^10*d^6 - 792*a^11*b^5*c^9*d^7 + 495*a^12*b^4*c^8*d^8 -
220*a^13*b^3*c^7*d^9 + 66*a^14*b^2*c^6*d^10)) - ((-(81*a^4*d^11 + 14641*b^
4*c^4*d^7 - 15972*a*b^3*c^3*d^8 + 6534*a^2*b^2*c^2*d^9 - 1188*a^3*b*c*d^10)
/(65536*b^12*c^19 + 65536*a^12*c^7*d^12 - 786432*a^11*b*c^8*d^11 + 4325376*
a^2*b^10*c^17*d^2 - 14417920*a^3*b^9*c^16*d^3 + 32440320*a^4*b^8*c^15*d^4 -
51904512*a^5*b^7*c^14*d^5 + 60555264*a^6*b^6*c^13*d^6 - 51904512*a^7*b^5*c
^12*d^7 + 32440320*a^8*b^4*c^11*d^8 - 14417920*a^9*b^3*c^10*d^9 + 4325376*a
^10*b^2*c^9*d^10 - 786432*a*b^11*c^18*d))^(1/4)*(3072*a^4*b^19*c^19*d^4 - 4
5056*a^5*b^18*c^18*d^5 + 292864*a^6*b^17*c^17*d^6 - 1115136*a^7*b^16*c^16*d
^7 + 2745344*a^8*b^15*c^15*d^8 - 4483072*a^9*b^14*c^14*d^9 + 4595712*a^10*b
^13*c^13*d^10 - 1993728*a^11*b^12*c^12*d^11 - 1993728*a^12*b^11*c^11*d^12 +
4595712*a^13*b^10*c^10*d^13 - 4483072*a^14*b^9*c^9*d^14 + 2745344*a^15*b^8
*c^8*d^15 - 1115136*a^16*b^7*c^7*d^16 + 292864*a^17*b^6*c^6*d^17 - 45056*a^
18*b^5*c^5*d^18 + 3072*a^19*b^4*c^4*d^19))/(a^4*b^8*c^12 + a^12*c^4*d^8 - 8
*a^5*b^7*c^11*d - 8*a^11*b*c^5*d^7 + 28*a^6*b^6*c^10*d^2 - 56*a^7*b^5*c^9*d
^3 + 70*a^8*b^4*c^8*d^4 - 56*a^9*b^3*c^7*d^5 + 28*a^10*b^2*c^6*d^6))*1i) +
(x*(9801*a^8*b^9*d^17 + 9801*b^17*c^8*d^9 - 149094*a*b^16*c^7*d^10 - 149094
*a^7*b^10*c*d^16 + 1001520*a^2*b^15*c^6*d^11 - 3484602*a^3*b^14*c^5*d^12 +
5769038*a^4*b^13*c^4*d^13 - 3484602*a^5*b^12*c^3*d^14 + 1001520*a^6*b^11*c^
2*d^15))/(1024*(a^4*b^12*c^16 + a^16*c^4*d^12 - 12*a^5*b^11*c^15*d - 12*a^1
5*b*c^5*d^11 + 66*a^6*b^10*c^14*d^2 - 220*a^7*b^9*c^13*d^3 + 495*a^8*b^8*c^
12*d^4 - 792*a^9*b^7*c^11*d^5 + 924*a^10*b^6*c^10*d^6 - 792*a^11*b^5*c^9*d^
7 + 495*a^12*b^4*c^8*d^8 - 220*a^13*b^3*c^7*d^9 + 66*a^14*b^2*c^6*d^10))))/
((-(81*a^4*d^11 + 14641*b^4*c^4*d^7 - 15972*a*b^3*c^3*d^8 + 6534*a^2*b^2*c^
2*d^9 - 1188*a^3*b*c*d^10)/(65536*b^12*c^19 + 65536*a^12*c^7*d^12 - 786432*
a^11*b*c^8*d^11 + 4325376*a^2*b^10*c^17*d^2 - 14417920*a^3*b^9*c^16*d^3 + 3
2440320*a^4*b^8*c^15*d^4 - 51904512*a^5*b^7*c^14*d^5 + 60555264*a^6*b^6*c^1
3*d^6 - 51904512*a^7*b^5*c^12*d^7 + 32440320*a^8*b^4*c^11*d^8 - 14417920*a^
9*b^3*c^10*d^9 + 4325376*a^10*b^2*c^9*d^10 - 786432*a*b^11*c^18*d))^(1/4)*(
(-(81*a^4*d^11 + 14641*b^4*c^4*d^7 - 15972*a*b^3*c^3*d^8 + 6534*a^2*b^2*c^2
*d^9 - 1188*a^3*b*c*d^10)/(65536*b^12*c^19 + 65536*a^12*c^7*d^12 - 786432*a
^11*b*c^8*d^11 + 4325376*a^2*b^10*c^17*d^2 - 14417920*a^3*b^9*c^16*d^3 + 32
440320*a^4*b^8*c^15*d^4 - 51904512*a^5*b^7*c^14*d^5 + 60555264*a^6*b^6*c^13
*d^6 - 51904512*a^7*b^5*c^12*d^7 + 32440320*a^8*b^4*c^11*d^8 - 14417920*a^9
*b^3*c^10*d^9 + 4325376*a^10*b^2*c^9*d^10 - 786432*a*b^11*c^18*d))^(1/4)*((
((891*a^8*b^7*d^15)/64 + (891*b^15*c^8*d^7)/64 - (3105*a*b^14*c^7*d^8)/16 -
(3105*a^7*b^8*c*d^14)/16 + (31509*a^2*b^13*c^6*d^9)/32 - (33069*a^3*b^12*c
^5*d^10)/16 + (60307*a^4*b^11*c^4*d^11)/32 - (33069*a^5*b^10*c^3*d^12)/16 +
(31509*a^6*b^9*c^2*d^13)/32)*1i)/(a^4*b^8*c^12 + a^12*c^4*d^8 - 8*a^5*b^7*
c^11*d - 8*a^11*b*c^5*d^7 + 28*a^6*b^6*c^10*d^2 - 56*a^7*b^5*c^9*d^3 + 70*a
^8*b^4*c^8*d^4 - 56*a^9*b^3*c^7*d^5 + 28*a^10*b^2*c^6*d^6) + (-(81*a^4*d^11
+ 14641*b^4*c^4*d^7 - 15972*a*b^3*c^3*d^8 + 6534*a^2*b^2*c^2*d^9 - 1188*a^
3*b*c*d^10)/(65536*b^12*c^19 + 65536*a^12*c^7*d^12 - 786432*a^11*b*c^8*d^11

+ 4325376*a^2*b^10*c^17*d^2 - 14417920*a^3*b^9*c^16*d^3 + 32440320*a^4*b^8
*c^15*d^4 - 51904512*a^5*b^7*c^14*d^5 + 60555264*a^6*b^6*c^13*d^6 - 5190451
2*a^7*b^5*c^12*d^7 + 32440320*a^8*b^4*c^11*d^8 - 14417920*a^9*b^3*c^10*d^9
+ 4325376*a^10*b^2*c^9*d^10 - 786432*a*b^11*c^18*d))^(3/4)*((x*(589824*a^2*
b^23*c^21*d^4 - 11403264*a^3*b^22*c^20*d^5 + 98762752*a^4*b^21*c^19*d^6 - 5
10394368*a^5*b^20*c^18*d^7 + 1766916096*a^6*b^19*c^17*d^8 - 4344840192*a^7*
b^18*c^16*d^9 + 7796490240*a^8*b^17*c^15*d^10 - 10168369152*a^9*b^16*c^14*d
^11 + 9007726592*a^10*b^15*c^13*d^12 - 3635478528*a^11*b^14*c^12*d^13 - 363
5478528*a^12*b^13*c^11*d^14 + 9007726592*a^13*b^12*c^10*d^15 - 10168369152*
a^14*b^11*c^9*d^16 + 7796490240*a^15*b^10*c^8*d^17 - 4344840192*a^16*b^9*c^
7*d^18 + 1766916096*a^17*b^8*c^6*d^19 - 510394368*a^18*b^7*c^5*d^20 + 98762
752*a^19*b^6*c^4*d^21 - 11403264*a^20*b^5*c^3*d^22 + 589824*a^21*b^4*c^2*d^
23)*1i)/(1024*(a^4*b^12*c^16 + a^16*c^4*d^12 - 12*a^5*b^11*c^15*d - 12*a^15
*b*c^5*d^11 + 66*a^6*b^10*c^14*d^2 - 220*a^7*b^9*c^13*d^3 + 495*a^8*b^8*c^1
2*d^4 - 792*a^9*b^7*c^11*d^5 + 924*a^10*b^6*c^10*d^6 - 792*a^11*b^5*c^9*d^7
+ 495*a^12*b^4*c^8*d^8 - 220*a^13*b^3*c^7*d^9 + 66*a^14*b^2*c^6*d^10)) + (
(-(81*a^4*d^11 + 14641*b^4*c^4*d^7 - 15972*a*b^3*c^3*d^8 + 6534*a^2*b^2*c^2
*d^9 - 1188*a^3*b*c*d^10)/(65536*b^12*c^19 + 65536*a^12*c^7*d^12 - 786432*a
^11*b*c^8*d^11 + 4325376*a^2*b^10*c^17*d^2 - 14417920*a^3*b^9*c^16*d^3 + 32
440320*a^4*b^8*c^15*d^4 - 51904512*a^5*b^7*c^14*d^5 + 60555264*a^6*b^6*c^13
*d^6 - 51904512*a^7*b^5*c^12*d^7 + 32440320*a^8*b^4*c^11*d^8 - 14417920*a^9
*b^3*c^10*d^9 + 4325376*a^10*b^2*c^9*d^10 - 786432*a*b^11*c^18*d))^(1/4)*(3
072*a^4*b^19*c^19*d^4 - 45056*a^5*b^18*c^18*d^5 + 292864*a^6*b^17*c^17*d^6
- 1115136*a^7*b^16*c^16*d^7 + 2745344*a^8*b^15*c^15*d^8 - 4483072*a^9*b^14*
c^14*d^9 + 4595712*a^10*b^13*c^13*d^10 - 1993728*a^11*b^12*c^12*d^11 - 1993
728*a^12*b^11*c^11*d^12 + 4595712*a^13*b^10*c^10*d^13 - 4483072*a^14*b^9*c^
9*d^14 + 2745344*a^15*b^8*c^8*d^15 - 1115136*a^16*b^7*c^7*d^16 + 292864*a^1
7*b^6*c^6*d^17 - 45056*a^18*b^5*c^5*d^18 + 3072*a^19*b^4*c^4*d^19))/(a^4*b^
8*c^12 + a^12*c^4*d^8 - 8*a^5*b^7*c^11*d - 8*a^11*b*c^5*d^7 + 28*a^6*b^6*c^
10*d^2 - 56*a^7*b^5*c^9*d^3 + 70*a^8*b^4*c^8*d^4 - 56*a^9*b^3*c^7*d^5 + 28*
a^10*b^2*c^6*d^6))*1i)*1i - (x*(9801*a^8*b^9*d^17 + 9801*b^17*c^8*d^9 - 149
094*a*b^16*c^7*d^10 - 149094*a^7*b^10*c*d^16 + 1001520*a^2*b^15*c^6*d^11 -
3484602*a^3*b^14*c^5*d^12 + 5769038*a^4*b^13*c^4*d^13 - 3484602*a^5*b^12*c^
3*d^14 + 1001520*a^6*b^11*c^2*d^15)*1i)/(1024*(a^4*b^12*c^16 + a^16*c^4*d^1
2 - 12*a^5*b^11*c^15*d - 12*a^15*b*c^5*d^11 + 66*a^6*b^10*c^14*d^2 - 220*a^
7*b^9*c^13*d^3 + 495*a^8*b^8*c^12*d^4 - 792*a^9*b^7*c^11*d^5 + 924*a^10*b^6
*c^10*d^6 - 792*a^11*b^5*c^9*d^7 + 495*a^12*b^4*c^8*d^8 - 220*a^13*b^3*c^7*
d^9 + 66*a^14*b^2*c^6*d^10))) + (-(81*a^4*d^11 + 14641*b^4*c^4*d^7 - 15972*
a*b^3*c^3*d^8 + 6534*a^2*b^2*c^2*d^9 - 1188*a^3*b*c*d^10)/(65536*b^12*c^19
+ 65536*a^12*c^7*d^12 - 786432*a^11*b*c^8*d^11 + 4325376*a^2*b^10*c^17*d^2
- 14417920*a^3*b^9*c^16*d^3 + 32440320*a^4*b^8*c^15*d^4 - 51904512*a^5*b^7*
c^14*d^5 + 60555264*a^6*b^6*c^13*d^6 - 51904512*a^7*b^5*c^12*d^7 + 32440320
*a^8*b^4*c^11*d^8 - 14417920*a^9*b^3*c^10*d^9 + 4325376*a^10*b^2*c^9*d^10 -
786432*a*b^11*c^18*d))^(1/4)*((-(81*a^4*d^11 + 14641*b^4*c^4*d^7 - 15972*a
*b^3*c^3*d^8 + 6534*a^2*b^2*c^2*d^9 - 1188*a^3*b*c*d^10)/(65536*b^12*c^19 +
65536*a^12*c^7*d^12 - 786432*a^11*b*c^8*d^11 + 4325376*a^2*b^10*c^17*d^2 -
14417920*a^3*b^9*c^16*d^3 + 32440320*a^4*b^8*c^15*d^4 - 51904512*a^5*b^7*c
^14*d^5 + 60555264*a^6*b^6*c^13*d^6 - 51904512*a^7*b^5*c^12*d^7 + 32440320*
a^8*b^4*c^11*d^8 - 14417920*a^9*b^3*c^10*d^9 + 4325376*a^10*b^2*c^9*d^10 -
786432*a*b^11*c^18*d))^(1/4)*((((891*a^8*b^7*d^15)/64 + (891*b^15*c^8*d^7)/
64 - (3105*a*b^14*c^7*d^8)/16 - (3105*a^7*b^8*c*d^14)/16 + (31509*a^2*b^13*
c^6*d^9)/32 - (33069*a^3*b^12*c^5*d^10)/16 + (60307*a^4*b^11*c^4*d^11)/32 -
(33069*a^5*b^10*c^3*d^12)/16 + (31509*a^6*b^9*c^2*d^13)/32)*1i)/(a^4*b^8*c
^12 + a^12*c^4*d^8 - 8*a^5*b^7*c^11*d - 8*a^11*b*c^5*d^7 + 28*a^6*b^6*c^10*
d^2 - 56*a^7*b^5*c^9*d^3 + 70*a^8*b^4*c^8*d^4 - 56*a^9*b^3*c^7*d^5 + 28*a^1
0*b^2*c^6*d^6) - (-(81*a^4*d^11 + 14641*b^4*c^4*d^7 - 15972*a*b^3*c^3*d^8 +
6534*a^2*b^2*c^2*d^9 - 1188*a^3*b*c*d^10)/(65536*b^12*c^19 + 65536*a^12*c^
7*d^12 - 786432*a^11*b*c^8*d^11 + 4325376*a^2*b^10*c^17*d^2 - 14417920*a^3*
b^9*c^16*d^3 + 32440320*a^4*b^8*c^15*d^4 - 51904512*a^5*b^7*c^14*d^5 + 6055

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5264*a^6*b^6*c^13*d^6 - 51904512*a^7*b^5*c^12*d^7 + 32440320*a^8*b^4*c^11*d
^8 - 14417920*a^9*b^3*c^10*d^9 + 4325376*a^10*b^2*c^9*d^10 - 786432*a*b^11*
c^18*d))^(3/4)*((x*(589824*a^2*b^23*c^21*d^4 - 11403264*a^3*b^22*c^20*d^5 +
 98762752*a^4*b^21*c^19*d^6 - 510394368*a^5*b^20*c^18*d^7 + 1766916096*a^6*
b^19*c^17*d^8 - 4344840192*a^7*b^18*c^16*d^9 + 7796490240*a^8*b^17*c^15*d^1
0 - 10168369152*a^9*b^16*c^14*d^11 + 9007726592*a^10*b^15*c^13*d^12 - 36354
78528*a^11*b^14*c^12*d^13 - 3635478528*a^12*b^13*c^11*d^14 + 9007726592*a^1
3*b^12*c^10*d^15 - 10168369152*a^14*b^11*c^9*d^16 + 7796490240*a^15*b^10*c^
8*d^17 - 4344840192*a^16*b^9*c^7*d^18 + 1766916096*a^17*b^8*c^6*d^19 - 5103
94368*a^18*b^7*c^5*d^20 + 98762752*a^19*b^6*c^4*d^21 - 11403264*a^20*b^5*c^
3*d^22 + 589824*a^21*b^4*c^2*d^23)*1i)/(1024*(a^4*b^12*c^16 + a^16*c^4*d^12
 - 12*a^5*b^11*c^15*d - 12*a^15*b*c^5*d^11 + 66*a^6*b^10*c^14*d^2 - 220*a^7
*b^9*c^13*d^3 + 495*a^8*b^8*c^12*d^4 - 792*a^9*b^7*c^11*d^5 + 924*a^10*b^6*
c^10*d^6 - 792*a^11*b^5*c^9*d^7 + 495*a^12*b^4*c^8*d^8 - 220*a^13*b^3*c^7*d
^9 + 66*a^14*b^2*c^6*d^10)) - ((-(81*a^4*d^11 + 14641*b^4*c^4*d^7 - 15972*a
*b^3*c^3*d^8 + 6534*a^2*b^2*c^2*d^9 - 1188*a^3*b*c*d^10)/(65536*b^12*c^19 +
 65536*a^12*c^7*d^12 - 786432*a^11*b*c^8*d^11 + 4325376*a^2*b^10*c^17*d^2 -
 14417920*a^3*b^9*c^16*d^3 + 32440320*a^4*b^8*c^15*d^4 - 51904512*a^5*b^7*c
^14*d^5 + 60555264*a^6*b^6*c^13*d^6 - 51904512*a^7*b^5*c^12*d^7 + 32440320*
a^8*b^4*c^11*d^8 - 14417920*a^9*b^3*c^10*d^9 + 4325376*a^10*b^2*c^9*d^10 -
786432*a*b^11*c^18*d))^(1/4)*(3072*a^4*b^19*c^19*d^4 - 45056*a^5*b^18*c^18*
d^5 + 292864*a^6*b^17*c^17*d^6 - 1115136*a^7*b^16*c^16*d^7 + 2745344*a^8*b^
15*c^15*d^8 - 4483072*a^9*b^14*c^14*d^9 + 4595712*a^10*b^13*c^13*d^10 - 199
3728*a^11*b^12*c^12*d^11 - 1993728*a^12*b^11*c^11*d^12 + 4595712*a^13*b^10*
c^10*d^13 - 4483072*a^14*b^9*c^9*d^14 + 2745344*a^15*b^8*c^8*d^15 - 1115136
*a^16*b^7*c^7*d^16 + 292864*a^17*b^6*c^6*d^17 - 45056*a^18*b^5*c^5*d^18 + 3
072*a^19*b^4*c^4*d^19))/(a^4*b^8*c^12 + a^12*c^4*d^8 - 8*a^5*b^7*c^11*d - 8
*a^11*b*c^5*d^7 + 28*a^6*b^6*c^10*d^2 - 56*a^7*b^5*c^9*d^3 + 70*a^8*b^4*c^8
*d^4 - 56*a^9*b^3*c^7*d^5 + 28*a^10*b^2*c^6*d^6))*1i)*1i + (x*(9801*a^8*b^9
*d^17 + 9801*b^17*c^8*d^9 - 149094*a*b^16*c^7*d^10 - 149094*a^7*b^10*c*d^16
 + 1001520*a^2*b^15*c^6*d^11 - 3484602*a^3*b^14*c^5*d^12 + 5769038*a^4*b^13
*c^4*d^13 - 3484602*a^5*b^12*c^3*d^14 + 1001520*a^6*b^11*c^2*d^15)*1i)/(102
4*(a^4*b^12*c^16 + a^16*c^4*d^12 - 12*a^5*b^11*c^15*d - 12*a^15*b*c^5*d^11
+ 66*a^6*b^10*c^14*d^2 - 220*a^7*b^9*c^13*d^3 + 495*a^8*b^8*c^12*d^4 - 792*
a^9*b^7*c^11*d^5 + 924*a^10*b^6*c^10*d^6 - 792*a^11*b^5*c^9*d^7 + 495*a^12*
b^4*c^8*d^8 - 220*a^13*b^3*c^7*d^9 + 66*a^14*b^2*c^6*d^10)))))*(-(81*a^4*d^
11 + 14641*b^4*c^4*d^7 - 15972*a*b^3*c^3*d^8 + 6534*a^2*b^2*c^2*d^9 - 1188*
a^3*b*c*d^10)/(65536*b^12*c^19 + 65536*a^12*c^7*d^12 - 786432*a^11*b*c^8*d^
11 + 4325376*a^2*b^10*c^17*d^2 - 14417920*a^3*b^9*c^16*d^3 + 32440320*a^4*b
^8*c^15*d^4 - 51904512*a^5*b^7*c^14*d^5 + 60555264*a^6*b^6*c^13*d^6 - 51904
512*a^7*b^5*c^12*d^7 + 32440320*a^8*b^4*c^11*d^8 - 14417920*a^9*b^3*c^10*d^
9 + 4325376*a^10*b^2*c^9*d^10 - 786432*a*b^11*c^18*d))^(1/4) - atan(((-(81*
b^11*c^4 + 14641*a^4*b^7*d^4 - 15972*a^3*b^8*c*d^3 + 6534*a^2*b^9*c^2*d^2 -
 1188*a*b^10*c^3*d)/(65536*a^19*d^12 + 65536*a^7*b^12*c^12 - 786432*a^8*b^1
1*c^11*d + 4325376*a^9*b^10*c^10*d^2 - 14417920*a^10*b^9*c^9*d^3 + 32440320
*a^11*b^8*c^8*d^4 - 51904512*a^12*b^7*c^7*d^5 + 60555264*a^13*b^6*c^6*d^6 -
 51904512*a^14*b^5*c^5*d^7 + 32440320*a^15*b^4*c^4*d^8 - 14417920*a^16*b^3*
c^3*d^9 + 4325376*a^17*b^2*c^2*d^10 - 786432*a^18*b*c*d^11))^(1/4)*((-(81*b
^11*c^4 + 14641*a^4*b^7*d^4 - 15972*a^3*b^8*c*d^3 + 6534*a^2*b^9*c^2*d^2 -
1188*a*b^10*c^3*d)/(65536*a^19*d^12 + 65536*a^7*b^12*c^12 - 786432*a^8*b^11
*c^11*d + 4325376*a^9*b^10*c^10*d^2 - 14417920*a^10*b^9*c^9*d^3 + 32440320*
a^11*b^8*c^8*d^4 - 51904512*a^12*b^7*c^7*d^5 + 60555264*a^13*b^6*c^6*d^6 -
51904512*a^14*b^5*c^5*d^7 + 32440320*a^15*b^4*c^4*d^8 - 14417920*a^16*b^3*c
^3*d^9 + 4325376*a^17*b^2*c^2*d^10 - 786432*a^18*b*c*d^11))^(1/4)*((-(81*b^
11*c^4 + 14641*a^4*b^7*d^4 - 15972*a^3*b^8*c*d^3 + 6534*a^2*b^9*c^2*d^2 - 1
188*a*b^10*c^3*d)/(65536*a^19*d^12 + 65536*a^7*b^12*c^12 - 786432*a^8*b^11*
c^11*d + 4325376*a^9*b^10*c^10*d^2 - 14417920*a^10*b^9*c^9*d^3 + 32440320*a
^11*b^8*c^8*d^4 - 51904512*a^12*b^7*c^7*d^5 + 60555264*a^13*b^6*c^6*d^6 - 5
1904512*a^14*b^5*c^5*d^7 + 32440320*a^15*b^4*c^4*d^8 - 14417920*a^16*b^3*c^
```

```
3*d^9 + 4325376*a^17*b^2*c^2*d^10 - 786432*a^18*b*c*d^11))^(3/4)*((x*(58982
4*a^2*b^23*c^21*d^4 - 11403264*a^3*b^22*c^20*d^5 + 98762752*a^4*b^21*c^19*d
^6 - 510394368*a^5*b^20*c^18*d^7 + 1766916096*a^6*b^19*c^17*d^8 - 434484019
2*a^7*b^18*c^16*d^9 + 7796490240*a^8*b^17*c^15*d^10 - 10168369152*a^9*b^16*
c^14*d^11 + 9007726592*a^10*b^15*c^13*d^12 - 3635478528*a^11*b^14*c^12*d^13
 - 3635478528*a^12*b^13*c^11*d^14 + 9007726592*a^13*b^12*c^10*d^15 - 101683
69152*a^14*b^11*c^9*d^16 + 7796490240*a^15*b^10*c^8*d^17 - 4344840192*a^16*
b^9*c^7*d^18 + 1766916096*a^17*b^8*c^6*d^19 - 510394368*a^18*b^7*c^5*d^20 +
 98762752*a^19*b^6*c^4*d^21 - 11403264*a^20*b^5*c^3*d^22 + 589824*a^21*b^4*
c^2*d^23))/(1024*(a^4*b^12*c^16 + a^16*c^4*d^12 - 12*a^5*b^11*c^15*d - 12*a
^15*b*c^5*d^11 + 66*a^6*b^10*c^14*d^2 - 220*a^7*b^9*c^13*d^3 + 495*a^8*b^8*
c^12*d^4 - 792*a^9*b^7*c^11*d^5 + 924*a^10*b^6*c^10*d^6 - 792*a^11*b^5*c^9*
d^7 + 495*a^12*b^4*c^8*d^8 - 220*a^13*b^3*c^7*d^9 + 66*a^14*b^2*c^6*d^10))
+ ((-(81*b^11*c^4 + 14641*a^4*b^7*d^4 - 15972*a^3*b^8*c*d^3 + 6534*a^2*b^9*
c^2*d^2 - 1188*a*b^10*c^3*d)/(65536*a^19*d^12 + 65536*a^7*b^12*c^12 - 78643
2*a^8*b^11*c^11*d + 4325376*a^9*b^10*c^10*d^2 - 14417920*a^10*b^9*c^9*d^3 +
 32440320*a^11*b^8*c^8*d^4 - 51904512*a^12*b^7*c^7*d^5 + 60555264*a^13*b^6*
c^6*d^6 - 51904512*a^14*b^5*c^5*d^7 + 32440320*a^15*b^4*c^4*d^8 - 14417920*
a^16*b^3*c^3*d^9 + 4325376*a^17*b^2*c^2*d^10 - 786432*a^18*b*c*d^11))^(1/4)
*(3072*a^4*b^19*c^19*d^4 - 45056*a^5*b^18*c^18*d^5 + 292864*a^6*b^17*c^17*d
^6 - 1115136*a^7*b^16*c^16*d^7 + 2745344*a^8*b^15*c^15*d^8 - 4483072*a^9*b^
14*c^14*d^9 + 4595712*a^10*b^13*c^13*d^10 - 1993728*a^11*b^12*c^12*d^11 - 1
993728*a^12*b^11*c^11*d^12 + 4595712*a^13*b^10*c^10*d^13 - 4483072*a^14*b^9
*c^9*d^14 + 2745344*a^15*b^8*c^8*d^15 - 1115136*a^16*b^7*c^7*d^16 + 292864*
a^17*b^6*c^6*d^17 - 45056*a^18*b^5*c^5*d^18 + 3072*a^19*b^4*c^4*d^19))/(a^4
*b^8*c^12 + a^12*c^4*d^8 - 8*a^5*b^7*c^11*d - 8*a^11*b*c^5*d^7 + 28*a^6*b^6
*c^10*d^2 - 56*a^7*b^5*c^9*d^3 + 70*a^8*b^4*c^8*d^4 - 56*a^9*b^3*c^7*d^5 +
28*a^10*b^2*c^6*d^6)) + ((891*a^8*b^7*d^15)/64 + (891*b^15*c^8*d^7)/64 - (3
105*a*b^14*c^7*d^8)/16 - (3105*a^7*b^8*c*d^14)/16 + (31509*a^2*b^13*c^6*d^9
)/32 - (33069*a^3*b^12*c^5*d^10)/16 + (60307*a^4*b^11*c^4*d^11)/32 - (33069
*a^5*b^10*c^3*d^12)/16 + (31509*a^6*b^9*c^2*d^13)/32)/(a^4*b^8*c^12 + a^12*
c^4*d^8 - 8*a^5*b^7*c^11*d - 8*a^11*b*c^5*d^7 + 28*a^6*b^6*c^10*d^2 - 56*a^
7*b^5*c^9*d^3 + 70*a^8*b^4*c^8*d^4 - 56*a^9*b^3*c^7*d^5 + 28*a^10*b^2*c^6*d
^6))*1i + (x*(9801*a^8*b^9*d^17 + 9801*b^17*c^8*d^9 - 149094*a*b^16*c^7*d^1
0 - 149094*a^7*b^10*c*d^16 + 1001520*a^2*b^15*c^6*d^11 - 3484602*a^3*b^14*c
^5*d^12 + 5769038*a^4*b^13*c^4*d^13 - 3484602*a^5*b^12*c^3*d^14 + 1001520*a
^6*b^11*c^2*d^15)*1i)/(1024*(a^4*b^12*c^16 + a^16*c^4*d^12 - 12*a^5*b^11*c^
15*d - 12*a^15*b*c^5*d^11 + 66*a^6*b^10*c^14*d^2 - 220*a^7*b^9*c^13*d^3 + 4
95*a^8*b^8*c^12*d^4 - 792*a^9*b^7*c^11*d^5 + 924*a^10*b^6*c^10*d^6 - 792*a^
11*b^5*c^9*d^7 + 495*a^12*b^4*c^8*d^8 - 220*a^13*b^3*c^7*d^9 + 66*a^14*b^2*
c^6*d^10))) + (-(81*b^11*c^4 + 14641*a^4*b^7*d^4 - 15972*a^3*b^8*c*d^3 + 65
34*a^2*b^9*c^2*d^2 - 1188*a*b^10*c^3*d)/(65536*a^19*d^12 + 65536*a^7*b^12*c
^12 - 786432*a^8*b^11*c^11*d + 4325376*a^9*b^10*c^10*d^2 - 14417920*a^10*b^
9*c^9*d^3 + 32440320*a^11*b^8*c^8*d^4 - 51904512*a^12*b^7*c^7*d^5 + 6055526
4*a^13*b^6*c^6*d^6 - 51904512*a^14*b^5*c^5*d^7 + 32440320*a^15*b^4*c^4*d^8
- 14417920*a^16*b^3*c^3*d^9 + 4325376*a^17*b^2*c^2*d^10 - 786432*a^18*b*c*d
^11))^(1/4)*((-(81*b^11*c^4 + 14641*a^4*b^7*d^4 - 15972*a^3*b^8*c*d^3 + 653
4*a^2*b^9*c^2*d^2 - 1188*a*b^10*c^3*d)/(65536*a^19*d^12 + 65536*a^7*b^12*c^
12 - 786432*a^8*b^11*c^11*d + 4325376*a^9*b^10*c^10*d^2 - 14417920*a^10*b^9
*c^9*d^3 + 32440320*a^11*b^8*c^8*d^4 - 51904512*a^12*b^7*c^7*d^5 + 60555264
*a^13*b^6*c^6*d^6 - 51904512*a^14*b^5*c^5*d^7 + 32440320*a^15*b^4*c^4*d^8 -
 14417920*a^16*b^3*c^3*d^9 + 4325376*a^17*b^2*c^2*d^10 - 786432*a^18*b*c*d^
11))^(1/4)*((-(81*b^11*c^4 + 14641*a^4*b^7*d^4 - 15972*a^3*b^8*c*d^3 + 6534
*a^2*b^9*c^2*d^2 - 1188*a*b^10*c^3*d)/(65536*a^19*d^12 + 65536*a^7*b^12*c^1
2 - 786432*a^8*b^11*c^11*d + 4325376*a^9*b^10*c^10*d^2 - 14417920*a^10*b^9*
c^9*d^3 + 32440320*a^11*b^8*c^8*d^4 - 51904512*a^12*b^7*c^7*d^5 + 60555264*
a^13*b^6*c^6*d^6 - 51904512*a^14*b^5*c^5*d^7 + 32440320*a^15*b^4*c^4*d^8 -
14417920*a^16*b^3*c^3*d^9 + 4325376*a^17*b^2*c^2*d^10 - 786432*a^18*b*c*d^1
1))^(3/4)*((x*(589824*a^2*b^23*c^21*d^4 - 11403264*a^3*b^22*c^20*d^5 + 9876
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2752*a^4*b^21*c^19*d^6 - 510394368*a^5*b^20*c^18*d^7 + 1766916096*a^6*b^19*
c^17*d^8 - 4344840192*a^7*b^18*c^16*d^9 + 7796490240*a^8*b^17*c^15*d^10 - 1
0168369152*a^9*b^16*c^14*d^11 + 9007726592*a^10*b^15*c^13*d^12 - 3635478528
*a^11*b^14*c^12*d^13 - 3635478528*a^12*b^13*c^11*d^14 + 9007726592*a^13*b^1
2*c^10*d^15 - 10168369152*a^14*b^11*c^9*d^16 + 7796490240*a^15*b^10*c^8*d^1
7 - 4344840192*a^16*b^9*c^7*d^18 + 1766916096*a^17*b^8*c^6*d^19 - 510394368
*a^18*b^7*c^5*d^20 + 98762752*a^19*b^6*c^4*d^21 - 11403264*a^20*b^5*c^3*d^2
2 + 589824*a^21*b^4*c^2*d^23))/(1024*(a^4*b^12*c^16 + a^16*c^4*d^12 - 12*a^
5*b^11*c^15*d - 12*a^15*b*c^5*d^11 + 66*a^6*b^10*c^14*d^2 - 220*a^7*b^9*c^1
3*d^3 + 495*a^8*b^8*c^12*d^4 - 792*a^9*b^7*c^11*d^5 + 924*a^10*b^6*c^10*d^6
 - 792*a^11*b^5*c^9*d^7 + 495*a^12*b^4*c^8*d^8 - 220*a^13*b^3*c^7*d^9 + 66*
a^14*b^2*c^6*d^10)) - ((-(81*b^11*c^4 + 14641*a^4*b^7*d^4 - 15972*a^3*b^8*c
*d^3 + 6534*a^2*b^9*c^2*d^2 - 1188*a*b^10*c^3*d)/(65536*a^19*d^12 + 65536*a
^7*b^12*c^12 - 786432*a^8*b^11*c^11*d + 4325376*a^9*b^10*c^10*d^2 - 1441792
0*a^10*b^9*c^9*d^3 + 32440320*a^11*b^8*c^8*d^4 - 51904512*a^12*b^7*c^7*d^5
+ 60555264*a^13*b^6*c^6*d^6 - 51904512*a^14*b^5*c^5*d^7 + 32440320*a^15*b^4
*c^4*d^8 - 14417920*a^16*b^3*c^3*d^9 + 4325376*a^17*b^2*c^2*d^10 - 786432*a
^18*b*c*d^11))^(1/4)*(3072*a^4*b^19*c^19*d^4 - 45056*a^5*b^18*c^18*d^5 + 29
2864*a^6*b^17*c^17*d^6 - 1115136*a^7*b^16*c^16*d^7 + 2745344*a^8*b^15*c^15*
d^8 - 4483072*a^9*b^14*c^14*d^9 + 4595712*a^10*b^13*c^13*d^10 - 1993728*a^1
1*b^12*c^12*d^11 - 1993728*a^12*b^11*c^11*d^12 + 4595712*a^13*b^10*c^10*d^1
3 - 4483072*a^14*b^9*c^9*d^14 + 2745344*a^15*b^8*c^8*d^15 - 1115136*a^16*b^
7*c^7*d^16 + 292864*a^17*b^6*c^6*d^17 - 45056*a^18*b^5*c^5*d^18 + 3072*a^19
*b^4*c^4*d^19))/(a^4*b^8*c^12 + a^12*c^4*d^8 - 8*a^5*b^7*c^11*d - 8*a^11*b*
c^5*d^7 + 28*a^6*b^6*c^10*d^2 - 56*a^7*b^5*c^9*d^3 + 70*a^8*b^4*c^8*d^4 - 5
6*a^9*b^3*c^7*d^5 + 28*a^10*b^2*c^6*d^6)) - ((891*a^8*b^7*d^15)/64 + (891*b
^15*c^8*d^7)/64 - (3105*a*b^14*c^7*d^8)/16 - (3105*a^7*b^8*c*d^14)/16 + (31
509*a^2*b^13*c^6*d^9)/32 - (33069*a^3*b^12*c^5*d^10)/16 + (60307*a^4*b^11*c
^4*d^11)/32 - (33069*a^5*b^10*c^3*d^12)/16 + (31509*a^6*b^9*c^2*d^13)/32)/(
a^4*b^8*c^12 + a^12*c^4*d^8 - 8*a^5*b^7*c^11*d - 8*a^11*b*c^5*d^7 + 28*a^6*
b^6*c^10*d^2 - 56*a^7*b^5*c^9*d^3 + 70*a^8*b^4*c^8*d^4 - 56*a^9*b^3*c^7*d^5
 + 28*a^10*b^2*c^6*d^6))*1i + (x*(9801*a^8*b^9*d^17 + 9801*b^17*c^8*d^9 - 1
49094*a*b^16*c^7*d^10 - 149094*a^7*b^10*c*d^16 + 1001520*a^2*b^15*c^6*d^11
- 3484602*a^3*b^14*c^5*d^12 + 5769038*a^4*b^13*c^4*d^13 - 3484602*a^5*b^12*
c^3*d^14 + 1001520*a^6*b^11*c^2*d^15)*1i)/(1024*(a^4*b^12*c^16 + a^16*c^4*d
^12 - 12*a^5*b^11*c^15*d - 12*a^15*b*c^5*d^11 + 66*a^6*b^10*c^14*d^2 - 220*
a^7*b^9*c^13*d^3 + 495*a^8*b^8*c^12*d^4 - 792*a^9*b^7*c^11*d^5 + 924*a^10*b
^6*c^10*d^6 - 792*a^11*b^5*c^9*d^7 + 495*a^12*b^4*c^8*d^8 - 220*a^13*b^3*c^
7*d^9 + 66*a^14*b^2*c^6*d^10))))/((-(81*b^11*c^4 + 14641*a^4*b^7*d^4 - 1597
2*a^3*b^8*c*d^3 + 6534*a^2*b^9*c^2*d^2 - 1188*a*b^10*c^3*d)/(65536*a^19*d^1
2 + 65536*a^7*b^12*c^12 - 786432*a^8*b^11*c^11*d + 4325376*a^9*b^10*c^10*d^
2 - 14417920*a^10*b^9*c^9*d^3 + 32440320*a^11*b^8*c^8*d^4 - 51904512*a^12*b
^7*c^7*d^5 + 60555264*a^13*b^6*c^6*d^6 - 51904512*a^14*b^5*c^5*d^7 + 324403
20*a^15*b^4*c^4*d^8 - 14417920*a^16*b^3*c^3*d^9 + 4325376*a^17*b^2*c^2*d^10
 - 786432*a^18*b*c*d^11))^(1/4)*((-(81*b^11*c^4 + 14641*a^4*b^7*d^4 - 15972
*a^3*b^8*c*d^3 + 6534*a^2*b^9*c^2*d^2 - 1188*a*b^10*c^3*d)/(65536*a^19*d^12
 + 65536*a^7*b^12*c^12 - 786432*a^8*b^11*c^11*d + 4325376*a^9*b^10*c^10*d^2
 - 14417920*a^10*b^9*c^9*d^3 + 32440320*a^11*b^8*c^8*d^4 - 51904512*a^12*b^
7*c^7*d^5 + 60555264*a^13*b^6*c^6*d^6 - 51904512*a^14*b^5*c^5*d^7 + 3244032
0*a^15*b^4*c^4*d^8 - 14417920*a^16*b^3*c^3*d^9 + 4325376*a^17*b^2*c^2*d^10
- 786432*a^18*b*c*d^11))^(1/4)*((-(81*b^11*c^4 + 14641*a^4*b^7*d^4 - 15972*
a^3*b^8*c*d^3 + 6534*a^2*b^9*c^2*d^2 - 1188*a*b^10*c^3*d)/(65536*a^19*d^12
+ 65536*a^7*b^12*c^12 - 786432*a^8*b^11*c^11*d + 4325376*a^9*b^10*c^10*d^2
- 14417920*a^10*b^9*c^9*d^3 + 32440320*a^11*b^8*c^8*d^4 - 51904512*a^12*b^7
*c^7*d^5 + 60555264*a^13*b^6*c^6*d^6 - 51904512*a^14*b^5*c^5*d^7 + 32440320
*a^15*b^4*c^4*d^8 - 14417920*a^16*b^3*c^3*d^9 + 4325376*a^17*b^2*c^2*d^10 -
 786432*a^18*b*c*d^11))^(3/4)*((x*(589824*a^2*b^23*c^21*d^4 - 11403264*a^3*
b^22*c^20*d^5 + 98762752*a^4*b^21*c^19*d^6 - 510394368*a^5*b^20*c^18*d^7 +
1766916096*a^6*b^19*c^17*d^8 - 4344840192*a^7*b^18*c^16*d^9 + 7796490240*a^

8*b^17*c^15*d^10 - 10168369152*a^9*b^16*c^14*d^11 + 9007726592*a^10*b^15*c^
13*d^12 - 3635478528*a^11*b^14*c^12*d^13 - 3635478528*a^12*b^13*c^11*d^14 +
 9007726592*a^13*b^12*c^10*d^15 - 10168369152*a^14*b^11*c^9*d^16 + 77964902
40*a^15*b^10*c^8*d^17 - 4344840192*a^16*b^9*c^7*d^18 + 1766916096*a^17*b^8*
c^6*d^19 - 510394368*a^18*b^7*c^5*d^20 + 98762752*a^19*b^6*c^4*d^21 - 11403
264*a^20*b^5*c^3*d^22 + 589824*a^21*b^4*c^2*d^23))/(1024*(a^4*b^12*c^16 + a
^16*c^4*d^12 - 12*a^5*b^11*c^15*d - 12*a^15*b*c^5*d^11 + 66*a^6*b^10*c^14*d
^2 - 220*a^7*b^9*c^13*d^3 + 495*a^8*b^8*c^12*d^4 - 792*a^9*b^7*c^11*d^5 + 9
24*a^10*b^6*c^10*d^6 - 792*a^11*b^5*c^9*d^7 + 495*a^12*b^4*c^8*d^8 - 220*a^
13*b^3*c^7*d^9 + 66*a^14*b^2*c^6*d^10)) + ((-(81*b^11*c^4 + 14641*a^4*b^7*d
^4 - 15972*a^3*b^8*c*d^3 + 6534*a^2*b^9*c^2*d^2 - 1188*a*b^10*c^3*d)/(65536
*a^19*d^12 + 65536*a^7*b^12*c^12 - 786432*a^8*b^11*c^11*d + 4325376*a^9*b^1
0*c^10*d^2 - 14417920*a^10*b^9*c^9*d^3 + 32440320*a^11*b^8*c^8*d^4 - 519045
12*a^12*b^7*c^7*d^5 + 60555264*a^13*b^6*c^6*d^6 - 51904512*a^14*b^5*c^5*d^7
 + 32440320*a^15*b^4*c^4*d^8 - 14417920*a^16*b^3*c^3*d^9 + 4325376*a^17*b^2
*c^2*d^10 - 786432*a^18*b*c*d^11))^(1/4)*(3072*a^4*b^19*c^19*d^4 - 45056*a^
5*b^18*c^18*d^5 + 292864*a^6*b^17*c^17*d^6 - 1115136*a^7*b^16*c^16*d^7 + 27
45344*a^8*b^15*c^15*d^8 - 4483072*a^9*b^14*c^14*d^9 + 4595712*a^10*b^13*c^1
3*d^10 - 1993728*a^11*b^12*c^12*d^11 - 1993728*a^12*b^11*c^11*d^12 + 459571
2*a^13*b^10*c^10*d^13 - 4483072*a^14*b^9*c^9*d^14 + 2745344*a^15*b^8*c^8*d^
15 - 1115136*a^16*b^7*c^7*d^16 + 292864*a^17*b^6*c^6*d^17 - 45056*a^18*b^5*
c^5*d^18 + 3072*a^19*b^4*c^4*d^19))/(a^4*b^8*c^12 + a^12*c^4*d^8 - 8*a^5*b^
7*c^11*d - 8*a^11*b*c^5*d^7 + 28*a^6*b^6*c^10*d^2 - 56*a^7*b^5*c^9*d^3 + 70
*a^8*b^4*c^8*d^4 - 56*a^9*b^3*c^7*d^5 + 28*a^10*b^2*c^6*d^6)) + ((891*a^8*b
^7*d^15)/64 + (891*b^15*c^8*d^7)/64 - (3105*a*b^14*c^7*d^8)/16 - (3105*a^7*
b^8*c*d^14)/16 + (31509*a^2*b^13*c^6*d^9)/32 - (33069*a^3*b^12*c^5*d^10)/16
 + (60307*a^4*b^11*c^4*d^11)/32 - (33069*a^5*b^10*c^3*d^12)/16 + (31509*a^6
*b^9*c^2*d^13)/32)/(a^4*b^8*c^12 + a^12*c^4*d^8 - 8*a^5*b^7*c^11*d - 8*a^11
*b*c^5*d^7 + 28*a^6*b^6*c^10*d^2 - 56*a^7*b^5*c^9*d^3 + 70*a^8*b^4*c^8*d^4
 - 56*a^9*b^3*c^7*d^5 + 28*a^10*b^2*c^6*d^6)) + (x*(9801*a^8*b^9*d^17 + 9801
*b^17*c^8*d^9 - 149094*a*b^16*c^7*d^10 - 149094*a^7*b^10*c*d^16 + 1001520*a
^2*b^15*c^6*d^11 - 3484602*a^3*b^14*c^5*d^12 + 5769038*a^4*b^13*c^4*d^13 -
3484602*a^5*b^12*c^3*d^14 + 1001520*a^6*b^11*c^2*d^15))/(1024*(a^4*b^12*c^1
6 + a^16*c^4*d^12 - 12*a^5*b^11*c^15*d - 12*a^15*b*c^5*d^11 + 66*a^6*b^10*c
^14*d^2 - 220*a^7*b^9*c^13*d^3 + 495*a^8*b^8*c^12*d^4 - 792*a^9*b^7*c^11*d^
5 + 924*a^10*b^6*c^10*d^6 - 792*a^11*b^5*c^9*d^7 + 495*a^12*b^4*c^8*d^8 - 2
20*a^13*b^3*c^7*d^9 + 66*a^14*b^2*c^6*d^10))) - (-(81*b^11*c^4 + 14641*a^4*
b^7*d^4 - 15972*a^3*b^8*c*d^3 + 6534*a^2*b^9*c^2*d^2 - 1188*a*b^10*c^3*d)/(
65536*a^19*d^12 + 65536*a^7*b^12*c^12 - 786432*a^8*b^11*c^11*d + 4325376*a^
9*b^10*c^10*d^2 - 14417920*a^10*b^9*c^9*d^3 + 32440320*a^11*b^8*c^8*d^4 - 5
1904512*a^12*b^7*c^7*d^5 + 60555264*a^13*b^6*c^6*d^6 - 51904512*a^14*b^5*c^
5*d^7 + 32440320*a^15*b^4*c^4*d^8 - 14417920*a^16*b^3*c^3*d^9 + 4325376*a^1
7*b^2*c^2*d^10 - 786432*a^18*b*c*d^11))^(1/4)*((-(81*b^11*c^4 + 14641*a^4*b
^7*d^4 - 15972*a^3*b^8*c*d^3 + 6534*a^2*b^9*c^2*d^2 - 1188*a*b^10*c^3*d)/(6
5536*a^19*d^12 + 65536*a^7*b^12*c^12 - 786432*a^8*b^11*c^11*d + 4325376*a^9
*b^10*c^10*d^2 - 14417920*a^10*b^9*c^9*d^3 + 32440320*a^11*b^8*c^8*d^4 - 51
904512*a^12*b^7*c^7*d^5 + 60555264*a^13*b^6*c^6*d^6 - 51904512*a^14*b^5*c^5
*d^7 + 32440320*a^15*b^4*c^4*d^8 - 14417920*a^16*b^3*c^3*d^9 + 4325376*a^17
*b^2*c^2*d^10 - 786432*a^18*b*c*d^11))^(1/4)*((-(81*b^11*c^4 + 14641*a^4*b^
7*d^4 - 15972*a^3*b^8*c*d^3 + 6534*a^2*b^9*c^2*d^2 - 1188*a*b^10*c^3*d)/(65
536*a^19*d^12 + 65536*a^7*b^12*c^12 - 786432*a^8*b^11*c^11*d + 4325376*a^9*
b^10*c^10*d^2 - 14417920*a^10*b^9*c^9*d^3 + 32440320*a^11*b^8*c^8*d^4 - 519
04512*a^12*b^7*c^7*d^5 + 60555264*a^13*b^6*c^6*d^6 - 51904512*a^14*b^5*c^5*
d^7 + 32440320*a^15*b^4*c^4*d^8 - 14417920*a^16*b^3*c^3*d^9 + 4325376*a^17*
b^2*c^2*d^10 - 786432*a^18*b*c*d^11))^(3/4)*((x*(589824*a^2*b^23*c^21*d^4 -
 11403264*a^3*b^22*c^20*d^5 + 98762752*a^4*b^21*c^19*d^6 - 510394368*a^5*b^
20*c^18*d^7 + 1766916096*a^6*b^19*c^17*d^8 - 4344840192*a^7*b^18*c^16*d^9 +
 7796490240*a^8*b^17*c^15*d^10 - 10168369152*a^9*b^16*c^14*d^11 + 900772659
2*a^10*b^15*c^13*d^12 - 3635478528*a^11*b^14*c^12*d^13 - 3635478528*a^12*b^

13*c^11*d^14 + 9007726592*a^13*b^12*c^10*d^15 - 10168369152*a^14*b^11*c^9*d
^16 + 7796490240*a^15*b^10*c^8*d^17 - 4344840192*a^16*b^9*c^7*d^18 + 176691
6096*a^17*b^8*c^6*d^19 - 510394368*a^18*b^7*c^5*d^20 + 98762752*a^19*b^6*c^
4*d^21 - 11403264*a^20*b^5*c^3*d^22 + 589824*a^21*b^4*c^2*d^23))/(1024*(a^4
*b^12*c^16 + a^16*c^4*d^12 - 12*a^5*b^11*c^15*d - 12*a^15*b*c^5*d^11 + 66*a
^6*b^10*c^14*d^2 - 220*a^7*b^9*c^13*d^3 + 495*a^8*b^8*c^12*d^4 - 792*a^9*b^
7*c^11*d^5 + 924*a^10*b^6*c^10*d^6 - 792*a^11*b^5*c^9*d^7 + 495*a^12*b^4*c^
8*d^8 - 220*a^13*b^3*c^7*d^9 + 66*a^14*b^2*c^6*d^10)) - ((-(81*b^11*c^4 + 1
4641*a^4*b^7*d^4 - 15972*a^3*b^8*c*d^3 + 6534*a^2*b^9*c^2*d^2 - 1188*a*b^10
*c^3*d)/(65536*a^19*d^12 + 65536*a^7*b^12*c^12 - 786432*a^8*b^11*c^11*d + 4
325376*a^9*b^10*c^10*d^2 - 14417920*a^10*b^9*c^9*d^3 + 32440320*a^11*b^8*c^
8*d^4 - 51904512*a^12*b^7*c^7*d^5 + 60555264*a^13*b^6*c^6*d^6 - 51904512*a^
14*b^5*c^5*d^7 + 32440320*a^15*b^4*c^4*d^8 - 14417920*a^16*b^3*c^3*d^9 + 43
25376*a^17*b^2*c^2*d^10 - 786432*a^18*b*c*d^11))^(1/4)*(3072*a^4*b^19*c^19*
d^4 - 45056*a^5*b^18*c^18*d^5 + 292864*a^6*b^17*c^17*d^6 - 1115136*a^7*b^16
*c^16*d^7 + 2745344*a^8*b^15*c^15*d^8 - 4483072*a^9*b^14*c^14*d^9 + 4595712
*a^10*b^13*c^13*d^10 - 1993728*a^11*b^12*c^12*d^11 - 1993728*a^12*b^11*c^11
*d^12 + 4595712*a^13*b^10*c^10*d^13 - 4483072*a^14*b^9*c^9*d^14 + 2745344*a
^15*b^8*c^8*d^15 - 1115136*a^16*b^7*c^7*d^16 + 292864*a^17*b^6*c^6*d^17 - 4
5056*a^18*b^5*c^5*d^18 + 3072*a^19*b^4*c^4*d^19))/(a^4*b^8*c^12 + a^12*c^4*
d^8 - 8*a^5*b^7*c^11*d - 8*a^11*b*c^5*d^7 + 28*a^6*b^6*c^10*d^2 - 56*a^7*b^
5*c^9*d^3 + 70*a^8*b^4*c^8*d^4 - 56*a^9*b^3*c^7*d^5 + 28*a^10*b^2*c^6*d^6))
 - ((891*a^8*b^7*d^15)/64 + (891*b^15*c^8*d^7)/64 - (3105*a*b^14*c^7*d^8)/1
6 - (3105*a^7*b^8*c*d^14)/16 + (31509*a^2*b^13*c^6*d^9)/32 - (33069*a^3*b^1
2*c^5*d^10)/16 + (60307*a^4*b^11*c^4*d^11)/32 - (33069*a^5*b^10*c^3*d^12)/1
6 + (31509*a^6*b^9*c^2*d^13)/32)/(a^4*b^8*c^12 + a^12*c^4*d^8 - 8*a^5*b^7*c
^11*d - 8*a^11*b*c^5*d^7 + 28*a^6*b^6*c^10*d^2 - 56*a^7*b^5*c^9*d^3 + 70*a^
8*b^4*c^8*d^4 - 56*a^9*b^3*c^7*d^5 + 28*a^10*b^2*c^6*d^6)) + (x*(9801*a^8*b
^9*d^17 + 9801*b^17*c^8*d^9 - 149094*a*b^16*c^7*d^10 - 149094*a^7*b^10*c*d^
16 + 1001520*a^2*b^15*c^6*d^11 - 3484602*a^3*b^14*c^5*d^12 + 5769038*a^4*b^
13*c^4*d^13 - 3484602*a^5*b^12*c^3*d^14 + 1001520*a^6*b^11*c^2*d^15))/(1024
*(a^4*b^12*c^16 + a^16*c^4*d^12 - 12*a^5*b^11*c^15*d - 12*a^15*b*c^5*d^11 +
 66*a^6*b^10*c^14*d^2 - 220*a^7*b^9*c^13*d^3 + 495*a^8*b^8*c^12*d^4 - 792*a
^9*b^7*c^11*d^5 + 924*a^10*b^6*c^10*d^6 - 792*a^11*b^5*c^9*d^7 + 495*a^12*b
^4*c^8*d^8 - 220*a^13*b^3*c^7*d^9 + 66*a^14*b^2*c^6*d^10)))))*(-(81*b^11*c^
4 + 14641*a^4*b^7*d^4 - 15972*a^3*b^8*c*d^3 + 6534*a^2*b^9*c^2*d^2 - 1188*a
*b^10*c^3*d)/(65536*a^19*d^12 + 65536*a^7*b^12*c^12 - 786432*a^8*b^11*c^11*
d + 4325376*a^9*b^10*c^10*d^2 - 14417920*a^10*b^9*c^9*d^3 + 32440320*a^11*b
^8*c^8*d^4 - 51904512*a^12*b^7*c^7*d^5 + 60555264*a^13*b^6*c^6*d^6 - 519045
12*a^14*b^5*c^5*d^7 + 32440320*a^15*b^4*c^4*d^8 - 14417920*a^16*b^3*c^3*d^9
 + 4325376*a^17*b^2*c^2*d^10 - 786432*a^18*b*c*d^11))^(1/4)*2i + 2*atan(((-
(81*b^11*c^4 + 14641*a^4*b^7*d^4 - 15972*a^3*b^8*c*d^3 + 6534*a^2*b^9*c^2*d
^2 - 1188*a*b^10*c^3*d)/(65536*a^19*d^12 + 65536*a^7*b^12*c^12 - 786432*a^8
*b^11*c^11*d + 4325376*a^9*b^10*c^10*d^2 - 14417920*a^10*b^9*c^9*d^3 + 3244
0320*a^11*b^8*c^8*d^4 - 51904512*a^12*b^7*c^7*d^5 + 60555264*a^13*b^6*c^6*d
^6 - 51904512*a^14*b^5*c^5*d^7 + 32440320*a^15*b^4*c^4*d^8 - 14417920*a^16*
b^3*c^3*d^9 + 4325376*a^17*b^2*c^2*d^10 - 786432*a^18*b*c*d^11))^(1/4)*((-(
81*b^11*c^4 + 14641*a^4*b^7*d^4 - 15972*a^3*b^8*c*d^3 + 6534*a^2*b^9*c^2*d^
2 - 1188*a*b^10*c^3*d)/(65536*a^19*d^12 + 65536*a^7*b^12*c^12 - 786432*a^8*
b^11*c^11*d + 4325376*a^9*b^10*c^10*d^2 - 14417920*a^10*b^9*c^9*d^3 + 32440
320*a^11*b^8*c^8*d^4 - 51904512*a^12*b^7*c^7*d^5 + 60555264*a^13*b^6*c^6*d^
6 - 51904512*a^14*b^5*c^5*d^7 + 32440320*a^15*b^4*c^4*d^8 - 14417920*a^16*b
^3*c^3*d^9 + 4325376*a^17*b^2*c^2*d^10 - 786432*a^18*b*c*d^11))^(1/4)*((-(8
1*b^11*c^4 + 14641*a^4*b^7*d^4 - 15972*a^3*b^8*c*d^3 + 6534*a^2*b^9*c^2*d^2
 - 1188*a*b^10*c^3*d)/(65536*a^19*d^12 + 65536*a^7*b^12*c^12 - 786432*a^8*b
^11*c^11*d + 4325376*a^9*b^10*c^10*d^2 - 14417920*a^10*b^9*c^9*d^3 + 324403
20*a^11*b^8*c^8*d^4 - 51904512*a^12*b^7*c^7*d^5 + 60555264*a^13*b^6*c^6*d^6
 - 51904512*a^14*b^5*c^5*d^7 + 32440320*a^15*b^4*c^4*d^8 - 14417920*a^16*b^
3*c^3*d^9 + 4325376*a^17*b^2*c^2*d^10 - 786432*a^18*b*c*d^11))^(3/4)*((x*(5

89824*a^2*b^23*c^21*d^4 - 11403264*a^3*b^22*c^20*d^5 + 98762752*a^4*b^21*c^19*d^6 - 510394368*a^5*b^20*c^18*d^7 + 1766916096*a^6*b^19*c^17*d^8 - 4344840192*a^7*b^18*c^16*d^9 + 7796490240*a^8*b^17*c^15*d^10 - 10168369152*a^9*b^16*c^14*d^11 + 9007726592*a^10*b^15*c^13*d^12 - 3635478528*a^11*b^14*c^12*d^13 - 3635478528*a^12*b^13*c^11*d^14 + 9007726592*a^13*b^12*c^10*d^15 - 10168369152*a^14*b^11*c^9*d^16 + 7796490240*a^15*b^10*c^8*d^17 - 4344840192*a^16*b^9*c^7*d^18 + 1766916096*a^17*b^8*c^6*d^19 - 510394368*a^18*b^7*c^5*d^20 + 98762752*a^19*b^6*c^4*d^21 - 11403264*a^20*b^5*c^3*d^22 + 589824*a^21*b^4*c^2*d^23)*1i)/(1024*(a^4*b^12*c^16 + a^16*c^4*d^12 - 12*a^5*b^11*c^15*d - 12*a^15*b*c^5*d^11 + 66*a^6*b^10*c^14*d^2 - 220*a^7*b^9*c^13*d^3 + 495*a^8*b^8*c^12*d^4 - 792*a^9*b^7*c^11*d^5 + 924*a^10*b^6*c^10*d^6 - 792*a^11*b^5*c^9*d^7 + 495*a^12*b^4*c^8*d^8 - 220*a^13*b^3*c^7*d^9 + 66*a^14*b^2*c^6*d^10)) + ((-(81*b^11*c^4 + 14641*a^4*b^7*d^4 - 15972*a^3*b^8*c*d^3 + 6534*a^2*b^9*c^2*d^2 - 1188*a*b^10*c^3*d)/(65536*a^19*d^12 + 65536*a^7*b^12*c^12 - 786432*a^8*b^11*c^11*d + 4325376*a^9*b^10*c^10*d^2 - 14417920*a^10*b^9*c^9*d^3 + 32440320*a^11*b^8*c^8*d^4 - 51904512*a^12*b^7*c^7*d^5 + 60555264*a^13*b^6*c^6*d^6 - 51904512*a^14*b^5*c^5*d^7 + 32440320*a^15*b^4*c^4*d^8 - 14417920*a^16*b^3*c^3*d^9 + 4325376*a^17*b^2*c^2*d^10 - 786432*a^18*b*c*d^11))^(1/4)*(3072*a^4*b^19*c^19*d^4 - 45056*a^5*b^18*c^18*d^5 + 292864*a^6*b^17*c^17*d^6 - 1115136*a^7*b^16*c^16*d^7 + 2745344*a^8*b^15*c^15*d^8 - 4483072*a^9*b^14*c^14*d^9 + 4595712*a^10*b^13*c^13*d^10 - 1993728*a^11*b^12*c^12*d^11 - 1993728*a^12*b^11*c^11*d^12 + 4595712*a^13*b^10*c^10*d^13 - 4483072*a^14*b^9*c^9*d^14 + 2745344*a^15*b^8*c^8*d^15 - 1115136*a^16*b^7*c^7*d^16 + 292864*a^17*b^6*c^6*d^17 - 45056*a^18*b^5*c^5*d^18 + 3072*a^19*b^4*c^4*d^19))/(a^4*b^8*c^12 + a^12*c^4*d^8 - 8*a^5*b^7*c^11*d - 8*a^11*b*c^5*d^7 + 28*a^6*b^6*c^10*d^2 - 56*a^7*b^5*c^9*d^3 + 70*a^8*b^4*c^8*d^4 - 56*a^9*b^3*c^7*d^5 + 28*a^10*b^2*c^6*d^6))*1i + (((891*a^8*b^7*d^15)/64 + (891*b^15*c^8*d^7)/64 - (3105*a*b^14*c^7*d^8)/16 - (3105*a^7*b^8*c*d^14)/16 + (31509*a^2*b^13*c^6*d^9)/32 - (33069*a^3*b^12*c^5*d^10)/16 + (60307*a^4*b^11*c^4*d^11)/32 - (33069*a^5*b^10*c^3*d^12)/16 + (31509*a^6*b^9*c^2*d^13)/32)*1i)/(a^4*b^8*c^12 + a^12*c^4*d^8 - 8*a^5*b^7*c^11*d - 8*a^11*b*c^5*d^7 + 28*a^6*b^6*c^10*d^2 - 56*a^7*b^5*c^9*d^3 + 70*a^8*b^4*c^8*d^4 - 56*a^9*b^3*c^7*d^5 + 28*a^10*b^2*c^6*d^6)) - (x*(9801*a^8*b^9*d^17 + 9801*b^17*c^8*d^9 - 149094*a*b^16*c^7*d^10 - 149094*a^7*b^10*c*d^16 + 1001520*a^2*b^15*c^6*d^11 - 3484602*a^3*b^14*c^5*d^12 + 5769038*a^4*b^13*c^4*d^13 - 3484602*a^5*b^12*c^3*d^14 + 1001520*a^6*b^11*c^2*d^15))/(1024*(a^4*b^12*c^16 + a^16*c^4*d^12 - 12*a^5*b^11*c^15*d - 12*a^15*b*c^5*d^11 + 66*a^6*b^10*c^14*d^2 - 220*a^7*b^9*c^13*d^3 + 495*a^8*b^8*c^12*d^4 - 792*a^9*b^7*c^11*d^5 + 924*a^10*b^6*c^10*d^6 - 792*a^11*b^5*c^9*d^7 + 495*a^12*b^4*c^8*d^8 - 220*a^13*b^3*c^7*d^9 + 66*a^14*b^2*c^6*d^10))) + (-(81*b^11*c^4 + 14641*a^4*b^7*d^4 - 15972*a^3*b^8*c*d^3 + 6534*a^2*b^9*c^2*d^2 - 1188*a*b^10*c^3*d)/(65536*a^19*d^12 + 65536*a^7*b^12*c^12 - 786432*a^8*b^11*c^11*d + 4325376*a^9*b^10*c^10*d^2 - 14417920*a^10*b^9*c^9*d^3 + 32440320*a^11*b^8*c^8*d^4 - 51904512*a^12*b^7*c^7*d^5 + 60555264*a^13*b^6*c^6*d^6 - 51904512*a^14*b^5*c^5*d^7 + 32440320*a^15*b^4*c^4*d^8 - 14417920*a^16*b^3*c^3*d^9 + 4325376*a^17*b^2*c^2*d^10 - 786432*a^18*b*c*d^11))^(1/4)*((-(81*b^11*c^4 + 14641*a^4*b^7*d^4 - 15972*a^3*b^8*c*d^3 + 6534*a^2*b^9*c^2*d^2 - 1188*a*b^10*c^3*d)/(65536*a^19*d^12 + 65536*a^7*b^12*c^12 - 786432*a^8*b^11*c^11*d + 4325376*a^9*b^10*c^10*d^2 - 14417920*a^10*b^9*c^9*d^3 + 32440320*a^11*b^8*c^8*d^4 - 51904512*a^12*b^7*c^7*d^5 + 60555264*a^13*b^6*c^6*d^6 - 51904512*a^14*b^5*c^5*d^7 + 32440320*a^15*b^4*c^4*d^8 - 14417920*a^16*b^3*c^3*d^9 + 4325376*a^17*b^2*c^2*d^10 - 786432*a^18*b*c*d^11))^(1/4)*((-(81*b^11*c^4 + 14641*a^4*b^7*d^4 - 15972*a^3*b^8*c*d^3 + 6534*a^2*b^9*c^2*d^2 - 1188*a*b^10*c^3*d)/(65536*a^19*d^12 + 65536*a^7*b^12*c^12 - 786432*a^8*b^11*c^11*d + 4325376*a^9*b^10*c^10*d^2 - 14417920*a^10*b^9*c^9*d^3 + 32440320*a^11*b^8*c^8*d^4 - 51904512*a^12*b^7*c^7*d^5 + 60555264*a^13*b^6*c^6*d^6 - 51904512*a^14*b^5*c^5*d^7 + 32440320*a^15*b^4*c^4*d^8 - 14417920*a^16*b^3*c^3*d^9 + 4325376*a^17*b^2*c^2*d^10 - 786432*a^18*b*c*d^11))^(3/4)*((x*(589824*a^2*b^23*c^21*d^4 - 11403264*a^3*b^22*c^20*d^5 + 98762752*a^4*b^21*c^19*d^6 - 510394368*a^5*b^20*c^18*d^7 + 1766916096* a^6*b^19*c^17*d^8 - 4344840192*a^7*b^18*c^16*d^9 + 7796490240*a^8*b^17*c^15*d^10 - 10168369152*a^9*b^16*c^14*d^11 + 9007726592*a^10*b^15*c^13*d^12 - 3635478528*a^11*b^14*c^12*d^13 - 3635478528*a^12*b^13*c^11*d^14 + 9007726592*a^13*b^12*c^10*d^15 - 10168369152*a^14*b^11*c^9*d^16 + 7796490240*a^15*b^10*c^8*d^17 - 4344840192*a^16*b^9*c^7*d^18 + 1766916096*a^17*b^8*c^6*d^19 - 510394368*a^18*b^7*c^5*d^20 + 98762752*a^19*b^6*c^4*d^21 - 11403264*a^20*b^5*c^3*d^22 + 589824*a^21*b^4*c^2*d^23)*1i)/(1024*(a^4*b^12*c^16 + a^16*c^4*d^12 - 12*a^5*b^11*c^15*d - 12*a^15*b*c^5*d^11 + 66*a^6*b^10*c^14*d^2 - 220*a^7*b^9*c^13*d^3 + 495*a^8*b^8*c^12*d^4 - 792*a^9*b^7*c^11*d^5 + 924*a^10*b^6*c^10*d^6 - 792*a^11*b^5*c^9*d^7 + 495*a^12*b^4*c^8*d^8 - 220*a^13*b^3*c^7*d^9 + 66*a^14*b^2*c^6*d^10)) - ((-(81*b^11*c^4 + 14641*a^4*b^7*d^4 - 15972*a^3*b^8*c*d^3 + 6534*a^2*b^9*c^2*d^2 - 1188*a*b^10*c^3*d)/(65536*a^19*d^12 + 65536*a^7*b^12*c^12 - 786432*a^8*b^11*c^11*d + 4325376*a^9*b^10*c^10*d^2 - 14417920*a^10*b^9*c^9*d^3 + 32440320*a^11*b^8*c^8*d^4 - 51904512*a^12*b^7*c^7*d^5 + 60555264*a^13*b^6*c^6*d^6 - 51904512*a^14*b^5*c^5*d^7 + 32440320*a^15*b^4*c^4*d^8 - 14417920*a^16*b^3*c^3*d^9 + 4325376*a^17*b^2*c^2*d^10 - 786432*a^18*b*c*d^11))^(1/4)*(3072*a^4*b^19*c^19*d^4 - 45056*a^5*b^18*c^18*d^5 + 292864*a^6*b^17*c^17*d^6 - 1115136*a^7*b^16*c^16*d^7 + 2745344*a^8*b^15*c^15*d^8 - 4483072*a^9*b^14*c^14*d^9 + 4595712*a^10*b^13*c^13*d^10 - 1993728*a^11*b^12*c^12*d^11 - 1993728*a^12*b^11*c^11*d^12 + 4595712*a^13*b^10*c^10*d^13 - 4483072*a^14*b^9*c^9*d^14 + 2745344*a^15*b^8*c^8*d^15 - 1115136*a^16*b^7*c^7*d^16 + 292864*a^17*b^6*c^6*d^17 - 45056*a^18*b^5*c^5*d^18 + 3072*a^19*b^4*c^4*d^19))/(a^4*b^8*c^12 + a^12*c^4*d^8 - 8*a^5*b^7*c^11*d - 8*a^11*b*c^5*d^7 + 28*a^6*b^6*c^10*d^2 - 56*a^7*b^5*c^9*d^3 + 70*a^8*b^4*c^8*d^4 - 56*a^9*b^3*c^7*d^5 + 28*a^10*b^2*c^6*d^6))*1i - (((891*a^8*b^7*d^15)/64 + (891*b^15*c^8*d^7)/64 - (3105*a*b^14*c^7*d^8)/16 - (3105*a^7*b^8*c*d^14)/16 + (31509*a^2*b^13*c^6*d^9)/32 - (33069*a^3*b^12*c^5*d^10)/16 + (60307*a^4*b^11*c^4*d^11)/32 - (33069*a^5*b^10*c^3*d^12)/16 + (31509*a^6*b^9*c^2*d^13)/32)*1i)/(a^4*b^8*c^12 + a^12*c^4*d^8 - 8*a^5*b^7*c^11*d - 8*a^11*b*c^5*d^7 + 28*a^6*b^6*c^10*d^2 - 56*a^7*b^5*c^9*d^3 + 70*a^8*b^4*c^8*d^4 - 56*a^9*b^3*c^7*d^5 + 28*a^10*b^2*c^6*d^6)) - (x*(9801*a^8*b^9*d^17 + 9801*b^17*c^8*d^9 - 149094*a*b^16*c^7*d^10 - 149094*a^7*b^10*c*d^16 + 1001520*a^2*b^15*c^6*d^11 - 3484602*a^3*b^14*c^5*d^12 + 5769038*a^4*b^13*c^4*d^13 - 3484602*a^5*b^12*c^3*d^14 + 1001520*a^6*b^11*c^2*d^15))/(1024*(a^4*b^12*c^16 + a^16*c^4*d^12 - 12*a^5*b^11*c^15*d - 12*a^15*b*c^5*d^11 + 66*a^6*b^10*c^14*d^2 - 220*a^7*b^9*c^13*d^3 + 495*a^8*b^8*c^12*d^4 - 792*a^9*b^7*c^11*d^5 + 924*a^10*b^6*c^10*d^6 - 792*a^11*b^5*c^9*d^7 + 495*a^12*b^4*c^8*d^8 - 220*a^13*b^3*c^7*d^9 + 66*a^14*b^2*c^6*d^10))))/((-(81*b^11*c^4 + 14641*a^4*b^7*d^4 - 15972*a^3*b^8*c*d^3 + 6534*a^2*b^9*c^2*d^2 - 1188*a*b^10*c^3*d)/(65536*a^19*d^12 + 65536*a^7*b^12*c^12 - 786432*a^8*b^11*c^11*d + 4325376*a^9*b^10*c^10*d^2 - 14417920*a^10*b^9*c^9*d^3 + 32440320*a^11*b^8*c^8*d^4 - 51904512*a^12*b^7*c^7*d^5 + 60555264*a^13*b^6*c^6*d^6 - 51904512*a^14*b^5*c^5*d^7 + 32440320*a^15*b^4*c^4*d^8 - 14417920*a^16*b^3*c^3*d^9 + 4325376*a^17*b^2*c^2*d^10 - 786432*a^18*b*c*d^11))^(1/4)*((-(81*b^11*c^4 + 14641*a^4*b^7*d^4 - 15972*a^3*b^8*c*d^3 + 6534*a^2*b^9*c^2*d^2 - 1188*a*b^10*c^3*d)/(65536*a^19*d^12 + 65536*a^7*b^12*c^12 - 786432*a^8*b^11*c^11*d + 4325376*a^9*b^10*c^10*d^2 - 14417920*a^10*b^9*c^9*d^3 + 32440320*a^11*b^8*c^8*d^4 - 51904512*a^12*b^7*c^7*d^5 + 60555264*a^13*b^6*c^6*d^6 - 51904512*a^14*b^5*c^5*d^7 + 32440320*a^15*b^4*c^4*d^8 - 14417920*a^16*b^3*c^3*d^9 + 4325376*a^17*b^2*c^2*d^10 - 786432*a^18*b*c*d^11))^(1/4)*((-(81*b^11*c^4 + 14641*a^4*b^7*d^4 - 15972*a^3*b^8*c*d^3 + 6534*a^2*b^9*c^2*d^2 - 1188*a*b^10*c^3*d)/(65536*a^19*d^12 + 65536*a^7*b^12*c^12 - 786432*a^8*b^11*c^11*d + 4325376*a^9*b^10*c^10*d^2 - 14417920*a^10*b^9*c^9*d^3 + 32440320*a^11*b^8*c^8*d^4 - 51904512*a^12*b^7*c^7*d^5 + 60555264*a^13*b^6*c^6*d^6 - 51904512*a^14*b^5*c^5*d^7 + 32440320*a^15*b^4*c^4*d^8 - 14417920*a^16*b^3*c^3*d^9 + 4325376*a^17*b^2*c^2*d^10 - 786432*a^18*b*c*d^11))^(3/4)*((x*(589824*a^2*b^23*c^21*d^4 - 11403264*a^3*b^22*c^20*d^5 + 98762752*a^4*b^21*c^19*d^6 - 510394368*a^5*b^20*c^18*d^7 + 1766916096*a^6*b^19*c^17*d^8 - 4344840192*a^7*b^18*c^16*d^9 + 7796490240*a^8*b^17*c^15*d^10 - 10168369152*a^9*b^16*c^14*d^11 + 900772659

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2*a^10*b^15*c^13*d^12 - 3635478528*a^11*b^14*c^12*d^13 - 3635478528*a^12*b^
13*c^11*d^14 + 9007726592*a^13*b^12*c^10*d^15 - 10168369152*a^14*b^11*c^9*d
^16 + 7796490240*a^15*b^10*c^8*d^17 - 4344840192*a^16*b^9*c^7*d^18 + 176691
6096*a^17*b^8*c^6*d^19 - 510394368*a^18*b^7*c^5*d^20 + 98762752*a^19*b^6*c^
4*d^21 - 11403264*a^20*b^5*c^3*d^22 + 589824*a^21*b^4*c^2*d^23)*1i)/(1024*(
a^4*b^12*c^16 + a^16*c^4*d^12 - 12*a^5*b^11*c^15*d - 12*a^15*b*c^5*d^11 + 6
6*a^6*b^10*c^14*d^2 - 220*a^7*b^9*c^13*d^3 + 495*a^8*b^8*c^12*d^4 - 792*a^9
*b^7*c^11*d^5 + 924*a^10*b^6*c^10*d^6 - 792*a^11*b^5*c^9*d^7 + 495*a^12*b^4
*c^8*d^8 - 220*a^13*b^3*c^7*d^9 + 66*a^14*b^2*c^6*d^10)) + ((-(81*b^11*c^4
+ 14641*a^4*b^7*d^4 - 15972*a^3*b^8*c*d^3 + 6534*a^2*b^9*c^2*d^2 - 1188*a*b
^10*c^3*d)/(65536*a^19*d^12 + 65536*a^7*b^12*c^12 - 786432*a^8*b^11*c^11*d
+ 4325376*a^9*b^10*c^10*d^2 - 14417920*a^10*b^9*c^9*d^3 + 32440320*a^11*b^8
*c^8*d^4 - 51904512*a^12*b^7*c^7*d^5 + 60555264*a^13*b^6*c^6*d^6 - 51904512
*a^14*b^5*c^5*d^7 + 32440320*a^15*b^4*c^4*d^8 - 14417920*a^16*b^3*c^3*d^9 +
 4325376*a^17*b^2*c^2*d^10 - 786432*a^18*b*c*d^11))^(1/4)*(3072*a^4*b^19*c^
19*d^4 - 45056*a^5*b^18*c^18*d^5 + 292864*a^6*b^17*c^17*d^6 - 1115136*a^7*b
^16*c^16*d^7 + 2745344*a^8*b^15*c^15*d^8 - 4483072*a^9*b^14*c^14*d^9 + 4595
712*a^10*b^13*c^13*d^10 - 1993728*a^11*b^12*c^12*d^11 - 1993728*a^12*b^11*c
^11*d^12 + 4595712*a^13*b^10*c^10*d^13 - 4483072*a^14*b^9*c^9*d^14 + 274534
4*a^15*b^8*c^8*d^15 - 1115136*a^16*b^7*c^7*d^16 + 292864*a^17*b^6*c^6*d^17
 - 45056*a^18*b^5*c^5*d^18 + 3072*a^19*b^4*c^4*d^19))/(a^4*b^8*c^12 + a^12*c
^4*d^8 - 8*a^5*b^7*c^11*d - 8*a^11*b*c^5*d^7 + 28*a^6*b^6*c^10*d^2 - 56*a^7
*b^5*c^9*d^3 + 70*a^8*b^4*c^8*d^4 - 56*a^9*b^3*c^7*d^5 + 28*a^10*b^2*c^6*d^
6))*1i + (((891*a^8*b^7*d^15)/64 + (891*b^15*c^8*d^7)/64 - (3105*a*b^14*c^7
*d^8)/16 - (3105*a^7*b^8*c*d^14)/16 + (31509*a^2*b^13*c^6*d^9)/32 - (33069*
a^3*b^12*c^5*d^10)/16 + (60307*a^4*b^11*c^4*d^11)/32 - (33069*a^5*b^10*c^3*
d^12)/16 + (31509*a^6*b^9*c^2*d^13)/32)*1i)/(a^4*b^8*c^12 + a^12*c^4*d^8 -
8*a^5*b^7*c^11*d - 8*a^11*b*c^5*d^7 + 28*a^6*b^6*c^10*d^2 - 56*a^7*b^5*c^9*
d^3 + 70*a^8*b^4*c^8*d^4 - 56*a^9*b^3*c^7*d^5 + 28*a^10*b^2*c^6*d^6))*1i -
(x*(9801*a^8*b^9*d^17 + 9801*b^17*c^8*d^9 - 149094*a*b^16*c^7*d^10 - 149094
*a^7*b^10*c*d^16 + 1001520*a^2*b^15*c^6*d^11 - 3484602*a^3*b^14*c^5*d^12 +
5769038*a^4*b^13*c^4*d^13 - 3484602*a^5*b^12*c^3*d^14 + 1001520*a^6*b^11*c^
2*d^15)*1i)/(1024*(a^4*b^12*c^16 + a^16*c^4*d^12 - 12*a^5*b^11*c^15*d - 12*
a^15*b*c^5*d^11 + 66*a^6*b^10*c^14*d^2 - 220*a^7*b^9*c^13*d^3 + 495*a^8*b^8
*c^12*d^4 - 792*a^9*b^7*c^11*d^5 + 924*a^10*b^6*c^10*d^6 - 792*a^11*b^5*c^9
*d^7 + 495*a^12*b^4*c^8*d^8 - 220*a^13*b^3*c^7*d^9 + 66*a^14*b^2*c^6*d^10))
) - (-(81*b^11*c^4 + 14641*a^4*b^7*d^4 - 15972*a^3*b^8*c*d^3 + 6534*a^2*b^9
*c^2*d^2 - 1188*a*b^10*c^3*d)/(65536*a^19*d^12 + 65536*a^7*b^12*c^12 - 7864
32*a^8*b^11*c^11*d + 4325376*a^9*b^10*c^10*d^2 - 14417920*a^10*b^9*c^9*d^3
+ 32440320*a^11*b^8*c^8*d^4 - 51904512*a^12*b^7*c^7*d^5 + 60555264*a^13*b^6
*c^6*d^6 - 51904512*a^14*b^5*c^5*d^7 + 32440320*a^15*b^4*c^4*d^8 - 14417920
*a^16*b^3*c^3*d^9 + 4325376*a^17*b^2*c^2*d^10 - 786432*a^18*b*c*d^11))^(1/4
)*((-(81*b^11*c^4 + 14641*a^4*b^7*d^4 - 15972*a^3*b^8*c*d^3 + 6534*a^2*b^9*
c^2*d^2 - 1188*a*b^10*c^3*d)/(65536*a^19*d^12 + 65536*a^7*b^12*c^12 - 78643
2*a^8*b^11*c^11*d + 4325376*a^9*b^10*c^10*d^2 - 14417920*a^10*b^9*c^9*d^3 +
 32440320*a^11*b^8*c^8*d^4 - 51904512*a^12*b^7*c^7*d^5 + 60555264*a^13*b^6*
c^6*d^6 - 51904512*a^14*b^5*c^5*d^7 + 32440320*a^15*b^4*c^4*d^8 - 14417920*
a^16*b^3*c^3*d^9 + 4325376*a^17*b^2*c^2*d^10 - 786432*a^18*b*c*d^11))^(1/4)
*((-(81*b^11*c^4 + 14641*a^4*b^7*d^4 - 15972*a^3*b^8*c*d^3 + 6534*a^2*b^9*c
^2*d^2 - 1188*a*b^10*c^3*d)/(65536*a^19*d^12 + 65536*a^7*b^12*c^12 - 786432
*a^8*b^11*c^11*d + 4325376*a^9*b^10*c^10*d^2 - 14417920*a^10*b^9*c^9*d^3 +
32440320*a^11*b^8*c^8*d^4 - 51904512*a^12*b^7*c^7*d^5 + 60555264*a^13*b^6*c
^6*d^6 - 51904512*a^14*b^5*c^5*d^7 + 32440320*a^15*b^4*c^4*d^8 - 14417920*a
^16*b^3*c^3*d^9 + 4325376*a^17*b^2*c^2*d^10 - 786432*a^18*b*c*d^11))^(3/4)*
((x*(589824*a^2*b^23*c^21*d^4 - 11403264*a^3*b^22*c^20*d^5 + 98762752*a^4*b
^21*c^19*d^6 - 510394368*a^5*b^20*c^18*d^7 + 1766916096*a^6*b^19*c^17*d^8 -
 4344840192*a^7*b^18*c^16*d^9 + 7796490240*a^8*b^17*c^15*d^10 - 10168369152
*a^9*b^16*c^14*d^11 + 9007726592*a^10*b^15*c^13*d^12 - 3635478528*a^11*b^14
*c^12*d^13 - 3635478528*a^12*b^13*c^11*d^14 + 9007726592*a^13*b^12*c^10*d^1
```

```
5 - 10168369152*a^14*b^11*c^9*d^16 + 7796490240*a^15*b^10*c^8*d^17 - 434484
0192*a^16*b^9*c^7*d^18 + 1766916096*a^17*b^8*c^6*d^19 - 510394368*a^18*b^7*
c^5*d^20 + 98762752*a^19*b^6*c^4*d^21 - 11403264*a^20*b^5*c^3*d^22 + 589824
*a^21*b^4*c^2*d^23)*1i)/(1024*(a^4*b^12*c^16 + a^16*c^4*d^12 - 12*a^5*b^11*
c^15*d - 12*a^15*b*c^5*d^11 + 66*a^6*b^10*c^14*d^2 - 220*a^7*b^9*c^13*d^3 +
 495*a^8*b^8*c^12*d^4 - 792*a^9*b^7*c^11*d^5 + 924*a^10*b^6*c^10*d^6 - 792*
a^11*b^5*c^9*d^7 + 495*a^12*b^4*c^8*d^8 - 220*a^13*b^3*c^7*d^9 + 66*a^14*b^
2*c^6*d^10)) - ((-(81*b^11*c^4 + 14641*a^4*b^7*d^4 - 15972*a^3*b^8*c*d^3 +
6534*a^2*b^9*c^2*d^2 - 1188*a*b^10*c^3*d)/(65536*a^19*d^12 + 65536*a^7*b^12
*c^12 - 786432*a^8*b^11*c^11*d + 4325376*a^9*b^10*c^10*d^2 - 14417920*a^10*
b^9*c^9*d^3 + 32440320*a^11*b^8*c^8*d^4 - 51904512*a^12*b^7*c^7*d^5 + 60555
264*a^13*b^6*c^6*d^6 - 51904512*a^14*b^5*c^5*d^7 + 32440320*a^15*b^4*c^4*d^
8 - 14417920*a^16*b^3*c^3*d^9 + 4325376*a^17*b^2*c^2*d^10 - 786432*a^18*b*c
*d^11))^(1/4)*(3072*a^4*b^19*c^19*d^4 - 45056*a^5*b^18*c^18*d^5 + 292864*a^
6*b^17*c^17*d^6 - 1115136*a^7*b^16*c^16*d^7 + 2745344*a^8*b^15*c^15*d^8 - 4
483072*a^9*b^14*c^14*d^9 + 4595712*a^10*b^13*c^13*d^10 - 1993728*a^11*b^12*
c^12*d^11 - 1993728*a^12*b^11*c^11*d^12 + 4595712*a^13*b^10*c^10*d^13 - 448
3072*a^14*b^9*c^9*d^14 + 2745344*a^15*b^8*c^8*d^15 - 1115136*a^16*b^7*c^7*d
^16 + 292864*a^17*b^6*c^6*d^17 - 45056*a^18*b^5*c^5*d^18 + 3072*a^19*b^4*c^
4*d^19))/(a^4*b^8*c^12 + a^12*c^4*d^8 - 8*a^5*b^7*c^11*d - 8*a^11*b*c^5*d^7
 + 28*a^6*b^6*c^10*d^2 - 56*a^7*b^5*c^9*d^3 + 70*a^8*b^4*c^8*d^4 - 56*a^9*b
^3*c^7*d^5 + 28*a^10*b^2*c^6*d^6))*1i - (((891*a^8*b^7*d^15)/64 + (891*b^15
*c^8*d^7)/64 - (3105*a*b^14*c^7*d^8)/16 - (3105*a^7*b^8*c*d^14)/16 + (31509
*a^2*b^13*c^6*d^9)/32 - (33069*a^3*b^12*c^5*d^10)/16 + (60307*a^4*b^11*c^4*
d^11)/32 - (33069*a^5*b^10*c^3*d^12)/16 + (31509*a^6*b^9*c^2*d^13)/32)*1i)/
(a^4*b^8*c^12 + a^12*c^4*d^8 - 8*a^5*b^7*c^11*d - 8*a^11*b*c^5*d^7 + 28*a^6
*b^6*c^10*d^2 - 56*a^7*b^5*c^9*d^3 + 70*a^8*b^4*c^8*d^4 - 56*a^9*b^3*c^7*d^
5 + 28*a^10*b^2*c^6*d^6))*1i - (x*(9801*a^8*b^9*d^17 + 9801*b^17*c^8*d^9 -
149094*a*b^16*c^7*d^10 - 149094*a^7*b^10*c*d^16 + 1001520*a^2*b^15*c^6*d^11
 - 3484602*a^3*b^14*c^5*d^12 + 5769038*a^4*b^13*c^4*d^13 - 3484602*a^5*b^12
*c^3*d^14 + 1001520*a^6*b^11*c^2*d^15)*1i)/(1024*(a^4*b^12*c^16 + a^16*c^4*
d^12 - 12*a^5*b^11*c^15*d - 12*a^15*b*c^5*d^11 + 66*a^6*b^10*c^14*d^2 - 220
*a^7*b^9*c^13*d^3 + 495*a^8*b^8*c^12*d^4 - 792*a^9*b^7*c^11*d^5 + 924*a^10*
b^6*c^10*d^6 - 792*a^11*b^5*c^9*d^7 + 495*a^12*b^4*c^8*d^8 - 220*a^13*b^3*c
^7*d^9 + 66*a^14*b^2*c^6*d^10)))))*(-(81*b^11*c^4 + 14641*a^4*b^7*d^4 - 159
72*a^3*b^8*c*d^3 + 6534*a^2*b^9*c^2*d^2 - 1188*a*b^10*c^3*d)/(65536*a^19*d^
12 + 65536*a^7*b^12*c^12 - 786432*a^8*b^11*c^11*d + 4325376*a^9*b^10*c^10*d
^2 - 14417920*a^10*b^9*c^9*d^3 + 32440320*a^11*b^8*c^8*d^4 - 51904512*a^12*
b^7*c^7*d^5 + 60555264*a^13*b^6*c^6*d^6 - 51904512*a^14*b^5*c^5*d^7 + 32440
320*a^15*b^4*c^4*d^8 - 14417920*a^16*b^3*c^3*d^9 + 4325376*a^17*b^2*c^2*d^1
0 - 786432*a^18*b*c*d^11))^(1/4)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**4+a)**2/(d*x**4+c)**2,x)
```

```
[Out] Timed out
```

3.110 $\int \frac{\sqrt{a+bx^4}}{ac-bcx^4} dx$

Optimal. Leaf size=103

$$\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x}{\sqrt{a+bx^4}}\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}c}+\frac{\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x}{\sqrt{a+bx^4}}\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}c}$$

Rubi [A] time = 0.06, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {404, 212, 206, 203}

$$\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x}{\sqrt{a+bx^4}}\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}c}+\frac{\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x}{\sqrt{a+bx^4}}\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}c}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x^4]/(a*c - b*c*x^4),x]
```

```
[Out] ArcTan[(Sqrt[2]*a^(1/4)*b^(1/4)*x)/Sqrt[a + b*x^4]]/(2*Sqrt[2]*a^(1/4)*b^(1
/4)*c) + ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*x)/Sqrt[a + b*x^4]]/(2*Sqrt[2]*a^
(1/4)*b^(1/4)*c)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 404

```
Int[Sqrt[(a_) + (b_.)*(x_)^4]/((c_) + (d_.)*(x_)^4), x_Symbol] :> Dist[a/c,
 Subst[Int[1/(1 - 4*a*b*x^4), x], x, x/Sqrt[a + b*x^4]], x] /; FreeQ[{a, b,
 c, d}, x] && EqQ[b*c + a*d, 0] && PosQ[a*b]
```

Rubi steps

$$\int \frac{\sqrt{a+bx^4}}{ac-bcx^4}\,dx = \frac{\text{Subst}\left(\int \frac{1}{1-4abx^4}\,dx, x, \frac{x}{\sqrt{a+bx^4}}\right)}{c}$$
$$= \frac{\text{Subst}\left(\int \frac{1}{1-2\sqrt{a}\sqrt{b}x^2}\,dx, x, \frac{x}{\sqrt{a+bx^4}}\right)}{2c} + \frac{\text{Subst}\left(\int \frac{1}{1+2\sqrt{a}\sqrt{b}x^2}\,dx, x, \frac{x}{\sqrt{a+bx^4}}\right)}{2c}$$
$$= \frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x}{\sqrt{a+bx^4}}\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}c} + \frac{\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x}{\sqrt{a+bx^4}}\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}c}$$

Mathematica [C] time = 0.17, size = 155, normalized size = 1.50

$$\frac{5ax\sqrt{a+bx^4}F_1\left(\frac{1}{4};-\frac{1}{2},1;\frac{5}{4};-\frac{bx^4}{a},\frac{bx^4}{a}\right)}{c\left(a-bx^4\right)\left(2bx^4\left(2F_1\left(\frac{5}{4};-\frac{1}{2},2;\frac{9}{4};-\frac{bx^4}{a},\frac{bx^4}{a}\right)+F_1\left(\frac{5}{4};\frac{1}{2},1;\frac{9}{4};-\frac{bx^4}{a},\frac{bx^4}{a}\right)\right)+5aF_1\left(\frac{1}{4};-\frac{1}{2},1;\frac{5}{4};-\frac{bx^4}{a},\frac{bx^4}{a}\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[a + b*x^4]/(a*c - b*c*x^4),x]
```

```
[Out] (5*a*x*Sqrt[a + b*x^4]*AppellF1[1/4, -1/2, 1, 5/4, -((b*x^4)/a), (b*x^4)/a]
)/(c*(a - b*x^4)*(5*a*AppellF1[1/4, -1/2, 1, 5/4, -((b*x^4)/a), (b*x^4)/a]
+ 2*b*x^4*(2*AppellF1[5/4, -1/2, 2, 9/4, -((b*x^4)/a), (b*x^4)/a] + AppellF
1[5/4, 1/2, 1, 9/4, -((b*x^4)/a), (b*x^4)/a])))
```

IntegrateAlgebraic [A] time = 0.43, size = 103, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x}{\sqrt{a+bx^4}}\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}c} + \frac{\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x}{\sqrt{a+bx^4}}\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}c}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[a + b*x^4]/(a*c - b*c*x^4),x]
```

```
[Out] ArcTan[(Sqrt[2]*a^(1/4)*b^(1/4)*x)/Sqrt[a + b*x^4]]/(2*Sqrt[2]*a^(1/4)*b^(1
/4)*c) + ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*x)/Sqrt[a + b*x^4]]/(2*Sqrt[2]*a^
(1/4)*b^(1/4)*c)
```

fricas [B] time = 3.92, size = 315, normalized size = 3.06

$$-\left(\frac{1}{4}\right)^{\frac{1}{4}}\left(\frac{1}{abc^4}\right)^{\frac{1}{4}}\arctan\left(\frac{\left(\frac{1}{4}\right)^{\frac{1}{4}}\sqrt{bx^4+a}\,c\left(\frac{1}{abc^4}\right)^{\frac{1}{4}}-\frac{2\left(\frac{1}{4}\right)^{\frac{3}{4}}abc^3\left(\frac{1}{abc^4}\right)^{\frac{3}{4}}+\left(\frac{1}{4}\right)^{\frac{1}{4}}bcx^2\left(\frac{1}{abc^4}\right)^{\frac{1}{4}}}{\sqrt{b}}}{x}\right)+\frac{1}{4}\left(\frac{1}{4}\right)^{\frac{1}{4}}\left(\frac{1}{abc^4}\right)^{\frac{1}{4}}\log\left(\frac{4\left(\frac{1}{4}\right)^{\frac{3}{4}}abc^3x^3\left(\frac{1}{abc^4}\right)^{\frac{3}{4}}+2\left(\frac{1}{4}\right)^{\frac{1}{4}}acx\left(\frac{1}{abc^4}\right)^{\frac{1}{4}}+\sqrt{bx^4+a}\left(ac^2\sqrt{\frac{1}{abc^4}}+x^2\right)}{bx^4-a}\right)-\frac{1}{4}\left(\frac{1}{4}\right)^{\frac{1}{4}}\left(\frac{1}{abc^4}\right)^{\frac{1}{4}}\log\left(-\frac{4\left(\frac{1}{4}\right)^{\frac{3}{4}}abc^3x^3\left(\frac{1}{abc^4}\right)^{\frac{3}{4}}+2\left(\frac{1}{4}\right)^{\frac{1}{4}}acx\left(\frac{1}{abc^4}\right)^{\frac{1}{4}}-\sqrt{bx^4+a}\left(ac^2\sqrt{\frac{1}{abc^4}}+x^2\right)}{bx^4-a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^(1/2)/(-b*c*x^4+a*c),x, algorithm="fricas")
```

```
[Out] -(1/4)^(1/4)*(1/(a*b*c^4))^(1/4)*arctan(((1/4)^(1/4)*sqrt(b*x^4 + a)*c*(1/(
a*b*c^4))^(1/4) - (2*(1/4)^(3/4)*a*b*c^3*(1/(a*b*c^4))^(3/4) + (1/4)^(1/4)*
b*c*x^2*(1/(a*b*c^4))^(1/4))/sqrt(b))/x) + 1/4*(1/4)^(1/4)*(1/(a*b*c^4))^(1
/4)*log((4*(1/4)^(3/4)*a*b*c^3*x^3*(1/(a*b*c^4))^(3/4) + 2*(1/4)^(1/4)*a*c*
x*(1/(a*b*c^4))^(1/4) + sqrt(b*x^4 + a)*(a*c^2*sqrt(1/(a*b*c^4)) + x^2))/(b
*x^4 - a)) - 1/4*(1/4)^(1/4)*(1/(a*b*c^4))^(1/4)*log(-(4*(1/4)^(3/4)*a*b*c^
3*x^3*(1/(a*b*c^4))^(3/4) + 2*(1/4)^(1/4)*a*c*x*(1/(a*b*c^4))^(1/4) - sqrt(
b*x^4 + a)*(a*c^2*sqrt(1/(a*b*c^4)) + x^2))/(b*x^4 - a))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{\sqrt{bx^4+a}}{bcx^4-ac}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^(1/2)/(-b*c*x^4+a*c),x, algorithm="giac")
[Out] integrate(-sqrt(b*x^4 + a)/(b*c*x^4 - a*c), x)
```

maple [A] time = 0.21, size = 103, normalized size = 1.00

$$-\frac{\sqrt{2}\,\arctan\left(\frac{\sqrt{b\,x^4+a}\,\sqrt{2}}{2(ab)^{\frac{1}{4}}x}\right)}{4\,(ab)^{\frac{1}{4}}\,c}+\frac{\sqrt{2}\,\ln\left(\frac{\frac{\sqrt{b\,x^4+a}\,\sqrt{2}}{2x}+(ab)^{\frac{1}{4}}}{\frac{\sqrt{b\,x^4+a}\,\sqrt{2}}{2x}-(ab)^{\frac{1}{4}}}\right)}{8\,(ab)^{\frac{1}{4}}\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^4+a)^(1/2)/(-b*c*x^4+a*c),x)
[Out] -1/4/c*2^(1/2)/(a*b)^(1/4)*arctan(1/2*(b*x^4+a)^(1/2)*2^(1/2)/x/(a*b)^(1/4)
)+1/8/c*2^(1/2)/(a*b)^(1/4)*ln((1/2*(b*x^4+a)^(1/2)*2^(1/2)/x+(a*b)^(1/4))/
(1/2*(b*x^4+a)^(1/2)*2^(1/2)/x-(a*b)^(1/4)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\sqrt{bx^4+a}}{bcx^4-ac}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^(1/2)/(-b*c*x^4+a*c),x, algorithm="maxima")
[Out] -integrate(sqrt(b*x^4 + a)/(b*c*x^4 - a*c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{b\,x^4+a}}{a\,c-b\,c\,x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^4)^(1/2)/(a*c - b*c*x^4),x)
[Out] int((a + b*x^4)^(1/2)/(a*c - b*c*x^4), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{\sqrt{a+bx^4}}{-a+bx^4}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**4+a)**(1/2)/(-b*c*x**4+a*c),x)
[Out] -Integral(sqrt(a + b*x**4)/(-a + b*x**4), x)/c
```

3.111 $\int \frac{\sqrt{a-bx^4}}{ac+bcx^4} dx$

Optimal. Leaf size=116

$$\frac{\tan^{-1}\left(\frac{\sqrt[4]{b}x\left(\sqrt{a}+\sqrt{b}x^2\right)}{\sqrt[4]{a}\sqrt{a-bx^4}}\right)}{2\sqrt[4]{a}\sqrt[4]{b}c}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{b}x\left(\sqrt{a}-\sqrt{b}x^2\right)}{\sqrt[4]{a}\sqrt{a-bx^4}}\right)}{2\sqrt[4]{a}\sqrt[4]{b}c}$$

Rubi [A] time = 0.02, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.040, Rules used = {405}

$$\frac{\tan^{-1}\left(\frac{\sqrt[4]{b}x\left(\sqrt{a}+\sqrt{b}x^2\right)}{\sqrt[4]{a}\sqrt{a-bx^4}}\right)}{2\sqrt[4]{a}\sqrt[4]{b}c}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{b}x\left(\sqrt{a}-\sqrt{b}x^2\right)}{\sqrt[4]{a}\sqrt{a-bx^4}}\right)}{2\sqrt[4]{a}\sqrt[4]{b}c}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a - b*x^4]/(a*c + b*c*x^4),x]
```

```
[Out] ArcTan[(b^(1/4)*x*(Sqrt[a] + Sqrt[b]*x^2))/(a^(1/4)*Sqrt[a - b*x^4])]/(2*a^
(1/4)*b^(1/4)*c) + ArcTanh[(b^(1/4)*x*(Sqrt[a] - Sqrt[b]*x^2))/(a^(1/4)*Sqr
t[a - b*x^4])]/(2*a^(1/4)*b^(1/4)*c)
```

Rule 405

```
Int[Sqrt[(a_) + (b_.)*(x_)^4]/((c_) + (d_.)*(x_)^4), x_Symbol] :> With[{q =
 Rt[-(a*b), 4]}, Simp[(a*ArcTan[(q*x*(a + q^2*x^2))/(a*Sqrt[a + b*x^4])])/(
2*c*q), x] + Simp[(a*ArcTanh[(q*x*(a - q^2*x^2))/(a*Sqrt[a + b*x^4])])/(2*c
*q), x]] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c + a*d, 0] && NegQ[a*b]
```

Rubi steps

$$\int \frac{\sqrt{a-bx^4}}{ac+bcx^4}\,dx=\frac{\tan^{-1}\left(\frac{\sqrt[4]{b}x\left(\sqrt{a}+\sqrt{b}x^2\right)}{\sqrt[4]{a}\sqrt{a-bx^4}}\right)}{2\sqrt[4]{a}\sqrt[4]{b}c}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{b}x\left(\sqrt{a}-\sqrt{b}x^2\right)}{\sqrt[4]{a}\sqrt{a-bx^4}}\right)}{2\sqrt[4]{a}\sqrt[4]{b}c}$$

Mathematica [C] time = 0.17, size = 155, normalized size = 1.34

$$\frac{5ax\sqrt{a-bx^4}F_1\left(\frac{1}{4};-\frac{1}{2},1;\frac{5}{4};\frac{bx^4}{a},-\frac{bx^4}{a}\right)}{c\left(a+bx^4\right)\left(5aF_1\left(\frac{1}{4};-\frac{1}{2},1;\frac{5}{4};\frac{bx^4}{a},-\frac{bx^4}{a}\right)-2bx^4\left(2F_1\left(\frac{5}{4};-\frac{1}{2},2;\frac{9}{4};\frac{bx^4}{a},-\frac{bx^4}{a}\right)+F_1\left(\frac{5}{4};\frac{1}{2},1;\frac{9}{4};\frac{bx^4}{a},-\frac{bx^4}{a}\right)\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[a - b*x^4]/(a*c + b*c*x^4),x]
```

```
[Out] (5*a*x*Sqrt[a - b*x^4]*AppellF1[1/4, -1/2, 1, 5/4, (b*x^4)/a, -((b*x^4)/a)]
)/(c*(a + b*x^4)*(5*a*AppellF1[1/4, -1/2, 1, 5/4, (b*x^4)/a, -((b*x^4)/a)]
- 2*b*x^4*(2*AppellF1[5/4, -1/2, 2, 9/4, (b*x^4)/a, -((b*x^4)/a)] + AppellF
1[5/4, 1/2, 1, 9/4, (b*x^4)/a, -((b*x^4)/a)])))
```

IntegrateAlgebraic [C] time = 0.42, size = 105, normalized size = 0.91

$$\frac{\left(\frac{1}{4}-\frac{i}{4}\right)\tan^{-1}\left(\frac{(1+i)\sqrt[4]{a}\sqrt[4]{b}x}{\sqrt{a-bx^4}}\right)}{\sqrt[4]{a}\sqrt[4]{b}c}-\frac{\left(\frac{1}{4}+\frac{i}{4}\right)\tan^{-1}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{a-bx^4}}{\sqrt[4]{a}\sqrt[4]{b}x}\right)}{\sqrt[4]{a}\sqrt[4]{b}c}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[a - b*x^4]/(a*c + b*c*x^4),x]
```

```
[Out] ((1/4 - I/4)*ArcTan[((1 + I)*a^(1/4)*b^(1/4)*x)/Sqrt[a - b*x^4]])/(a^(1/4)*
b^(1/4)*c) - ((1/4 + I/4)*ArcTan[((1/2 + I/2)*Sqrt[a - b*x^4])/(a^(1/4)*b^(
1/4)*x)]])/(a^(1/4)*b^(1/4)*c)
```

fricas [B] time = 3.54, size = 339, normalized size = 2.92

$$-\left(\frac{1}{4}\right)^{\frac{1}{4}} \left(-\frac{1}{abc^4}\right)^{\frac{1}{4}} \arctan\left(\frac{2 \left(\frac{1}{4}\right)^{\frac{3}{4}} abc^3 \sqrt{-\frac{1}{b}} \left(-\frac{1}{abc^4}\right)^{\frac{3}{4}} + \left(\frac{1}{4}\right)^{\frac{1}{4}} \left(bcx^2 \sqrt{-\frac{1}{b}} + \sqrt{-bx^4 + a}\, c\right) \left(-\frac{1}{abc^4}\right)^{\frac{1}{4}}}{x}\right) - \frac{1}{4} \left(\frac{1}{4}\right)^{\frac{1}{4}} \left(-\frac{1}{abc^4}\right)^{\frac{1}{4}} \log\left(-\frac{4 \left(\frac{1}{4}\right)^{\frac{3}{4}} abc^3x^3 \left(-\frac{1}{abc^4}\right)^{\frac{3}{4}} + \sqrt{-bx^4 + a}\, ac^2 \sqrt{-\frac{1}{abc^4}} - 2 \left(\frac{1}{4}\right)^{\frac{1}{4}} acx \left(-\frac{1}{abc^4}\right)^{\frac{1}{4}} + \sqrt{-bx^4 + a}\, x^2}{bx^4 + a}\right) + \frac{1}{4} \left(\frac{1}{4}\right)^{\frac{1}{4}} \left(-\frac{1}{abc^4}\right)^{\frac{1}{4}} \log\left(\frac{4 \left(\frac{1}{4}\right)^{\frac{3}{4}} abc^3x^3 \left(-\frac{1}{abc^4}\right)^{\frac{3}{4}} - \sqrt{-bx^4 + a}\, ac^2 \sqrt{-\frac{1}{abc^4}} - 2 \left(\frac{1}{4}\right)^{\frac{1}{4}} acx \left(-\frac{1}{abc^4}\right)^{\frac{1}{4}} - \sqrt{-bx^4 + a}\, x^2}{bx^4 + a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^4+a)^(1/2)/(b*c*x^4+a*c),x, algorithm="fricas")
```

```
[Out] -(1/4)^(1/4)*(-1/(a*b*c^4))^(1/4)*arctan((2*(1/4)^(3/4)*a*b*c^3*sqrt(-1/b)*
(-1/(a*b*c^4))^(3/4) + (1/4)^(1/4)*(b*c*x^2*sqrt(-1/b) + sqrt(-b*x^4 + a)*c
)*(-1/(a*b*c^4))^(1/4))/x) - 1/4*(1/4)^(1/4)*(-1/(a*b*c^4))^(1/4)*log(-(4*(
1/4)^(3/4)*a*b*c^3*x^3*(-1/(a*b*c^4))^(3/4) + sqrt(-b*x^4 + a)*a*c^2*sqrt(-
1/(a*b*c^4)) - 2*(1/4)^(1/4)*a*c*x*(-1/(a*b*c^4))^(1/4) + sqrt(-b*x^4 + a)*
x^2)/(b*x^4 + a)) + 1/4*(1/4)^(1/4)*(-1/(a*b*c^4))^(1/4)*log((4*(1/4)^(3/4)
*a*b*c^3*x^3*(-1/(a*b*c^4))^(3/4) - sqrt(-b*x^4 + a)*a*c^2*sqrt(-1/(a*b*c^4
)) - 2*(1/4)^(1/4)*a*c*x*(-1/(a*b*c^4))^(1/4) - sqrt(-b*x^4 + a)*x^2)/(b*x^
4 + a))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-bx^4 + a}}{bcx^4 + ac}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^4+a)^(1/2)/(b*c*x^4+a*c),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-b*x^4 + a)/(b*c*x^4 + a*c), x)
```

maple [A] time = 0.22, size = 158, normalized size = 1.36

$$\frac{\arctan\left(-\frac{\sqrt{-b\,x^4+a}}{(ab)^{\frac{1}{4}}\,x}+1\right)}{4\,(ab)^{\frac{1}{4}}\,c} - \frac{\arctan\left(\frac{\sqrt{-b\,x^4+a}}{(ab)^{\frac{1}{4}}\,x}+1\right)}{4\,(ab)^{\frac{1}{4}}\,c} - \frac{\ln\left(\frac{-\frac{(ab)^{\frac{1}{4}}\,\sqrt{-b\,x^4+a}}{x}+\frac{-b\,x^4+a}{2x^2}+\sqrt{ab}}{\frac{(ab)^{\frac{1}{4}}\,\sqrt{-b\,x^4+a}}{x}+\frac{-b\,x^4+a}{2x^2}+\sqrt{ab}}\right)}{8\,(ab)^{\frac{1}{4}}\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x^4+a)^(1/2)/(b*c*x^4+a*c),x)
```

```
[Out] -1/8/c/(a*b)^(1/4)*ln((1/2*(-b*x^4+a)/x^2-(a*b)^(1/4)*(-b*x^4+a)^(1/2)/x+(a
*b)^(1/2))/(1/2*(-b*x^4+a)/x^2+(a*b)^(1/4)*(-b*x^4+a)^(1/2)/x+(a*b)^(1/2)))
-1/4/c/(a*b)^(1/4)*arctan(1/(a*b)^(1/4)*(-b*x^4+a)^(1/2)/x+1)+1/4/c/(a*b)^(
1/4)*arctan(-1/(a*b)^(1/4)*(-b*x^4+a)^(1/2)/x+1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-bx^4 + a}}{bcx^4 + ac}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^4+a)^(1/2)/(b*c*x^4+a*c),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-b*x^4 + a)/(b*c*x^4 + a*c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{a-b\,x^4}}{b\,c\,x^4+a\,c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - b*x^4)^(1/2)/(a*c + b*c*x^4),x)
```

```
[Out] int((a - b*x^4)^(1/2)/(a*c + b*c*x^4), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sqrt{a-bx^4}}{a+bx^4}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x**4+a)**(1/2)/(b*c*x**4+a*c),x)
```

```
[Out] Integral(sqrt(a - b*x**4)/(a + b*x**4), x)/c
```

3.112 $\int \frac{\left(a+bx^4\right)^{7/4}}{c+dx^4}\,dx$

Optimal. Leaf size=211

$$-\frac{b^{3/4}(4bc-7ad)\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{8d^2}-\frac{b^{3/4}(4bc-7ad)\tanh^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{8d^2}+\frac{(bc-ad)^{7/4}\tan^{-1}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}d^2}+\frac{(bc-ad)}{}$$

Rubi [A] time = 0.22, antiderivative size = 211, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {416, 530, 240, 212, 206, 203, 377, 208, 205}

$$-\frac{b^{3/4}(4bc-7ad)\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{8d^2}-\frac{b^{3/4}(4bc-7ad)\tanh^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{8d^2}+\frac{(bc-ad)^{7/4}\tan^{-1}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}d^2}+\frac{(bc-ad)^{7/4}\tanh^{-1}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}d^2}+\frac{bx\left(a+bx^4\right)^{3/4}}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^4)^(7/4)/(c + d*x^4),x]
```

```
[Out] (b*x*(a + b*x^4)^(3/4))/(4*d) - (b^(3/4)*(4*b*c - 7*a*d)*ArcTan[(b^(1/4)*x)
/(a + b*x^4)^(1/4)])/(8*d^2) + ((b*c - a*d)^(7/4)*ArcTan[((b*c - a*d)^(1/4)
*x)/(c^(1/4)*(a + b*x^4)^(1/4))])/(2*c^(3/4)*d^2) - (b^(3/4)*(4*b*c - 7*a*d
)*ArcTanh[(b^(1/4)*x)/(a + b*x^4)^(1/4)])/(8*d^2) + ((b*c - a*d)^(7/4)*ArcT
anh[((b*c - a*d)^(1/4)*x)/(c^(1/4)*(a + b*x^4)^(1/4))])/(2*c^(3/4)*d^2)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
```

```
n]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 416

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(d*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(b*(n*(p + q) + 1)),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 530

```
Int[(((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n_)))/((c_) + (d_.)*
(x_)^(n_)), x_Symbol] :> Dist[f/d, Int[(a + b*x^n)^p, x], x] + Dist[(d*e -
c*f)/d, Int[(a + b*x^n)^p/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, p
, n}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a+bx^4\right)^{7/4}}{c+dx^4}\,dx &= \frac{bx\left(a+bx^4\right)^{3/4}}{4d}+\frac{\int \frac{-a(bc-4ad)-b(4bc-7ad)x^4}{\sqrt[4]{a+bx^4}\left(c+dx^4\right)}\,dx}{4d}\\
&= \frac{bx\left(a+bx^4\right)^{3/4}}{4d}-\frac{(b(4bc-7ad))\int \frac{1}{\sqrt[4]{a+bx^4}}\,dx}{4d^2}+\frac{(bc-ad)^2\int \frac{1}{\sqrt[4]{a+bx^4}\left(c+dx^4\right)}\,dx}{d^2}\\
&= \frac{bx\left(a+bx^4\right)^{3/4}}{4d}-\frac{(b(4bc-7ad))\,\text{Subst}\left(\int \frac{1}{1-bx^4}\,dx,x,\frac{x}{\sqrt[4]{a+bx^4}}\right)}{4d^2}+\frac{(bc-ad)^2\,\text{Subst}\left(\int \frac{1}{c-(bc-a\ldots}\right.}{d^2}\\
&= \frac{bx\left(a+bx^4\right)^{3/4}}{4d}-\frac{(b(4bc-7ad))\,\text{Subst}\left(\int \frac{1}{1-\sqrt{b}x^2}\,dx,x,\frac{x}{\sqrt[4]{a+bx^4}}\right)}{8d^2}-\frac{(b(4bc-7ad))\,\text{Subst}\left(\int \ldots\right.}{8d^2}\\
&= \frac{bx\left(a+bx^4\right)^{3/4}}{4d}-\frac{b^{3/4}(4bc-7ad)\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{8d^2}+\frac{(bc-ad)^{7/4}\tan^{-1}\left(\frac{\sqrt[4]{bc-ad}\,x}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}d^2}-\frac{b^{3/4}(4b\ldots}{}
\end{aligned}$$

Mathematica [C] time = 0.63, size = 364, normalized size = 1.73

$$\frac{5\sqrt[4]{c}\left(4a^2d\sqrt[4]{a+bx^4}\log\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{ax^4+b}}+\sqrt[4]{c}\right)+4b^2c^{3/4}x^5\sqrt[4]{bc-ad}+4abc^{3/4}x\sqrt[4]{bc-ad}+a\sqrt[4]{a+bx^4}(bc-4ad)\log\left(\sqrt[4]{c}-\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{ax^4+b}}\right)-abc\sqrt[4]{a+bx^4}\log\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{ax^4+b}}+\sqrt[4]{c}\right)+2a\sqrt[4]{a+bx^4}(4ad-bc)\tan^{-1}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{ax^4+b}}\right)\right)+4bx^5\sqrt[4]{\frac{bx^4}{a}+1}\sqrt[4]{bc-ad}(7ad-4bc)F_1\left(\frac{5}{4};\frac{1}{4},1;\frac{9}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{80cd\sqrt[4]{a+bx^4}\sqrt[4]{bc-ad}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^4)^(7/4)/(c + d*x^4),x]
```

```
[Out] (4*b*(b*c - a*d)^(1/4)*(-4*b*c + 7*a*d)*x^5*(1 + (b*x^4)/a)^(1/4)*AppellF1[
5/4, 1/4, 1, 9/4, -((b*x^4)/a), -((d*x^4)/c)] + 5*c^(1/4)*(4*a*b*c^(3/4)*(b
*c - a*d)^(1/4)*x + 4*b^2*c^(3/4)*(b*c - a*d)^(1/4)*x^5 + 2*a*(-(b*c) + 4*a
*d)*(a + b*x^4)^(1/4)*ArcTan[((b*c - a*d)^(1/4)*x)/(c^(1/4)*(b + a*x^4)^(1/
4))] + a*(b*c - 4*a*d)*(a + b*x^4)^(1/4)*Log[c^(1/4) - ((b*c - a*d)^(1/4)*x
```

```
)/(b + a*x^4)^(1/4)] - a*b*c*(a + b*x^4)^(1/4)*Log[c^(1/4) + ((b*c - a*d)^(
1/4)*x)/(b + a*x^4)^(1/4)] + 4*a^2*d*(a + b*x^4)^(1/4)*Log[c^(1/4) + ((b*c
- a*d)^(1/4)*x)/(b + a*x^4)^(1/4)]))/(80*c*d*(b*c - a*d)^(1/4)*(a + b*x^4)^
(1/4))
```

IntegrateAlgebraic [C] time = 1.19, size = 327, normalized size = 1.55

$$\frac{\left(7ab^{3/4}d-4b^{7/4}c\right)\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a+bx^4}}\right)}{8d^2}+\frac{\left(7ab^{3/4}d-4b^{7/4}c\right)\tanh^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a+bx^4}}\right)}{8d^2}+\frac{\left(\frac{1}{4}+\frac{i}{4}\right)(bc-ad)^{7/4}\tan^{-1}\left(\frac{\frac{\left(\frac{1}{2}-\frac{i}{2}\right)x^2\sqrt[4]{bc-ad}}{\sqrt[4]{c}}-\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt[4]{c}\sqrt{a+bx^4}}{\sqrt[4]{bc-ad}}}{x\sqrt[4]{a+bx^4}}\right)}{c^{3/4}d^2}+\frac{\left(\frac{1}{4}+\frac{i}{4}\right)(bc-ad)^{7/4}\tanh^{-1}\left(\frac{\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt[4]{c}\sqrt{a+bx^4}}{\sqrt[4]{bc-ad}}+\frac{\left(\frac{1}{2}-\frac{i}{2}\right)x^2\sqrt[4]{bc-ad}}{\sqrt[4]{c}}}{x\sqrt[4]{a+bx^4}}\right)}{c^{3/4}d^2}+\frac{bx\left(a+bx^4\right)^{3/4}}{4d}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x^4)^(7/4)/(c + d*x^4),x]
```

```
[Out] (b*x*(a + b*x^4)^(3/4))/(4*d) + ((-4*b^(7/4)*c + 7*a*b^(3/4)*d)*ArcTan[(b^(
1/4)*x)/(a + b*x^4)^(1/4)])/(8*d^2) + ((1/4 + I/4)*(b*c - a*d)^(7/4)*ArcTan
[(((1/2 - I/2)*(b*c - a*d)^(1/4)*x^2)/c^(1/4) - ((1/2 + I/2)*c^(1/4)*Sqrt[a
 + b*x^4])/(b*c - a*d)^(1/4))/(x*(a + b*x^4)^(1/4))]])/(c^(3/4)*d^2) + ((-4*
b^(7/4)*c + 7*a*b^(3/4)*d)*ArcTanh[(b^(1/4)*x)/(a + b*x^4)^(1/4)])/(8*d^2)
+ ((1/4 + I/4)*(b*c - a*d)^(7/4)*ArcTanh[(((1/2 - I/2)*(b*c - a*d)^(1/4)*x^
2)/c^(1/4) + ((1/2 + I/2)*c^(1/4)*Sqrt[a + b*x^4])/(b*c - a*d)^(1/4))/(x*(a
 + b*x^4)^(1/4))]])/(c^(3/4)*d^2)
```

fricas [B] time = 10.96, size = 2381, normalized size = 11.28

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^(7/4)/(d*x^4+c),x, algorithm="fricas")
```

```
[Out] 1/16*(4*(b*x^4 + a)^(3/4)*b*x + 16*d*((b^7*c^7 - 7*a*b^6*c^6*d + 21*a^2*b^5
*c^5*d^2 - 35*a^3*b^4*c^4*d^3 + 35*a^4*b^3*c^3*d^4 - 21*a^5*b^2*c^2*d^5 + 7
*a^6*b*c*d^6 - a^7*d^7)/(c^3*d^8))^(1/4)*arctan(-(c*d^2*x*sqrt(((b^7*c^8*d^
4 - 7*a*b^6*c^7*d^5 + 21*a^2*b^5*c^6*d^6 - 35*a^3*b^4*c^5*d^7 + 35*a^4*b^3*
c^4*d^8 - 21*a^5*b^2*c^3*d^9 + 7*a^6*b*c^2*d^10 - a^7*c*d^11)*x^2*sqrt((b^7
*c^7 - 7*a*b^6*c^6*d + 21*a^2*b^5*c^5*d^2 - 35*a^3*b^4*c^4*d^3 + 35*a^4*b^3
*c^3*d^4 - 21*a^5*b^2*c^2*d^5 + 7*a^6*b*c*d^6 - a^7*d^7)/(c^3*d^8)) + (b^10
*c^10 - 10*a*b^9*c^9*d + 45*a^2*b^8*c^8*d^2 - 120*a^3*b^7*c^7*d^3 + 210*a^4
*b^6*c^6*d^4 - 252*a^5*b^5*c^5*d^5 + 210*a^6*b^4*c^4*d^6 - 120*a^7*b^3*c^3*
d^7 + 45*a^8*b^2*c^2*d^8 - 10*a^9*b*c*d^9 + a^10*d^10)*sqrt(b*x^4 + a))/x^2
)*((b^7*c^7 - 7*a*b^6*c^6*d + 21*a^2*b^5*c^5*d^2 - 35*a^3*b^4*c^4*d^3 + 35*
a^4*b^3*c^3*d^4 - 21*a^5*b^2*c^2*d^5 + 7*a^6*b*c*d^6 - a^7*d^7)/(c^3*d^8))^
(1/4) + (b^5*c^6*d^2 - 5*a*b^4*c^5*d^3 + 10*a^2*b^3*c^4*d^4 - 10*a^3*b^2*c^
3*d^5 + 5*a^4*b*c^2*d^6 - a^5*c*d^7)*(b*x^4 + a)^(1/4)*((b^7*c^7 - 7*a*b^6*
c^6*d + 21*a^2*b^5*c^5*d^2 - 35*a^3*b^4*c^4*d^3 + 35*a^4*b^3*c^3*d^4 - 21*a
^5*b^2*c^2*d^5 + 7*a^6*b*c*d^6 - a^7*d^7)/(c^3*d^8))^(1/4))/((b^7*c^7 - 7*a
*b^6*c^6*d + 21*a^2*b^5*c^5*d^2 - 35*a^3*b^4*c^4*d^3 + 35*a^4*b^3*c^3*d^4 -
 21*a^5*b^2*c^2*d^5 + 7*a^6*b*c*d^6 - a^7*d^7)*x)) + 4*d*((256*b^7*c^4 - 17
92*a*b^6*c^3*d + 4704*a^2*b^5*c^2*d^2 - 5488*a^3*b^4*c*d^3 + 2401*a^4*b^3*d
^4)/d^8)^(1/4)*arctan((d^2*x*sqrt(((256*b^7*c^4*d^4 - 1792*a*b^6*c^3*d^5 +
4704*a^2*b^5*c^2*d^6 - 5488*a^3*b^4*c*d^7 + 2401*a^4*b^3*d^8)*x^2*sqrt((256
*b^7*c^4 - 1792*a*b^6*c^3*d + 4704*a^2*b^5*c^2*d^2 - 5488*a^3*b^4*c*d^3 + 2
401*a^4*b^3*d^4)/d^8) + (4096*b^10*c^6 - 43008*a*b^9*c^5*d + 188160*a^2*b^8
*c^4*d^2 - 439040*a^3*b^7*c^3*d^3 + 576240*a^4*b^6*c^2*d^4 - 403368*a^5*b^5
*c*d^5 + 117649*a^6*b^4*d^6)*sqrt(b*x^4 + a))/x^2)*((256*b^7*c^4 - 1792*a*b
^6*c^3*d + 4704*a^2*b^5*c^2*d^2 - 5488*a^3*b^4*c*d^3 + 2401*a^4*b^3*d^4)/d^
8)^(1/4) + (64*b^5*c^3*d^2 - 336*a*b^4*c^2*d^3 + 588*a^2*b^3*c*d^4 - 343*a^
3*b^2*d^5)*(b*x^4 + a)^(1/4)*((256*b^7*c^4 - 1792*a*b^6*c^3*d + 4704*a^2*b^
5*c^2*d^2 - 5488*a^3*b^4*c*d^3 + 2401*a^4*b^3*d^4)/d^8)^(1/4))/((256*b^7*c^
4 - 1792*a*b^6*c^3*d + 4704*a^2*b^5*c^2*d^2 - 5488*a^3*b^4*c*d^3 + 2401*a^4
*b^3*d^4)*x)) + 4*d*((b^7*c^7 - 7*a*b^6*c^6*d + 21*a^2*b^5*c^5*d^2 - 35*a^3
```

```
*b^4*c^4*d^3 + 35*a^4*b^3*c^3*d^4 - 21*a^5*b^2*c^2*d^5 + 7*a^6*b*c*d^6 - a^
7*d^7)/(c^3*d^8))^(1/4)*log(-(c^2*d^6*x*((b^7*c^7 - 7*a*b^6*c^6*d + 21*a^2*
b^5*c^5*d^2 - 35*a^3*b^4*c^4*d^3 + 35*a^4*b^3*c^3*d^4 - 21*a^5*b^2*c^2*d^5
+ 7*a^6*b*c*d^6 - a^7*d^7)/(c^3*d^8))^(3/4) + (b^5*c^5 - 5*a*b^4*c^4*d + 10
*a^2*b^3*c^3*d^2 - 10*a^3*b^2*c^2*d^3 + 5*a^4*b*c*d^4 - a^5*d^5)*(b*x^4 + a
)^(1/4))/x) - 4*d*((b^7*c^7 - 7*a*b^6*c^6*d + 21*a^2*b^5*c^5*d^2 - 35*a^3*b
^4*c^4*d^3 + 35*a^4*b^3*c^3*d^4 - 21*a^5*b^2*c^2*d^5 + 7*a^6*b*c*d^6 - a^7*
d^7)/(c^3*d^8))^(1/4)*log((c^2*d^6*x*((b^7*c^7 - 7*a*b^6*c^6*d + 21*a^2*b^5
*c^5*d^2 - 35*a^3*b^4*c^4*d^3 + 35*a^4*b^3*c^3*d^4 - 21*a^5*b^2*c^2*d^5 + 7
*a^6*b*c*d^6 - a^7*d^7)/(c^3*d^8))^(3/4) - (b^5*c^5 - 5*a*b^4*c^4*d + 10*a^
2*b^3*c^3*d^2 - 10*a^3*b^2*c^2*d^3 + 5*a^4*b*c*d^4 - a^5*d^5)*(b*x^4 + a)^(
1/4))/x) - d*((256*b^7*c^4 - 1792*a*b^6*c^3*d + 4704*a^2*b^5*c^2*d^2 - 5488
*a^3*b^4*c*d^3 + 2401*a^4*b^3*d^4)/d^8)^(1/4)*log(-(d^6*x*((256*b^7*c^4 - 1
792*a*b^6*c^3*d + 4704*a^2*b^5*c^2*d^2 - 5488*a^3*b^4*c*d^3 + 2401*a^4*b^3*
d^4)/d^8)^(3/4) + (64*b^5*c^3 - 336*a*b^4*c^2*d + 588*a^2*b^3*c*d^2 - 343*a
^3*b^2*d^3)*(b*x^4 + a)^(1/4))/x) + d*((256*b^7*c^4 - 1792*a*b^6*c^3*d + 47
04*a^2*b^5*c^2*d^2 - 5488*a^3*b^4*c*d^3 + 2401*a^4*b^3*d^4)/d^8)^(1/4)*log(
(d^6*x*((256*b^7*c^4 - 1792*a*b^6*c^3*d + 4704*a^2*b^5*c^2*d^2 - 5488*a^3*b
^4*c*d^3 + 2401*a^4*b^3*d^4)/d^8)^(3/4) - (64*b^5*c^3 - 336*a*b^4*c^2*d + 5
88*a^2*b^3*c*d^2 - 343*a^3*b^2*d^3)*(b*x^4 + a)^(1/4))/x))/d
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^4 + a\right)^{\frac{7}{4}}}{dx^4 + c}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^(7/4)/(d*x^4+c),x, algorithm="giac")
```

```
[Out] integrate((b*x^4 + a)^(7/4)/(d*x^4 + c), x)
```

maple [F] time = 0.59, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^4 + a\right)^{\frac{7}{4}}}{d\,x^4 + c}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^4+a)^(7/4)/(d*x^4+c),x)
```

```
[Out] int((b*x^4+a)^(7/4)/(d*x^4+c),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^4 + a\right)^{\frac{7}{4}}}{dx^4 + c}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^(7/4)/(d*x^4+c),x, algorithm="maxima")
```

```
[Out] integrate((b*x^4 + a)^(7/4)/(d*x^4 + c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(b\,x^4 + a\right)^{7/4}}{d\,x^4 + c}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^4)^(7/4)/(c + d*x^4),x)
```

```
[Out] int((a + b*x^4)^(7/4)/(c + d*x^4), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+bx^4\right)^{\frac{7}{4}}}{c+dx^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**4+a)**(7/4)/(d*x**4+c),x)
```

```
[Out] Integral((a + b*x**4)**(7/4)/(c + d*x**4), x)
```

3.113 $\int \frac{(a+bx^4)^{3/4}}{c+dx^4}\,dx$

Optimal. Leaf size=173

$$\frac{b^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a+bx^4}}\right)}{2d}+\frac{b^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a+bx^4}}\right)}{2d}-\frac{(bc-ad)^{3/4}\tan^{-1}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\,\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}d}-\frac{(bc-ad)^{3/4}\tanh^{-1}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\,\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}d}$$

Rubi [A] time = 0.10, antiderivative size = 173, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {408, 240, 212, 206, 203, 377, 208, 205}

$$\frac{b^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a+bx^4}}\right)}{2d}+\frac{b^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a+bx^4}}\right)}{2d}-\frac{(bc-ad)^{3/4}\tan^{-1}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\,\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}d}-\frac{(bc-ad)^{3/4}\tanh^{-1}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\,\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^4)^(3/4)/(c + d*x^4),x]
```

```
[Out] (b^(3/4)*ArcTan[(b^(1/4)*x)/(a + b*x^4)^(1/4)])/(2*d) - ((b*c - a*d)^(3/4)*
ArcTan[((b*c - a*d)^(1/4)*x)/(c^(1/4)*(a + b*x^4)^(1/4))])/(2*c^(3/4)*d) +
(b^(3/4)*ArcTanh[(b^(1/4)*x)/(a + b*x^4)^(1/4)])/(2*d) - ((b*c - a*d)^(3/4)
*ArcTanh[((b*c - a*d)^(1/4)*x)/(c^(1/4)*(a + b*x^4)^(1/4))])/(2*c^(3/4)*d)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
```

```
n]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 408

```
Int[((a_) + (b_.)*(x_)^4)^(p_)/((c_) + (d_.)*(x_)^4), x_Symbol] :> Dist[b/d
, Int[(a + b*x^4)^(p - 1), x], x] - Dist[(b*c - a*d)/d, Int[(a + b*x^4)^(p
- 1)/(c + d*x^4), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] &&
(EqQ[p, 3/4] || EqQ[p, 5/4])
```

Rule steps

$$\begin{aligned}\int \frac{\left(a+bx^4\right)^{3/4}}{c+dx^4}\,dx &= \frac{b\int \frac{1}{\sqrt[4]{a+bx^4}}\,dx}{d}-\frac{(bc-ad)\int \frac{1}{\sqrt[4]{a+bx^4}\left(c+dx^4\right)}\,dx}{d}\\ &= \frac{b\,\mathrm{Subst}\left(\int \frac{1}{1-bx^4}\,dx,x,\frac{x}{\sqrt[4]{a+bx^4}}\right)}{d}-\frac{(bc-ad)\,\mathrm{Subst}\left(\int \frac{1}{c-(bc-ad)x^4}\,dx,x,\frac{x}{\sqrt[4]{a+bx^4}}\right)}{d}\\ &= \frac{b\,\mathrm{Subst}\left(\int \frac{1}{1-\sqrt{b}\,x^2}\,dx,x,\frac{x}{\sqrt[4]{a+bx^4}}\right)}{2d}+\frac{b\,\mathrm{Subst}\left(\int \frac{1}{1+\sqrt{b}\,x^2}\,dx,x,\frac{x}{\sqrt[4]{a+bx^4}}\right)}{2d}-\frac{(bc-ad)\,\mathrm{Subst}\left(\right.}{}\\ &= \frac{b^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a+bx^4}}\right)}{2d}-\frac{(bc-ad)^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{bc-ad}\,x}{\sqrt[4]{c}\,\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}d}+\frac{b^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a+bx^4}}\right)}{2d}-\frac{(bc-ad)}{}\end{aligned}$$

Mathematica [C] time = 0.18, size = 161, normalized size = 0.93

$$\frac{5acx\left(a+bx^4\right)^{3/4}F_1\left(\frac{1}{4};-\frac{3}{4},1;\frac{5}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{\left(c+dx^4\right)\left(x^4\left(3bcF_1\left(\frac{5}{4};\frac{1}{4},1;\frac{9}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)-4adF_1\left(\frac{5}{4};-\frac{3}{4},2;\frac{9}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)\right)+5acF_1\left(\frac{1}{4};-\frac{3}{4},1;\frac{5}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^4)^(3/4)/(c + d*x^4),x]
```

```
[Out] (5*a*c*x*(a + b*x^4)^(3/4)*AppellF1[1/4, -3/4, 1, 5/4, -((b*x^4)/a), -((d*x
^4)/c)])/((c + d*x^4)*(5*a*c*AppellF1[1/4, -3/4, 1, 5/4, -((b*x^4)/a), -((d
*x^4)/c)] + x^4*(-4*a*d*AppellF1[5/4, -3/4, 2, 9/4, -((b*x^4)/a), -((d*x^4)
/c)] + 3*b*c*AppellF1[5/4, 1/4, 1, 9/4, -((b*x^4)/a), -((d*x^4)/c)])))
```

IntegrateAlgebraic [C] time = 0.90, size = 281, normalized size = 1.62

$$\frac{b^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a+bx^4}}\right)}{2d}+\frac{b^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a+bx^4}}\right)}{2d}-\frac{\left(\frac{1}{4}+\frac{i}{4}\right)(bc-ad)^{3/4}\tan^{-1}\left(\frac{\frac{\left(\frac{1}{2}-\frac{i}{2}\right)x^2\sqrt[4]{bc-ad}}{\sqrt[4]{c}}-\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt[4]{c}\,\sqrt{a+bx^4}}{\sqrt[4]{bc-ad}}}{x\sqrt[4]{a+bx^4}}\right)}{c^{3/4}d}-\frac{\left(\frac{1}{4}+\frac{i}{4}\right)(bc-ad)^{3/4}\tanh^{-1}\left(\frac{\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt[4]{c}\,\sqrt{a+bx^4}}{\sqrt[4]{bc-ad}}+\frac{\left(\frac{1}{2}-\frac{i}{2}\right)x^2\sqrt[4]{bc-ad}}{\sqrt[4]{c}}}{x\sqrt[4]{a+bx^4}}\right)}{c^{3/4}d}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x^4)^(3/4)/(c + d*x^4),x]
```

```
[Out] (b^(3/4)*ArcTan[(b^(1/4)*x)/(a + b*x^4)^(1/4)])/(2*d) - ((1/4 + I/4)*(b*c -
 a*d)^(3/4)*ArcTan[(((1/2 - I/2)*(b*c - a*d)^(1/4)*x^2)/c^(1/4) - ((1/2 + I
```

```
/2)*c^(1/4)*Sqrt[a + b*x^4])/(b*c - a*d)^(1/4))/(x*(a + b*x^4)^(1/4))])/(c^
(3/4)*d) + (b^(3/4)*ArcTanh[(b^(1/4)*x)/(a + b*x^4)^(1/4)])/(2*d) - ((1/4 +
 I/4)*(b*c - a*d)^(3/4)*ArcTanh[(((1/2 - I/2)*(b*c - a*d)^(1/4)*x^2)/c^(1/4
) + ((1/2 + I/2)*c^(1/4)*Sqrt[a + b*x^4])/(b*c - a*d)^(1/4))/(x*(a + b*x^4)
^(1/4))])/(c^(3/4)*d)
```

fricas [B] time = 1.83, size = 844, normalized size = 4.88

$$\left(\frac{b^{3} c^{3} - 3 a b^{2} c^{2} d + 3 a^{2} b c d^{2} - a^{3} d^{3}}{c^{3} d^{4}}\right)^{\frac{1}{4}} \arctan\left(-\frac{c d x \sqrt{\frac{\left(b^{3} c^{4} d^{2} - 3 a b^{2} c^{3} d^{3} + 3 a^{2} b c^{2} d^{4} - a^{3} c d^{5}\right) x^{2} \sqrt{\frac{b^{3} c^{3} - 3 a b^{2} c^{2} d + 3 a^{2} b c d^{2} - a^{3} d^{3}}{c^{3} d^{4}}} + \left(b^{4} c^{4} - 4 a b^{3} c^{3} d + 6 a^{2} b^{2} c^{2} d^{2} - 4 a^{3} b c d^{3} + a^{4} d^{4}\right) \sqrt{b x^{4} + a}}{x^{2}}} \left(\frac{b^{3} c^{3} - 3 a b^{2} c^{2} d + 3 a^{2} b c d^{2} - a^{3} d^{3}}{c^{3} d^{4}}\right)^{\frac{1}{4}} - \left(b^{2} c^{3} d - 2 a b c^{2} d^{2} + a^{2} c d^{3}\right) \left(b x^{4} + a\right)^{\frac{1}{4}} \left(\frac{b^{3} c^{3} - 3 a b^{2} c^{2} d + 3 a^{2} b c d^{2} - a^{3} d^{3}}{c^{3} d^{4}}\right)^{\frac{1}{4}}}{\left(b^{3} c^{3} - 3 a b^{2} c^{2} d + 3 a^{2} b c d^{2} - a^{3} d^{3}\right) x}\right) + \left(\frac{b^{3}}{d^{4}}\right)^{\frac{1}{4}} \arctan\left(-\frac{\left(b x^{4} + a\right)^{\frac{1}{4}} b^{2} d \left(\frac{b^{3}}{d^{4}}\right)^{\frac{1}{4}} - d x \left(\frac{b^{3}}{d^{4}}\right)^{\frac{1}{4}} \sqrt{\frac{b^{3} d^{2} x^{2} \sqrt{\frac{b^{3}}{d^{4}}} + \sqrt{b x^{4} + a} b^{4}}{x^{2}}}}{b^{3} x}\right) - \frac{1}{4} \left(\frac{b^{3} c^{3} - 3 a b^{2} c^{2} d + 3 a^{2} b c d^{2} - a^{3} d^{3}}{c^{3} d^{4}}\right)^{\frac{1}{4}} \log\left(\frac{c^{2} d^{3} x \left(\frac{b^{3} c^{3} - 3 a b^{2} c^{2} d + 3 a^{2} b c d^{2} - a^{3} d^{3}}{c^{3} d^{4}}\right)^{\frac{3}{4}} + \left(b x^{4} + a\right)^{\frac{1}{4}} \left(b^{2} c^{2} - 2 a b c d + a^{2} d^{2}\right)}{x}\right) + \frac{1}{4} \left(\frac{b^{3} c^{3} - 3 a b^{2} c^{2} d + 3 a^{2} b c d^{2} - a^{3} d^{3}}{c^{3} d^{4}}\right)^{\frac{1}{4}} \log\left(-\frac{c^{2} d^{3} x \left(\frac{b^{3} c^{3} - 3 a b^{2} c^{2} d + 3 a^{2} b c d^{2} - a^{3} d^{3}}{c^{3} d^{4}}\right)^{\frac{3}{4}} - \left(b x^{4} + a\right)^{\frac{1}{4}} \left(b^{2} c^{2} - 2 a b c d + a^{2} d^{2}\right)}{x}\right) + \frac{1}{4} \left(\frac{b^{3}}{d^{4}}\right)^{\frac{1}{4}} \log\left(\frac{d^{3} x \left(\frac{b^{3}}{d^{4}}\right)^{\frac{3}{4}} + \left(b x^{4} + a\right)^{\frac{1}{4}} b^{2}}{x}\right) - \frac{1}{4} \left(\frac{b^{3}}{d^{4}}\right)^{\frac{1}{4}} \log\left(-\frac{d^{3} x \left(\frac{b^{3}}{d^{4}}\right)^{\frac{3}{4}} - \left(b x^{4} + a\right)^{\frac{1}{4}} b^{2}}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^(3/4)/(d*x^4+c),x, algorithm="fricas")
```

```
[Out] ((b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)/(c^3*d^4))^(1/4)*arcta
n(-(c*d*x*sqrt(((b^3*c^4*d^2 - 3*a*b^2*c^3*d^3 + 3*a^2*b*c^2*d^4 - a^3*c*d^
5)*x^2*sqrt((b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)/(c^3*d^4))
+ (b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*s
qrt(b*x^4 + a))/x^2)*((b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)/(
c^3*d^4))^(1/4) - (b^2*c^3*d - 2*a*b*c^2*d^2 + a^2*c*d^3)*(b*x^4 + a)^(1/4)
*((b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)/(c^3*d^4))^(1/4))/((b
^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*x)) + (b^3/d^4)^(1/4)*arc
tan(-((b*x^4 + a)^(1/4)*b^2*d*(b^3/d^4)^(1/4) - d*x*(b^3/d^4)^(1/4)*sqrt((b
^3*d^2*x^2*sqrt(b^3/d^4) + sqrt(b*x^4 + a)*b^4)/x^2))/(b^3*x)) - 1/4*((b^3*
c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)/(c^3*d^4))^(1/4)*log((c^2*d^
3*x*((b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)/(c^3*d^4))^(3/4) +
 (b*x^4 + a)^(1/4)*(b^2*c^2 - 2*a*b*c*d + a^2*d^2))/x) + 1/4*((b^3*c^3 - 3*
a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)/(c^3*d^4))^(1/4)*log(-(c^2*d^3*x*((b
^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)/(c^3*d^4))^(3/4) - (b*x^4
 + a)^(1/4)*(b^2*c^2 - 2*a*b*c*d + a^2*d^2))/x) + 1/4*(b^3/d^4)^(1/4)*log((
d^3*x*(b^3/d^4)^(3/4) + (b*x^4 + a)^(1/4)*b^2)/x) - 1/4*(b^3/d^4)^(1/4)*log
(-(d^3*x*(b^3/d^4)^(3/4) - (b*x^4 + a)^(1/4)*b^2)/x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^4 + a\right)^{\frac{3}{4}}}{dx^4 + c}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^(3/4)/(d*x^4+c),x, algorithm="giac")
```

```
[Out] integrate((b*x^4 + a)^(3/4)/(d*x^4 + c), x)
```

maple [F] time = 0.58, size = 0, normalized size = 0.00

$$\int \frac{\left(b\, x^4 + a\right)^{\frac{3}{4}}}{d\, x^4 + c}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^4+a)^(3/4)/(d*x^4+c),x)
```

```
[Out] int((b*x^4+a)^(3/4)/(d*x^4+c),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^4 + a\right)^{\frac{3}{4}}}{dx^4 + c}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^(3/4)/(d*x^4+c),x, algorithm="maxima")
```

```
[Out] integrate((b*x^4 + a)^(3/4)/(d*x^4 + c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(b\,x^4+a\right)^{3/4}}{d\,x^4+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^4)^(3/4)/(c + d*x^4),x)
```

```
[Out] int((a + b*x^4)^(3/4)/(c + d*x^4), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+bx^4\right)^{\frac{3}{4}}}{c+dx^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**4+a)**(3/4)/(d*x**4+c),x)
```

```
[Out] Integral((a + b*x**4)**(3/4)/(c + d*x**4), x)
```

3.114 $\int \frac{1}{\sqrt[4]{a+bx^4}\left(c+dx^4\right)}\,dx$

Optimal. Leaf size=105

$$\frac{\tan^{-1}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}\sqrt[4]{bc-ad}}+\frac{\tanh^{-1}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}\sqrt[4]{bc-ad}}$$

Rubi [A] time = 0.06, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {377, 212, 208, 205}

$$\frac{\tan^{-1}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}\sqrt[4]{bc-ad}}+\frac{\tanh^{-1}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}\sqrt[4]{bc-ad}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^4)^(1/4)*(c + d*x^4)),x]
```

```
[Out] ArcTan[((b*c - a*d)^(1/4)*x)/(c^(1/4)*(a + b*x^4)^(1/4))]/(2*c^(3/4)*(b*c -
 a*d)^(1/4)) + ArcTanh[((b*c - a*d)^(1/4)*x)/(c^(1/4)*(a + b*x^4)^(1/4))]/(
2*c^(3/4)*(b*c - a*d)^(1/4))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt[4]{a+bx^4}\left(c+dx^4\right)}\,dx &= \text{Subst}\left(\int \frac{1}{c-(bc-ad)x^4}\,dx,x,\frac{x}{\sqrt[4]{a+bx^4}}\right)\\ &= \frac{\text{Subst}\left(\int \frac{1}{\sqrt{c}-\sqrt{bc-ad}\,x^2}\,dx,x,\frac{x}{\sqrt[4]{a+bx^4}}\right)}{2\sqrt{c}}+\frac{\text{Subst}\left(\int \frac{1}{\sqrt{c}+\sqrt{bc-ad}\,x^2}\,dx,x,\frac{x}{\sqrt[4]{a+bx^4}}\right)}{2\sqrt{c}}\\ &= \frac{\tan^{-1}\left(\frac{\sqrt[4]{bc-ad}\,x}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}\sqrt[4]{bc-ad}}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{bc-ad}\,x}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}\sqrt[4]{bc-ad}}\end{aligned}$$

Mathematica [A] time = 0.05, size = 84, normalized size = 0.80

$$\frac{\tan^{-1}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)+\tanh^{-1}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}\sqrt[4]{bc-ad}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x^4)^(1/4)*(c + d*x^4)),x]
```

```
[Out] (ArcTan[((b*c - a*d)^(1/4)*x)/(c^(1/4)*(a + b*x^4)^(1/4))] + ArcTanh[((b*c
- a*d)^(1/4)*x)/(c^(1/4)*(a + b*x^4)^(1/4))])/(2*c^(3/4)*(b*c - a*d)^(1/4))
```

IntegrateAlgebraic [C] time = 0.83, size = 213, normalized size = 2.03

$$\frac{\left(\frac{1}{4}+\frac{i}{4}\right)\tan^{-1}\left(\frac{\frac{\left(\frac{1}{2}-\frac{i}{2}\right)x^2\sqrt[4]{bc-ad}}{\sqrt[4]{c}}-\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt[4]{c}\sqrt{a+bx^4}}{\sqrt[4]{bc-ad}}}{x\sqrt[4]{a+bx^4}}\right)}{c^{3/4}\sqrt[4]{bc-ad}}+\frac{\left(\frac{1}{4}+\frac{i}{4}\right)\tanh^{-1}\left(\frac{\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt[4]{c}\sqrt{a+bx^4}}{\sqrt[4]{bc-ad}}+\frac{\left(\frac{1}{2}-\frac{i}{2}\right)x^2\sqrt[4]{bc-ad}}{\sqrt[4]{c}}}{x\sqrt[4]{a+bx^4}}\right)}{c^{3/4}\sqrt[4]{bc-ad}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((a + b*x^4)^(1/4)*(c + d*x^4)),x]
```

```
[Out] ((1/4 + I/4)*ArcTan[(((1/2 - I/2)*(b*c - a*d)^(1/4)*x^2)/c^(1/4) - ((1/2 +
I/2)*c^(1/4)*Sqrt[a + b*x^4])/(b*c - a*d)^(1/4))/(x*(a + b*x^4)^(1/4))]])/(c
^(3/4)*(b*c - a*d)^(1/4)) + ((1/4 + I/4)*ArcTanh[(((1/2 - I/2)*(b*c - a*d)^
(1/4)*x^2)/c^(1/4) + ((1/2 + I/2)*c^(1/4)*Sqrt[a + b*x^4])/(b*c - a*d)^(1/4
))/(x*(a + b*x^4)^(1/4))]])/(c^(3/4)*(b*c - a*d)^(1/4))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a)^(1/4)/(d*x^4+c),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^4+a\right)^{\frac{1}{4}}\left(dx^4+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a)^(1/4)/(d*x^4+c),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^4 + a)^(1/4)*(d*x^4 + c)), x)
```

maple [F] time = 0.57, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,x^4+a\right)^{\frac{1}{4}}\left(d\,x^4+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^4+a)^(1/4)/(d*x^4+c),x)
```

```
[Out] int(1/(b*x^4+a)^(1/4)/(d*x^4+c),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^4+a\right)^{\frac{1}{4}}\left(dx^4+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a)^(1/4)/(d*x^4+c),x, algorithm="maxima")
[Out] integrate(1/((b*x^4 + a)^(1/4)*(d*x^4 + c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(b\,x^4+a\right)^{1/4}\,\left(d\,x^4+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^4)^(1/4)*(c + d*x^4)),x)
[Out] int(1/((a + b*x^4)^(1/4)*(c + d*x^4)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[4]{a+bx^4}\,\left(c+dx^4\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**4+a)**(1/4)/(d*x**4+c),x)
[Out] Integral(1/((a + b*x**4)**(1/4)*(c + d*x**4)), x)
```

3.115 $\int \frac{1}{(a+bx^4)^{5/4}(c+dx^4)} dx$

Optimal. Leaf size=134

$$-\frac{d\tan^{-1}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}(bc-ad)^{5/4}}-\frac{d\tanh^{-1}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}(bc-ad)^{5/4}}+\frac{bx}{a\sqrt[4]{a+bx^4}\,(bc-ad)}$$

Rubi [A] time = 0.10, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {382, 377, 212, 208, 205}

$$-\frac{d\tan^{-1}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}(bc-ad)^{5/4}}-\frac{d\tanh^{-1}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}(bc-ad)^{5/4}}+\frac{bx}{a\sqrt[4]{a+bx^4}\,(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^4)^(5/4)*(c + d*x^4)),x]
```

```
[Out] (b*x)/(a*(b*c - a*d)*(a + b*x^4)^(1/4)) - (d*ArcTan[((b*c - a*d)^(1/4)*x)/(
c^(1/4)*(a + b*x^4)^(1/4))])/(2*c^(3/4)*(b*c - a*d)^(5/4)) - (d*ArcTanh[((b
*c - a*d)^(1/4)*x)/(c^(1/4)*(a + b*x^4)^(1/4))])/(2*c^(3/4)*(b*c - a*d)^(5/
4))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)), I
nt[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q}, x
] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !LtQ
[q, -1]) && NeQ[p, -1]
```

Rubi steps

$$\int \frac{1}{\left(a+bx^4\right)^{5/4}\left(c+dx^4\right)}\,dx = \frac{bx}{a(bc-ad)\sqrt[4]{a+bx^4}} - \frac{d\int \frac{1}{\sqrt[4]{a+bx^4}\left(c+dx^4\right)}\,dx}{bc-ad}$$

$$= \frac{bx}{a(bc-ad)\sqrt[4]{a+bx^4}} - \frac{d\,\text{Subst}\left(\int \frac{1}{c-(bc-ad)x^4}\,dx, x, \frac{x}{\sqrt[4]{a+bx^4}}\right)}{bc-ad}$$

$$= \frac{bx}{a(bc-ad)\sqrt[4]{a+bx^4}} - \frac{d\,\text{Subst}\left(\int \frac{1}{\sqrt{c}-\sqrt{bc-ad}\,x^2}\,dx, x, \frac{x}{\sqrt[4]{a+bx^4}}\right)}{2\sqrt{c}\,(bc-ad)} - \frac{d\,\text{Subst}\left(\int \frac{1}{\sqrt{c}+\sqrt{b}\cdots}\right.}{2\sqrt{c}\,(\cdots}$$

$$= \frac{bx}{a(bc-ad)\sqrt[4]{a+bx^4}} - \frac{d\tan^{-1}\left(\frac{\sqrt[4]{bc-ad}\,x}{\sqrt[4]{c}\,\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}(bc-ad)^{5/4}} - \frac{d\tanh^{-1}\left(\frac{\sqrt[4]{bc-ad}\,x}{\sqrt[4]{c}\,\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}(bc-ad)^{5/4}}$$

Mathematica [C] time = 0.58, size = 256, normalized size = 1.91

$$-\frac{45c^3\left(a+bx^4\right)^2 {}_2F_1\left(\frac{1}{4},1;\frac{5}{4};\frac{(bc-ad)x^4}{c\left(bx^4+a\right)}\right)-45c^3\left(a+bx^4\right)^2+36c^2dx^4\left(a+bx^4\right)^2 {}_2F_1\left(\frac{1}{4},1;\frac{5}{4};\frac{(bc-ad)x^4}{c\left(bx^4+a\right)}\right)-36c^2dx^4\left(a+bx^4\right)^2+4dx^{12}(bc-ad)^2 {}_2F_1\left(2,\frac{9}{4};\frac{13}{4};\frac{(bc-ad)x^4}{c\left(bx^4+a\right)}\right)+4cx^8(bc-ad)^2 {}_2F_1\left(2,\frac{9}{4};\frac{13}{4};\frac{(bc-ad)x^4}{c\left(bx^4+a\right)}\right)}{9c^3x^3\left(a+bx^4\right)^{9/4}(ad-bc)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + b*x^4)^(5/4)*(c + d*x^4)),x]
```

```
[Out] -1/9*(-45*c^3*(a + b*x^4)^2 - 36*c^2*d*x^4*(a + b*x^4)^2 + 45*c^3*(a + b*x^
4)^2*Hypergeometric2F1[1/4, 1, 5/4, ((b*c - a*d)*x^4)/(c*(a + b*x^4))] + 36
*c^2*d*x^4*(a + b*x^4)^2*Hypergeometric2F1[1/4, 1, 5/4, ((b*c - a*d)*x^4)/(
c*(a + b*x^4))] + 4*c*(b*c - a*d)^2*x^8*Hypergeometric2F1[2, 9/4, 13/4, ((b
*c - a*d)*x^4)/(c*(a + b*x^4))] + 4*d*(b*c - a*d)^2*x^12*Hypergeometric2F1[
2, 9/4, 13/4, ((b*c - a*d)*x^4)/(c*(a + b*x^4))])/(c^3*(-(b*c) + a*d)*x^3*(
a + b*x^4)^(9/4))
```

IntegrateAlgebraic [C] time = 1.58, size = 243, normalized size = 1.81

$$-\frac{\left(\frac{1}{4}+\frac{i}{4}\right)d\tan^{-1}\left(\frac{\frac{\left(\frac{1}{2}-\frac{i}{2}\right)x^2\sqrt[4]{bc-ad}}{\sqrt[4]{c}}-\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt[4]{c}\,\sqrt{a+bx^4}}{\sqrt[4]{bc-ad}}}{x\sqrt[4]{a+bx^4}}\right)}{c^{3/4}(bc-ad)^{5/4}}-\frac{\left(\frac{1}{4}+\frac{i}{4}\right)d\tanh^{-1}\left(\frac{\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt[4]{c}\,\sqrt{a+bx^4}}{\sqrt[4]{bc-ad}}+\frac{\left(\frac{1}{2}-\frac{i}{2}\right)x^2\sqrt[4]{bc-ad}}{\sqrt[4]{c}}}{x\sqrt[4]{a+bx^4}}\right)}{c^{3/4}(bc-ad)^{5/4}}-\frac{bx}{a\sqrt[4]{a+bx^4}\,(ad-bc)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((a + b*x^4)^(5/4)*(c + d*x^4)),x]
```

```
[Out] -((b*x)/(a*(-(b*c) + a*d)*(a + b*x^4)^(1/4))) - ((1/4 + I/4)*d*ArcTan[(((1/
2 - I/2)*(b*c - a*d)^(1/4)*x^2)/c^(1/4) - ((1/2 + I/2)*c^(1/4)*Sqrt[a + b*x
^4])/(b*c - a*d)^(1/4))/(x*(a + b*x^4)^(1/4))])/(c^(3/4)*(b*c - a*d)^(5/4))
 - ((1/4 + I/4)*d*ArcTanh[(((1/2 - I/2)*(b*c - a*d)^(1/4)*x^2)/c^(1/4) + ((
1/2 + I/2)*c^(1/4)*Sqrt[a + b*x^4])/(b*c - a*d)^(1/4))/(x*(a + b*x^4)^(1/4)
)])/(c^(3/4)*(b*c - a*d)^(5/4))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a)^(5/4)/(d*x^4+c),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^4+a\right)^{\frac{5}{4}}\left(dx^4+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a)^(5/4)/(d*x^4+c),x, algorithm="giac")
[Out] integrate(1/((b*x^4 + a)^(5/4)*(d*x^4 + c)), x)
```

maple [F] time = 0.60, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,x^4+a\right)^{\frac{5}{4}}\left(d\,x^4+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^4+a)^(5/4)/(d*x^4+c),x)
[Out] int(1/(b*x^4+a)^(5/4)/(d*x^4+c),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^4+a\right)^{\frac{5}{4}}\left(dx^4+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a)^(5/4)/(d*x^4+c),x, algorithm="maxima")
[Out] integrate(1/((b*x^4 + a)^(5/4)*(d*x^4 + c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(b\,x^4+a\right)}^{5/4}\,\left(d\,x^4+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^4)^(5/4)*(c + d*x^4)),x)
[Out] int(1/((a + b*x^4)^(5/4)*(c + d*x^4)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+bx^4\right)^{\frac{5}{4}}\left(c+dx^4\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**4+a)**(5/4)/(d*x**4+c),x)
[Out] Integral(1/((a + b*x**4)**(5/4)*(c + d*x**4)), x)
```

3.116 $\int \frac{1}{\left(a+bx^4\right)^{9/4}\left(c+dx^4\right)}\,dx$

Optimal. Leaf size=180

$$\frac{bx(4bc-9ad)}{5a^2\sqrt[4]{a+bx^4}\,(bc-ad)^2}+\frac{d^2\tan^{-1}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}(bc-ad)^{9/4}}+\frac{d^2\tanh^{-1}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}(bc-ad)^{9/4}}+\frac{bx}{5a\left(a+bx^4\right)^{5/4}(bc-ad)}$$

Rubi [A] time = 0.20, antiderivative size = 180, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {414, 527, 12, 377, 212, 208, 205}

$$\frac{bx(4bc-9ad)}{5a^2\sqrt[4]{a+bx^4}\,(bc-ad)^2}+\frac{d^2\tan^{-1}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}(bc-ad)^{9/4}}+\frac{d^2\tanh^{-1}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}(bc-ad)^{9/4}}+\frac{bx}{5a\left(a+bx^4\right)^{5/4}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^4)^(9/4)*(c + d*x^4)),x]
```

```
[Out] (b*x)/(5*a*(b*c - a*d)*(a + b*x^4)^(5/4)) + (b*(4*b*c - 9*a*d)*x)/(5*a^2*(b
*c - a*d)^2*(a + b*x^4)^(1/4)) + (d^2*ArcTan[((b*c - a*d)^(1/4)*x)/(c^(1/4)
*(a + b*x^4)^(1/4))])/(2*c^(3/4)*(b*c - a*d)^(9/4)) + (d^2*ArcTanh[((b*c -
a*d)^(1/4)*x)/(c^(1/4)*(a + b*x^4)^(1/4))])/(2*c^(3/4)*(b*c - a*d)^(9/4))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
```

```
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{1}{\left(a+bx^4\right)^{9/4}\left(c+dx^4\right)}\,dx = \frac{bx}{5a(bc-ad)\left(a+bx^4\right)^{5/4}} - \frac{\int \frac{-4bc+5ad-4bdx^4}{\left(a+bx^4\right)^{5/4}\left(c+dx^4\right)}\,dx}{5a(bc-ad)}$$

$$= \frac{bx}{5a(bc-ad)\left(a+bx^4\right)^{5/4}} + \frac{b(4bc-9ad)x}{5a^2(bc-ad)^2\sqrt[4]{a+bx^4}} + \frac{\int \frac{5a^2d^2}{\sqrt[4]{a+bx^4}\left(c+dx^4\right)}\,dx}{5a^2(bc-ad)^2}$$

$$= \frac{bx}{5a(bc-ad)\left(a+bx^4\right)^{5/4}} + \frac{b(4bc-9ad)x}{5a^2(bc-ad)^2\sqrt[4]{a+bx^4}} + \frac{d^2\int \frac{1}{\sqrt[4]{a+bx^4}\left(c+dx^4\right)}\,dx}{(bc-ad)^2}$$

$$= \frac{bx}{5a(bc-ad)\left(a+bx^4\right)^{5/4}} + \frac{b(4bc-9ad)x}{5a^2(bc-ad)^2\sqrt[4]{a+bx^4}} + \frac{d^2\,\text{Subst}\left(\int \frac{1}{c-(bc-ad)x^4}\,dx, \ldots\right.}{(bc-ad)^2}$$

$$= \frac{bx}{5a(bc-ad)\left(a+bx^4\right)^{5/4}} + \frac{b(4bc-9ad)x}{5a^2(bc-ad)^2\sqrt[4]{a+bx^4}} + \frac{d^2\,\text{Subst}\left(\int \frac{1}{\sqrt{c}-\sqrt{bc-ad}\,x^2}\ldots\right.}{2\sqrt{c}\,(bc-ad)\ldots}$$

$$= \frac{bx}{5a(bc-ad)\left(a+bx^4\right)^{5/4}} + \frac{b(4bc-9ad)x}{5a^2(bc-ad)^2\sqrt[4]{a+bx^4}} + \frac{d^2\tan^{-1}\left(\frac{\sqrt[4]{bc-ad}\,x}{\sqrt[4]{c}\,\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}(bc-ad)^{9/4}} + \ldots$$

Mathematica [C] time = 2.37, size = 621, normalized size = 3.45

$$\frac{80c^2x^{12}(bc-ad)^3\,{}_3F_2\left(2,2,\frac{13}{4};1,\frac{17}{4};\frac{(bc-ad)x^4}{c\left(bx^4+a\right)}\right)+80d^2x^{20}(bc-ad)^3\,{}_3F_2\left(2,2,\frac{13}{4};1,\frac{17}{4};\frac{(bc-ad)x^4}{c\left(bx^4+a\right)}\right)+160cdx^{16}(bc-ad)^3\,{}_3F_2\left(2,2,\frac{13}{4};1,\frac{17}{4};\frac{(bc-ad)x^4}{c\left(bx^4+a\right)}\right)+2925c^5\left(a+bx^4\right)^3\,{}_2F_1\left(\frac{1}{4},1;\frac{5}{4};\frac{(bc-ad)x^4}{c\left(bx^4+a\right)}\right)-2925c^5\left(a+bx^4\right)^3+4680c^4dx^4\left(a+bx^4\right)^3\,{}_2F_1\left(\frac{1}{4},1;\frac{5}{4};\frac{(bc-ad)x^4}{c\left(bx^4+a\right)}\right)-4680c^4dx^4\left(a+bx^4\right)^3-585c^4x^4\left(a+bx^4\right)^2(bc-ad)+2080c^3d^2x^8\left(a+bx^4\right)^3\,{}_2F_1\left(\frac{1}{4},1;\frac{5}{4};\frac{(bc-ad)x^4}{c\left(bx^4+a\right)}\right)-2080c^3d^2x^8\left(a+bx^4\right)^3-936c^3dx^8\left(a+bx^4\right)^2(bc-ad)-416c^2d^2x^{12}\left(a+bx^4\right)^2(bc-ad)+280c^2x^{12}(bc-ad)^3\,{}_2F_1\left(2,\frac{13}{4};\frac{17}{4};\frac{(bc-ad)x^4}{c\left(bx^4+a\right)}\right)+240d^2x^{20}(bc-ad)^3\,{}_2F_1\left(2,\frac{13}{4};\frac{17}{4};\frac{(bc-ad)x^4}{c\left(bx^4+a\right)}\right)+520cdx^{16}(bc-ad)^3\,{}_2F_1\left(2,\frac{13}{4};\frac{17}{4};\frac{(bc-ad)x^4}{c\left(bx^4+a\right)}\right)}{325c^4x^7\left(a+bx^4\right)^{13/4}(bc-ad)^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + b*x^4)^(9/4)*(c + d*x^4)),x]
```

```
[Out] (-585*c^4*(b*c - a*d)*x^4*(a + b*x^4)^2 - 936*c^3*d*(b*c - a*d)*x^8*(a + b*
x^4)^2 - 416*c^2*d^2*(b*c - a*d)*x^12*(a + b*x^4)^2 - 2925*c^5*(a + b*x^4)^
3 - 4680*c^4*d*x^4*(a + b*x^4)^3 - 2080*c^3*d^2*x^8*(a + b*x^4)^3 + 2925*c^
5*(a + b*x^4)^3*Hypergeometric2F1[1/4, 1, 5/4, ((b*c - a*d)*x^4)/(c*(a + b*
x^4))] + 4680*c^4*d*x^4*(a + b*x^4)^3*Hypergeometric2F1[1/4, 1, 5/4, ((b*c
- a*d)*x^4)/(c*(a + b*x^4))] + 2080*c^3*d^2*x^8*(a + b*x^4)^3*Hypergeometri
c2F1[1/4, 1, 5/4, ((b*c - a*d)*x^4)/(c*(a + b*x^4))] + 280*c^2*(b*c - a*d)^
3*x^12*Hypergeometric2F1[2, 13/4, 17/4, ((b*c - a*d)*x^4)/(c*(a + b*x^4))]
+ 520*c*d*(b*c - a*d)^3*x^16*Hypergeometric2F1[2, 13/4, 17/4, ((b*c - a*d)*
```

```
x^4)/(c*(a + b*x^4))] + 240*d^2*(b*c - a*d)^3*x^20*Hypergeometric2F1[2, 13/
4, 17/4, ((b*c - a*d)*x^4)/(c*(a + b*x^4))] + 80*c^2*(b*c - a*d)^3*x^12*Hyp
ergeometricPFQ[{2, 2, 13/4}, {1, 17/4}, ((b*c - a*d)*x^4)/(c*(a + b*x^4))]
+ 160*c*d*(b*c - a*d)^3*x^16*HypergeometricPFQ[{2, 2, 13/4}, {1, 17/4}, ((b
*c - a*d)*x^4)/(c*(a + b*x^4))] + 80*d^2*(b*c - a*d)^3*x^20*HypergeometricP
FQ[{2, 2, 13/4}, {1, 17/4}, ((b*c - a*d)*x^4)/(c*(a + b*x^4))])/(325*c^4*(b
*c - a*d)^2*x^7*(a + b*x^4)^(13/4))
```

IntegrateAlgebraic [C] time = 3.49, size = 283, normalized size = 1.57

$$\frac{-10a^2bdx+5ab^2cx-9ab^2dx^5+4b^3cx^5}{5a^2\left(a+bx^4\right)^{5/4}(ad-bc)^2}+\frac{\left(\frac{1}{4}+\frac{i}{4}\right)d^2\tan^{-1}\left(\frac{\frac{\left(\frac{1}{2}-\frac{i}{2}\right)x^2\sqrt[4]{bc-ad}}{\sqrt[4]{c}}-\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt[4]{c}\sqrt{a+bx^4}}{\sqrt[4]{bc-ad}}}{x\sqrt[4]{a+bx^4}}\right)}{c^{3/4}(bc-ad)^{9/4}}+\frac{\left(\frac{1}{4}+\frac{i}{4}\right)d^2\tanh^{-1}\left(\frac{\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt[4]{c}\sqrt{a+bx^4}}{\sqrt[4]{bc-ad}}+\frac{\left(\frac{1}{2}-\frac{i}{2}\right)x^2\sqrt[4]{bc-ad}}{\sqrt[4]{c}}}{x\sqrt[4]{a+bx^4}}\right)}{c^{3/4}(bc-ad)^{9/4}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((a + b*x^4)^(9/4)*(c + d*x^4)),x]
```

```
[Out] (5*a*b^2*c*x - 10*a^2*b*d*x + 4*b^3*c*x^5 - 9*a*b^2*d*x^5)/(5*a^2*(-(b*c) +
 a*d)^2*(a + b*x^4)^(5/4)) + ((1/4 + I/4)*d^2*ArcTan[(((1/2 - I/2)*(b*c - a
*d)^(1/4)*x^2)/c^(1/4) - ((1/2 + I/2)*c^(1/4)*Sqrt[a + b*x^4])/(b*c - a*d)^
(1/4))/(x*(a + b*x^4)^(1/4))])/(c^(3/4)*(b*c - a*d)^(9/4)) + ((1/4 + I/4)*d
^2*ArcTanh[(((1/2 - I/2)*(b*c - a*d)^(1/4)*x^2)/c^(1/4) + ((1/2 + I/2)*c^(1
/4)*Sqrt[a + b*x^4])/(b*c - a*d)^(1/4))/(x*(a + b*x^4)^(1/4))])/(c^(3/4)*(b
*c - a*d)^(9/4))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a)^(9/4)/(d*x^4+c),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^4+a\right)^{\frac{9}{4}}\left(dx^4+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a)^(9/4)/(d*x^4+c),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^4 + a)^(9/4)*(d*x^4 + c)), x)
```

maple [F] time = 0.60, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,x^4+a\right)^{\frac{9}{4}}\left(d\,x^4+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^4+a)^(9/4)/(d*x^4+c),x)
```

```
[Out] int(1/(b*x^4+a)^(9/4)/(d*x^4+c),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^4+a\right)^{\frac{9}{4}}\left(dx^4+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a)^(9/4)/(d*x^4+c),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^4 + a)^(9/4)*(d*x^4 + c)), x)
```

mupad [F]   time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(b\,x^4+a\right)^{9/4}\,\left(d\,x^4+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^4)^(9/4)*(c + d*x^4)),x)
```

```
[Out] int(1/((a + b*x^4)^(9/4)*(c + d*x^4)), x)
```

sympy [F]   time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+bx^{4}\right)^{\frac{9}{4}}\left(c+dx^{4}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**4+a)**(9/4)/(d*x**4+c),x)
```

```
[Out] Integral(1/((a + b*x**4)**(9/4)*(c + d*x**4)), x)
```

3.117 $\int \frac{1}{\left(a+bx^4\right)^{13/4}\left(c+dx^4\right)}\,dx$

Optimal. Leaf size=233

$$\frac{bx(8bc-17ad)}{45a^2\left(a+bx^4\right)^{5/4}(bc-ad)^2}+\frac{bx\left(113a^2d^2-100abcd+32b^2c^2\right)}{45a^3\sqrt[4]{a+bx^4}\,(bc-ad)^3}-\frac{d^3\tan^{-1}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}(bc-ad)^{13/4}}-\frac{d^3\tanh^{-1}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}(bc-ad)^{13/4}}+\frac{}{9}$$

Rubi [A] time = 0.29, antiderivative size = 233, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {414, 527, 12, 377, 212, 208, 205}

$$\frac{bx\left(113a^2d^2-100abcd+32b^2c^2\right)}{45a^3\sqrt[4]{a+bx^4}\,(bc-ad)^3}+\frac{bx(8bc-17ad)}{45a^2\left(a+bx^4\right)^{5/4}(bc-ad)^2}-\frac{d^3\tan^{-1}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}(bc-ad)^{13/4}}-\frac{d^3\tanh^{-1}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}(bc-ad)^{13/4}}+\frac{bx}{9a\left(a+bx^4\right)^{9/4}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^4)^(13/4)*(c + d*x^4)),x]
```

```
[Out] (b*x)/(9*a*(b*c - a*d)*(a + b*x^4)^(9/4)) + (b*(8*b*c - 17*a*d)*x)/(45*a^2*
(b*c - a*d)^2*(a + b*x^4)^(5/4)) + (b*(32*b^2*c^2 - 100*a*b*c*d + 113*a^2*d
^2)*x)/(45*a^3*(b*c - a*d)^3*(a + b*x^4)^(1/4)) - (d^3*ArcTan[((b*c - a*d)^
(1/4)*x)/(c^(1/4)*(a + b*x^4)^(1/4))])/(2*c^(3/4)*(b*c - a*d)^(13/4)) - (d^
3*ArcTanh[((b*c - a*d)^(1/4)*x)/(c^(1/4)*(a + b*x^4)^(1/4))])/(2*c^(3/4)*(b
*c - a*d)^(13/4))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
```

```
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{1}{\left(a+bx^4\right)^{13/4}\left(c+dx^4\right)}\,dx = \frac{bx}{9a(bc-ad)\left(a+bx^4\right)^{9/4}} - \frac{\int \frac{-8bc+9ad-8bdx^4}{\left(a+bx^4\right)^{9/4}\left(c+dx^4\right)}\,dx}{9a(bc-ad)}$$

$$= \frac{bx}{9a(bc-ad)\left(a+bx^4\right)^{9/4}} + \frac{b(8bc-17ad)x}{45a^2(bc-ad)^2\left(a+bx^4\right)^{5/4}} + \frac{\int \frac{32b^2c^2-68abcd+45a^2d^2+ \cdots}{\left(a+bx^4\right)^{5/4}(c \cdots}}{45a^2(bc- \cdots}$$

$$= \frac{bx}{9a(bc-ad)\left(a+bx^4\right)^{9/4}} + \frac{b(8bc-17ad)x}{45a^2(bc-ad)^2\left(a+bx^4\right)^{5/4}} + \frac{b\left(32b^2c^2-100abcd \cdots\right.}{45a^3(bc-ad)^3\sqrt[4]{\cdots}}$$

$$= \frac{bx}{9a(bc-ad)\left(a+bx^4\right)^{9/4}} + \frac{b(8bc-17ad)x}{45a^2(bc-ad)^2\left(a+bx^4\right)^{5/4}} + \frac{b\left(32b^2c^2-100abcd \cdots\right.}{45a^3(bc-ad)^3\sqrt[4]{\cdots}}$$

$$= \frac{bx}{9a(bc-ad)\left(a+bx^4\right)^{9/4}} + \frac{b(8bc-17ad)x}{45a^2(bc-ad)^2\left(a+bx^4\right)^{5/4}} + \frac{b\left(32b^2c^2-100abcd \cdots\right.}{45a^3(bc-ad)^3\sqrt[4]{\cdots}}$$

$$= \frac{bx}{9a(bc-ad)\left(a+bx^4\right)^{9/4}} + \frac{b(8bc-17ad)x}{45a^2(bc-ad)^2\left(a+bx^4\right)^{5/4}} + \frac{b\left(32b^2c^2-100abcd \cdots\right.}{45a^3(bc-ad)^3\sqrt[4]{\cdots}}$$

$$= \frac{bx}{9a(bc-ad)\left(a+bx^4\right)^{9/4}} + \frac{b(8bc-17ad)x}{45a^2(bc-ad)^2\left(a+bx^4\right)^{5/4}} + \frac{b\left(32b^2c^2-100abcd \cdots\right.}{45a^3(bc-ad)^3\sqrt[4]{\cdots}}$$

Mathematica [A] time = 5.44, size = 231, normalized size = 0.99

$$\frac{bx\left(\left(a+bx^4\right)^2\left(113a^2d^2-100abcd+32b^2c^2\right)+5a^2(bc-ad)^2+a\left(a+bx^4\right)(ad-bc)(17ad-8bc)\right)}{45a^3\left(a+bx^4\right)^{9/4}(bc-ad)^3} - \frac{d^3\left(-\log\left(\sqrt[4]{c}-\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{ax^4+b}}\right)+\log\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{ax^4+b}}+\sqrt[4]{c}\right)+2\tan^{-1}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{ax^4+b}}\right)\right)}{4c^{3/4}(bc-ad)^{13/4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + b*x^4)^(13/4)*(c + d*x^4)),x]
```

```
[Out] (b*x*(5*a^2*(b*c - a*d)^2 + a*(-(b*c) + a*d)*(-8*b*c + 17*a*d)*(a + b*x^4)
+ (32*b^2*c^2 - 100*a*b*c*d + 113*a^2*d^2)*(a + b*x^4)^2))/(45*a^3*(b*c - a
*d)^3*(a + b*x^4)^(9/4)) - (d^3*(2*ArcTan[((b*c - a*d)^(1/4)*x)/(c^(1/4)*(b
```

```
 + a*x^4)^(1/4))] - Log[c^(1/4) - ((b*c - a*d)^(1/4)*x)/(b + a*x^4)^(1/4)]
+ Log[c^(1/4) + ((b*c - a*d)^(1/4)*x)/(b + a*x^4)^(1/4)]))/(4*c^(3/4)*(b*c
- a*d)^(13/4))
```

IntegrateAlgebraic [C] time = 5.18, size = 356, normalized size = 1.53

$$\frac{-135a^4bd^2x+135a^3b^2cdx-243a^3b^2d^2x^5-45a^2b^3c^2x+225a^2b^3cdx^5-113a^2b^3d^2x^9-72ab^4c^2x^5+100ab^4cdx^9-32b^5c^2x^9}{45a^3\left(a+bx^4\right)^{9/4}(ad-bc)^3}-\frac{\left(\frac{1}{4}+\frac{i}{4}\right)d^3\tan^{-1}\left(\frac{\frac{\left(\frac{1}{2}-\frac{i}{2}\right)x^2\sqrt[4]{bc-ad}}{\sqrt[4]{c}}-\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt[4]{c}\sqrt{a+bx^4}}{\sqrt[4]{bc-ad}}}{x\sqrt[4]{a+bx^4}}\right)}{c^{3/4}(bc-ad)^{13/4}}-\frac{\left(\frac{1}{4}+\frac{i}{4}\right)d^3\tanh^{-1}\left(\frac{\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt[4]{c}\sqrt{a+bx^4}}{\sqrt[4]{bc-ad}}+\frac{\left(\frac{1}{2}-\frac{i}{2}\right)x^2\sqrt[4]{bc-ad}}{\sqrt[4]{c}}}{x\sqrt[4]{a+bx^4}}\right)}{c^{3/4}(bc-ad)^{13/4}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((a + b*x^4)^(13/4)*(c + d*x^4)),x]
```

```
[Out] (-45*a^2*b^3*c^2*x + 135*a^3*b^2*c*d*x - 135*a^4*b*d^2*x - 72*a*b^4*c^2*x^5
 + 225*a^2*b^3*c*d*x^5 - 243*a^3*b^2*d^2*x^5 - 32*b^5*c^2*x^9 + 100*a*b^4*c
*d*x^9 - 113*a^2*b^3*d^2*x^9)/(45*a^3*(-(b*c) + a*d)^3*(a + b*x^4)^(9/4)) -
 ((1/4 + I/4)*d^3*ArcTan[(((1/2 - I/2)*(b*c - a*d)^(1/4)*x^2)/c^(1/4) - ((1
/2 + I/2)*c^(1/4)*Sqrt[a + b*x^4])/(b*c - a*d)^(1/4))/(x*(a + b*x^4)^(1/4))
])/(c^(3/4)*(b*c - a*d)^(13/4)) - ((1/4 + I/4)*d^3*ArcTanh[(((1/2 - I/2)*(b
*c - a*d)^(1/4)*x^2)/c^(1/4) + ((1/2 + I/2)*c^(1/4)*Sqrt[a + b*x^4])/(b*c -
 a*d)^(1/4))/(x*(a + b*x^4)^(1/4))])/(c^(3/4)*(b*c - a*d)^(13/4))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a)^(13/4)/(d*x^4+c),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^4+a\right)^{\frac{13}{4}}\left(dx^4+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a)^(13/4)/(d*x^4+c),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^4 + a)^(13/4)*(d*x^4 + c)), x)
```

maple [F] time = 0.62, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,x^4+a\right)^{\frac{13}{4}}\left(d\,x^4+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^4+a)^(13/4)/(d*x^4+c),x)
```

```
[Out] int(1/(b*x^4+a)^(13/4)/(d*x^4+c),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^4+a\right)^{\frac{13}{4}}\left(dx^4+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a)^(13/4)/(d*x^4+c),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^4 + a)^(13/4)*(d*x^4 + c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(b\,x^4+a\right)^{13/4}\,\left(d\,x^4+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^4)^(13/4)*(c + d*x^4)),x)
```

```
[Out] int(1/((a + b*x^4)^(13/4)*(c + d*x^4)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a + bx^{4}\right)^{\frac{13}{4}} \left(c + dx^{4}\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**4+a)**(13/4)/(d*x**4+c),x)
```

```
[Out] Integral(1/((a + b*x**4)**(13/4)*(c + d*x**4)), x)
```

3.118 $\int \frac{(a+bx^4)^{11/4}}{(c+dx^4)^2} dx$

Optimal. Leaf size=280

$$-\frac{b^{7/4}(8bc-11ad)\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{8d^3}-\frac{b^{7/4}(8bc-11ad)\tanh^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{8d^3}+\frac{(bc-ad)^{7/4}(3ad+8bc)\tan^{-1}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{8c^{7/4}d^3}$$

Rubi [A] time = 0.36, antiderivative size = 280, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.476, Rules used = {413, 528, 530, 240, 212, 206, 203, 377, 208, 205}

$$-\frac{b^{7/4}(8bc-11ad)\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{8d^3}-\frac{b^{7/4}(8bc-11ad)\tanh^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{8d^3}+\frac{(bc-ad)^{7/4}(3ad+8bc)\tan^{-1}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{8c^{7/4}d^3}+\frac{(bc-ad)^{7/4}(3ad+8bc)\tanh^{-1}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{8c^{7/4}d^3}+\frac{bx\left(a+bx^4\right)^{3/4}(2bc-ad)}{4cd^2}-\frac{x\left(a+bx^4\right)^{7/4}(bc-ad)}{4cd\left(c+dx^4\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^4)^(11/4)/(c + d*x^4)^2,x]
```

```
[Out] (b*(2*b*c - a*d)*x*(a + b*x^4)^(3/4))/(4*c*d^2) - ((b*c - a*d)*x*(a + b*x^4
)^(7/4))/(4*c*d*(c + d*x^4)) - (b^(7/4)*(8*b*c - 11*a*d)*ArcTan[(b^(1/4)*x)
/(a + b*x^4)^(1/4)])/(8*d^3) + ((b*c - a*d)^(7/4)*(8*b*c + 3*a*d)*ArcTan[((
b*c - a*d)^(1/4)*x)/(c^(1/4)*(a + b*x^4)^(1/4))])/(8*c^(7/4)*d^3) - (b^(7/4
)*(8*b*c - 11*a*d)*ArcTanh[(b^(1/4)*x)/(a + b*x^4)^(1/4)])/(8*d^3) + ((b*c
- a*d)^(7/4)*(8*b*c + 3*a*d)*ArcTanh[((b*c - a*d)^(1/4)*x)/(c^(1/4)*(a + b*
x^4)^(1/4))])/(8*c^(7/4)*d^3)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 413

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[((a*d - c*b)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*n*(p +
 1)), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 528

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/
(b*(n*(p + q + 1) + 1)), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 530

```
Int[(((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n_)))/((c_) + (d_.)*
(x_)^(n_)), x_Symbol] :> Dist[f/d, Int[(a + b*x^n)^p, x], x] + Dist[(d*e -
c*f)/d, Int[(a + b*x^n)^p/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, p
, n}, x]
```

Rubi steps

$$\int \frac{\left(a+bx^4\right)^{11/4}}{\left(c+dx^4\right)^2}\,dx = -\frac{(bc-ad)x\left(a+bx^4\right)^{7/4}}{4cd\left(c+dx^4\right)} + \frac{\int \frac{\left(a+bx^4\right)^{3/4}\left(a(bc+3ad)+4b(2bc-ad)x^4\right)}{c+dx^4}dx}{4cd}$$

$$= \frac{b(2bc-ad)x\left(a+bx^4\right)^{3/4}}{4cd^2} - \frac{(bc-ad)x\left(a+bx^4\right)^{7/4}}{4cd\left(c+dx^4\right)} + \frac{\int \frac{-4a\left(2b^2c^2-2abcd-3a^2d^2\right)-4b^2c(8bc-11ad)x^4}{\sqrt[4]{a+bx^4}\left(c+dx^4\right)}}{16cd^2}$$

$$= \frac{b(2bc-ad)x\left(a+bx^4\right)^{3/4}}{4cd^2} - \frac{(bc-ad)x\left(a+bx^4\right)^{7/4}}{4cd\left(c+dx^4\right)} - \frac{\left(b^2(8bc-11ad)\right)\int \frac{1}{\sqrt[4]{a+bx^4}}dx}{4d^3} + \frac{((bc}{}$$

$$= \frac{b(2bc-ad)x\left(a+bx^4\right)^{3/4}}{4cd^2} - \frac{(bc-ad)x\left(a+bx^4\right)^{7/4}}{4cd\left(c+dx^4\right)} - \frac{\left(b^2(8bc-11ad)\right)\text{Subst}\left(\int \frac{1}{1-bx^4}dx, x,\right.}{4d^3}$$

$$= \frac{b(2bc-ad)x\left(a+bx^4\right)^{3/4}}{4cd^2} - \frac{(bc-ad)x\left(a+bx^4\right)^{7/4}}{4cd\left(c+dx^4\right)} - \frac{\left(b^2(8bc-11ad)\right)\text{Subst}\left(\int \frac{1}{1-\sqrt{b}x^2}dx,\right.}{8d^3}$$

$$= \frac{b(2bc-ad)x\left(a+bx^4\right)^{3/4}}{4cd^2} - \frac{(bc-ad)x\left(a+bx^4\right)^{7/4}}{4cd\left(c+dx^4\right)} - \frac{b^{7/4}(8bc-11ad)\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{8d^3} + \frac{(b}{}$$

Mathematica [C] time = 0.96, size = 560, normalized size = 2.00

$$\frac{1}{80}\left(\frac{15a^3\left(-\log\left(\sqrt[4]{c}-\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{ax^4+b}}\right)+\log\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{ax^4+b}}+\sqrt[4]{c}\right)+2\tan^{-1}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{ax^4+b}}\right)\right)}{c^{7/4}\sqrt[4]{bc-ad}}+\frac{10a^2b\left(-\log\left(\sqrt[4]{c}-\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{ax^4+b}}\right)+\log\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{ax^4+b}}+\sqrt[4]{c}\right)+2\tan^{-1}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{ax^4+b}}\right)\right)}{c^{3/4}d\sqrt[4]{bc-ad}}-\frac{32b^3x^5\sqrt[4]{\frac{bx^4}{a}+1}F_1\left(\frac{5}{4};\frac{1}{4},1;\frac{9}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{d^2\sqrt[4]{a+bx^4}}+\frac{44ab^2x^5\sqrt[4]{\frac{bx^4}{a}+1}F_1\left(\frac{5}{4};\frac{1}{4},1;\frac{9}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{cd\sqrt[4]{a+bx^4}}+\frac{20x\left(a+bx^4\right)^{3/4}\left(\frac{(bc-ad)^2}{c\left(c+dx^4\right)}+b^2\right)}{d^2}-\frac{10ab^2\sqrt[4]{c}\left(-\log\left(\sqrt[4]{c}-\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{ax^4+b}}\right)+\log\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{ax^4+b}}+\sqrt[4]{c}\right)+2\tan^{-1}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{ax^4+b}}\right)\right)}{d^2\sqrt[4]{bc-ad}}\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^4)^(11/4)/(c + d*x^4)^2,x]
```

```
[Out] ((20*x*(a + b*x^4)^(3/4)*(b^2 + (b*c - a*d)^2/(c*(c + d*x^4))))/d^2 - (32*b
^3*x^5*(1 + (b*x^4)/a)^(1/4)*AppellF1[5/4, 1/4, 1, 9/4, -((b*x^4)/a), -((d*
x^4)/c)])/(d^2*(a + b*x^4)^(1/4)) + (44*a*b^2*x^5*(1 + (b*x^4)/a)^(1/4)*App
ellF1[5/4, 1/4, 1, 9/4, -((b*x^4)/a), -((d*x^4)/c)])/(c*d*(a + b*x^4)^(1/4)
) + (15*a^3*(2*ArcTan[((b*c - a*d)^(1/4)*x)/(c^(1/4)*(b + a*x^4)^(1/4))] -
Log[c^(1/4) - ((b*c - a*d)^(1/4)*x)/(b + a*x^4)^(1/4)] + Log[c^(1/4) + ((b*
c - a*d)^(1/4)*x)/(b + a*x^4)^(1/4)]))/(c^(7/4)*(b*c - a*d)^(1/4)) - (10*a*
b^2*c^(1/4)*(2*ArcTan[((b*c - a*d)^(1/4)*x)/(c^(1/4)*(b + a*x^4)^(1/4))] -
Log[c^(1/4) - ((b*c - a*d)^(1/4)*x)/(b + a*x^4)^(1/4)] + Log[c^(1/4) + ((b*
c - a*d)^(1/4)*x)/(b + a*x^4)^(1/4)]))/(d^2*(b*c - a*d)^(1/4)) + (10*a^2*b*
(2*ArcTan[((b*c - a*d)^(1/4)*x)/(c^(1/4)*(b + a*x^4)^(1/4))] - Log[c^(1/4)
- ((b*c - a*d)^(1/4)*x)/(b + a*x^4)^(1/4)] + Log[c^(1/4) + ((b*c - a*d)^(1/
4)*x)/(b + a*x^4)^(1/4)]))/(c^(3/4)*d*(b*c - a*d)^(1/4)))/80
```

IntegrateAlgebraic [C] time = 2.76, size = 379, normalized size = 1.35

$$\frac{\left(a+bx^4\right)^{3/4}\left(a^2d^2x-2abcdx+2b^2c^2x+b^2cdx^5\right)}{4cd^2\left(c+dx^4\right)}+\frac{\left(11ab^{7/4}d-8b^{11/4}c\right)\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{8d^3}+\frac{\left(11ab^{7/4}d-8b^{11/4}c\right)\tanh^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{8d^3}+\frac{\left(\frac{1}{16}+\frac{i}{16}\right)(3ad+8bc)(bc-ad)^{7/4}\tan^{-1}\left(\frac{(1-i)x^2\sqrt{bc-ad}-(1+i)\sqrt{c}\sqrt{a+bx^4}}{2\sqrt[4]{c}x\sqrt[4]{a+bx^4}\sqrt[4]{bc-ad}}\right)}{c^{7/4}d^3}+\frac{\left(\frac{1}{16}+\frac{i}{16}\right)(3ad+8bc)(bc-ad)^{7/4}\tanh^{-1}\left(\frac{(1-i)x^2\sqrt{bc-ad}+(1+i)\sqrt{c}\sqrt{a+bx^4}}{2\sqrt[4]{c}x\sqrt[4]{a+bx^4}\sqrt[4]{bc-ad}}\right)}{c^{7/4}d^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x^4)^(11/4)/(c + d*x^4)^2,x]
```

```
[Out] ((a + b*x^4)^(3/4)*(2*b^2*c^2*x - 2*a*b*c*d*x + a^2*d^2*x + b^2*c*d*x^5))/(
4*c*d^2*(c + d*x^4)) + ((-8*b^(11/4)*c + 11*a*b^(7/4)*d)*ArcTan[(b^(1/4)*x)
/(a + b*x^4)^(1/4)])/(8*d^3) + ((1/16 + I/16)*(b*c - a*d)^(7/4)*(8*b*c + 3*
a*d)*ArcTan[((1 - I)*Sqrt[b*c - a*d]*x^2 - (1 + I)*Sqrt[c]*Sqrt[a + b*x^4])
/(2*c^(1/4)*(b*c - a*d)^(1/4)*x*(a + b*x^4)^(1/4))])/(c^(7/4)*d^3) + ((-8*b
```

```
^(11/4)*c + 11*a*b^(7/4)*d)*ArcTanh[(b^(1/4)*x)/(a + b*x^4)^(1/4)])/(8*d^3)
 + ((1/16 + I/16)*(b*c - a*d)^(7/4)*(8*b*c + 3*a*d)*ArcTanh[((1 - I)*Sqrt[b
*c - a*d]*x^2 + (1 + I)*Sqrt[c]*Sqrt[a + b*x^4])/(2*c^(1/4)*(b*c - a*d)^(1/
4)*x*(a + b*x^4)^(1/4))]])/(c^(7/4)*d^3)
```

fricas [**B**] time = 47.32, size = 3308, normalized size = 11.81

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^(11/4)/(d*x^4+c)^2,x, algorithm="fricas")
```

```
[Out] 1/16*(4*(c*d^3*x^4 + c^2*d^2)*((4096*b^11*c^11 - 22528*a*b^10*c^10*d + 4646
4*a^2*b^9*c^9*d^2 - 37664*a^3*b^8*c^8*d^3 - 5071*a^4*b^7*c^7*d^4 + 25641*a^
5*b^6*c^6*d^5 - 7931*a^6*b^5*c^5*d^6 - 6259*a^7*b^4*c^4*d^7 + 2739*a^8*b^3*
c^3*d^8 + 891*a^9*b^2*c^2*d^9 - 297*a^10*b*c*d^10 - 81*a^11*d^11)/(c^7*d^12
))^(1/4)*arctan(-(c^2*d^3*x*sqrt(((4096*b^11*c^14*d^6 - 22528*a*b^10*c^13*d
^7 + 46464*a^2*b^9*c^12*d^8 - 37664*a^3*b^8*c^11*d^9 - 5071*a^4*b^7*c^10*d^
10 + 25641*a^5*b^6*c^9*d^11 - 7931*a^6*b^5*c^8*d^12 - 6259*a^7*b^4*c^7*d^13
 + 2739*a^8*b^3*c^6*d^14 + 891*a^9*b^2*c^5*d^15 - 297*a^10*b*c^4*d^16 - 81*
a^11*c^3*d^17)*x^2*sqrt((4096*b^11*c^11 - 22528*a*b^10*c^10*d + 46464*a^2*b
^9*c^9*d^2 - 37664*a^3*b^8*c^8*d^3 - 5071*a^4*b^7*c^7*d^4 + 25641*a^5*b^6*c
^6*d^5 - 7931*a^6*b^5*c^5*d^6 - 6259*a^7*b^4*c^4*d^7 + 2739*a^8*b^3*c^3*d^8
 + 891*a^9*b^2*c^2*d^9 - 297*a^10*b*c*d^10 - 81*a^11*d^11)/(c^7*d^12)) + (2
62144*b^16*c^16 - 2031616*a*b^15*c^15*d + 6451200*a^2*b^14*c^14*d^2 - 10168
320*a^3*b^13*c^13*d^3 + 6467520*a^4*b^12*c^12*d^4 + 3123216*a^5*b^11*c^11*d
^5 - 7258119*a^6*b^10*c^10*d^6 + 2307030*a^7*b^9*c^9*d^7 + 2428965*a^8*b^8*
c^8*d^8 - 1607320*a^9*b^7*c^7*d^9 - 387134*a^10*b^6*c^6*d^10 + 436356*a^11*
b^5*c^5*d^11 + 40770*a^12*b^4*c^4*d^12 - 63720*a^13*b^3*c^3*d^13 - 6075*a^1
4*b^2*c^2*d^14 + 4374*a^15*b*c*d^15 + 729*a^16*d^16)*sqrt(b*x^4 + a))/x^2)*
((4096*b^11*c^11 - 22528*a*b^10*c^10*d + 46464*a^2*b^9*c^9*d^2 - 37664*a^3*
b^8*c^8*d^3 - 5071*a^4*b^7*c^7*d^4 + 25641*a^5*b^6*c^6*d^5 - 7931*a^6*b^5*c
^5*d^6 - 6259*a^7*b^4*c^4*d^7 + 2739*a^8*b^3*c^3*d^8 + 891*a^9*b^2*c^2*d^9
 - 297*a^10*b*c*d^10 - 81*a^11*d^11)/(c^7*d^12))^(1/4) + (512*b^8*c^10*d^3 -
 1984*a*b^7*c^9*d^4 + 2456*a^2*b^6*c^8*d^5 - 413*a^3*b^5*c^7*d^6 - 1175*a^4
*b^4*c^6*d^7 + 478*a^5*b^3*c^5*d^8 + 234*a^6*b^2*c^4*d^9 - 81*a^7*b*c^3*d^1
0 - 27*a^8*c^2*d^11)*(b*x^4 + a)^(1/4)*((4096*b^11*c^11 - 22528*a*b^10*c^10
*d + 46464*a^2*b^9*c^9*d^2 - 37664*a^3*b^8*c^8*d^3 - 5071*a^4*b^7*c^7*d^4 +
 25641*a^5*b^6*c^6*d^5 - 7931*a^6*b^5*c^5*d^6 - 6259*a^7*b^4*c^4*d^7 + 2739
*a^8*b^3*c^3*d^8 + 891*a^9*b^2*c^2*d^9 - 297*a^10*b*c*d^10 - 81*a^11*d^11)/
(c^7*d^12))^(1/4))/((4096*b^11*c^11 - 22528*a*b^10*c^10*d + 46464*a^2*b^9*c
^9*d^2 - 37664*a^3*b^8*c^8*d^3 - 5071*a^4*b^7*c^7*d^4 + 25641*a^5*b^6*c^6*d
^5 - 7931*a^6*b^5*c^5*d^6 - 6259*a^7*b^4*c^4*d^7 + 2739*a^8*b^3*c^3*d^8 + 8
91*a^9*b^2*c^2*d^9 - 297*a^10*b*c*d^10 - 81*a^11*d^11)*x)) + 4*(c*d^3*x^4 +
 c^2*d^2)*((4096*b^11*c^4 - 22528*a*b^10*c^3*d + 46464*a^2*b^9*c^2*d^2 - 42
592*a^3*b^8*c*d^3 + 14641*a^4*b^7*d^4)/d^12)^(1/4)*arctan((d^3*x*sqrt(((409
6*b^11*c^4*d^6 - 22528*a*b^10*c^3*d^7 + 46464*a^2*b^9*c^2*d^8 - 42592*a^3*b
^8*c*d^9 + 14641*a^4*b^7*d^10)*x^2*sqrt((4096*b^11*c^4 - 22528*a*b^10*c^3*d
 + 46464*a^2*b^9*c^2*d^2 - 42592*a^3*b^8*c*d^3 + 14641*a^4*b^7*d^4)/d^12) +
 (262144*b^16*c^6 - 2162688*a*b^15*c^5*d + 7434240*a^2*b^14*c^4*d^2 - 13629
440*a^3*b^13*c^3*d^3 + 14055360*a^4*b^12*c^2*d^4 - 7730448*a^5*b^11*c*d^5 +
 1771561*a^6*b^10*d^6)*sqrt(b*x^4 + a))/x^2)*((4096*b^11*c^4 - 22528*a*b^10
*c^3*d + 46464*a^2*b^9*c^2*d^2 - 42592*a^3*b^8*c*d^3 + 14641*a^4*b^7*d^4)/d
^12)^(1/4) + (512*b^8*c^3*d^3 - 2112*a*b^7*c^2*d^4 + 2904*a^2*b^6*c*d^5 - 1
331*a^3*b^5*d^6)*(b*x^4 + a)^(1/4)*((4096*b^11*c^4 - 22528*a*b^10*c^3*d + 4
6464*a^2*b^9*c^2*d^2 - 42592*a^3*b^8*c*d^3 + 14641*a^4*b^7*d^4)/d^12)^(1/4)
)/((4096*b^11*c^4 - 22528*a*b^10*c^3*d + 46464*a^2*b^9*c^2*d^2 - 42592*a^3*
b^8*c*d^3 + 14641*a^4*b^7*d^4)*x)) + (c*d^3*x^4 + c^2*d^2)*((4096*b^11*c^11
 - 22528*a*b^10*c^10*d + 46464*a^2*b^9*c^9*d^2 - 37664*a^3*b^8*c^8*d^3 - 50
71*a^4*b^7*c^7*d^4 + 25641*a^5*b^6*c^6*d^5 - 7931*a^6*b^5*c^5*d^6 - 6259*a^
```

```
7*b^4*c^4*d^7 + 2739*a^8*b^3*c^3*d^8 + 891*a^9*b^2*c^2*d^9 - 297*a^10*b*c*d
^10 - 81*a^11*d^11)/(c^7*d^12))^(1/4)*log(-(c^5*d^9*x*((4096*b^11*c^11 - 22
528*a*b^10*c^10*d + 46464*a^2*b^9*c^9*d^2 - 37664*a^3*b^8*c^8*d^3 - 5071*a^
4*b^7*c^7*d^4 + 25641*a^5*b^6*c^6*d^5 - 7931*a^6*b^5*c^5*d^6 - 6259*a^7*b^4
*c^4*d^7 + 2739*a^8*b^3*c^3*d^8 + 891*a^9*b^2*c^2*d^9 - 297*a^10*b*c*d^10 -
 81*a^11*d^11)/(c^7*d^12))^(3/4) + (512*b^8*c^8 - 1984*a*b^7*c^7*d + 2456*a
^2*b^6*c^6*d^2 - 413*a^3*b^5*c^5*d^3 - 1175*a^4*b^4*c^4*d^4 + 478*a^5*b^3*c
^3*d^5 + 234*a^6*b^2*c^2*d^6 - 81*a^7*b*c*d^7 - 27*a^8*d^8)*(b*x^4 + a)^(1/
4))/x) - (c*d^3*x^4 + c^2*d^2)*((4096*b^11*c^11 - 22528*a*b^10*c^10*d + 464
64*a^2*b^9*c^9*d^2 - 37664*a^3*b^8*c^8*d^3 - 5071*a^4*b^7*c^7*d^4 + 25641*a
^5*b^6*c^6*d^5 - 7931*a^6*b^5*c^5*d^6 - 6259*a^7*b^4*c^4*d^7 + 2739*a^8*b^3
*c^3*d^8 + 891*a^9*b^2*c^2*d^9 - 297*a^10*b*c*d^10 - 81*a^11*d^11)/(c^7*d^1
2))^(1/4)*log((c^5*d^9*x*((4096*b^11*c^11 - 22528*a*b^10*c^10*d + 46464*a^2
*b^9*c^9*d^2 - 37664*a^3*b^8*c^8*d^3 - 5071*a^4*b^7*c^7*d^4 + 25641*a^5*b^6
*c^6*d^5 - 7931*a^6*b^5*c^5*d^6 - 6259*a^7*b^4*c^4*d^7 + 2739*a^8*b^3*c^3*d
^8 + 891*a^9*b^2*c^2*d^9 - 297*a^10*b*c*d^10 - 81*a^11*d^11)/(c^7*d^12))^(3
/4) - (512*b^8*c^8 - 1984*a*b^7*c^7*d + 2456*a^2*b^6*c^6*d^2 - 413*a^3*b^5*
c^5*d^3 - 1175*a^4*b^4*c^4*d^4 + 478*a^5*b^3*c^3*d^5 + 234*a^6*b^2*c^2*d^6
- 81*a^7*b*c*d^7 - 27*a^8*d^8)*(b*x^4 + a)^(1/4))/x) - (c*d^3*x^4 + c^2*d^2
)*((4096*b^11*c^4 - 22528*a*b^10*c^3*d + 46464*a^2*b^9*c^2*d^2 - 42592*a^3*
b^8*c*d^3 + 14641*a^4*b^7*d^4)/d^12)^(1/4)*log(-(d^9*x*((4096*b^11*c^4 - 22
528*a*b^10*c^3*d + 46464*a^2*b^9*c^2*d^2 - 42592*a^3*b^8*c*d^3 + 14641*a^4*
b^7*d^4)/d^12)^(3/4) + (512*b^8*c^3 - 2112*a*b^7*c^2*d + 2904*a^2*b^6*c*d^2
 - 1331*a^3*b^5*d^3)*(b*x^4 + a)^(1/4))/x) + (c*d^3*x^4 + c^2*d^2)*((4096*b
^11*c^4 - 22528*a*b^10*c^3*d + 46464*a^2*b^9*c^2*d^2 - 42592*a^3*b^8*c*d^3
+ 14641*a^4*b^7*d^4)/d^12)^(1/4)*log((d^9*x*((4096*b^11*c^4 - 22528*a*b^10*
c^3*d + 46464*a^2*b^9*c^2*d^2 - 42592*a^3*b^8*c*d^3 + 14641*a^4*b^7*d^4)/d^
12)^(3/4) - (512*b^8*c^3 - 2112*a*b^7*c^2*d + 2904*a^2*b^6*c*d^2 - 1331*a^3
*b^5*d^3)*(b*x^4 + a)^(1/4))/x) + 4*(b^2*c*d*x^5 + (2*b^2*c^2 - 2*a*b*c*d +
 a^2*d^2)*x)*(b*x^4 + a)^(3/4))/(c*d^3*x^4 + c^2*d^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^4+a\right)^{\frac{11}{4}}}{\left(dx^4+c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^(11/4)/(d*x^4+c)^2,x, algorithm="giac")
[Out] integrate((b*x^4 + a)^(11/4)/(d*x^4 + c)^2, x)
```

maple [F] time = 0.43, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^4+a\right)^{\frac{11}{4}}}{\left(d\,x^4+c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^4+a)^(11/4)/(d*x^4+c)^2,x)
[Out] int((b*x^4+a)^(11/4)/(d*x^4+c)^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^4+a\right)^{\frac{11}{4}}}{\left(dx^4+c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^(11/4)/(d*x^4+c)^2,x, algorithm="maxima")
```

```
[Out] integrate((b*x^4 + a)^(11/4)/(d*x^4 + c)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(b\,x^4+a\right)^{11/4}}{\left(d\,x^4+c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^4)^(11/4)/(c + d*x^4)^2,x)
```

```
[Out] int((a + b*x^4)^(11/4)/(c + d*x^4)^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**4+a)**(11/4)/(d*x**4+c)**2,x)
```

```
[Out] Timed out
```

3.119 $\int \frac{(a+bx^4)^{7/4}}{(c+dx^4)^2}\,dx$

Optimal. Leaf size=230

$$\frac{b^{7/4}\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a+bx^4}}\right)}{2d^2}+\frac{b^{7/4}\tanh^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a+bx^4}}\right)}{2d^2}-\frac{(bc-ad)^{3/4}(3ad+4bc)\tan^{-1}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\,\sqrt[4]{a+bx^4}}\right)}{8c^{7/4}d^2}-\frac{(bc-ad)^{3/4}(3ad+4bc)\,\mathrm{ta}}{8c^{7/4}d^2}$$

Rubi [A] time = 0.18, antiderivative size = 230, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {413, 530, 240, 212, 206, 203, 377, 208, 205}

$$\frac{b^{7/4}\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a+bx^4}}\right)}{2d^2}+\frac{b^{7/4}\tanh^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a+bx^4}}\right)}{2d^2}-\frac{(bc-ad)^{3/4}(3ad+4bc)\tan^{-1}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\,\sqrt[4]{a+bx^4}}\right)}{8c^{7/4}d^2}-\frac{(bc-ad)^{3/4}(3ad+4bc)\tanh^{-1}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\,\sqrt[4]{a+bx^4}}\right)}{8c^{7/4}d^2}-\frac{x\left(a+bx^4\right)^{3/4}(bc-ad)}{4cd\left(c+dx^4\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^4)^(7/4)/(c + d*x^4)^2,x]
```

```
[Out] -((b*c - a*d)*x*(a + b*x^4)^(3/4))/(4*c*d*(c + d*x^4)) + (b^(7/4)*ArcTan[(b
^(1/4)*x)/(a + b*x^4)^(1/4)])/(2*d^2) - ((b*c - a*d)^(3/4)*(4*b*c + 3*a*d)*
ArcTan[((b*c - a*d)^(1/4)*x)/(c^(1/4)*(a + b*x^4)^(1/4))])/(8*c^(7/4)*d^2)
+ (b^(7/4)*ArcTanh[(b^(1/4)*x)/(a + b*x^4)^(1/4)])/(2*d^2) - ((b*c - a*d)^(
3/4)*(4*b*c + 3*a*d)*ArcTanh[((b*c - a*d)^(1/4)*x)/(c^(1/4)*(a + b*x^4)^(1/
4))])/(8*c^(7/4)*d^2)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
```

```
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 413

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[((a*d - c*b)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*n*(p +
 1)), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 530

```
Int[(((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n_)))/((c_) + (d_.)*
(x_)^(n_)), x_Symbol] :> Dist[f/d, Int[(a + b*x^n)^p, x], x] + Dist[(d*e -
c*f)/d, Int[(a + b*x^n)^p/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, p
, n}, x]
```

Rubi steps

$$\int \frac{\left(a+bx^4\right)^{7/4}}{\left(c+dx^4\right)^2}\,dx = -\frac{(bc-ad)x\left(a+bx^4\right)^{3/4}}{4cd\left(c+dx^4\right)}+\frac{\int \frac{a(bc+3ad)+4b^2cx^4}{\sqrt[4]{a+bx^4}\left(c+dx^4\right)}\,dx}{4cd}$$

$$= -\frac{(bc-ad)x\left(a+bx^4\right)^{3/4}}{4cd\left(c+dx^4\right)}+\frac{b^2\int \frac{1}{\sqrt[4]{a+bx^4}}\,dx}{d^2}-\frac{((bc-ad)(4bc+3ad))\int \frac{1}{\sqrt[4]{a+bx^4}\left(c+dx^4\right)}\,dx}{4cd^2}$$

$$= -\frac{(bc-ad)x\left(a+bx^4\right)^{3/4}}{4cd\left(c+dx^4\right)}+\frac{b^2\,\mathrm{Subst}\left(\int \frac{1}{1-bx^4}\,dx,x,\frac{x}{\sqrt[4]{a+bx^4}}\right)}{d^2}-\frac{((bc-ad)(4bc+3ad))\,\mathrm{Sub}}{4}$$

$$= -\frac{(bc-ad)x\left(a+bx^4\right)^{3/4}}{4cd\left(c+dx^4\right)}+\frac{b^2\,\mathrm{Subst}\left(\int \frac{1}{1-\sqrt{b}\,x^2}\,dx,x,\frac{x}{\sqrt[4]{a+bx^4}}\right)}{2d^2}+\frac{b^2\,\mathrm{Subst}\left(\int \frac{1}{1+\sqrt{b}\,x^2}\,dx,x}{2d^2}$$

$$= -\frac{(bc-ad)x\left(a+bx^4\right)^{3/4}}{4cd\left(c+dx^4\right)}+\frac{b^{7/4}\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a+bx^4}}\right)}{2d^2}-\frac{(bc-ad)^{3/4}(4bc+3ad)\tan^{-1}\left(\frac{\sqrt[4]{bc-ad}\,x}{\sqrt[4]{c}\,\sqrt[4]{a+bx}}\right.}{8c^{7/4}d^2}$$

Mathematica [C] time = 0.64, size = 358, normalized size = 1.56

$$\frac{\frac{15a^2\left(-\log\left(\sqrt[4]{c}-\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{ax^4+b}}\right)+\log\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{ax^4+b}}+\sqrt[4]{c}\right)+2\tan^{-1}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\,\sqrt[4]{ax^4+b}}\right)\right)}{\sqrt[4]{bc-ad}}+\frac{16b^2c^{3/4}x^5\sqrt[4]{\frac{bx^4}{a}+1}\,F_1\left(\frac{5}{4};\frac{1}{4},1;\frac{9}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{d\sqrt[4]{a+bx^4}}-\frac{20c^{3/4}x\left(a+bx^4\right)^{3/4}(bc-ad)}{d\left(c+dx^4\right)}+\frac{5abc\left(-\log\left(\sqrt[4]{c}-\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{ax^4+b}}\right)+\log\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{ax^4+b}}+\sqrt[4]{c}\right)+2\tan^{-1}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\,\sqrt[4]{ax^4+b}}\right)\right)}{d\sqrt[4]{bc-ad}}}{80c^{7/4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^4)^(7/4)/(c + d*x^4)^2,x]
```

```
[Out] ((-20*c^(3/4)*(b*c - a*d)*x*(a + b*x^4)^(3/4))/(d*(c + d*x^4)) + (16*b^2*c^
(3/4)*x^5*(1 + (b*x^4)/a)^(1/4)*AppellF1[5/4, 1/4, 1, 9/4, -((b*x^4)/a), -(
```

```
(d*x^4)/c)])/(d*(a + b*x^4)^(1/4)) + (15*a^2*(2*ArcTan[((b*c - a*d)^(1/4)*x
)/(c^(1/4)*(b + a*x^4)^(1/4))] - Log[c^(1/4) - ((b*c - a*d)^(1/4)*x)/(b + a
*x^4)^(1/4)] + Log[c^(1/4) + ((b*c - a*d)^(1/4)*x)/(b + a*x^4)^(1/4)]))/(b*
c - a*d)^(1/4) + (5*a*b*c*(2*ArcTan[((b*c - a*d)^(1/4)*x)/(c^(1/4)*(b + a*x
^4)^(1/4))] - Log[c^(1/4) - ((b*c - a*d)^(1/4)*x)/(b + a*x^4)^(1/4)] + Log[
c^(1/4) + ((b*c - a*d)^(1/4)*x)/(b + a*x^4)^(1/4)]))/(d*(b*c - a*d)^(1/4)))
/(80*c^(7/4))
```

IntegrateAlgebraic [C] time = 1.68, size = 356, normalized size = 1.55

$$-\frac{\left(\frac{1}{16}+\frac{i}{16}\right)\left(-3a^2d^2-abcd+4b^2c^2\right)\tan^{-1}\left(\frac{(1-i)x^2\sqrt{bc-ad}-(1+i)\sqrt{c}\sqrt{a+bx^4}}{2\sqrt[4]{c}\,x\sqrt[4]{a+bx^4}\sqrt[4]{bc-ad}}\right)}{c^{7/4}d^2\sqrt[4]{bc-ad}}-\frac{\left(\frac{1}{16}+\frac{i}{16}\right)\left(-3a^2d^2-abcd+4b^2c^2\right)\tanh^{-1}\left(\frac{(1-i)x^2\sqrt{bc-ad}+(1+i)\sqrt{c}\sqrt{a+bx^4}}{2\sqrt[4]{c}\,x\sqrt[4]{a+bx^4}\sqrt[4]{bc-ad}}\right)}{c^{7/4}d^2\sqrt[4]{bc-ad}}+\frac{b^{7/4}\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{2d^2}+\frac{b^{7/4}\tanh^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{2d^2}+\frac{x\left(a+bx^4\right)^{3/4}(ad-bc)}{4cd\left(c+dx^4\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x^4)^(7/4)/(c + d*x^4)^2,x]
```

```
[Out] ((-(b*c) + a*d)*x*(a + b*x^4)^(3/4))/(4*c*d*(c + d*x^4)) + (b^(7/4)*ArcTan[
(b^(1/4)*x)/(a + b*x^4)^(1/4)])/(2*d^2) - ((1/16 + I/16)*(4*b^2*c^2 - a*b*c
*d - 3*a^2*d^2)*ArcTan[((1 - I)*Sqrt[b*c - a*d]*x^2 - (1 + I)*Sqrt[c]*Sqrt[
a + b*x^4])/(2*c^(1/4)*(b*c - a*d)^(1/4)*x*(a + b*x^4)^(1/4))])/(c^(7/4)*d^
2*(b*c - a*d)^(1/4)) + (b^(7/4)*ArcTanh[(b^(1/4)*x)/(a + b*x^4)^(1/4)])/(2*
d^2) - ((1/16 + I/16)*(4*b^2*c^2 - a*b*c*d - 3*a^2*d^2)*ArcTanh[((1 - I)*Sq
rt[b*c - a*d]*x^2 + (1 + I)*Sqrt[c]*Sqrt[a + b*x^4])/(2*c^(1/4)*(b*c - a*d)
^(1/4)*x*(a + b*x^4)^(1/4))])/(c^(7/4)*d^2*(b*c - a*d)^(1/4))
```

fricas [B] time = 3.51, size = 1667, normalized size = 7.25

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^(7/4)/(d*x^4+c)^2,x, algorithm="fricas")
```

```
[Out] -1/16*(4*(b*x^4 + a)^(3/4)*(b*c - a*d)*x - 4*(c*d^2*x^4 + c^2*d)*((256*b^7*
c^7 - 672*a^2*b^5*c^5*d^2 - 112*a^3*b^4*c^4*d^3 + 609*a^4*b^3*c^3*d^4 + 189
*a^5*b^2*c^2*d^5 - 189*a^6*b*c*d^6 - 81*a^7*d^7)/(c^7*d^8))^(1/4)*arctan(-(
c^2*d^2*x*sqrt(((256*b^7*c^10*d^4 - 672*a^2*b^5*c^8*d^6 - 112*a^3*b^4*c^7*d
^7 + 609*a^4*b^3*c^6*d^8 + 189*a^5*b^2*c^5*d^9 - 189*a^6*b*c^4*d^10 - 81*a^
7*c^3*d^11)*x^2*sqrt((256*b^7*c^7 - 672*a^2*b^5*c^5*d^2 - 112*a^3*b^4*c^4*d
^3 + 609*a^4*b^3*c^3*d^4 + 189*a^5*b^2*c^2*d^5 - 189*a^6*b*c*d^6 - 81*a^7*d
^7)/(c^7*d^8)) + (4096*b^10*c^10 + 2048*a*b^9*c^9*d - 14592*a^2*b^8*c^8*d^2
 - 9472*a^3*b^7*c^7*d^3 + 18928*a^4*b^6*c^6*d^4 + 15624*a^5*b^5*c^5*d^5 - 9
639*a^6*b^4*c^4*d^6 - 11124*a^7*b^3*c^3*d^7 + 486*a^8*b^2*c^2*d^8 + 2916*a^
9*b*c*d^9 + 729*a^10*d^10)*sqrt(b*x^4 + a))/x^2)*((256*b^7*c^7 - 672*a^2*b^
5*c^5*d^2 - 112*a^3*b^4*c^4*d^3 + 609*a^4*b^3*c^3*d^4 + 189*a^5*b^2*c^2*d^5
 - 189*a^6*b*c*d^6 - 81*a^7*d^7)/(c^7*d^8))^(1/4) - (64*b^5*c^7*d^2 + 16*a*
b^4*c^6*d^3 - 116*a^2*b^3*c^5*d^4 - 45*a^3*b^2*c^4*d^5 + 54*a^4*b*c^3*d^6 +
 27*a^5*c^2*d^7)*(b*x^4 + a)^(1/4)*((256*b^7*c^7 - 672*a^2*b^5*c^5*d^2 - 11
2*a^3*b^4*c^4*d^3 + 609*a^4*b^3*c^3*d^4 + 189*a^5*b^2*c^2*d^5 - 189*a^6*b*c
*d^6 - 81*a^7*d^7)/(c^7*d^8))^(1/4))/((256*b^7*c^7 - 672*a^2*b^5*c^5*d^2 -
112*a^3*b^4*c^4*d^3 + 609*a^4*b^3*c^3*d^4 + 189*a^5*b^2*c^2*d^5 - 189*a^6*b
*c*d^6 - 81*a^7*d^7)*x)) - 16*(c*d^2*x^4 + c^2*d)*(b^7/d^8)^(1/4)*arctan(-(
(b*x^4 + a)^(1/4)*b^5*d^2*(b^7/d^8)^(1/4) - d^2*x*(b^7/d^8)^(1/4)*sqrt((b^7
*d^4*x^2*sqrt(b^7/d^8) + sqrt(b*x^4 + a)*b^10)/x^2))/(b^7*x)) + (c*d^2*x^4
+ c^2*d)*((256*b^7*c^7 - 672*a^2*b^5*c^5*d^2 - 112*a^3*b^4*c^4*d^3 + 609*a^
4*b^3*c^3*d^4 + 189*a^5*b^2*c^2*d^5 - 189*a^6*b*c*d^6 - 81*a^7*d^7)/(c^7*d^
8))^(1/4)*log((c^5*d^6*x*((256*b^7*c^7 - 672*a^2*b^5*c^5*d^2 - 112*a^3*b^4*
c^4*d^3 + 609*a^4*b^3*c^3*d^4 + 189*a^5*b^2*c^2*d^5 - 189*a^6*b*c*d^6 - 81*
a^7*d^7)/(c^7*d^8))^(3/4) + (64*b^5*c^5 + 16*a*b^4*c^4*d - 116*a^2*b^3*c^3*
d^2 - 45*a^3*b^2*c^2*d^3 + 54*a^4*b*c*d^4 + 27*a^5*d^5)*(b*x^4 + a)^(1/4))/
x) - (c*d^2*x^4 + c^2*d)*((256*b^7*c^7 - 672*a^2*b^5*c^5*d^2 - 112*a^3*b^4*
c^4*d^3 + 609*a^4*b^3*c^3*d^4 + 189*a^5*b^2*c^2*d^5 - 189*a^6*b*c*d^6 - 81*
```

```
a^7*d^7)/(c^7*d^8))^(1/4)*log(-(c^5*d^6*x*((256*b^7*c^7 - 672*a^2*b^5*c^5*d
^2 - 112*a^3*b^4*c^4*d^3 + 609*a^4*b^3*c^3*d^4 + 189*a^5*b^2*c^2*d^5 - 189*
a^6*b*c*d^6 - 81*a^7*d^7)/(c^7*d^8))^(3/4) - (64*b^5*c^5 + 16*a*b^4*c^4*d -
 116*a^2*b^3*c^3*d^2 - 45*a^3*b^2*c^2*d^3 + 54*a^4*b*c*d^4 + 27*a^5*d^5)*(b
*x^4 + a)^(1/4))/x) - 4*(c*d^2*x^4 + c^2*d)*(b^7/d^8)^(1/4)*log((d^6*x*(b^7
/d^8)^(3/4) + (b*x^4 + a)^(1/4)*b^5)/x) + 4*(c*d^2*x^4 + c^2*d)*(b^7/d^8)^(
1/4)*log(-(d^6*x*(b^7/d^8)^(3/4) - (b*x^4 + a)^(1/4)*b^5)/x))/(c*d^2*x^4 +
c^2*d)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^4+a\right)^{\frac{7}{4}}}{\left(dx^4+c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^(7/4)/(d*x^4+c)^2,x, algorithm="giac")
[Out] integrate((b*x^4 + a)^(7/4)/(d*x^4 + c)^2, x)
```

maple [F] time = 0.65, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^4+a\right)^{\frac{7}{4}}}{\left(d\,x^4+c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^4+a)^(7/4)/(d*x^4+c)^2,x)
[Out] int((b*x^4+a)^(7/4)/(d*x^4+c)^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^4+a\right)^{\frac{7}{4}}}{\left(dx^4+c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^(7/4)/(d*x^4+c)^2,x, algorithm="maxima")
[Out] integrate((b*x^4 + a)^(7/4)/(d*x^4 + c)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(b\,x^4+a\right)^{7/4}}{\left(d\,x^4+c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^4)^(7/4)/(c + d*x^4)^2,x)
[Out] int((a + b*x^4)^(7/4)/(c + d*x^4)^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**4+a)**(7/4)/(d*x**4+c)**2,x)
[Out] Timed out
```

3.120 $\int \frac{(a+bx^4)^{3/4}}{(c+dx^4)^2}\,dx$

Optimal. Leaf size=135

$$\frac{3a\tan^{-1}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{8c^{7/4}\sqrt[4]{bc-ad}}+\frac{3a\tanh^{-1}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{8c^{7/4}\sqrt[4]{bc-ad}}+\frac{x\left(a+bx^4\right)^{3/4}}{4c\left(c+dx^4\right)}$$

Rubi [A] time = 0.08, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {378, 377, 212, 208, 205}

$$\frac{3a\tan^{-1}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{8c^{7/4}\sqrt[4]{bc-ad}}+\frac{3a\tanh^{-1}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{8c^{7/4}\sqrt[4]{bc-ad}}+\frac{x\left(a+bx^4\right)^{3/4}}{4c\left(c+dx^4\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^4)^(3/4)/(c + d*x^4)^2,x]
```

```
[Out] (x*(a + b*x^4)^(3/4))/(4*c*(c + d*x^4)) + (3*a*ArcTan[((b*c - a*d)^(1/4)*x)
/(c^(1/4)*(a + b*x^4)^(1/4))])/(8*c^(7/4)*(b*c - a*d)^(1/4)) + (3*a*ArcTanh
[((b*c - a*d)^(1/4)*x)/(c^(1/4)*(a + b*x^4)^(1/4))])/(8*c^(7/4)*(b*c - a*d)
^(1/4))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 378

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> -Simp[(x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*n*(p + 1)), x] - Dist[(c
*q)/(a*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1, 0]
 && GtQ[q, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{\left(a+bx^4\right)^{3/4}}{\left(c+dx^4\right)^2}\,dx = \frac{x\left(a+bx^4\right)^{3/4}}{4c\left(c+dx^4\right)} + \frac{(3a)\int \frac{1}{\sqrt[4]{a+bx^4}\left(c+dx^4\right)}\,dx}{4c}$$

$$= \frac{x\left(a+bx^4\right)^{3/4}}{4c\left(c+dx^4\right)} + \frac{(3a)\,\text{Subst}\left(\int \frac{1}{c-(bc-ad)x^4}\,dx, x, \frac{x}{\sqrt[4]{a+bx^4}}\right)}{4c}$$

$$= \frac{x\left(a+bx^4\right)^{3/4}}{4c\left(c+dx^4\right)} + \frac{(3a)\,\text{Subst}\left(\int \frac{1}{\sqrt{c}-\sqrt{bc-ad}\,x^2}\,dx, x, \frac{x}{\sqrt[4]{a+bx^4}}\right)}{8c^{3/2}} + \frac{(3a)\,\text{Subst}\left(\int \frac{1}{\sqrt{c}+\sqrt{bc-ad}\,x^2}\,d\right.}{8c^{3/2}}$$

$$= \frac{x\left(a+bx^4\right)^{3/4}}{4c\left(c+dx^4\right)} + \frac{3a\tan^{-1}\left(\frac{\sqrt[4]{bc-ad}\,x}{\sqrt[4]{c}\,\sqrt[4]{a+bx^4}}\right)}{8c^{7/4}\sqrt[4]{bc-ad}} + \frac{3a\tanh^{-1}\left(\frac{\sqrt[4]{bc-ad}\,x}{\sqrt[4]{c}\,\sqrt[4]{a+bx^4}}\right)}{8c^{7/4}\sqrt[4]{bc-ad}}$$

Mathematica [C] time = 0.06, size = 78, normalized size = 0.58

$$\frac{x\left(a+bx^4\right)^{3/4}\,{}_2F_1\left(-\frac{3}{4},\frac{1}{4};\frac{5}{4};\frac{(ad-bc)x^4}{a\left(dx^4+c\right)}\right)}{c^2\left(\frac{bx^4}{a}+1\right)^{3/4}\sqrt[4]{\frac{dx^4}{c}+1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^4)^(3/4)/(c + d*x^4)^2,x]
```

```
[Out] (x*(a + b*x^4)^(3/4)*Hypergeometric2F1[-3/4, 1/4, 5/4, ((-(b*c) + a*d)*x^4)
/(a*(c + d*x^4))])/(c^2*(1 + (b*x^4)/a)^(3/4)*(1 + (d*x^4)/c)^(1/4))
```

IntegrateAlgebraic [C] time = 1.13, size = 243, normalized size = 1.80

$$\frac{\left(\frac{3}{16}+\frac{3i}{16}\right)a\tan^{-1}\left(\frac{\frac{\left(\frac{1}{2}-\frac{i}{2}\right)x^2\sqrt[4]{bc-ad}}{\sqrt[4]{c}}-\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt[4]{c}\sqrt{a+bx^4}}{\sqrt[4]{bc-ad}}}{x\sqrt[4]{a+bx^4}}\right)}{c^{7/4}\sqrt[4]{bc-ad}} + \frac{\left(\frac{3}{16}+\frac{3i}{16}\right)a\tanh^{-1}\left(\frac{\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt[4]{c}\sqrt{a+bx^4}}{\sqrt[4]{bc-ad}}+\frac{\left(\frac{1}{2}-\frac{i}{2}\right)x^2\sqrt[4]{bc-ad}}{\sqrt[4]{c}}}{x\sqrt[4]{a+bx^4}}\right)}{c^{7/4}\sqrt[4]{bc-ad}} + \frac{x\left(a+bx^4\right)^{3/4}}{4c\left(c+dx^4\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x^4)^(3/4)/(c + d*x^4)^2,x]
```

```
[Out] (x*(a + b*x^4)^(3/4))/(4*c*(c + d*x^4)) + ((3/16 + (3*I)/16)*a*ArcTan[(((1/
2 - I/2)*(b*c - a*d)^(1/4)*x^2)/c^(1/4) - ((1/2 + I/2)*c^(1/4)*Sqrt[a + b*x
^4])/(b*c - a*d)^(1/4))/(x*(a + b*x^4)^(1/4))])/(c^(7/4)*(b*c - a*d)^(1/4))
 + ((3/16 + (3*I)/16)*a*ArcTanh[(((1/2 - I/2)*(b*c - a*d)^(1/4)*x^2)/c^(1/4
) + ((1/2 + I/2)*c^(1/4)*Sqrt[a + b*x^4])/(b*c - a*d)^(1/4))/(x*(a + b*x^4)
^(1/4))])/(c^(7/4)*(b*c - a*d)^(1/4))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^(3/4)/(d*x^4+c)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^4+a\right)^{\frac{3}{4}}}{\left(dx^4+c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^(3/4)/(d*x^4+c)^2,x, algorithm="giac")
[Out] integrate((b*x^4 + a)^(3/4)/(d*x^4 + c)^2, x)
```

maple [F] time = 0.61, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^4+a\right)^{\frac{3}{4}}}{\left(d\,x^4+c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^4+a)^(3/4)/(d*x^4+c)^2,x)
[Out] int((b*x^4+a)^(3/4)/(d*x^4+c)^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^4+a\right)^{\frac{3}{4}}}{\left(dx^4+c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^(3/4)/(d*x^4+c)^2,x, algorithm="maxima")
[Out] integrate((b*x^4 + a)^(3/4)/(d*x^4 + c)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(b\,x^4+a\right)^{3/4}}{\left(d\,x^4+c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^4)^(3/4)/(c + d*x^4)^2,x)
[Out] int((a + b*x^4)^(3/4)/(c + d*x^4)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+bx^4\right)^{\frac{3}{4}}}{\left(c+dx^4\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**4+a)**(3/4)/(d*x**4+c)**2,x)
[Out] Integral((a + b*x**4)**(3/4)/(c + d*x**4)**2, x)
```

3.121 $\int \frac{1}{\sqrt[4]{a+bx^4}\left(c+dx^4\right)^2}\,dx$

Optimal. Leaf size=162

$$\frac{(4bc-3ad)\tan^{-1}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{8c^{7/4}(bc-ad)^{5/4}}+\frac{(4bc-3ad)\tanh^{-1}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{8c^{7/4}(bc-ad)^{5/4}}-\frac{dx\left(a+bx^4\right)^{3/4}}{4c\left(c+dx^4\right)(bc-ad)}$$

Rubi [A] time = 0.11, antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {382, 377, 212, 208, 205}

$$\frac{(4bc-3ad)\tan^{-1}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{8c^{7/4}(bc-ad)^{5/4}}+\frac{(4bc-3ad)\tanh^{-1}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{8c^{7/4}(bc-ad)^{5/4}}-\frac{dx\left(a+bx^4\right)^{3/4}}{4c\left(c+dx^4\right)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^4)^(1/4)*(c + d*x^4)^2),x]
```

```
[Out] -(d*x*(a + b*x^4)^(3/4))/(4*c*(b*c - a*d)*(c + d*x^4)) + ((4*b*c - 3*a*d)*A
rcTan[((b*c - a*d)^(1/4)*x)/(c^(1/4)*(a + b*x^4)^(1/4))])/(8*c^(7/4)*(b*c -
 a*d)^(5/4)) + ((4*b*c - 3*a*d)*ArcTanh[((b*c - a*d)^(1/4)*x)/(c^(1/4)*(a +
 b*x^4)^(1/4))])/(8*c^(7/4)*(b*c - a*d)^(5/4))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)), I
nt[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q}, x
] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !LtQ
[q, -1]) && NeQ[p, -1]
```

Rubi steps

$$\int \frac{1}{\sqrt[4]{a+bx^4}\left(c+dx^4\right)^2}\,dx = -\frac{dx\left(a+bx^4\right)^{3/4}}{4c(bc-ad)\left(c+dx^4\right)} + \frac{(4bc-3ad)\int \frac{1}{\sqrt[4]{a+bx^4}\left(c+dx^4\right)}\,dx}{4c(bc-ad)}$$

$$= -\frac{dx\left(a+bx^4\right)^{3/4}}{4c(bc-ad)\left(c+dx^4\right)} + \frac{(4bc-3ad)\,\text{Subst}\left(\int \frac{1}{c-(bc-ad)x^4}\,dx, x, \frac{x}{\sqrt[4]{a+bx^4}}\right)}{4c(bc-ad)}$$

$$= -\frac{dx\left(a+bx^4\right)^{3/4}}{4c(bc-ad)\left(c+dx^4\right)} + \frac{(4bc-3ad)\,\text{Subst}\left(\int \frac{1}{\sqrt{c}-\sqrt{bc-ad}\,x^2}\,dx, x, \frac{x}{\sqrt[4]{a+bx^4}}\right)}{8c^{3/2}(bc-ad)} + \frac{(4bc-$$

$$= -\frac{dx\left(a+bx^4\right)^{3/4}}{4c(bc-ad)\left(c+dx^4\right)} + \frac{(4bc-3ad)\tan^{-1}\left(\frac{\sqrt[4]{bc-ad}\,x}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{8c^{7/4}(bc-ad)^{5/4}} + \frac{(4bc-3ad)\tanh^{-1}\left(\frac{\sqrt[4]{b}}{\sqrt[4]{c}}\right.}{8c^{7/4}(bc-ad)^{5/4}}$$

Mathematica [C] time = 0.14, size = 99, normalized size = 0.61

$$\frac{x\left(\left(c+dx^4\right)(4bc-3ad)\,{}_2F_1\left(\frac{1}{4},1;\frac{5}{4};\frac{(bc-ad)x^4}{c\left(bx^4+a\right)}\right)-cd\left(a+bx^4\right)\right)}{4c^2\sqrt[4]{a+bx^4}\left(c+dx^4\right)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x^4)^(1/4)*(c + d*x^4)^2),x]
```

```
[Out] (x*(-(c*d*(a + b*x^4)) + (4*b*c - 3*a*d)*(c + d*x^4)*Hypergeometric2F1[1/4,
 1, 5/4, ((b*c - a*d)*x^4)/(c*(a + b*x^4))]))/(4*c^2*(b*c - a*d)*(a + b*x^4
)^(1/4)*(c + d*x^4))
```

IntegrateAlgebraic [C] time = 2.12, size = 260, normalized size = 1.60

$$\frac{\left(\frac{1}{16}+\frac{i}{16}\right)(4bc-3ad)\tan^{-1}\left(\frac{(1-i)x^2\sqrt{bc-ad}-(1+i)\sqrt{c}\sqrt{a+bx^4}}{2\sqrt[4]{c}\,x\sqrt[4]{a+bx^4}\sqrt[4]{bc-ad}}\right)}{c^{7/4}(bc-ad)^{5/4}}+\frac{\left(\frac{1}{16}+\frac{i}{16}\right)(4bc-3ad)\tanh^{-1}\left(\frac{(1-i)x^2\sqrt{bc-ad}+(1+i)\sqrt{c}\sqrt{a+bx^4}}{2\sqrt[4]{c}\,x\sqrt[4]{a+bx^4}\sqrt[4]{bc-ad}}\right)}{c^{7/4}(bc-ad)^{5/4}}-\frac{dx\left(a+bx^4\right)^{3/4}}{4c\left(c+dx^4\right)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((a + b*x^4)^(1/4)*(c + d*x^4)^2),x]
```

```
[Out] -1/4*(d*x*(a + b*x^4)^(3/4))/(c*(b*c - a*d)*(c + d*x^4)) + ((1/16 + I/16)*(
4*b*c - 3*a*d)*ArcTan[((1 - I)*Sqrt[b*c - a*d]*x^2 - (1 + I)*Sqrt[c]*Sqrt[a
 + b*x^4])/(2*c^(1/4)*(b*c - a*d)^(1/4)*x*(a + b*x^4)^(1/4))])/(c^(7/4)*(b*
c - a*d)^(5/4)) + ((1/16 + I/16)*(4*b*c - 3*a*d)*ArcTanh[((1 - I)*Sqrt[b*c
- a*d]*x^2 + (1 + I)*Sqrt[c]*Sqrt[a + b*x^4])/(2*c^(1/4)*(b*c - a*d)^(1/4)*
x*(a + b*x^4)^(1/4))])/(c^(7/4)*(b*c - a*d)^(5/4))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a)^(1/4)/(d*x^4+c)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^4+a\right)^{\frac{1}{4}}\left(dx^4+c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a)^(1/4)/(d*x^4+c)^2,x, algorithm="giac")
[Out] integrate(1/((b*x^4 + a)^(1/4)*(d*x^4 + c)^2), x)
```

maple [F] time = 0.60, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,x^{4}+a\right)^{\frac{1}{4}}\left(d\,x^{4}+c\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^4+a)^(1/4)/(d*x^4+c)^2,x)
[Out] int(1/(b*x^4+a)^(1/4)/(d*x^4+c)^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^{4}+a\right)^{\frac{1}{4}}\left(dx^{4}+c\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a)^(1/4)/(d*x^4+c)^2,x, algorithm="maxima")
[Out] integrate(1/((b*x^4 + a)^(1/4)*(d*x^4 + c)^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(b\,x^4+a\right)}^{1/4}\,{\left(d\,x^4+c\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^4)^(1/4)*(c + d*x^4)^2),x)
[Out] int(1/((a + b*x^4)^(1/4)*(c + d*x^4)^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[4]{a + bx^{4}}\left(c + dx^{4}\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**4+a)**(1/4)/(d*x**4+c)**2,x)
[Out] Integral(1/((a + b*x**4)**(1/4)*(c + d*x**4)**2), x)
```

3.122 $\int \frac{1}{\left(a+bx^4\right)^{5/4}\left(c+dx^4\right)^2}\,dx$

Optimal. Leaf size=205

$$-\frac{d(8bc-3ad)\tan^{-1}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{8c^{7/4}(bc-ad)^{9/4}}-\frac{d(8bc-3ad)\tanh^{-1}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{8c^{7/4}(bc-ad)^{9/4}}+\frac{bx(ad+4bc)}{4ac\sqrt[4]{a+bx^4}(bc-ad)^2}-\frac{dx}{4c\sqrt[4]{a+bx^4}\left(c+d\right.}$$

Rubi [A] time = 0.18, antiderivative size = 205, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {414, 527, 12, 377, 212, 208, 205}

$$-\frac{d(8bc-3ad)\tan^{-1}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{8c^{7/4}(bc-ad)^{9/4}}-\frac{d(8bc-3ad)\tanh^{-1}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{8c^{7/4}(bc-ad)^{9/4}}+\frac{bx(ad+4bc)}{4ac\sqrt[4]{a+bx^4}(bc-ad)^2}-\frac{dx}{4c\sqrt[4]{a+bx^4}\left(c+dx^4\right)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^4)^(5/4)*(c + d*x^4)^2),x]
```

```
[Out] (b*(4*b*c + a*d)*x)/(4*a*c*(b*c - a*d)^2*(a + b*x^4)^(1/4)) - (d*x)/(4*c*(b
*c - a*d)*(a + b*x^4)^(1/4)*(c + d*x^4)) - (d*(8*b*c - 3*a*d)*ArcTan[((b*c
- a*d)^(1/4)*x)/(c^(1/4)*(a + b*x^4)^(1/4))])/(8*c^(7/4)*(b*c - a*d)^(9/4))
 - (d*(8*b*c - 3*a*d)*ArcTanh[((b*c - a*d)^(1/4)*x)/(c^(1/4)*(a + b*x^4)^(1
/4))])/(8*c^(7/4)*(b*c - a*d)^(9/4))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
```

```
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{1}{\left(a+bx^4\right)^{5/4}\left(c+dx^4\right)^2}\,dx = -\frac{dx}{4c(bc-ad)\sqrt[4]{a+bx^4}\left(c+dx^4\right)} + \frac{\int \frac{4bc-3ad-4bdx^4}{\left(a+bx^4\right)^{5/4}\left(c+dx^4\right)}\,dx}{4c(bc-ad)}$$

$$= \frac{b(4bc+ad)x}{4ac(bc-ad)^2\sqrt[4]{a+bx^4}} - \frac{dx}{4c(bc-ad)\sqrt[4]{a+bx^4}\left(c+dx^4\right)} - \frac{\int \frac{ad(8bc-3ad)}{\sqrt[4]{a+bx^4}\left(c+dx^4\right)}\,dx}{4ac(bc-ad)^2}$$

$$= \frac{b(4bc+ad)x}{4ac(bc-ad)^2\sqrt[4]{a+bx^4}} - \frac{dx}{4c(bc-ad)\sqrt[4]{a+bx^4}\left(c+dx^4\right)} - \frac{(d(8bc-3ad))\int \ldots}{4c(bc- \ldots}$$

$$= \frac{b(4bc+ad)x}{4ac(bc-ad)^2\sqrt[4]{a+bx^4}} - \frac{dx}{4c(bc-ad)\sqrt[4]{a+bx^4}\left(c+dx^4\right)} - \frac{(d(8bc-3ad))\,\mathrm{Su}\ldots}{\ldots}$$

$$= \frac{b(4bc+ad)x}{4ac(bc-ad)^2\sqrt[4]{a+bx^4}} - \frac{dx}{4c(bc-ad)\sqrt[4]{a+bx^4}\left(c+dx^4\right)} - \frac{(d(8bc-3ad))\,\mathrm{Su}\ldots}{\ldots}$$

$$= \frac{b(4bc+ad)x}{4ac(bc-ad)^2\sqrt[4]{a+bx^4}} - \frac{dx}{4c(bc-ad)\sqrt[4]{a+bx^4}\left(c+dx^4\right)} - \frac{d(8bc-3ad)\tan^{-}\ldots}{8c^{7/4}(bc- \ldots}$$

Mathematica [C] time = 2.27, size = 625, normalized size = 3.05

$$\frac{c\left(a+bx^4\right)^{3/4}\left(\frac{320d^2x^{20}(bc-ad)^3\,{}_3F_2\left(2,2,\frac{9}{4};1,\frac{17}{4};\frac{(bc-ad)x^4}{c(bx^4+a)}\right)}{c^5\left(a+bx^4\right)^3}+\frac{640dx^{16}(bc-ad)^3\,{}_3F_2\left(2,2,\frac{9}{4};1,\frac{17}{4};\frac{(bc-ad)x^4}{c(bx^4+a)}\right)}{c^4\left(a+bx^4\right)^3}+\frac{320x^{12}(bc-ad)^3\,{}_3F_2\left(2,2,\frac{9}{4};1,\frac{17}{4};\frac{(bc-ad)x^4}{c(bx^4+a)}\right)}{c^3\left(a+bx^4\right)^3}+\frac{16380d^2x^{12}(ad-bc)\,{}_2F_1\left(\frac{1}{4},1;\frac{5}{4};\frac{(bc-ad)x^4}{c(bx^4+a)}\right)}{c^3\left(a+bx^4\right)}+\frac{7488d^2x^{12}(bc-ad)}{c^3\left(a+bx^4\right)}+\frac{44460d^2x^8\,{}_2F_1\left(\frac{1}{4},1;\frac{5}{4};\frac{(bc-ad)x^4}{c(bx^4+a)}\right)}{c^2}+\frac{33930dx^8(ad-bc)\,{}_2F_1\left(\frac{1}{4},1;\frac{5}{4};\frac{(bc-ad)x^4}{c(bx^4+a)}\right)}{c^2\left(a+bx^4\right)}+\frac{14976dx^8(bc-ad)}{c^2\left(a+bx^4\right)}+\frac{94770dx^4\,{}_2F_1\left(\frac{1}{4},1;\frac{5}{4};\frac{(bc-ad)x^4}{c(bx^4+a)}\right)}{c}-\frac{14625x^4(bc-ad)\,{}_2F_1\left(\frac{1}{4},1;\frac{5}{4};\frac{(bc-ad)x^4}{c(bx^4+a)}\right)}{c\left(a+bx^4\right)}+47385\,{}_2F_1\left(\frac{1}{4},1;\frac{5}{4};\frac{(bc-ad)x^4}{c(bx^4+a)}\right)+\frac{5148x^4(bc-ad)}{c\left(a+bx^4\right)}-\frac{44460d^2x^8}{c^2}-\frac{94770dx^4}{c}-47385\right)}{2340x^7\left(c+dx^4\right)(bc-ad)^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + b*x^4)^(5/4)*(c + d*x^4)^2),x]
```

```
[Out] (c*(a + b*x^4)^(3/4)*(-47385 - (94770*d*x^4)/c - (44460*d^2*x^8)/c^2 + (514
8*(b*c - a*d)*x^4)/(c*(a + b*x^4)) + (14976*d*(b*c - a*d)*x^8)/(c^2*(a + b*
x^4)) + (7488*d^2*(b*c - a*d)*x^12)/(c^3*(a + b*x^4)) + 47385*Hypergeometri
c2F1[1/4, 1, 5/4, ((b*c - a*d)*x^4)/(c*(a + b*x^4))] + (94770*d*x^4*Hyperge
ometric2F1[1/4, 1, 5/4, ((b*c - a*d)*x^4)/(c*(a + b*x^4))])/c + (44460*d^2*
x^8*Hypergeometric2F1[1/4, 1, 5/4, ((b*c - a*d)*x^4)/(c*(a + b*x^4))])/c^2
- (14625*(b*c - a*d)*x^4*Hypergeometric2F1[1/4, 1, 5/4, ((b*c - a*d)*x^4)/(
c*(a + b*x^4))])/(c*(a + b*x^4)) + (33930*d*(-(b*c) + a*d)*x^8*Hypergeometr
ic2F1[1/4, 1, 5/4, ((b*c - a*d)*x^4)/(c*(a + b*x^4))])/(c^2*(a + b*x^4)) +
(16380*d^2*(-(b*c) + a*d)*x^12*Hypergeometric2F1[1/4, 1, 5/4, ((b*c - a*d)*
```

```
x^4)/(c*(a + b*x^4))])/(c^3*(a + b*x^4)) + (320*(b*c - a*d)^3*x^12*Hypergeo
metricPFQ[{2, 2, 9/4}, {1, 17/4}, ((b*c - a*d)*x^4)/(c*(a + b*x^4))])/(c^3*
(a + b*x^4)^3) + (640*d*(b*c - a*d)^3*x^16*HypergeometricPFQ[{2, 2, 9/4}, {
1, 17/4}, ((b*c - a*d)*x^4)/(c*(a + b*x^4))])/(c^4*(a + b*x^4)^3) + (320*d^
2*(b*c - a*d)^3*x^20*HypergeometricPFQ[{2, 2, 9/4}, {1, 17/4}, ((b*c - a*d)
*x^4)/(c*(a + b*x^4))])/(c^5*(a + b*x^4)^3)))/(2340*(b*c - a*d)^2*x^7*(c +
d*x^4))
```

IntegrateAlgebraic [C] time = 3.92, size = 304, normalized size = 1.48

$$\frac{a^2d^2x+abd^2x^5+4b^2c^2x+4b^2cdx^5}{4ac\sqrt[4]{a+bx^4}\left(c+dx^4\right)(ad-bc)^2}-\frac{\left(\frac{1}{16}+\frac{i}{16}\right)\left(8bcd-3ad^2\right)\tan^{-1}\left(\frac{(1-i)x^2\sqrt{bc-ad}-(1+i)\sqrt{c}\sqrt{a+bx^4}}{2\sqrt[4]{c}x\sqrt[4]{a+bx^4}\sqrt[4]{bc-ad}}\right)}{c^{7/4}(bc-ad)^{9/4}}-\frac{\left(\frac{1}{16}+\frac{i}{16}\right)\left(8bcd-3ad^2\right)\tanh^{-1}\left(\frac{(1-i)x^2\sqrt{bc-ad}+(1+i)\sqrt{c}\sqrt{a+bx^4}}{2\sqrt[4]{c}x\sqrt[4]{a+bx^4}\sqrt[4]{bc-ad}}\right)}{c^{7/4}(bc-ad)^{9/4}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((a + b*x^4)^(5/4)*(c + d*x^4)^2),x]
```

```
[Out] (4*b^2*c^2*x + a^2*d^2*x + 4*b^2*c*d*x^5 + a*b*d^2*x^5)/(4*a*c*(-(b*c) + a*
d)^2*(a + b*x^4)^(1/4)*(c + d*x^4)) - ((1/16 + I/16)*(8*b*c*d - 3*a*d^2)*Ar
cTan[((1 - I)*Sqrt[b*c - a*d]*x^2 - (1 + I)*Sqrt[c]*Sqrt[a + b*x^4])/(2*c^(
1/4)*(b*c - a*d)^(1/4)*x*(a + b*x^4)^(1/4))])/(c^(7/4)*(b*c - a*d)^(9/4)) -
 ((1/16 + I/16)*(8*b*c*d - 3*a*d^2)*ArcTanh[((1 - I)*Sqrt[b*c - a*d]*x^2 +
(1 + I)*Sqrt[c]*Sqrt[a + b*x^4])/(2*c^(1/4)*(b*c - a*d)^(1/4)*x*(a + b*x^4)
^(1/4))])/(c^(7/4)*(b*c - a*d)^(9/4))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a)^(5/4)/(d*x^4+c)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^4+a\right)^{\frac{5}{4}}\left(dx^4+c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a)^(5/4)/(d*x^4+c)^2,x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^4 + a)^(5/4)*(d*x^4 + c)^2), x)
```

maple [F] time = 0.60, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,x^4+a\right)^{\frac{5}{4}}\left(d\,x^4+c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^4+a)^(5/4)/(d*x^4+c)^2,x)
```

```
[Out] int(1/(b*x^4+a)^(5/4)/(d*x^4+c)^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^4+a\right)^{\frac{5}{4}}\left(dx^4+c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a)^(5/4)/(d*x^4+c)^2,x, algorithm="maxima")
[Out] integrate(1/((b*x^4 + a)^(5/4)*(d*x^4 + c)^2), x)
```

mupad [F]   time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(b\,x^4+a\right)^{5/4}\,\left(d\,x^4+c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^4)^(5/4)*(c + d*x^4)^2),x)
[Out] int(1/((a + b*x^4)^(5/4)*(c + d*x^4)^2), x)
```

sympy [F]   time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+bx^4\right)^{\frac{5}{4}}\left(c+dx^4\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**4+a)**(5/4)/(d*x**4+c)**2,x)
[Out] Integral(1/((a + b*x**4)**(5/4)*(c + d*x**4)**2), x)
```

3.123 $\int \frac{1}{\left(a+bx^4\right)^{9/4}\left(c+dx^4\right)^2}\,dx$

Optimal. Leaf size=266

$$\frac{bx\left(-5a^2d^2-56abcd+16b^2c^2\right)}{20a^2c\sqrt[4]{a+bx^4}\,(bc-ad)^3}+\frac{3d^2(4bc-ad)\tan^{-1}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{8c^{7/4}(bc-ad)^{13/4}}+\frac{3d^2(4bc-ad)\tanh^{-1}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{8c^{7/4}(bc-ad)^{13/4}}-\frac{}{4c\,(a+bx}$$

Rubi [A] time = 0.29, antiderivative size = 266, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {414, 527, 12, 377, 212, 208, 205}

$$\frac{bx\left(-5a^2d^2-56abcd+16b^2c^2\right)}{20a^2c\sqrt[4]{a+bx^4}\,(bc-ad)^3}+\frac{3d^2(4bc-ad)\tan^{-1}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{8c^{7/4}(bc-ad)^{13/4}}+\frac{3d^2(4bc-ad)\tanh^{-1}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{8c^{7/4}(bc-ad)^{13/4}}-\frac{dx}{4c\left(a+bx^4\right)^{5/4}\left(c+dx^4\right)(bc-ad)}+\frac{bx(5ad+4bc)}{20ac\left(a+bx^4\right)^{5/4}(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^4)^(9/4)*(c + d*x^4)^2),x]
```

```
[Out] (b*(4*b*c + 5*a*d)*x)/(20*a*c*(b*c - a*d)^2*(a + b*x^4)^(5/4)) + (b*(16*b^2
*c^2 - 56*a*b*c*d - 5*a^2*d^2)*x)/(20*a^2*c*(b*c - a*d)^3*(a + b*x^4)^(1/4)
) - (d*x)/(4*c*(b*c - a*d)*(a + b*x^4)^(5/4)*(c + d*x^4)) + (3*d^2*(4*b*c -
 a*d)*ArcTan[((b*c - a*d)^(1/4)*x)/(c^(1/4)*(a + b*x^4)^(1/4))])/(8*c^(7/4)
*(b*c - a*d)^(13/4)) + (3*d^2*(4*b*c - a*d)*ArcTanh[((b*c - a*d)^(1/4)*x)/(
c^(1/4)*(a + b*x^4)^(1/4))])/(8*c^(7/4)*(b*c - a*d)^(13/4))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
```

```
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{1}{\left(a+bx^4\right)^{9/4}\left(c+dx^4\right)^2}\,dx = -\frac{dx}{4c(bc-ad)\left(a+bx^4\right)^{5/4}\left(c+dx^4\right)} + \frac{\int \frac{4bc-3ad-8bdx^4}{\left(a+bx^4\right)^{9/4}\left(c+dx^4\right)}\,dx}{4c(bc-ad)}$$

$$= \frac{b(4bc+5ad)x}{20ac(bc-ad)^2\left(a+bx^4\right)^{5/4}} - \frac{dx}{4c(bc-ad)\left(a+bx^4\right)^{5/4}\left(c+dx^4\right)} - \frac{\int \frac{-16b^2c^2+40\cdots}{\cdots}}{\cdots}$$

$$= \frac{b(4bc+5ad)x}{20ac(bc-ad)^2\left(a+bx^4\right)^{5/4}} + \frac{b\left(16b^2c^2-56abcd-5a^2d^2\right)x}{20a^2c(bc-ad)^3\sqrt[4]{a+bx^4}} - \frac{\cdots}{4c(bc-ad)\left(a\cdots\right.}$$

$$= \frac{b(4bc+5ad)x}{20ac(bc-ad)^2\left(a+bx^4\right)^{5/4}} + \frac{b\left(16b^2c^2-56abcd-5a^2d^2\right)x}{20a^2c(bc-ad)^3\sqrt[4]{a+bx^4}} - \frac{\cdots}{4c(bc-ad)\left(a\cdots\right.}$$

$$= \frac{b(4bc+5ad)x}{20ac(bc-ad)^2\left(a+bx^4\right)^{5/4}} + \frac{b\left(16b^2c^2-56abcd-5a^2d^2\right)x}{20a^2c(bc-ad)^3\sqrt[4]{a+bx^4}} - \frac{\cdots}{4c(bc-ad)\left(a\cdots\right.}$$

$$= \frac{b(4bc+5ad)x}{20ac(bc-ad)^2\left(a+bx^4\right)^{5/4}} + \frac{b\left(16b^2c^2-56abcd-5a^2d^2\right)x}{20a^2c(bc-ad)^3\sqrt[4]{a+bx^4}} - \frac{\cdots}{4c(bc-ad)\left(a\cdots\right.}$$

$$= \frac{b(4bc+5ad)x}{20ac(bc-ad)^2\left(a+bx^4\right)^{5/4}} + \frac{b\left(16b^2c^2-56abcd-5a^2d^2\right)x}{20a^2c(bc-ad)^3\sqrt[4]{a+bx^4}} - \frac{\cdots}{4c(bc-ad)\left(a\cdots\right.}$$

Mathematica [C] time = 5.01, size = 1216, normalized size = 4.57

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + b*x^4)^(9/4)*(c + d*x^4)^2),x]
```

```
[Out] -1/198900*(285532*c^5*(b*c - a*d)^2*x^8*(a + b*x^4)^2 + 933504*c^4*d*(b*c -
 a*d)^2*x^12*(a + b*x^4)^2 + 891072*c^3*d^2*(b*c - a*d)^2*x^16*(a + b*x^4)^
2 + 282880*c^2*d^3*(b*c - a*d)^2*x^20*(a + b*x^4)^2 + 9793836*c^6*(b*c - a*
d)*x^4*(a + b*x^4)^3 + 27973296*c^5*d*(b*c - a*d)*x^8*(a + b*x^4)^3 + 25968
384*c^4*d^2*(b*c - a*d)*x^12*(a + b*x^4)^3 + 8146944*c^3*d^3*(b*c - a*d)*x^
```

```
16*(a + b*x^4)^3 - 23529870*c^7*(a + b*x^4)^4 - 65547495*c^6*d*x^4*(a + b*x
^4)^4 - 60505380*c^5*d^2*x^8*(a + b*x^4)^4 - 18935280*c^4*d^3*x^12*(a + b*x
^4)^4 - 14499810*c^6*(b*c - a*d)*x^4*(a + b*x^4)^3*Hypergeometric2F1[1/4, 1
, 5/4, ((b*c - a*d)*x^4)/(c*(a + b*x^4))] - 41082795*c^5*d*(b*c - a*d)*x^8*
(a + b*x^4)^3*Hypergeometric2F1[1/4, 1, 5/4, ((b*c - a*d)*x^4)/(c*(a + b*x^
4))] - 38069460*c^4*d^2*(b*c - a*d)*x^12*(a + b*x^4)^3*Hypergeometric2F1[1/
4, 1, 5/4, ((b*c - a*d)*x^4)/(c*(a + b*x^4))] - 11934000*c^3*d^3*(b*c - a*d
)*x^16*(a + b*x^4)^3*Hypergeometric2F1[1/4, 1, 5/4, ((b*c - a*d)*x^4)/(c*(a
 + b*x^4))] + 23529870*c^7*(a + b*x^4)^4*Hypergeometric2F1[1/4, 1, 5/4, ((b
*c - a*d)*x^4)/(c*(a + b*x^4))] + 65547495*c^6*d*x^4*(a + b*x^4)^4*Hypergeo
metric2F1[1/4, 1, 5/4, ((b*c - a*d)*x^4)/(c*(a + b*x^4))] + 60505380*c^5*d^
2*x^8*(a + b*x^4)^4*Hypergeometric2F1[1/4, 1, 5/4, ((b*c - a*d)*x^4)/(c*(a
+ b*x^4))] + 18935280*c^4*d^3*x^12*(a + b*x^4)^4*Hypergeometric2F1[1/4, 1,
5/4, ((b*c - a*d)*x^4)/(c*(a + b*x^4))] + 77760*c^3*(b*c - a*d)^4*x^16*Hype
rgeometricPFQ[{2, 2, 13/4}, {1, 21/4}, ((b*c - a*d)*x^4)/(c*(a + b*x^4))] +
 224640*c^2*d*(b*c - a*d)^4*x^20*HypergeometricPFQ[{2, 2, 13/4}, {1, 21/4},
 ((b*c - a*d)*x^4)/(c*(a + b*x^4))] + 216000*c*d^2*(b*c - a*d)^4*x^24*Hyper
geometricPFQ[{2, 2, 13/4}, {1, 21/4}, ((b*c - a*d)*x^4)/(c*(a + b*x^4))] +
69120*d^3*(b*c - a*d)^4*x^28*HypergeometricPFQ[{2, 2, 13/4}, {1, 21/4}, ((b
*c - a*d)*x^4)/(c*(a + b*x^4))] + 11520*c^3*(b*c - a*d)^4*x^16*Hypergeometr
icPFQ[{2, 2, 2, 13/4}, {1, 1, 21/4}, ((b*c - a*d)*x^4)/(c*(a + b*x^4))] + 3
4560*c^2*d*(b*c - a*d)^4*x^20*HypergeometricPFQ[{2, 2, 2, 13/4}, {1, 1, 21/
4}, ((b*c - a*d)*x^4)/(c*(a + b*x^4))] + 34560*c*d^2*(b*c - a*d)^4*x^24*Hyp
ergeometricPFQ[{2, 2, 2, 13/4}, {1, 1, 21/4}, ((b*c - a*d)*x^4)/(c*(a + b*x
^4))] + 11520*d^3*(b*c - a*d)^4*x^28*HypergeometricPFQ[{2, 2, 2, 13/4}, {1,
 1, 21/4}, ((b*c - a*d)*x^4)/(c*(a + b*x^4))])/(c^8*(-b + (a*d)/c)^3*x^11*(
a + b*x^4)^(13/4)*(c + d*x^4))
```

IntegrateAlgebraic [C] time = 6.85, size = 394, normalized size = 1.48

$$\frac{5a^4d^3x+10a^3bd^3x^5+60a^2b^2c^2dx+60a^2b^2cd^2x^5+5a^2b^2d^3x^9-20ab^3c^3x+36ab^3c^2dx^5+56ab^3cd^2x^9-16b^4c^3x^5-16b^4c^2dx^9}{20a^2c\left(a+bx^4\right)^{5/4}\left(c+dx^4\right)(ad-bc)^3}+\frac{\left(\frac{3}{16}+\frac{3i}{16}\right)\left(4bcd^2-ad^3\right)\tan^{-1}\left(\frac{(1-i)x^2\sqrt{bc-ad}-(1+i)\sqrt{c}\sqrt{a+bx^4}}{2\sqrt[4]{c}\,x\sqrt[4]{a+bx^4}\sqrt[4]{bc-ad}}\right)}{c^{7/4}(bc-ad)^{13/4}}+\frac{\left(\frac{3}{16}+\frac{3i}{16}\right)\left(4bcd^2-ad^3\right)\tanh^{-1}\left(\frac{(1-i)x^2\sqrt{bc-ad}+(1+i)\sqrt{c}\sqrt{a+bx^4}}{2\sqrt[4]{c}\,x\sqrt[4]{a+bx^4}\sqrt[4]{bc-ad}}\right)}{c^{7/4}(bc-ad)^{13/4}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((a + b*x^4)^(9/4)*(c + d*x^4)^2),x]
```

```
[Out] (-20*a*b^3*c^3*x + 60*a^2*b^2*c^2*d*x + 5*a^4*d^3*x - 16*b^4*c^3*x^5 + 36*a
*b^3*c^2*d*x^5 + 60*a^2*b^2*c*d^2*x^5 + 10*a^3*b*d^3*x^5 - 16*b^4*c^2*d*x^9
 + 56*a*b^3*c*d^2*x^9 + 5*a^2*b^2*d^3*x^9)/(20*a^2*c*(-(b*c) + a*d)^3*(a +
b*x^4)^(5/4)*(c + d*x^4)) + ((3/16 + (3*I)/16)*(4*b*c*d^2 - a*d^3)*ArcTan[(
(1 - I)*Sqrt[b*c - a*d]*x^2 - (1 + I)*Sqrt[c]*Sqrt[a + b*x^4])/(2*c^(1/4)*(
b*c - a*d)^(1/4)*x*(a + b*x^4)^(1/4))])/(c^(7/4)*(b*c - a*d)^(13/4)) + ((3/
16 + (3*I)/16)*(4*b*c*d^2 - a*d^3)*ArcTanh[((1 - I)*Sqrt[b*c - a*d]*x^2 + (
1 + I)*Sqrt[c]*Sqrt[a + b*x^4])/(2*c^(1/4)*(b*c - a*d)^(1/4)*x*(a + b*x^4)^
(1/4))])/(c^(7/4)*(b*c - a*d)^(13/4))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a)^(9/4)/(d*x^4+c)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^4+a\right)^{\frac{9}{4}}\left(dx^4+c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a)^(9/4)/(d*x^4+c)^2,x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^4 + a)^(9/4)*(d*x^4 + c)^2), x)
```

maple [F] time = 0.61, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,x^{4}+a\right)^{\frac{9}{4}} \left(d\,x^{4}+c\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^4+a)^(9/4)/(d*x^4+c)^2,x)
```

```
[Out] int(1/(b*x^4+a)^(9/4)/(d*x^4+c)^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^{4}+a\right)^{\frac{9}{4}} \left(dx^{4}+c\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a)^(9/4)/(d*x^4+c)^2,x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^4 + a)^(9/4)*(d*x^4 + c)^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(b\,x^4+a\right)^{9/4}\,\left(d\,x^4+c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^4)^(9/4)*(c + d*x^4)^2),x)
```

```
[Out] int(1/((a + b*x^4)^(9/4)*(c + d*x^4)^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+bx^{4}\right)^{\frac{9}{4}} \left(c+dx^{4}\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**4+a)**(9/4)/(d*x**4+c)**2,x)
```

```
[Out] Integral(1/((a + b*x**4)**(9/4)*(c + d*x**4)**2), x)
```

3.124 $\int \frac{1}{\sqrt[4]{1+x^4}\,(2+x^4)}\,dx$

Optimal. Leaf size=53

$$\frac{\tan^{-1}\left(\frac{x}{\sqrt[4]{2}\sqrt[4]{x^4+1}}\right)}{2\,2^{3/4}}+\frac{\tanh^{-1}\left(\frac{x}{\sqrt[4]{2}\sqrt[4]{x^4+1}}\right)}{2\,2^{3/4}}$$

Rubi [A] time = 0.02, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {377, 212, 206, 203}

$$\frac{\tan^{-1}\left(\frac{x}{\sqrt[4]{2}\sqrt[4]{x^4+1}}\right)}{2\,2^{3/4}}+\frac{\tanh^{-1}\left(\frac{x}{\sqrt[4]{2}\sqrt[4]{x^4+1}}\right)}{2\,2^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 + x^4)^(1/4)*(2 + x^4)),x]
```

```
[Out] ArcTan[x/(2^(1/4)*(1 + x^4)^(1/4))]/(2*2^(3/4)) + ArcTanh[x/(2^(1/4)*(1 + x
^4)^(1/4))]/(2*2^(3/4))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rubi steps

$$\int \frac{1}{\sqrt[4]{1+x^4}\,(2+x^4)}\,dx = \text{Subst}\left(\int \frac{1}{2-x^4}\,dx, x, \frac{x}{\sqrt[4]{1+x^4}}\right)$$

$$= \frac{\text{Subst}\left(\int \frac{1}{\sqrt{2}-x^2}\,dx, x, \frac{x}{\sqrt[4]{1+x^4}}\right)}{2\sqrt{2}} + \frac{\text{Subst}\left(\int \frac{1}{\sqrt{2}+x^2}\,dx, x, \frac{x}{\sqrt[4]{1+x^4}}\right)}{2\sqrt{2}}$$

$$= \frac{\tan^{-1}\left(\frac{x}{\sqrt[4]{2}\,\sqrt[4]{1+x^4}}\right)}{2\,2^{3/4}} + \frac{\tanh^{-1}\left(\frac{x}{\sqrt[4]{2}\,\sqrt[4]{1+x^4}}\right)}{2\,2^{3/4}}$$

Mathematica [**A**] time = 0.02, size = 44, normalized size = 0.83

$$\frac{\tan^{-1}\left(\frac{x}{\sqrt[4]{2}\,\sqrt[4]{x^4+1}}\right)+\tanh^{-1}\left(\frac{x}{\sqrt[4]{2}\,\sqrt[4]{x^4+1}}\right)}{2\,2^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 + x^4)^(1/4)*(2 + x^4)),x]
```

```
[Out] (ArcTan[x/(2^(1/4)*(1 + x^4)^(1/4))] + ArcTanh[x/(2^(1/4)*(1 + x^4)^(1/4))]
)/(2*2^(3/4))
```

IntegrateAlgebraic [**A**] time = 0.23, size = 53, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{x}{\sqrt[4]{2}\,\sqrt[4]{x^4+1}}\right)}{2\,2^{3/4}} + \frac{\tanh^{-1}\left(\frac{x}{\sqrt[4]{2}\,\sqrt[4]{x^4+1}}\right)}{2\,2^{3/4}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((1 + x^4)^(1/4)*(2 + x^4)),x]
```

```
[Out] ArcTan[x/(2^(1/4)*(1 + x^4)^(1/4))]/(2*2^(3/4)) + ArcTanh[x/(2^(1/4)*(1 + x
^4)^(1/4))]/(2*2^(3/4))
```

fricas [**B**] time = 16.48, size = 208, normalized size = 3.92

$$-\frac{1}{16}\cdot 8^{\frac{3}{4}}\arctan\left(-\frac{8^{\frac{3}{4}}\left(x^4+1\right)^{\frac{1}{4}}x^3+4\cdot 8^{\frac{1}{4}}\left(x^4+1\right)^{\frac{3}{4}}x-2^{\frac{1}{4}}\left(8^{\frac{3}{4}}\sqrt{x^4+1}\,x^2+8^{\frac{1}{4}}\left(3x^4+2\right)\right)}{2\left(x^4+2\right)}\right)+\frac{1}{64}\cdot 8^{\frac{3}{4}}\log\left(\frac{8\sqrt{2}\left(x^4+1\right)^{\frac{1}{4}}x^3+8\cdot 8^{\frac{1}{4}}\sqrt{x^4+1}\,x^2+8^{\frac{3}{4}}\left(3x^4+2\right)+16\left(x^4+1\right)^{\frac{3}{4}}x}{x^4+2}\right)-\frac{1}{64}\cdot 8^{\frac{3}{4}}\log\left(\frac{8\sqrt{2}\left(x^4+1\right)^{\frac{1}{4}}x^3-8\cdot 8^{\frac{1}{4}}\sqrt{x^4+1}\,x^2-8^{\frac{3}{4}}\left(3x^4+2\right)+16\left(x^4+1\right)^{\frac{3}{4}}x}{x^4+2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4+1)^(1/4)/(x^4+2),x, algorithm="fricas")
```

```
[Out] -1/16*8^(3/4)*arctan(-1/2*(8^(3/4)*(x^4 + 1)^(1/4)*x^3 + 4*8^(1/4)*(x^4 + 1
)^(3/4)*x - 2^(1/4)*(8^(3/4)*sqrt(x^4 + 1)*x^2 + 8^(1/4)*(3*x^4 + 2)))/(x^4
 + 2)) + 1/64*8^(3/4)*log((8*sqrt(2)*(x^4 + 1)^(1/4)*x^3 + 8*8^(1/4)*sqrt(x
^4 + 1)*x^2 + 8^(3/4)*(3*x^4 + 2) + 16*(x^4 + 1)^(3/4)*x)/(x^4 + 2)) - 1/64
*8^(3/4)*log((8*sqrt(2)*(x^4 + 1)^(1/4)*x^3 - 8*8^(1/4)*sqrt(x^4 + 1)*x^2 -
 8^(3/4)*(3*x^4 + 2) + 16*(x^4 + 1)^(3/4)*x)/(x^4 + 2))
```

giac [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^4+2\right)\left(x^4+1\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4+1)^(1/4)/(x^4+2),x, algorithm="giac")
```

```
[Out] integrate(1/((x^4 + 2)*(x^4 + 1)^(1/4)), x)
```

maple [C] time = 3.24, size = 211, normalized size = 3.98

$$-\frac{\mathrm{RootOf}\left(\_Z^2+\mathrm{RootOf}\left(\_Z^4-2\right)^2\right)\ln\left(-\frac{-3x^4\,\mathrm{RootOf}\left(\_Z^2+\mathrm{RootOf}\left(\_Z^4-2\right)^2\right)-2\left(x^4+1\right)^{\frac{1}{4}}x^3\,\mathrm{RootOf}\left(\_Z^4-2\right)^2+2\sqrt{x^4+1}\,x^2\,\mathrm{RootOf}\left(\_Z^2+\mathrm{RootOf}\left(\_Z^4-2\right)^2\right)\mathrm{RootOf}\left(\_Z^4-2\right)^2+4\left(x^4+1\right)^{\frac{3}{4}}x-2\,\mathrm{RootOf}\left(\_Z^2+\mathrm{RootOf}\left(\_Z^4-2\right)^2\right)}{x^4+2}\right)}{8}+\frac{\mathrm{RootOf}\left(\_Z^4-2\right)\ln\left(\frac{3x^4\,\mathrm{RootOf}\left(\_Z^4-2\right)+2\left(x^4+1\right)^{\frac{1}{4}}x^3\,\mathrm{RootOf}\left(\_Z^4-2\right)^2+2\sqrt{x^4+1}\,x^2\,\mathrm{RootOf}\left(\_Z^4-2\right)^3+4\left(x^4+1\right)^{\frac{3}{4}}x+2\,\mathrm{RootOf}\left(\_Z^4-2\right)}{x^4+2}\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4+1)^(1/4)/(x^4+2),x)
```

```
[Out] -1/8*RootOf(_Z^2+RootOf(_Z^4-2)^2)*ln(-(2*(x^4+1)^(1/2)*RootOf(_Z^4-2)^2*Ro
otOf(_Z^2+RootOf(_Z^4-2)^2)*x^2-2*(x^4+1)^(1/4)*RootOf(_Z^4-2)^2*x^3-3*Root
Of(_Z^2+RootOf(_Z^4-2)^2)*x^4+4*(x^4+1)^(3/4)*x-2*RootOf(_Z^2+RootOf(_Z^4-2
)^2))/(x^4+2))+1/8*RootOf(_Z^4-2)*ln((2*(x^4+1)^(1/2)*RootOf(_Z^4-2)^3*x^2+
2*(x^4+1)^(1/4)*RootOf(_Z^4-2)^2*x^3+3*RootOf(_Z^4-2)*x^4+4*(x^4+1)^(3/4)*x
+2*RootOf(_Z^4-2))/(x^4+2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^4+2\right)\left(x^4+1\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4+1)^(1/4)/(x^4+2),x, algorithm="maxima")
```

```
[Out] integrate(1/((x^4 + 2)*(x^4 + 1)^(1/4)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{{\left(x^4+1\right)}^{1/4}\,\left(x^4+2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((x^4 + 1)^(1/4)*(x^4 + 2)),x)
```

```
[Out] int(1/((x^4 + 1)^(1/4)*(x^4 + 2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[4]{x^{4}+1}\,\left(x^{4}+2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**4+1)**(1/4)/(x**4+2),x)
```

```
[Out] Integral(1/((x**4 + 1)**(1/4)*(x**4 + 2)), x)
```

3.125 $\int \frac{1}{\left(a-(a-b)x^4\right)\sqrt[4]{a+bx^4}}\,dx$

Optimal. Leaf size=57

$$\frac{\tan^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{a+bx^4}}\right)}{2a^{5/4}}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{a+bx^4}}\right)}{2a^{5/4}}$$

Rubi [A] time = 0.02, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {377, 212, 206, 203}

$$\frac{\tan^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{a+bx^4}}\right)}{2a^{5/4}}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{a+bx^4}}\right)}{2a^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a - (a - b)*x^4)*(a + b*x^4)^(1/4)),x]
```

```
[Out] ArcTan[(a^(1/4)*x)/(a + b*x^4)^(1/4)]/(2*a^(5/4)) + ArcTanh[(a^(1/4)*x)/(a
+ b*x^4)^(1/4)]/(2*a^(5/4))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(a-(a-b)x^4\right)\sqrt[4]{a+bx^4}}\,dx &= \text{Subst}\left(\int \frac{1}{a-(ab-a(-a+b))x^4}\,dx,x,\frac{x}{\sqrt[4]{a+bx^4}}\right)\\ &= \frac{\text{Subst}\left(\int \frac{1}{1-\sqrt{a}\,x^2}\,dx,x,\frac{x}{\sqrt[4]{a+bx^4}}\right)}{2a}+\frac{\text{Subst}\left(\int \frac{1}{1+\sqrt{a}\,x^2}\,dx,x,\frac{x}{\sqrt[4]{a+bx^4}}\right)}{2a}\\ &= \frac{\tan^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{a+bx^4}}\right)}{2a^{5/4}}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{a}\,x}{\sqrt[4]{a+bx^4}}\right)}{2a^{5/4}}\end{aligned}$$

Mathematica [A] time = 0.03, size = 48, normalized size = 0.84

$$\frac{\tan^{-1}\left(\frac{\sqrt[4]{a}x}{\sqrt[4]{a+bx^4}}\right)+\tanh^{-1}\left(\frac{\sqrt[4]{a}x}{\sqrt[4]{a+bx^4}}\right)}{2a^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a - (a - b)*x^4)*(a + b*x^4)^(1/4)),x]
```

```
[Out] (ArcTan[(a^(1/4)*x)/(a + b*x^4)^(1/4)] + ArcTanh[(a^(1/4)*x)/(a + b*x^4)^(1
/4)])/(2*a^(5/4))
```

IntegrateAlgebraic [A] time = 0.44, size = 57, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{\sqrt[4]{a}x}{\sqrt[4]{a+bx^4}}\right)}{2a^{5/4}}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{a}x}{\sqrt[4]{a+bx^4}}\right)}{2a^{5/4}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((a - (a - b)*x^4)*(a + b*x^4)^(1/4)),x]
```

```
[Out] ArcTan[(a^(1/4)*x)/(a + b*x^4)^(1/4)]/(2*a^(5/4)) + ArcTanh[(a^(1/4)*x)/(a
+ b*x^4)^(1/4)]/(2*a^(5/4))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-(a-b)*x^4)/(b*x^4+a)^(1/4),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{1}{\left((a-b)x^4-a\right)\left(bx^4+a\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-(a-b)*x^4)/(b*x^4+a)^(1/4),x, algorithm="giac")
```

```
[Out] integrate(-1/(((a - b)*x^4 - a)*(b*x^4 + a)^(1/4)), x)
```

maple [F] time = 0.62, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-\left(a-b\right)x^{4}+a\right)\left(b\,x^{4}+a\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a-(a-b)*x^4)/(b*x^4+a)^(1/4),x)
```

```
[Out] int(1/(a-(a-b)*x^4)/(b*x^4+a)^(1/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{\left((a-b)x^4-a\right)\left(bx^4+a\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-(a-b)*x^4)/(b*x^4+a)^(1/4),x, algorithm="maxima")
```

```
[Out] -integrate(1/(((a - b)*x^4 - a)*(b*x^4 + a)^(1/4)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\left(b\,x^4+a\right)^{1/4}\,\left(a-x^4\,\left(a-b\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^4)^(1/4)*(a - x^4*(a - b))),x)
```

```
[Out] int(1/((a + b*x^4)^(1/4)*(a - x^4*(a - b))), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{ax^4\sqrt[4]{a+bx^4}-a\sqrt[4]{a+bx^4}-bx^4\sqrt[4]{a+bx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-(a-b)*x**4)/(b*x**4+a)**(1/4),x)
```

```
[Out] -Integral(1/(a*x**4*(a + b*x**4)**(1/4) - a*(a + b*x**4)**(1/4) - b*x**4*(a
 + b*x**4)**(1/4)), x)
```

3.126 $\int \frac{1}{\sqrt[5]{a+bx^5}\left(c+dx^5\right)}\,dx$

Optimal. Leaf size=545

$$-\frac{\log\left(\sqrt[5]{c}-\frac{x\sqrt[5]{bc-ad}}{\sqrt[5]{a+bx^5}}\right)}{5c^{4/5}\sqrt[5]{bc-ad}}-\frac{\sqrt{\frac{1}{2}\left(5+\sqrt{5}\right)}\tan^{-1}\left(\sqrt{\frac{1}{5}\left(5-2\sqrt{5}\right)}-\frac{2\sqrt{\frac{2}{5+\sqrt{5}}}\,x\sqrt[5]{bc-ad}}{\sqrt[5]{c}\sqrt[5]{a+bx^5}}\right)}{5c^{4/5}\sqrt[5]{bc-ad}}+\frac{\sqrt{\frac{1}{2}\left(5-\sqrt{5}\right)}\tan^{-1}\left(\frac{\sqrt{\frac{2}{5}\left(5+\sqrt{5}\right)}\ldots}{\sqrt[5]{c}\sqrt[5]{a+\ldots}}\ldots\right.}{5c^{4/5}\sqrt[5]{bc\ldots}}$$

Rubi [A] time = 1.09, antiderivative size = 545, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {377, 202, 634, 618, 204, 628, 31}

$$-\frac{\log\left(\sqrt[5]{c}-\frac{x\sqrt[5]{bc-ad}}{\sqrt[5]{a+bx^5}}\right)}{5c^{4/5}\sqrt[5]{bc-ad}}+\frac{\left(1-\sqrt{5}\right)\log\left(\frac{2c^{2/5}\left(a+bx^5\right)^{2/5}+2x^2(bc-ad)^{2/5}-\sqrt{5}\sqrt[5]{c}\,x\sqrt[5]{a+bx^5}\sqrt[5]{bc-ad}+\sqrt[5]{c}\,x\sqrt[5]{a+bx^5}\sqrt[5]{bc-ad}}{\left(a+bx^5\right)^{2/5}}\right)}{20c^{4/5}\sqrt[5]{bc-ad}}+\frac{\left(1+\sqrt{5}\right)\log\left(\frac{2c^{2/5}\left(a+bx^5\right)^{2/5}+2x^2(bc-ad)^{2/5}+\sqrt{5}\sqrt[5]{c}\,x\sqrt[5]{a+bx^5}\sqrt[5]{bc-ad}+\sqrt[5]{c}\,x\sqrt[5]{a+bx^5}\sqrt[5]{bc-ad}}{\left(a+bx^5\right)^{2/5}}\right)}{20c^{4/5}\sqrt[5]{bc-ad}}-\frac{\sqrt{\frac{1}{2}\left(5+\sqrt{5}\right)}\tan^{-1}\left(\sqrt{\frac{1}{5}\left(5-2\sqrt{5}\right)}-\frac{2\sqrt{\frac{2}{5+\sqrt{5}}}\,x\sqrt[5]{bc-ad}}{\sqrt[5]{c}\sqrt[5]{a+bx^5}}\right)}{5c^{4/5}\sqrt[5]{bc-ad}}+\frac{\sqrt{\frac{1}{2}\left(5-\sqrt{5}\right)}\tan^{-1}\left(\frac{\sqrt{\frac{2}{5}\left(5+\sqrt{5}\right)}\,x\sqrt[5]{bc-ad}}{\sqrt[5]{c}\sqrt[5]{a+bx^5}}+\sqrt{\frac{1}{5}\left(5+2\sqrt{5}\right)}\right)}{5c^{4/5}\sqrt[5]{bc-ad}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^5)^(1/5)*(c + d*x^5)),x]
```

```
[Out] -(Sqrt[(5 + Sqrt[5])/2]*ArcTan[Sqrt[(5 - 2*Sqrt[5])/5] - (2*Sqrt[2/(5 + Sqr
t[5])]*(b*c - a*d)^(1/5)*x)/(c^(1/5)*(a + b*x^5)^(1/5))])/(5*c^(4/5)*(b*c -
 a*d)^(1/5)) + (Sqrt[(5 - Sqrt[5])/2]*ArcTan[Sqrt[(5 + 2*Sqrt[5])/5] + (Sqr
t[(2*(5 + Sqrt[5]))/5]*(b*c - a*d)^(1/5)*x)/(c^(1/5)*(a + b*x^5)^(1/5))])/(
5*c^(4/5)*(b*c - a*d)^(1/5)) - Log[c^(1/5) - ((b*c - a*d)^(1/5)*x)/(a + b*x
^5)^(1/5)]/(5*c^(4/5)*(b*c - a*d)^(1/5)) + ((1 - Sqrt[5])*Log[(2*(b*c - a*d
)^(2/5)*x^2 + c^(1/5)*(b*c - a*d)^(1/5)*x*(a + b*x^5)^(1/5) - Sqrt[5]*c^(1/
5)*(b*c - a*d)^(1/5)*x*(a + b*x^5)^(1/5) + 2*c^(2/5)*(a + b*x^5)^(2/5))/(a
+ b*x^5)^(2/5)])/(20*c^(4/5)*(b*c - a*d)^(1/5)) + ((1 + Sqrt[5])*Log[(2*(b*
c - a*d)^(2/5)*x^2 + c^(1/5)*(b*c - a*d)^(1/5)*x*(a + b*x^5)^(1/5) + Sqrt[5
]*c^(1/5)*(b*c - a*d)^(1/5)*x*(a + b*x^5)^(1/5) + 2*c^(2/5)*(a + b*x^5)^(2/
5))/(a + b*x^5)^(2/5)])/(20*c^(4/5)*(b*c - a*d)^(1/5))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 202

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[-(
a/b), n]], s = Denominator[Rt[-(a/b), n]], k, u}, Simp[u = Int[(r + s*Cos[(
(2*k - 1)*Pi)/n]*x)/(r^2 + 2*r*s*Cos[((2*k - 1)*Pi)/n]*x + s^2*x^2), x]; (r
*Int[1/(r - s*x), x])/(a*n) + Dist[(2*r)/(a*n), Sum[u, {k, 1, (n - 1)/2}],
x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 3)/2, 0] && NegQ[a/b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{1}{\sqrt[5]{a+bx^5}\left(c+dx^5\right)}\,dx = \text{Subst}\left(\int \frac{1}{c-(bc-ad)x^5}\,dx, x, \frac{x}{\sqrt[5]{a+bx^5}}\right)$$

$$= \frac{2\,\text{Subst}\left(\int \frac{\sqrt[5]{c}+\frac{1}{4}\left(1-\sqrt{5}\right)\sqrt[5]{bc-ad}\,x}{c^{2/5}+\frac{1}{2}\left(1-\sqrt{5}\right)\sqrt[5]{c}\,\sqrt[5]{bc-ad}\,x+(bc-ad)^{2/5}x^2}\,dx, x, \frac{x}{\sqrt[5]{a+bx^5}}\right)}{5c^{4/5}} + \frac{2\,\text{Subst}\left(\int \frac{\cdots}{c^{2/5}+\frac{1}{2}\left(1+\cdots\right.}\cdots\right.}{\cdots}$$

$$= -\frac{\log\left(\sqrt[5]{c}-\frac{\sqrt[5]{bc-ad}\,x}{\sqrt[5]{a+bx^5}}\right)}{5c^{4/5}\sqrt[5]{bc-ad}} + \frac{\left(5-\sqrt{5}\right)\text{Subst}\left(\int \frac{1}{c^{2/5}+\frac{1}{2}\left(1+\sqrt{5}\right)\sqrt[5]{c}\,\sqrt[5]{bc-ad}\,x+(bc-ad)^{2/5}x^2}\,dx, \cdots\right.}{20c^{3/5}}$$

$$= -\frac{\log\left(\sqrt[5]{c}-\frac{\sqrt[5]{bc-ad}\,x}{\sqrt[5]{a+bx^5}}\right)}{5c^{4/5}\sqrt[5]{bc-ad}} + \frac{\left(1-\sqrt{5}\right)\log\left(2c^{2/5}+\frac{2(bc-ad)^{2/5}x^2}{\left(a+bx^5\right)^{2/5}}+\frac{\sqrt[5]{c}\,\sqrt[5]{bc-ad}\,x}{\sqrt[5]{a+bx^5}}-\frac{\sqrt{5}\,\sqrt[5]{c}\,\cdots}{\sqrt[5]{a+\cdots}}\right.}{20c^{4/5}\sqrt[5]{bc-ad}}$$

$$= \frac{\sqrt{\frac{1}{2}\left(5+\sqrt{5}\right)}\,\tan^{-1}\left(\frac{\left(1-\sqrt{5}\right)\sqrt[5]{c}+\frac{4\sqrt[5]{bc-ad}\,x}{\sqrt[5]{a+bx^5}}}{\sqrt{2\left(5+\sqrt{5}\right)}\,\sqrt[5]{c}}\right)}{5c^{4/5}\sqrt[5]{bc-ad}} + \frac{\sqrt{\frac{1}{2}\left(5-\sqrt{5}\right)}\,\tan^{-1}\left(\frac{\sqrt{5+\sqrt{5}}\left(\left(1+\sqrt{5}\right)\cdots\right.}{2\sqrt{10}\,\cdots}\right.}{5c^{4/5}\sqrt[5]{bc-ad}}$$

Mathematica [C] time = 0.02, size = 48, normalized size = 0.09

$$\frac{x\,{}_2F_1\left(\frac{1}{5},1;\frac{6}{5};\frac{(bc-ad)x^5}{c\left(bx^5+a\right)}\right)}{c\sqrt[5]{a+bx^5}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x^5)^(1/5)*(c + d*x^5)),x]
```

```
[Out] (x*Hypergeometric2F1[1/5, 1, 6/5, ((b*c - a*d)*x^5)/(c*(a + b*x^5))])/(c*(a
 + b*x^5)^(1/5))
```

IntegrateAlgebraic [F] time = 180.01, size = 0, normalized size = 0.00

$Aborted

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/((a + b*x^5)^(1/5)*(c + d*x^5)),x]
```

```
[Out] $Aborted
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^5+a)^(1/5)/(d*x^5+c),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (trace 0)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^5+a\right)^{\frac{1}{5}}\left(dx^5+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^5+a)^(1/5)/(d*x^5+c),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^5 + a)^(1/5)*(d*x^5 + c)), x)
```

maple [F] time = 0.57, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,x^5+a\right)^{\frac{1}{5}}\left(d\,x^5+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^5+a)^(1/5)/(d*x^5+c),x)
```

```
[Out] int(1/(b*x^5+a)^(1/5)/(d*x^5+c),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^5+a\right)^{\frac{1}{5}}\left(dx^5+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^5+a)^(1/5)/(d*x^5+c),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^5 + a)^(1/5)*(d*x^5 + c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(b\,x^5+a\right)}^{1/5}\,\left(d\,x^5+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^5)^(1/5)*(c + d*x^5)),x)
```

```
[Out] int(1/((a + b*x^5)^(1/5)*(c + d*x^5)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[5]{a+bx^5}\,\left(c+dx^5\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**5+a)**(1/5)/(d*x**5+c),x)
```

```
[Out] Integral(1/((a + b*x**5)**(1/5)*(c + d*x**5)), x)
```

3.127 $\int \sqrt{a+\frac{b}{x}} \left(c+\frac{d}{x}\right)^3 \, dx$

Optimal. Leaf size=143

$$-\frac{d\sqrt{a+\frac{b}{x}}\left(2\left(-2a^2d^2+15abcd+57b^2c^2\right)+\frac{bd(2ad+33bc)}{x}\right)}{15b^2}+\frac{c^2(6ad+bc)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{\sqrt{a}}+x\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^3-\frac{7}{5}d$$

Rubi [A] time = 0.12, antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {375, 97, 153, 147, 63, 208}

$$-\frac{d\sqrt{a+\frac{b}{x}}\left(2\left(-2a^2d^2+15abcd+57b^2c^2\right)+\frac{bd(2ad+33bc)}{x}\right)}{15b^2}+\frac{c^2(6ad+bc)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{\sqrt{a}}+x\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^3-\frac{7}{5}d\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^2$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b/x]*(c + d/x)^3,x]
```

```
[Out] (-7*d*Sqrt[a + b/x]*(c + d/x)^2)/5 - (d*Sqrt[a + b/x]*(2*(57*b^2*c^2 + 15*a
*b*c*d - 2*a^2*d^2) + (b*d*(33*b*c + 2*a*d))/x))/(15*b^2) + Sqrt[a + b/x]*(
c + d/x)^3*x + (c^2*(b*c + 6*a*d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/Sqrt[a]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 97

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p)/(b*
(m + 1)), x] - Dist[1/(b*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*
(e + f*x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ
[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 147

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> -Simp[((a*d*f*h*(n + 2) + b*c*f*h*(m
+ 2) - b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1))/(b^2*d^2*(m + n + 2)*(m + n + 3)), x] + Dist[(a^2*d^2
*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m +
n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3
) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3)), In
t[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n},
x] && NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

Rule 153

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(h*(a + b*x)^m*(c + d*x)^(n
+ 1)*(e + f*x)^(p + 1))/(d*f*(m + n + p + 2)), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
```

```
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegerQ[m]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 375

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[((a + b/x^n)^p*(c + d/x^n)^q)/x^2, x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rubi steps

$$\int \sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^3 dx = -\text{Subst}\left(\int \frac{\sqrt{a+bx}\,(c+dx)^3}{x^2}\,dx, x, \frac{1}{x}\right)$$

$$= \sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^3 x - \text{Subst}\left(\int \frac{(c+dx)^2\left(\frac{1}{2}(bc+6ad)+\frac{7bdx}{2}\right)}{x\sqrt{a+bx}}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{7}{5}d\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^2 + \sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^3 x - \frac{2\,\text{Subst}\left(\int \frac{(c+dx)\left(\frac{5}{4}bc(bc+6ad)+\frac{1}{4}bd(33bc\cdots}{x\sqrt{a+bx}}\right.}{5b}$$

$$= -\frac{7}{5}d\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^2 - \frac{d\sqrt{a+\frac{b}{x}}\left(2\left(57b^2c^2+15abcd-2a^2d^2\right)+\frac{bd(33bc+2ad)}{x}\right)}{15b^2} + \cdots$$

$$= -\frac{7}{5}d\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^2 - \frac{d\sqrt{a+\frac{b}{x}}\left(2\left(57b^2c^2+15abcd-2a^2d^2\right)+\frac{bd(33bc+2ad)}{x}\right)}{15b^2} + \cdots$$

$$= -\frac{7}{5}d\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^2 - \frac{d\sqrt{a+\frac{b}{x}}\left(2\left(57b^2c^2+15abcd-2a^2d^2\right)+\frac{bd(33bc+2ad)}{x}\right)}{15b^2} + \cdots$$

Mathematica [A] time = 0.19, size = 118, normalized size = 0.83

$$\frac{\sqrt{a+\frac{b}{x}}\left(4a^2d^3x^2-2abd^2x(15cx+d)-3b^2\left(-5c^3x^3+30c^2dx^2+10cd^2x+2d^3\right)\right)}{15b^2x^2}+\frac{c^2(6ad+bc)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b/x]*(c + d/x)^3,x]
```

```
[Out] (Sqrt[a + b/x]*(4*a^2*d^3*x^2 - 2*a*b*d^2*x*(d + 15*c*x) - 3*b^2*(2*d^3 + 1
0*c*d^2*x + 30*c^2*d*x^2 - 5*c^3*x^3)))/(15*b^2*x^2) + (c^2*(b*c + 6*a*d)*A
rcTanh[Sqrt[a + b/x]/Sqrt[a]])/Sqrt[a]
```

IntegrateAlgebraic [A] time = 0.19, size = 135, normalized size = 0.94

$$\frac{\sqrt{\frac{ax+b}{x}}\left(4a^2d^3x^2-30abcd^2x^2-2abd^3x+15b^2c^3x^3-90b^2c^2dx^2-30b^2cd^2x-6b^2d^3\right)}{15b^2x^2}+\frac{\left(6ac^2d+bc^3\right)\tanh^{-1}\left(\frac{\sqrt{\frac{ax+b}{x}}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[a + b/x]*(c + d/x)^3,x]
```

```
[Out] (Sqrt[(b + a*x)/x]*(-6*b^2*d^3 - 30*b^2*c*d^2*x - 2*a*b*d^3*x - 90*b^2*c^2*
d*x^2 - 30*a*b*c*d^2*x^2 + 4*a^2*d^3*x^2 + 15*b^2*c^3*x^3))/(15*b^2*x^2) +
((b*c^3 + 6*a*c^2*d)*ArcTanh[Sqrt[(b + a*x)/x]/Sqrt[a]])/Sqrt[a]
```

fricas [A] time = 0.86, size = 306, normalized size = 2.14

$$\left[\frac{15\left(b^3c^3+6ab^2c^2d\right)\sqrt{a}x^2\log\left(2ax+2\sqrt{a}x\sqrt{\frac{ax+b}{x}}+b\right)+2\left(15ab^2c^3x^3-6ab^2d^3-2\left(45ab^2c^2d+15a^2bcd^2-2a^3d^3\right)x^2-2\left(15ab^2cd^2+a^2bd^3\right)x\right)\sqrt{\frac{ax+b}{x}}}{30ab^2x^2},-\frac{15\left(b^3c^3+6ab^2c^2d\right)\sqrt{-a}x^2\arctan\left(\frac{\sqrt{-a}\sqrt{\frac{ax+b}{x}}}{a}\right)-\left(15ab^2c^3x^3-6ab^2d^3-2\left(45ab^2c^2d+15a^2bcd^2-2a^3d^3\right)x^2-2\left(15ab^2cd^2+a^2bd^3\right)x\right)\sqrt{\frac{ax+b}{x}}}{15ab^2x^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d/x)^3*(a+b/x)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/30*(15*(b^3*c^3 + 6*a*b^2*c^2*d)*sqrt(a)*x^2*log(2*a*x + 2*sqrt(a)*x*sqr
t((a*x + b)/x) + b) + 2*(15*a*b^2*c^3*x^3 - 6*a*b^2*d^3 - 2*(45*a*b^2*c^2*d
 + 15*a^2*b*c*d^2 - 2*a^3*d^3)*x^2 - 2*(15*a*b^2*c*d^2 + a^2*b*d^3)*x)*sqrt
((a*x + b)/x))/(a*b^2*x^2), -1/15*(15*(b^3*c^3 + 6*a*b^2*c^2*d)*sqrt(-a)*x^
2*arctan(sqrt(-a)*sqrt((a*x + b)/x)/a) - (15*a*b^2*c^3*x^3 - 6*a*b^2*d^3 -
2*(45*a*b^2*c^2*d + 15*a^2*b*c*d^2 - 2*a^3*d^3)*x^2 - 2*(15*a*b^2*c*d^2 + a
^2*b*d^3)*x)*sqrt((a*x + b)/x))/(a*b^2*x^2)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d/x)^3*(a+b/x)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(x)]
Warning, choosing root of [1,0,%%%{-2,[1,2,0]%%%}+%%%{-2,[1,0,0]%%%}+%%%{-2
,[0,1,1]%%%},0,%%%{1,[2,4,0]%%%}+%%%{-2,[2,2,0]%%%}+%%%{1,[2,0,0]%%%}+%%%{2
,[1,3,1]%%%}+%%%{-2,[1,1,1]%%%}+%%%{1,[0,2,2]%%%}] at parameters values [86
,-97,-82]Warning, choosing root of [1,0,%%%{-2,[1,2,0]%%%}+%%%{-2,[1,0,0]%%
%}+%%%{-2,[0,1,1]%%%},0,%%%{1,[2,4,0]%%%}+%%%{-2,[2,2,0]%%%}+%%%{1,[2,0,0]%
%%}+%%%{2,[1,3,1]%%%}+%%%{-2,[1,1,1]%%%}+%%%{1,[0,2,2]%%%}] at parameters v
alues [7,-27,26]Warning, choosing root of [1,0,%%%{-2,[1,2,0]%%%}+%%%{-2,[1
,0,0]%%%}+%%%{-2,[0,1,1]%%%},0,%%%{1,[2,4,0]%%%}+%%%{-2,[2,2,0]%%%}+%%%{1,[
2,0,0]%%%}+%%%{2,[1,3,1]%%%}+%%%{-2,[1,1,1]%%%}+%%%{1,[0,2,2]%%%}] at param
eters values [-89,63,-49]Warning, choosing root of [1,0,%%%{-4,[1,0,0]%%%}+
%%%{-2,[0,1,1]%%%},0,%%%{1,[0,2,2]%%%}] at parameters values [61.7937478349
,-30,70]Warning, choosing root of [1,0,%%%{-4,[1,0,0]%%%}+%%%{-2,[0,1,1]%%%
},0,%%%{1,[0,2,2]%%%}] at parameters values [73.519035968,-9,-13]Warning, c
hoosing root of [1,0,%%%{-2,[1,0,1]%%%}+%%%{-4,[0,1,0]%%%},0,%%%{1,[2,0,2]%
%%}] at parameters values [18,15.451549686,-33]Sign error (%%%{-b,0%%%}+%%%
{2*sqrt(a)*sqrt(b),1/2%%%}+%%%{-2*a,1%%%}+%%%{a*sqrt(a)*sqrt(b)/b,3/2%%%}+%
%%{-a^2*sqrt(a)*sqrt(b)/(4*b^2),5/2%%%}+%%%{undef,7/2%%%})Limit: Max order
reached or unable to make series expansion Error: Bad Argument Value
```

maple [A] time = 0.06, size = 248, normalized size = 1.73

$$\frac{\sqrt{\frac{ax+b}{x}}\left(90a\,b^2c^2d\,x^4\ln\left(\frac{2ax+b+2\sqrt{a\,x^2+bx}\,\sqrt{a}}{2\sqrt{a}}\right)+15b^3c^3x^4\ln\left(\frac{2ax+b+2\sqrt{a\,x^2+bx}\,\sqrt{a}}{2\sqrt{a}}\right)+180\sqrt{a\,x^2+bx}\,a^{\frac{3}{2}}b\,c^2d\,x^4+30\sqrt{a\,x^2+bx}\,\sqrt{a}\,b^2c^3x^4-180\left(a\,x^2+bx\right)^{\frac{3}{2}}\sqrt{a}\,b\,c^2d\,x^2+8\left(a\,x^2+bx\right)^{\frac{3}{2}}a^{\frac{3}{2}}d^3x-60\left(a\,x^2+bx\right)^{\frac{3}{2}}\sqrt{a}\,bc\,d^2x-12\left(a\,x^2+bx\right)^{\frac{3}{2}}\sqrt{a}\,b\,d^3\right)}{30\sqrt{(ax+b)\,x}\,\sqrt{a}\,b^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d/x)^3*(a+b/x)^(1/2),x)
```

```
[Out] 1/30*((a*x+b)/x)^(1/2)*(180*(a*x^2+b*x)^(1/2)*a^(3/2)*x^4*b*c^2*d+30*(a*x^2
+b*x)^(1/2)*a^(1/2)*x^4*b^2*c^3+90*ln(1/2*(2*a*x+b+2*(a*x^2+b*x)^(1/2)*a^(1
/2))/a^(1/2))*x^4*a*b^2*c^2*d+15*ln(1/2*(2*a*x+b+2*(a*x^2+b*x)^(1/2)*a^(1/2
))/a^(1/2))*x^4*b^3*c^3+8*(a*x^2+b*x)^(3/2)*a^(3/2)*x*d^3-180*(a*x^2+b*x)^(
3/2)*a^(1/2)*x^2*b*c^2*d-60*d^2*c*(a*x^2+b*x)^(3/2)*x*b*a^(1/2)-12*(a*x^2+b
*x)^(3/2)*a^(1/2)*b*d^3)/x^3/((a*x+b)*x)^(1/2)/b^2/a^(1/2)
```

maxima [A] time = 1.45, size = 164, normalized size = 1.15

$$\frac{1}{2}\left(2\sqrt{a+\frac{b}{x}}\,x-\frac{b\log\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{a+\frac{b}{x}}+\sqrt{a}}\right)}{\sqrt{a}}\right)c^3-3\left(\sqrt{a}\,\log\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{a+\frac{b}{x}}+\sqrt{a}}\right)+2\sqrt{a+\frac{b}{x}}\right)c^2d-\frac{2}{15}\,d^3\left(\frac{3\left(a+\frac{b}{x}\right)^{\frac{5}{2}}}{b^2}-\frac{5\left(a+\frac{b}{x}\right)^{\frac{3}{2}}a}{b^2}\right)-\frac{2\left(a+\frac{b}{x}\right)^{\frac{3}{2}}cd^2}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d/x)^3*(a+b/x)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*(2*sqrt(a + b/x)*x - b*log((sqrt(a + b/x) - sqrt(a))/(sqrt(a + b/x) + s
qrt(a)))/sqrt(a))*c^3 - 3*(sqrt(a)*log((sqrt(a + b/x) - sqrt(a))/(sqrt(a +
b/x) + sqrt(a))) + 2*sqrt(a + b/x))*c^2*d - 2/15*d^3*(3*(a + b/x)^(5/2)/b^2
 - 5*(a + b/x)^(3/2)*a/b^2) - 2*(a + b/x)^(3/2)*c*d^2/b
```

mupad [B] time = 2.59, size = 173, normalized size = 1.21

$$\left(a+\frac{b}{x}\right)^{3/2}\left(\frac{6\,a\,d^3-6\,b\,c\,d^2}{3\,b^2}-\frac{4\,a\,d^3}{3\,b^2}\right)+\sqrt{a+\frac{b}{x}}\,\left(2\,a\,\left(\frac{6\,a\,d^3-6\,b\,c\,d^2}{b^2}-\frac{4\,a\,d^3}{b^2}\right)-\frac{6\,d\,{\left(a\,d-b\,c\right)}^2}{b^2}+\frac{2\,a^2\,d^3}{b^2}\right)+c^3\,x\,\sqrt{a+\frac{b}{x}}-\frac{2\,d^3\,{\left(a+\frac{b}{x}\right)}^{5/2}}{5\,b^2}-\frac{c^2\,\mathrm{atan}\left(\frac{\sqrt{a+\frac{b}{x}}\,1{}\mathrm{i}}{\sqrt{a}}\right)\,\left(6\,a\,d+b\,c\right)\,1{}\mathrm{i}}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x)^(1/2)*(c + d/x)^3,x)
```

```
[Out] (a + b/x)^(3/2)*((6*a*d^3 - 6*b*c*d^2)/(3*b^2) - (4*a*d^3)/(3*b^2)) + (a +
b/x)^(1/2)*(2*a*((6*a*d^3 - 6*b*c*d^2)/b^2 - (4*a*d^3)/b^2) - (6*d*(a*d - b
*c)^2)/b^2 + (2*a^2*d^3)/b^2) + c^3*x*(a + b/x)^(1/2) - (2*d^3*(a + b/x)^(5
/2))/(5*b^2) - (c^2*atan(((a + b/x)^(1/2)*1i)/a^(1/2))*(6*a*d + b*c)*1i)/a^
(1/2)
```

sympy [A] time = 57.22, size = 454, normalized size = 3.17

$$\frac{4a^{\frac{11}{2}}b^{\frac{3}{2}}d^3x^3\sqrt{\frac{ax}{b}+1}}{15a^{\frac{7}{2}}b^3x^{\frac{7}{2}}+15a^{\frac{5}{2}}b^4x^{\frac{5}{2}}}+\frac{2a^{\frac{9}{2}}b^{\frac{5}{2}}d^3x^2\sqrt{\frac{ax}{b}+1}}{15a^{\frac{7}{2}}b^3x^{\frac{7}{2}}+15a^{\frac{5}{2}}b^4x^{\frac{5}{2}}}-\frac{8a^{\frac{7}{2}}b^{\frac{7}{2}}d^3x\sqrt{\frac{ax}{b}+1}}{15a^{\frac{7}{2}}b^3x^{\frac{7}{2}}+15a^{\frac{5}{2}}b^4x^{\frac{5}{2}}}-\frac{6a^{\frac{5}{2}}b^{\frac{9}{2}}d^3\sqrt{\frac{ax}{b}+1}}{15a^{\frac{7}{2}}b^3x^{\frac{7}{2}}+15a^{\frac{5}{2}}b^4x^{\frac{5}{2}}}-\frac{4a^6bd^3x^{\frac{7}{2}}}{15a^{\frac{7}{2}}b^3x^{\frac{7}{2}}+15a^{\frac{5}{2}}b^4x^{\frac{5}{2}}}-\frac{4a^5b^2d^3x^{\frac{5}{2}}}{15a^{\frac{7}{2}}b^3x^{\frac{7}{2}}+15a^{\frac{5}{2}}b^4x^{\frac{5}{2}}}-\frac{6ac^2d\,\mathrm{atan}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{-a}}\right)}{\sqrt{-a}}+\sqrt{b}c^3\sqrt{x}\sqrt{\frac{ax}{b}+1}-6c^2d\sqrt{a+\frac{b}{x}}+3cd^2\left(\begin{cases}-\frac{\sqrt{a}}{x} & \text{for}\ b=0\\ -\frac{2\left(a+\frac{b}{x}\right)^{\frac{3}{2}}}{3b} & \text{otherwise}\end{cases}\right)+\frac{bc^3\,\mathrm{asinh}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d/x)**3*(a+b/x)**(1/2),x)
```

```
[Out] 4*a**(11/2)*b**(3/2)*d**3*x**3*sqrt(a*x/b + 1)/(15*a**(7/2)*b**3*x**(7/2) +
 15*a**(5/2)*b**4*x**(5/2)) + 2*a**(9/2)*b**(5/2)*d**3*x**2*sqrt(a*x/b + 1)
/(15*a**(7/2)*b**3*x**(7/2) + 15*a**(5/2)*b**4*x**(5/2)) - 8*a**(7/2)*b**(7
/2)*d**3*x*sqrt(a*x/b + 1)/(15*a**(7/2)*b**3*x**(7/2) + 15*a**(5/2)*b**4*x*
*(5/2)) - 6*a**(5/2)*b**(9/2)*d**3*sqrt(a*x/b + 1)/(15*a**(7/2)*b**3*x**(7/
2) + 15*a**(5/2)*b**4*x**(5/2)) - 4*a**6*b*d**3*x**(7/2)/(15*a**(7/2)*b**3*
x**(7/2) + 15*a**(5/2)*b**4*x**(5/2)) - 4*a**5*b**2*d**3*x**(5/2)/(15*a**(7
/2)*b**3*x**(7/2) + 15*a**(5/2)*b**4*x**(5/2)) - 6*a*c**2*d*atan(sqrt(a + b
/x)/sqrt(-a))/sqrt(-a) + sqrt(b)*c**3*sqrt(x)*sqrt(a*x/b + 1) - 6*c**2*d*sq
rt(a + b/x) + 3*c*d**2*Piecewise((-sqrt(a)/x, Eq(b, 0)), (-2*(a + b/x)**(3/
2)/(3*b), True)) + b*c**3*asinh(sqrt(a)*sqrt(x)/sqrt(b))/sqrt(a)
```

3.128 $\int \sqrt{a+\frac{b}{x}} \left(c+\frac{d}{x}\right)^2 \, dx$

Optimal. Leaf size=99

$$\frac{c^2 x \left(a+\frac{b}{x}\right)^{3/2}}{a}-\frac{c\sqrt{a+\frac{b}{x}}\,(4ad+bc)}{a}+\frac{c(4ad+bc)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{\sqrt{a}}-\frac{2d^2\left(a+\frac{b}{x}\right)^{3/2}}{3b}$$

Rubi [A] time = 0.07, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {375, 89, 80, 50, 63, 208}

$$\frac{c^2 x \left(a+\frac{b}{x}\right)^{3/2}}{a}-\frac{c\sqrt{a+\frac{b}{x}}\,(4ad+bc)}{a}+\frac{c(4ad+bc)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{\sqrt{a}}-\frac{2d^2\left(a+\frac{b}{x}\right)^{3/2}}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b/x]*(c + d/x)^2,x]
```

```
[Out] -((c*(b*c + 4*a*d)*Sqrt[a + b/x])/a) - (2*d^2*(a + b/x)^(3/2))/(3*b) + (c^2
*(a + b/x)^(3/2)*x)/a + (c*(b*c + 4*a*d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/Sq
rt[a]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
```

```
1])))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 375

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[((a + b/x^n)^p*(c + d/x^n)^q)/x^2, x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rubi steps

$$\int \sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^2 dx = -\text{Subst}\left(\int \frac{\sqrt{a+bx}\,(c+dx)^2}{x^2}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{c^2\left(a+\frac{b}{x}\right)^{3/2}x}{a} - \frac{\text{Subst}\left(\int \frac{\sqrt{a+bx}\left(\frac{1}{2}c(bc+4ad)+ad^2x\right)}{x}\,dx, x, \frac{1}{x}\right)}{a}$$

$$= -\frac{2d^2\left(a+\frac{b}{x}\right)^{3/2}}{3b} + \frac{c^2\left(a+\frac{b}{x}\right)^{3/2}x}{a} - \frac{(c(bc+4ad))\,\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x}\,dx, x, \frac{1}{x}\right)}{2a}$$

$$= -\frac{c(bc+4ad)\sqrt{a+\frac{b}{x}}}{a} - \frac{2d^2\left(a+\frac{b}{x}\right)^{3/2}}{3b} + \frac{c^2\left(a+\frac{b}{x}\right)^{3/2}x}{a} - \frac{1}{2}(c(bc+4ad))\,\text{Subst}\left(\int\right.$$

$$= -\frac{c(bc+4ad)\sqrt{a+\frac{b}{x}}}{a} - \frac{2d^2\left(a+\frac{b}{x}\right)^{3/2}}{3b} + \frac{c^2\left(a+\frac{b}{x}\right)^{3/2}x}{a} - \frac{(c(bc+4ad))\,\text{Subst}\left(\int\right.}{b}$$

$$= -\frac{c(bc+4ad)\sqrt{a+\frac{b}{x}}}{a} - \frac{2d^2\left(a+\frac{b}{x}\right)^{3/2}}{3b} + \frac{c^2\left(a+\frac{b}{x}\right)^{3/2}x}{a} + \frac{c(bc+4ad)\tanh^{-1}\left(\frac{\sqrt{a+}}{\sqrt{a}}\right.}{\sqrt{a}}$$

Mathematica [A] time = 0.16, size = 84, normalized size = 0.85

$$\frac{\sqrt{a+\frac{b}{x}}\left(b\left(3c^2x^2-12cdx-2d^2\right)-2ad^2x\right)}{3bx} + \frac{c(4ad+bc)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b/x]*(c + d/x)^2,x]
```

```
[Out] (Sqrt[a + b/x]*(-2*a*d^2*x + b*(-2*d^2 - 12*c*d*x + 3*c^2*x^2)))/(3*b*x) +
(c*(b*c + 4*a*d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/Sqrt[a]
```

IntegrateAlgebraic [A] time = 0.17, size = 90, normalized size = 0.91

$$\frac{\sqrt{\frac{ax+b}{x}}\left(-2ad^2x+3bc^2x^2-12bcdx-2bd^2\right)}{3bx} + \frac{\left(4acd+bc^2\right)\tanh^{-1}\left(\frac{\sqrt{\frac{ax+b}{x}}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[a + b/x]*(c + d/x)^2,x]
```

```
[Out] (Sqrt[(b + a*x)/x]*(-2*b*d^2 - 12*b*c*d*x - 2*a*d^2*x + 3*b*c^2*x^2))/(3*b*
x) + ((b*c^2 + 4*a*c*d)*ArcTanh[Sqrt[(b + a*x)/x]/Sqrt[a]])/Sqrt[a]
```

fricas [A] time = 0.76, size = 208, normalized size = 2.10

$$\left[\frac{3\left(b^{2}c^{2}+4\,abcd\right)\sqrt{a}\,x\log\left(2\,ax+2\,\sqrt{a}\,x\sqrt{\frac{ax+b}{x}}+b\right)+2\left(3\,abc^{2}x^{2}-2\,abd^{2}-2\left(6\,abcd+a^{2}d^{2}\right)x\right)\sqrt{\frac{ax+b}{x}}}{6\,abx},-\frac{3\left(b^{2}c^{2}+4\,abcd\right)\sqrt{-a}\,x\arctan\left(\frac{\sqrt{-a}\sqrt{\frac{ax+b}{x}}}{a}\right)-\left(3\,abc^{2}x^{2}-2\,abd^{2}-2\left(6\,abcd+a^{2}d^{2}\right)x\right)\sqrt{\frac{ax+b}{x}}}{3\,abx}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d/x)^2*(a+b/x)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/6*(3*(b^2*c^2 + 4*a*b*c*d)*sqrt(a)*x*log(2*a*x + 2*sqrt(a)*x*sqrt((a*x +
 b)/x) + b) + 2*(3*a*b*c^2*x^2 - 2*a*b*d^2 - 2*(6*a*b*c*d + a^2*d^2)*x)*sqr
t((a*x + b)/x))/(a*b*x), -1/3*(3*(b^2*c^2 + 4*a*b*c*d)*sqrt(-a)*x*arctan(sq
rt(-a)*sqrt((a*x + b)/x)/a) - (3*a*b*c^2*x^2 - 2*a*b*d^2 - 2*(6*a*b*c*d + a
^2*d^2)*x)*sqrt((a*x + b)/x))/(a*b*x)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d/x)^2*(a+b/x)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(x)]
Warning, choosing root of [1,0,%%%{-2,[1,2,0]%%%}+%%%{-2,[1,0,0]%%%}+%%%{-2
,[0,1,1]%%%},0,%%%{1,[2,4,0]%%%}+%%%{-2,[2,2,0]%%%}+%%%{1,[2,0,0]%%%}+%%%{2
,[1,3,1]%%%}+%%%{-2,[1,1,1]%%%}+%%%{1,[0,2,2]%%%}] at parameters values [86
,-97,-82]Warning, choosing root of [1,0,%%%{-2,[1,2,0]%%%}+%%%{-2,[1,0,0]%%
%}+%%%{-2,[0,1,1]%%%},0,%%%{1,[2,4,0]%%%}+%%%{-2,[2,2,0]%%%}+%%%{1,[2,0,0]%
%%}+%%%{2,[1,3,1]%%%}+%%%{-2,[1,1,1]%%%}+%%%{1,[0,2,2]%%%}] at parameters v
alues [7,-27,26]Warning, choosing root of [1,0,%%%{-2,[1,2,0]%%%}+%%%{-2,[1
,0,0]%%%}+%%%{-2,[0,1,1]%%%},0,%%%{1,[2,4,0]%%%}+%%%{-2,[2,2,0]%%%}+%%%{1,[
2,0,0]%%%}+%%%{2,[1,3,1]%%%}+%%%{-2,[1,1,1]%%%}+%%%{1,[0,2,2]%%%}] at param
eters values [-89,63,-49]Warning, choosing root of [1,0,%%%{-4,[1,0,0]%%%}+
%%%{-2,[0,1,1]%%%},0,%%%{1,[0,2,2]%%%}] at parameters values [61.7937478349
,-30,70]Warning, choosing root of [1,0,%%%{-4,[1,0,0]%%%}+%%%{-2,[0,1,1]%%%
},0,%%%{1,[0,2,2]%%%}] at parameters values [73.519035968,-9,-13]Warning, c
hoosing root of [1,0,%%%{-2,[1,0,1]%%%}+%%%{-4,[0,1,0]%%%},0,%%%{1,[2,0,2]%
%%}] at parameters values [18,15.451549686,-33]Sign error (%%%{-b,0%%%}+%%%
{2*sqrt(a)*sqrt(b),1/2%%%}+%%%{-2*a,1%%%}+%%%{a*sqrt(a)*sqrt(b)/b,3/2%%%}+%
%%{-a^2*sqrt(a)*sqrt(b)/(4*b^2),5/2%%%}+%%%{undef,7/2%%%})Limit: Max order
reached or unable to make series expansion Error: Bad Argument Value
```

maple [B] time = 0.06, size = 191, normalized size = 1.93

$$-\frac{\sqrt{\frac{ax+b}{x}}\left(-12abcd\,x^{3}\ln\left(\frac{2ax+b+2\sqrt{a\,x^{2}+bx}\,\sqrt{a}}{2\sqrt{a}}\right)-3b^{2}c^{2}x^{3}\ln\left(\frac{2ax+b+2\sqrt{a\,x^{2}+bx}\,\sqrt{a}}{2\sqrt{a}}\right)-24\sqrt{a\,x^{2}+bx}\,a^{\frac{3}{2}}cd\,x^{3}-6\sqrt{a\,x^{2}+bx}\,\sqrt{a}\,b\,c^{2}x^{3}+24\left(a\,x^{2}+bx\right)^{\frac{3}{2}}\sqrt{a}\,cdx+4\left(a\,x^{2}+bx\right)^{\frac{3}{2}}\sqrt{a}\,d^{2}\right)}{6\sqrt{\left(ax+b\right)x}\,\sqrt{a}\,b\,x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d/x)^2*(a+b/x)^(1/2),x)
```

```
[Out] -1/6*((a*x+b)/x)^(1/2)/x^2*(-24*(a*x^2+b*x)^(1/2)*a^(3/2)*x^3*c*d-6*(a*x^2+
b*x)^(1/2)*a^(1/2)*x^3*b*c^2-12*ln(1/2*(2*a*x+b+2*(a*x^2+b*x)^(1/2)*a^(1/2)
```

```
)/a^(1/2))*x^3*a*b*c*d-3*ln(1/2*(2*a*x+b+2*(a*x^2+b*x)^(1/2)*a^(1/2))/a^(1/
2))*x^3*b^2*c^2+24*(a*x^2+b*x)^(3/2)*a^(1/2)*x*c*d+4*d^2*(a*x^2+b*x)^(3/2)*
a^(1/2))/((a*x+b)*x)^(1/2)/b/a^(1/2)
```

maxima [A] time = 1.19, size = 126, normalized size = 1.27

$$\frac{1}{2}\left(2\sqrt{a+\frac{b}{x}}\,x-\frac{b\log\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{a+\frac{b}{x}}+\sqrt{a}}\right)}{\sqrt{a}}\right)c^2-2\left(\sqrt{a}\,\log\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{a+\frac{b}{x}}+\sqrt{a}}\right)+2\sqrt{a+\frac{b}{x}}\right)cd-\frac{2\left(a+\frac{b}{x}\right)^{\frac{3}{2}}d^2}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d/x)^2*(a+b/x)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*(2*sqrt(a + b/x)*x - b*log((sqrt(a + b/x) - sqrt(a))/(sqrt(a + b/x) + s
qrt(a)))/sqrt(a))*c^2 - 2*(sqrt(a)*log((sqrt(a + b/x) - sqrt(a))/(sqrt(a +
b/x) + sqrt(a))) + 2*sqrt(a + b/x))*c*d - 2/3*(a + b/x)^(3/2)*d^2/b
```

mupad [B] time = 1.90, size = 99, normalized size = 1.00

$$\left(\frac{4\,a\,d^2-4\,b\,c\,d}{b}-\frac{4\,a\,d^2}{b}\right)\sqrt{a+\frac{b}{x}}+c^2\,x\,\sqrt{a+\frac{b}{x}}-\frac{2\,d^2\,{\left(a+\frac{b}{x}\right)}^{3/2}}{3\,b}-\frac{c\,\mathrm{atan}\left(\frac{\sqrt{a+\frac{b}{x}}\,1\mathrm{i}}{\sqrt{a}}\right)\,\left(4\,a\,d+b\,c\right)\,1\mathrm{i}}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x)^(1/2)*(c + d/x)^2,x)
```

```
[Out] ((4*a*d^2 - 4*b*c*d)/b - (4*a*d^2)/b)*(a + b/x)^(1/2) + c^2*x*(a + b/x)^(1/
2) - (2*d^2*(a + b/x)^(3/2))/(3*b) - (c*atan(((a + b/x)^(1/2)*1i)/a^(1/2))*
(4*a*d + b*c)*1i)/a^(1/2)
```

sympy [A] time = 36.58, size = 121, normalized size = 1.22

$$-\frac{4acd\,\mathrm{atan}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{-a}}\right)}{\sqrt{-a}}+\sqrt{b}\,c^2\sqrt{x}\sqrt{\frac{ax}{b}+1}-4cd\sqrt{a+\frac{b}{x}}+d^2\left(\begin{cases}-\frac{\sqrt{a}}{x} & \text{for } b=0\\ -\frac{2\left(a+\frac{b}{x}\right)^{\frac{3}{2}}}{3b} & \text{otherwise}\end{cases}\right)+\frac{bc^2\,\mathrm{asinh}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d/x)**2*(a+b/x)**(1/2),x)
```

```
[Out] -4*a*c*d*atan(sqrt(a + b/x)/sqrt(-a))/sqrt(-a) + sqrt(b)*c**2*sqrt(x)*sqrt(
a*x/b + 1) - 4*c*d*sqrt(a + b/x) + d**2*Piecewise((-sqrt(a)/x, Eq(b, 0)), (
-2*(a + b/x)**(3/2)/(3*b), True)) + b*c**2*asinh(sqrt(a)*sqrt(x)/sqrt(b))/s
qrt(a)
```

3.129 $\int \sqrt{a+\frac{b}{x}} \left(c+\frac{d}{x}\right) \, dx$

Optimal. Leaf size=74

$$-\frac{\sqrt{a+\frac{b}{x}}\,(2ad+bc)}{a}+\frac{(2ad+bc)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{\sqrt{a}}+\frac{cx\left(a+\frac{b}{x}\right)^{3/2}}{a}$$

Rubi [A] time = 0.05, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {375, 78, 50, 63, 208}

$$-\frac{\sqrt{a+\frac{b}{x}}\,(2ad+bc)}{a}+\frac{(2ad+bc)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{\sqrt{a}}+\frac{cx\left(a+\frac{b}{x}\right)^{3/2}}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b/x]*(c + d/x),x]
```

```
[Out] -(((b*c + 2*a*d)*Sqrt[a + b/x])/a) + (c*(a + b/x)^(3/2)*x)/a + ((b*c + 2*a*
d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/Sqrt[a]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 375

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[((a + b/x^n)^p*(c + d/x^n)^q)/x^2, x], x, 1/x] /; FreeQ[{a,
```

```
b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)dx &= -\text{Subst}\left(\int \frac{\sqrt{a+bx}\,(c+dx)}{x^2}\,dx, x, \frac{1}{x}\right)\\
&= \frac{c\left(a+\frac{b}{x}\right)^{3/2}x}{a} - \frac{\left(\frac{bc}{2}+ad\right)\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x}\,dx, x, \frac{1}{x}\right)}{a}\\
&= -\frac{(bc+2ad)\sqrt{a+\frac{b}{x}}}{a} + \frac{c\left(a+\frac{b}{x}\right)^{3/2}x}{a} - \frac{1}{2}(bc+2ad)\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \frac{1}{x}\right)\\
&= -\frac{(bc+2ad)\sqrt{a+\frac{b}{x}}}{a} + \frac{c\left(a+\frac{b}{x}\right)^{3/2}x}{a} - \frac{(bc+2ad)\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+\frac{b}{x}}\right)}{b}\\
&= -\frac{(bc+2ad)\sqrt{a+\frac{b}{x}}}{a} + \frac{c\left(a+\frac{b}{x}\right)^{3/2}x}{a} + \frac{(bc+2ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{\sqrt{a}}
\end{aligned}$$

Mathematica [A] time = 0.04, size = 52, normalized size = 0.70

$$\sqrt{a+\frac{b}{x}}\,(cx-2d) + \frac{(2ad+bc)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b/x]*(c + d/x),x]
```

```
[Out] Sqrt[a + b/x]*(-2*d + c*x) + ((b*c + 2*a*d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])
/Sqrt[a]
```

IntegrateAlgebraic [A] time = 0.11, size = 56, normalized size = 0.76

$$\sqrt{\frac{ax+b}{x}}\,(cx-2d) + \frac{(2ad+bc)\tanh^{-1}\left(\frac{\sqrt{\frac{ax+b}{x}}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[a + b/x]*(c + d/x),x]
```

```
[Out] Sqrt[(b + a*x)/x]*(-2*d + c*x) + ((b*c + 2*a*d)*ArcTanh[Sqrt[(b + a*x)/x]/S
qrt[a]])/Sqrt[a]
```

fricas [A] time = 0.85, size = 128, normalized size = 1.73

$$\left[\frac{(bc+2\,ad)\sqrt{a}\,\log\left(2\,ax+2\,\sqrt{a}\,x\sqrt{\frac{ax+b}{x}}+b\right)+2\,(acx-2\,ad)\sqrt{\frac{ax+b}{x}}}{2\,a}, -\frac{(bc+2\,ad)\sqrt{-a}\,\arctan\left(\frac{\sqrt{-a}\sqrt{\frac{ax+b}{x}}}{a}\right)-(acx-2\,ad)\sqrt{\frac{ax+b}{x}}}{a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d/x)*(a+b/x)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*((b*c + 2*a*d)*sqrt(a)*log(2*a*x + 2*sqrt(a)*x*sqrt((a*x + b)/x) + b)
+ 2*(a*c*x - 2*a*d)*sqrt((a*x + b)/x))/a, -((b*c + 2*a*d)*sqrt(-a)*arctan(s
qrt(-a)*sqrt((a*x + b)/x)/a) - (a*c*x - 2*a*d)*sqrt((a*x + b)/x))/a]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d/x)*(a+b/x)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(x)]
Warning, choosing root of [1,0,%%%{-2,[1,2,0]%%%}+%%%{-2,[1,0,0]%%%}+%%%{-2
,[0,1,1]%%%},0,%%%{1,[2,4,0]%%%}+%%%{-2,[2,2,0]%%%}+%%%{1,[2,0,0]%%%}+%%%{2
,[1,3,1]%%%}+%%%{-2,[1,1,1]%%%}+%%%{1,[0,2,2]%%%}] at parameters values [86
,-97,-82]Warning, choosing root of [1,0,%%%{-2,[1,2,0]%%%}+%%%{-2,[1,0,0]%%
%}+%%%{-2,[0,1,1]%%%},0,%%%{1,[2,4,0]%%%}+%%%{-2,[2,2,0]%%%}+%%%{1,[2,0,0]%
%%}+%%%{2,[1,3,1]%%%}+%%%{-2,[1,1,1]%%%}+%%%{1,[0,2,2]%%%}] at parameters v
alues [7,-27,26]Warning, choosing root of [1,0,%%%{-2,[1,2,0]%%%}+%%%{-2,[1
,0,0]%%%}+%%%{-2,[0,1,1]%%%},0,%%%{1,[2,4,0]%%%}+%%%{-2,[2,2,0]%%%}+%%%{1,[
2,0,0]%%%}+%%%{2,[1,3,1]%%%}+%%%{-2,[1,1,1]%%%}+%%%{1,[0,2,2]%%%}] at param
eters values [-89,63,-49]Warning, choosing root of [1,0,%%%{-4,[1,0,0]%%%}+
%%%{-2,[0,1,1]%%%},0,%%%{1,[0,2,2]%%%}] at parameters values [61.7937478349
,-30,70]Warning, choosing root of [1,0,%%%{-4,[1,0,0]%%%}+%%%{-2,[0,1,1]%%%
},0,%%%{1,[0,2,2]%%%}] at parameters values [73.519035968,-9,-13]Warning, c
hoosing root of [1,0,%%%{-2,[1,0,1]%%%}+%%%{-4,[0,1,0]%%%},0,%%%{1,[2,0,2]%
%%}] at parameters values [18,15.451549686,-33]Sign error (%%%{-b,0%%%}+%%%
{2*sqrt(a)*sqrt(b),1/2%%%}+%%%{-2*a,1%%%}+%%%{a*sqrt(a)*sqrt(b)/b,3/2%%%}+%
%%{-a^2*sqrt(a)*sqrt(b)/(4*b^2),5/2%%%}+%%%{undef,7/2%%%})Limit: Max order
reached or unable to make series expansion Error: Bad Argument Value
```

maple [B] time = 0.06, size = 163, normalized size = 2.20

$$\frac{\sqrt{\frac{ax+b}{x}}\,\left(2abd\,x^{2}\ln\left(\frac{2ax+b+2\sqrt{a\,x^{2}+bx}\,\sqrt{a}}{2\sqrt{a}}\right)+b^{2}c\,x^{2}\ln\left(\frac{2ax+b+2\sqrt{a\,x^{2}+bx}\,\sqrt{a}}{2\sqrt{a}}\right)+4\sqrt{a\,x^{2}+bx}\,a^{\frac{3}{2}}d\,x^{2}+2\sqrt{a\,x^{2}+bx}\,\sqrt{a}\,bc\,x^{2}-4\left(a\,x^{2}+bx\right)^{\frac{3}{2}}\sqrt{a}\,d\right)}{2\sqrt{(ax+b)\,x}\,\sqrt{a}\,bx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d/x)*(a+b/x)^(1/2),x)
```

```
[Out] 1/2*((a*x+b)/x)^(1/2)*(4*a^(3/2)*(a*x^2+b*x)^(1/2)*x^2*d+2*a^(1/2)*(a*x^2+b
*x)^(1/2)*x^2*b*c+2*ln(1/2*(2*a*x+b+2*(a*x^2+b*x)^(1/2)*a^(1/2))/a^(1/2))*x
^2*a*b*d+ln(1/2*(2*a*x+b+2*(a*x^2+b*x)^(1/2)*a^(1/2))/a^(1/2))*x^2*b^2*c-4*
a^(1/2)*(a*x^2+b*x)^(3/2)*d)/x/((a*x+b)*x)^(1/2)/b/a^(1/2)
```

maxima [A] time = 1.37, size = 106, normalized size = 1.43

$$\frac{1}{2}\left(2\sqrt{a+\frac{b}{x}}\,x-\frac{b\log\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{a+\frac{b}{x}}+\sqrt{a}}\right)}{\sqrt{a}}\right)c-\left(\sqrt{a}\,\log\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{a+\frac{b}{x}}+\sqrt{a}}\right)+2\sqrt{a+\frac{b}{x}}\right)d$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d/x)*(a+b/x)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*(2*sqrt(a + b/x)*x - b*log((sqrt(a + b/x) - sqrt(a))/(sqrt(a + b/x) + s
qrt(a)))/sqrt(a))*c - (sqrt(a)*log((sqrt(a + b/x) - sqrt(a))/(sqrt(a + b/x)
 + sqrt(a))) + 2*sqrt(a + b/x))*d
```

mupad [B] time = 1.96, size = 92, normalized size = 1.24

$$2\sqrt{a}\,d\,\mathrm{atanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)-2\,d\,\sqrt{a+\frac{b}{x}}+c\,x\,\sqrt{a\,x^2+b\,x}\,\sqrt{\frac{1}{x^2}}+\frac{b\,c\,x\,\ln\left(\frac{\frac{b}{2}+a\,x+\sqrt{a}\,\sqrt{a\,x^2+b\,x}}{\sqrt{a}}\right)\sqrt{\frac{1}{x^2}}}{2\,\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x)^(1/2)*(c + d/x),x)
```

```
[Out] 2*a^(1/2)*d*atanh((a + b/x)^(1/2)/a^(1/2)) - 2*d*(a + b/x)^(1/2) + c*x*(b*x
 + a*x^2)^(1/2)*(1/x^2)^(1/2) + (b*c*x*log((b/2 + a*x + a^(1/2)*(b*x + a*x^
2)^(1/2))/a^(1/2))*(1/x^2)^(1/2))/(2*a^(1/2))
```

sympy [A] time = 41.31, size = 87, normalized size = 1.18

$$-\frac{2ad\,\mathrm{atan}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{-a}}\right)}{\sqrt{-a}}+\sqrt{b}\,c\sqrt{x}\sqrt{\frac{ax}{b}+1}-2d\sqrt{a+\frac{b}{x}}+\frac{bc\,\mathrm{asinh}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d/x)*(a+b/x)**(1/2),x)
```

```
[Out] -2*a*d*atan(sqrt(a + b/x)/sqrt(-a))/sqrt(-a) + sqrt(b)*c*sqrt(x)*sqrt(a*x/b
 + 1) - 2*d*sqrt(a + b/x) + b*c*asinh(sqrt(a)*sqrt(x)/sqrt(b))/sqrt(a)
```

3.130 $\int \sqrt{a+\frac{b}{x}}\,dx$

Optimal. Leaf size=39

$$x\sqrt{a+\frac{b}{x}}+\frac{b\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Rubi [A] time = 0.02, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {242, 47, 63, 208}

$$x\sqrt{a+\frac{b}{x}}+\frac{b\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b/x],x]
```

```
[Out] Sqrt[a + b/x]*x + (b*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/Sqrt[a]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 242

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a + b/x^n)^p/x^
2, x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\int \sqrt{a+\frac{b}{x}}\,dx &= -\operatorname{Subst}\left(\int \frac{\sqrt{a+bx}}{x^2}\,dx, x, \frac{1}{x}\right)\\
&= \sqrt{a+\frac{b}{x}}\,x - \frac{1}{2} b \operatorname{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \frac{1}{x}\right)\\
&= \sqrt{a+\frac{b}{x}}\,x - \operatorname{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+\frac{b}{x}}\right)\\
&= \sqrt{a+\frac{b}{x}}\,x + \frac{b\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{\sqrt{a}}
\end{aligned}
$$

Mathematica [A] time = 0.07, size = 39, normalized size = 1.00

$$
x\sqrt{a+\frac{b}{x}} + \frac{b\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{\sqrt{a}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b/x],x]
[Out] Sqrt[a + b/x]*x + (b*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/Sqrt[a]
```

IntegrateAlgebraic [A] time = 0.00, size = 43, normalized size = 1.10

$$
x\sqrt{\frac{ax+b}{x}} + \frac{b\tanh^{-1}\left(\frac{\sqrt{\frac{ax+b}{x}}}{\sqrt{a}}\right)}{\sqrt{a}}
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[a + b/x],x]
[Out] x*Sqrt[(b + a*x)/x] + (b*ArcTanh[Sqrt[(b + a*x)/x]/Sqrt[a]])/Sqrt[a]
```

fricas [A] time = 0.75, size = 99, normalized size = 2.54

$$
\left[\frac{2\,ax\sqrt{\frac{ax+b}{x}} + \sqrt{a}\,b\log\left(2\,ax + 2\,\sqrt{a}\,x\sqrt{\frac{ax+b}{x}} + b\right)}{2\,a}, \frac{ax\sqrt{\frac{ax+b}{x}} - \sqrt{-a}\,b\arctan\left(\frac{\sqrt{-a}\sqrt{\frac{ax+b}{x}}}{a}\right)}{a}\right]
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(1/2),x, algorithm="fricas")
[Out] [1/2*(2*a*x*sqrt((a*x + b)/x) + sqrt(a)*b*log(2*a*x + 2*sqrt(a)*x*sqrt((a*x
 + b)/x) + b))/a, (a*x*sqrt((a*x + b)/x) - sqrt(-a)*b*arctan(sqrt(-a)*sqrt(
(a*x + b)/x)/a))/a]
```

giac [B] time = 0.20, size = 64, normalized size = 1.64

$$
-\frac{b\log\left(\left|-2\left(\sqrt{a}\,x - \sqrt{ax^2+bx}\right)\sqrt{a} - b\right|\right)\operatorname{sgn}(x)}{2\sqrt{a}} + \frac{b\log\left(|b|\right)\operatorname{sgn}(x)}{2\sqrt{a}} + \sqrt{ax^2+bx}\operatorname{sgn}(x)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*b*log(abs(-2*(sqrt(a)*x - sqrt(a*x^2 + b*x))*sqrt(a) - b))*sgn(x)/sqrt
(a) + 1/2*b*log(abs(b))*sgn(x)/sqrt(a) + sqrt(a*x^2 + b*x)*sgn(x)
```

maple [B] time = 0.05, size = 74, normalized size = 1.90

$$\frac{\sqrt{\frac{ax+b}{x}}\,\left(b\ln\left(\frac{2ax+b+2\sqrt{a\,x^2+bx}\,\sqrt{a}}{2\sqrt{a}}\right)+2\sqrt{a\,x^2+bx}\,\sqrt{a}\right)x}{2\sqrt{(ax+b)\,x}\,\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x)^(1/2),x)
```

```
[Out] 1/2*((a*x+b)/x)^(1/2)*x*(2*(a*x^2+b*x)^(1/2)*a^(1/2)+b*ln(1/2*(2*a*x+b+2*(a
*x^2+b*x)^(1/2)*a^(1/2))/a^(1/2)))/((a*x+b)*x)^(1/2)/a^(1/2)
```

maxima [A] time = 1.20, size = 50, normalized size = 1.28

$$\sqrt{a+\frac{b}{x}}\,x-\frac{b\log\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{a+\frac{b}{x}}+\sqrt{a}}\right)}{2\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(1/2),x, algorithm="maxima")
```

```
[Out] sqrt(a + b/x)*x - 1/2*b*log((sqrt(a + b/x) - sqrt(a))/(sqrt(a + b/x) + sqrt
(a)))/sqrt(a)
```

mupad [B] time = 0.08, size = 58, normalized size = 1.49

$$x\sqrt{a\,x^2+b\,x}\,\sqrt{\frac{1}{x^2}}+\frac{b\,x\,\ln\left(\frac{\frac{b}{2}+a\,x+\sqrt{a}\,\sqrt{a\,x^2+b\,x}}{\sqrt{a}}\right)\sqrt{\frac{1}{x^2}}}{2\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x)^(1/2),x)
```

```
[Out] x*(b*x + a*x^2)^(1/2)*(1/x^2)^(1/2) + (b*x*log((b/2 + a*x + a^(1/2)*(b*x +
a*x^2)^(1/2))/a^(1/2))*(1/x^2)^(1/2))/(2*a^(1/2))
```

sympy [A] time = 2.19, size = 42, normalized size = 1.08

$$\sqrt{b}\,\sqrt{x}\,\sqrt{\frac{ax}{b}+1}+\frac{b\,\operatorname{asinh}\left(\frac{\sqrt{a}\,\sqrt{x}}{\sqrt{b}}\right)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)**(1/2),x)
```

```
[Out] sqrt(b)*sqrt(x)*sqrt(a*x/b + 1) + b*asinh(sqrt(a)*sqrt(x)/sqrt(b))/sqrt(a)
```

3.131 $\int \frac{\sqrt{a+\frac{b}{x}}}{c+\frac{d}{x}} \, dx$

Optimal. Leaf size=104

$$\frac{2\sqrt{d}\sqrt{bc-ad}\tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c^2}+\frac{(bc-2ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{\sqrt{a}\,c^2}+\frac{x\sqrt{a+\frac{b}{x}}}{c}$$

Rubi [A] time = 0.11, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {375, 99, 156, 63, 208, 205}

$$\frac{2\sqrt{d}\sqrt{bc-ad}\tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c^2}+\frac{(bc-2ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{\sqrt{a}\,c^2}+\frac{x\sqrt{a+\frac{b}{x}}}{c}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b/x]/(c + d/x),x]
```

```
[Out] (Sqrt[a + b/x]*x)/c + (2*Sqrt[d]*Sqrt[b*c - a*d]*ArcTan[(Sqrt[d]*Sqrt[a + b
/x])/Sqrt[b*c - a*d]])/c^2 + ((b*c - 2*a*d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])
/(Sqrt[a]*c^2)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 99

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[1/((m + 1)*(b*e - a*f)), Int[(a + b*x)^
(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m
 + n + p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1
] && GtQ[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || Integ
ersQ[p, m + n])
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 375

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[((a + b/x^n)^p*(c + d/x^n)^q)/x^2, x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rubi steps

$$\int \frac{\sqrt{a+\frac{b}{x}}}{c+\frac{d}{x}}\,dx = -\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x^2(c+dx)}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{\sqrt{a+\frac{b}{x}}\,x}{c} - \frac{\text{Subst}\left(\int \frac{\frac{1}{2}(bc-2ad)-\frac{bdx}{2}}{x\sqrt{a+bx}\,(c+dx)}\,dx, x, \frac{1}{x}\right)}{c}$$

$$= \frac{\sqrt{a+\frac{b}{x}}\,x}{c} - \frac{(bc-2ad)\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \frac{1}{x}\right)}{2c^2} + \frac{(d(bc-ad))\,\text{Subst}\left(\int \frac{1}{\sqrt{a+bx}\,(c+dx)}\,dx, x, \frac{1}{x}\right)}{c^2}$$

$$= \frac{\sqrt{a+\frac{b}{x}}\,x}{c} - \frac{(bc-2ad)\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+\frac{b}{x}}\right)}{bc^2} + \frac{(2d(bc-ad))\,\text{Subst}\left(\int \frac{1}{c-\frac{ad}{b}+\frac{dx^2}{b}}\,dx, x, \cdots\right.}{bc^2}$$

$$= \frac{\sqrt{a+\frac{b}{x}}\,x}{c} + \frac{2\sqrt{d}\,\sqrt{bc-ad}\,\tan^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c^2} + \frac{(bc-2ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{\sqrt{a}\,c^2}$$

Mathematica [A] time = 0.24, size = 100, normalized size = 0.96

$$\frac{2\sqrt{d}\,\sqrt{bc-ad}\,\tan^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right) + \frac{(bc-2ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{\sqrt{a}} + cx\sqrt{a+\frac{b}{x}}}{c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b/x]/(c + d/x),x]
```

```
[Out] (c*Sqrt[a + b/x]*x + 2*Sqrt[d]*Sqrt[b*c - a*d]*ArcTan[(Sqrt[d]*Sqrt[a + b/x
])/Sqrt[b*c - a*d]] + ((b*c - 2*a*d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/Sqrt[a
])/c^2
```

IntegrateAlgebraic [A] time = 0.23, size = 110, normalized size = 1.06

$$\frac{2\sqrt{d}\,\sqrt{bc-ad}\,\tan^{-1}\left(\frac{\sqrt{d}\,\sqrt{\frac{ax+b}{x}}}{\sqrt{bc-ad}}\right)}{c^2} + \frac{(bc-2ad)\tanh^{-1}\left(\frac{\sqrt{\frac{ax+b}{x}}}{\sqrt{a}}\right)}{\sqrt{a}\,c^2} + \frac{x\sqrt{\frac{ax+b}{x}}}{c}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[a + b/x]/(c + d/x),x]
```

```
[Out] (x*Sqrt[(b + a*x)/x])/c + (2*Sqrt[d]*Sqrt[b*c - a*d]*ArcTan[(Sqrt[d]*Sqrt[(
b + a*x)/x])/Sqrt[b*c - a*d]])/c^2 + ((b*c - 2*a*d)*ArcTanh[Sqrt[(b + a*x)/
x]/Sqrt[a]])/(Sqrt[a]*c^2)
```

fricas [A] time = 0.94, size = 482, normalized size = 4.63

$$\left[\frac{2\,acx\sqrt{\frac{ax+b}{x}} - (bc-2\,ad)\sqrt{a}\,\log\left(2\,ax - 2\,\sqrt{a}\,x\sqrt{\frac{ax+b}{x}} + b\right) + 2\,\sqrt{-bcd+ad^2}\,a\log\left(\frac{bd-(bc-2\,ad)x+2\,\sqrt{-bcd+ad^2}\,x\sqrt{\frac{ax+b}{x}}}{cx+d}\right)}{2\,ac^2}, \frac{2\,acx\sqrt{\frac{ax+b}{x}} - 4\,\sqrt{bcd-ad^2}\,a\arctan\left(\frac{\sqrt{bcd-ad^2}\,x\sqrt{\frac{ax+b}{x}}}{adx+bd}\right) - (bc-2\,ad)\sqrt{a}\,\log\left(2\,ax - 2\,\sqrt{a}\,x\sqrt{\frac{ax+b}{x}} + b\right)}{2\,ac^2}, \frac{acx\sqrt{\frac{ax+b}{x}} - (bc-2\,ad)\sqrt{-a}\,\arctan\left(\frac{\sqrt{-a}\,\sqrt{\frac{ax+b}{x}}}{a}\right) + \sqrt{-bcd+ad^2}\,a\log\left(\frac{bd-(bc-2\,ad)x+2\,\sqrt{-bcd+ad^2}\,x\sqrt{\frac{ax+b}{x}}}{cx+d}\right)}{ac^2}, \frac{acx\sqrt{\frac{ax+b}{x}} - 2\,\sqrt{bcd-ad^2}\,a\arctan\left(\frac{\sqrt{bcd-ad^2}\,x\sqrt{\frac{ax+b}{x}}}{adx+bd}\right) - (bc-2\,ad)\sqrt{-a}\,\arctan\left(\frac{\sqrt{-a}\,\sqrt{\frac{ax+b}{x}}}{a}\right)}{ac^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(1/2)/(c+d/x),x, algorithm="fricas")
```

```
[Out] [1/2*(2*a*c*x*sqrt((a*x + b)/x) - (b*c - 2*a*d)*sqrt(a)*log(2*a*x - 2*sqrt(
a)*x*sqrt((a*x + b)/x) + b) + 2*sqrt(-b*c*d + a*d^2)*a*log((b*d - (b*c - 2*
a*d)*x + 2*sqrt(-b*c*d + a*d^2)*x*sqrt((a*x + b)/x))/(c*x + d)))/(a*c^2), 1
/2*(2*a*c*x*sqrt((a*x + b)/x) - 4*sqrt(b*c*d - a*d^2)*a*arctan(sqrt(b*c*d -
 a*d^2)*x*sqrt((a*x + b)/x)/(a*d*x + b*d)) - (b*c - 2*a*d)*sqrt(a)*log(2*a*
x - 2*sqrt(a)*x*sqrt((a*x + b)/x) + b))/(a*c^2), (a*c*x*sqrt((a*x + b)/x) -
 (b*c - 2*a*d)*sqrt(-a)*arctan(sqrt(-a)*sqrt((a*x + b)/x)/a) + sqrt(-b*c*d
+ a*d^2)*a*log((b*d - (b*c - 2*a*d)*x + 2*sqrt(-b*c*d + a*d^2)*x*sqrt((a*x
+ b)/x))/(c*x + d)))/(a*c^2), (a*c*x*sqrt((a*x + b)/x) - 2*sqrt(b*c*d - a*d
^2)*a*arctan(sqrt(b*c*d - a*d^2)*x*sqrt((a*x + b)/x)/(a*d*x + b*d)) - (b*c
- 2*a*d)*sqrt(-a)*arctan(sqrt(-a)*sqrt((a*x + b)/x)/a))/(a*c^2)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(1/2)/(c+d/x),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(x)]
Error: Bad Argument Type
```

maple [B] time = 0.11, size = 287, normalized size = 2.76

$$-\frac{\sqrt{\frac{ax+b}{x}}\left(2a^{\frac{3}{2}}d^2\ln\left(\frac{-2adx+bcx-bd+2\sqrt{\frac{(ad-bc)d}{c^2}}\,\sqrt{(ax+b)x}\,c}{cx+d}\right) - 2\sqrt{a}\,bcd\ln\left(\frac{-2adx+bcx-bd+2\sqrt{\frac{(ad-bc)d}{c^2}}\,\sqrt{(ax+b)x}\,c}{cx+d}\right) + 2\sqrt{\frac{(ad-bc)d}{c^2}}\,acd\ln\left(\frac{2ax+b+2\sqrt{(ax+b)x}\,\sqrt{a}}{2\sqrt{a}}\right) - \sqrt{\frac{(ad-bc)d}{c^2}}\,b\,c^2\ln\left(\frac{2ax+b+2\sqrt{(ax+b)x}\,\sqrt{a}}{2\sqrt{a}}\right) - 2\sqrt{(ax+b)\,x}\,\sqrt{\frac{(ad-bc)d}{c^2}}\,\sqrt{a}\,c^2\right)x}{2\sqrt{(ax+b)\,x}\,\sqrt{\frac{(ad-bc)d}{c^2}}\,\sqrt{a}\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x)^(1/2)/(c+d/x),x)
```

```
[Out] -1/2*((a*x+b)/x)^(1/2)*x*(2*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^
(1/2))*(d*(a*d-b*c)/c^2)^(1/2)*a*c*d-ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^
(1/2))/a^(1/2))*(d*(a*d-b*c)/c^2)^(1/2)*b*c^2+2*ln((2*(d*(a*d-b*c)/c^2)^(1/
2)*((a*x+b)*x)^(1/2)*c-2*a*d*x+b*c*x-b*d)/(c*x+d))*a^(3/2)*d^2-2*ln((2*(d*(
a*d-b*c)/c^2)^(1/2)*((a*x+b)*x)^(1/2)*c-2*a*d*x+b*c*x-b*d)/(c*x+d))*a^(1/2)
*b*c*d-2*((a*x+b)*x)^(1/2)*c^2*a^(1/2)*(d*(a*d-b*c)/c^2)^(1/2))/((a*x+b)*x)
^(1/2)/c^3/a^(1/2)/(d*(a*d-b*c)/c^2)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a + \frac{b}{x}}}{c + \frac{d}{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(1/2)/(c+d/x),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a + b/x)/(c + d/x), x)
```

mupad [B] time = 1.63, size = 149, normalized size = 1.43

$$\frac{x\sqrt{a+\frac{b}{x}}}{c}+\frac{\ln\left(\sqrt{a+\frac{b}{x}}-\sqrt{a}\right)\left(a\,d-\frac{b\,c}{2}\right)}{\sqrt{a}\,c^2}-\frac{\ln\left(\sqrt{a+\frac{b}{x}}+\sqrt{a}\right)(2\,a\,d-b\,c)}{2\,\sqrt{a}\,c^2}-\frac{\mathrm{atan}\left(\frac{b^4\,d^3\,\sqrt{a+\frac{b}{x}}\,\sqrt{a\,d^2-b\,c\,d}\,4\mathrm{i}}{4\,a\,b^4\,d^4-4\,b^5\,c\,d^3}\right)\sqrt{a\,d^2-b\,c\,d}\,2\mathrm{i}}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x)^(1/2)/(c + d/x),x)
```

```
[Out] (x*(a + b/x)^(1/2))/c - (atan((b^4*d^3*(a + b/x)^(1/2)*(a*d^2 - b*c*d)^(1/2
)*4i)/(4*a*b^4*d^4 - 4*b^5*c*d^3))*(a*d^2 - b*c*d)^(1/2)*2i)/c^2 + (log((a
+ b/x)^(1/2) - a^(1/2))*(a*d - (b*c)/2))/(a^(1/2)*c^2) - (log((a + b/x)^(1/
2) + a^(1/2))*(2*a*d - b*c))/(2*a^(1/2)*c^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\sqrt{a+\frac{b}{x}}}{cx+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)**(1/2)/(c+d/x),x)
```

```
[Out] Integral(x*sqrt(a + b/x)/(c*x + d), x)
```

3.132 $\int \frac{\sqrt{a+\frac{b}{x}}}{\left(c+\frac{d}{x}\right)^2}\,dx$

Optimal. Leaf size=147

$$\frac{\sqrt{d}\,(3bc-4ad)\tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c^3\sqrt{bc-ad}}+\frac{(bc-4ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{\sqrt{a}\,c^3}+\frac{2d\sqrt{a+\frac{b}{x}}}{c^2\left(c+\frac{d}{x}\right)}+\frac{x\sqrt{a+\frac{b}{x}}}{c\left(c+\frac{d}{x}\right)}$$

Rubi [A] time = 0.21, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {375, 99, 151, 156, 63, 208, 205}

$$\frac{2d\sqrt{a+\frac{b}{x}}}{c^2\left(c+\frac{d}{x}\right)}+\frac{\sqrt{d}\,(3bc-4ad)\tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c^3\sqrt{bc-ad}}+\frac{(bc-4ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{\sqrt{a}\,c^3}+\frac{x\sqrt{a+\frac{b}{x}}}{c\left(c+\frac{d}{x}\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b/x]/(c + d/x)^2,x]
```

```
[Out] (2*d*Sqrt[a + b/x])/(c^2*(c + d/x)) + (Sqrt[a + b/x]*x)/(c*(c + d/x)) + (Sq
rt[d]*(3*b*c - 4*a*d)*ArcTan[(Sqrt[d]*Sqrt[a + b/x])/Sqrt[b*c - a*d]])/(c^3
*Sqrt[b*c - a*d]) + ((b*c - 4*a*d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/(Sqrt[a]
*c^3)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 99

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[1/((m + 1)*(b*e - a*f)), Int[(a + b*x)^
(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m
 + n + p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1
] && GtQ[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || Integ
ersQ[p, m + n])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 375

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[((a + b/x^n)^p*(c + d/x^n)^q)/x^2, x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rubi steps

$$\int \frac{\sqrt{a+\frac{b}{x}}}{\left(c+\frac{d}{x}\right)^2}\,dx = -\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x^2(c+dx)^2}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{\sqrt{a+\frac{b}{x}}\,x}{c\left(c+\frac{d}{x}\right)} - \frac{\text{Subst}\left(\int \frac{\frac{1}{2}(bc-4ad)-\frac{3bdx}{2}}{x\sqrt{a+bx}\,(c+dx)^2}\,dx, x, \frac{1}{x}\right)}{c}$$

$$= \frac{2d\sqrt{a+\frac{b}{x}}}{c^2\left(c+\frac{d}{x}\right)} + \frac{\sqrt{a+\frac{b}{x}}\,x}{c\left(c+\frac{d}{x}\right)} + \frac{\text{Subst}\left(\int \frac{-\frac{1}{2}(bc-4ad)(bc-ad)+bd(bc-ad)x}{x\sqrt{a+bx}\,(c+dx)}\,dx, x, \frac{1}{x}\right)}{c^2(bc-ad)}$$

$$= \frac{2d\sqrt{a+\frac{b}{x}}}{c^2\left(c+\frac{d}{x}\right)} + \frac{\sqrt{a+\frac{b}{x}}\,x}{c\left(c+\frac{d}{x}\right)} - \frac{(bc-4ad)\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \frac{1}{x}\right)}{2c^3} + \frac{(d(3bc-4ad))\,\text{Subst}\left(\int \frac{}{\sqrt{a+bx}}\right.}{2c^3}$$

$$= \frac{2d\sqrt{a+\frac{b}{x}}}{c^2\left(c+\frac{d}{x}\right)} + \frac{\sqrt{a+\frac{b}{x}}\,x}{c\left(c+\frac{d}{x}\right)} - \frac{(bc-4ad)\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+\frac{b}{x}}\right)}{bc^3} + \frac{(d(3bc-4ad))\,\text{Subst}\left(\int\right.}{bc}$$

$$= \frac{2d\sqrt{a+\frac{b}{x}}}{c^2\left(c+\frac{d}{x}\right)} + \frac{\sqrt{a+\frac{b}{x}}\,x}{c\left(c+\frac{d}{x}\right)} + \frac{\sqrt{d}\,(3bc-4ad)\tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c^3\sqrt{bc-ad}} + \frac{(bc-4ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{\sqrt{a}\,c^3}$$

Mathematica [A] time = 0.39, size = 122, normalized size = 0.83

$$\frac{\frac{cx\sqrt{a+\frac{b}{x}}\,(cx+2d)}{cx+d} + \frac{\sqrt{d}\,(3bc-4ad)\tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{\sqrt{bc-ad}} + \frac{(bc-4ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{\sqrt{a}}}{c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b/x]/(c + d/x)^2,x]
```

```
[Out] ((c*Sqrt[a + b/x]*x*(2*d + c*x))/(d + c*x) + (Sqrt[d]*(3*b*c - 4*a*d)*ArcTa
n[(Sqrt[d]*Sqrt[a + b/x])/Sqrt[b*c - a*d]])/Sqrt[b*c - a*d] + ((b*c - 4*a*d
)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/Sqrt[a])/c^3
```

IntegrateAlgebraic [**A**] time = 0.39, size = 138, normalized size = 0.94

$$\frac{\left(3bc\sqrt{d}-4ad^{3/2}\right)\tan^{-1}\left(\frac{\sqrt{d}\sqrt{\frac{ax+b}{x}}}{\sqrt{bc-ad}}\right)}{c^3\sqrt{bc-ad}}+\frac{(bc-4ad)\tanh^{-1}\left(\frac{\sqrt{\frac{ax+b}{x}}}{\sqrt{a}}\right)}{\sqrt{a}\,c^3}+\frac{\sqrt{\frac{ax+b}{x}}\left(cx^2+2dx\right)}{c^2(cx+d)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[a + b/x]/(c + d/x)^2,x]
```

```
[Out] (Sqrt[(b + a*x)/x]*(2*d*x + c*x^2))/(c^2*(d + c*x)) + ((3*b*c*Sqrt[d] - 4*a
*d^(3/2))*ArcTan[(Sqrt[d]*Sqrt[(b + a*x)/x])/Sqrt[b*c - a*d]])/(c^3*Sqrt[b*
c - a*d]) + ((b*c - 4*a*d)*ArcTanh[Sqrt[(b + a*x)/x]/Sqrt[a]])/(Sqrt[a]*c^3
)
```

fricas [**A**] time = 0.91, size = 801, normalized size = 5.45

$$\left[-\frac{\left(bcd-4ad^2+\left(bc^2-4acd\right)x\right)\sqrt{a}\log\left(2ax-2\sqrt{a}x\sqrt{\frac{ax+b}{x}}+b\right)+\left(3abcd-4a^2d^2+\left(3abc^2-4a^2cd\right)x\right)\sqrt{-\frac{d}{bc-ad}}\log\left(-\frac{2(bc-ad)x\sqrt{-\frac{d}{bc-ad}}\sqrt{\frac{ax+b}{x}}-bd+(bc-2ad)x}{cx+d}\right)-2\left(ac^2x^2+2acdx\right)\sqrt{\frac{ax+b}{x}}}{2\left(ac^4x+ac^3d\right)},\ -\frac{2\left(bcd-4ad^2+\left(bc^2-4acd\right)x\right)\sqrt{-a}\arctan\left(\frac{\sqrt{-a}\sqrt{\frac{ax+b}{x}}}{a}\right)+\left(3abcd-4a^2d^2+\left(3abc^2-4a^2cd\right)x\right)\sqrt{-\frac{d}{bc-ad}}\log\left(-\frac{2(bc-ad)x\sqrt{-\frac{d}{bc-ad}}\sqrt{\frac{ax+b}{x}}-bd+(bc-2ad)x}{cx+d}\right)-2\left(ac^2x^2+2acdx\right)\sqrt{\frac{ax+b}{x}}}{2\left(ac^4x+ac^3d\right)},\ \frac{2\left(3abcd-4a^2d^2+\left(3abc^2-4a^2cd\right)x\right)\sqrt{\frac{d}{bc-ad}}\arctan\left(-\frac{(bc-ad)x\sqrt{\frac{d}{bc-ad}}\sqrt{\frac{ax+b}{x}}}{adx+bd}\right)-\left(bcd-4ad^2+\left(bc^2-4acd\right)x\right)\sqrt{a}\log\left(2ax-2\sqrt{a}x\sqrt{\frac{ax+b}{x}}+b\right)+2\left(ac^2x^2+2acdx\right)\sqrt{\frac{ax+b}{x}}}{2\left(ac^4x+ac^3d\right)},\ \frac{\left(3abcd-4a^2d^2+\left(3abc^2-4a^2cd\right)x\right)\sqrt{\frac{d}{bc-ad}}\arctan\left(-\frac{(bc-ad)x\sqrt{\frac{d}{bc-ad}}\sqrt{\frac{ax+b}{x}}}{adx+bd}\right)-\left(bcd-4ad^2+\left(bc^2-4acd\right)x\right)\sqrt{-a}\arctan\left(\frac{\sqrt{-a}\sqrt{\frac{ax+b}{x}}}{a}\right)+\left(ac^2x^2+2acdx\right)\sqrt{\frac{ax+b}{x}}}{ac^4x+ac^3d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(1/2)/(c+d/x)^2,x, algorithm="fricas")
```

```
[Out] [-1/2*((b*c*d - 4*a*d^2 + (b*c^2 - 4*a*c*d)*x)*sqrt(a)*log(2*a*x - 2*sqrt(a
)*x*sqrt((a*x + b)/x) + b) + (3*a*b*c*d - 4*a^2*d^2 + (3*a*b*c^2 - 4*a^2*c*
d)*x)*sqrt(-d/(b*c - a*d))*log(-(2*(b*c - a*d)*x*sqrt(-d/(b*c - a*d))*sqrt(
(a*x + b)/x) - b*d + (b*c - 2*a*d)*x)/(c*x + d)) - 2*(a*c^2*x^2 + 2*a*c*d*x
)*sqrt((a*x + b)/x))/(a*c^4*x + a*c^3*d), -1/2*(2*(b*c*d - 4*a*d^2 + (b*c^2
 - 4*a*c*d)*x)*sqrt(-a)*arctan(sqrt(-a)*sqrt((a*x + b)/x)/a) + (3*a*b*c*d -
 4*a^2*d^2 + (3*a*b*c^2 - 4*a^2*c*d)*x)*sqrt(-d/(b*c - a*d))*log(-(2*(b*c -
 a*d)*x*sqrt(-d/(b*c - a*d))*sqrt((a*x + b)/x) - b*d + (b*c - 2*a*d)*x)/(c*
x + d)) - 2*(a*c^2*x^2 + 2*a*c*d*x)*sqrt((a*x + b)/x))/(a*c^4*x + a*c^3*d),
 1/2*(2*(3*a*b*c*d - 4*a^2*d^2 + (3*a*b*c^2 - 4*a^2*c*d)*x)*sqrt(d/(b*c - a
*d))*arctan(-(b*c - a*d)*x*sqrt(d/(b*c - a*d))*sqrt((a*x + b)/x)/(a*d*x + b
*d)) - (b*c*d - 4*a*d^2 + (b*c^2 - 4*a*c*d)*x)*sqrt(a)*log(2*a*x - 2*sqrt(a
)*x*sqrt((a*x + b)/x) + b) + 2*(a*c^2*x^2 + 2*a*c*d*x)*sqrt((a*x + b)/x))/(
a*c^4*x + a*c^3*d), ((3*a*b*c*d - 4*a^2*d^2 + (3*a*b*c^2 - 4*a^2*c*d)*x)*sq
rt(d/(b*c - a*d))*arctan(-(b*c - a*d)*x*sqrt(d/(b*c - a*d))*sqrt((a*x + b)/
x)/(a*d*x + b*d)) - (b*c*d - 4*a*d^2 + (b*c^2 - 4*a*c*d)*x)*sqrt(-a)*arctan
(sqrt(-a)*sqrt((a*x + b)/x)/a) + (a*c^2*x^2 + 2*a*c*d*x)*sqrt((a*x + b)/x))
/(a*c^4*x + a*c^3*d)]
```

giac [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(1/2)/(c+d/x)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(x)]
Unable to divide, perhaps due to rounding error%%%{%%{[-2,0]:[1,0,%%%{-1,[1
```

```
]%%%}]%%},[4,6,4,0]%%%}+%%%{%%%{8,[1]%%%},[3,5,4,1]%%%}+%%%{%%{[-4,0]:[1,0,
%%%{-1,[1]%%%}]%%},[2,5,5,1]%%%}+%%%{%%{[%%%{-8,[1]%%%},0]:[1,0,%%%{-1,[1]%
%%}]%%},[2,4,4,2]%%%}+%%%{%%%{8,[1]%%%},[1,4,5,2]%%%}+%%%{%%{[-2,0]:[1,0,%%
%{-1,[1]%%%}]%%},[0,4,6,2]%%%} / %%%{%%%{1,[1]%%%},[4,2,0,0]%%%}+%%%{%%{pol
y1[%%%{-4,[1]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[3,1,0,1]%%%}+%%%{%%%{2,[1]%%%
},[2,1,1,1]%%%}+%%%{%%%{4,[2]%%%},[2,0,0,2]%%%}+%%%{%%{poly1[%%%{-4,[1]%%%}
,0]:[1,0,%%%{-1,[1]%%%}]%%},[1,0,1,2]%%%}+%%%{%%%{1,[1]%%%},[0,0,2,2]%%%} E
rror: Bad Argument Value
```

maple [B] time = 0.07, size = 943, normalized size = 6.41

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x)^(1/2)/(c+d/x)^2,x)
```

```
[Out] -1/2*((a*x+b)/x)^(1/2)*x*(4*a^(7/2)*ln((-2*a*d*x+b*c*x-b*d+2*((a*d-b*c)/c^2
*d)^(1/2)*((a*x+b)*x)^(1/2)*c)/(c*x+d))*x*c*d^3+2*a^(5/2)*((a*x+b)*x)^(1/2)
*((a*d-b*c)/c^2*d)^(1/2)*x^2*c^4+4*a^(7/2)*ln((-2*a*d*x+b*c*x-b*d+2*((a*d-b
*c)/c^2*d)^(1/2)*((a*x+b)*x)^(1/2)*c)/(c*x+d))*d^4-2*a^(5/2)*((a*x+b)*x)^(1
/2)*((a*d-b*c)/c^2*d)^(1/2)*x*c^3*d-7*a^(5/2)*ln((-2*a*d*x+b*c*x-b*d+2*((a*
d-b*c)/c^2*d)^(1/2)*((a*x+b)*x)^(1/2)*c)/(c*x+d))*x*b*c^2*d^2-4*a^(5/2)*((a
*x+b)*x)^(1/2)*((a*d-b*c)/c^2*d)^(1/2)*c^2*d^2-7*a^(5/2)*ln((-2*a*d*x+b*c*x
-b*d+2*((a*d-b*c)/c^2*d)^(1/2)*((a*x+b)*x)^(1/2)*c)/(c*x+d))*b*c*d^3-2*c^4*
((a*x+b)*x)^(3/2)*a^(3/2)*((a*d-b*c)/c^2*d)^(1/2)+4*a^(3/2)*((a*x+b)*x)^(1/
2)*((a*d-b*c)/c^2*d)^(1/2)*x*b*c^4+3*a^(3/2)*ln((-2*a*d*x+b*c*x-b*d+2*((a*d
-b*c)/c^2*d)^(1/2)*((a*x+b)*x)^(1/2)*c)/(c*x+d))*x*b^2*c^3*d+4*a^(3/2)*((a*
x+b)*x)^(1/2)*((a*d-b*c)/c^2*d)^(1/2)*b*c^3*d+3*a^(3/2)*ln((-2*a*d*x+b*c*x-
b*d+2*((a*d-b*c)/c^2*d)^(1/2)*((a*x+b)*x)^(1/2)*c)/(c*x+d))*b^2*c^2*d^2+4*a
^3*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))*((a*d-b*c)/c^2*d)^
(1/2)*x*c^2*d^2-5*a^2*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))
*((a*d-b*c)/c^2*d)^(1/2)*x*b*c^3*d+a*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^
(1/2))/a^(1/2))*((a*d-b*c)/c^2*d)^(1/2)*x*b^2*c^4+4*a^3*ln(1/2*(2*a*x+b+2*(
(a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))*((a*d-b*c)/c^2*d)^(1/2)*c*d^3-5*a^2*ln(1
/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))*((a*d-b*c)/c^2*d)^(1/2)*b
*c^2*d^2+a*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))*((a*d-b*c)
/c^2*d)^(1/2)*b^2*c^3*d)/c^4/((a*x+b)*x)^(1/2)/(a*d-b*c)/(c*x+d)/a^(3/2)/((
a*d-b*c)/c^2*d)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+\frac{b}{x}}}{\left(c+\frac{d}{x}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(1/2)/(c+d/x)^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a + b/x)/(c + d/x)^2, x)
```

mupad [B] time = 2.26, size = 1195, normalized size = 8.13

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x)^(1/2)/(c + d/x)^2,x)
```

```
[Out] - ((2*b*d*(a + b/x)^(3/2))/c^2 - (b*(a + b/x)^(1/2)*(2*a*d - b*c))/c^2)/((a
 + b/x)*(2*a*d - b*c) - d*(a + b/x)^2 - a^2*d + a*b*c) - (atanh((8*b^5*d^3*
```

```
(a + b/x)^(1/2))/(a^(1/2)*(8*b^5*d^3 - (2*b^6*c*d^2)/a)) + (2*b^6*d^2*(a +
b/x)^(1/2))/(a^(3/2)*((2*b^6*d^2)/a - (8*b^5*d^3)/c)))*(4*a*d - b*c))/(a^(1
/2)*c^3) - (atan((((d*(a*d - b*c))^(1/2)*((4*(a + b/x)^(1/2)*(16*a^2*b^2*d^
5 + 5*b^4*c^2*d^3 - 16*a*b^3*c*d^4))/c^4 - (((2*(2*b^4*c^7*d^2 - 4*a*b^3*c^
6*d^3))/c^6 - (2*(2*b^3*c^7*d^2 - 4*a*b^2*c^6*d^3)*(a + b/x)^(1/2)*(d*(a*d
- b*c))^(1/2)*(4*a*d - 3*b*c))/(c^4*(b*c^4 - a*c^3*d)))*(d*(a*d - b*c))^(1/
2)*(4*a*d - 3*b*c))/(2*(b*c^4 - a*c^3*d)))*(4*a*d - 3*b*c)*1i)/(2*(b*c^4 -
a*c^3*d)) + ((d*(a*d - b*c))^(1/2)*((4*(a + b/x)^(1/2)*(16*a^2*b^2*d^5 + 5*
b^4*c^2*d^3 - 16*a*b^3*c*d^4))/c^4 + (((2*(2*b^4*c^7*d^2 - 4*a*b^3*c^6*d^3)
)/c^6 + (2*(2*b^3*c^7*d^2 - 4*a*b^2*c^6*d^3)*(a + b/x)^(1/2)*(d*(a*d - b*c)
)^(1/2)*(4*a*d - 3*b*c))/(c^4*(b*c^4 - a*c^3*d)))*(d*(a*d - b*c))^(1/2)*(4*
a*d - 3*b*c))/(2*(b*c^4 - a*c^3*d)))*(4*a*d - 3*b*c)*1i)/(2*(b*c^4 - a*c^3*
d)))/((4*(16*a^2*b^3*d^5 + 3*b^5*c^2*d^3 - 16*a*b^4*c*d^4))/c^6 - ((d*(a*d
- b*c))^(1/2)*((4*(a + b/x)^(1/2)*(16*a^2*b^2*d^5 + 5*b^4*c^2*d^3 - 16*a*b^
3*c*d^4))/c^4 - (((2*(2*b^4*c^7*d^2 - 4*a*b^3*c^6*d^3))/c^6 - (2*(2*b^3*c^7
*d^2 - 4*a*b^2*c^6*d^3)*(a + b/x)^(1/2)*(d*(a*d - b*c))^(1/2)*(4*a*d - 3*b*
c))/(c^4*(b*c^4 - a*c^3*d)))*(d*(a*d - b*c))^(1/2)*(4*a*d - 3*b*c))/(2*(b*c
^4 - a*c^3*d)))*(4*a*d - 3*b*c))/(2*(b*c^4 - a*c^3*d)) + ((d*(a*d - b*c))^(
1/2)*((4*(a + b/x)^(1/2)*(16*a^2*b^2*d^5 + 5*b^4*c^2*d^3 - 16*a*b^3*c*d^4))
/c^4 + (((2*(2*b^4*c^7*d^2 - 4*a*b^3*c^6*d^3))/c^6 + (2*(2*b^3*c^7*d^2 - 4*
a*b^2*c^6*d^3)*(a + b/x)^(1/2)*(d*(a*d - b*c))^(1/2)*(4*a*d - 3*b*c))/(c^4*
(b*c^4 - a*c^3*d)))*(d*(a*d - b*c))^(1/2)*(4*a*d - 3*b*c))/(2*(b*c^4 - a*c^
3*d)))*(4*a*d - 3*b*c))/(2*(b*c^4 - a*c^3*d))))*(d*(a*d - b*c))^(1/2)*(4*a*
d - 3*b*c)*1i)/(b*c^4 - a*c^3*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2\sqrt{a+\frac{b}{x}}}{(cx+d)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)**(1/2)/(c+d/x)**2,x)
```

```
[Out] Integral(x**2*sqrt(a + b/x)/(c*x + d)**2, x)
```

3.133 $\int \frac{\sqrt{a+\frac{b}{x}}}{\left(c+\frac{d}{x}\right)^3}\,dx$

Optimal. Leaf size=213

$$\frac{\sqrt{d}\left(24a^2d^2-40abcd+15b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{4c^4(bc-ad)^{3/2}}+\frac{(bc-6ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{\sqrt{a}\,c^4}+\frac{d\sqrt{a+\frac{b}{x}}\,(11bc-12ad)}{4c^3\left(c+\frac{d}{x}\right)(bc-ad)}+\frac{3d\sqrt{a+}}{2c^2\left(c+\right.}$$

Rubi [A] time = 0.34, antiderivative size = 213, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {375, 99, 151, 156, 63, 208, 205}

$$\frac{\sqrt{d}\left(24a^2d^2-40abcd+15b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{4c^4(bc-ad)^{3/2}}+\frac{d\sqrt{a+\frac{b}{x}}\,(11bc-12ad)}{4c^3\left(c+\frac{d}{x}\right)(bc-ad)}+\frac{3d\sqrt{a+\frac{b}{x}}}{2c^2\left(c+\frac{d}{x}\right)^2}+\frac{(bc-6ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{\sqrt{a}\,c^4}+\frac{x\sqrt{a+\frac{b}{x}}}{c\left(c+\frac{d}{x}\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b/x]/(c + d/x)^3,x]
```

```
[Out] (3*d*Sqrt[a + b/x])/(2*c^2*(c + d/x)^2) + (d*(11*b*c - 12*a*d)*Sqrt[a + b/x
])/(4*c^3*(b*c - a*d)*(c + d/x)) + (Sqrt[a + b/x]*x)/(c*(c + d/x)^2) + (Sqr
t[d]*(15*b^2*c^2 - 40*a*b*c*d + 24*a^2*d^2)*ArcTan[(Sqrt[d]*Sqrt[a + b/x])/
Sqrt[b*c - a*d]])/(4*c^4*(b*c - a*d)^(3/2)) + ((b*c - 6*a*d)*ArcTanh[Sqrt[a
 + b/x]/Sqrt[a]])/(Sqrt[a]*c^4)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 99

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[1/((m + 1)*(b*e - a*f)), Int[(a + b*x)^
(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m
 + n + p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1
] && GtQ[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || Integ
ersQ[p, m + n])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 375

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[((a + b/x^n)^p*(c + d/x^n)^q)/x^2, x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rubi steps

$$\int \frac{\sqrt{a+\frac{b}{x}}}{\left(c+\frac{d}{x}\right)^3}\,dx = -\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x^2(c+dx)^3}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{\sqrt{a+\frac{b}{x}}\,x}{c\left(c+\frac{d}{x}\right)^2} - \frac{\text{Subst}\left(\int \frac{\frac{1}{2}(bc-6ad)-\frac{5bdx}{2}}{x\sqrt{a+bx}\,(c+dx)^3}\,dx, x, \frac{1}{x}\right)}{c}$$

$$= \frac{3d\sqrt{a+\frac{b}{x}}}{2c^2\left(c+\frac{d}{x}\right)^2} + \frac{\sqrt{a+\frac{b}{x}}\,x}{c\left(c+\frac{d}{x}\right)^2} + \frac{\text{Subst}\left(\int \frac{-((bc-6ad)(bc-ad))+\frac{9}{2}bd(bc-ad)x}{x\sqrt{a+bx}\,(c+dx)^2}\,dx, x, \frac{1}{x}\right)}{2c^2(bc-ad)}$$

$$= \frac{3d\sqrt{a+\frac{b}{x}}}{2c^2\left(c+\frac{d}{x}\right)^2} + \frac{d(11bc-12ad)\sqrt{a+\frac{b}{x}}}{4c^3(bc-ad)\left(c+\frac{d}{x}\right)} + \frac{\sqrt{a+\frac{b}{x}}\,x}{c\left(c+\frac{d}{x}\right)^2} - \frac{\text{Subst}\left(\int \frac{(bc-6ad)(bc-ad)^2-\frac{1}{4}bd(11bc-12ad)(b}{x\sqrt{a+bx}\,(c+dx)}\right.}{2c^3(bc-ad)^2}$$

$$= \frac{3d\sqrt{a+\frac{b}{x}}}{2c^2\left(c+\frac{d}{x}\right)^2} + \frac{d(11bc-12ad)\sqrt{a+\frac{b}{x}}}{4c^3(bc-ad)\left(c+\frac{d}{x}\right)} + \frac{\sqrt{a+\frac{b}{x}}\,x}{c\left(c+\frac{d}{x}\right)^2} - \frac{(bc-6ad)\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \frac{1}{x}\right)}{2c^4} +$$

$$= \frac{3d\sqrt{a+\frac{b}{x}}}{2c^2\left(c+\frac{d}{x}\right)^2} + \frac{d(11bc-12ad)\sqrt{a+\frac{b}{x}}}{4c^3(bc-ad)\left(c+\frac{d}{x}\right)} + \frac{\sqrt{a+\frac{b}{x}}\,x}{c\left(c+\frac{d}{x}\right)^2} - \frac{(bc-6ad)\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+\right.}{bc^4}$$

$$= \frac{3d\sqrt{a+\frac{b}{x}}}{2c^2\left(c+\frac{d}{x}\right)^2} + \frac{d(11bc-12ad)\sqrt{a+\frac{b}{x}}}{4c^3(bc-ad)\left(c+\frac{d}{x}\right)} + \frac{\sqrt{a+\frac{b}{x}}\,x}{c\left(c+\frac{d}{x}\right)^2} + \frac{\sqrt{d}\left(15b^2c^2-40abcd+24a^2d^2\right)\tan^{-1}}{4c^4(bc-ad)^{3/2}}$$

Mathematica [A] time = 0.82, size = 330, normalized size = 1.55

$$\frac{(cx+d)\left(\frac{1}{2}cd^{5/2}\sqrt{a+\frac{b}{x}}(ax+b)\left(12a^2d^2-17abcd+4b^2c^2\right)+(cx+d)\left(-\frac{1}{2}ad^2\left(24a^2d^2-40abcd+15b^2c^2\right)\left(\sqrt{d}\sqrt{a+\frac{b}{x}}-\sqrt{bc-ad}\tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)\right)-d^{3/2}(bc-6ad)(bc-ad)^2\left(2\sqrt{a+\frac{b}{x}}-2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)\right)\right)\right)+2c^3d^{3/2}x^3\left(a+\frac{b}{x}\right)^{3/2}(bc-ad)^2+c^2d^{5/2}x\sqrt{a+\frac{b}{x}}(ax+b)(2bc-3ad)(bc-ad)}{2ac^4d^{3/2}(cx+d)^2(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b/x]/(c + d/x)^3,x]
```

```
[Out] (2*c^3*d^(3/2)*(b*c - a*d)^2*(a + b/x)^(3/2)*x^3 + c^2*d^(5/2)*(2*b*c - 3*a
*d)*(b*c - a*d)*Sqrt[a + b/x]*x*(b + a*x) + (d + c*x)*((c*d^(5/2)*(4*b^2*c^
2 - 17*a*b*c*d + 12*a^2*d^2)*Sqrt[a + b/x]*(b + a*x))/2 + (d + c*x)*(-1/2*(
a*d^2*(15*b^2*c^2 - 40*a*b*c*d + 24*a^2*d^2)*(Sqrt[d]*Sqrt[a + b/x] - Sqrt[
b*c - a*d]*ArcTan[(Sqrt[d]*Sqrt[a + b/x])/Sqrt[b*c - a*d]])) - d^(3/2)*(b*c
 - 6*a*d)*(b*c - a*d)^2*(2*Sqrt[a + b/x] - 2*Sqrt[a]*ArcTanh[Sqrt[a + b/x]/
Sqrt[a]]))))/(2*a*c^4*d^(3/2)*(b*c - a*d)^2*(d + c*x)^2)
```

IntegrateAlgebraic [A] time = 1.35, size = 215, normalized size = 1.01

$$\frac{\left(24a^2d^{5/2}-40abcd^{3/2}+15b^2c^2\sqrt{d}\right)\tan^{-1}\left(\frac{\sqrt{d}\sqrt{\frac{ax+b}{x}}}{\sqrt{bc-ad}}\right)}{4c^4(bc-ad)^{3/2}}+\frac{(bc-6ad)\tanh^{-1}\left(\frac{\sqrt{\frac{ax+b}{x}}}{\sqrt{a}}\right)}{\sqrt{a}\,c^4}+\frac{\sqrt{\frac{ax+b}{x}}\left(-4ac^2dx^3-18acd^2x^2-12ad^3x+4bc^3x^3+17bc^2dx^2+11bcd^2x\right)}{4c^3(cx+d)^2(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[a + b/x]/(c + d/x)^3,x]
```

```
[Out] (Sqrt[(b + a*x)/x]*(11*b*c*d^2*x - 12*a*d^3*x + 17*b*c^2*d*x^2 - 18*a*c*d^2
*x^2 + 4*b*c^3*x^3 - 4*a*c^2*d*x^3))/(4*c^3*(b*c - a*d)*(d + c*x)^2) + ((15
*b^2*c^2*Sqrt[d] - 40*a*b*c*d^(3/2) + 24*a^2*d^(5/2))*ArcTan[(Sqrt[d]*Sqrt[
(b + a*x)/x])/Sqrt[b*c - a*d]])/(4*c^4*(b*c - a*d)^(3/2)) + ((b*c - 6*a*d)*
ArcTanh[Sqrt[(b + a*x)/x]/Sqrt[a]])/(Sqrt[a]*c^4)
```

fricas [B] time = 1.08, size = 1749, normalized size = 8.21

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(1/2)/(c+d/x)^3,x, algorithm="fricas")
```

```
[Out] [-1/8*(4*(b^2*c^2*d^2 - 7*a*b*c*d^3 + 6*a^2*d^4 + (b^2*c^4 - 7*a*b*c^3*d +
6*a^2*c^2*d^2)*x^2 + 2*(b^2*c^3*d - 7*a*b*c^2*d^2 + 6*a^2*c*d^3)*x)*sqrt(a)
*log(2*a*x - 2*sqrt(a)*x*sqrt((a*x + b)/x) + b) + (15*a*b^2*c^2*d^2 - 40*a^
2*b*c*d^3 + 24*a^3*d^4 + (15*a*b^2*c^4 - 40*a^2*b*c^3*d + 24*a^3*c^2*d^2)*x
^2 + 2*(15*a*b^2*c^3*d - 40*a^2*b*c^2*d^2 + 24*a^3*c*d^3)*x)*sqrt(-d/(b*c -
 a*d))*log(-(2*(b*c - a*d)*x*sqrt(-d/(b*c - a*d))*sqrt((a*x + b)/x) - b*d +
 (b*c - 2*a*d)*x)/(c*x + d)) - 2*(4*(a*b*c^4 - a^2*c^3*d)*x^3 + (17*a*b*c^3
*d - 18*a^2*c^2*d^2)*x^2 + (11*a*b*c^2*d^2 - 12*a^2*c*d^3)*x)*sqrt((a*x + b
)/x))/(a*b*c^5*d^2 - a^2*c^4*d^3 + (a*b*c^7 - a^2*c^6*d)*x^2 + 2*(a*b*c^6*d
 - a^2*c^5*d^2)*x), 1/4*((15*a*b^2*c^2*d^2 - 40*a^2*b*c*d^3 + 24*a^3*d^4 +
(15*a*b^2*c^4 - 40*a^2*b*c^3*d + 24*a^3*c^2*d^2)*x^2 + 2*(15*a*b^2*c^3*d -
40*a^2*b*c^2*d^2 + 24*a^3*c*d^3)*x)*sqrt(d/(b*c - a*d))*arctan(-(b*c - a*d)
*x*sqrt(d/(b*c - a*d))*sqrt((a*x + b)/x)/(a*d*x + b*d)) - 2*(b^2*c^2*d^2 -
7*a*b*c*d^3 + 6*a^2*d^4 + (b^2*c^4 - 7*a*b*c^3*d + 6*a^2*c^2*d^2)*x^2 + 2*(
b^2*c^3*d - 7*a*b*c^2*d^2 + 6*a^2*c*d^3)*x)*sqrt(a)*log(2*a*x - 2*sqrt(a)*x
*sqrt((a*x + b)/x) + b) + (4*(a*b*c^4 - a^2*c^3*d)*x^3 + (17*a*b*c^3*d - 18
*a^2*c^2*d^2)*x^2 + (11*a*b*c^2*d^2 - 12*a^2*c*d^3)*x)*sqrt((a*x + b)/x))/(
a*b*c^5*d^2 - a^2*c^4*d^3 + (a*b*c^7 - a^2*c^6*d)*x^2 + 2*(a*b*c^6*d - a^2*
c^5*d^2)*x), -1/8*(8*(b^2*c^2*d^2 - 7*a*b*c*d^3 + 6*a^2*d^4 + (b^2*c^4 - 7*
a*b*c^3*d + 6*a^2*c^2*d^2)*x^2 + 2*(b^2*c^3*d - 7*a*b*c^2*d^2 + 6*a^2*c*d^3
)*x)*sqrt(-a)*arctan(sqrt(-a)*sqrt((a*x + b)/x)/a) + (15*a*b^2*c^2*d^2 - 40
*a^2*b*c*d^3 + 24*a^3*d^4 + (15*a*b^2*c^4 - 40*a^2*b*c^3*d + 24*a^3*c^2*d^2
)*x^2 + 2*(15*a*b^2*c^3*d - 40*a^2*b*c^2*d^2 + 24*a^3*c*d^3)*x)*sqrt(-d/(b*
```

```
c - a*d))*log(-(2*(b*c - a*d)*x*sqrt(-d/(b*c - a*d))*sqrt((a*x + b)/x) - b*
d + (b*c - 2*a*d)*x)/(c*x + d)) - 2*(4*(a*b*c^4 - a^2*c^3*d)*x^3 + (17*a*b*
c^3*d - 18*a^2*c^2*d^2)*x^2 + (11*a*b*c^2*d^2 - 12*a^2*c*d^3)*x)*sqrt((a*x
+ b)/x))/(a*b*c^5*d^2 - a^2*c^4*d^3 + (a*b*c^7 - a^2*c^6*d)*x^2 + 2*(a*b*c^
6*d - a^2*c^5*d^2)*x), 1/4*((15*a*b^2*c^2*d^2 - 40*a^2*b*c*d^3 + 24*a^3*d^4
 + (15*a*b^2*c^4 - 40*a^2*b*c^3*d + 24*a^3*c^2*d^2)*x^2 + 2*(15*a*b^2*c^3*d
 - 40*a^2*b*c^2*d^2 + 24*a^3*c*d^3)*x)*sqrt(d/(b*c - a*d))*arctan(-(b*c - a
*d)*x*sqrt(d/(b*c - a*d))*sqrt((a*x + b)/x)/(a*d*x + b*d)) - 4*(b^2*c^2*d^2
 - 7*a*b*c*d^3 + 6*a^2*d^4 + (b^2*c^4 - 7*a*b*c^3*d + 6*a^2*c^2*d^2)*x^2 +
2*(b^2*c^3*d - 7*a*b*c^2*d^2 + 6*a^2*c*d^3)*x)*sqrt(-a)*arctan(sqrt(-a)*sqr
t((a*x + b)/x)/a) + (4*(a*b*c^4 - a^2*c^3*d)*x^3 + (17*a*b*c^3*d - 18*a^2*c
^2*d^2)*x^2 + (11*a*b*c^2*d^2 - 12*a^2*c*d^3)*x)*sqrt((a*x + b)/x))/(a*b*c^
5*d^2 - a^2*c^4*d^3 + (a*b*c^7 - a^2*c^6*d)*x^2 + 2*(a*b*c^6*d - a^2*c^5*d^
2)*x)]
```

giac [B] time = 0.41, size = 820, normalized size = 3.85

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(1/2)/(c+d/x)^3,x, algorithm="giac")
```

```
[Out] -1/4*(15*sqrt(a)*b^2*c^2*d*arctan(sqrt(a)*d/sqrt(b*c*d - a*d^2)) - 40*a^(3/
2)*b*c*d^2*arctan(sqrt(a)*d/sqrt(b*c*d - a*d^2)) + 24*a^(5/2)*d^3*arctan(sq
rt(a)*d/sqrt(b*c*d - a*d^2)) - 2*sqrt(b*c*d - a*d^2)*b^2*c^2*log(abs(b)) +
14*sqrt(b*c*d - a*d^2)*a*b*c*d*log(abs(b)) - 12*sqrt(b*c*d - a*d^2)*a^2*d^2
*log(abs(b)) + 9*sqrt(b*c*d - a*d^2)*a*b*c*d - 10*sqrt(b*c*d - a*d^2)*a^2*d
^2)*sgn(x)/(sqrt(b*c*d - a*d^2)*sqrt(a)*b*c^5 - sqrt(b*c*d - a*d^2)*a^(3/2)
*c^4*d) - 1/4*(15*b^2*c^2*d*sgn(x) - 40*a*b*c*d^2*sgn(x) + 24*a^2*d^3*sgn(x
))*arctan(-((sqrt(a)*x - sqrt(a*x^2 + b*x))*c + sqrt(a)*d)/sqrt(b*c*d - a*d
^2))/((b*c^5 - a*c^4*d)*sqrt(b*c*d - a*d^2)) + sqrt(a*x^2 + b*x)*sgn(x)/c^3
 - 1/4*(9*(sqrt(a)*x - sqrt(a*x^2 + b*x))^3*sqrt(a)*b^2*c^3*d*sgn(x) - 32*(
sqrt(a)*x - sqrt(a*x^2 + b*x))^3*a^(3/2)*b*c^2*d^2*sgn(x) + 24*(sqrt(a)*x -
 sqrt(a*x^2 + b*x))^3*a^(5/2)*c*d^3*sgn(x) + 3*(sqrt(a)*x - sqrt(a*x^2 + b*
x))^2*a*b^2*c^2*d^2*sgn(x) - 40*(sqrt(a)*x - sqrt(a*x^2 + b*x))^2*a^2*b*c*d
^3*sgn(x) + 40*(sqrt(a)*x - sqrt(a*x^2 + b*x))^2*a^3*d^4*sgn(x) + 7*(sqrt(a
)*x - sqrt(a*x^2 + b*x))*sqrt(a)*b^3*c^2*d^2*sgn(x) - 44*(sqrt(a)*x - sqrt(
a*x^2 + b*x))*a^(3/2)*b^2*c*d^3*sgn(x) + 40*(sqrt(a)*x - sqrt(a*x^2 + b*x))
*a^(5/2)*b*d^4*sgn(x) - 9*a*b^3*c*d^3*sgn(x) + 10*a^2*b^2*d^4*sgn(x))/((sqr
t(a)*b*c^5 - a^(3/2)*c^4*d)*((sqrt(a)*x - sqrt(a*x^2 + b*x))^2*c + 2*(sqrt(
a)*x - sqrt(a*x^2 + b*x))*sqrt(a)*d + b*d)^2) - 1/2*(b*c*sgn(x) - 6*a*d*sgn
(x))*log(abs(2*(sqrt(a)*x - sqrt(a*x^2 + b*x))*sqrt(a) + b))/(sqrt(a)*c^4)
```

maple [B] time = 0.07, size = 1972, normalized size = 9.26

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x)^(1/2)/(c+d/x)^3,x)
```

```
[Out] -1/8*((a*x+b)/x)^(1/2)*x*(55*a^(5/2)*ln((-2*a*d*x+b*c*x-b*d+2*((a*d-b*c)/c^
2*d)^(1/2)*((a*x+b)*x)^(1/2)*c)/(c*x+d))*x^2*b^2*c^4*d^2+24*a^4*ln(1/2*(2*a
*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))*((a*d-b*c)/c^2*d)^(1/2)*x^2*c^3*
d^3-36*a^(7/2)*((a*x+b)*x)^(1/2)*((a*d-b*c)/c^2*d)^(1/2)*x*c^3*d^3-128*a^(7
/2)*ln((-2*a*d*x+b*c*x-b*d+2*((a*d-b*c)/c^2*d)^(1/2)*((a*x+b)*x)^(1/2)*c)/(
c*x+d))*x*b*c^2*d^4+110*a^(5/2)*ln((-2*a*d*x+b*c*x-b*d+2*((a*d-b*c)/c^2*d)^
(1/2)*((a*x+b)*x)^(1/2)*c)/(c*x+d))*x*b^2*c^3*d^3+48*a^4*ln(1/2*(2*a*x+b+2*
((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))*((a*d-b*c)/c^2*d)^(1/2)*x*c^2*d^4+46*a^
(5/2)*((a*x+b)*x)^(1/2)*((a*d-b*c)/c^2*d)^(1/2)*b*c^3*d^3-52*a^3*ln(1/2*(2*
a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))*((a*d-b*c)/c^2*d)^(1/2)*b*c^2*d
```

```
^4+32*a^2*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))*((a*d-b*c)/
c^2*d)^(1/2)*b^2*c^3*d^3-22*a^(3/2)*((a*x+b)*x)^(1/2)*((a*d-b*c)/c^2*d)^(1/
2)*b^2*c^4*d^2-4*a*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))*((
a*d-b*c)/c^2*d)^(1/2)*b^3*c^4*d^2+14*a^(3/2)*((a*x+b)*x)^(3/2)*((a*d-b*c)/c
^2*d)^(1/2)*x*b*c^6-22*a^(3/2)*((a*x+b)*x)^(1/2)*((a*d-b*c)/c^2*d)^(1/2)*x^
2*b^2*c^6-15*a^(3/2)*ln((-2*a*d*x+b*c*x-b*d+2*((a*d-b*c)/c^2*d)^(1/2)*((a*x
+b)*x)^(1/2)*c)/(c*x+d))*x^2*b^3*c^5*d-4*a*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1
/2)*a^(1/2))/a^(1/2))*((a*d-b*c)/c^2*d)^(1/2)*x^2*b^3*c^6+10*a^(3/2)*((a*x+
b)*x)^(3/2)*((a*d-b*c)/c^2*d)^(1/2)*b*c^5*d+12*a^(7/2)*((a*x+b)*x)^(1/2)*((
a*d-b*c)/c^2*d)^(1/2)*x^3*c^5*d-14*a^(5/2)*((a*x+b)*x)^(1/2)*((a*d-b*c)/c^2
*d)^(1/2)*x^3*b*c^6-12*a^(5/2)*((a*x+b)*x)^(3/2)*((a*d-b*c)/c^2*d)^(1/2)*x*
c^5*d-64*a^(7/2)*ln((-2*a*d*x+b*c*x-b*d+2*((a*d-b*c)/c^2*d)^(1/2)*((a*x+b)*
x)^(1/2)*c)/(c*x+d))*x^2*b*c^3*d^3-30*a^(3/2)*ln((-2*a*d*x+b*c*x-b*d+2*((a*
d-b*c)/c^2*d)^(1/2)*((a*x+b)*x)^(1/2)*c)/(c*x+d))*x*b^3*c^4*d^2+24*a^(9/2)*
ln((-2*a*d*x+b*c*x-b*d+2*((a*d-b*c)/c^2*d)^(1/2)*((a*x+b)*x)^(1/2)*c)/(c*x+
d))*d^6+64*a^2*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))*((a*d-
b*c)/c^2*d)^(1/2)*x*b^2*c^4*d^2+78*a^(5/2)*((a*x+b)*x)^(1/2)*((a*d-b*c)/c^2
*d)^(1/2)*x*b*c^4*d^2-104*a^3*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/
a^(1/2))*((a*d-b*c)/c^2*d)^(1/2)*x*b*c^3*d^3+18*a^(5/2)*((a*x+b)*x)^(1/2)*(
(a*d-b*c)/c^2*d)^(1/2)*x^2*b*c^5*d-52*a^3*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/
2)*a^(1/2))/a^(1/2))*((a*d-b*c)/c^2*d)^(1/2)*x^2*b*c^4*d^2+32*a^2*ln(1/2*(2
*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))*((a*d-b*c)/c^2*d)^(1/2)*x^2*b^
2*c^5*d-44*a^(3/2)*((a*x+b)*x)^(1/2)*((a*d-b*c)/c^2*d)^(1/2)*x*b^2*c^5*d-8*
a*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))*((a*d-b*c)/c^2*d)^(
1/2)*x*b^3*c^5*d+48*a^(9/2)*ln((-2*a*d*x+b*c*x-b*d+2*((a*d-b*c)/c^2*d)^(1/2
)*((a*x+b)*x)^(1/2)*c)/(c*x+d))*x*c*d^5-24*a^(7/2)*((a*x+b)*x)^(1/2)*((a*d-
b*c)/c^2*d)^(1/2)*c^2*d^4-64*a^(7/2)*ln((-2*a*d*x+b*c*x-b*d+2*((a*d-b*c)/c^
2*d)^(1/2)*((a*x+b)*x)^(1/2)*c)/(c*x+d))*b*c*d^5+55*a^(5/2)*ln((-2*a*d*x+b*
c*x-b*d+2*((a*d-b*c)/c^2*d)^(1/2)*((a*x+b)*x)^(1/2)*c)/(c*x+d))*b^2*c^2*d^4
+24*a^4*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))*((a*d-b*c)/c^
2*d)^(1/2)*c*d^5+24*a^(9/2)*ln((-2*a*d*x+b*c*x-b*d+2*((a*d-b*c)/c^2*d)^(1/2
)*((a*x+b)*x)^(1/2)*c)/(c*x+d))*x^2*c^2*d^4-8*a^(5/2)*((a*x+b)*x)^(3/2)*((a
*d-b*c)/c^2*d)^(1/2)*c^4*d^2-15*a^(3/2)*ln((-2*a*d*x+b*c*x-b*d+2*((a*d-b*c)
/c^2*d)^(1/2)*((a*x+b)*x)^(1/2)*c)/(c*x+d))*b^3*c^3*d^3)/c^5/((a*x+b)*x)^(1
/2)/(a*d-b*c)^2/(c*x+d)^2/a^(3/2)/((a*d-b*c)/c^2*d)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a + \frac{b}{x}}}{\left(c + \frac{d}{x}\right)^3} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(1/2)/(c+d/x)^3,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a + b/x)/(c + d/x)^3, x)
```

mupad [B] time = 3.73, size = 1895, normalized size = 8.90

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x)^(1/2)/(c + d/x)^3,x)
```

```
[Out] (log((a + b/x)^(1/2)*(d*(a*d - b*c)^3)^(1/2) - a^2*d^2 - b^2*c^2 + 2*a*b*c*
d)*(d*(a*d - b*c)^3)^(1/2)*(3*a^2*d^2 + (15*b^2*c^2)/8 - 5*a*b*c*d))/(b^3*c
^7 - a^3*c^4*d^3 + 3*a^2*b*c^5*d^2 - 3*a*b^2*c^6*d) - ((b*(a + b/x)^(1/2)*(
12*a^2*d^2 + 4*b^2*c^2 - 17*a*b*c*d))/(4*c^3) + (b*(a + b/x)^(5/2)*(12*a*d^
3 - 11*b*c*d^2))/(4*c^3*(a*d - b*c)) - (d*(a + b/x)^(3/2)*(17*b^3*c^2 + 24*
a^2*b*d^2 - 40*a*b^2*c*d))/(4*c^3*(a*d - b*c)))/((a + b/x)^2*(3*a*d^2 - 2*b
```

```
*c*d) - (a + b/x)*(3*a^2*d^2 + b^2*c^2 - 4*a*b*c*d) - d^2*(a + b/x)^3 + a^3
*d^2 + a*b^2*c^2 - 2*a^2*b*c*d) - (log((a + b/x)^(1/2)*(d*(a*d - b*c)^3)^(1
/2) + a^2*d^2 + b^2*c^2 - 2*a*b*c*d)*(d*(a*d - b*c)^3)^(1/2)*(24*a^2*d^2 +
15*b^2*c^2 - 40*a*b*c*d))/(8*(b^3*c^7 - a^3*c^4*d^3 + 3*a^2*b*c^5*d^2 - 3*a
*b^2*c^6*d)) - (atan((((((a + b/x)^(1/2)*(1152*a^4*b^2*d^7 + 241*b^6*c^4*d^
3 - 1424*a*b^5*c^3*d^4 - 3264*a^3*b^3*c*d^6 + 3296*a^2*b^4*c^2*d^5))/(8*(b^
2*c^8 + a^2*c^6*d^2 - 2*a*b*c^7*d)) - ((6*a*d - b*c)*((4*b^6*c^11*d^2 - 21*
a*b^5*c^10*d^3 + 29*a^2*b^4*c^9*d^4 - 12*a^3*b^3*c^8*d^5)/(b^2*c^11 + a^2*c
^9*d^2 - 2*a*b*c^10*d) - ((a + b/x)^(1/2)*(6*a*d - b*c)*(64*b^5*c^11*d^2 -
256*a*b^4*c^10*d^3 + 320*a^2*b^3*c^9*d^4 - 128*a^3*b^2*c^8*d^5))/(16*a^(1/2
)*c^4*(b^2*c^8 + a^2*c^6*d^2 - 2*a*b*c^7*d))))/(2*a^(1/2)*c^4))*(6*a*d - b*
c)*1i)/(2*a^(1/2)*c^4) + ((((a + b/x)^(1/2)*(1152*a^4*b^2*d^7 + 241*b^6*c^4
*d^3 - 1424*a*b^5*c^3*d^4 - 3264*a^3*b^3*c*d^6 + 3296*a^2*b^4*c^2*d^5))/(8*
(b^2*c^8 + a^2*c^6*d^2 - 2*a*b*c^7*d)) + ((6*a*d - b*c)*((4*b^6*c^11*d^2 -
21*a*b^5*c^10*d^3 + 29*a^2*b^4*c^9*d^4 - 12*a^3*b^3*c^8*d^5)/(b^2*c^11 + a^
2*c^9*d^2 - 2*a*b*c^10*d) + ((a + b/x)^(1/2)*(6*a*d - b*c)*(64*b^5*c^11*d^2
 - 256*a*b^4*c^10*d^3 + 320*a^2*b^3*c^9*d^4 - 128*a^3*b^2*c^8*d^5))/(16*a^(
1/2)*c^4*(b^2*c^8 + a^2*c^6*d^2 - 2*a*b*c^7*d))))/(2*a^(1/2)*c^4))*(6*a*d -
 b*c)*1i)/(2*a^(1/2)*c^4))/((216*a^4*b^3*d^7 + (165*b^7*c^4*d^3)/8 - (805*a
*b^6*c^3*d^4)/4 - 594*a^3*b^4*c*d^6 + 558*a^2*b^5*c^2*d^5)/(b^2*c^11 + a^2*
c^9*d^2 - 2*a*b*c^10*d) - ((((a + b/x)^(1/2)*(1152*a^4*b^2*d^7 + 241*b^6*c^
4*d^3 - 1424*a*b^5*c^3*d^4 - 3264*a^3*b^3*c*d^6 + 3296*a^2*b^4*c^2*d^5))/(8
*(b^2*c^8 + a^2*c^6*d^2 - 2*a*b*c^7*d)) - ((6*a*d - b*c)*((4*b^6*c^11*d^2 -
 21*a*b^5*c^10*d^3 + 29*a^2*b^4*c^9*d^4 - 12*a^3*b^3*c^8*d^5)/(b^2*c^11 + a
^2*c^9*d^2 - 2*a*b*c^10*d) - ((a + b/x)^(1/2)*(6*a*d - b*c)*(64*b^5*c^11*d^
2 - 256*a*b^4*c^10*d^3 + 320*a^2*b^3*c^9*d^4 - 128*a^3*b^2*c^8*d^5))/(16*a^
(1/2)*c^4*(b^2*c^8 + a^2*c^6*d^2 - 2*a*b*c^7*d))))/(2*a^(1/2)*c^4))*(6*a*d
- b*c))/(2*a^(1/2)*c^4) + ((((a + b/x)^(1/2)*(1152*a^4*b^2*d^7 + 241*b^6*c^
4*d^3 - 1424*a*b^5*c^3*d^4 - 3264*a^3*b^3*c*d^6 + 3296*a^2*b^4*c^2*d^5))/(8
*(b^2*c^8 + a^2*c^6*d^2 - 2*a*b*c^7*d)) + ((6*a*d - b*c)*((4*b^6*c^11*d^2 -
 21*a*b^5*c^10*d^3 + 29*a^2*b^4*c^9*d^4 - 12*a^3*b^3*c^8*d^5)/(b^2*c^11 + a
^2*c^9*d^2 - 2*a*b*c^10*d) + ((a + b/x)^(1/2)*(6*a*d - b*c)*(64*b^5*c^11*d^
2 - 256*a*b^4*c^10*d^3 + 320*a^2*b^3*c^9*d^4 - 128*a^3*b^2*c^8*d^5))/(16*a^
(1/2)*c^4*(b^2*c^8 + a^2*c^6*d^2 - 2*a*b*c^7*d))))/(2*a^(1/2)*c^4))*(6*a*d
- b*c))/(2*a^(1/2)*c^4)))*(6*a*d - b*c)*1i)/(a^(1/2)*c^4)
```

sympy [**F(-1)**]   time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)**(1/2)/(c+d/x)**3,x)
```

```
[Out] Timed out
```

3.134 $\int \left(a+\frac{b}{x}\right)^{3/2} \left(c+\frac{d}{x}\right)^3 \, dx$

Optimal. Leaf size=164

$$-\frac{d\left(a+\frac{b}{x}\right)^{3/2}\left(\frac{3bd(2ad+19bc)}{x}+2(13bc-ad)(2ad+5bc)\right)}{35b^2}-3c^2\sqrt{a+\frac{b}{x}}\,(2ad+bc)+3\sqrt{a}\,c^2(2ad+bc)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)$$

Rubi [A] time = 0.14, antiderivative size = 164, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {375, 97, 153, 147, 50, 63, 208}

$$-\frac{d\left(a+\frac{b}{x}\right)^{3/2}\left(\frac{3bd(2ad+19bc)}{x}+2(13bc-ad)(2ad+5bc)\right)}{35b^2}-3c^2\sqrt{a+\frac{b}{x}}\,(2ad+bc)+3\sqrt{a}\,c^2(2ad+bc)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)+x\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)^3-\frac{9}{7}d\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)^2$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x)^(3/2)*(c + d/x)^3,x]
```

```
[Out] -3*c^2*(b*c + 2*a*d)*Sqrt[a + b/x] - (9*d*(a + b/x)^(3/2)*(c + d/x)^2)/7 -
(d*(a + b/x)^(3/2)*(2*(13*b*c - a*d)*(5*b*c + 2*a*d) + (3*b*d*(19*b*c + 2*a
*d))/x))/(35*b^2) + (a + b/x)^(3/2)*(c + d/x)^3*x + 3*Sqrt[a]*c^2*(b*c + 2*
a*d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 97

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p)/(b*
(m + 1)), x] - Dist[1/(b*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*
(e + f*x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ
[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 147

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> -Simp[((a*d*f*h*(n + 2) + b*c*f*h*(m
+ 2) - b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1))/(b^2*d^2*(m + n + 2)*(m + n + 3)), x] + Dist[(a^2*d^2
*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m +
n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3
) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3)), In
```

```
t[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n},
x] && NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

Rule 153

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(h*(a + b*x)^m*(c + d*x)^(n
+ 1)*(e + f*x)^(p + 1))/(d*f*(m + n + p + 2)), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegerQ[m]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 375

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[((a + b/x^n)^p*(c + d/x^n)^q)/x^2, x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\int \left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)^3 dx &= -\operatorname{Subst}\left(\int \frac{(a+bx)^{3/2}(c+dx)^3}{x^2}\,dx, x, \frac{1}{x}\right)\\
&= \left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)^3 x - \operatorname{Subst}\left(\int \frac{\sqrt{a+bx}\,(c+dx)^2\left(\frac{3}{2}(bc+2ad)+\frac{9bdx}{2}\right)}{x}\,dx, x, \frac{1}{x}\right.\\
&= -\frac{9}{7}d\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)^2+\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)^3 x-\frac{2\operatorname{Subst}\left(\int \frac{\sqrt{a+bx}\,(c+dx)\left(\frac{21}{4}bc(bc+\cdots\right.}{x}\right.}{7b\cdots}\\
&= -\frac{9}{7}d\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)^2-\frac{d\left(a+\frac{b}{x}\right)^{3/2}\left(2(13bc-ad)(5bc+2ad)+\frac{3bd(19bc+2ad)}{x}\right)}{35b^2}\cdots\\
&= -3c^2(bc+2ad)\sqrt{a+\frac{b}{x}}-\frac{9}{7}d\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)^2-\frac{d\left(a+\frac{b}{x}\right)^{3/2}\left(2(13bc-ad)(5b\cdots\right.}{35b^2}\\
&= -3c^2(bc+2ad)\sqrt{a+\frac{b}{x}}-\frac{9}{7}d\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)^2-\frac{d\left(a+\frac{b}{x}\right)^{3/2}\left(2(13bc-ad)(5b\cdots\right.}{35b^2}\\
&= -3c^2(bc+2ad)\sqrt{a+\frac{b}{x}}-\frac{9}{7}d\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)^2-\frac{d\left(a+\frac{b}{x}\right)^{3/2}\left(2(13bc-ad)(5b\cdots\right.}{35b^2}
\end{aligned}
$$

Mathematica [A] time = 0.21, size = 159, normalized size = 0.97

$$
\frac{\sqrt{a+\frac{b}{x}}\left(4a^3d^3x^3-2a^2bd^2x^2(21cx+d)+ab^2x\left(35c^3x^3-280c^2dx^2-84cd^2x-16d^3\right)-2b^3\left(35c^3x^3+35c^2dx^2+21cd^2x+5d^3\right)\right)}{35b^2x^3}+3\sqrt{a}\,c^2(2ad+bc)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x)^(3/2)*(c + d/x)^3,x]
```

```
[Out] (Sqrt[a + b/x]*(4*a^3*d^3*x^3 - 2*a^2*b*d^2*x^2*(d + 21*c*x) + a*b^2*x*(-16
*d^3 - 84*c*d^2*x - 280*c^2*d*x^2 + 35*c^3*x^3) - 2*b^3*(5*d^3 + 21*c*d^2*x
 + 35*c^2*d*x^2 + 35*c^3*x^3)))/(35*b^2*x^3) + 3*Sqrt[a]*c^2*(b*c + 2*a*d)*
ArcTanh[Sqrt[a + b/x]/Sqrt[a]]
```

IntegrateAlgebraic [A] time = 0.24, size = 194, normalized size = 1.18

$$3\left(2a^{3/2}c^2d+\sqrt{a}\,bc^3\right)\tanh^{-1}\left(\frac{\sqrt{\frac{ax+b}{x}}}{\sqrt{a}}\right)+\frac{\sqrt{\frac{ax+b}{x}}\left(4a^3d^3x^3-42a^2bcd^2x^3-2a^2bd^3x^2+35ab^2c^3x^4-280ab^2c^2dx^3-84ab^2cd^2x^2-16ab^2d^3x-70b^3c^3x^3-70b^3c^2dx^2-42b^3cd^2x-10b^3d^3\right)}{35b^2x^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b/x)^(3/2)*(c + d/x)^3,x]
```

```
[Out] (Sqrt[(b + a*x)/x]*(-10*b^3*d^3 - 42*b^3*c*d^2*x - 16*a*b^2*d^3*x - 70*b^3*
c^2*d*x^2 - 84*a*b^2*c*d^2*x^2 - 2*a^2*b*d^3*x^2 - 70*b^3*c^3*x^3 - 280*a*b
^2*c^2*d*x^3 - 42*a^2*b*c*d^2*x^3 + 4*a^3*d^3*x^3 + 35*a*b^2*c^3*x^4))/(35*
b^2*x^3) + 3*(Sqrt[a]*b*c^3 + 2*a^(3/2)*c^2*d)*ArcTanh[Sqrt[(b + a*x)/x]/Sq
rt[a]]
```

fricas [A] time = 0.76, size = 380, normalized size = 2.32

$$\left[\frac{105\left(b^3c^3+2\,ab^2c^2d\right)\sqrt{a}\,x^3\log\left(2\,ax+2\,\sqrt{a}\,x\sqrt{\frac{ax+b}{x}}+b\right)+2\left(35\,ab^2c^3x^4-10\,b^3d^3-2\left(35\,b^3c^3+140\,ab^2c^2d+21\,a^2bcd^2-2\,a^3d^3\right)x^3-2\left(35\,b^3c^2d+42\,ab^2cd^2+a^2bd^3\right)x^2-2\left(21\,b^3cd^2+8\,ab^2d^3\right)x\right)\sqrt{\frac{ax+b}{x}}}{70\,b^2x^3},\ -\frac{105\left(b^3c^3+2\,ab^2c^2d\right)\sqrt{-a}\,x^3\arctan\left(\frac{\sqrt{-a}\sqrt{\frac{ax+b}{x}}}{a}\right)-\left(35\,ab^2c^3x^4-10\,b^3d^3-2\left(35\,b^3c^3+140\,ab^2c^2d+21\,a^2bcd^2-2\,a^3d^3\right)x^3-2\left(35\,b^3c^2d+42\,ab^2cd^2+a^2bd^3\right)x^2-2\left(21\,b^3cd^2+8\,ab^2d^3\right)x\right)\sqrt{\frac{ax+b}{x}}}{35\,b^2x^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(3/2)*(c+d/x)^3,x, algorithm="fricas")
```

```
[Out] [1/70*(105*(b^3*c^3 + 2*a*b^2*c^2*d)*sqrt(a)*x^3*log(2*a*x + 2*sqrt(a)*x*sq
rt((a*x + b)/x) + b) + 2*(35*a*b^2*c^3*x^4 - 10*b^3*d^3 - 2*(35*b^3*c^3 + 1
40*a*b^2*c^2*d + 21*a^2*b*c*d^2 - 2*a^3*d^3)*x^3 - 2*(35*b^3*c^2*d + 42*a*b
^2*c*d^2 + a^2*b*d^3)*x^2 - 2*(21*b^3*c*d^2 + 8*a*b^2*d^3)*x)*sqrt((a*x + b
)/x))/(b^2*x^3), -1/35*(105*(b^3*c^3 + 2*a*b^2*c^2*d)*sqrt(-a)*x^3*arctan(s
qrt(-a)*sqrt((a*x + b)/x)/a) - (35*a*b^2*c^3*x^4 - 10*b^3*d^3 - 2*(35*b^3*c
^3 + 140*a*b^2*c^2*d + 21*a^2*b*c*d^2 - 2*a^3*d^3)*x^3 - 2*(35*b^3*c^2*d +
42*a*b^2*c*d^2 + a^2*b*d^3)*x^2 - 2*(21*b^3*c*d^2 + 8*a*b^2*d^3)*x)*sqrt((a
*x + b)/x))/(b^2*x^3)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(3/2)*(c+d/x)^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(x)]
Warning, choosing root of [1,0,%%%{-2,[1,2,0]%%%}+%%%{-2,[1,0,0]%%%}+%%%{-2
,[0,1,1]%%%},0,%%%{1,[2,4,0]%%%}+%%%{-2,[2,2,0]%%%}+%%%{1,[2,0,0]%%%}+%%%{2
,[1,3,1]%%%}+%%%{-2,[1,1,1]%%%}+%%%{1,[0,2,2]%%%}] at parameters values [86
,-97,-82]Warning, choosing root of [1,0,%%%{-2,[1,2,0]%%%}+%%%{-2,[1,0,0]%%
%}+%%%{-2,[0,1,1]%%%},0,%%%{1,[2,4,0]%%%}+%%%{-2,[2,2,0]%%%}+%%%{1,[2,0,0]%
%%}+%%%{2,[1,3,1]%%%}+%%%{-2,[1,1,1]%%%}+%%%{1,[0,2,2]%%%}] at parameters v
alues [7,-27,26]Warning, choosing root of [1,0,%%%{-4,[1,0,0]%%%}+%%%{-2,[0
,1,1]%%%},0,%%%{1,[0,2,2]%%%}] at parameters values [18.6420984049,-49,-86]
Warning, choosing root of [1,0,%%%{-4,[1,0,0]%%%}+%%%{-2,[0,1,1]%%%},0,%%%{
1,[0,2,2]%%%}] at parameters values [78.6493344628,22,42]Warning, choosing
```

```
root of [1,0,%%%{-2,[1,0,1]%%%}+%%%{-4,[0,1,0]%%%},0,%%%{1,[2,0,2]%%%}] at
parameters values [-13,74.7709350525,24]Sign error (%%%{-b,0%%%}+%%%{2*sqrt
(a)*sqrt(b),1/2%%%}+%%%{-2*a,1%%%}+%%%{a*sqrt(a)*sqrt(b)/b,3/2%%%}+%%%{-a^2
*sqrt(a)*sqrt(b)/(4*b^2),5/2%%%}+%%%{undef,7/2%%%})Evaluation time: 0.8Limi
t: Max order reached or unable to make series expansion Error: Bad Argument
 Value
```

maple [B] time = 0.06, size = 353, normalized size = 2.15

$$\frac{\sqrt{\frac{ax+b}{x}}\left(210a^2b^2c^2d\,x^5\ln\left(\frac{2ax+b+2\sqrt{a\,x^2+bx}\,\sqrt{a}}{2\sqrt{a}}\right)+105a\,b^3c^3x^5\ln\left(\frac{2ax+b+2\sqrt{a\,x^2+bx}\,\sqrt{a}}{2\sqrt{a}}\right)+420\sqrt{a\,x^2+bx}\,a^{\frac{5}{2}}b\,c^2d\,x^5+210\sqrt{a\,x^2+bx}\,a^{\frac{3}{2}}b^2c^3x^5-420\left(a\,x^2+bx\right)^{\frac{3}{2}}a^{\frac{3}{2}}b\,c^2d\,x^3-140\left(a\,x^2+bx\right)^{\frac{3}{2}}\sqrt{a}\,b^2c^3x^3+8\left(a\,x^2+bx\right)^{\frac{3}{2}}a^{\frac{5}{2}}d^3x^2-84\left(a\,x^2+bx\right)^{\frac{3}{2}}a^{\frac{3}{2}}bc\,d^2x^2-140\left(a\,x^2+bx\right)^{\frac{3}{2}}\sqrt{a}\,b^2c^2d\,x^2-12\left(a\,x^2+bx\right)^{\frac{3}{2}}a^{\frac{3}{2}}b\,d^3x-84\left(a\,x^2+bx\right)^{\frac{3}{2}}\sqrt{a}\,b^2c\,d^2x-20\left(a\,x^2+bx\right)^{\frac{3}{2}}\sqrt{a}\,b^2d^3\right)}{70\sqrt{(ax+b)\,x}\,\sqrt{a}\,b^2x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x)^(3/2)*(c+d/x)^3,x)
```

```
[Out] 1/70*((a*x+b)/x)^(1/2)*(420*(a*x^2+b*x)^(1/2)*a^(5/2)*x^5*b*c^2*d+210*(a*x^
2+b*x)^(1/2)*a^(3/2)*x^5*b^2*c^3+8*(a*x^2+b*x)^(3/2)*a^(5/2)*x^2*d^3-420*(a
*x^2+b*x)^(3/2)*a^(3/2)*x^3*b*c^2*d+210*ln(1/2*(2*a*x+b+2*(a*x^2+b*x)^(1/2)
*a^(1/2))/a^(1/2))*x^5*a^2*b^2*c^2*d+105*ln(1/2*(2*a*x+b+2*(a*x^2+b*x)^(1/2
)*a^(1/2))/a^(1/2))*x^5*a*b^3*c^3-84*(a*x^2+b*x)^(3/2)*a^(3/2)*x^2*b*c*d^2-
140*(a*x^2+b*x)^(3/2)*a^(1/2)*x^3*b^2*c^3-12*(a*x^2+b*x)^(3/2)*a^(3/2)*x*b*
d^3-140*(a*x^2+b*x)^(3/2)*a^(1/2)*x^2*b^2*c^2*d-84*(a*x^2+b*x)^(3/2)*a^(1/2
)*x*b^2*c*d^2-20*(a*x^2+b*x)^(3/2)*a^(1/2)*b^2*d^3)/x^4/b^2/((a*x+b)*x)^(1/
2)/a^(1/2)
```

maxima [A] time = 1.49, size = 190, normalized size = 1.16

$$-\frac{6\left(a+\frac{b}{x}\right)^{\frac{5}{2}}cd^2}{5\,b}+\frac{1}{2}\left(2\sqrt{a+\frac{b}{x}}\,ax-3\sqrt{a}\,b\log\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{a+\frac{b}{x}}+\sqrt{a}}\right)-4\sqrt{a+\frac{b}{x}}\,b\right)c^3-\left(3\,a^{\frac{3}{2}}\log\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{a+\frac{b}{x}}+\sqrt{a}}\right)+2\left(a+\frac{b}{x}\right)^{\frac{3}{2}}+6\sqrt{a+\frac{b}{x}}\,a\right)c^2d-\frac{2}{35}\left(\frac{5\left(a+\frac{b}{x}\right)^{\frac{7}{2}}}{b^2}-\frac{7\left(a+\frac{b}{x}\right)^{\frac{5}{2}}a}{b^2}\right)d^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(3/2)*(c+d/x)^3,x, algorithm="maxima")
```

```
[Out] -6/5*(a + b/x)^(5/2)*c*d^2/b + 1/2*(2*sqrt(a + b/x)*a*x - 3*sqrt(a)*b*log((
sqrt(a + b/x) - sqrt(a))/(sqrt(a + b/x) + sqrt(a))) - 4*sqrt(a + b/x)*b)*c^
3 - (3*a^(3/2)*log((sqrt(a + b/x) - sqrt(a))/(sqrt(a + b/x) + sqrt(a))) + 2
*(a + b/x)^(3/2) + 6*sqrt(a + b/x)*a)*c^2*d - 2/35*(5*(a + b/x)^(7/2)/b^2 -
 7*(a + b/x)^(5/2)*a/b^2)*d^3
```

mupad [B] time = 3.88, size = 327, normalized size = 1.99

$$\left(a+\frac{b}{x}\right)^{5/2}\left(\frac{6\,a\,d^3-6\,b\,c\,d^2}{5\,b^2}-\frac{4\,a\,d^3}{5\,b^2}\right)+\sqrt{a+\frac{b}{x}}\left(\frac{2\,(a\,d-b\,c)^3}{b^2}+2\,a\left(2\,a\left(\frac{6\,a\,d^3-6\,b\,c\,d^2}{b^2}-\frac{4\,a\,d^3}{b^2}\right)-\frac{6\,d\,(a\,d-b\,c)^2}{b^2}+\frac{2\,a^2\,d^3}{b^2}\right)-a^2\left(\frac{6\,a\,d^3-6\,b\,c\,d^2}{b^2}-\frac{4\,a\,d^3}{b^2}\right)\right)+\left(a+\frac{b}{x}\right)^{3/2}\left(\frac{2\,a\left(\frac{6\,a\,d^3-6\,b\,c\,d^2}{b^2}-\frac{4\,a\,d^3}{b^2}\right)}{3}-\frac{2\,d\,(a\,d-b\,c)^2}{b^2}+\frac{2\,a^2\,d^3}{3\,b^2}\right)-\frac{2\,d^3\left(a+\frac{b}{x}\right)^{7/2}}{7\,b^2}+a\,c^3\,x\sqrt{a+\frac{b}{x}}-2\,c^2\,\mathrm{atan}\left(\frac{2\,c^2\sqrt{a+\frac{b}{x}}\,(2\,a\,d+b\,c)\sqrt{-\frac{9\,a}{4}}}{6\,d\,a^2\,c^2+3\,b\,a\,c^3}\right)(2\,a\,d+b\,c)\sqrt{-\frac{9\,a}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x)^(3/2)*(c + d/x)^3,x)
```

```
[Out] (a + b/x)^(5/2)*((6*a*d^3 - 6*b*c*d^2)/(5*b^2) - (4*a*d^3)/(5*b^2)) + (a +
b/x)^(1/2)*((2*(a*d - b*c)^3)/b^2 + 2*a*(2*a*((6*a*d^3 - 6*b*c*d^2)/b^2 - (
4*a*d^3)/b^2) - (6*d*(a*d - b*c)^2)/b^2 + (2*a^2*d^3)/b^2) - a^2*((6*a*d^3
- 6*b*c*d^2)/b^2 - (4*a*d^3)/b^2)) + (a + b/x)^(3/2)*((2*a*((6*a*d^3 - 6*b*
c*d^2)/b^2 - (4*a*d^3)/b^2))/3 - (2*d*(a*d - b*c)^2)/b^2 + (2*a^2*d^3)/(3*b
^2)) - (2*d^3*(a + b/x)^(7/2))/(7*b^2) + a*c^3*x*(a + b/x)^(1/2) - 2*c^2*at
an((2*c^2*(a + b/x)^(1/2)*(2*a*d + b*c)*(-(9*a)/4)^(1/2))/(6*a^2*c^2*d + 3*
a*b*c^3))*(2*a*d + b*c)*(-(9*a)/4)^(1/2)
```

sympy [A] time = 123.48, size = 1817, normalized size = 11.08

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b/x)**(3/2)*(c+d/x)**3,x)

```
[Out] -16*a**(19/2)*b**(11/2)*d**3*x**6*sqrt(a*x/b + 1)/(105*a**(13/2)*b**7*x**(1
3/2) + 315*a**(11/2)*b**8*x**(11/2) + 315*a**(9/2)*b**9*x**(9/2) + 105*a**(
7/2)*b**10*x**(7/2)) - 40*a**(17/2)*b**(13/2)*d**3*x**5*sqrt(a*x/b + 1)/(10
5*a**(13/2)*b**7*x**(13/2) + 315*a**(11/2)*b**8*x**(11/2) + 315*a**(9/2)*b*
*9*x**(9/2) + 105*a**(7/2)*b**10*x**(7/2)) - 30*a**(15/2)*b**(15/2)*d**3*x*
*4*sqrt(a*x/b + 1)/(105*a**(13/2)*b**7*x**(13/2) + 315*a**(11/2)*b**8*x**(1
1/2) + 315*a**(9/2)*b**9*x**(9/2) + 105*a**(7/2)*b**10*x**(7/2)) - 40*a**(1
3/2)*b**(17/2)*d**3*x**3*sqrt(a*x/b + 1)/(105*a**(13/2)*b**7*x**(13/2) + 31
5*a**(11/2)*b**8*x**(11/2) + 315*a**(9/2)*b**9*x**(9/2) + 105*a**(7/2)*b**1
0*x**(7/2)) + 4*a**(13/2)*b**(3/2)*d**3*x**3*sqrt(a*x/b + 1)/(15*a**(7/2)*b
**3*x**(7/2) + 15*a**(5/2)*b**4*x**(5/2)) - 100*a**(11/2)*b**(19/2)*d**3*x*
*2*sqrt(a*x/b + 1)/(105*a**(13/2)*b**7*x**(13/2) + 315*a**(11/2)*b**8*x**(1
1/2) + 315*a**(9/2)*b**9*x**(9/2) + 105*a**(7/2)*b**10*x**(7/2)) + 12*a**(1
1/2)*b**(5/2)*c*d**2*x**3*sqrt(a*x/b + 1)/(15*a**(7/2)*b**3*x**(7/2) + 15*a
**(5/2)*b**4*x**(5/2)) + 2*a**(11/2)*b**(5/2)*d**3*x**2*sqrt(a*x/b + 1)/(15
*a**(7/2)*b**3*x**(7/2) + 15*a**(5/2)*b**4*x**(5/2)) - 96*a**(9/2)*b**(21/2
)*d**3*x*sqrt(a*x/b + 1)/(105*a**(13/2)*b**7*x**(13/2) + 315*a**(11/2)*b**8
*x**(11/2) + 315*a**(9/2)*b**9*x**(9/2) + 105*a**(7/2)*b**10*x**(7/2)) + 6*
a**(9/2)*b**(7/2)*c*d**2*x**2*sqrt(a*x/b + 1)/(15*a**(7/2)*b**3*x**(7/2) +
15*a**(5/2)*b**4*x**(5/2)) - 8*a**(9/2)*b**(7/2)*d**3*x*sqrt(a*x/b + 1)/(15
*a**(7/2)*b**3*x**(7/2) + 15*a**(5/2)*b**4*x**(5/2)) - 30*a**(7/2)*b**(23/2
)*d**3*sqrt(a*x/b + 1)/(105*a**(13/2)*b**7*x**(13/2) + 315*a**(11/2)*b**8*x
**(11/2) + 315*a**(9/2)*b**9*x**(9/2) + 105*a**(7/2)*b**10*x**(7/2)) - 24*a
**(7/2)*b**(9/2)*c*d**2*x*sqrt(a*x/b + 1)/(15*a**(7/2)*b**3*x**(7/2) + 15*a
**(5/2)*b**4*x**(5/2)) - 6*a**(7/2)*b**(9/2)*d**3*sqrt(a*x/b + 1)/(15*a**(7
/2)*b**3*x**(7/2) + 15*a**(5/2)*b**4*x**(5/2)) - 18*a**(5/2)*b**(11/2)*c*d*
*2*sqrt(a*x/b + 1)/(15*a**(7/2)*b**3*x**(7/2) + 15*a**(5/2)*b**4*x**(5/2))
+ sqrt(a)*b*c**3*asinh(sqrt(a)*sqrt(x)/sqrt(b)) + 16*a**10*b**5*d**3*x**(13
/2)/(105*a**(13/2)*b**7*x**(13/2) + 315*a**(11/2)*b**8*x**(11/2) + 315*a**(
9/2)*b**9*x**(9/2) + 105*a**(7/2)*b**10*x**(7/2)) + 48*a**9*b**6*d**3*x**(1
1/2)/(105*a**(13/2)*b**7*x**(13/2) + 315*a**(11/2)*b**8*x**(11/2) + 315*a**
(9/2)*b**9*x**(9/2) + 105*a**(7/2)*b**10*x**(7/2)) + 48*a**8*b**7*d**3*x**(
9/2)/(105*a**(13/2)*b**7*x**(13/2) + 315*a**(11/2)*b**8*x**(11/2) + 315*a**
(9/2)*b**9*x**(9/2) + 105*a**(7/2)*b**10*x**(7/2)) + 16*a**7*b**8*d**3*x**(
7/2)/(105*a**(13/2)*b**7*x**(13/2) + 315*a**(11/2)*b**8*x**(11/2) + 315*a**
(9/2)*b**9*x**(9/2) + 105*a**(7/2)*b**10*x**(7/2)) - 4*a**7*b*d**3*x**(7/2)
/(15*a**(7/2)*b**3*x**(7/2) + 15*a**(5/2)*b**4*x**(5/2)) - 12*a**6*b**2*c*d
**2*x**(7/2)/(15*a**(7/2)*b**3*x**(7/2) + 15*a**(5/2)*b**4*x**(5/2)) - 4*a*
*6*b**2*d**3*x**(5/2)/(15*a**(7/2)*b**3*x**(7/2) + 15*a**(5/2)*b**4*x**(5/2
)) - 12*a**5*b**3*c*d**2*x**(5/2)/(15*a**(7/2)*b**3*x**(7/2) + 15*a**(5/2)*
b**4*x**(5/2)) - 6*a**2*c**2*d*atan(sqrt(a + b/x)/sqrt(-a))/sqrt(-a) + a*sq
rt(b)*c**3*sqrt(x)*sqrt(a*x/b + 1) - 2*a*b*c**3*atan(sqrt(a + b/x)/sqrt(-a)
)/sqrt(-a) - 6*a*c**2*d*sqrt(a + b/x) + 3*a*c*d**2*Piecewise((-sqrt(a)/x, E
q(b, 0)), (-2*(a + b/x)**(3/2)/(3*b), True)) - 2*b*c**3*sqrt(a + b/x) + 3*b
*c**2*d*Piecewise((-sqrt(a)/x, Eq(b, 0)), (-2*(a + b/x)**(3/2)/(3*b), True)
)
```

3.135 $\int \left(a+\frac{b}{x}\right)^{3/2} \left(c+\frac{d}{x}\right)^2 dx$

Optimal. Leaf size=126

$$\frac{c^2 x \left(a+\frac{b}{x}\right)^{5/2}}{a}-\frac{c \left(a+\frac{b}{x}\right)^{3/2} (4ad+3bc)}{3a}-c\sqrt{a+\frac{b}{x}}\,(4ad+3bc)+\sqrt{a}\,c(4ad+3bc)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)-\frac{2d^2 \left(a+\frac{b}{x}\right)^{5/2}}{5b}$$

Rubi [A] time = 0.08, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {375, 89, 80, 50, 63, 208}

$$\frac{c^2 x \left(a+\frac{b}{x}\right)^{5/2}}{a}-\frac{c \left(a+\frac{b}{x}\right)^{3/2} (4ad+3bc)}{3a}-c\sqrt{a+\frac{b}{x}}\,(4ad+3bc)+\sqrt{a}\,c(4ad+3bc)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)-\frac{2d^2 \left(a+\frac{b}{x}\right)^{5/2}}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x)^(3/2)*(c + d/x)^2,x]
```

```
[Out] -(c*(3*b*c + 4*a*d)*Sqrt[a + b/x]) - (c*(3*b*c + 4*a*d)*(a + b/x)^(3/2))/(3
*a) - (2*d^2*(a + b/x)^(5/2))/(5*b) + (c^2*(a + b/x)^(5/2)*x)/a + Sqrt[a]*c
*(3*b*c + 4*a*d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
```

```
1])))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 375

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[((a + b/x^n)^p*(c + d/x^n)^q)/x^2, x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)^2 dx &= -\text{Subst}\left(\int \frac{(a+bx)^{3/2}(c+dx)^2}{x^2}\,dx, x, \frac{1}{x}\right)\\
&= \frac{c^2\left(a+\frac{b}{x}\right)^{5/2} x}{a} - \frac{\text{Subst}\left(\int \frac{(a+bx)^{3/2}\left(\frac{1}{2}c(3bc+4ad)+ad^2x\right)}{x}\,dx, x, \frac{1}{x}\right)}{a}\\
&= -\frac{2d^2\left(a+\frac{b}{x}\right)^{5/2}}{5b} + \frac{c^2\left(a+\frac{b}{x}\right)^{5/2} x}{a} - \frac{(c(3bc+4ad))\,\text{Subst}\left(\int \frac{(a+bx)^{3/2}}{x}\,dx, x, \frac{1}{x}\right)}{2a}\\
&= -\frac{c(3bc+4ad)\left(a+\frac{b}{x}\right)^{3/2}}{3a} - \frac{2d^2\left(a+\frac{b}{x}\right)^{5/2}}{5b} + \frac{c^2\left(a+\frac{b}{x}\right)^{5/2} x}{a} - \frac{1}{2}(c(3bc+4ad))\,\text{Subst}\\
&= -c(3bc+4ad)\sqrt{a+\frac{b}{x}} - \frac{c(3bc+4ad)\left(a+\frac{b}{x}\right)^{3/2}}{3a} - \frac{2d^2\left(a+\frac{b}{x}\right)^{5/2}}{5b} + \frac{c^2\left(a+\frac{b}{x}\right)^{5/2} x}{a}\\
&= -c(3bc+4ad)\sqrt{a+\frac{b}{x}} - \frac{c(3bc+4ad)\left(a+\frac{b}{x}\right)^{3/2}}{3a} - \frac{2d^2\left(a+\frac{b}{x}\right)^{5/2}}{5b} + \frac{c^2\left(a+\frac{b}{x}\right)^{5/2} x}{a}\\
&= -c(3bc+4ad)\sqrt{a+\frac{b}{x}} - \frac{c(3bc+4ad)\left(a+\frac{b}{x}\right)^{3/2}}{3a} - \frac{2d^2\left(a+\frac{b}{x}\right)^{5/2}}{5b} + \frac{c^2\left(a+\frac{b}{x}\right)^{5/2} x}{a}
\end{aligned}$$

Mathematica [A] time = 0.26, size = 106, normalized size = 0.84

$$-\frac{c(4ad+3bc)\left(\sqrt{a+\frac{b}{x}}\,(4ax+b)-3a^{3/2}x\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)\right)}{3ax}+\frac{c^2x\left(a+\frac{b}{x}\right)^{5/2}}{a}-\frac{2d^2\left(a+\frac{b}{x}\right)^{5/2}}{5b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x)^(3/2)*(c + d/x)^2,x]
```

```
[Out] (-2*d^2*(a + b/x)^(5/2))/(5*b) + (c^2*(a + b/x)^(5/2)*x)/a - (c*(3*b*c + 4*
a*d)*(Sqrt[a + b/x]*(b + 4*a*x) - 3*a^(3/2)*x*ArcTanh[Sqrt[a + b/x]/Sqrt[a]
]))/(3*a*x)
```

IntegrateAlgebraic [A] time = 0.20, size = 132, normalized size = 1.05

$$\left(4a^{3/2}cd+3\sqrt{a}\,bc^2\right)\tanh^{-1}\left(\frac{\sqrt{\frac{ax+b}{x}}}{\sqrt{a}}\right)+\frac{\sqrt{\frac{ax+b}{x}}\left(-6a^2d^2x^2+15abc^2x^3-80abcdx^2-12abd^2x-30b^2c^2x^2-20b^2cdx-6b^2d^2\right)}{15bx^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b/x)^(3/2)*(c + d/x)^2,x]
```

```
[Out] (Sqrt[(b + a*x)/x]*(-6*b^2*d^2 - 20*b^2*c*d*x - 12*a*b*d^2*x - 30*b^2*c^2*x
^2 - 80*a*b*c*d*x^2 - 6*a^2*d^2*x^2 + 15*a*b*c^2*x^3))/(15*b*x^2) + (3*Sqrt
[a]*b*c^2 + 4*a^(3/2)*c*d)*ArcTanh[Sqrt[(b + a*x)/x]/Sqrt[a]]
```

fricas [A] time = 0.61, size = 268, normalized size = 2.13

$$\left[\frac{15\left(3\,b^{2}c^{2}+4\,abcd\right)\sqrt{a}\,x^{2}\log\left(2\,ax+2\,\sqrt{a}\,x\sqrt{\frac{ax+b}{x}}+b\right)+2\left(15\,abc^{2}x^{3}-6\,b^{2}d^{2}-2\left(15\,b^{2}c^{2}+40\,abcd+3\,a^{2}d^{2}\right)x^{2}-4\left(5\,b^{2}cd+3\,abd^{2}\right)x\right)\sqrt{\frac{ax+b}{x}}}{30\,bx^{2}}, -\frac{15\left(3\,b^{2}c^{2}+4\,abcd\right)\sqrt{-a}\,x^{2}\arctan\left(\frac{\sqrt{-a}\sqrt{\frac{ax+b}{x}}}{a}\right)-\left(15\,abc^{2}x^{3}-6\,b^{2}d^{2}-2\left(15\,b^{2}c^{2}+40\,abcd+3\,a^{2}d^{2}\right)x^{2}-4\left(5\,b^{2}cd+3\,abd^{2}\right)x\right)\sqrt{\frac{ax+b}{x}}}{15\,bx^{2}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(3/2)*(c+d/x)^2,x, algorithm="fricas")
```

```
[Out] [1/30*(15*(3*b^2*c^2 + 4*a*b*c*d)*sqrt(a)*x^2*log(2*a*x + 2*sqrt(a)*x*sqrt(
(a*x + b)/x) + b) + 2*(15*a*b*c^2*x^3 - 6*b^2*d^2 - 2*(15*b^2*c^2 + 40*a*b*
c*d + 3*a^2*d^2)*x^2 - 4*(5*b^2*c*d + 3*a*b*d^2)*x)*sqrt((a*x + b)/x))/(b*x
^2), -1/15*(15*(3*b^2*c^2 + 4*a*b*c*d)*sqrt(-a)*x^2*arctan(sqrt(-a)*sqrt((a
*x + b)/x)/a) - (15*a*b*c^2*x^3 - 6*b^2*d^2 - 2*(15*b^2*c^2 + 40*a*b*c*d +
3*a^2*d^2)*x^2 - 4*(5*b^2*c*d + 3*a*b*d^2)*x)*sqrt((a*x + b)/x))/(b*x^2)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(3/2)*(c+d/x)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(x)]
Warning, choosing root of [1,0,%%%{-2,[1,2,0]%%%}+%%%{-2,[1,0,0]%%%}+%%%{-2
,[0,1,1]%%%},0,%%%{1,[2,4,0]%%%}+%%%{-2,[2,2,0]%%%}+%%%{1,[2,0,0]%%%}+%%%{2
,[1,3,1]%%%}+%%%{-2,[1,1,1]%%%}+%%%{1,[0,2,2]%%%}] at parameters values [86
,-97,-82]Warning, choosing root of [1,0,%%%{-2,[1,2,0]%%%}+%%%{-2,[1,0,0]%%
%}+%%%{-2,[0,1,1]%%%},0,%%%{1,[2,4,0]%%%}+%%%{-2,[2,2,0]%%%}+%%%{1,[2,0,0]%
%%}+%%%{2,[1,3,1]%%%}+%%%{-2,[1,1,1]%%%}+%%%{1,[0,2,2]%%%}] at parameters v
alues [7,-27,26]Warning, choosing root of [1,0,%%%{-4,[1,0,0]%%%}+%%%{-2,[0
,1,1]%%%},0,%%%{1,[0,2,2]%%%}] at parameters values [18.6420984049,-49,-86]
Warning, choosing root of [1,0,%%%{-4,[1,0,0]%%%}+%%%{-2,[0,1,1]%%%},0,%%%{
1,[0,2,2]%%%}] at parameters values [78.6493344628,22,42]Warning, choosing
root of [1,0,%%%{-2,[1,0,1]%%%}+%%%{-4,[0,1,0]%%%},0,%%%{1,[2,0,2]%%%}] at
parameters values [-13,74.7709350525,24]Sign error (%%%{-b,0%%%}+%%%{2*sqrt
(a)*sqrt(b),1/2%%%}+%%%{-2*a,1%%%}+%%%{a*sqrt(a)*sqrt(b)/b,3/2%%%}+%%%{-a^2
*sqrt(a)*sqrt(b)/(4*b^2),5/2%%%}+%%%{undef,7/2%%%})Evaluation time: 0.43Lim
it: Max order reached or unable to make series expansion Error: Bad Argumen
t Value
```

maple [B] time = 0.06, size = 260, normalized size = 2.06

$$-\frac{\sqrt{\frac{ax+b}{x}}\,\left(-60a^{2}bcd\,x^{4}\ln\left(\frac{2ax+b+2\sqrt{a\,x^{2}+bx}\,\sqrt{a}}{2\sqrt{a}}\right)-45a\,b^{2}c^{2}x^{4}\ln\left(\frac{2ax+b+2\sqrt{a\,x^{2}+bx}\,\sqrt{a}}{2\sqrt{a}}\right)-120\sqrt{a\,x^{2}+bx}\,a^{\frac{5}{2}}cd\,x^{4}-90\sqrt{a\,x^{2}+bx}\,a^{\frac{3}{2}}b\,c^{2}x^{4}+120\left(a\,x^{2}+bx\right)^{\frac{3}{2}}a^{\frac{3}{2}}cd\,x^{2}+60\left(a\,x^{2}+bx\right)^{\frac{3}{2}}\sqrt{a}\,b\,c^{2}x^{2}+12\left(a\,x^{2}+bx\right)^{\frac{3}{2}}a^{\frac{3}{2}}d^{2}x+40\left(a\,x^{2}+bx\right)^{\frac{3}{2}}\sqrt{a}\,bcdx+12\left(a\,x^{2}+bx\right)^{\frac{3}{2}}\sqrt{a}\,b\,d^{2}\right)}{30\sqrt{(ax+b)\,x}\,\sqrt{a}\,b\,x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x)^(3/2)*(c+d/x)^2,x)
```

```
[Out] -1/30*((a*x+b)/x)^(1/2)/x^3/b*(-120*(a*x^2+b*x)^(1/2)*a^(5/2)*x^4*c*d-90*(a
*x^2+b*x)^(1/2)*a^(3/2)*x^4*b*c^2-60*ln(1/2*(2*a*x+b+2*(a*x^2+b*x)^(1/2)*a^
(1/2))/a^(1/2))*x^4*a^2*b*c*d-45*ln(1/2*(2*a*x+b+2*(a*x^2+b*x)^(1/2)*a^(1/2
```

```
))/a^(1/2))*x^4*a*b^2*c^2+120*(a*x^2+b*x)^(3/2)*a^(3/2)*x^2*c*d+60*(a*x^2+b
*x)^(3/2)*a^(1/2)*x^2*b*c^2+12*(a*x^2+b*x)^(3/2)*a^(3/2)*x*d^2+40*(a*x^2+b*
x)^(3/2)*a^(1/2)*x*b*c*d+12*(a*x^2+b*x)^(3/2)*a^(1/2)*b*d^2)/((a*x+b)*x)^(1
/2)/a^(1/2)
```

maxima [A] time = 1.16, size = 152, normalized size = 1.21

$$-\frac{2\left(a+\frac{b}{x}\right)^{\frac{5}{2}}d^2}{5\,b}+\frac{1}{2}\left(2\sqrt{a+\frac{b}{x}}\,ax-3\sqrt{a}\,b\log\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{a+\frac{b}{x}}+\sqrt{a}}\right)-4\sqrt{a+\frac{b}{x}}\,b\right)c^2-\frac{2}{3}\left(3\,a^{\frac{3}{2}}\log\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{a+\frac{b}{x}}+\sqrt{a}}\right)+2\left(a+\frac{b}{x}\right)^{\frac{3}{2}}+6\sqrt{a+\frac{b}{x}}\,a\right)cd$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(3/2)*(c+d/x)^2,x, algorithm="maxima")
```

```
[Out] -2/5*(a + b/x)^(5/2)*d^2/b + 1/2*(2*sqrt(a + b/x)*a*x - 3*sqrt(a)*b*log((sq
rt(a + b/x) - sqrt(a))/(sqrt(a + b/x) + sqrt(a))) - 4*sqrt(a + b/x)*b)*c^2
- 2/3*(3*a^(3/2)*log((sqrt(a + b/x) - sqrt(a))/(sqrt(a + b/x) + sqrt(a))) +
 2*(a + b/x)^(3/2) + 6*sqrt(a + b/x)*a)*c*d
```

mupad [B] time = 2.58, size = 197, normalized size = 1.56

$$\sqrt{a+\frac{b}{x}}\left(2\,a\left(\frac{4\,a\,d^2-4\,b\,c\,d}{b}-\frac{4\,a\,d^2}{b}\right)-\frac{2\,(a\,d-b\,c)^2}{b}+\frac{2\,a^2\,d^2}{b}\right)+\left(\frac{4\,a\,d^2-4\,b\,c\,d}{3\,b}-\frac{4\,a\,d^2}{3\,b}\right)\left(a+\frac{b}{x}\right)^{3/2}-\frac{2\,d^2\left(a+\frac{b}{x}\right)^{5/2}}{5\,b}+a\,c^2\,x\,\sqrt{a+\frac{b}{x}}-2\,c\,\mathrm{atan}\left(\frac{2\,c\,\sqrt{a+\frac{b}{x}}\,(4\,a\,d+3\,b\,c)\,\sqrt{-\frac{a}{4}}}{4\,d\,a^2\,c+3\,b\,a\,c^2}\right)(4\,a\,d+3\,b\,c)\,\sqrt{-\frac{a}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x)^(3/2)*(c + d/x)^2,x)
```

```
[Out] (a + b/x)^(1/2)*(2*a*((4*a*d^2 - 4*b*c*d)/b - (4*a*d^2)/b) - (2*(a*d - b*c)
^2)/b + (2*a^2*d^2)/b) + ((4*a*d^2 - 4*b*c*d)/(3*b) - (4*a*d^2)/(3*b))*(a +
 b/x)^(3/2) - (2*d^2*(a + b/x)^(5/2))/(5*b) + a*c^2*x*(a + b/x)^(1/2) - 2*c
*atan((2*c*(a + b/x)^(1/2)*(4*a*d + 3*b*c)*(-a/4)^(1/2))/(3*a*b*c^2 + 4*a^2
*c*d))*(4*a*d + 3*b*c)*(-a/4)^(1/2)
```

sympy [A] time = 94.07, size = 534, normalized size = 4.24

$$\frac{4a^{\frac{11}{2}}b^{\frac{5}{2}}d^2x^3\sqrt{\frac{ax}{b}+1}}{15a^{\frac{7}{2}}b^3x^{\frac{7}{2}}+15a^{\frac{5}{2}}b^4x^{\frac{5}{2}}}+\frac{2a^{\frac{9}{2}}b^{\frac{7}{2}}d^2x^2\sqrt{\frac{ax}{b}+1}}{15a^{\frac{7}{2}}b^3x^{\frac{7}{2}}+15a^{\frac{5}{2}}b^4x^{\frac{5}{2}}}-\frac{8a^{\frac{7}{2}}b^{\frac{9}{2}}d^2x\sqrt{\frac{ax}{b}+1}}{15a^{\frac{7}{2}}b^3x^{\frac{7}{2}}+15a^{\frac{5}{2}}b^4x^{\frac{5}{2}}}-\frac{6a^{\frac{5}{2}}b^{\frac{11}{2}}d^2\sqrt{\frac{ax}{b}+1}}{15a^{\frac{7}{2}}b^3x^{\frac{7}{2}}+15a^{\frac{5}{2}}b^4x^{\frac{5}{2}}}+\sqrt{a}\,bc^2\operatorname{asinh}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)-\frac{4a^6b^2d^2x^{\frac{7}{2}}}{15a^{\frac{7}{2}}b^3x^{\frac{7}{2}}+15a^{\frac{5}{2}}b^4x^{\frac{5}{2}}}-\frac{4a^5b^3d^2x^{\frac{5}{2}}}{15a^{\frac{7}{2}}b^3x^{\frac{7}{2}}+15a^{\frac{5}{2}}b^4x^{\frac{5}{2}}}-\frac{4a^2cd\operatorname{atan}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{-a}}\right)}{\sqrt{-a}}+a\sqrt{b}c^2\sqrt{x}\sqrt{\frac{ax}{b}+1}-\frac{2abc^2\operatorname{atan}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{-a}}\right)}{\sqrt{-a}}-4acd\sqrt{a+\frac{b}{x}}+ad^2\left(\begin{cases}-\frac{\sqrt{a}}{x} & \text{for}\: b=0\\-\frac{2\left(a+\frac{b}{x}\right)^{\frac{3}{2}}}{3b} & \text{otherwise}\end{cases}\right)-2bc^2\sqrt{a+\frac{b}{x}}+2bcd\left(\begin{cases}-\frac{\sqrt{a}}{x} & \text{for}\: b=0\\-\frac{2\left(a+\frac{b}{x}\right)^{\frac{3}{2}}}{3b} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)**(3/2)*(c+d/x)**2,x)
```

```
[Out] 4*a**(11/2)*b**(5/2)*d**2*x**3*sqrt(a*x/b + 1)/(15*a**(7/2)*b**3*x**(7/2) +
 15*a**(5/2)*b**4*x**(5/2)) + 2*a**(9/2)*b**(7/2)*d**2*x**2*sqrt(a*x/b + 1)
/(15*a**(7/2)*b**3*x**(7/2) + 15*a**(5/2)*b**4*x**(5/2)) - 8*a**(7/2)*b**(9
/2)*d**2*x*sqrt(a*x/b + 1)/(15*a**(7/2)*b**3*x**(7/2) + 15*a**(5/2)*b**4*x*
*(5/2)) - 6*a**(5/2)*b**(11/2)*d**2*sqrt(a*x/b + 1)/(15*a**(7/2)*b**3*x**(7
/2) + 15*a**(5/2)*b**4*x**(5/2)) + sqrt(a)*b*c**2*asinh(sqrt(a)*sqrt(x)/sqr
t(b)) - 4*a**6*b**2*d**2*x**(7/2)/(15*a**(7/2)*b**3*x**(7/2) + 15*a**(5/2)*
b**4*x**(5/2)) - 4*a**5*b**3*d**2*x**(5/2)/(15*a**(7/2)*b**3*x**(7/2) + 15*
a**(5/2)*b**4*x**(5/2)) - 4*a**2*c*d*atan(sqrt(a + b/x)/sqrt(-a))/sqrt(-a)
+ a*sqrt(b)*c**2*sqrt(x)*sqrt(a*x/b + 1) - 2*a*b*c**2*atan(sqrt(a + b/x)/sq
rt(-a))/sqrt(-a) - 4*a*c*d*sqrt(a + b/x) + a*d**2*Piecewise((-sqrt(a)/x, Eq
(b, 0)), (-2*(a + b/x)**(3/2)/(3*b), True)) - 2*b*c**2*sqrt(a + b/x) + 2*b*
c*d*Piecewise((-sqrt(a)/x, Eq(b, 0)), (-2*(a + b/x)**(3/2)/(3*b), True))
```

3.136 $\int \left(a+\frac{b}{x}\right)^{3/2} \left(c+\frac{d}{x}\right) dx$

Optimal. Leaf size=100

$$-\frac{\left(a+\frac{b}{x}\right)^{3/2}(2ad+3bc)}{3a}-\sqrt{a+\frac{b}{x}}\,(2ad+3bc)+\sqrt{a}\,(2ad+3bc)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)+\frac{cx\left(a+\frac{b}{x}\right)^{5/2}}{a}$$

Rubi [A] time = 0.06, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {375, 78, 50, 63, 208}

$$-\frac{\left(a+\frac{b}{x}\right)^{3/2}(2ad+3bc)}{3a}-\sqrt{a+\frac{b}{x}}\,(2ad+3bc)+\sqrt{a}\,(2ad+3bc)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)+\frac{cx\left(a+\frac{b}{x}\right)^{5/2}}{a}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x)^(3/2)*(c + d/x),x]
```

```
[Out] -((3*b*c + 2*a*d)*Sqrt[a + b/x]) - ((3*b*c + 2*a*d)*(a + b/x)^(3/2))/(3*a)
+ (c*(a + b/x)^(5/2)*x)/a + Sqrt[a]*(3*b*c + 2*a*d)*ArcTanh[Sqrt[a + b/x]/S
qrt[a]]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 375

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[((a + b/x^n)^p*(c + d/x^n)^q)/x^2, x], x, 1/x] /; FreeQ[{a,
```

```
b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int \left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)dx &= -\text{Subst}\left(\int \frac{(a+bx)^{3/2}(c+dx)}{x^2}\,dx, x, \frac{1}{x}\right)\\
&= \frac{c\left(a+\frac{b}{x}\right)^{5/2}x}{a} - \frac{\left(\frac{3bc}{2}+ad\right)\text{Subst}\left(\int \frac{(a+bx)^{3/2}}{x}\,dx, x, \frac{1}{x}\right)}{a}\\
&= -\frac{(3bc+2ad)\left(a+\frac{b}{x}\right)^{3/2}}{3a} + \frac{c\left(a+\frac{b}{x}\right)^{5/2}x}{a} - \frac{1}{2}(3bc+2ad)\,\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x}\,dx, x, \frac{1}{x}\right)\\
&= -\left((3bc+2ad)\sqrt{a+\frac{b}{x}}\right) - \frac{(3bc+2ad)\left(a+\frac{b}{x}\right)^{3/2}}{3a} + \frac{c\left(a+\frac{b}{x}\right)^{5/2}x}{a} - \frac{1}{2}(a(3bc+2ad))\,\text{S}\\
&= -\left((3bc+2ad)\sqrt{a+\frac{b}{x}}\right) - \frac{(3bc+2ad)\left(a+\frac{b}{x}\right)^{3/2}}{3a} + \frac{c\left(a+\frac{b}{x}\right)^{5/2}x}{a} - \frac{(a(3bc+2ad))\,\text{Su}}{}\\
&= -\left((3bc+2ad)\sqrt{a+\frac{b}{x}}\right) - \frac{(3bc+2ad)\left(a+\frac{b}{x}\right)^{3/2}}{3a} + \frac{c\left(a+\frac{b}{x}\right)^{5/2}x}{a} + \sqrt{a}\,(3bc+2ad)\,\text{ta}
\end{aligned}$$

Mathematica [A] time = 0.09, size = 73, normalized size = 0.73

$$\frac{\sqrt{a+\frac{b}{x}}\,(ax(3cx-8d)-2b(3cx+d))}{3x}+\sqrt{a}\,(2ad+3bc)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x)^(3/2)*(c + d/x),x]
```

```
[Out] (Sqrt[a + b/x]*(a*x*(-8*d + 3*c*x) - 2*b*(d + 3*c*x)))/(3*x) + Sqrt[a]*(3*b
*c + 2*a*d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]]
```

IntegrateAlgebraic [A] time = 0.16, size = 82, normalized size = 0.82

$$\left(2a^{3/2}d+3\sqrt{a}\,bc\right)\tanh^{-1}\left(\frac{\sqrt{\frac{ax+b}{x}}}{\sqrt{a}}\right)+\frac{\sqrt{\frac{ax+b}{x}}\left(3acx^2-8adx-6bcx-2bd\right)}{3x}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b/x)^(3/2)*(c + d/x),x]
```

```
[Out] (Sqrt[(b + a*x)/x]*(-2*b*d - 6*b*c*x - 8*a*d*x + 3*a*c*x^2))/(3*x) + (3*Sqr
t[a]*b*c + 2*a^(3/2)*d)*ArcTanh[Sqrt[(b + a*x)/x]/Sqrt[a]]
```

fricas [A] time = 0.87, size = 164, normalized size = 1.64

$$\left[\frac{3\,(3\,bc+2\,ad)\sqrt{a}\,x\log\left(2\,ax+2\,\sqrt{a}\,x\sqrt{\frac{ax+b}{x}}+b\right)+2\left(3\,acx^2-2\,bd-2\,(3\,bc+4\,ad)x\right)\sqrt{\frac{ax+b}{x}}}{6\,x}, -\frac{3\,(3\,bc+2\,ad)\sqrt{-a}\,x\arctan\left(\frac{\sqrt{-a}\,\sqrt{\frac{ax+b}{x}}}{a}\right)-\left(3\,acx^2-2\,bd-2\,(3\,bc+4\,ad)x\right)\sqrt{\frac{ax+b}{x}}}{3\,x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(3/2)*(c+d/x),x, algorithm="fricas")
```

```
[Out] [1/6*(3*(3*b*c + 2*a*d)*sqrt(a)*x*log(2*a*x + 2*sqrt(a)*x*sqrt((a*x + b)/x)
 + b) + 2*(3*a*c*x^2 - 2*b*d - 2*(3*b*c + 4*a*d)*x)*sqrt((a*x + b)/x))/x, -
1/3*(3*(3*b*c + 2*a*d)*sqrt(-a)*x*arctan(sqrt(-a)*sqrt((a*x + b)/x)/a) - (3
*a*c*x^2 - 2*b*d - 2*(3*b*c + 4*a*d)*x)*sqrt((a*x + b)/x))/x]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(3/2)*(c+d/x),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(x)]
Warning, choosing root of [1,0,%%%{-2,[1,2,0]%%%}+%%%{-2,[1,0,0]%%%}+%%%{-2
,[0,1,1]%%%},0,%%%{1,[2,4,0]%%%}+%%%{-2,[2,2,0]%%%}+%%%{1,[2,0,0]%%%}+%%%{2
,[1,3,1]%%%}+%%%{-2,[1,1,1]%%%}+%%%{1,[0,2,2]%%%}] at parameters values [86
,-97,-82]Warning, choosing root of [1,0,%%%{-2,[1,2,0]%%%}+%%%{-2,[1,0,0]%%
%}+%%%{-2,[0,1,1]%%%},0,%%%{1,[2,4,0]%%%}+%%%{-2,[2,2,0]%%%}+%%%{1,[2,0,0]%
%%}+%%%{2,[1,3,1]%%%}+%%%{-2,[1,1,1]%%%}+%%%{1,[0,2,2]%%%}] at parameters v
alues [7,-27,26]Warning, choosing root of [1,0,%%%{-4,[1,0,0]%%%}+%%%{-2,[0
,1,1]%%%},0,%%%{1,[0,2,2]%%%}] at parameters values [18.6420984049,-49,-86]
Warning, choosing root of [1,0,%%%{-4,[1,0,0]%%%}+%%%{-2,[0,1,1]%%%},0,%%%{
1,[0,2,2]%%%}] at parameters values [78.6493344628,22,42]Warning, choosing
root of [1,0,%%%{-2,[1,0,1]%%%}+%%%{-4,[0,1,0]%%%},0,%%%{1,[2,0,2]%%%}] at
parameters values [-13,74.7709350525,24]Sign error (%%%{-b,0%%%}+%%%{2*sqrt
(a)*sqrt(b),1/2%%%}+%%%{-2*a,1%%%}+%%%{a*sqrt(a)*sqrt(b)/b,3/2%%%}+%%%{-a^2
*sqrt(a)*sqrt(b)/(4*b^2),5/2%%%}+%%%{undef,7/2%%%})Limit: Max order reached
 or unable to make series expansion Error: Bad Argument Value
```

maple [B] time = 0.06, size = 205, normalized size = 2.05

$$\frac{\sqrt{\frac{ax+b}{x}}\,\left(6a^{2}bd\,x^{3}\ln\left(\frac{2ax+b+2\sqrt{a\,x^{2}+bx}\,\sqrt{a}}{2\sqrt{a}}\right)+9a\,b^{2}c\,x^{3}\ln\left(\frac{2ax+b+2\sqrt{a\,x^{2}+bx}\,\sqrt{a}}{2\sqrt{a}}\right)+12\sqrt{a\,x^{2}+bx}\,a^{\frac{5}{2}}d\,x^{3}+18\sqrt{a\,x^{2}+bx}\,a^{\frac{3}{2}}bc\,x^{3}-12\left(a\,x^{2}+bx\right)^{\frac{3}{2}}a^{\frac{3}{2}}dx-12\left(a\,x^{2}+bx\right)^{\frac{3}{2}}\sqrt{a}\,bcx-4\left(a\,x^{2}+bx\right)^{\frac{3}{2}}\sqrt{a}\,bd\right)}{6\sqrt{(ax+b)\,x}\,\sqrt{a}\,b\,x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x)^(3/2)*(c+d/x),x)
```

```
[Out] 1/6*((a*x+b)/x)^(1/2)*(12*a^(5/2)*(a*x^2+b*x)^(1/2)*x^3*d+18*a^(3/2)*(a*x^2
+b*x)^(1/2)*x^3*b*c-12*a^(3/2)*(a*x^2+b*x)^(3/2)*x*d+6*ln(1/2*(2*a*x+b+2*(a
*x^2+b*x)^(1/2)*a^(1/2))/a^(1/2))*x^3*a^2*b*d+9*ln(1/2*(2*a*x+b+2*(a*x^2+b*
x)^(1/2)*a^(1/2))/a^(1/2))*x^3*a*b^2*c-12*a^(1/2)*(a*x^2+b*x)^(3/2)*x*b*c-4
*d*(a*x^2+b*x)^(3/2)*b*a^(1/2))/x^2/((a*x+b)*x)^(1/2)/b/a^(1/2)
```

maxima [A] time = 1.33, size = 132, normalized size = 1.32

$$\frac{1}{2}\left(2\sqrt{a+\frac{b}{x}}\,ax-3\sqrt{a}\,b\log\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{a+\frac{b}{x}}+\sqrt{a}}\right)-4\sqrt{a+\frac{b}{x}}\,b\right)c-\frac{1}{3}\left(3\,a^{\frac{3}{2}}\log\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{a+\frac{b}{x}}+\sqrt{a}}\right)+2\left(a+\frac{b}{x}\right)^{\frac{3}{2}}+6\sqrt{a+\frac{b}{x}}\,a\right)d$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(3/2)*(c+d/x),x, algorithm="maxima")
```

```
[Out] 1/2*(2*sqrt(a + b/x)*a*x - 3*sqrt(a)*b*log((sqrt(a + b/x) - sqrt(a))/(sqrt(
a + b/x) + sqrt(a))) - 4*sqrt(a + b/x)*b)*c - 1/3*(3*a^(3/2)*log((sqrt(a +
```

```
b/x) - sqrt(a))/(sqrt(a + b/x) + sqrt(a))) + 2*(a + b/x)^(3/2) + 6*sqrt(a +
 b/x)*a)*d
```

mupad [B] time = 2.51, size = 81, normalized size = 0.81

$$2\,a^{3/2}\,d\,\mathrm{atanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)-\frac{2\,d\,{\left(a+\frac{b}{x}\right)}^{3/2}}{3}-2\,a\,d\,\sqrt{a+\frac{b}{x}}-\frac{2\,c\,x\,{\left(a+\frac{b}{x}\right)}^{3/2}\,{}_2\mathrm{F}_1\left(-\frac{3}{2},-\frac{1}{2};\ \frac{1}{2};\ -\frac{a\,x}{b}\right)}{{\left(\frac{a\,x}{b}+1\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x)^(3/2)*(c + d/x),x)
```

```
[Out] 2*a^(3/2)*d*atanh((a + b/x)^(1/2)/a^(1/2)) - (2*d*(a + b/x)^(3/2))/3 - 2*a*
d*(a + b/x)^(1/2) - (2*c*x*(a + b/x)^(3/2)*hypergeom([-3/2, -1/2], 1/2, -(a
*x)/b))/((a*x)/b + 1)^(3/2)
```

sympy [A] time = 56.15, size = 163, normalized size = 1.63

$$\sqrt{a}\,bc\,\mathrm{asinh}\left(\frac{\sqrt{a}\,\sqrt{x}}{\sqrt{b}}\right)-\frac{2a^2d\,\mathrm{atan}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{-a}}\right)}{\sqrt{-a}}+a\sqrt{b}\,c\sqrt{x}\,\sqrt{\frac{ax}{b}+1}-\frac{2abc\,\mathrm{atan}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{-a}}\right)}{\sqrt{-a}}-2ad\sqrt{a+\frac{b}{x}}-2bc\sqrt{a+\frac{b}{x}}+bd\left(\begin{cases}-\frac{\sqrt{a}}{x} & \text{for } b=0\\ -\frac{2\left(a+\frac{b}{x}\right)^{\frac{3}{2}}}{3b} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)**(3/2)*(c+d/x),x)
```

```
[Out] sqrt(a)*b*c*asinh(sqrt(a)*sqrt(x)/sqrt(b)) - 2*a**2*d*atan(sqrt(a + b/x)/sq
rt(-a))/sqrt(-a) + a*sqrt(b)*c*sqrt(x)*sqrt(a*x/b + 1) - 2*a*b*c*atan(sqrt(
a + b/x)/sqrt(-a))/sqrt(-a) - 2*a*d*sqrt(a + b/x) - 2*b*c*sqrt(a + b/x) + b
*d*Piecewise((-sqrt(a)/x, Eq(b, 0)), (-2*(a + b/x)**(3/2)/(3*b), True))
```

3.137 $\int \left(a+\frac{b}{x}\right)^{3/2} dx$

Optimal. Leaf size=54

$$x\left(a+\frac{b}{x}\right)^{3/2}-3b\sqrt{a+\frac{b}{x}}+3\sqrt{a}\,b\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)$$

Rubi [A] time = 0.03, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.454, Rules used = {242, 47, 50, 63, 208}

$$x\left(a+\frac{b}{x}\right)^{3/2}-3b\sqrt{a+\frac{b}{x}}+3\sqrt{a}\,b\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x)^(3/2),x]
```

```
[Out] -3*b*Sqrt[a + b/x] + (a + b/x)^(3/2)*x + 3*Sqrt[a]*b*ArcTanh[Sqrt[a + b/x]/
Sqrt[a]]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 242

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a + b/x^n)^p/x^
2, x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\int \left(a+\frac{b}{x}\right)^{3/2} dx &= -\text{Subst}\left(\int \frac{(a+bx)^{3/2}}{x^2} dx, x, \frac{1}{x}\right)\\
&= \left(a+\frac{b}{x}\right)^{3/2} x - \frac{1}{2}(3b)\,\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x} dx, x, \frac{1}{x}\right)\\
&= -3b\sqrt{a+\frac{b}{x}} + \left(a+\frac{b}{x}\right)^{3/2} x - \frac{1}{2}(3ab)\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}} dx, x, \frac{1}{x}\right)\\
&= -3b\sqrt{a+\frac{b}{x}} + \left(a+\frac{b}{x}\right)^{3/2} x - (3a)\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}} dx, x, \sqrt{a+\frac{b}{x}}\right)\\
&= -3b\sqrt{a+\frac{b}{x}} + \left(a+\frac{b}{x}\right)^{3/2} x + 3\sqrt{a}\,b\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)
\end{aligned}
$$

Mathematica [A] time = 0.03, size = 46, normalized size = 0.85

$$
\sqrt{a+\frac{b}{x}}\,(ax-2b) + 3\sqrt{a}\,b\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x)^(3/2),x]
```

```
[Out] Sqrt[a + b/x]*(-2*b + a*x) + 3*Sqrt[a]*b*ArcTanh[Sqrt[a + b/x]/Sqrt[a]]
```

IntegrateAlgebraic [A] time = 0.00, size = 50, normalized size = 0.93

$$
\sqrt{\frac{ax+b}{x}}\,(ax-2b) + 3\sqrt{a}\,b\tanh^{-1}\left(\frac{\sqrt{\frac{ax+b}{x}}}{\sqrt{a}}\right)
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b/x)^(3/2),x]
```

```
[Out] (-2*b + a*x)*Sqrt[(b + a*x)/x] + 3*Sqrt[a]*b*ArcTanh[Sqrt[(b + a*x)/x]/Sqrt
[a]]
```

fricas [A] time = 0.80, size = 100, normalized size = 1.85

$$
\left[\frac{3}{2}\sqrt{a}\,b\log\left(2\,ax+2\,\sqrt{a}\,x\sqrt{\frac{ax+b}{x}}+b\right)+(ax-2\,b)\sqrt{\frac{ax+b}{x}},\,-3\,\sqrt{-a}\,b\arctan\left(\frac{\sqrt{-a}\,\sqrt{\frac{ax+b}{x}}}{a}\right)+(ax-2\,b)\sqrt{\frac{ax+b}{x}}\right]
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(3/2),x, algorithm="fricas")
```

```
[Out] [3/2*sqrt(a)*b*log(2*a*x + 2*sqrt(a)*x*sqrt((a*x + b)/x) + b) + (a*x - 2*b)
*sqrt((a*x + b)/x), -3*sqrt(-a)*b*arctan(sqrt(-a)*sqrt((a*x + b)/x)/a) + (a
*x - 2*b)*sqrt((a*x + b)/x)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(x)]
Warning, choosing root of [1,0,%%%{-2,[1,2,0]%%%}+%%%{-2,[1,0,0]%%%}+%%%{-2
,[0,1,1]%%%},0,%%%{1,[2,4,0]%%%}+%%%{-2,[2,2,0]%%%}+%%%{1,[2,0,0]%%%}+%%%{2
,[1,3,1]%%%}+%%%{-2,[1,1,1]%%%}+%%%{1,[0,2,2]%%%}] at parameters values [86
,-97,-82]Warning, choosing root of [1,0,%%%{-4,[1,0,0]%%%}+%%%{-2,[0,1,1]%%
%},0,%%%{1,[0,2,2]%%%}] at parameters values [82.1195442914,26,-89]Warning,
 choosing root of [1,0,%%%{-4,[1,0,0]%%%}+%%%{-2,[0,1,1]%%%},0,%%%{1,[0,2,2
]%%%}] at parameters values [85.3561567818,-64,-30]Warning, choosing root o
f [1,0,%%%{-2,[1,0,1]%%%}+%%%{-4,[0,1,0]%%%},0,%%%{1,[2,0,2]%%%}] at parame
ters values [42,43.9628838282,-9]Sign error (%%%{-b,0%%%}+%%%{2*sqrt(a)*sqr
t(b),1/2%%%}+%%%{-2*a,1%%%}+%%%{a*sqrt(a)*sqrt(b)/b,3/2%%%}+%%%{-a^2*sqrt(a
)*sqrt(b)/(4*b^2),5/2%%%}+%%%{undef,7/2%%%})Limit: Max order reached or una
ble to make series expansion Error: Bad Argument Value
```

maple [B] time = 0.05, size = 100, normalized size = 1.85

$$\frac{\sqrt{\frac{ax+b}{x}}\,\left(3ab\,x^{2}\ln\left(\frac{2ax+b+2\sqrt{a\,x^{2}+bx}\,\sqrt{a}}{2\sqrt{a}}\right)+6\sqrt{a\,x^{2}+bx}\,a^{\frac{3}{2}}x^{2}-4\left(a\,x^{2}+bx\right)^{\frac{3}{2}}\sqrt{a}\right)}{2\sqrt{\left(ax+b\right)x}\,\sqrt{a}\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x)^(3/2),x)
```

```
[Out] 1/2*((a*x+b)/x)^(1/2)*(6*a^(3/2)*(a*x^2+b*x)^(1/2)*x^2+3*ln(1/2*(2*a*x+b+2*
(a*x^2+b*x)^(1/2)*a^(1/2))/a^(1/2))*x^2*a*b-4*(a*x^2+b*x)^(3/2)*a^(1/2))/x/
((a*x+b)*x)^(1/2)/a^(1/2)
```

maxima [A] time = 1.20, size = 63, normalized size = 1.17

$$\sqrt{a+\frac{b}{x}}\,ax-\frac{3}{2}\,\sqrt{a}\,b\log\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{a+\frac{b}{x}}+\sqrt{a}}\right)-2\,\sqrt{a+\frac{b}{x}}\,b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(3/2),x, algorithm="maxima")
```

```
[Out] sqrt(a + b/x)*a*x - 3/2*sqrt(a)*b*log((sqrt(a + b/x) - sqrt(a))/(sqrt(a + b
/x) + sqrt(a))) - 2*sqrt(a + b/x)*b
```

mupad [B] time = 1.50, size = 34, normalized size = 0.63

$$-\frac{2\,x\,{\left(a+\frac{b}{x}\right)}^{3/2}\,{}_2\mathrm{F}_1\left(-\frac{3}{2},-\frac{1}{2};\ \frac{1}{2};\ -\frac{a\,x}{b}\right)}{{\left(\frac{a\,x}{b}+1\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x)^(3/2),x)
```

```
[Out] -(2*x*(a + b/x)^(3/2)*hypergeom([-3/2, -1/2], 1/2, -(a*x)/b))/((a*x)/b + 1)
^(3/2)
```

sympy [B] time = 2.69, size = 92, normalized size = 1.70

$$3\sqrt{a}\,b\,\mathrm{asinh}\left(\frac{\sqrt{a}\,\sqrt{x}}{\sqrt{b}}\right)+\frac{a^2x^{\frac{3}{2}}}{\sqrt{b}\,\sqrt{\frac{ax}{b}+1}}-\frac{a\sqrt{b}\,\sqrt{x}}{\sqrt{\frac{ax}{b}+1}}-\frac{2b^{\frac{3}{2}}}{\sqrt{x}\,\sqrt{\frac{ax}{b}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)**(3/2),x)
```

```
[Out] 3*sqrt(a)*b*asinh(sqrt(a)*sqrt(x)/sqrt(b)) + a**2*x**(3/2)/(sqrt(b)*sqrt(a*
x/b + 1)) - a*sqrt(b)*sqrt(x)/sqrt(a*x/b + 1) - 2*b**(3/2)/(sqrt(x)*sqrt(a*
x/b + 1))
```

3.138 $\int \frac{\left(a+\frac{b}{x}\right)^{3/2}}{c+\frac{d}{x}} dx$

Optimal. Leaf size=106

$$-\frac{2(bc-ad)^{3/2}\tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c^2\sqrt{d}}+\frac{\sqrt{a}\,(3bc-2ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{c^2}+\frac{ax\sqrt{a+\frac{b}{x}}}{c}$$

Rubi [A] time = 0.13, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {375, 98, 156, 63, 208, 205}

$$-\frac{2(bc-ad)^{3/2}\tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c^2\sqrt{d}}+\frac{\sqrt{a}\,(3bc-2ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{c^2}+\frac{ax\sqrt{a+\frac{b}{x}}}{c}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x)^(3/2)/(c + d/x),x]
```

```
[Out] (a*Sqrt[a + b/x]*x)/c - (2*(b*c - a*d)^(3/2)*ArcTan[(Sqrt[d]*Sqrt[a + b/x])
/Sqrt[b*c - a*d]])/(c^2*Sqrt[d]) + (Sqrt[a]*(3*b*c - 2*a*d)*ArcTanh[Sqrt[a
+ b/x]/Sqrt[a]])/c^2
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 375

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[((a + b/x^n)^p*(c + d/x^n)^q)/x^2, x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rubi steps

$$\int \frac{\left(a+\frac{b}{x}\right)^{3/2}}{c+\frac{d}{x}}\,dx = -\text{Subst}\left(\int \frac{(a+bx)^{3/2}}{x^2(c+dx)}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{a\sqrt{a+\frac{b}{x}}\,x}{c} + \frac{\text{Subst}\left(\int \frac{-\frac{1}{2}a(3bc-2ad)-\frac{1}{2}b(2bc-ad)x}{x\sqrt{a+bx}\,(c+dx)}\,dx, x, \frac{1}{x}\right)}{c}$$

$$= \frac{a\sqrt{a+\frac{b}{x}}\,x}{c} - \frac{(a(3bc-2ad))\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \frac{1}{x}\right)}{2c^2} - \frac{(bc-ad)^2\,\text{Subst}\left(\int \frac{1}{\sqrt{a+bx}\,(c+dx)}\,dx, x,\right.}{c^2}$$

$$= \frac{a\sqrt{a+\frac{b}{x}}\,x}{c} - \frac{(a(3bc-2ad))\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+\frac{b}{x}}\right)}{bc^2} - \frac{\left(2(bc-ad)^2\right)\text{Subst}\left(\int \frac{1}{c-\frac{ad}{b}+\frac{dx^2}{b}}\right.}{bc^2}$$

$$= \frac{a\sqrt{a+\frac{b}{x}}\,x}{c} - \frac{2(bc-ad)^{3/2}\tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c^2\sqrt{d}} + \frac{\sqrt{a}\,(3bc-2ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{c^2}$$

Mathematica [A] time = 0.24, size = 102, normalized size = 0.96

$$\frac{-\frac{2(bc-ad)^{3/2}\tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{\sqrt{d}} + \sqrt{a}\,(3bc-2ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right) + acx\sqrt{a+\frac{b}{x}}}{c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x)^(3/2)/(c + d/x),x]
```

```
[Out] (a*c*Sqrt[a + b/x]*x - (2*(b*c - a*d)^(3/2)*ArcTan[(Sqrt[d]*Sqrt[a + b/x])/
Sqrt[b*c - a*d]])/Sqrt[d] + Sqrt[a]*(3*b*c - 2*a*d)*ArcTanh[Sqrt[a + b/x]/S
qrt[a]])/c^2
```

IntegrateAlgebraic [A] time = 0.28, size = 116, normalized size = 1.09

$$\frac{\left(3\sqrt{a}\,bc-2a^{3/2}d\right)\tanh^{-1}\left(\frac{\sqrt{\frac{ax+b}{x}}}{\sqrt{a}}\right)}{c^2} - \frac{2(bc-ad)^{3/2}\tan^{-1}\left(\frac{\sqrt{d}\sqrt{\frac{ax+b}{x}}}{\sqrt{bc-ad}}\right)}{c^2\sqrt{d}} + \frac{ax\sqrt{\frac{ax+b}{x}}}{c}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b/x)^(3/2)/(c + d/x),x]
```

```
[Out] (a*x*Sqrt[(b + a*x)/x])/c - (2*(b*c - a*d)^(3/2)*ArcTan[(Sqrt[d]*Sqrt[(b +
a*x)/x])/Sqrt[b*c - a*d]])/(c^2*Sqrt[d]) + ((3*Sqrt[a]*b*c - 2*a^(3/2)*d)*A
rcTanh[Sqrt[(b + a*x)/x]/Sqrt[a]])/c^2
```

fricas [A] time = 0.90, size = 519, normalized size = 4.90

$$\left[\frac{2\,acx\sqrt{\frac{ax+b}{x}}-(3\,bc-2\,ad)\sqrt{a}\log\left(2\,ax-2\,\sqrt{a}x\sqrt{\frac{ax+b}{x}}+b\right)-2\,(bc-ad)\sqrt{-\frac{bc-ad}{d}}\log\left(\frac{2\,dx\sqrt{-\frac{bc-ad}{d}}\sqrt{\frac{ax+b}{x}}+bd-(bc-2\,ad)x}{cx+d}\right)}{2\,c^2},\ \frac{acx\sqrt{\frac{ax+b}{x}}-(3\,bc-2\,ad)\sqrt{-a}\arctan\left(\frac{\sqrt{-a}\sqrt{\frac{ax+b}{x}}}{a}\right)-(bc-ad)\sqrt{-\frac{bc-ad}{d}}\log\left(\frac{2\,dx\sqrt{-\frac{bc-ad}{d}}\sqrt{\frac{ax+b}{x}}+bd-(bc-2\,ad)x}{cx+d}\right)}{c^2},\ \frac{2\,acx\sqrt{\frac{ax+b}{x}}+4\,(bc-ad)\sqrt{\frac{bc-ad}{d}}\arctan\left(-\frac{d\sqrt{\frac{bc-ad}{d}}\sqrt{\frac{ax+b}{x}}}{bc-ad}\right)-(3\,bc-2\,ad)\sqrt{a}\log\left(2\,ax-2\,\sqrt{a}x\sqrt{\frac{ax+b}{x}}+b\right)}{2\,c^2},\ \frac{acx\sqrt{\frac{ax+b}{x}}+2\,(bc-ad)\sqrt{\frac{bc-ad}{d}}\arctan\left(-\frac{d\sqrt{\frac{bc-ad}{d}}\sqrt{\frac{ax+b}{x}}}{bc-ad}\right)-(3\,bc-2\,ad)\sqrt{-a}\arctan\left(\frac{\sqrt{-a}\sqrt{\frac{ax+b}{x}}}{a}\right)}{c^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(3/2)/(c+d/x),x, algorithm="fricas")
```

```
[Out] [1/2*(2*a*c*x*sqrt((a*x + b)/x) - (3*b*c - 2*a*d)*sqrt(a)*log(2*a*x - 2*sqr
t(a)*x*sqrt((a*x + b)/x) + b) - 2*(b*c - a*d)*sqrt(-(b*c - a*d)/d)*log((2*d
*x*sqrt(-(b*c - a*d)/d)*sqrt((a*x + b)/x) + b*d - (b*c - 2*a*d)*x)/(c*x + d
)))/c^2, (a*c*x*sqrt((a*x + b)/x) - (3*b*c - 2*a*d)*sqrt(-a)*arctan(sqrt(-a
)*sqrt((a*x + b)/x)/a) - (b*c - a*d)*sqrt(-(b*c - a*d)/d)*log((2*d*x*sqrt(-
(b*c - a*d)/d)*sqrt((a*x + b)/x) + b*d - (b*c - 2*a*d)*x)/(c*x + d)))/c^2,
1/2*(2*a*c*x*sqrt((a*x + b)/x) + 4*(b*c - a*d)*sqrt((b*c - a*d)/d)*arctan(-
d*sqrt((b*c - a*d)/d)*sqrt((a*x + b)/x)/(b*c - a*d)) - (3*b*c - 2*a*d)*sqrt
(a)*log(2*a*x - 2*sqrt(a)*x*sqrt((a*x + b)/x) + b))/c^2, (a*c*x*sqrt((a*x +
 b)/x) + 2*(b*c - a*d)*sqrt((b*c - a*d)/d)*arctan(-d*sqrt((b*c - a*d)/d)*sq
rt((a*x + b)/x)/(b*c - a*d)) - (3*b*c - 2*a*d)*sqrt(-a)*arctan(sqrt(-a)*sqr
t((a*x + b)/x)/a))/c^2]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(3/2)/(c+d/x),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(x)]
Error: Bad Argument Type
```

maple [B] time = 0.06, size = 528, normalized size = 4.98

$$-\frac{\sqrt{\frac{ax+b}{x}}\left(2a^{\frac{5}{2}}d^3\ln\left(\frac{-2adx+bcx-bd+2\sqrt{\frac{(ad-bc)d}{c^2}}\,\sqrt{(ax+b)x}\,c}{cx+d}\right)-4a^{\frac{3}{2}}bc\,d^2\ln\left(\frac{-2adx+bcx-bd+2\sqrt{\frac{(ad-bc)d}{c^2}}\,\sqrt{(ax+b)x}\,c}{cx+d}\right)+2\sqrt{a}\,b^2c^2d\ln\left(\frac{-2adx+bcx-bd+2\sqrt{\frac{(ad-bc)d}{c^2}}\,\sqrt{(ax+b)x}\,c}{cx+d}\right)+2\sqrt{\frac{(ad-bc)d}{c^2}}\,a^2c\,d^2\ln\left(\frac{2ax+b+2\sqrt{(ax+b)x}\,\sqrt{a}}{2\sqrt{a}}\right)-3\sqrt{\frac{(ad-bc)d}{c^2}}\,ab\,c^2d\ln\left(\frac{2ax+b+2\sqrt{(ax+b)x}\,\sqrt{a}}{2\sqrt{a}}\right)+\sqrt{\frac{(ad-bc)d}{c^2}}\,b^2c^3\ln\left(\frac{2ax+b+2\sqrt{(ax+b)x}\,\sqrt{a}}{2\sqrt{a}}\right)-\sqrt{\frac{(ad-bc)d}{c^2}}\,b^2c^3\ln\left(\frac{2ax+b+2\sqrt{a\,x^2+bx}\,\sqrt{a}}{2\sqrt{a}}\right)-2\sqrt{(ax+b)\,x}\,\sqrt{\frac{(ad-bc)d}{c^2}}\,a^{\frac{3}{2}}c^2d+2\sqrt{(ax+b)\,x}\,\sqrt{\frac{(ad-bc)d}{c^2}}\,\sqrt{a}\,b\,c^3-2\sqrt{\frac{(ad-bc)d}{c^2}}\,\sqrt{a\,x^2+bx}\,\sqrt{a}\,b\,c^3\right)x}{2\sqrt{(ax+b)\,x}\,\sqrt{\frac{(ad-bc)d}{c^2}}\,\sqrt{a}\,c^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x)^(3/2)/(c+d/x),x)
```

```
[Out] -1/2*((a*x+b)/x)^(1/2)*x*(2*a^(5/2)*ln((-2*a*d*x+b*c*x-b*d+2*((a*d-b*c)/c^2
*d)^(1/2)*((a*x+b)*x)^(1/2)*c)/(c*x+d))*d^3-2*a^(3/2)*((a*x+b)*x)^(1/2)*((a
*d-b*c)/c^2*d)^(1/2)*c^2*d-4*a^(3/2)*ln((-2*a*d*x+b*c*x-b*d+2*((a*d-b*c)/c^
2*d)^(1/2)*((a*x+b)*x)^(1/2)*c)/(c*x+d))*b*c*d^2+2*a^(1/2)*((a*x+b)*x)^(1/2
)*((a*d-b*c)/c^2*d)^(1/2)*b*c^3-2*a^(1/2)*((a*d-b*c)/c^2*d)^(1/2)*(a*x^2+b*
x)^(1/2)*b*c^3+2*a^(1/2)*ln((-2*a*d*x+b*c*x-b*d+2*((a*d-b*c)/c^2*d)^(1/2)*(
(a*x+b)*x)^(1/2)*c)/(c*x+d))*b^2*c^2*d+2*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2
)*a^(1/2))/a^(1/2))*((a*d-b*c)/c^2*d)^(1/2)*a^2*c*d^2-3*ln(1/2*(2*a*x+b+2*(
(a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))*((a*d-b*c)/c^2*d)^(1/2)*a*b*c^2*d+ln(1/2
*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))*((a*d-b*c)/c^2*d)^(1/2)*b^2
*c^3-((a*d-b*c)/c^2*d)^(1/2)*ln(1/2*(2*a*x+b+2*(a*x^2+b*x)^(1/2)*a^(1/2))/a
^(1/2))*b^2*c^3)/((a*x+b)*x)^(1/2)/d/c^3/a^(1/2)/((a*d-b*c)/c^2*d)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + \frac{b}{x}\right)^{\frac{3}{2}}}{c + \frac{d}{x}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(3/2)/(c+d/x),x, algorithm="maxima")
```

```
[Out] integrate((a + b/x)^(3/2)/(c + d/x), x)
```

mupad [B] time = 1.68, size = 556, normalized size = 5.25

$$\frac{a\,x\,\sqrt{a+\frac{b}{x}}}{c}-\frac{\sqrt{a}\,\mathrm{atanh}\left(\frac{58\,a^{3/2}\,b^6\,d^2\,\sqrt{a+\frac{b}{x}}}{58\,a^2\,b^6\,d^2-24\,a\,b^7\,c\,d-\frac{46\,a^3\,b^5\,d^3}{c}+\frac{12\,a^4\,b^4\,d^4}{c^2}}+\frac{46\,a^{5/2}\,b^5\,d^3\,\sqrt{a+\frac{b}{x}}}{46\,a^3\,b^5\,d^3-58\,a^2\,b^6\,c\,d^2-\frac{12\,a^4\,b^4\,d^4}{c}+24\,a\,b^7\,c^2\,d}+\frac{12\,a^{7/2}\,b^4\,d^4\,\sqrt{a+\frac{b}{x}}}{12\,a^4\,b^4\,d^4-46\,a^3\,b^5\,c\,d^3+58\,a^2\,b^6\,c^2\,d^2-24\,a\,b^7\,c^3\,d}-\frac{24\,\sqrt{a}\,b^7\,c\,d\,\sqrt{a+\frac{b}{x}}}{58\,a^2\,b^6\,d^2-24\,a\,b^7\,c\,d-\frac{46\,a^3\,b^5\,d^3}{c}+\frac{12\,a^4\,b^4\,d^4}{c^2}}\right)\,\left(2\,a\,d-3\,b\,c\right)}{c^2}+\frac{2\,\mathrm{atanh}\left(\frac{12\,a^2\,b^4\,d^2\,\sqrt{a+\frac{b}{x}}\,\sqrt{a^3\,d^4-3\,a^2\,b\,c\,d^3+3\,a\,b^2\,c^2\,d^2-b^3\,c^3\,d}}{12\,a^4\,b^4\,d^4-40\,a^3\,b^5\,c\,d^3+44\,a^2\,b^6\,c^2\,d^2-16\,a\,b^7\,c^3\,d}+\frac{16\,a\,b^5\,d\,\sqrt{a+\frac{b}{x}}\,\sqrt{a^3\,d^4-3\,a^2\,b\,c\,d^3+3\,a\,b^2\,c^2\,d^2-b^3\,c^3\,d}}{40\,a^3\,b^5\,d^3-44\,a^2\,b^6\,c\,d^2-\frac{12\,a^4\,b^4\,d^4}{c}+16\,a\,b^7\,c^2\,d}\right)\,\sqrt{d\,{\left(a\,d-b\,c\right)}^3}}{c^2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x)^(3/2)/(c + d/x),x)
```

```
[Out] (a*x*(a + b/x)^(1/2))/c - (a^(1/2)*atanh((58*a^(3/2)*b^6*d^2*(a + b/x)^(1/2
))/(58*a^2*b^6*d^2 - 24*a*b^7*c*d - (46*a^3*b^5*d^3)/c + (12*a^4*b^4*d^4)/c
^2) + (46*a^(5/2)*b^5*d^3*(a + b/x)^(1/2))/(46*a^3*b^5*d^3 - 58*a^2*b^6*c*d
^2 - (12*a^4*b^4*d^4)/c + 24*a*b^7*c^2*d) + (12*a^(7/2)*b^4*d^4*(a + b/x)^(
1/2))/(12*a^4*b^4*d^4 - 46*a^3*b^5*c*d^3 + 58*a^2*b^6*c^2*d^2 - 24*a*b^7*c^
3*d) - (24*a^(1/2)*b^7*c*d*(a + b/x)^(1/2))/(58*a^2*b^6*d^2 - 24*a*b^7*c*d
- (46*a^3*b^5*d^3)/c + (12*a^4*b^4*d^4)/c^2))*(2*a*d - 3*b*c))/c^2 + (2*ata
nh((12*a^2*b^4*d^2*(a + b/x)^(1/2)*(a^3*d^4 - b^3*c^3*d + 3*a*b^2*c^2*d^2 -
 3*a^2*b*c*d^3)^(1/2))/(12*a^4*b^4*d^4 - 40*a^3*b^5*c*d^3 + 44*a^2*b^6*c^2*
d^2 - 16*a*b^7*c^3*d) + (16*a*b^5*d*(a + b/x)^(1/2)*(a^3*d^4 - b^3*c^3*d +
3*a*b^2*c^2*d^2 - 3*a^2*b*c*d^3)^(1/2))/(40*a^3*b^5*d^3 - 44*a^2*b^6*c*d^2
- (12*a^4*b^4*d^4)/c + 16*a*b^7*c^2*d))*(d*(a*d - b*c)^3)^(1/2))/(c^2*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\left(a+\frac{b}{x}\right)^{\frac{3}{2}}}{cx+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)**(3/2)/(c+d/x),x)
```

```
[Out] Integral(x*(a + b/x)**(3/2)/(c*x + d), x)
```

3.139 $\int \frac{\left(a+\frac{b}{x}\right)^{3/2}}{\left(c+\frac{d}{x}\right)^2} \, dx$

Optimal. Leaf size=156

$$-\frac{(bc-4ad)\sqrt{bc-ad}\,\tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c^3\sqrt{d}}+\frac{\sqrt{a}\,(3bc-4ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{c^3}-\frac{\sqrt{a+\frac{b}{x}}\,(bc-2ad)}{c^2\left(c+\frac{d}{x}\right)}+\frac{ax\sqrt{a+\frac{b}{x}}}{c\left(c+\frac{d}{x}\right)}$$

Rubi [A] time = 0.22, antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {375, 98, 151, 156, 63, 208, 205}

$$-\frac{\sqrt{a+\frac{b}{x}}\,(bc-2ad)}{c^2\left(c+\frac{d}{x}\right)}-\frac{(bc-4ad)\sqrt{bc-ad}\,\tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c^3\sqrt{d}}+\frac{\sqrt{a}\,(3bc-4ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{c^3}+\frac{ax\sqrt{a+\frac{b}{x}}}{c\left(c+\frac{d}{x}\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x)^(3/2)/(c + d/x)^2,x]
```

```
[Out] -(((b*c - 2*a*d)*Sqrt[a + b/x])/(c^2*(c + d/x))) + (a*Sqrt[a + b/x]*x)/(c*(
c + d/x)) - ((b*c - 4*a*d)*Sqrt[b*c - a*d]*ArcTan[(Sqrt[d]*Sqrt[a + b/x])/S
qrt[b*c - a*d]])/(c^3*Sqrt[d]) + (Sqrt[a]*(3*b*c - 4*a*d)*ArcTanh[Sqrt[a +
b/x]/Sqrt[a]])/c^3
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 375

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[((a + b/x^n)^p*(c + d/x^n)^q)/x^2, x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rubi steps

$$\int \frac{\left(a+\frac{b}{x}\right)^{3/2}}{\left(c+\frac{d}{x}\right)^2}\,dx = -\text{Subst}\left(\int \frac{(a+bx)^{3/2}}{x^2(c+dx)^2}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{a\sqrt{a+\frac{b}{x}}\,x}{c\left(c+\frac{d}{x}\right)} + \frac{\text{Subst}\left(\int \frac{-\frac{1}{2}a(3bc-4ad)-\frac{1}{2}b(2bc-3ad)x}{x\sqrt{a+bx}\,(c+dx)^2}\,dx, x, \frac{1}{x}\right)}{c}$$

$$= -\frac{(bc-2ad)\sqrt{a+\frac{b}{x}}}{c^2\left(c+\frac{d}{x}\right)} + \frac{a\sqrt{a+\frac{b}{x}}\,x}{c\left(c+\frac{d}{x}\right)} - \frac{\text{Subst}\left(\int \frac{\frac{1}{2}a(3bc-4ad)(bc-ad)+\frac{1}{2}b(bc-2ad)(bc-ad)x}{x\sqrt{a+bx}\,(c+dx)}\,dx, x, \frac{1}{x}\right)}{c^2(bc-ad)}$$

$$= -\frac{(bc-2ad)\sqrt{a+\frac{b}{x}}}{c^2\left(c+\frac{d}{x}\right)} + \frac{a\sqrt{a+\frac{b}{x}}\,x}{c\left(c+\frac{d}{x}\right)} - \frac{(a(3bc-4ad))\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \frac{1}{x}\right)}{2c^3} - \frac{((bc-4ad)(bc}{}$$

$$= -\frac{(bc-2ad)\sqrt{a+\frac{b}{x}}}{c^2\left(c+\frac{d}{x}\right)} + \frac{a\sqrt{a+\frac{b}{x}}\,x}{c\left(c+\frac{d}{x}\right)} - \frac{(a(3bc-4ad))\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+\frac{b}{x}}\right)}{bc^3} - \frac{((bc-4a}{}$$

$$= -\frac{(bc-2ad)\sqrt{a+\frac{b}{x}}}{c^2\left(c+\frac{d}{x}\right)} + \frac{a\sqrt{a+\frac{b}{x}}\,x}{c\left(c+\frac{d}{x}\right)} - \frac{(bc-4ad)\sqrt{bc-ad}\,\tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c^3\sqrt{d}} + \frac{\sqrt{a}\,(3bc-4ad)\,\text{ta}}{c^3}$$

Mathematica [A] time = 0.39, size = 143, normalized size = 0.92

$$\frac{-\frac{\left(4a^2d^2-5abcd+b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{\sqrt{d}\sqrt{bc-ad}}+\frac{cx\sqrt{a+\frac{b}{x}}(acx+2ad-bc)}{cx+d}+\sqrt{a}(3bc-4ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x)^(3/2)/(c + d/x)^2,x]
```

```
[Out] ((c*Sqrt[a + b/x]*x*(-(b*c) + 2*a*d + a*c*x))/(d + c*x) - ((b^2*c^2 - 5*a*b
*c*d + 4*a^2*d^2)*ArcTan[(Sqrt[d]*Sqrt[a + b/x])/Sqrt[b*c - a*d]])/(Sqrt[d]
*Sqrt[b*c - a*d]) + Sqrt[a]*(3*b*c - 4*a*d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])
/c^3
```

IntegrateAlgebraic [A] time = 0.43, size = 160, normalized size = 1.03

$$\frac{\left(3\sqrt{a}\,bc-4a^{3/2}d\right)\tanh^{-1}\left(\frac{\sqrt{\frac{ax+b}{x}}}{\sqrt{a}}\right)}{c^3}+\frac{\left(-4a^2d^2+5abcd-b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{d}\sqrt{\frac{ax+b}{x}}}{\sqrt{bc-ad}}\right)}{c^3\sqrt{d}\sqrt{bc-ad}}+\frac{\sqrt{\frac{ax+b}{x}}\left(acx^2+2adx-bcx\right)}{c^2(cx+d)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b/x)^(3/2)/(c + d/x)^2,x]
```

```
[Out] (Sqrt[(b + a*x)/x]*(-(b*c*x) + 2*a*d*x + a*c*x^2))/(c^2*(d + c*x)) + ((-(b^
2*c^2) + 5*a*b*c*d - 4*a^2*d^2)*ArcTan[(Sqrt[d]*Sqrt[(b + a*x)/x])/Sqrt[b*c
 - a*d]])/(c^3*Sqrt[d]*Sqrt[b*c - a*d]) + ((3*Sqrt[a]*b*c - 4*a^(3/2)*d)*Ar
cTanh[Sqrt[(b + a*x)/x]/Sqrt[a]])/c^3
```

fricas [A] time = 0.93, size = 769, normalized size = 4.93

$$\left[-\frac{\left(3bcd-4ad^2+\left(3bc^2-4acd\right)x\right)\sqrt{a}\log\left(2ax-2\sqrt{a}x\sqrt{\frac{ax+b}{x}}+b\right)+\left(bcd-4ad^2+\left(bc^2-4acd\right)x\right)\sqrt{-\frac{bc-ad}{d}}\log\left(\frac{2dx\sqrt{-\frac{bc-ad}{d}}\sqrt{\frac{ax+b}{x}}+bd-(bc-2ad)x}{cx+d}\right)-2\left(ac^2x^2-\left(bc^2-2acd\right)x\right)\sqrt{\frac{ax+b}{x}}}{2\left(c^4x+c^3d\right)},\ \frac{2\left(bcd-4ad^2+\left(bc^2-4acd\right)x\right)\sqrt{\frac{bc-ad}{d}}\arctan\left(-\frac{d\sqrt{\frac{bc-ad}{d}}\sqrt{\frac{ax+b}{x}}}{bc-ad}\right)-\left(3bcd-4ad^2+\left(3bc^2-4acd\right)x\right)\sqrt{a}\log\left(2ax-2\sqrt{a}x\sqrt{\frac{ax+b}{x}}+b\right)+2\left(ac^2x^2-\left(bc^2-2acd\right)x\right)\sqrt{\frac{ax+b}{x}}}{2\left(c^4x+c^3d\right)},\ -\frac{2\left(3bcd-4ad^2+\left(3bc^2-4acd\right)x\right)\sqrt{-a}\arctan\left(\frac{\sqrt{-a}\sqrt{\frac{ax+b}{x}}}{a}\right)+\left(bcd-4ad^2+\left(bc^2-4acd\right)x\right)\sqrt{-\frac{bc-ad}{d}}\log\left(\frac{2dx\sqrt{-\frac{bc-ad}{d}}\sqrt{\frac{ax+b}{x}}+bd-(bc-2ad)x}{cx+d}\right)-2\left(ac^2x^2-\left(bc^2-2acd\right)x\right)\sqrt{\frac{ax+b}{x}}}{2\left(c^4x+c^3d\right)},\ \frac{\left(bcd-4ad^2+\left(bc^2-4acd\right)x\right)\sqrt{\frac{bc-ad}{d}}\arctan\left(-\frac{d\sqrt{\frac{bc-ad}{d}}\sqrt{\frac{ax+b}{x}}}{bc-ad}\right)-\left(3bcd-4ad^2+\left(3bc^2-4acd\right)x\right)\sqrt{-a}\arctan\left(\frac{\sqrt{-a}\sqrt{\frac{ax+b}{x}}}{a}\right)+\left(ac^2x^2-\left(bc^2-2acd\right)x\right)\sqrt{\frac{ax+b}{x}}}{c^4x+c^3d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(3/2)/(c+d/x)^2,x, algorithm="fricas")
```

```
[Out] [-1/2*((3*b*c*d - 4*a*d^2 + (3*b*c^2 - 4*a*c*d)*x)*sqrt(a)*log(2*a*x - 2*sq
rt(a)*x*sqrt((a*x + b)/x) + b) + (b*c*d - 4*a*d^2 + (b*c^2 - 4*a*c*d)*x)*sq
rt(-(b*c - a*d)/d)*log((2*d*x*sqrt(-(b*c - a*d)/d)*sqrt((a*x + b)/x) + b*d
- (b*c - 2*a*d)*x)/(c*x + d)) - 2*(a*c^2*x^2 - (b*c^2 - 2*a*c*d)*x)*sqrt((a
*x + b)/x))/(c^4*x + c^3*d), 1/2*(2*(b*c*d - 4*a*d^2 + (b*c^2 - 4*a*c*d)*x)
*sqrt((b*c - a*d)/d)*arctan(-d*sqrt((b*c - a*d)/d)*sqrt((a*x + b)/x)/(b*c -
 a*d)) - (3*b*c*d - 4*a*d^2 + (3*b*c^2 - 4*a*c*d)*x)*sqrt(a)*log(2*a*x - 2*
sqrt(a)*x*sqrt((a*x + b)/x) + b) + 2*(a*c^2*x^2 - (b*c^2 - 2*a*c*d)*x)*sqrt
((a*x + b)/x))/(c^4*x + c^3*d), -1/2*(2*(3*b*c*d - 4*a*d^2 + (3*b*c^2 - 4*a
*c*d)*x)*sqrt(-a)*arctan(sqrt(-a)*sqrt((a*x + b)/x)/a) + (b*c*d - 4*a*d^2 +
 (b*c^2 - 4*a*c*d)*x)*sqrt(-(b*c - a*d)/d)*log((2*d*x*sqrt(-(b*c - a*d)/d)*
sqrt((a*x + b)/x) + b*d - (b*c - 2*a*d)*x)/(c*x + d)) - 2*(a*c^2*x^2 - (b*c
^2 - 2*a*c*d)*x)*sqrt((a*x + b)/x))/(c^4*x + c^3*d), ((b*c*d - 4*a*d^2 + (b
*c^2 - 4*a*c*d)*x)*sqrt((b*c - a*d)/d)*arctan(-d*sqrt((b*c - a*d)/d)*sqrt((
a*x + b)/x)/(b*c - a*d)) - (3*b*c*d - 4*a*d^2 + (3*b*c^2 - 4*a*c*d)*x)*sqrt
(-a)*arctan(sqrt(-a)*sqrt((a*x + b)/x)/a) + (a*c^2*x^2 - (b*c^2 - 2*a*c*d)*
x)*sqrt((a*x + b)/x))/(c^4*x + c^3*d)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(3/2)/(c+d/x)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(x)]
Unable to divide, perhaps due to rounding error%%%{%%{[-2,0]:[1,0,%%%{-1,[1
]%%%}]%%},[4,6,4,0]%%%}+%%%{%%%{8,[1]%%%},[3,5,4,1]%%%}+%%%{%%{[-4,0]:[1,0,
%%%{-1,[1]%%%}]%%},[2,5,5,1]%%%}+%%%{%%{[%%%{-8,[1]%%%},0]:[1,0,%%%{-1,[1]%
%%}]%%},[2,4,4,2]%%%}+%%%{%%%{8,[1]%%%},[1,4,5,2]%%%}+%%%{%%{[-2,0]:[1,0,%%
%{-1,[1]%%%}]%%},[0,4,6,2]%%%} / %%%{%%%{1,[1]%%%},[4,2,0,0]%%%}+%%%{%%{pol
y1[%%%{-4,[1]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[3,1,0,1]%%%}+%%%{%%%{2,[1]%%%
},[2,1,1,1]%%%}+%%%{%%%{4,[2]%%%},[2,0,0,2]%%%}+%%%{%%{poly1[%%%{-4,[1]%%%}
,0]:[1,0,%%%{-1,[1]%%%}]%%},[1,0,1,2]%%%}+%%%{%%%{1,[1]%%%},[0,0,2,2]%%%} E
rror: Bad Argument Value
```

maple [B] time = 0.06, size = 834, normalized size = 5.35

$$-\frac{\left(4a^{\frac{7}{2}}c\,d^{3}x\ln\left(\frac{-2adx+bcx-bd+2\sqrt{\frac{(ad-bc)d}{c^2}}\sqrt{(ax+b)x}\,c}{cx+d}\right)-5a^{\frac{5}{2}}b\,c^{2}d^{2}x\ln\left(\frac{-2adx+bcx-bd+2\sqrt{\frac{(ad-bc)d}{c^2}}\sqrt{(ax+b)x}\,c}{cx+d}\right)+a^{\frac{3}{2}}b^{2}c^{3}dx\ln\left(\frac{-2adx+bcx-bd+2\sqrt{\frac{(ad-bc)d}{c^2}}\sqrt{(ax+b)x}\,c}{cx+d}\right)+4\sqrt{\frac{(ad-bc)d}{c^2}}\,a^{3}c^{2}d^{2}x\ln\left(\frac{2ax+b+2\sqrt{(ax+b)x}\sqrt{a}}{2\sqrt{a}}\right)-3\sqrt{\frac{(ad-bc)d}{c^2}}\,a^{2}b\,c^{3}dx\ln\left(\frac{2ax+b+2\sqrt{(ax+b)x}\sqrt{a}}{2\sqrt{a}}\right)+4a^{\frac{7}{2}}d^{4}\ln\left(\frac{-2adx+bcx-bd+2\sqrt{\frac{(ad-bc)d}{c^2}}\sqrt{(ax+b)x}\,c}{cx+d}\right)-5a^{\frac{5}{2}}b\,c\,d^{3}\ln\left(\frac{-2adx+bcx-bd+2\sqrt{\frac{(ad-bc)d}{c^2}}\sqrt{(ax+b)x}\,c}{cx+d}\right)+2\sqrt{(ax+b)x}\,\sqrt{\frac{(ad-bc)d}{c^2}}\,a^{\frac{5}{2}}c^{4}x^{2}+a^{\frac{3}{2}}b^{2}c^{2}d^{2}\ln\left(\frac{-2adx+bcx-bd+2\sqrt{\frac{(ad-bc)d}{c^2}}\sqrt{(ax+b)x}\,c}{cx+d}\right)+4\sqrt{\frac{(ad-bc)d}{c^2}}\,a^{3}c\,d^{3}\ln\left(\frac{2ax+b+2\sqrt{(ax+b)x}\sqrt{a}}{2\sqrt{a}}\right)-3\sqrt{\frac{(ad-bc)d}{c^2}}\,a^{2}b\,c^{2}d^{2}\ln\left(\frac{2ax+b+2\sqrt{(ax+b)x}\sqrt{a}}{2\sqrt{a}}\right)-2\sqrt{(ax+b)x}\,\sqrt{\frac{(ad-bc)d}{c^2}}\,a^{\frac{5}{2}}c^{3}dx+2\sqrt{(ax+b)x}\,\sqrt{\frac{(ad-bc)d}{c^2}}\,a^{\frac{3}{2}}b\,c^{4}x-4\sqrt{(ax+b)x}\,\sqrt{\frac{(ad-bc)d}{c^2}}\,a^{\frac{5}{2}}c^{2}d^{2}+2\sqrt{(ax+b)x}\,\sqrt{\frac{(ad-bc)d}{c^2}}\,a^{\frac{3}{2}}b\,c^{3}d-2\left((ax+b)x\right)^{\frac{3}{2}}\sqrt{\frac{(ad-bc)d}{c^2}}\,a^{\frac{3}{2}}c^{4}\right)\sqrt{\frac{ax+b}{x}}\,x}{2\sqrt{\frac{(ad-bc)d}{c^2}}\,(cx+d)\sqrt{(ax+b)x}\,a^{\frac{3}{2}}c^{4}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x)^(3/2)/(c+d/x)^2,x)
```

```
[Out] -1/2*(4*a^(7/2)*c*d^3*x*ln((-2*a*d*x+b*c*x-b*d+2*((a*d-b*c)/c^2*d)^(1/2)*((
a*x+b)*x)^(1/2)*c)/(c*x+d))+2*((a*x+b)*x)^(1/2)*((a*d-b*c)/c^2*d)^(1/2)*a^(
5/2)*c^4*x^2+4*a^(7/2)*d^4*ln((-2*a*d*x+b*c*x-b*d+2*((a*d-b*c)/c^2*d)^(1/2)
*((a*x+b)*x)^(1/2)*c)/(c*x+d))-2*((a*x+b)*x)^(1/2)*((a*d-b*c)/c^2*d)^(1/2)*
a^(5/2)*c^3*d*x-5*a^(5/2)*b*c^2*d^2*x*ln((-2*a*d*x+b*c*x-b*d+2*((a*d-b*c)/c
^2*d)^(1/2)*((a*x+b)*x)^(1/2)*c)/(c*x+d))-4*((a*x+b)*x)^(1/2)*((a*d-b*c)/c^
2*d)^(1/2)*a^(5/2)*c^2*d^2-5*a^(5/2)*b*c*d^3*ln((-2*a*d*x+b*c*x-b*d+2*((a*d
-b*c)/c^2*d)^(1/2)*((a*x+b)*x)^(1/2)*c)/(c*x+d))-2*((a*x+b)*x)^(3/2)*((a*d-
b*c)/c^2*d)^(1/2)*a^(3/2)*c^4+2*((a*x+b)*x)^(1/2)*((a*d-b*c)/c^2*d)^(1/2)*a
^(3/2)*b*c^4*x+a^(3/2)*b^2*c^3*d*x*ln((-2*a*d*x+b*c*x-b*d+2*((a*d-b*c)/c^2*
d)^(1/2)*((a*x+b)*x)^(1/2)*c)/(c*x+d))+2*((a*x+b)*x)^(1/2)*((a*d-b*c)/c^2*d
)^(1/2)*a^(3/2)*b*c^3*d+a^(3/2)*b^2*c^2*d^2*ln((-2*a*d*x+b*c*x-b*d+2*((a*d-
b*c)/c^2*d)^(1/2)*((a*x+b)*x)^(1/2)*c)/(c*x+d))+4*((a*d-b*c)/c^2*d)^(1/2)*a
^3*c^2*d^2*x*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))-3*((a*d-
b*c)/c^2*d)^(1/2)*a^2*b*c^3*d*x*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2)
)/a^(1/2))+4*((a*d-b*c)/c^2*d)^(1/2)*a^3*c*d^3*ln(1/2*(2*a*x+b+2*((a*x+b)*x
)^(1/2)*a^(1/2))/a^(1/2))-3*((a*d-b*c)/c^2*d)^(1/2)*a^2*b*c^2*d^2*ln(1/2*(2
*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2)))*x*((a*x+b)/x)^(1/2)/c^4/((a*d
-b*c)/c^2*d)^(1/2)/a^(3/2)/(c*x+d)/d/((a*x+b)*x)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + \frac{b}{x}\right)^{\frac{3}{2}}}{\left(c + \frac{d}{x}\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(3/2)/(c+d/x)^2,x, algorithm="maxima")
```

```
[Out] integrate((a + b/x)^(3/2)/(c + d/x)^2, x)
```

mupad [B] time = 2.16, size = 448, normalized size = 2.87

$$\frac{\mathrm{atanh}\left(\frac{8\,a^2\,b^5\,d^2\,\sqrt{a+\frac{b}{x}}\,\sqrt{a\,d^2-b\,c\,d}}{8\,a^3\,b^5\,d^3-10\,a^2\,b^6\,c\,d^2+2\,a\,b^7\,c^2\,d}-\frac{2\,a\,b^6\,d\,\sqrt{a+\frac{b}{x}}\,\sqrt{a\,d^2-b\,c\,d}}{2\,a\,b^7\,c\,d-10\,a^2\,b^6\,d^2+\frac{8\,a^3\,b^5\,d^3}{c}}\right)\,\sqrt{d\,\left(a\,d-b\,c\right)}\,\left(4\,a\,d-b\,c\right)}{c^3\,d}-\frac{\sqrt{a}\,\mathrm{atanh}\left(\frac{6\,\sqrt{a}\,b^7\,d\,\sqrt{a+\frac{b}{x}}}{6\,a\,b^7\,d-\frac{14\,a^2\,b^6\,d^2}{c}+\frac{8\,a^3\,b^5\,d^3}{c^2}}-\frac{14\,a^{3/2}\,b^6\,d^2\,\sqrt{a+\frac{b}{x}}}{6\,a\,b^7\,c\,d-14\,a^2\,b^6\,d^2+\frac{8\,a^3\,b^5\,d^3}{c}}+\frac{8\,a^{5/2}\,b^5\,d^3\,\sqrt{a+\frac{b}{x}}}{8\,a^3\,b^5\,d^3-14\,a^2\,b^6\,c\,d^2+6\,a\,b^7\,c^2\,d}\right)\,\left(4\,a\,d-3\,b\,c\right)}{c^3}-\frac{\frac{2\,\left(a\,b^2\,c-a^2\,b\,d\right)\,\sqrt{a+\frac{b}{x}}}{c^2}+\frac{b\,{\left(a+\frac{b}{x}\right)}^{3/2}\,\left(2\,a\,d-b\,c\right)}{c^2}}{\left(a+\frac{b}{x}\right)\,\left(2\,a\,d-b\,c\right)-d\,{\left(a+\frac{b}{x}\right)}^2-a^2\,d+a\,b\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x)^(3/2)/(c + d/x)^2,x)
```

```
[Out] (atanh((8*a^2*b^5*d^2*(a + b/x)^(1/2)*(a*d^2 - b*c*d)^(1/2))/(8*a^3*b^5*d^3
 - 10*a^2*b^6*c*d^2 + 2*a*b^7*c^2*d) - (2*a*b^6*d*(a + b/x)^(1/2)*(a*d^2 -
b*c*d)^(1/2))/(2*a*b^7*c*d - 10*a^2*b^6*d^2 + (8*a^3*b^5*d^3)/c))*(d*(a*d -
 b*c))^(1/2)*(4*a*d - b*c))/(c^3*d) - (a^(1/2)*atanh((6*a^(1/2)*b^7*d*(a +
b/x)^(1/2))/(6*a*b^7*d - (14*a^2*b^6*d^2)/c + (8*a^3*b^5*d^3)/c^2) - (14*a^
(3/2)*b^6*d^2*(a + b/x)^(1/2))/(6*a*b^7*c*d - 14*a^2*b^6*d^2 + (8*a^3*b^5*d
^3)/c) + (8*a^(5/2)*b^5*d^3*(a + b/x)^(1/2))/(8*a^3*b^5*d^3 - 14*a^2*b^6*c*
d^2 + 6*a*b^7*c^2*d))*(4*a*d - 3*b*c))/c^3 - ((2*(a*b^2*c - a^2*b*d)*(a + b
/x)^(1/2))/c^2 + (b*(a + b/x)^(3/2)*(2*a*d - b*c))/c^2)/((a + b/x)*(2*a*d -
 b*c) - d*(a + b/x)^2 - a^2*d + a*b*c)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)**(3/2)/(c+d/x)**2,x)
```

```
[Out] Timed out
```

3.140 $\int \frac{\left(a+\frac{b}{x}\right)^{3/2}}{\left(c+\frac{d}{x}\right)^3} dx$

Optimal. Leaf size=209

$$-\frac{3\left(8a^2d^2-8abcd+b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{4c^4\sqrt{d}\sqrt{bc-ad}}+\frac{3\sqrt{a}\,(bc-2ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{c^4}-\frac{3\sqrt{a+\frac{b}{x}}\,(bc-4ad)}{4c^3\left(c+\frac{d}{x}\right)}-\frac{\sqrt{a+\frac{b}{x}}\,(bc-}{2c^2\left(c+\frac{d}{x}\right.}$$

Rubi [A] time = 0.34, antiderivative size = 209, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {375, 98, 151, 156, 63, 208, 205}

$$-\frac{3\left(8a^2d^2-8abcd+b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{4c^4\sqrt{d}\sqrt{bc-ad}}-\frac{3\sqrt{a+\frac{b}{x}}\,(bc-4ad)}{4c^3\left(c+\frac{d}{x}\right)}-\frac{\sqrt{a+\frac{b}{x}}\,(bc-3ad)}{2c^2\left(c+\frac{d}{x}\right)^2}+\frac{3\sqrt{a}\,(bc-2ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{c^4}+\frac{ax\sqrt{a+\frac{b}{x}}}{c\left(c+\frac{d}{x}\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x)^(3/2)/(c + d/x)^3,x]
```

```
[Out] -((b*c - 3*a*d)*Sqrt[a + b/x])/(2*c^2*(c + d/x)^2) - (3*(b*c - 4*a*d)*Sqrt[
a + b/x])/(4*c^3*(c + d/x)) + (a*Sqrt[a + b/x]*x)/(c*(c + d/x)^2) - (3*(b^2
*c^2 - 8*a*b*c*d + 8*a^2*d^2)*ArcTan[(Sqrt[d]*Sqrt[a + b/x])/Sqrt[b*c - a*d
]])/(4*c^4*Sqrt[d]*Sqrt[b*c - a*d]) + (3*Sqrt[a]*(b*c - 2*a*d)*ArcTanh[Sqrt
[a + b/x]/Sqrt[a]])/c^4
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 375

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[((a + b/x^n)^p*(c + d/x^n)^q)/x^2, x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rubi steps

$$\int \frac{\left(a+\frac{b}{x}\right)^{3/2}}{\left(c+\frac{d}{x}\right)^3}\,dx = -\text{Subst}\left(\int \frac{(a+bx)^{3/2}}{x^2(c+dx)^3}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{a\sqrt{a+\frac{b}{x}}\,x}{c\left(c+\frac{d}{x}\right)^2} + \frac{\text{Subst}\left(\int \frac{-\frac{3}{2}a(bc-2ad)-\frac{1}{2}b(2bc-5ad)x}{x\sqrt{a+bx}\,(c+dx)^3}\,dx, x, \frac{1}{x}\right)}{c}$$

$$= -\frac{(bc-3ad)\sqrt{a+\frac{b}{x}}}{2c^2\left(c+\frac{d}{x}\right)^2} + \frac{a\sqrt{a+\frac{b}{x}}\,x}{c\left(c+\frac{d}{x}\right)^2} - \frac{\text{Subst}\left(\int \frac{3a(bc-2ad)(bc-ad)+\frac{3}{2}b(bc-3ad)(bc-ad)x}{x\sqrt{a+bx}\,(c+dx)^2}\,dx, x, \frac{1}{x}\right)}{2c^2(bc-ad)}$$

$$= -\frac{(bc-3ad)\sqrt{a+\frac{b}{x}}}{2c^2\left(c+\frac{d}{x}\right)^2} - \frac{3(bc-4ad)\sqrt{a+\frac{b}{x}}}{4c^3\left(c+\frac{d}{x}\right)} + \frac{a\sqrt{a+\frac{b}{x}}\,x}{c\left(c+\frac{d}{x}\right)^2} + \frac{\text{Subst}\left(\int \frac{-3a(bc-2ad)(bc-ad)^2-\frac{3}{4}b(bc}{x\sqrt{a+bx}\,(c+dx}\right.}{2c^3(bc-ad}$$

$$= -\frac{(bc-3ad)\sqrt{a+\frac{b}{x}}}{2c^2\left(c+\frac{d}{x}\right)^2} - \frac{3(bc-4ad)\sqrt{a+\frac{b}{x}}}{4c^3\left(c+\frac{d}{x}\right)} + \frac{a\sqrt{a+\frac{b}{x}}\,x}{c\left(c+\frac{d}{x}\right)^2} - \frac{(3a(bc-2ad))\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\right.}{2c^4}$$

$$= -\frac{(bc-3ad)\sqrt{a+\frac{b}{x}}}{2c^2\left(c+\frac{d}{x}\right)^2} - \frac{3(bc-4ad)\sqrt{a+\frac{b}{x}}}{4c^3\left(c+\frac{d}{x}\right)} + \frac{a\sqrt{a+\frac{b}{x}}\,x}{c\left(c+\frac{d}{x}\right)^2} - \frac{(3a(bc-2ad))\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\right.}{bc^4}$$

$$= -\frac{(bc-3ad)\sqrt{a+\frac{b}{x}}}{2c^2\left(c+\frac{d}{x}\right)^2} - \frac{3(bc-4ad)\sqrt{a+\frac{b}{x}}}{4c^3\left(c+\frac{d}{x}\right)} + \frac{a\sqrt{a+\frac{b}{x}}\,x}{c\left(c+\frac{d}{x}\right)^2} - \frac{3\left(b^2c^2-8abcd+8a^2d^2\right)\tan^{-1}}{4c^4\sqrt{d}\,\sqrt{bc-ad}}$$

Mathematica [A] time = 0.60, size = 168, normalized size = 0.80

$$\frac{-\frac{3\left(8a^2d^2-8abcd+b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{\sqrt{d}\sqrt{bc-ad}}+\frac{cx\sqrt{a+\frac{b}{x}}\left(2a\left(2c^2x^2+9cdx+6d^2\right)-bc(5cx+3d)\right)}{(cx+d)^2}+12\sqrt{a}\left(bc-2ad\right)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{4c^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x)^(3/2)/(c + d/x)^3,x]
```

```
[Out] ((c*Sqrt[a + b/x]*x*(-(b*c*(3*d + 5*c*x)) + 2*a*(6*d^2 + 9*c*d*x + 2*c^2*x^
2)))/(d + c*x)^2 - (3*(b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*ArcTan[(Sqrt[d]*Sqr
t[a + b/x])/Sqrt[b*c - a*d]])/(Sqrt[d]*Sqrt[b*c - a*d]) + 12*Sqrt[a]*(b*c -
 2*a*d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/(4*c^4)
```

IntegrateAlgebraic [A] time = 0.57, size = 189, normalized size = 0.90

$$-\frac{3\left(2a^{3/2}d-\sqrt{a}\,bc\right)\tanh^{-1}\left(\frac{\sqrt{\frac{ax+b}{x}}}{\sqrt{a}}\right)}{c^4}-\frac{3\left(8a^2d^2-8abcd+b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{d}\sqrt{\frac{ax+b}{x}}}{\sqrt{bc-ad}}\right)}{4c^4\sqrt{d}\sqrt{bc-ad}}+\frac{\sqrt{\frac{ax+b}{x}}\left(4ac^2x^3+18acdx^2+12ad^2x-5bc^2x^2-3bcdx\right)}{4c^3(cx+d)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b/x)^(3/2)/(c + d/x)^3,x]
```

```
[Out] (Sqrt[(b + a*x)/x]*(-3*b*c*d*x + 12*a*d^2*x - 5*b*c^2*x^2 + 18*a*c*d*x^2 +
4*a*c^2*x^3))/(4*c^3*(d + c*x)^2) - (3*(b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*Ar
cTan[(Sqrt[d]*Sqrt[(b + a*x)/x])/Sqrt[b*c - a*d]])/(4*c^4*Sqrt[d]*Sqrt[b*c
- a*d]) - (3*(-(Sqrt[a]*b*c) + 2*a^(3/2)*d)*ArcTanh[Sqrt[(b + a*x)/x]/Sqrt[
a]])/c^4
```

fricas [B] time = 1.06, size = 1765, normalized size = 8.44

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(3/2)/(c+d/x)^3,x, algorithm="fricas")
```

```
[Out] [-1/8*(12*(b^2*c^2*d^3 - 3*a*b*c*d^4 + 2*a^2*d^5 + (b^2*c^4*d - 3*a*b*c^3*d
^2 + 2*a^2*c^2*d^3)*x^2 + 2*(b^2*c^3*d^2 - 3*a*b*c^2*d^3 + 2*a^2*c*d^4)*x)*
sqrt(a)*log(2*a*x - 2*sqrt(a)*x*sqrt((a*x + b)/x) + b) + 3*(b^2*c^2*d^2 - 8
*a*b*c*d^3 + 8*a^2*d^4 + (b^2*c^4 - 8*a*b*c^3*d + 8*a^2*c^2*d^2)*x^2 + 2*(b
^2*c^3*d - 8*a*b*c^2*d^2 + 8*a^2*c*d^3)*x)*sqrt(-b*c*d + a*d^2)*log((b*d -
(b*c - 2*a*d)*x + 2*sqrt(-b*c*d + a*d^2)*x*sqrt((a*x + b)/x))/(c*x + d)) -
2*(4*(a*b*c^4*d - a^2*c^3*d^2)*x^3 - (5*b^2*c^4*d - 23*a*b*c^3*d^2 + 18*a^2
*c^2*d^3)*x^2 - 3*(b^2*c^3*d^2 - 5*a*b*c^2*d^3 + 4*a^2*c*d^4)*x)*sqrt((a*x
+ b)/x))/(b*c^5*d^3 - a*c^4*d^4 + (b*c^7*d - a*c^6*d^2)*x^2 + 2*(b*c^6*d^2
- a*c^5*d^3)*x), -1/8*(24*(b^2*c^2*d^3 - 3*a*b*c*d^4 + 2*a^2*d^5 + (b^2*c^4
*d - 3*a*b*c^3*d^2 + 2*a^2*c^2*d^3)*x^2 + 2*(b^2*c^3*d^2 - 3*a*b*c^2*d^3 +
2*a^2*c*d^4)*x)*sqrt(-a)*arctan(sqrt(-a)*sqrt((a*x + b)/x)/a) + 3*(b^2*c^2*
d^2 - 8*a*b*c*d^3 + 8*a^2*d^4 + (b^2*c^4 - 8*a*b*c^3*d + 8*a^2*c^2*d^2)*x^2
 + 2*(b^2*c^3*d - 8*a*b*c^2*d^2 + 8*a^2*c*d^3)*x)*sqrt(-b*c*d + a*d^2)*log(
(b*d - (b*c - 2*a*d)*x + 2*sqrt(-b*c*d + a*d^2)*x*sqrt((a*x + b)/x))/(c*x +
 d)) - 2*(4*(a*b*c^4*d - a^2*c^3*d^2)*x^3 - (5*b^2*c^4*d - 23*a*b*c^3*d^2 +
 18*a^2*c^2*d^3)*x^2 - 3*(b^2*c^3*d^2 - 5*a*b*c^2*d^3 + 4*a^2*c*d^4)*x)*sqr
t((a*x + b)/x))/(b*c^5*d^3 - a*c^4*d^4 + (b*c^7*d - a*c^6*d^2)*x^2 + 2*(b*c
^6*d^2 - a*c^5*d^3)*x), 1/4*(3*(b^2*c^2*d^2 - 8*a*b*c*d^3 + 8*a^2*d^4 + (b^
2*c^4 - 8*a*b*c^3*d + 8*a^2*c^2*d^2)*x^2 + 2*(b^2*c^3*d - 8*a*b*c^2*d^2 + 8
*a^2*c*d^3)*x)*sqrt(b*c*d - a*d^2)*arctan(sqrt(b*c*d - a*d^2)*x*sqrt((a*x +
 b)/x)/(a*d*x + b*d)) - 6*(b^2*c^2*d^3 - 3*a*b*c*d^4 + 2*a^2*d^5 + (b^2*c^4
*d - 3*a*b*c^3*d^2 + 2*a^2*c^2*d^3)*x^2 + 2*(b^2*c^3*d^2 - 3*a*b*c^2*d^3 +
```

```
2*a^2*c*d^4)*x)*sqrt(a)*log(2*a*x - 2*sqrt(a)*x*sqrt((a*x + b)/x) + b) + (4
*(a*b*c^4*d - a^2*c^3*d^2)*x^3 - (5*b^2*c^4*d - 23*a*b*c^3*d^2 + 18*a^2*c^2
*d^3)*x^2 - 3*(b^2*c^3*d^2 - 5*a*b*c^2*d^3 + 4*a^2*c*d^4)*x)*sqrt((a*x + b)
/x))/(b*c^5*d^3 - a*c^4*d^4 + (b*c^7*d - a*c^6*d^2)*x^2 + 2*(b*c^6*d^2 - a*
c^5*d^3)*x), 1/4*(3*(b^2*c^2*d^2 - 8*a*b*c*d^3 + 8*a^2*d^4 + (b^2*c^4 - 8*a
*b*c^3*d + 8*a^2*c^2*d^2)*x^2 + 2*(b^2*c^3*d - 8*a*b*c^2*d^2 + 8*a^2*c*d^3)
*x)*sqrt(b*c*d - a*d^2)*arctan(sqrt(b*c*d - a*d^2)*x*sqrt((a*x + b)/x)/(a*d
*x + b*d)) - 12*(b^2*c^2*d^3 - 3*a*b*c*d^4 + 2*a^2*d^5 + (b^2*c^4*d - 3*a*b
*c^3*d^2 + 2*a^2*c^2*d^3)*x^2 + 2*(b^2*c^3*d^2 - 3*a*b*c^2*d^3 + 2*a^2*c*d^
4)*x)*sqrt(-a)*arctan(sqrt(-a)*sqrt((a*x + b)/x)/a) + (4*(a*b*c^4*d - a^2*c
^3*d^2)*x^3 - (5*b^2*c^4*d - 23*a*b*c^3*d^2 + 18*a^2*c^2*d^3)*x^2 - 3*(b^2*
c^3*d^2 - 5*a*b*c^2*d^3 + 4*a^2*c*d^4)*x)*sqrt((a*x + b)/x))/(b*c^5*d^3 - a
*c^4*d^4 + (b*c^7*d - a*c^6*d^2)*x^2 + 2*(b*c^6*d^2 - a*c^5*d^3)*x)]
```

giac [B] time = 0.61, size = 727, normalized size = 3.48

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(3/2)/(c+d/x)^3,x, algorithm="giac")
```

```
[Out] sqrt(a*x^2 + b*x)*a*sgn(x)/c^3 + 3/4*(b^2*c^2*sgn(x) - 8*a*b*c*d*sgn(x) + 8
*a^2*d^2*sgn(x))*arctan(-((sqrt(a)*x - sqrt(a*x^2 + b*x))*c + sqrt(a)*d)/sq
rt(b*c*d - a*d^2))/(sqrt(b*c*d - a*d^2)*c^4) - 3/2*(a*b*c*sgn(x) - 2*a^2*d*
sgn(x))*log(abs(2*(sqrt(a)*x - sqrt(a*x^2 + b*x))*sqrt(a) + b))/(sqrt(a)*c^
4) + 1/4*(3*sqrt(a)*b^2*c^2*arctan(sqrt(a)*d/sqrt(b*c*d - a*d^2)) - 24*a^(3
/2)*b*c*d*arctan(sqrt(a)*d/sqrt(b*c*d - a*d^2)) + 24*a^(5/2)*d^2*arctan(sqr
t(a)*d/sqrt(b*c*d - a*d^2)) + 6*sqrt(b*c*d - a*d^2)*a*b*c*log(abs(b)) - 12*
sqrt(b*c*d - a*d^2)*a^2*d*log(abs(b)) + 5*sqrt(b*c*d - a*d^2)*a*b*c - 10*sq
rt(b*c*d - a*d^2)*a^2*d)*sgn(x)/(sqrt(b*c*d - a*d^2)*sqrt(a)*c^4) + 1/4*(5*
(sqrt(a)*x - sqrt(a*x^2 + b*x))^3*sqrt(a)*b^2*c^3*sgn(x) - 24*(sqrt(a)*x -
sqrt(a*x^2 + b*x))^3*a^(3/2)*b*c^2*d*sgn(x) + 24*(sqrt(a)*x - sqrt(a*x^2 +
b*x))^3*a^(5/2)*c*d^2*sgn(x) - (sqrt(a)*x - sqrt(a*x^2 + b*x))^2*a*b^2*c^2*
d*sgn(x) - 24*(sqrt(a)*x - sqrt(a*x^2 + b*x))^2*a^2*b*c*d^2*sgn(x) + 40*(sq
rt(a)*x - sqrt(a*x^2 + b*x))^2*a^3*d^3*sgn(x) + 3*(sqrt(a)*x - sqrt(a*x^2 +
 b*x))*sqrt(a)*b^3*c^2*d*sgn(x) - 28*(sqrt(a)*x - sqrt(a*x^2 + b*x))*a^(3/2
)*b^2*c*d^2*sgn(x) + 40*(sqrt(a)*x - sqrt(a*x^2 + b*x))*a^(5/2)*b*d^3*sgn(x
) - 5*a*b^3*c*d^2*sgn(x) + 10*a^2*b^2*d^3*sgn(x))/(((sqrt(a)*x - sqrt(a*x^2
 + b*x))^2*c + 2*(sqrt(a)*x - sqrt(a*x^2 + b*x))*sqrt(a)*d + b*d)^2*sqrt(a)
*c^4)
```

maple [B] time = 0.07, size = 1817, normalized size = 8.69

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x)^(3/2)/(c+d/x)^3,x)
```

```
[Out] -1/8*(27*a^(5/2)*b^2*c^4*d^2*x^2*ln((-2*a*d*x+b*c*x-b*d+2*((a*d-b*c)/c^2*d)
^(1/2)*((a*x+b)*x)^(1/2)*c)/(c*x+d))+24*((a*d-b*c)/c^2*d)^(1/2)*a^4*c^3*d^3
*x^2*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))-36*((a*x+b)*x)^(
1/2)*((a*d-b*c)/c^2*d)^(1/2)*a^(7/2)*c^3*d^3*x-96*a^(7/2)*b*c^2*d^4*x*ln((-
2*a*d*x+b*c*x-b*d+2*((a*d-b*c)/c^2*d)^(1/2)*((a*x+b)*x)^(1/2)*c)/(c*x+d))+5
4*a^(5/2)*b^2*c^3*d^3*x*ln((-2*a*d*x+b*c*x-b*d+2*((a*d-b*c)/c^2*d)^(1/2)*((
a*x+b)*x)^(1/2)*c)/(c*x+d))+48*((a*d-b*c)/c^2*d)^(1/2)*a^4*c^2*d^4*x*ln(1/2
*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))+30*((a*x+b)*x)^(1/2)*((a*d-
b*c)/c^2*d)^(1/2)*a^(5/2)*b*c^3*d^3-36*((a*d-b*c)/c^2*d)^(1/2)*a^3*b*c^2*d^
4*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))+12*((a*d-b*c)/c^2*d
)^(1/2)*a^2*b^2*c^3*d^3*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2
))-6*((a*x+b)*x)^(1/2)*((a*d-b*c)/c^2*d)^(1/2)*a^(3/2)*b^2*c^4*d^2+6*((a*x+
```

```
b)*x)^(3/2)*((a*d-b*c)/c^2*d)^(1/2)*a^(3/2)*b*c^6*x-6*((a*x+b)*x)^(1/2)*((a
*d-b*c)/c^2*d)^(1/2)*a^(3/2)*b^2*c^6*x^2-3*a^(3/2)*b^3*c^5*d*x^2*ln((-2*a*d
*x+b*c*x-b*d+2*((a*d-b*c)/c^2*d)^(1/2)*((a*x+b)*x)^(1/2)*c)/(c*x+d))+2*((a*
x+b)*x)^(3/2)*((a*d-b*c)/c^2*d)^(1/2)*a^(3/2)*b*c^5*d+12*((a*x+b)*x)^(1/2)*
((a*d-b*c)/c^2*d)^(1/2)*a^(7/2)*c^5*d*x^3-6*((a*x+b)*x)^(1/2)*((a*d-b*c)/c^
2*d)^(1/2)*a^(5/2)*b*c^6*x^3-12*((a*x+b)*x)^(3/2)*((a*d-b*c)/c^2*d)^(1/2)*a
^(5/2)*c^5*d*x-48*a^(7/2)*b*c^3*d^3*x^2*ln((-2*a*d*x+b*c*x-b*d+2*((a*d-b*c)
/c^2*d)^(1/2)*((a*x+b)*x)^(1/2)*c)/(c*x+d))-6*a^(3/2)*b^3*c^4*d^2*x*ln((-2*
a*d*x+b*c*x-b*d+2*((a*d-b*c)/c^2*d)^(1/2)*((a*x+b)*x)^(1/2)*c)/(c*x+d))+24*
a^(9/2)*d^6*ln((-2*a*d*x+b*c*x-b*d+2*((a*d-b*c)/c^2*d)^(1/2)*((a*x+b)*x)^(1
/2)*c)/(c*x+d))+24*((a*d-b*c)/c^2*d)^(1/2)*a^2*b^2*c^4*d^2*x*ln(1/2*(2*a*x+
b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))+54*((a*x+b)*x)^(1/2)*((a*d-b*c)/c^2
*d)^(1/2)*a^(5/2)*b*c^4*d^2*x-72*((a*d-b*c)/c^2*d)^(1/2)*a^3*b*c^3*d^3*x*ln
(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))+18*((a*x+b)*x)^(1/2)*((
a*d-b*c)/c^2*d)^(1/2)*a^(5/2)*b*c^5*d*x^2-36*((a*d-b*c)/c^2*d)^(1/2)*a^3*b*
c^4*d^2*x^2*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))+12*((a*d-
b*c)/c^2*d)^(1/2)*a^2*b^2*c^5*d*x^2*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(
1/2))/a^(1/2))-12*((a*x+b)*x)^(1/2)*((a*d-b*c)/c^2*d)^(1/2)*a^(3/2)*b^2*c^5
*d*x+48*a^(9/2)*c*d^5*x*ln((-2*a*d*x+b*c*x-b*d+2*((a*d-b*c)/c^2*d)^(1/2)*((
a*x+b)*x)^(1/2)*c)/(c*x+d))-24*((a*x+b)*x)^(1/2)*((a*d-b*c)/c^2*d)^(1/2)*a^
(7/2)*c^2*d^4-48*a^(7/2)*b*c*d^5*ln((-2*a*d*x+b*c*x-b*d+2*((a*d-b*c)/c^2*d)
^(1/2)*((a*x+b)*x)^(1/2)*c)/(c*x+d))+27*a^(5/2)*b^2*c^2*d^4*ln((-2*a*d*x+b*
c*x-b*d+2*((a*d-b*c)/c^2*d)^(1/2)*((a*x+b)*x)^(1/2)*c)/(c*x+d))+24*((a*d-b*
c)/c^2*d)^(1/2)*a^4*c*d^5*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1
/2))+24*a^(9/2)*c^2*d^4*x^2*ln((-2*a*d*x+b*c*x-b*d+2*((a*d-b*c)/c^2*d)^(1/2
)*((a*x+b)*x)^(1/2)*c)/(c*x+d))-8*((a*x+b)*x)^(3/2)*((a*d-b*c)/c^2*d)^(1/2)
*a^(5/2)*c^4*d^2-3*a^(3/2)*b^3*c^3*d^3*ln((-2*a*d*x+b*c*x-b*d+2*((a*d-b*c)/
c^2*d)^(1/2)*((a*x+b)*x)^(1/2)*c)/(c*x+d)))*x*((a*x+b)/x)^(1/2)/c^5/d/((a*d
-b*c)/c^2*d)^(1/2)/a^(3/2)/(c*x+d)^2/(a*d-b*c)/((a*x+b)*x)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+\frac{b}{x}\right)^{\frac{3}{2}}}{\left(c+\frac{d}{x}\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(3/2)/(c+d/x)^3,x, algorithm="maxima")
```

```
[Out] integrate((a + b/x)^(3/2)/(c + d/x)^3, x)
```

mupad [B] time = 3.47, size = 1664, normalized size = 7.96

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x)^(3/2)/(c + d/x)^3,x)
```

```
[Out] - ((3*(a + b/x)^(1/2)*(3*a*b^3*c^2 + 4*a^3*b*d^2 - 7*a^2*b^2*c*d))/(4*c^3)
- ((a + b/x)^(3/2)*(5*b^3*c^2 + 24*a^2*b*d^2 - 24*a*b^2*c*d))/(4*c^3) + (3*
b*(a + b/x)^(5/2)*(4*a*d^2 - b*c*d))/(4*c^3))/((a + b/x)^2*(3*a*d^2 - 2*b*c
*d) - (a + b/x)*(3*a^2*d^2 + b^2*c^2 - 4*a*b*c*d) - d^2*(a + b/x)^3 + a^3*d
^2 + a*b^2*c^2 - 2*a^2*b*c*d) - (3*a^(1/2)*atanh((27*a^(1/2)*b^7*d*(a + b/x
)^(1/2))/(8*((27*a*b^7*d)/8 - (27*a^2*b^6*d^2)/(4*c))) + (27*a^(3/2)*b^6*d^
2*(a + b/x)^(1/2))/(4*((27*a^2*b^6*d^2)/4 - (27*a*b^7*c*d)/8)))*(2*a*d - b*
c))/c^4 - (atan((((((a + b/x)^(1/2)*(9*b^6*c^4*d + 1152*a^4*b^2*d^5 - 144*a
*b^5*c^3*d^2 - 1728*a^3*b^3*c*d^4 + 864*a^2*b^4*c^2*d^3))/(8*c^6) - (3*(d*(
a*d - b*c))^(1/2)*((9*a*b^4*c^9*d^2 - 12*a^2*b^3*c^8*d^3)/c^9 - (3*(64*b^3*
c^9*d^2 - 128*a*b^2*c^8*d^3)*(a + b/x)^(1/2)*(d*(a*d - b*c))^(1/2)*(8*a^2*d
```

```
^2 + b^2*c^2 - 8*a*b*c*d))/(64*c^6*(a*c^4*d^2 - b*c^5*d)))*(8*a^2*d^2 + b^2
*c^2 - 8*a*b*c*d))/(8*(a*c^4*d^2 - b*c^5*d)))*(d*(a*d - b*c))^(1/2)*(8*a^2*
d^2 + b^2*c^2 - 8*a*b*c*d)*3i)/(8*(a*c^4*d^2 - b*c^5*d)) + ((((a + b/x)^(1/
2)*(9*b^6*c^4*d + 1152*a^4*b^2*d^5 - 144*a*b^5*c^3*d^2 - 1728*a^3*b^3*c*d^4
 + 864*a^2*b^4*c^2*d^3))/(8*c^6) + (3*(d*(a*d - b*c))^(1/2)*((9*a*b^4*c^9*d
^2 - 12*a^2*b^3*c^8*d^3)/c^9 + (3*(64*b^3*c^9*d^2 - 128*a*b^2*c^8*d^3)*(a +
 b/x)^(1/2)*(d*(a*d - b*c))^(1/2)*(8*a^2*d^2 + b^2*c^2 - 8*a*b*c*d))/(64*c^
6*(a*c^4*d^2 - b*c^5*d)))*(8*a^2*d^2 + b^2*c^2 - 8*a*b*c*d))/(8*(a*c^4*d^2
- b*c^5*d)))*(d*(a*d - b*c))^(1/2)*(8*a^2*d^2 + b^2*c^2 - 8*a*b*c*d)*3i)/(8
*(a*c^4*d^2 - b*c^5*d)))/((216*a^5*b^3*d^5 - 378*a^4*b^4*c*d^4 - (189*a^2*b
^6*c^3*d^2)/4 + 216*a^3*b^5*c^2*d^3 + (27*a*b^7*c^4*d)/8)/c^9 - (3*(((a + b
/x)^(1/2)*(9*b^6*c^4*d + 1152*a^4*b^2*d^5 - 144*a*b^5*c^3*d^2 - 1728*a^3*b^
3*c*d^4 + 864*a^2*b^4*c^2*d^3))/(8*c^6) - (3*(d*(a*d - b*c))^(1/2)*((9*a*b^
4*c^9*d^2 - 12*a^2*b^3*c^8*d^3)/c^9 - (3*(64*b^3*c^9*d^2 - 128*a*b^2*c^8*d^
3)*(a + b/x)^(1/2)*(d*(a*d - b*c))^(1/2)*(8*a^2*d^2 + b^2*c^2 - 8*a*b*c*d))
/(64*c^6*(a*c^4*d^2 - b*c^5*d)))*(8*a^2*d^2 + b^2*c^2 - 8*a*b*c*d))/(8*(a*c
^4*d^2 - b*c^5*d)))*(d*(a*d - b*c))^(1/2)*(8*a^2*d^2 + b^2*c^2 - 8*a*b*c*d)
)/(8*(a*c^4*d^2 - b*c^5*d)) + (3*(((a + b/x)^(1/2)*(9*b^6*c^4*d + 1152*a^4*
b^2*d^5 - 144*a*b^5*c^3*d^2 - 1728*a^3*b^3*c*d^4 + 864*a^2*b^4*c^2*d^3))/(8
*c^6) + (3*(d*(a*d - b*c))^(1/2)*((9*a*b^4*c^9*d^2 - 12*a^2*b^3*c^8*d^3)/c^
9 + (3*(64*b^3*c^9*d^2 - 128*a*b^2*c^8*d^3)*(a + b/x)^(1/2)*(d*(a*d - b*c))
^(1/2)*(8*a^2*d^2 + b^2*c^2 - 8*a*b*c*d))/(64*c^6*(a*c^4*d^2 - b*c^5*d)))*(
8*a^2*d^2 + b^2*c^2 - 8*a*b*c*d))/(8*(a*c^4*d^2 - b*c^5*d)))*(d*(a*d - b*c)
)^(1/2)*(8*a^2*d^2 + b^2*c^2 - 8*a*b*c*d))/(8*(a*c^4*d^2 - b*c^5*d))))*(d*(
a*d - b*c))^(1/2)*(8*a^2*d^2 + b^2*c^2 - 8*a*b*c*d)*3i)/(4*(a*c^4*d^2 - b*c
^5*d))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)**(3/2)/(c+d/x)**3,x)
```

```
[Out] Timed out
```

3.141 $\int \left(a+\frac{b}{x}\right)^{5/2} \left(c+\frac{d}{x}\right)^3 \, dx$

Optimal. Leaf size=198

$$a^{3/2}c^2(6ad+5bc)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)-\frac{d\left(a+\frac{b}{x}\right)^{5/2}\left(2\left(-10a^2d^2+135abcd+469b^2c^2\right)+\frac{5bd(10ad+89bc)}{x}\right)}{315b^2}-\frac{1}{3}c^2\left(a+\frac{b}{x}\right.$$

Rubi [A] time = 0.16, antiderivative size = 198, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {375, 97, 153, 147, 50, 63, 208}

$$-\frac{d\left(a+\frac{b}{x}\right)^{5/2}\left(2\left(-10a^2d^2+135abcd+469b^2c^2\right)+\frac{5bd(10ad+89bc)}{x}\right)}{315b^2}+a^{3/2}c^2(6ad+5bc)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)-\frac{1}{3}c^2\left(a+\frac{b}{x}\right)^{3/2}(6ad+5bc)-ac^2\sqrt{a+\frac{b}{x}}(6ad+5bc)+x\left(a+\frac{b}{x}\right)^{5/2}\left(c+\frac{d}{x}\right)^3-\frac{11}{9}d\left(a+\frac{b}{x}\right)^{5/2}\left(c+\frac{d}{x}\right)^2$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x)^(5/2)*(c + d/x)^3,x]
```

```
[Out] -(a*c^2*(5*b*c + 6*a*d)*Sqrt[a + b/x]) - (c^2*(5*b*c + 6*a*d)*(a + b/x)^(3/
2))/3 - (11*d*(a + b/x)^(5/2)*(c + d/x)^2)/9 - (d*(a + b/x)^(5/2)*(2*(469*b
^2*c^2 + 135*a*b*c*d - 10*a^2*d^2) + (5*b*d*(89*b*c + 10*a*d))/x))/(315*b^2
) + (a + b/x)^(5/2)*(c + d/x)^3*x + a^(3/2)*c^2*(5*b*c + 6*a*d)*ArcTanh[Sqr
t[a + b/x]/Sqrt[a]]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 97

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p)/(b*
(m + 1)), x] - Dist[1/(b*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*
(e + f*x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ
[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 147

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> -Simp[((a*d*f*h*(n + 2) + b*c*f*h*(m
+ 2) - b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1))/(b^2*d^2*(m + n + 2)*(m + n + 3)), x] + Dist[(a^2*d^2
*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m +
n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3
```

```
) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3)), In
t[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n},
x] && NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

Rule 153

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(h*(a + b*x)^m*(c + d*x)^(n
+ 1)*(e + f*x)^(p + 1))/(d*f*(m + n + p + 2)), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegerQ[m]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 375

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[((a + b/x^n)^p*(c + d/x^n)^q)/x^2, x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rubi steps

$$\int \left(a+\frac{b}{x}\right)^{5/2}\left(c+\frac{d}{x}\right)^3 dx = -\text{Subst}\left(\int \frac{(a+bx)^{5/2}(c+dx)^3}{x^2}\,dx, x, \frac{1}{x}\right)$$

$$= \left(a+\frac{b}{x}\right)^{5/2}\left(c+\frac{d}{x}\right)^3 x - \text{Subst}\left(\int \frac{(a+bx)^{3/2}(c+dx)^2\left(\frac{1}{2}(5bc+6ad)+\frac{11bdx}{2}\right)}{x}\,dx,\right.$$

$$= -\frac{11}{9}d\left(a+\frac{b}{x}\right)^{5/2}\left(c+\frac{d}{x}\right)^2 + \left(a+\frac{b}{x}\right)^{5/2}\left(c+\frac{d}{x}\right)^3 x - \frac{2\,\text{Subst}\left(\int \frac{(a+bx)^{3/2}(c+dx)\left(\frac{9}{4}bc(5\right.}{}\right.}{9}$$

$$= -\frac{11}{9}d\left(a+\frac{b}{x}\right)^{5/2}\left(c+\frac{d}{x}\right)^2 - \frac{d\left(a+\frac{b}{x}\right)^{5/2}\left(2\left(469b^2c^2+135abcd-10a^2d^2\right)+\frac{5bd(8}{}\right.}{315b^2}$$

$$= -\frac{1}{3}c^2(5bc+6ad)\left(a+\frac{b}{x}\right)^{3/2} - \frac{11}{9}d\left(a+\frac{b}{x}\right)^{5/2}\left(c+\frac{d}{x}\right)^2 - \frac{d\left(a+\frac{b}{x}\right)^{5/2}\left(2\left(469b^2c^2\right.\right.}{}$$

$$= -ac^2(5bc+6ad)\sqrt{a+\frac{b}{x}} - \frac{1}{3}c^2(5bc+6ad)\left(a+\frac{b}{x}\right)^{3/2} - \frac{11}{9}d\left(a+\frac{b}{x}\right)^{5/2}\left(c+\frac{d}{x}\right)^2$$

$$= -ac^2(5bc+6ad)\sqrt{a+\frac{b}{x}} - \frac{1}{3}c^2(5bc+6ad)\left(a+\frac{b}{x}\right)^{3/2} - \frac{11}{9}d\left(a+\frac{b}{x}\right)^{5/2}\left(c+\frac{d}{x}\right)^2$$

$$= -ac^2(5bc+6ad)\sqrt{a+\frac{b}{x}} - \frac{1}{3}c^2(5bc+6ad)\left(a+\frac{b}{x}\right)^{3/2} - \frac{11}{9}d\left(a+\frac{b}{x}\right)^{5/2}\left(c+\frac{d}{x}\right)^2$$

Mathematica [A] time = 0.25, size = 201, normalized size = 1.02

$$a^{3/2}c^2(6ad+5bc)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)+\frac{\sqrt{a+\frac{b}{x}}\left(20a^4d^3x^4-10a^3bd^2x^3(27cx+d)-3a^2b^2x^2\left(-105c^3x^3+966c^2dx^2+270cd^2x+50d^3\right)-2ab^3x\left(735c^3x^3+693c^2dx^2+405cd^2x+95d^3\right)-2b^4\left(105c^3x^3+189c^2dx^2+135cd^2x+35d^3\right)\right)}{315b^2x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x)^(5/2)*(c + d/x)^3,x]
```

```
[Out] (Sqrt[a + b/x]*(20*a^4*d^3*x^4 - 10*a^3*b*d^2*x^3*(d + 27*c*x) - 3*a^2*b^2*
x^2*(50*d^3 + 270*c*d^2*x + 966*c^2*d*x^2 - 105*c^3*x^3) - 2*b^4*(35*d^3 +
135*c*d^2*x + 189*c^2*d*x^2 + 105*c^3*x^3) - 2*a*b^3*x*(95*d^3 + 405*c*d^2*
x + 693*c^2*d*x^2 + 735*c^3*x^3)))/(315*b^2*x^4) + a^(3/2)*c^2*(5*b*c + 6*a
*d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]]
```

IntegrateAlgebraic [A] time = 0.28, size = 252, normalized size = 1.27

$$\left(5a^{3/2}bc^3+6a^{5/2}c^2d\right)\tanh^{-1}\left(\frac{\sqrt{\frac{ax+b}{x}}}{\sqrt{a}}\right)+\frac{\sqrt{\frac{ax+b}{x}}\left(20a^4d^3x^4-270a^3bcd^2x^4-10a^3bd^3x^3+315a^2b^2c^3x^5-2898a^2b^2c^2dx^4-810a^2b^2cd^2x^3-150a^2b^2d^3x^2-1470ab^3c^3x^4-1386ab^3c^2dx^3-810ab^3cd^2x^2-190ab^3d^3x-210b^4c^3x^3-378b^4c^2dx^2-270b^4cd^2x-70b^4d^3\right)}{315b^2x^4}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b/x)^(5/2)*(c + d/x)^3,x]
```

```
[Out] (Sqrt[(b + a*x)/x]*(-70*b^4*d^3 - 270*b^4*c*d^2*x - 190*a*b^3*d^3*x - 378*b
^4*c^2*d*x^2 - 810*a*b^3*c*d^2*x^2 - 150*a^2*b^2*d^3*x^2 - 210*b^4*c^3*x^3
- 1386*a*b^3*c^2*d*x^3 - 810*a^2*b^2*c*d^2*x^3 - 10*a^3*b*d^3*x^3 - 1470*a*
b^3*c^3*x^4 - 2898*a^2*b^2*c^2*d*x^4 - 270*a^3*b*c*d^2*x^4 + 20*a^4*d^3*x^4
 + 315*a^2*b^2*c^3*x^5))/(315*b^2*x^4) + (5*a^(3/2)*b*c^3 + 6*a^(5/2)*c^2*d
)*ArcTanh[Sqrt[(b + a*x)/x]/Sqrt[a]]
```

fricas [A] time = 0.86, size = 494, normalized size = 2.49

$$\left[\frac{315\left(5ab^3c^3+6a^2b^2c^2d\right)\sqrt{a}x^4\log\left(2ax+2\sqrt{a}x\sqrt{\frac{ax+b}{x}}+b\right)+2\left(315a^2b^2c^3x^5-70b^4d^3-2\left(735ab^3c^3+1449a^2b^2c^2d+135a^3bcd^2-10a^4d^3\right)x^4-2\left(105b^4c^3+693ab^3c^2d+405a^2b^2cd^2+5a^3bd^3\right)x^3-6\left(63b^4c^2d+135ab^3cd^2+25a^2b^2d^3\right)x^2-10\left(27b^4cd^2+19ab^3d^3\right)x\right)\sqrt{\frac{ax+b}{x}}}{630b^2x^4},\ -\frac{315\left(5ab^3c^3+6a^2b^2c^2d\right)\sqrt{-a}x^4\arctan\left(\frac{\sqrt{-a}\sqrt{\frac{ax+b}{x}}}{a}\right)-\left(315a^2b^2c^3x^5-70b^4d^3-2\left(735ab^3c^3+1449a^2b^2c^2d+135a^3bcd^2-10a^4d^3\right)x^4-2\left(105b^4c^3+693ab^3c^2d+405a^2b^2cd^2+5a^3bd^3\right)x^3-6\left(63b^4c^2d+135ab^3cd^2+25a^2b^2d^3\right)x^2-10\left(27b^4cd^2+19ab^3d^3\right)x\right)\sqrt{\frac{ax+b}{x}}}{315b^2x^4}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(5/2)*(c+d/x)^3,x, algorithm="fricas")
```

```
[Out] [1/630*(315*(5*a*b^3*c^3 + 6*a^2*b^2*c^2*d)*sqrt(a)*x^4*log(2*a*x + 2*sqrt(
a)*x*sqrt((a*x + b)/x) + b) + 2*(315*a^2*b^2*c^3*x^5 - 70*b^4*d^3 - 2*(735*
a*b^3*c^3 + 1449*a^2*b^2*c^2*d + 135*a^3*b*c*d^2 - 10*a^4*d^3)*x^4 - 2*(105
*b^4*c^3 + 693*a*b^3*c^2*d + 405*a^2*b^2*c*d^2 + 5*a^3*b*d^3)*x^3 - 6*(63*b
^4*c^2*d + 135*a*b^3*c*d^2 + 25*a^2*b^2*d^3)*x^2 - 10*(27*b^4*c*d^2 + 19*a*
b^3*d^3)*x)*sqrt((a*x + b)/x))/(b^2*x^4), -1/315*(315*(5*a*b^3*c^3 + 6*a^2*
b^2*c^2*d)*sqrt(-a)*x^4*arctan(sqrt(-a)*sqrt((a*x + b)/x)/a) - (315*a^2*b^2
*c^3*x^5 - 70*b^4*d^3 - 2*(735*a*b^3*c^3 + 1449*a^2*b^2*c^2*d + 135*a^3*b*c
*d^2 - 10*a^4*d^3)*x^4 - 2*(105*b^4*c^3 + 693*a*b^3*c^2*d + 405*a^2*b^2*c*d
^2 + 5*a^3*b*d^3)*x^3 - 6*(63*b^4*c^2*d + 135*a*b^3*c*d^2 + 25*a^2*b^2*d^3)
*x^2 - 10*(27*b^4*c*d^2 + 19*a*b^3*d^3)*x)*sqrt((a*x + b)/x))/(b^2*x^4)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(5/2)*(c+d/x)^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(x)]
Warning, choosing root of [1,0,%%%{-2,[1,2,0]%%%}+%%%{-2,[1,0,0]%%%}+%%%{-2
,[0,1,1]%%%},0,%%%{1,[2,4,0]%%%}+%%%{-2,[2,2,0]%%%}+%%%{1,[2,0,0]%%%}+%%%{2
```

```
,[1,3,1]%%%}+%%%{-2,[1,1,1]%%%}+%%%{1,[0,2,2]%%%}] at parameters values [86
,-97,-82]Warning, choosing root of [1,0,%%%{-2,[1,2,0]%%%}+%%%{-2,[1,0,0]%%
%}+%%%{-2,[0,1,1]%%%},0,%%%{1,[2,4,0]%%%}+%%%{-2,[2,2,0]%%%}+%%%{1,[2,0,0]%
%%}+%%%{2,[1,3,1]%%%}+%%%{-2,[1,1,1]%%%}+%%%{1,[0,2,2]%%%}] at parameters v
alues [7,-27,26]Warning, choosing root of [1,0,%%%{-4,[1,0,0]%%%}+%%%{-2,[0
,1,1]%%%},0,%%%{1,[0,2,2]%%%}] at parameters values [18.6420984049,-49,-86]
Warning, choosing root of [1,0,%%%{-4,[1,0,0]%%%}+%%%{-2,[0,1,1]%%%},0,%%%{
1,[0,2,2]%%%}] at parameters values [78.6493344628,22,42]Warning, choosing
root of [1,0,%%%{-2,[1,0,1]%%%}+%%%{-4,[0,1,0]%%%},0,%%%{1,[2,0,2]%%%}] at
parameters values [-13,74.7709350525,24]Sign error (%%%{-b,0%%%}+%%%{2*sqrt
(a)*sqrt(b),1/2%%%}+%%%{-2*a,1%%%}+%%%{a*sqrt(a)*sqrt(b)/b,3/2%%%}+%%%{-a^2
*sqrt(a)*sqrt(b)/(4*b^2),5/2%%%}+%%%{undef,7/2%%%})Evaluation time: 1.79Lim
it: Max order reached or unable to make series expansion Error: Bad Argumen
t Value
```

maple [B] time = 0.07, size = 457, normalized size = 2.31

$$\frac{\sqrt{\frac{ax+b}{x}}\left(1890a^{3}b^{2}c^{2}d\,x^{6}\ln\left(\frac{2ax+b+2\sqrt{a\,x^{2}+bx}\,\sqrt{a}}{2\sqrt{a}}\right)+1575a^{2}b^{3}c^{3}x^{6}\ln\left(\frac{2ax+b+2\sqrt{a\,x^{2}+bx}\,\sqrt{a}}{2\sqrt{a}}\right)+3780\sqrt{a\,x^{2}+bx}\,a^{\frac{7}{2}}b\,c^{2}d\,x^{6}+3150\sqrt{a\,x^{2}+bx}\,a^{\frac{5}{2}}b^{2}c^{3}x^{6}-3780\left(a\,x^{2}+bx\right)^{\frac{3}{2}}a^{\frac{5}{2}}b\,c^{2}d\,x^{4}-2520\left(a\,x^{2}+bx\right)^{\frac{3}{2}}a^{\frac{3}{2}}b^{2}c^{3}x^{4}+40\left(a\,x^{2}+bx\right)^{\frac{3}{2}}a^{\frac{7}{2}}d^{3}x^{3}-540\left(a\,x^{2}+bx\right)^{\frac{3}{2}}a^{\frac{5}{2}}bc\,d^{2}x^{3}-2016\left(a\,x^{2}+bx\right)^{\frac{3}{2}}a^{\frac{3}{2}}b^{2}c^{2}d\,x^{3}-420\left(a\,x^{2}+bx\right)^{\frac{3}{2}}\sqrt{a}\,b^{3}c^{3}x^{3}-60\left(a\,x^{2}+bx\right)^{\frac{3}{2}}a^{\frac{5}{2}}b\,d^{3}x^{2}-1080\left(a\,x^{2}+bx\right)^{\frac{3}{2}}a^{\frac{3}{2}}b^{2}c\,d^{2}x^{2}-756\left(a\,x^{2}+bx\right)^{\frac{3}{2}}\sqrt{a}\,b^{3}c^{2}d\,x^{2}-240\left(a\,x^{2}+bx\right)^{\frac{3}{2}}a^{\frac{3}{2}}b^{2}d^{3}x-540\left(a\,x^{2}+bx\right)^{\frac{3}{2}}\sqrt{a}\,b^{3}c\,d^{2}x-140\left(a\,x^{2}+bx\right)^{\frac{3}{2}}\sqrt{a}\,b^{3}d^{3}\right)}{630\sqrt{(ax+b)\,x}\,\sqrt{a}\,b^{2}x^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x)^(5/2)*(c+d/x)^3,x)
```

```
[Out] 1/630*((a*x+b)/x)^(1/2)*(3780*(a*x^2+b*x)^(1/2)*a^(7/2)*x^6*b*c^2*d+3150*(a
*x^2+b*x)^(1/2)*a^(5/2)*x^6*b^2*c^3+40*(a*x^2+b*x)^(3/2)*a^(7/2)*x^3*d^3-37
80*(a*x^2+b*x)^(3/2)*a^(5/2)*x^4*b*c^2*d-540*(a*x^2+b*x)^(3/2)*a^(5/2)*x^3*
b*c*d^2-2520*(a*x^2+b*x)^(3/2)*a^(3/2)*x^4*b^2*c^3+1890*ln(1/2*(2*a*x+b+2*(
a*x^2+b*x)^(1/2)*a^(1/2))/a^(1/2))*x^6*a^3*b^2*c^2*d+1575*ln(1/2*(2*a*x+b+2
*(a*x^2+b*x)^(1/2)*a^(1/2))/a^(1/2))*x^6*a^2*b^3*c^3-60*(a*x^2+b*x)^(3/2)*a
^(5/2)*x^2*b*d^3-2016*(a*x^2+b*x)^(3/2)*a^(3/2)*x^3*b^2*c^2*d-1080*(a*x^2+b
*x)^(3/2)*a^(3/2)*x^2*b^2*c*d^2-420*(a*x^2+b*x)^(3/2)*a^(1/2)*x^3*b^3*c^3-2
40*(a*x^2+b*x)^(3/2)*a^(3/2)*x*b^2*d^3-756*(a*x^2+b*x)^(3/2)*a^(1/2)*x^2*b^
3*c^2*d-540*(a*x^2+b*x)^(3/2)*a^(1/2)*x*b^3*c*d^2-140*(a*x^2+b*x)^(3/2)*a^(
1/2)*b^3*d^3)/x^5/b^2/((a*x+b)*x)^(1/2)/a^(1/2)
```

maxima [A] time = 1.21, size = 219, normalized size = 1.11

$$-\frac{6\left(a+\frac{b}{x}\right)^{\frac{7}{2}}cd^{2}}{7b}+\frac{1}{6}\left(6\sqrt{a+\frac{b}{x}}\,a^{2}x-15a^{\frac{3}{2}}b\log\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{a+\frac{b}{x}}+\sqrt{a}}\right)-4\left(a+\frac{b}{x}\right)^{\frac{3}{2}}b-24\sqrt{a+\frac{b}{x}}\,ab\right)c^{3}-\frac{1}{5}\left(15a^{\frac{5}{2}}\log\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{a+\frac{b}{x}}+\sqrt{a}}\right)+6\left(a+\frac{b}{x}\right)^{\frac{5}{2}}+10\left(a+\frac{b}{x}\right)^{\frac{3}{2}}a+30\sqrt{a+\frac{b}{x}}\,a^{2}\right)c^{2}d-\frac{2}{63}\left(\frac{7\left(a+\frac{b}{x}\right)^{\frac{9}{2}}}{b^{2}}-\frac{9\left(a+\frac{b}{x}\right)^{\frac{7}{2}}a}{b^{2}}\right)d^{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(5/2)*(c+d/x)^3,x, algorithm="maxima")
```

```
[Out] -6/7*(a + b/x)^(7/2)*c*d^2/b + 1/6*(6*sqrt(a + b/x)*a^2*x - 15*a^(3/2)*b*lo
g((sqrt(a + b/x) - sqrt(a))/(sqrt(a + b/x) + sqrt(a))) - 4*(a + b/x)^(3/2)*
b - 24*sqrt(a + b/x)*a*b)*c^3 - 1/5*(15*a^(5/2)*log((sqrt(a + b/x) - sqrt(a
))/(sqrt(a + b/x) + sqrt(a))) + 6*(a + b/x)^(5/2) + 10*(a + b/x)^(3/2)*a +
30*sqrt(a + b/x)*a^2)*c^2*d - 2/63*(7*(a + b/x)^(9/2)/b^2 - 9*(a + b/x)^(7/
2)*a/b^2)*d^3
```

mupad [B] time = 6.05, size = 487, normalized size = 2.46

$$\left(a+\frac{b}{x}\right)^{7/2}\left(\frac{6\,a\,d^{3}-6\,b\,c\,d^{2}}{7\,b^{2}}-\frac{4\,a\,d^{3}}{7\,b^{2}}\right)-\sqrt{a+\frac{b}{x}}\left(a^{2}\left(2\,a\left(\frac{6\,a\,d^{3}-6\,b\,c\,d^{2}}{b^{2}}-\frac{4\,a\,d^{3}}{b^{2}}\right)-\frac{6\,d\,{\left(a\,d-b\,c\right)}^{2}}{b^{2}}+\frac{2\,a^{2}\,d^{3}}{b^{2}}\right)-2\,a\left(\frac{2\,{\left(a\,d-b\,c\right)}^{3}}{b^{2}}+2\,a\left(2\,a\left(\frac{6\,a\,d^{3}-6\,b\,c\,d^{2}}{b^{2}}-\frac{4\,a\,d^{3}}{b^{2}}\right)-\frac{6\,d\,{\left(a\,d-b\,c\right)}^{2}}{b^{2}}+\frac{2\,a^{2}\,d^{3}}{b^{2}}\right)-a^{2}\left(\frac{6\,a\,d^{3}-6\,b\,c\,d^{2}}{b^{2}}-\frac{4\,a\,d^{3}}{b^{2}}\right)\right)\right)+\left(a+\frac{b}{x}\right)^{3/2}\left(\frac{2\,{\left(a\,d-b\,c\right)}^{3}}{3\,b^{2}}+\frac{2\,a\left(2\,a\left(\frac{6\,a\,d^{3}-6\,b\,c\,d^{2}}{b^{2}}-\frac{4\,a\,d^{3}}{b^{2}}\right)-\frac{6\,d\,{\left(a\,d-b\,c\right)}^{2}}{b^{2}}+\frac{2\,a^{2}\,d^{3}}{b^{2}}\right)}{3}-\frac{a^{2}\left(\frac{6\,a\,d^{3}-6\,b\,c\,d^{2}}{b^{2}}-\frac{4\,a\,d^{3}}{b^{2}}\right)}{3}\right)+\left(a+\frac{b}{x}\right)^{5/2}\left(\frac{2\,a\left(\frac{6\,a\,d^{3}-6\,b\,c\,d^{2}}{b^{2}}-\frac{4\,a\,d^{3}}{b^{2}}\right)}{5}-\frac{6\,d\,{\left(a\,d-b\,c\right)}^{2}}{5\,b^{2}}+\frac{2\,a^{2}\,d^{3}}{5\,b^{2}}\right)-\frac{2\,d^{3}\,{\left(a+\frac{b}{x}\right)}^{9/2}}{9\,b^{2}}+a^{2}\,c^{3}\,x\,\sqrt{a+\frac{b}{x}}-a^{3/2}\,c^{2}\,\mathrm{atan}\left(\frac{\sqrt{a+\frac{b}{x}}\,1{}\mathrm{i}}{\sqrt{a}}\right)\,\left(6\,a\,d+5\,b\,c\right)\,1{}\mathrm{i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x)^(5/2)*(c + d/x)^3,x)
```

```
[Out] (a + b/x)^(7/2)*((6*a*d^3 - 6*b*c*d^2)/(7*b^2) - (4*a*d^3)/(7*b^2)) - (a +
b/x)^(1/2)*(a^2*(2*a*((6*a*d^3 - 6*b*c*d^2)/b^2 - (4*a*d^3)/b^2) - (6*d*(a*
d - b*c)^2)/b^2 + (2*a^2*d^3)/b^2) - 2*a*((2*(a*d - b*c)^3)/b^2 + 2*a*(2*a*
```

```
((6*a*d^3 - 6*b*c*d^2)/b^2 - (4*a*d^3)/b^2) - (6*d*(a*d - b*c)^2)/b^2 + (2*
a^2*d^3)/b^2) - a^2*((6*a*d^3 - 6*b*c*d^2)/b^2 - (4*a*d^3)/b^2))) + (a + b/
x)^(3/2)*((2*(a*d - b*c)^3)/(3*b^2) + (2*a*(2*a*((6*a*d^3 - 6*b*c*d^2)/b^2
- (4*a*d^3)/b^2) - (6*d*(a*d - b*c)^2)/b^2 + (2*a^2*d^3)/b^2))/3 - (a^2*((6
*a*d^3 - 6*b*c*d^2)/b^2 - (4*a*d^3)/b^2))/3) + (a + b/x)^(5/2)*((2*a*((6*a*
d^3 - 6*b*c*d^2)/b^2 - (4*a*d^3)/b^2))/5 - (6*d*(a*d - b*c)^2)/(5*b^2) + (2
*a^2*d^3)/(5*b^2)) - (2*d^3*(a + b/x)^(9/2))/(9*b^2) + a^2*c^3*x*(a + b/x)^
(1/2) - a^(3/2)*c^2*atan(((a + b/x)^(1/2)*1i)/a^(1/2))*(6*a*d + 5*b*c)*1i
```

sympy [A] time = 158.70, size = 5513, normalized size = 27.84

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)**(5/2)*(c+d/x)**3,x)
```

```
[Out] 32*a**(29/2)*b**(27/2)*d**3*x**10*sqrt(a*x/b + 1)/(315*a**(21/2)*b**15*x**(
21/2) + 1890*a**(19/2)*b**16*x**(19/2) + 4725*a**(17/2)*b**17*x**(17/2) + 6
300*a**(15/2)*b**18*x**(15/2) + 4725*a**(13/2)*b**19*x**(13/2) + 1890*a**(1
1/2)*b**20*x**(11/2) + 315*a**(9/2)*b**21*x**(9/2)) + 176*a**(27/2)*b**(29/
2)*d**3*x**9*sqrt(a*x/b + 1)/(315*a**(21/2)*b**15*x**(21/2) + 1890*a**(19/2
)*b**16*x**(19/2) + 4725*a**(17/2)*b**17*x**(17/2) + 6300*a**(15/2)*b**18*x
**(15/2) + 4725*a**(13/2)*b**19*x**(13/2) + 1890*a**(11/2)*b**20*x**(11/2)
+ 315*a**(9/2)*b**21*x**(9/2)) + 396*a**(25/2)*b**(31/2)*d**3*x**8*sqrt(a*x
/b + 1)/(315*a**(21/2)*b**15*x**(21/2) + 1890*a**(19/2)*b**16*x**(19/2) + 4
725*a**(17/2)*b**17*x**(17/2) + 6300*a**(15/2)*b**18*x**(15/2) + 4725*a**(1
3/2)*b**19*x**(13/2) + 1890*a**(11/2)*b**20*x**(11/2) + 315*a**(9/2)*b**21*
x**(9/2)) + 462*a**(23/2)*b**(33/2)*d**3*x**7*sqrt(a*x/b + 1)/(315*a**(21/2
)*b**15*x**(21/2) + 1890*a**(19/2)*b**16*x**(19/2) + 4725*a**(17/2)*b**17*x
**(17/2) + 6300*a**(15/2)*b**18*x**(15/2) + 4725*a**(13/2)*b**19*x**(13/2)
+ 1890*a**(11/2)*b**20*x**(11/2) + 315*a**(9/2)*b**21*x**(9/2)) + 210*a**(2
1/2)*b**(35/2)*d**3*x**6*sqrt(a*x/b + 1)/(315*a**(21/2)*b**15*x**(21/2) + 1
890*a**(19/2)*b**16*x**(19/2) + 4725*a**(17/2)*b**17*x**(17/2) + 6300*a**(1
5/2)*b**18*x**(15/2) + 4725*a**(13/2)*b**19*x**(13/2) + 1890*a**(11/2)*b**2
0*x**(11/2) + 315*a**(9/2)*b**21*x**(9/2)) - 32*a**(21/2)*b**(11/2)*d**3*x*
*6*sqrt(a*x/b + 1)/(105*a**(13/2)*b**7*x**(13/2) + 315*a**(11/2)*b**8*x**(1
1/2) + 315*a**(9/2)*b**9*x**(9/2) + 105*a**(7/2)*b**10*x**(7/2)) - 378*a**(
19/2)*b**(37/2)*d**3*x**5*sqrt(a*x/b + 1)/(315*a**(21/2)*b**15*x**(21/2) +
1890*a**(19/2)*b**16*x**(19/2) + 4725*a**(17/2)*b**17*x**(17/2) + 6300*a**(
15/2)*b**18*x**(15/2) + 4725*a**(13/2)*b**19*x**(13/2) + 1890*a**(11/2)*b**
20*x**(11/2) + 315*a**(9/2)*b**21*x**(9/2)) - 48*a**(19/2)*b**(13/2)*c*d**2
*x**6*sqrt(a*x/b + 1)/(105*a**(13/2)*b**7*x**(13/2) + 315*a**(11/2)*b**8*x*
*(11/2) + 315*a**(9/2)*b**9*x**(9/2) + 105*a**(7/2)*b**10*x**(7/2)) - 80*a*
*(19/2)*b**(13/2)*d**3*x**5*sqrt(a*x/b + 1)/(105*a**(13/2)*b**7*x**(13/2) +
 315*a**(11/2)*b**8*x**(11/2) + 315*a**(9/2)*b**9*x**(9/2) + 105*a**(7/2)*b
**10*x**(7/2)) - 1134*a**(17/2)*b**(39/2)*d**3*x**4*sqrt(a*x/b + 1)/(315*a*
*(21/2)*b**15*x**(21/2) + 1890*a**(19/2)*b**16*x**(19/2) + 4725*a**(17/2)*b
**17*x**(17/2) + 6300*a**(15/2)*b**18*x**(15/2) + 4725*a**(13/2)*b**19*x**(
13/2) + 1890*a**(11/2)*b**20*x**(11/2) + 315*a**(9/2)*b**21*x**(9/2)) - 120
*a**(17/2)*b**(15/2)*c*d**2*x**5*sqrt(a*x/b + 1)/(105*a**(13/2)*b**7*x**(13
/2) + 315*a**(11/2)*b**8*x**(11/2) + 315*a**(9/2)*b**9*x**(9/2) + 105*a**(7
/2)*b**10*x**(7/2)) - 60*a**(17/2)*b**(15/2)*d**3*x**4*sqrt(a*x/b + 1)/(105
*a**(13/2)*b**7*x**(13/2) + 315*a**(11/2)*b**8*x**(11/2) + 315*a**(9/2)*b**
9*x**(9/2) + 105*a**(7/2)*b**10*x**(7/2)) - 1494*a**(15/2)*b**(41/2)*d**3*x
**3*sqrt(a*x/b + 1)/(315*a**(21/2)*b**15*x**(21/2) + 1890*a**(19/2)*b**16*x
**(19/2) + 4725*a**(17/2)*b**17*x**(17/2) + 6300*a**(15/2)*b**18*x**(15/2)
+ 4725*a**(13/2)*b**19*x**(13/2) + 1890*a**(11/2)*b**20*x**(11/2) + 315*a**
(9/2)*b**21*x**(9/2)) - 90*a**(15/2)*b**(17/2)*c*d**2*x**4*sqrt(a*x/b + 1)/
(105*a**(13/2)*b**7*x**(13/2) + 315*a**(11/2)*b**8*x**(11/2) + 315*a**(9/2)
*b**9*x**(9/2) + 105*a**(7/2)*b**10*x**(7/2)) - 80*a**(15/2)*b**(17/2)*d**3
```

```
*x**3*sqrt(a*x/b + 1)/(105*a**(13/2)*b**7*x**(13/2) + 315*a**(11/2)*b**8*x*
*(11/2) + 315*a**(9/2)*b**9*x**(9/2) + 105*a**(7/2)*b**10*x**(7/2)) + 4*a**
(15/2)*b**(3/2)*d**3*x**3*sqrt(a*x/b + 1)/(15*a**(7/2)*b**3*x**(7/2) + 15*a
**(5/2)*b**4*x**(5/2)) - 1098*a**(13/2)*b**(43/2)*d**3*x**2*sqrt(a*x/b + 1)
/(315*a**(21/2)*b**15*x**(21/2) + 1890*a**(19/2)*b**16*x**(19/2) + 4725*a**
(17/2)*b**17*x**(17/2) + 6300*a**(15/2)*b**18*x**(15/2) + 4725*a**(13/2)*b*
*19*x**(13/2) + 1890*a**(11/2)*b**20*x**(11/2) + 315*a**(9/2)*b**21*x**(9/2
)) - 120*a**(13/2)*b**(19/2)*c*d**2*x**3*sqrt(a*x/b + 1)/(105*a**(13/2)*b**
7*x**(13/2) + 315*a**(11/2)*b**8*x**(11/2) + 315*a**(9/2)*b**9*x**(9/2) + 1
05*a**(7/2)*b**10*x**(7/2)) - 200*a**(13/2)*b**(19/2)*d**3*x**2*sqrt(a*x/b
+ 1)/(105*a**(13/2)*b**7*x**(13/2) + 315*a**(11/2)*b**8*x**(11/2) + 315*a**
(9/2)*b**9*x**(9/2) + 105*a**(7/2)*b**10*x**(7/2)) + 24*a**(13/2)*b**(5/2)*
c*d**2*x**3*sqrt(a*x/b + 1)/(15*a**(7/2)*b**3*x**(7/2) + 15*a**(5/2)*b**4*x
**(5/2)) + 2*a**(13/2)*b**(5/2)*d**3*x**2*sqrt(a*x/b + 1)/(15*a**(7/2)*b**3
*x**(7/2) + 15*a**(5/2)*b**4*x**(5/2)) - 430*a**(11/2)*b**(45/2)*d**3*x*sqr
t(a*x/b + 1)/(315*a**(21/2)*b**15*x**(21/2) + 1890*a**(19/2)*b**16*x**(19/2
) + 4725*a**(17/2)*b**17*x**(17/2) + 6300*a**(15/2)*b**18*x**(15/2) + 4725*
a**(13/2)*b**19*x**(13/2) + 1890*a**(11/2)*b**20*x**(11/2) + 315*a**(9/2)*b
**21*x**(9/2)) - 300*a**(11/2)*b**(21/2)*c*d**2*x**2*sqrt(a*x/b + 1)/(105*a
**(13/2)*b**7*x**(13/2) + 315*a**(11/2)*b**8*x**(11/2) + 315*a**(9/2)*b**9*
x**(9/2) + 105*a**(7/2)*b**10*x**(7/2)) - 192*a**(11/2)*b**(21/2)*d**3*x*sq
rt(a*x/b + 1)/(105*a**(13/2)*b**7*x**(13/2) + 315*a**(11/2)*b**8*x**(11/2)
+ 315*a**(9/2)*b**9*x**(9/2) + 105*a**(7/2)*b**10*x**(7/2)) + 12*a**(11/2)*
b**(7/2)*c**2*d*x**3*sqrt(a*x/b + 1)/(15*a**(7/2)*b**3*x**(7/2) + 15*a**(5/
2)*b**4*x**(5/2)) + 12*a**(11/2)*b**(7/2)*c*d**2*x**2*sqrt(a*x/b + 1)/(15*a
**(7/2)*b**3*x**(7/2) + 15*a**(5/2)*b**4*x**(5/2)) - 8*a**(11/2)*b**(7/2)*d
**3*x*sqrt(a*x/b + 1)/(15*a**(7/2)*b**3*x**(7/2) + 15*a**(5/2)*b**4*x**(5/2
)) - 70*a**(9/2)*b**(47/2)*d**3*sqrt(a*x/b + 1)/(315*a**(21/2)*b**15*x**(21
/2) + 1890*a**(19/2)*b**16*x**(19/2) + 4725*a**(17/2)*b**17*x**(17/2) + 630
0*a**(15/2)*b**18*x**(15/2) + 4725*a**(13/2)*b**19*x**(13/2) + 1890*a**(11/
2)*b**20*x**(11/2) + 315*a**(9/2)*b**21*x**(9/2)) - 288*a**(9/2)*b**(23/2)*
c*d**2*x*sqrt(a*x/b + 1)/(105*a**(13/2)*b**7*x**(13/2) + 315*a**(11/2)*b**8
*x**(11/2) + 315*a**(9/2)*b**9*x**(9/2) + 105*a**(7/2)*b**10*x**(7/2)) - 60
*a**(9/2)*b**(23/2)*d**3*sqrt(a*x/b + 1)/(105*a**(13/2)*b**7*x**(13/2) + 31
5*a**(11/2)*b**8*x**(11/2) + 315*a**(9/2)*b**9*x**(9/2) + 105*a**(7/2)*b**1
0*x**(7/2)) + 6*a**(9/2)*b**(9/2)*c**2*d*x**2*sqrt(a*x/b + 1)/(15*a**(7/2)*
b**3*x**(7/2) + 15*a**(5/2)*b**4*x**(5/2)) - 48*a**(9/2)*b**(9/2)*c*d**2*x*
sqrt(a*x/b + 1)/(15*a**(7/2)*b**3*x**(7/2) + 15*a**(5/2)*b**4*x**(5/2)) - 6
*a**(9/2)*b**(9/2)*d**3*sqrt(a*x/b + 1)/(15*a**(7/2)*b**3*x**(7/2) + 15*a**
(5/2)*b**4*x**(5/2)) - 90*a**(7/2)*b**(25/2)*c*d**2*sqrt(a*x/b + 1)/(105*a*
*(13/2)*b**7*x**(13/2) + 315*a**(11/2)*b**8*x**(11/2) + 315*a**(9/2)*b**9*x
**(9/2) + 105*a**(7/2)*b**10*x**(7/2)) - 24*a**(7/2)*b**(11/2)*c**2*d*x*sqr
t(a*x/b + 1)/(15*a**(7/2)*b**3*x**(7/2) + 15*a**(5/2)*b**4*x**(5/2)) - 36*a
**(7/2)*b**(11/2)*c*d**2*sqrt(a*x/b + 1)/(15*a**(7/2)*b**3*x**(7/2) + 15*a*
*(5/2)*b**4*x**(5/2)) - 18*a**(5/2)*b**(13/2)*c**2*d*sqrt(a*x/b + 1)/(15*a*
*(7/2)*b**3*x**(7/2) + 15*a**(5/2)*b**4*x**(5/2)) + a**(3/2)*b*c**3*asinh(s
qrt(a)*sqrt(x)/sqrt(b)) - 32*a**15*b**13*d**3*x**(21/2)/(315*a**(21/2)*b**1
5*x**(21/2) + 1890*a**(19/2)*b**16*x**(19/2) + 4725*a**(17/2)*b**17*x**(17/
2) + 6300*a**(15/2)*b**18*x**(15/2) + 4725*a**(13/2)*b**19*x**(13/2) + 1890
*a**(11/2)*b**20*x**(11/2) + 315*a**(9/2)*b**21*x**(9/2)) - 192*a**14*b**14
*d**3*x**(19/2)/(315*a**(21/2)*b**15*x**(21/2) + 1890*a**(19/2)*b**16*x**(1
9/2) + 4725*a**(17/2)*b**17*x**(17/2) + 6300*a**(15/2)*b**18*x**(15/2) + 47
25*a**(13/2)*b**19*x**(13/2) + 1890*a**(11/2)*b**20*x**(11/2) + 315*a**(9/2
)*b**21*x**(9/2)) - 480*a**13*b**15*d**3*x**(17/2)/(315*a**(21/2)*b**15*x**
(21/2) + 1890*a**(19/2)*b**16*x**(19/2) + 4725*a**(17/2)*b**17*x**(17/2) +
6300*a**(15/2)*b**18*x**(15/2) + 4725*a**(13/2)*b**19*x**(13/2) + 1890*a**(
11/2)*b**20*x**(11/2) + 315*a**(9/2)*b**21*x**(9/2)) - 640*a**12*b**16*d**3
*x**(15/2)/(315*a**(21/2)*b**15*x**(21/2) + 1890*a**(19/2)*b**16*x**(19/2)
+ 4725*a**(17/2)*b**17*x**(17/2) + 6300*a**(15/2)*b**18*x**(15/2) + 4725*a*
```

```
*(13/2)*b**19*x**(13/2) + 1890*a**(11/2)*b**20*x**(11/2) + 315*a**(9/2)*b**
21*x**(9/2)) - 480*a**11*b**17*d**3*x**(13/2)/(315*a**(21/2)*b**15*x**(21/2
) + 1890*a**(19/2)*b**16*x**(19/2) + 4725*a**(17/2)*b**17*x**(17/2) + 6300*
a**(15/2)*b**18*x**(15/2) + 4725*a**(13/2)*b**19*x**(13/2) + 1890*a**(11/2)
*b**20*x**(11/2) + 315*a**(9/2)*b**21*x**(9/2)) + 32*a**11*b**5*d**3*x**(13
/2)/(105*a**(13/2)*b**7*x**(13/2) + 315*a**(11/2)*b**8*x**(11/2) + 315*a**(
9/2)*b**9*x**(9/2) + 105*a**(7/2)*b**10*x**(7/2)) - 192*a**10*b**18*d**3*x*
*(11/2)/(315*a**(21/2)*b**15*x**(21/2) + 1890*a**(19/2)*b**16*x**(19/2) + 4
725*a**(17/2)*b**17*x**(17/2) + 6300*a**(15/2)*b**18*x**(15/2) + 4725*a**(1
3/2)*b**19*x**(13/2) + 1890*a**(11/2)*b**20*x**(11/2) + 315*a**(9/2)*b**21*
x**(9/2)) + 48*a**10*b**6*c*d**2*x**(13/2)/(105*a**(13/2)*b**7*x**(13/2) +
315*a**(11/2)*b**8*x**(11/2) + 315*a**(9/2)*b**9*x**(9/2) + 105*a**(7/2)*b*
*10*x**(7/2)) + 96*a**10*b**6*d**3*x**(11/2)/(105*a**(13/2)*b**7*x**(13/2)
+ 315*a**(11/2)*b**8*x**(11/2) + 315*a**(9/2)*b**9*x**(9/2) + 105*a**(7/2)*
b**10*x**(7/2)) - 32*a**9*b**19*d**3*x**(9/2)/(315*a**(21/2)*b**15*x**(21/2
) + 1890*a**(19/2)*b**16*x**(19/2) + 4725*a**(17/2)*b**17*x**(17/2) + 6300*
a**(15/2)*b**18*x**(15/2) + 4725*a**(13/2)*b**19*x**(13/2) + 1890*a**(11/2)
*b**20*x**(11/2) + 315*a**(9/2)*b**21*x**(9/2)) + 144*a**9*b**7*c*d**2*x**(
11/2)/(105*a**(13/2)*b**7*x**(13/2) + 315*a**(11/2)*b**8*x**(11/2) + 315*a*
*(9/2)*b**9*x**(9/2) + 105*a**(7/2)*b**10*x**(7/2)) + 96*a**9*b**7*d**3*x**
(9/2)/(105*a**(13/2)*b**7*x**(13/2) + 315*a**(11/2)*b**8*x**(11/2) + 315*a*
*(9/2)*b**9*x**(9/2) + 105*a**(7/2)*b**10*x**(7/2)) + 144*a**8*b**8*c*d**2*
x**(9/2)/(105*a**(13/2)*b**7*x**(13/2) + 315*a**(11/2)*b**8*x**(11/2) + 315
*a**(9/2)*b**9*x**(9/2) + 105*a**(7/2)*b**10*x**(7/2)) + 32*a**8*b**8*d**3*
x**(7/2)/(105*a**(13/2)*b**7*x**(13/2) + 315*a**(11/2)*b**8*x**(11/2) + 315
*a**(9/2)*b**9*x**(9/2) + 105*a**(7/2)*b**10*x**(7/2)) - 4*a**8*b*d**3*x**(
7/2)/(15*a**(7/2)*b**3*x**(7/2) + 15*a**(5/2)*b**4*x**(5/2)) + 48*a**7*b**9
*c*d**2*x**(7/2)/(105*a**(13/2)*b**7*x**(13/2) + 315*a**(11/2)*b**8*x**(11/
2) + 315*a**(9/2)*b**9*x**(9/2) + 105*a**(7/2)*b**10*x**(7/2)) - 24*a**7*b*
*2*c*d**2*x**(7/2)/(15*a**(7/2)*b**3*x**(7/2) + 15*a**(5/2)*b**4*x**(5/2))
- 4*a**7*b**2*d**3*x**(5/2)/(15*a**(7/2)*b**3*x**(7/2) + 15*a**(5/2)*b**4*x
**(5/2)) - 12*a**6*b**3*c**2*d*x**(7/2)/(15*a**(7/2)*b**3*x**(7/2) + 15*a**
(5/2)*b**4*x**(5/2)) - 24*a**6*b**3*c*d**2*x**(5/2)/(15*a**(7/2)*b**3*x**(7
/2) + 15*a**(5/2)*b**4*x**(5/2)) - 12*a**5*b**4*c**2*d*x**(5/2)/(15*a**(7/2
)*b**3*x**(7/2) + 15*a**(5/2)*b**4*x**(5/2)) - 6*a**3*c**2*d*atan(sqrt(a +
b/x)/sqrt(-a))/sqrt(-a) + a**2*sqrt(b)*c**3*sqrt(x)*sqrt(a*x/b + 1) - 4*a**
2*b*c**3*atan(sqrt(a + b/x)/sqrt(-a))/sqrt(-a) - 6*a**2*c**2*d*sqrt(a + b/x
) + 3*a**2*c*d**2*Piecewise((-sqrt(a)/x, Eq(b, 0)), (-2*(a + b/x)**(3/2)/(3
*b), True)) - 4*a*b*c**3*sqrt(a + b/x) + 6*a*b*c**2*d*Piecewise((-sqrt(a)/x
, Eq(b, 0)), (-2*(a + b/x)**(3/2)/(3*b), True)) + b**2*c**3*Piecewise((-sqr
t(a)/x, Eq(b, 0)), (-2*(a + b/x)**(3/2)/(3*b), True))
```

3.142 $\int \left(a+\frac{b}{x}\right)^{5/2} \left(c+\frac{d}{x}\right)^2 dx$

Optimal. Leaf size=152

$$a^{3/2}c(4ad+5bc)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)+\frac{c^2x\left(a+\frac{b}{x}\right)^{7/2}}{a}-\frac{c\left(a+\frac{b}{x}\right)^{5/2}(4ad+5bc)}{5a}-\frac{1}{3}c\left(a+\frac{b}{x}\right)^{3/2}(4ad+5bc)-ac\sqrt{a+$$

Rubi [A] time = 0.10, antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {375, 89, 80, 50, 63, 208}

$$a^{3/2}c(4ad+5bc)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)+\frac{c^2x\left(a+\frac{b}{x}\right)^{7/2}}{a}-\frac{c\left(a+\frac{b}{x}\right)^{5/2}(4ad+5bc)}{5a}-\frac{1}{3}c\left(a+\frac{b}{x}\right)^{3/2}(4ad+5bc)-ac\sqrt{a+\frac{b}{x}}(4ad+5bc)-\frac{2d^2\left(a+\frac{b}{x}\right)^{7/2}}{7b}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x)^(5/2)*(c + d/x)^2,x]
```

```
[Out] -(a*c*(5*b*c + 4*a*d)*Sqrt[a + b/x]) - (c*(5*b*c + 4*a*d)*(a + b/x)^(3/2))/
3 - (c*(5*b*c + 4*a*d)*(a + b/x)^(5/2))/(5*a) - (2*d^2*(a + b/x)^(7/2))/(7*
b) + (c^2*(a + b/x)^(7/2)*x)/a + a^(3/2)*c*(5*b*c + 4*a*d)*ArcTanh[Sqrt[a +
 b/x]/Sqrt[a]]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
```

```
1])))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 375

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[((a + b/x^n)^p*(c + d/x^n)^q)/x^2, x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rubi steps

$$\int \left(a+\frac{b}{x}\right)^{5/2}\left(c+\frac{d}{x}\right)^2 dx = -\text{Subst}\left(\int \frac{(a+bx)^{5/2}(c+dx)^2}{x^2}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{c^2\left(a+\frac{b}{x}\right)^{7/2}x}{a} - \frac{\text{Subst}\left(\int \frac{(a+bx)^{5/2}\left(\frac{1}{2}c(5bc+4ad)+ad^2x\right)}{x}\,dx, x, \frac{1}{x}\right)}{a}$$

$$= -\frac{2d^2\left(a+\frac{b}{x}\right)^{7/2}}{7b} + \frac{c^2\left(a+\frac{b}{x}\right)^{7/2}x}{a} - \frac{(c(5bc+4ad))\,\text{Subst}\left(\int \frac{(a+bx)^{5/2}}{x}\,dx, x, \frac{1}{x}\right)}{2a}$$

$$= -\frac{c(5bc+4ad)\left(a+\frac{b}{x}\right)^{5/2}}{5a} - \frac{2d^2\left(a+\frac{b}{x}\right)^{7/2}}{7b} + \frac{c^2\left(a+\frac{b}{x}\right)^{7/2}x}{a} - \frac{1}{2}(c(5bc+4ad))\,\text{Subst}$$

$$= -\frac{1}{3}c(5bc+4ad)\left(a+\frac{b}{x}\right)^{3/2} - \frac{c(5bc+4ad)\left(a+\frac{b}{x}\right)^{5/2}}{5a} - \frac{2d^2\left(a+\frac{b}{x}\right)^{7/2}}{7b} + \frac{c^2\left(a+\frac{b}{x}\right)^{7/2}}{a}$$

$$= -ac(5bc+4ad)\sqrt{a+\frac{b}{x}} - \frac{1}{3}c(5bc+4ad)\left(a+\frac{b}{x}\right)^{3/2} - \frac{c(5bc+4ad)\left(a+\frac{b}{x}\right)^{5/2}}{5a} - \frac{2d^2}{}$$

$$= -ac(5bc+4ad)\sqrt{a+\frac{b}{x}} - \frac{1}{3}c(5bc+4ad)\left(a+\frac{b}{x}\right)^{3/2} - \frac{c(5bc+4ad)\left(a+\frac{b}{x}\right)^{5/2}}{5a} - \frac{2d^2}{}$$

$$= -ac(5bc+4ad)\sqrt{a+\frac{b}{x}} - \frac{1}{3}c(5bc+4ad)\left(a+\frac{b}{x}\right)^{3/2} - \frac{c(5bc+4ad)\left(a+\frac{b}{x}\right)^{5/2}}{5a} - \frac{2d^2}{}$$

Mathematica [A] time = 0.16, size = 121, normalized size = 0.80

$$-\frac{c(4ad+5bc)\left(\sqrt{a+\frac{b}{x}}\left(23a^2x^2+11abx+3b^2\right)-15a^{5/2}x^2\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)\right)}{15ax^2}+\frac{c^2x\left(a+\frac{b}{x}\right)^{7/2}}{a}-\frac{2d^2\left(a+\frac{b}{x}\right)^{7/2}}{7b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x)^(5/2)*(c + d/x)^2,x]
```

```
[Out] (-2*d^2*(a + b/x)^(7/2))/(7*b) + (c^2*(a + b/x)^(7/2)*x)/a - (c*(5*b*c + 4*
a*d)*(Sqrt[a + b/x]*(3*b^2 + 11*a*b*x + 23*a^2*x^2) - 15*a^(5/2)*x^2*ArcTan
h[Sqrt[a + b/x]/Sqrt[a]]))/(15*a*x^2)
```

IntegrateAlgebraic [A] time = 0.26, size = 173, normalized size = 1.14

$$\left(5a^{3/2}bc^2+4a^{5/2}cd\right)\tanh^{-1}\left(\frac{\sqrt{\frac{ax+b}{x}}}{\sqrt{a}}\right)+\frac{\sqrt{\frac{ax+b}{x}}\left(-30a^3d^2x^3+105a^2bc^2x^4-644a^2bcdx^3-90a^2bd^2x^2-490ab^2c^2x^3-308ab^2cdx^2-90ab^2d^2x-70b^3c^2x^2-84b^3cdx-30b^3d^2\right)}{105bx^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b/x)^(5/2)*(c + d/x)^2,x]
```

```
[Out] (Sqrt[(b + a*x)/x]*(-30*b^3*d^2 - 84*b^3*c*d*x - 90*a*b^2*d^2*x - 70*b^3*c^
2*x^2 - 308*a*b^2*c*d*x^2 - 90*a^2*b*d^2*x^2 - 490*a*b^2*c^2*x^3 - 644*a^2*
b*c*d*x^3 - 30*a^3*d^2*x^3 + 105*a^2*b*c^2*x^4))/(105*b*x^3) + (5*a^(3/2)*b
*c^2 + 4*a^(5/2)*c*d)*ArcTanh[Sqrt[(b + a*x)/x]/Sqrt[a]]
```

fricas [A] time = 0.82, size = 350, normalized size = 2.30

$$\left[\frac{105\left(5ab^2c^2+4a^2bcd\right)\sqrt{a}x^3\log\left(2ax+2\sqrt{a}x\sqrt{\frac{ax+b}{x}}+b\right)+2\left(105a^2bc^2x^4-30b^3d^2-2\left(245ab^2c^2+322a^2bcd+15a^3d^2\right)x^3-2\left(35b^3c^2+154ab^2cd+45a^2bd^2\right)x^2-6\left(14b^3cd+15ab^2d^2\right)x\right)\sqrt{\frac{ax+b}{x}}}{210bx^3},-\frac{105\left(5ab^2c^2+4a^2bcd\right)\sqrt{-a}x^3\arctan\left(\frac{\sqrt{-a}\sqrt{\frac{ax+b}{x}}}{a}\right)-\left(105a^2bc^2x^4-30b^3d^2-2\left(245ab^2c^2+322a^2bcd+15a^3d^2\right)x^3-2\left(35b^3c^2+154ab^2cd+45a^2bd^2\right)x^2-6\left(14b^3cd+15ab^2d^2\right)x\right)\sqrt{\frac{ax+b}{x}}}{105bx^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(5/2)*(c+d/x)^2,x, algorithm="fricas")
```

```
[Out] [1/210*(105*(5*a*b^2*c^2 + 4*a^2*b*c*d)*sqrt(a)*x^3*log(2*a*x + 2*sqrt(a)*x
*sqrt((a*x + b)/x) + b) + 2*(105*a^2*b*c^2*x^4 - 30*b^3*d^2 - 2*(245*a*b^2*
c^2 + 322*a^2*b*c*d + 15*a^3*d^2)*x^3 - 2*(35*b^3*c^2 + 154*a*b^2*c*d + 45*
a^2*b*d^2)*x^2 - 6*(14*b^3*c*d + 15*a*b^2*d^2)*x)*sqrt((a*x + b)/x))/(b*x^3
), -1/105*(105*(5*a*b^2*c^2 + 4*a^2*b*c*d)*sqrt(-a)*x^3*arctan(sqrt(-a)*sqr
t((a*x + b)/x)/a) - (105*a^2*b*c^2*x^4 - 30*b^3*d^2 - 2*(245*a*b^2*c^2 + 32
2*a^2*b*c*d + 15*a^3*d^2)*x^3 - 2*(35*b^3*c^2 + 154*a*b^2*c*d + 45*a^2*b*d^
2)*x^2 - 6*(14*b^3*c*d + 15*a*b^2*d^2)*x)*sqrt((a*x + b)/x))/(b*x^3)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(5/2)*(c+d/x)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(x)]
Warning, choosing root of [1,0,%%%{-2,[1,2,0]%%%}+%%%{-2,[1,0,0]%%%}+%%%{-2
,[0,1,1]%%%},0,%%%{1,[2,4,0]%%%}+%%%{-2,[2,2,0]%%%}+%%%{1,[2,0,0]%%%}+%%%{2
,[1,3,1]%%%}+%%%{-2,[1,1,1]%%%}+%%%{1,[0,2,2]%%%}] at parameters values [86
,-97,-82]Warning, choosing root of [1,0,%%%{-2,[1,2,0]%%%}+%%%{-2,[1,0,0]%%
%}+%%%{-2,[0,1,1]%%%},0,%%%{1,[2,4,0]%%%}+%%%{-2,[2,2,0]%%%}+%%%{1,[2,0,0]%
%%}+%%%{2,[1,3,1]%%%}+%%%{-2,[1,1,1]%%%}+%%%{1,[0,2,2]%%%}] at parameters v
alues [7,-27,26]Warning, choosing root of [1,0,%%%{-4,[1,0,0]%%%}+%%%{-2,[0
,1,1]%%%},0,%%%{1,[0,2,2]%%%}] at parameters values [18.6420984049,-49,-86]
Warning, choosing root of [1,0,%%%{-4,[1,0,0]%%%}+%%%{-2,[0,1,1]%%%},0,%%%{
1,[0,2,2]%%%}] at parameters values [78.6493344628,22,42]Warning, choosing
root of [1,0,%%%{-2,[1,0,1]%%%}+%%%{-4,[0,1,0]%%%},0,%%%{1,[2,0,2]%%%}] at
parameters values [-13,74.7709350525,24]Sign error (%%%{-b,0%%%}+%%%{2*sqrt
(a)*sqrt(b),1/2%%%}+%%%{-2*a,1%%%}+%%%{a*sqrt(a)*sqrt(b)/b,3/2%%%}+%%%{-a^2
*sqrt(a)*sqrt(b)/(4*b^2),5/2%%%}+%%%{undef,7/2%%%})Evaluation time: 0.81Lim
it: Max order reached or unable to make series expansion Error: Bad Argumen
t Value
```

maple [B] time = 0.06, size = 336, normalized size = 2.21

$$-\frac{\sqrt{\frac{ax+b}{x}}\left(-420a^3bcdx^5\ln\left(\frac{2ax+b+2\sqrt{ax^2+bx}\,\sqrt{a}}{2\sqrt{a}}\right)-525a^2b^2c^2x^5\ln\left(\frac{2ax+b+2\sqrt{ax^2+bx}\,\sqrt{a}}{2\sqrt{a}}\right)-840\sqrt{ax^2+bx}\,a^{\frac{7}{2}}cdx^5-1050\sqrt{ax^2+bx}\,a^{\frac{5}{2}}bc^2x^5+840\left(ax^2+bx\right)^{\frac{3}{2}}a^{\frac{5}{2}}cdx^3+840\left(ax^2+bx\right)^{\frac{3}{2}}a^{\frac{3}{2}}bc^2x^3+60\left(ax^2+bx\right)^{\frac{3}{2}}a^{\frac{5}{2}}d^2x^2+448\left(ax^2+bx\right)^{\frac{3}{2}}a^{\frac{3}{2}}bcdx^2+140\left(ax^2+bx\right)^{\frac{3}{2}}\sqrt{a}\,b^2c^2x^2+120\left(ax^2+bx\right)^{\frac{3}{2}}a^{\frac{3}{2}}bd^2x+168\left(ax^2+bx\right)^{\frac{3}{2}}\sqrt{a}\,b^2cdx+60\left(ax^2+bx\right)^{\frac{3}{2}}\sqrt{a}\,b^2d^2\right)}{210\sqrt{(ax+b)x}\,\sqrt{a}\,bx^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x)^(5/2)*(c+d/x)^2,x)
```

```
[Out] -1/210*((a*x+b)/x)^(1/2)/x^4/b*(-840*(a*x^2+b*x)^(1/2)*a^(7/2)*x^5*c*d-1050
*(a*x^2+b*x)^(1/2)*a^(5/2)*x^5*b*c^2-420*ln(1/2*(2*a*x+b+2*(a*x^2+b*x)^(1/2
)*a^(1/2))/a^(1/2))*x^5*a^3*b*c*d-525*ln(1/2*(2*a*x+b+2*(a*x^2+b*x)^(1/2)*a
^(1/2))/a^(1/2))*x^5*a^2*b^2*c^2+840*(a*x^2+b*x)^(3/2)*a^(5/2)*x^3*c*d+840*
(a*x^2+b*x)^(3/2)*a^(3/2)*x^3*b*c^2+60*(a*x^2+b*x)^(3/2)*a^(5/2)*x^2*d^2+44
8*(a*x^2+b*x)^(3/2)*a^(3/2)*x^2*b*c*d+140*(a*x^2+b*x)^(3/2)*a^(1/2)*x^2*b^2
*c^2+120*(a*x^2+b*x)^(3/2)*a^(3/2)*x*b*d^2+168*(a*x^2+b*x)^(3/2)*a^(1/2)*x*
b^2*c*d+60*(a*x^2+b*x)^(3/2)*a^(1/2)*b^2*d^2)/((a*x+b)*x)^(1/2)/a^(1/2)
```

maxima [A] time = 1.25, size = 181, normalized size = 1.19

$$-\frac{2\left(a+\frac{b}{x}\right)^{\frac{7}{2}}d^2}{7b}+\frac{1}{6}\left(6\sqrt{a+\frac{b}{x}}\,a^2x-15a^{\frac{3}{2}}b\log\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{a+\frac{b}{x}}+\sqrt{a}}\right)-4\left(a+\frac{b}{x}\right)^{\frac{3}{2}}b-24\sqrt{a+\frac{b}{x}}\,ab\right)c^2-\frac{2}{15}\left(15a^{\frac{5}{2}}\log\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{a+\frac{b}{x}}+\sqrt{a}}\right)+6\left(a+\frac{b}{x}\right)^{\frac{5}{2}}+10\left(a+\frac{b}{x}\right)^{\frac{3}{2}}a+30\sqrt{a+\frac{b}{x}}\,a^2\right)cd$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(5/2)*(c+d/x)^2,x, algorithm="maxima")
```

```
[Out] -2/7*(a + b/x)^(7/2)*d^2/b + 1/6*(6*sqrt(a + b/x)*a^2*x - 15*a^(3/2)*b*log(
(sqrt(a + b/x) - sqrt(a))/(sqrt(a + b/x) + sqrt(a))) - 4*(a + b/x)^(3/2)*b
- 24*sqrt(a + b/x)*a*b)*c^2 - 2/15*(15*a^(5/2)*log((sqrt(a + b/x) - sqrt(a)
)/(sqrt(a + b/x) + sqrt(a))) + 6*(a + b/x)^(5/2) + 10*(a + b/x)^(3/2)*a + 3
0*sqrt(a + b/x)*a^2)*c*d
```

mupad [B] time = 3.79, size = 271, normalized size = 1.78

$$\left(a+\frac{b}{x}\right)^{3/2}\left(\frac{2a\left(\frac{4ad^2-4bcd}{b}-\frac{4ad^2}{b}\right)}{3}-\frac{2(ad-bc)^2}{3b}+\frac{2a^2d^2}{3b}\right)+\left(\frac{4ad^2-4bcd}{5b}-\frac{4ad^2}{5b}\right)\left(a+\frac{b}{x}\right)^{5/2}-\sqrt{a+\frac{b}{x}}\left(a^2\left(\frac{4ad^2-4bcd}{b}-\frac{4ad^2}{b}\right)-2a\left(2a\left(\frac{4ad^2-4bcd}{b}-\frac{4ad^2}{b}\right)-\frac{2(ad-bc)^2}{b}+\frac{2a^2d^2}{b}\right)\right)-\frac{2d^2\left(a+\frac{b}{x}\right)^{7/2}}{7b}+a^2c^2x\sqrt{a+\frac{b}{x}}-a^{3/2}c\,\mathrm{atan}\left(\frac{\sqrt{a+\frac{b}{x}}\,1i}{\sqrt{a}}\right)(4ad+5bc)\,1i$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x)^(5/2)*(c + d/x)^2,x)
```

```
[Out] (a + b/x)^(3/2)*((2*a*((4*a*d^2 - 4*b*c*d)/b - (4*a*d^2)/b))/3 - (2*(a*d -
b*c)^2)/(3*b) + (2*a^2*d^2)/(3*b)) + ((4*a*d^2 - 4*b*c*d)/(5*b) - (4*a*d^2)
/(5*b))*(a + b/x)^(5/2) - (a + b/x)^(1/2)*(a^2*((4*a*d^2 - 4*b*c*d)/b - (4*
a*d^2)/b) - 2*a*(2*a*((4*a*d^2 - 4*b*c*d)/b - (4*a*d^2)/b) - (2*(a*d - b*c)
^2)/b + (2*a^2*d^2)/b)) - (2*d^2*(a + b/x)^(7/2))/(7*b) + a^2*c^2*x*(a + b/
x)^(1/2) - a^(3/2)*c*atan(((a + b/x)^(1/2)*1i)/a^(1/2))*(4*a*d + 5*b*c)*1i
```

sympy [A] time = 112.03, size = 1841, normalized size = 12.11

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)**(5/2)*(c+d/x)**2,x)
```

```
[Out] -16*a**(19/2)*b**(13/2)*d**2*x**6*sqrt(a*x/b + 1)/(105*a**(13/2)*b**7*x**(1
3/2) + 315*a**(11/2)*b**8*x**(11/2) + 315*a**(9/2)*b**9*x**(9/2) + 105*a**(
7/2)*b**10*x**(7/2)) - 40*a**(17/2)*b**(15/2)*d**2*x**5*sqrt(a*x/b + 1)/(10
5*a**(13/2)*b**7*x**(13/2) + 315*a**(11/2)*b**8*x**(11/2) + 315*a**(9/2)*b*
*9*x**(9/2) + 105*a**(7/2)*b**10*x**(7/2)) - 30*a**(15/2)*b**(17/2)*d**2*x*
*4*sqrt(a*x/b + 1)/(105*a**(13/2)*b**7*x**(13/2) + 315*a**(11/2)*b**8*x**(1
1/2) + 315*a**(9/2)*b**9*x**(9/2) + 105*a**(7/2)*b**10*x**(7/2)) - 40*a**(1
3/2)*b**(19/2)*d**2*x**3*sqrt(a*x/b + 1)/(105*a**(13/2)*b**7*x**(13/2) + 31
5*a**(11/2)*b**8*x**(11/2) + 315*a**(9/2)*b**9*x**(9/2) + 105*a**(7/2)*b**1
0*x**(7/2)) + 8*a**(13/2)*b**(5/2)*d**2*x**3*sqrt(a*x/b + 1)/(15*a**(7/2)*b
**3*x**(7/2) + 15*a**(5/2)*b**4*x**(5/2)) - 100*a**(11/2)*b**(21/2)*d**2*x*
*2*sqrt(a*x/b + 1)/(105*a**(13/2)*b**7*x**(13/2) + 315*a**(11/2)*b**8*x**(1
```

```
1/2) + 315*a**(9/2)*b**9*x**(9/2) + 105*a**(7/2)*b**10*x**(7/2)) + 8*a**(11
/2)*b**(7/2)*c*d*x**3*sqrt(a*x/b + 1)/(15*a**(7/2)*b**3*x**(7/2) + 15*a**(5
/2)*b**4*x**(5/2)) + 4*a**(11/2)*b**(7/2)*d**2*x**2*sqrt(a*x/b + 1)/(15*a**
(7/2)*b**3*x**(7/2) + 15*a**(5/2)*b**4*x**(5/2)) - 96*a**(9/2)*b**(23/2)*d*
*2*x*sqrt(a*x/b + 1)/(105*a**(13/2)*b**7*x**(13/2) + 315*a**(11/2)*b**8*x**
(11/2) + 315*a**(9/2)*b**9*x**(9/2) + 105*a**(7/2)*b**10*x**(7/2)) + 4*a**(
9/2)*b**(9/2)*c*d*x**2*sqrt(a*x/b + 1)/(15*a**(7/2)*b**3*x**(7/2) + 15*a**(
5/2)*b**4*x**(5/2)) - 16*a**(9/2)*b**(9/2)*d**2*x*sqrt(a*x/b + 1)/(15*a**(7
/2)*b**3*x**(7/2) + 15*a**(5/2)*b**4*x**(5/2)) - 30*a**(7/2)*b**(25/2)*d**2
*sqrt(a*x/b + 1)/(105*a**(13/2)*b**7*x**(13/2) + 315*a**(11/2)*b**8*x**(11/
2) + 315*a**(9/2)*b**9*x**(9/2) + 105*a**(7/2)*b**10*x**(7/2)) - 16*a**(7/2
)*b**(11/2)*c*d*x*sqrt(a*x/b + 1)/(15*a**(7/2)*b**3*x**(7/2) + 15*a**(5/2)*
b**4*x**(5/2)) - 12*a**(7/2)*b**(11/2)*d**2*sqrt(a*x/b + 1)/(15*a**(7/2)*b*
*3*x**(7/2) + 15*a**(5/2)*b**4*x**(5/2)) - 12*a**(5/2)*b**(13/2)*c*d*sqrt(a
*x/b + 1)/(15*a**(7/2)*b**3*x**(7/2) + 15*a**(5/2)*b**4*x**(5/2)) + a**(3/2
)*b*c**2*asinh(sqrt(a)*sqrt(x)/sqrt(b)) + 16*a**10*b**6*d**2*x**(13/2)/(105
*a**(13/2)*b**7*x**(13/2) + 315*a**(11/2)*b**8*x**(11/2) + 315*a**(9/2)*b**
9*x**(9/2) + 105*a**(7/2)*b**10*x**(7/2)) + 48*a**9*b**7*d**2*x**(11/2)/(10
5*a**(13/2)*b**7*x**(13/2) + 315*a**(11/2)*b**8*x**(11/2) + 315*a**(9/2)*b*
*9*x**(9/2) + 105*a**(7/2)*b**10*x**(7/2)) + 48*a**8*b**8*d**2*x**(9/2)/(10
5*a**(13/2)*b**7*x**(13/2) + 315*a**(11/2)*b**8*x**(11/2) + 315*a**(9/2)*b*
*9*x**(9/2) + 105*a**(7/2)*b**10*x**(7/2)) + 16*a**7*b**9*d**2*x**(7/2)/(10
5*a**(13/2)*b**7*x**(13/2) + 315*a**(11/2)*b**8*x**(11/2) + 315*a**(9/2)*b*
*9*x**(9/2) + 105*a**(7/2)*b**10*x**(7/2)) - 8*a**7*b**2*d**2*x**(7/2)/(15*
a**(7/2)*b**3*x**(7/2) + 15*a**(5/2)*b**4*x**(5/2)) - 8*a**6*b**3*c*d*x**(7
/2)/(15*a**(7/2)*b**3*x**(7/2) + 15*a**(5/2)*b**4*x**(5/2)) - 8*a**6*b**3*d
**2*x**(5/2)/(15*a**(7/2)*b**3*x**(7/2) + 15*a**(5/2)*b**4*x**(5/2)) - 8*a*
*5*b**4*c*d*x**(5/2)/(15*a**(7/2)*b**3*x**(7/2) + 15*a**(5/2)*b**4*x**(5/2)
) - 4*a**3*c*d*atan(sqrt(a + b/x)/sqrt(-a))/sqrt(-a) + a**2*sqrt(b)*c**2*sq
rt(x)*sqrt(a*x/b + 1) - 4*a**2*b*c**2*atan(sqrt(a + b/x)/sqrt(-a))/sqrt(-a)
 - 4*a**2*c*d*sqrt(a + b/x) + a**2*d**2*Piecewise((-sqrt(a)/x, Eq(b, 0)), (
-2*(a + b/x)**(3/2)/(3*b), True)) - 4*a*b*c**2*sqrt(a + b/x) + 4*a*b*c*d*Pi
ecewise((-sqrt(a)/x, Eq(b, 0)), (-2*(a + b/x)**(3/2)/(3*b), True)) + b**2*c
**2*Piecewise((-sqrt(a)/x, Eq(b, 0)), (-2*(a + b/x)**(3/2)/(3*b), True))
```

3.143 $\int \left(a+\frac{b}{x}\right)^{5/2} \left(c+\frac{d}{x}\right) dx$

Optimal. Leaf size=125

$$a^{3/2}(2ad+5bc)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)-\frac{\left(a+\frac{b}{x}\right)^{5/2}(2ad+5bc)}{5a}-\frac{1}{3}\left(a+\frac{b}{x}\right)^{3/2}(2ad+5bc)-a\sqrt{a+\frac{b}{x}}(2ad+5bc)+\frac{cx\left(a+\right.}{a}$$

Rubi [A] time = 0.08, antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {375, 78, 50, 63, 208}

$$a^{3/2}(2ad+5bc)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)-\frac{\left(a+\frac{b}{x}\right)^{5/2}(2ad+5bc)}{5a}-\frac{1}{3}\left(a+\frac{b}{x}\right)^{3/2}(2ad+5bc)-a\sqrt{a+\frac{b}{x}}(2ad+5bc)+\frac{cx\left(a+\frac{b}{x}\right)^{7/2}}{a}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x)^(5/2)*(c + d/x),x]
```

```
[Out] -(a*(5*b*c + 2*a*d)*Sqrt[a + b/x]) - ((5*b*c + 2*a*d)*(a + b/x)^(3/2))/3 -
((5*b*c + 2*a*d)*(a + b/x)^(5/2))/(5*a) + (c*(a + b/x)^(7/2)*x)/a + a^(3/2)
*(5*b*c + 2*a*d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 375

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[((a + b/x^n)^p*(c + d/x^n)^q)/x^2, x], x, 1/x] /; FreeQ[{a,
```

```
b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rubi steps

$$\int \left(a+\frac{b}{x}\right)^{5/2}\left(c+\frac{d}{x}\right)dx = -\text{Subst}\left(\int \frac{(a+bx)^{5/2}(c+dx)}{x^2}dx, x, \frac{1}{x}\right)$$
$$= \frac{c\left(a+\frac{b}{x}\right)^{7/2}x}{a} - \frac{\left(\frac{5bc}{2}+ad\right)\text{Subst}\left(\int \frac{(a+bx)^{5/2}}{x}dx, x, \frac{1}{x}\right)}{a}$$
$$= -\frac{(5bc+2ad)\left(a+\frac{b}{x}\right)^{5/2}}{5a} + \frac{c\left(a+\frac{b}{x}\right)^{7/2}x}{a} - \frac{1}{2}(5bc+2ad)\,\text{Subst}\left(\int \frac{(a+bx)^{3/2}}{x}dx,\right.$$
$$= -\frac{1}{3}(5bc+2ad)\left(a+\frac{b}{x}\right)^{3/2} - \frac{(5bc+2ad)\left(a+\frac{b}{x}\right)^{5/2}}{5a} + \frac{c\left(a+\frac{b}{x}\right)^{7/2}x}{a} - \frac{1}{2}(a(5bc+2a$$
$$= -a(5bc+2ad)\sqrt{a+\frac{b}{x}} - \frac{1}{3}(5bc+2ad)\left(a+\frac{b}{x}\right)^{3/2} - \frac{(5bc+2ad)\left(a+\frac{b}{x}\right)^{5/2}}{5a} + \frac{c\left(a\right.}{}$$
$$= -a(5bc+2ad)\sqrt{a+\frac{b}{x}} - \frac{1}{3}(5bc+2ad)\left(a+\frac{b}{x}\right)^{3/2} - \frac{(5bc+2ad)\left(a+\frac{b}{x}\right)^{5/2}}{5a} + \frac{c\left(a\right.}{}$$
$$= -a(5bc+2ad)\sqrt{a+\frac{b}{x}} - \frac{1}{3}(5bc+2ad)\left(a+\frac{b}{x}\right)^{3/2} - \frac{(5bc+2ad)\left(a+\frac{b}{x}\right)^{5/2}}{5a} + \frac{c\left(a\right.}{}$$

Mathematica [A] time = 0.11, size = 94, normalized size = 0.75

$$a^{3/2}(2ad+5bc)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right) + \frac{\sqrt{a+\frac{b}{x}}\left(a^2x^2(15cx-46d)-2abx(35cx+11d)-2b^2(5cx+3d)\right)}{15x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x)^(5/2)*(c + d/x),x]
```

```
[Out] (Sqrt[a + b/x]*(-2*b^2*(3*d + 5*c*x) + a^2*x^2*(-46*d + 15*c*x) - 2*a*b*x*(
11*d + 35*c*x)))/(15*x^2) + a^(3/2)*(5*b*c + 2*a*d)*ArcTanh[Sqrt[a + b/x]/S
qrt[a]]
```

IntegrateAlgebraic [A] time = 0.18, size = 106, normalized size = 0.85

$$\left(5a^{3/2}bc+2a^{5/2}d\right)\tanh^{-1}\left(\frac{\sqrt{\frac{ax+b}{x}}}{\sqrt{a}}\right) + \frac{\sqrt{\frac{ax+b}{x}}\left(15a^2cx^3-46a^2dx^2-70abcx^2-22abdx-10b^2cx-6b^2d\right)}{15x^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b/x)^(5/2)*(c + d/x),x]
```

```
[Out] (Sqrt[(b + a*x)/x]*(-6*b^2*d - 10*b^2*c*x - 22*a*b*d*x - 70*a*b*c*x^2 - 46*
a^2*d*x^2 + 15*a^2*c*x^3))/(15*x^2) + (5*a^(3/2)*b*c + 2*a^(5/2)*d)*ArcTanh
[Sqrt[(b + a*x)/x]/Sqrt[a]]
```

fricas [A] time = 0.66, size = 222, normalized size = 1.78

$$\left[\frac{15\left(5\,abc+2\,a^{2}d\right)\sqrt{a}\,x^{2}\log\left(2\,ax+2\,\sqrt{a}\,x\sqrt{\frac{ax+b}{x}}+b\right)+2\left(15\,a^{2}cx^{3}-6\,b^{2}d-2\left(35\,abc+23\,a^{2}d\right)x^{2}-2\left(5\,b^{2}c+11\,abd\right)x\right)\sqrt{\frac{ax+b}{x}}}{30\,x^{2}},\ -\frac{15\left(5\,abc+2\,a^{2}d\right)\sqrt{-a}\,x^{2}\arctan\left(\frac{\sqrt{-a}\sqrt{\frac{ax+b}{x}}}{a}\right)-\left(15\,a^{2}cx^{3}-6\,b^{2}d-2\left(35\,abc+23\,a^{2}d\right)x^{2}-2\left(5\,b^{2}c+11\,abd\right)x\right)\sqrt{\frac{ax+b}{x}}}{15\,x^{2}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(5/2)*(c+d/x),x, algorithm="fricas")
```

```
[Out] [1/30*(15*(5*a*b*c + 2*a^2*d)*sqrt(a)*x^2*log(2*a*x + 2*sqrt(a)*x*sqrt((a*x
 + b)/x) + b) + 2*(15*a^2*c*x^3 - 6*b^2*d - 2*(35*a*b*c + 23*a^2*d)*x^2 - 2
*(5*b^2*c + 11*a*b*d)*x)*sqrt((a*x + b)/x))/x^2, -1/15*(15*(5*a*b*c + 2*a^2
*d)*sqrt(-a)*x^2*arctan(sqrt(-a)*sqrt((a*x + b)/x)/a) - (15*a^2*c*x^3 - 6*b
^2*d - 2*(35*a*b*c + 23*a^2*d)*x^2 - 2*(5*b^2*c + 11*a*b*d)*x)*sqrt((a*x +
b)/x))/x^2]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(5/2)*(c+d/x),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(x)]
Warning, choosing root of [1,0,%%%{-2,[1,2,0]%%%}+%%%{-2,[1,0,0]%%%}+%%%{-2
,[0,1,1]%%%},0,%%%{1,[2,4,0]%%%}+%%%{-2,[2,2,0]%%%}+%%%{1,[2,0,0]%%%}+%%%{2
,[1,3,1]%%%}+%%%{-2,[1,1,1]%%%}+%%%{1,[0,2,2]%%%}] at parameters values [86
,-97,-82]Warning, choosing root of [1,0,%%%{-2,[1,2,0]%%%}+%%%{-2,[1,0,0]%%
%}+%%%{-2,[0,1,1]%%%},0,%%%{1,[2,4,0]%%%}+%%%{-2,[2,2,0]%%%}+%%%{1,[2,0,0]%
%%}+%%%{2,[1,3,1]%%%}+%%%{-2,[1,1,1]%%%}+%%%{1,[0,2,2]%%%}] at parameters v
alues [7,-27,26]Warning, choosing root of [1,0,%%%{-4,[1,0,0]%%%}+%%%{-2,[0
,1,1]%%%},0,%%%{1,[0,2,2]%%%}] at parameters values [18.6420984049,-49,-86]
Warning, choosing root of [1,0,%%%{-4,[1,0,0]%%%}+%%%{-2,[0,1,1]%%%},0,%%%{
1,[0,2,2]%%%}] at parameters values [78.6493344628,22,42]Warning, choosing
root of [1,0,%%%{-2,[1,0,1]%%%}+%%%{-4,[0,1,0]%%%},0,%%%{1,[2,0,2]%%%}] at
parameters values [-13,74.7709350525,24]Sign error (%%%{-b,0%%%}+%%%{2*sqrt
(a)*sqrt(b),1/2%%%}+%%%{-2*a,1%%%}+%%%{a*sqrt(a)*sqrt(b)/b,3/2%%%}+%%%{-a^2
*sqrt(a)*sqrt(b)/(4*b^2),5/2%%%}+%%%{undef,7/2%%%})Limit: Max order reached
 or unable to make series expansion Error: Bad Argument Value
```

maple [B] time = 0.06, size = 253, normalized size = 2.02

$$-\frac{\sqrt{\frac{ax+b}{x}}\left(-30a^{3}bd\,x^{4}\ln\left(\frac{2ax+b+2\sqrt{a\,x^{2}+bx}\,\sqrt{a}}{2\sqrt{a}}\right)-75a^{2}b^{2}c\,x^{4}\ln\left(\frac{2ax+b+2\sqrt{a\,x^{2}+bx}\,\sqrt{a}}{2\sqrt{a}}\right)-60\sqrt{a\,x^{2}+bx}\,a^{\frac{7}{2}}d\,x^{4}-150\sqrt{a\,x^{2}+bx}\,a^{\frac{5}{2}}bc\,x^{4}+60\left(a\,x^{2}+bx\right)^{\frac{3}{2}}a^{\frac{5}{2}}d\,x^{2}+120\left(a\,x^{2}+bx\right)^{\frac{3}{2}}a^{\frac{3}{2}}bc\,x^{2}+32\left(a\,x^{2}+bx\right)^{\frac{3}{2}}a^{\frac{3}{2}}bdx+20\left(a\,x^{2}+bx\right)^{\frac{3}{2}}\sqrt{a}\,b^{2}cx+12\left(a\,x^{2}+bx\right)^{\frac{3}{2}}\sqrt{a}\,b^{2}d\right)}{30\sqrt{(ax+b)\,x}\,\sqrt{a}\,b\,x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x)^(5/2)*(c+d/x),x)
```

```
[Out] -1/30*((a*x+b)/x)^(1/2)/x^3/b*(-60*(a*x^2+b*x)^(1/2)*a^(7/2)*x^4*d-150*(a*x
^2+b*x)^(1/2)*a^(5/2)*x^4*b*c-30*ln(1/2*(2*a*x+b+2*(a*x^2+b*x)^(1/2)*a^(1/2
))/a^(1/2))*x^4*a^3*b*d-75*ln(1/2*(2*a*x+b+2*(a*x^2+b*x)^(1/2)*a^(1/2))/a^(
1/2))*x^4*a^2*b^2*c+60*(a*x^2+b*x)^(3/2)*a^(5/2)*x^2*d+120*(a*x^2+b*x)^(3/2
)*a^(3/2)*x^2*b*c+32*(a*x^2+b*x)^(3/2)*a^(3/2)*x*b*d+20*(a*x^2+b*x)^(3/2)*a
^(1/2)*x*b^2*c+12*(a*x^2+b*x)^(3/2)*a^(1/2)*b^2*d)/((a*x+b)*x)^(1/2)/a^(1/2
)
```

maxima [A] time = 1.51, size = 161, normalized size = 1.29

$$\frac{1}{6}\left(6\sqrt{a+\frac{b}{x}}\,a^{2}x-15\,a^{\frac{3}{2}}b\log\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{a+\frac{b}{x}}+\sqrt{a}}\right)-4\left(a+\frac{b}{x}\right)^{\frac{3}{2}}b-24\sqrt{a+\frac{b}{x}}\,ab\right)c-\frac{1}{15}\left(15\,a^{\frac{5}{2}}\log\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{a+\frac{b}{x}}+\sqrt{a}}\right)+6\left(a+\frac{b}{x}\right)^{\frac{5}{2}}+10\left(a+\frac{b}{x}\right)^{\frac{3}{2}}a+30\sqrt{a+\frac{b}{x}}\,a^{2}\right)d$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(5/2)*(c+d/x),x, algorithm="maxima")
```

```
[Out] 1/6*(6*sqrt(a + b/x)*a^2*x - 15*a^(3/2)*b*log((sqrt(a + b/x) - sqrt(a))/(sq
rt(a + b/x) + sqrt(a))) - 4*(a + b/x)^(3/2)*b - 24*sqrt(a + b/x)*a*b)*c - 1
/15*(15*a^(5/2)*log((sqrt(a + b/x) - sqrt(a))/(sqrt(a + b/x) + sqrt(a))) +
6*(a + b/x)^(5/2) + 10*(a + b/x)^(3/2)*a + 30*sqrt(a + b/x)*a^2)*d
```

mupad [B] time = 3.48, size = 99, normalized size = 0.79

$$-\frac{2\,d\,\left(a+\frac{b}{x}\right)^{5/2}}{5}-2\,a^2\,d\,\sqrt{a+\frac{b}{x}}-\frac{2\,a\,d\,\left(a+\frac{b}{x}\right)^{3/2}}{3}-\frac{2\,c\,x\,\left(a+\frac{b}{x}\right)^{5/2}\,{}_2\mathrm{F}_1\left(-\frac{5}{2},-\frac{3}{2};\ -\frac{1}{2};\ -\frac{a\,x}{b}\right)}{3\,{\left(\frac{a\,x}{b}+1\right)}^{5/2}}-a^{5/2}\,d\,\mathrm{atan}\left(\frac{\sqrt{a+\frac{b}{x}}\,1\mathrm{i}}{\sqrt{a}}\right)\,2\mathrm{i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x)^(5/2)*(c + d/x),x)
```

```
[Out] - (2*d*(a + b/x)^(5/2))/5 - 2*a^2*d*(a + b/x)^(1/2) - a^(5/2)*d*atan(((a +
b/x)^(1/2)*1i)/a^(1/2))*2i - (2*a*d*(a + b/x)^(3/2))/3 - (2*c*x*(a + b/x)^(
5/2)*hypergeom([-5/2, -3/2], -1/2, -(a*x)/b))/(3*((a*x)/b + 1)^(5/2))
```

sympy [A] time = 82.77, size = 520, normalized size = 4.16

$$\frac{4a^{\frac{11}{2}}b^{\frac{7}{2}}dx^3\sqrt{\frac{ax}{b}+1}}{15a^{\frac{7}{2}}b^3x^{\frac{7}{2}}+15a^{\frac{5}{2}}b^4x^{\frac{5}{2}}}+\frac{2a^{\frac{9}{2}}b^{\frac{9}{2}}dx^2\sqrt{\frac{ax}{b}+1}}{15a^{\frac{7}{2}}b^3x^{\frac{7}{2}}+15a^{\frac{5}{2}}b^4x^{\frac{5}{2}}}-\frac{8a^{\frac{7}{2}}b^{\frac{11}{2}}dx\sqrt{\frac{ax}{b}+1}}{15a^{\frac{7}{2}}b^3x^{\frac{7}{2}}+15a^{\frac{5}{2}}b^4x^{\frac{5}{2}}}-\frac{6a^{\frac{5}{2}}b^{\frac{13}{2}}d\sqrt{\frac{ax}{b}+1}}{15a^{\frac{7}{2}}b^3x^{\frac{7}{2}}+15a^{\frac{5}{2}}b^4x^{\frac{5}{2}}}+a^{\frac{3}{2}}bc\,\mathrm{asinh}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)-\frac{4a^6b^3dx^{\frac{7}{2}}}{15a^{\frac{7}{2}}b^3x^{\frac{7}{2}}+15a^{\frac{5}{2}}b^4x^{\frac{5}{2}}}-\frac{4a^5b^4dx^{\frac{5}{2}}}{15a^{\frac{7}{2}}b^3x^{\frac{7}{2}}+15a^{\frac{5}{2}}b^4x^{\frac{5}{2}}}-\frac{2a^3d\,\mathrm{atan}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{-a}}\right)}{\sqrt{-a}}+a^2\sqrt{b}c\sqrt{x}\sqrt{\frac{ax}{b}+1}-\frac{4a^2bc\,\mathrm{atan}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{-a}}\right)}{\sqrt{-a}}-2a^2d\sqrt{a+\frac{b}{x}}-4abc\sqrt{a+\frac{b}{x}}+2abd\left(\begin{cases}-\frac{\sqrt{a}}{x} & \text{for}\ b=0\\-\frac{2\left(a+\frac{b}{x}\right)^{\frac{3}{2}}}{3b} & \text{otherwise}\end{cases}\right)+b^2c\left(\begin{cases}-\frac{\sqrt{a}}{x} & \text{for}\ b=0\\-\frac{2\left(a+\frac{b}{x}\right)^{\frac{3}{2}}}{3b} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)**(5/2)*(c+d/x),x)
```

```
[Out] 4*a**(11/2)*b**(7/2)*d*x**3*sqrt(a*x/b + 1)/(15*a**(7/2)*b**3*x**(7/2) + 15
*a**(5/2)*b**4*x**(5/2)) + 2*a**(9/2)*b**(9/2)*d*x**2*sqrt(a*x/b + 1)/(15*a
**(7/2)*b**3*x**(7/2) + 15*a**(5/2)*b**4*x**(5/2)) - 8*a**(7/2)*b**(11/2)*d
*x*sqrt(a*x/b + 1)/(15*a**(7/2)*b**3*x**(7/2) + 15*a**(5/2)*b**4*x**(5/2))
- 6*a**(5/2)*b**(13/2)*d*sqrt(a*x/b + 1)/(15*a**(7/2)*b**3*x**(7/2) + 15*a*
*(5/2)*b**4*x**(5/2)) + a**(3/2)*b*c*asinh(sqrt(a)*sqrt(x)/sqrt(b)) - 4*a**
6*b**3*d*x**(7/2)/(15*a**(7/2)*b**3*x**(7/2) + 15*a**(5/2)*b**4*x**(5/2)) -
 4*a**5*b**4*d*x**(5/2)/(15*a**(7/2)*b**3*x**(7/2) + 15*a**(5/2)*b**4*x**(5
/2)) - 2*a**3*d*atan(sqrt(a + b/x)/sqrt(-a))/sqrt(-a) + a**2*sqrt(b)*c*sqrt
(x)*sqrt(a*x/b + 1) - 4*a**2*b*c*atan(sqrt(a + b/x)/sqrt(-a))/sqrt(-a) - 2*
a**2*d*sqrt(a + b/x) - 4*a*b*c*sqrt(a + b/x) + 2*a*b*d*Piecewise((-sqrt(a)/
x, Eq(b, 0)), (-2*(a + b/x)**(3/2)/(3*b), True)) + b**2*c*Piecewise((-sqrt(
a)/x, Eq(b, 0)), (-2*(a + b/x)**(3/2)/(3*b), True))
```

3.144 $\int \left(a+\frac{b}{x}\right)^{5/2} dx$

Optimal. Leaf size=71

$$5a^{3/2}b\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)+x\left(a+\frac{b}{x}\right)^{5/2}-\frac{5}{3}b\left(a+\frac{b}{x}\right)^{3/2}-5ab\sqrt{a+\frac{b}{x}}$$

Rubi [A] time = 0.03, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.454, Rules used = {242, 47, 50, 63, 208}

$$5a^{3/2}b\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)+x\left(a+\frac{b}{x}\right)^{5/2}-\frac{5}{3}b\left(a+\frac{b}{x}\right)^{3/2}-5ab\sqrt{a+\frac{b}{x}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x)^(5/2),x]
```

```
[Out] -5*a*b*Sqrt[a + b/x] - (5*b*(a + b/x)^(3/2))/3 + (a + b/x)^(5/2)*x + 5*a^(3
/2)*b*ArcTanh[Sqrt[a + b/x]/Sqrt[a]]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] && !(IntegerQ[n] && !Intege
rQ[m]) && !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 242

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a + b/x^n)^p/x^
2, x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\int \left(a+\frac{b}{x}\right)^{5/2} dx &= -\text{Subst}\left(\int \frac{(a+bx)^{5/2}}{x^2} dx, x, \frac{1}{x}\right)\\
&= \left(a+\frac{b}{x}\right)^{5/2} x - \frac{1}{2}(5b)\,\text{Subst}\left(\int \frac{(a+bx)^{3/2}}{x} dx, x, \frac{1}{x}\right)\\
&= -\frac{5}{3}b\left(a+\frac{b}{x}\right)^{3/2} + \left(a+\frac{b}{x}\right)^{5/2} x - \frac{1}{2}(5ab)\,\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x} dx, x, \frac{1}{x}\right)\\
&= -5ab\sqrt{a+\frac{b}{x}} - \frac{5}{3}b\left(a+\frac{b}{x}\right)^{3/2} + \left(a+\frac{b}{x}\right)^{5/2} x - \frac{1}{2}\left(5a^2b\right)\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}} dx, x, \frac{1}{x}\right)\\
&= -5ab\sqrt{a+\frac{b}{x}} - \frac{5}{3}b\left(a+\frac{b}{x}\right)^{3/2} + \left(a+\frac{b}{x}\right)^{5/2} x - \left(5a^2\right)\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}} dx, x, \sqrt{a+\frac{b}{x}}\right)\\
&= -5ab\sqrt{a+\frac{b}{x}} - \frac{5}{3}b\left(a+\frac{b}{x}\right)^{3/2} + \left(a+\frac{b}{x}\right)^{5/2} x + 5a^{3/2}b\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)
\end{aligned}
$$

Mathematica [A] time = 0.06, size = 64, normalized size = 0.90

$$5a^{3/2}b\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right) + \frac{\sqrt{a+\frac{b}{x}}\left(3a^2x^2-14abx-2b^2\right)}{3x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x)^(5/2),x]
[Out] (Sqrt[a + b/x]*(-2*b^2 - 14*a*b*x + 3*a^2*x^2))/(3*x) + 5*a^(3/2)*b*ArcTanh
[Sqrt[a + b/x]/Sqrt[a]]
```

IntegrateAlgebraic [A] time = 0.00, size = 68, normalized size = 0.96

$$5a^{3/2}b\tanh^{-1}\left(\frac{\sqrt{\frac{ax+b}{x}}}{\sqrt{a}}\right) + \frac{\sqrt{\frac{ax+b}{x}}\left(3a^2x^2-14abx-2b^2\right)}{3x}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b/x)^(5/2),x]
[Out] (Sqrt[(b + a*x)/x]*(-2*b^2 - 14*a*b*x + 3*a^2*x^2))/(3*x) + 5*a^(3/2)*b*Arc
Tanh[Sqrt[(b + a*x)/x]/Sqrt[a]]
```

fricas [A] time = 0.88, size = 139, normalized size = 1.96

$$\left[\frac{15\,a^{\frac{3}{2}}bx\log\left(2\,ax+2\,\sqrt{a}\,x\sqrt{\frac{ax+b}{x}}+b\right)+2\left(3\,a^2x^2-14\,abx-2\,b^2\right)\sqrt{\frac{ax+b}{x}}}{6\,x},\ -\frac{15\,\sqrt{-a}\,abx\arctan\left(\frac{\sqrt{-a}\,\sqrt{\frac{ax+b}{x}}}{a}\right)-\left(3\,a^2x^2-14\,abx-2\,b^2\right)\sqrt{\frac{ax+b}{x}}}{3\,x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(5/2),x, algorithm="fricas")
[Out] [1/6*(15*a^(3/2)*b*x*log(2*a*x + 2*sqrt(a)*x*sqrt((a*x + b)/x) + b) + 2*(3*
a^2*x^2 - 14*a*b*x - 2*b^2)*sqrt((a*x + b)/x))/x, -1/3*(15*sqrt(-a)*a*b*x*a
```

```
rctan(sqrt(-a)*sqrt((a*x + b)/x)/a) - (3*a^2*x^2 - 14*a*b*x - 2*b^2)*sqrt((
a*x + b)/x))/x]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(x)]
Warning, choosing root of [1,0,%%%{-2,[1,2,0]%%%}+%%%{-2,[1,0,0]%%%}+%%%{-2
,[0,1,1]%%%},0,%%%{1,[2,4,0]%%%}+%%%{-2,[2,2,0]%%%}+%%%{1,[2,0,0]%%%}+%%%{2
,[1,3,1]%%%}+%%%{-2,[1,1,1]%%%}+%%%{1,[0,2,2]%%%}] at parameters values [86
,-97,-82]Warning, choosing root of [1,0,%%%{-4,[1,0,0]%%%}+%%%{-2,[0,1,1]%%
%},0,%%%{1,[0,2,2]%%%}] at parameters values [82.1195442914,26,-89]Warning,
 choosing root of [1,0,%%%{-4,[1,0,0]%%%}+%%%{-2,[0,1,1]%%%},0,%%%{1,[0,2,2
]%%%}] at parameters values [85.3561567818,-64,-30]Warning, choosing root o
f [1,0,%%%{-2,[1,0,1]%%%}+%%%{-4,[0,1,0]%%%},0,%%%{1,[2,0,2]%%%}] at parame
ters values [42,43.9628838282,-9]Sign error (%%%{-b,0%%%}+%%%{2*sqrt(a)*sqr
t(b),1/2%%%}+%%%{-2*a,1%%%}+%%%{a*sqrt(a)*sqrt(b)/b,3/2%%%}+%%%{-a^2*sqrt(a
)*sqrt(b)/(4*b^2),5/2%%%}+%%%{undef,7/2%%%})Limit: Max order reached or una
ble to make series expansion Error: Bad Argument Value
```

maple [B] time = 0.06, size = 120, normalized size = 1.69

$$\frac{\sqrt{\frac{ax+b}{x}}\,\left(15a^{2}b\,x^{3}\ln\left(\frac{2ax+b+2\sqrt{a\,x^{2}+bx}\,\sqrt{a}}{2\sqrt{a}}\right)+30\sqrt{a\,x^{2}+bx}\;a^{\frac{5}{2}}x^{3}-24\left(a\,x^{2}+bx\right)^{\frac{3}{2}}a^{\frac{3}{2}}x-4\left(a\,x^{2}+bx\right)^{\frac{3}{2}}\sqrt{a}\;b\right)}{6\sqrt{\left(ax+b\right)x}\;\sqrt{a}\;x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x)^(5/2),x)
```

```
[Out] 1/6*((a*x+b)/x)^(1/2)*(30*a^(5/2)*(a*x^2+b*x)^(1/2)*x^3-24*a^(3/2)*(a*x^2+b
*x)^(3/2)*x+15*ln(1/2*(2*a*x+b+2*(a*x^2+b*x)^(1/2)*a^(1/2))/a^(1/2))*x^3*a^
2*b-4*b*(a*x^2+b*x)^(3/2)*a^(1/2))/x^2/((a*x+b)*x)^(1/2)/a^(1/2)
```

maxima [A] time = 1.25, size = 78, normalized size = 1.10

$$\sqrt{a+\frac{b}{x}}\,a^{2}x-\frac{5}{2}\,a^{\frac{3}{2}}b\log\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{a+\frac{b}{x}}+\sqrt{a}}\right)-\frac{2}{3}\left(a+\frac{b}{x}\right)^{\frac{3}{2}}b-4\sqrt{a+\frac{b}{x}}\,ab$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(5/2),x, algorithm="maxima")
```

```
[Out] sqrt(a + b/x)*a^2*x - 5/2*a^(3/2)*b*log((sqrt(a + b/x) - sqrt(a))/(sqrt(a +
 b/x) + sqrt(a))) - 2/3*(a + b/x)^(3/2)*b - 4*sqrt(a + b/x)*a*b
```

mupad [B] time = 1.63, size = 34, normalized size = 0.48

$$-\frac{2\,x\,\left(a+\frac{b}{x}\right)^{5/2}\,{}_2\mathrm{F}_1\left(-\frac{5}{2},-\frac{3}{2};\,-\frac{1}{2};\,-\frac{a\,x}{b}\right)}{3\,\left(\frac{a\,x}{b}+1\right)^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x)^(5/2),x)
```

```
[Out] -(2*x*(a + b/x)^(5/2)*hypergeom([-5/2, -3/2], -1/2, -(a*x)/b))/(3*((a*x)/b
+ 1)^(5/2))
```

sympy [A] time = 4.36, size = 99, normalized size = 1.39

$$a^{\frac{5}{2}}x\sqrt{1+\frac{b}{ax}}-\frac{14a^{\frac{3}{2}}b\sqrt{1+\frac{b}{ax}}}{3}-\frac{5a^{\frac{3}{2}}b\log\left(\frac{b}{ax}\right)}{2}+5a^{\frac{3}{2}}b\log\left(\sqrt{1+\frac{b}{ax}}+1\right)-\frac{2\sqrt{a}\,b^{2}\sqrt{1+\frac{b}{ax}}}{3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)**(5/2),x)
```

```
[Out] a**(5/2)*x*sqrt(1 + b/(a*x)) - 14*a**(3/2)*b*sqrt(1 + b/(a*x))/3 - 5*a**(3/
2)*b*log(b/(a*x))/2 + 5*a**(3/2)*b*log(sqrt(1 + b/(a*x)) + 1) - 2*sqrt(a)*b
**2*sqrt(1 + b/(a*x))/(3*x)
```

3.145 $\int \frac{\left(a+\frac{b}{x}\right)^{5/2}}{c+\frac{d}{x}}\,dx$

Optimal. Leaf size=134

$$\frac{a^{3/2}(5bc-2ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{c^2}+\frac{2(bc-ad)^{5/2}\tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c^2d^{3/2}}-\frac{b\sqrt{a+\frac{b}{x}}(ad+2bc)}{cd}+\frac{ax\left(a+\frac{b}{x}\right)^{3/2}}{c}$$

Rubi [A] time = 0.22, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {375, 98, 154, 156, 63, 208, 205}

$$\frac{a^{3/2}(5bc-2ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{c^2}+\frac{2(bc-ad)^{5/2}\tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c^2d^{3/2}}-\frac{b\sqrt{a+\frac{b}{x}}(ad+2bc)}{cd}+\frac{ax\left(a+\frac{b}{x}\right)^{3/2}}{c}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x)^(5/2)/(c + d/x),x]
```

```
[Out] -((b*(2*b*c + a*d)*Sqrt[a + b/x])/(c*d)) + (a*(a + b/x)^(3/2)*x)/c + (2*(b*
c - a*d)^(5/2)*ArcTan[(Sqrt[d]*Sqrt[a + b/x])/Sqrt[b*c - a*d]])/(c^2*d^(3/2
)) + (a^(3/2)*(5*b*c - 2*a*d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/c^2
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 154

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(h*(a + b*x)^m*(c + d*x)^(n
+ 1)*(e + f*x)^(p + 1))/(d*f*(m + n + p + 2)), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
```

```
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 375

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[((a + b/x^n)^p*(c + d/x^n)^q)/x^2, x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rubi steps

$$\int \frac{\left(a+\frac{b}{x}\right)^{5/2}}{c+\frac{d}{x}}\,dx = -\text{Subst}\left(\int \frac{(a+bx)^{5/2}}{x^2(c+dx)}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{a\left(a+\frac{b}{x}\right)^{3/2}x}{c} + \frac{\text{Subst}\left(\int \frac{\sqrt{a+bx}\left(-\frac{1}{2}a(5bc-2ad)-\frac{1}{2}b(2bc+ad)x\right)}{x(c+dx)}\,dx, x, \frac{1}{x}\right)}{c}$$

$$= -\frac{b(2bc+ad)\sqrt{a+\frac{b}{x}}}{cd} + \frac{a\left(a+\frac{b}{x}\right)^{3/2}x}{c} + \frac{2\,\text{Subst}\left(\int \frac{-\frac{1}{4}a^2d(5bc-2ad)+\frac{1}{4}b\left(2b^2c^2-6abcd+a^2d^2\right)x}{x\sqrt{a+bx}\,(c+dx)}\,dx, x, \ldots\right)}{cd}$$

$$= -\frac{b(2bc+ad)\sqrt{a+\frac{b}{x}}}{cd} + \frac{a\left(a+\frac{b}{x}\right)^{3/2}x}{c} - \frac{\left(a^2(5bc-2ad)\right)\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \frac{1}{x}\right)}{2c^2} + \frac{(bc-\ldots}{\ldots}$$

$$= -\frac{b(2bc+ad)\sqrt{a+\frac{b}{x}}}{cd} + \frac{a\left(a+\frac{b}{x}\right)^{3/2}x}{c} - \frac{\left(a^2(5bc-2ad)\right)\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+\frac{b}{x}}\right)}{bc^2} + \ldots$$

$$= -\frac{b(2bc+ad)\sqrt{a+\frac{b}{x}}}{cd} + \frac{a\left(a+\frac{b}{x}\right)^{3/2}x}{c} + \frac{2(bc-ad)^{5/2}\tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c^2d^{3/2}} + \frac{a^{3/2}(5bc-2ad)\,\text{t}\ldots}{c^2\ldots}$$

Mathematica [A] time = 0.27, size = 116, normalized size = 0.87

$$\frac{a^{3/2}(5bc-2ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)+\frac{c\sqrt{a+\frac{b}{x}}\left(a^2dx-2b^2c\right)}{d}+\frac{2(bc-ad)^{5/2}\tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{d^{3/2}}}{c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x)^(5/2)/(c + d/x),x]
```

```
[Out] ((c*Sqrt[a + b/x]*(-2*b^2*c + a^2*d*x))/d + (2*(b*c - a*d)^(5/2)*ArcTan[(Sq
rt[d]*Sqrt[a + b/x])/Sqrt[b*c - a*d]])/d^(3/2) + a^(3/2)*(5*b*c - 2*a*d)*Ar
cTanh[Sqrt[a + b/x]/Sqrt[a]])/c^2
```

IntegrateAlgebraic [A] time = 0.30, size = 130, normalized size = 0.97

$$\frac{\left(5a^{3/2}bc-2a^{5/2}d\right)\tanh^{-1}\left(\frac{\sqrt{\frac{ax+b}{x}}}{\sqrt{a}}\right)}{c^2}+\frac{\sqrt{\frac{ax+b}{x}}\left(a^2dx-2b^2c\right)}{cd}+\frac{2(bc-ad)^{5/2}\tan^{-1}\left(\frac{\sqrt{d}\sqrt{\frac{ax+b}{x}}}{\sqrt{bc-ad}}\right)}{c^2d^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b/x)^(5/2)/(c + d/x),x]
```

```
[Out] (Sqrt[(b + a*x)/x]*(-2*b^2*c + a^2*d*x))/(c*d) + (2*(b*c - a*d)^(5/2)*ArcTa
n[(Sqrt[d]*Sqrt[(b + a*x)/x])/Sqrt[b*c - a*d]])/(c^2*d^(3/2)) + ((5*a^(3/2)
*b*c - 2*a^(5/2)*d)*ArcTanh[Sqrt[(b + a*x)/x]/Sqrt[a]])/c^2
```

fricas [A] time = 1.15, size = 659, normalized size = 4.92

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(5/2)/(c+d/x),x, algorithm="fricas")
```

```
[Out] [-1/2*((5*a*b*c*d - 2*a^2*d^2)*sqrt(a)*log(2*a*x - 2*sqrt(a)*x*sqrt((a*x +
b)/x) + b) - 2*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(-(b*c - a*d)/d)*log((2*
d*x*sqrt(-(b*c - a*d)/d)*sqrt((a*x + b)/x) + b*d - (b*c - 2*a*d)*x)/(c*x +
d)) - 2*(a^2*c*d*x - 2*b^2*c^2)*sqrt((a*x + b)/x))/(c^2*d), -((5*a*b*c*d -
2*a^2*d^2)*sqrt(-a)*arctan(sqrt(-a)*sqrt((a*x + b)/x)/a) - (b^2*c^2 - 2*a*b
*c*d + a^2*d^2)*sqrt(-(b*c - a*d)/d)*log((2*d*x*sqrt(-(b*c - a*d)/d)*sqrt((
a*x + b)/x) + b*d - (b*c - 2*a*d)*x)/(c*x + d)) - (a^2*c*d*x - 2*b^2*c^2)*s
qrt((a*x + b)/x))/(c^2*d), -1/2*(4*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt((b*
c - a*d)/d)*arctan(-d*sqrt((b*c - a*d)/d)*sqrt((a*x + b)/x)/(b*c - a*d)) +
(5*a*b*c*d - 2*a^2*d^2)*sqrt(a)*log(2*a*x - 2*sqrt(a)*x*sqrt((a*x + b)/x) +
 b) - 2*(a^2*c*d*x - 2*b^2*c^2)*sqrt((a*x + b)/x))/(c^2*d), -(2*(b^2*c^2 -
2*a*b*c*d + a^2*d^2)*sqrt((b*c - a*d)/d)*arctan(-d*sqrt((b*c - a*d)/d)*sqrt
((a*x + b)/x)/(b*c - a*d)) + (5*a*b*c*d - 2*a^2*d^2)*sqrt(-a)*arctan(sqrt(-
a)*sqrt((a*x + b)/x)/a) - (a^2*c*d*x - 2*b^2*c^2)*sqrt((a*x + b)/x))/(c^2*d
)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(5/2)/(c+d/x),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(x)]
Error: Bad Argument Type
```

maple [B] time = 0.06, size = 859, normalized size = 6.41

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x)^(5/2)/(c+d/x),x)
```

```
[Out] -1/2*((a*x+b)/x)^(1/2)/x*(2*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^
(1/2))*((a*d-b*c)/c^2*d)^(1/2)*x^2*a^3*c*d^3-5*ln(1/2*(2*a*x+b+2*((a*x+b)*x
)^(1/2)*a^(1/2))/a^(1/2))*((a*d-b*c)/c^2*d)^(1/2)*x^2*a^2*b*c^2*d^2+4*ln(1/
```

```
2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))*((a*d-b*c)/c^2*d)^(1/2)*x^
2*a*b^2*c^3*d-ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))*((a*d-b
*c)/c^2*d)^(1/2)*x^2*b^3*c^4-8*((a*d-b*c)/c^2*d)^(1/2)*(a*x^2+b*x)^(1/2)*a^
(3/2)*x^2*b*c^3*d+2*((a*d-b*c)/c^2*d)^(1/2)*(a*x^2+b*x)^(1/2)*a^(1/2)*x^2*b
^2*c^4-4*((a*d-b*c)/c^2*d)^(1/2)*ln(1/2*(2*a*x+b+2*(a*x^2+b*x)^(1/2)*a^(1/2
))/a^(1/2))*x^2*a*b^2*c^3*d+((a*d-b*c)/c^2*d)^(1/2)*ln(1/2*(2*a*x+b+2*(a*x^
2+b*x)^(1/2)*a^(1/2))/a^(1/2))*x^2*b^3*c^4-2*((a*d-b*c)/c^2*d)^(1/2)*a^(5/2
)*((a*x+b)*x)^(1/2)*x^2*c^2*d^2+4*((a*d-b*c)/c^2*d)^(1/2)*a^(3/2)*((a*x+b)*
x)^(1/2)*x^2*b*c^3*d-2*((a*d-b*c)/c^2*d)^(1/2)*a^(1/2)*((a*x+b)*x)^(1/2)*x^
2*b^2*c^4+2*ln((-2*a*d*x+b*c*x-b*d+2*((a*d-b*c)/c^2*d)^(1/2)*((a*x+b)*x)^(1
/2)*c)/(c*x+d))*a^(7/2)*x^2*d^4-6*ln((-2*a*d*x+b*c*x-b*d+2*((a*d-b*c)/c^2*d
)^(1/2)*((a*x+b)*x)^(1/2)*c)/(c*x+d))*a^(5/2)*x^2*b*c*d^3+6*ln((-2*a*d*x+b*
c*x-b*d+2*((a*d-b*c)/c^2*d)^(1/2)*((a*x+b)*x)^(1/2)*c)/(c*x+d))*a^(3/2)*x^2
*b^2*c^2*d^2-2*ln((-2*a*d*x+b*c*x-b*d+2*((a*d-b*c)/c^2*d)^(1/2)*((a*x+b)*x)
^(1/2)*c)/(c*x+d))*a^(1/2)*x^2*b^3*c^3*d+4*((a*d-b*c)/c^2*d)^(1/2)*(a*x^2+b
*x)^(3/2)*a^(1/2)*b*c^3*d)/((a*x+b)*x)^(1/2)/d^2/c^3/a^(1/2)/((a*d-b*c)/c^2
*d)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+\frac{b}{x}\right)^{\frac{5}{2}}}{c+\frac{d}{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(5/2)/(c+d/x),x, algorithm="maxima")
[Out] integrate((a + b/x)^(5/2)/(c + d/x), x)
```

mupad [B] time = 2.16, size = 1427, normalized size = 10.65

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x)^(5/2)/(c + d/x),x)
[Out] (atan((a^3*b^5*(a + b/x)^(1/2)*(a^5*d^8 - b^5*c^5*d^3 + 5*a*b^4*c^4*d^4 - 1
0*a^2*b^3*c^3*d^5 + 10*a^3*b^2*c^2*d^6 - 5*a^4*b*c*d^7)^(1/2)*160i)/(448*a^
3*b^8*c^3*d - 340*a^6*b^5*d^4 - 128*a^2*b^9*c^4 + 740*a^5*b^6*c*d^3 + (16*a
*b^10*c^5)/d - 796*a^4*b^7*c^2*d^2 + (60*a^7*b^4*d^5)/c) - (a^2*b^6*(a + b/
x)^(1/2)*(a^5*d^8 - b^5*c^5*d^3 + 5*a*b^4*c^4*d^4 - 10*a^2*b^3*c^3*d^5 + 10
*a^3*b^2*c^2*d^6 - 5*a^4*b*c*d^7)^(1/2)*80i)/(16*a*b^10*c^4 + 740*a^5*b^6*d
^4 - 128*a^2*b^9*c^3*d - 796*a^4*b^7*c*d^3 + 448*a^3*b^8*c^2*d^2 - (340*a^6
*b^5*d^5)/c + (60*a^7*b^4*d^6)/c^2) - (a^4*b^4*(a + b/x)^(1/2)*(a^5*d^8 - b
^5*c^5*d^3 + 5*a*b^4*c^4*d^4 - 10*a^2*b^3*c^3*d^5 + 10*a^3*b^2*c^2*d^6 - 5*
a^4*b*c*d^7)^(1/2)*60i)/(448*a^3*b^8*c^4 + 60*a^7*b^4*d^4 - 796*a^4*b^7*c^3
*d - 340*a^6*b^5*c*d^3 + (16*a*b^10*c^6)/d^2 + 740*a^5*b^6*c^2*d^2 - (128*a
^2*b^9*c^5)/d) + (a*b^7*c*(a + b/x)^(1/2)*(a^5*d^8 - b^5*c^5*d^3 + 5*a*b^4*
c^4*d^4 - 10*a^2*b^3*c^3*d^5 + 10*a^3*b^2*c^2*d^6 - 5*a^4*b*c*d^7)^(1/2)*16
i)/(740*a^5*b^6*d^5 - 796*a^4*b^7*c*d^4 - 128*a^2*b^9*c^3*d^2 + 448*a^3*b^8
*c^2*d^3 - (340*a^6*b^5*d^6)/c + (60*a^7*b^4*d^7)/c^2 + 16*a*b^10*c^4*d))*(
d^3*(a*d - b*c)^5)^(1/2)*2i)/(c^2*d^3) - (2*b^2*(a + b/x)^(1/2))/d + (atan(
(b^9*c^3*(a + b/x)^(1/2)*(a^3)^(1/2)*40i)/(40*a^2*b^9*c^3 - 790*a^5*b^6*d^3
 - 256*a^3*b^8*c^2*d + 696*a^4*b^7*c*d^2 + (370*a^6*b^5*d^4)/c - (60*a^7*b^
4*d^5)/c^2) + (a*b^8*c^2*(a + b/x)^(1/2)*(a^3)^(1/2)*256i)/(256*a^3*b^8*c^2
 + 790*a^5*b^6*d^2 - (40*a^2*b^9*c^3)/d - (370*a^6*b^5*d^3)/c + (60*a^7*b^4
*d^4)/c^2 - 696*a^4*b^7*c*d) + (a^3*b^6*d^2*(a + b/x)^(1/2)*(a^3)^(1/2)*790
i)/(256*a^3*b^8*c^2 + 790*a^5*b^6*d^2 - (40*a^2*b^9*c^3)/d - (370*a^6*b^5*d
^3)/c + (60*a^7*b^4*d^4)/c^2 - 696*a^4*b^7*c*d) - (a^4*b^5*d^3*(a + b/x)^(1
```

```
/2)*(a^3)^(1/2)*370i)/(256*a^3*b^8*c^3 - 370*a^6*b^5*d^3 - 696*a^4*b^7*c^2*
d + 790*a^5*b^6*c*d^2 - (40*a^2*b^9*c^4)/d + (60*a^7*b^4*d^4)/c) + (a^5*b^4
*d^4*(a + b/x)^(1/2)*(a^3)^(1/2)*60i)/(256*a^3*b^8*c^4 + 60*a^7*b^4*d^4 - 6
96*a^4*b^7*c^3*d - 370*a^6*b^5*c*d^3 + 790*a^5*b^6*c^2*d^2 - (40*a^2*b^9*c^
5)/d) - (a^2*b^7*c*d*(a + b/x)^(1/2)*(a^3)^(1/2)*696i)/(256*a^3*b^8*c^2 + 7
90*a^5*b^6*d^2 - (40*a^2*b^9*c^3)/d - (370*a^6*b^5*d^3)/c + (60*a^7*b^4*d^4
)/c^2 - 696*a^4*b^7*c*d))*(2*a*d - 5*b*c)*(a^3)^(1/2)*1i)/c^2 + (a^2*b*d*(a
 + b/x)^(1/2))/(c*(d*(a + b/x) - a*d))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)**(5/2)/(c+d/x),x)
```

```
[Out] Timed out
```

3.146 $\int \frac{\left(a+\frac{b}{x}\right)^{5/2}}{\left(c+\frac{d}{x}\right)^2} dx$

Optimal. Leaf size=166

$$\frac{a^{3/2}(5bc-4ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{c^3}-\frac{(bc-ad)^{3/2}(4ad+bc)\tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c^3d^{3/2}}+\frac{\sqrt{a+\frac{b}{x}}(bc-2ad)(bc-ad)}{c^2d\left(c+\frac{d}{x}\right)}+\frac{ax\left(a+\right.}{c\left(c\right.}$$

Rubi [A] time = 0.23, antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {375, 98, 149, 156, 63, 208, 205}

$$\frac{a^{3/2}(5bc-4ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{c^3}-\frac{(bc-ad)^{3/2}(4ad+bc)\tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c^3d^{3/2}}+\frac{\sqrt{a+\frac{b}{x}}(bc-2ad)(bc-ad)}{c^2d\left(c+\frac{d}{x}\right)}+\frac{ax\left(a+\frac{b}{x}\right)^{3/2}}{c\left(c+\frac{d}{x}\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x)^(5/2)/(c + d/x)^2,x]
```

```
[Out] ((b*c - 2*a*d)*(b*c - a*d)*Sqrt[a + b/x])/(c^2*d*(c + d/x)) + (a*(a + b/x)^
(3/2)*x)/(c*(c + d/x)) - ((b*c - a*d)^(3/2)*(b*c + 4*a*d)*ArcTan[(Sqrt[d]*S
qrt[a + b/x])/Sqrt[b*c - a*d]])/(c^3*d^(3/2)) + (a^(3/2)*(5*b*c - 4*a*d)*Ar
cTanh[Sqrt[a + b/x]/Sqrt[a]])/c^3
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 149

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^n*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] - Dist[1/(b*(
b*e - a*f)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Si
mp[b*c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g
- e*h)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, p}, x] && LtQ[m, -1] && GtQ[n, 0] && IntegerQ[m]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 375

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[((a + b/x^n)^p*(c + d/x^n)^q)/x^2, x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rubi steps

$$\int \frac{\left(a+\frac{b}{x}\right)^{5/2}}{\left(c+\frac{d}{x}\right)^2}\,dx = -\text{Subst}\left(\int \frac{(a+bx)^{5/2}}{x^2(c+dx)^2}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{a\left(a+\frac{b}{x}\right)^{3/2}x}{c\left(c+\frac{d}{x}\right)} + \frac{\text{Subst}\left(\int \frac{\sqrt{a+bx}\left(-\frac{1}{2}a(5bc-4ad)-\frac{1}{2}b(2bc-ad)x\right)}{x(c+dx)^2}\,dx, x, \frac{1}{x}\right)}{c}$$

$$= \frac{(bc-2ad)(bc-ad)\sqrt{a+\frac{b}{x}}}{c^2d\left(c+\frac{d}{x}\right)} + \frac{a\left(a+\frac{b}{x}\right)^{3/2}x}{c\left(c+\frac{d}{x}\right)} - \frac{\text{Subst}\left(\int \frac{\frac{1}{2}a^2d(5bc-4ad)+\frac{1}{2}b\left(b^2c^2+2abcd-2a^2d^2\right)x}{x\sqrt{a+bx}\,(c+dx)}\,dx, x\right.}{c^2d}$$

$$= \frac{(bc-2ad)(bc-ad)\sqrt{a+\frac{b}{x}}}{c^2d\left(c+\frac{d}{x}\right)} + \frac{a\left(a+\frac{b}{x}\right)^{3/2}x}{c\left(c+\frac{d}{x}\right)} - \frac{\left(a^2(5bc-4ad)\right)\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \frac{1}{x}\right)}{2c^3} - \frac{((}{}$$

$$= \frac{(bc-2ad)(bc-ad)\sqrt{a+\frac{b}{x}}}{c^2d\left(c+\frac{d}{x}\right)} + \frac{a\left(a+\frac{b}{x}\right)^{3/2}x}{c\left(c+\frac{d}{x}\right)} - \frac{\left(a^2(5bc-4ad)\right)\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+\frac{b}{x}}\right)}{bc^3}$$

$$= \frac{(bc-2ad)(bc-ad)\sqrt{a+\frac{b}{x}}}{c^2d\left(c+\frac{d}{x}\right)} + \frac{a\left(a+\frac{b}{x}\right)^{3/2}x}{c\left(c+\frac{d}{x}\right)} - \frac{(bc-ad)^{3/2}(bc+4ad)\tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c^3d^{3/2}} + \frac{a^{3/2}(}{}$$

Mathematica [A] time = 0.43, size = 145, normalized size = 0.87

$$\frac{a^{3/2}(5bc-4ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right) + \frac{cx\sqrt{a+\frac{b}{x}}\left(a^2d(cx+2d)-2abcd+b^2c^2\right)}{d(cx+d)} - \frac{(bc-ad)^{3/2}(4ad+bc)\tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{d^{3/2}}}{c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x)^(5/2)/(c + d/x)^2,x]
```

```
[Out] ((c*Sqrt[a + b/x]*x*(b^2*c^2 - 2*a*b*c*d + a^2*d*(2*d + c*x)))/(d*(d + c*x)
) - ((b*c - a*d)^(3/2)*(b*c + 4*a*d)*ArcTan[(Sqrt[d]*Sqrt[a + b/x])/Sqrt[b*
c - a*d]])/d^(3/2) + a^(3/2)*(5*b*c - 4*a*d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]]
)/c^3
```

IntegrateAlgebraic [A] time = 0.39, size = 166, normalized size = 1.00

$$\frac{\left(5a^{3/2}bc-4a^{5/2}d\right)\tanh^{-1}\left(\frac{\sqrt{\frac{ax+b}{x}}}{\sqrt{a}}\right)}{c^3}+\frac{\sqrt{\frac{ax+b}{x}}\left(a^2cdx^2+2a^2d^2x-2abcdx+b^2c^2x\right)}{c^2d(cx+d)}-\frac{(4ad+bc)(bc-ad)^{3/2}\tan^{-1}\left(\frac{\sqrt{d}\sqrt{\frac{ax+b}{x}}}{\sqrt{bc-ad}}\right)}{c^3d^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b/x)^(5/2)/(c + d/x)^2,x]
```

```
[Out] (Sqrt[(b + a*x)/x]*(b^2*c^2*x - 2*a*b*c*d*x + 2*a^2*d^2*x + a^2*c*d*x^2))/(
c^2*d*(d + c*x)) - ((b*c - a*d)^(3/2)*(b*c + 4*a*d)*ArcTan[(Sqrt[d]*Sqrt[(b
 + a*x)/x])/Sqrt[b*c - a*d]])/(c^3*d^(3/2)) + ((5*a^(3/2)*b*c - 4*a^(5/2)*d
)*ArcTanh[Sqrt[(b + a*x)/x]/Sqrt[a]])/c^3
```

fricas [A] time = 1.15, size = 1001, normalized size = 6.03

$$\left[-\frac{\left(5abcd^2-4a^2d^3+\left(5abc^2d-4a^2cd^2\right)x\right)\sqrt{a}\log\left(2ax-2\sqrt{a}x\sqrt{\frac{ax+b}{x}}+b\right)+\left(b^2c^2d+3abcd^2-4a^2d^3+\left(b^2c^3+3abc^2d-4a^2cd^2\right)x\right)\sqrt{-\frac{bc-ad}{d}}\log\left(\frac{2dx\sqrt{-\frac{bc-ad}{d}}\sqrt{\frac{ax+b}{x}}+bd-(bc-2ad)x}{cx+d}\right)-2\left(a^2c^2dx^2+\left(b^2c^3-2abc^2d+2a^2cd^2\right)x\right)\sqrt{\frac{ax+b}{x}}}{2\left(c^4dx+c^3d^2\right)},\ -\frac{2\left(5abcd^2-4a^2d^3+\left(5abc^2d-4a^2cd^2\right)x\right)\sqrt{-a}\arctan\left(\frac{\sqrt{-a}\sqrt{\frac{ax+b}{x}}}{a}\right)+\left(b^2c^2d+3abcd^2-4a^2d^3+\left(b^2c^3+3abc^2d-4a^2cd^2\right)x\right)\sqrt{-\frac{bc-ad}{d}}\log\left(\frac{2dx\sqrt{-\frac{bc-ad}{d}}\sqrt{\frac{ax+b}{x}}+bd-(bc-2ad)x}{cx+d}\right)-2\left(a^2c^2dx^2+\left(b^2c^3-2abc^2d+2a^2cd^2\right)x\right)\sqrt{\frac{ax+b}{x}}}{2\left(c^4dx+c^3d^2\right)},\ \frac{2\left(b^2c^2d+3abcd^2-4a^2d^3+\left(b^2c^3+3abc^2d-4a^2cd^2\right)x\right)\sqrt{\frac{bc-ad}{d}}\arctan\left(-\frac{d\sqrt{\frac{bc-ad}{d}}\sqrt{\frac{ax+b}{x}}}{bc-ad}\right)-\left(5abcd^2-4a^2d^3+\left(5abc^2d-4a^2cd^2\right)x\right)\sqrt{a}\log\left(2ax-2\sqrt{a}x\sqrt{\frac{ax+b}{x}}+b\right)+2\left(a^2c^2dx^2+\left(b^2c^3-2abc^2d+2a^2cd^2\right)x\right)\sqrt{\frac{ax+b}{x}}}{2\left(c^4dx+c^3d^2\right)},\ \frac{\left(b^2c^2d+3abcd^2-4a^2d^3+\left(b^2c^3+3abc^2d-4a^2cd^2\right)x\right)\sqrt{\frac{bc-ad}{d}}\arctan\left(-\frac{d\sqrt{\frac{bc-ad}{d}}\sqrt{\frac{ax+b}{x}}}{bc-ad}\right)-\left(5abcd^2-4a^2d^3+\left(5abc^2d-4a^2cd^2\right)x\right)\sqrt{-a}\arctan\left(\frac{\sqrt{-a}\sqrt{\frac{ax+b}{x}}}{a}\right)+\left(a^2c^2dx^2+\left(b^2c^3-2abc^2d+2a^2cd^2\right)x\right)\sqrt{\frac{ax+b}{x}}}{c^4dx+c^3d^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(5/2)/(c+d/x)^2,x, algorithm="fricas")
```

```
[Out] [-1/2*((5*a*b*c*d^2 - 4*a^2*d^3 + (5*a*b*c^2*d - 4*a^2*c*d^2)*x)*sqrt(a)*lo
g(2*a*x - 2*sqrt(a)*x*sqrt((a*x + b)/x) + b) + (b^2*c^2*d + 3*a*b*c*d^2 - 4
*a^2*d^3 + (b^2*c^3 + 3*a*b*c^2*d - 4*a^2*c*d^2)*x)*sqrt(-(b*c - a*d)/d)*lo
g((2*d*x*sqrt(-(b*c - a*d)/d)*sqrt((a*x + b)/x) + b*d - (b*c - 2*a*d)*x)/(c
*x + d)) - 2*(a^2*c^2*d*x^2 + (b^2*c^3 - 2*a*b*c^2*d + 2*a^2*c*d^2)*x)*sqrt
((a*x + b)/x))/(c^4*d*x + c^3*d^2), -1/2*(2*(5*a*b*c*d^2 - 4*a^2*d^3 + (5*a
*b*c^2*d - 4*a^2*c*d^2)*x)*sqrt(-a)*arctan(sqrt(-a)*sqrt((a*x + b)/x)/a) +
(b^2*c^2*d + 3*a*b*c*d^2 - 4*a^2*d^3 + (b^2*c^3 + 3*a*b*c^2*d - 4*a^2*c*d^2
)*x)*sqrt(-(b*c - a*d)/d)*log((2*d*x*sqrt(-(b*c - a*d)/d)*sqrt((a*x + b)/x)
 + b*d - (b*c - 2*a*d)*x)/(c*x + d)) - 2*(a^2*c^2*d*x^2 + (b^2*c^3 - 2*a*b*
c^2*d + 2*a^2*c*d^2)*x)*sqrt((a*x + b)/x))/(c^4*d*x + c^3*d^2), 1/2*(2*(b^2
*c^2*d + 3*a*b*c*d^2 - 4*a^2*d^3 + (b^2*c^3 + 3*a*b*c^2*d - 4*a^2*c*d^2)*x)
*sqrt((b*c - a*d)/d)*arctan(-d*sqrt((b*c - a*d)/d)*sqrt((a*x + b)/x)/(b*c -
 a*d)) - (5*a*b*c*d^2 - 4*a^2*d^3 + (5*a*b*c^2*d - 4*a^2*c*d^2)*x)*sqrt(a)*
log(2*a*x - 2*sqrt(a)*x*sqrt((a*x + b)/x) + b) + 2*(a^2*c^2*d*x^2 + (b^2*c^
3 - 2*a*b*c^2*d + 2*a^2*c*d^2)*x)*sqrt((a*x + b)/x))/(c^4*d*x + c^3*d^2), (
(b^2*c^2*d + 3*a*b*c*d^2 - 4*a^2*d^3 + (b^2*c^3 + 3*a*b*c^2*d - 4*a^2*c*d^2
)*x)*sqrt((b*c - a*d)/d)*arctan(-d*sqrt((b*c - a*d)/d)*sqrt((a*x + b)/x)/(b
*c - a*d)) - (5*a*b*c*d^2 - 4*a^2*d^3 + (5*a*b*c^2*d - 4*a^2*c*d^2)*x)*sqrt
(-a)*arctan(sqrt(-a)*sqrt((a*x + b)/x)/a) + (a^2*c^2*d*x^2 + (b^2*c^3 - 2*a
*b*c^2*d + 2*a^2*c*d^2)*x)*sqrt((a*x + b)/x))/(c^4*d*x + c^3*d^2)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(5/2)/(c+d/x)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x)::;OUTPUT:Warning, integration of abs or sign assumes
```

```
constant sign by intervals (correct if the argument is real):Check [abs(x)]
Unable to divide, perhaps due to rounding error%%%{%%{[-2,0]:[1,0,%%%{-1,[1
]%%%}]%%},[4,6,4,0]%%%}+%%%{%%%{8,[1]%%%},[3,5,4,1]%%%}+%%%{%%{[-4,0]:[1,0,
%%%{-1,[1]%%%}]%%},[2,5,5,1]%%%}+%%%{%%{[%%%{-8,[1]%%%},0]:[1,0,%%%{-1,[1]%
%%}]%%},[2,4,4,2]%%%}+%%%{%%%{8,[1]%%%},[1,4,5,2]%%%}+%%%{%%{[-2,0]:[1,0,%%
%{-1,[1]%%%}]%%},[0,4,6,2]%%%} / %%%{%%%{1,[1]%%%},[4,2,0,0]%%%}+%%%{%%{pol
y1[%%%{-4,[1]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[3,1,0,1]%%%}+%%%{%%%{2,[1]%%%
},[2,1,1,1]%%%}+%%%{%%%{4,[2]%%%},[2,0,0,2]%%%}+%%%{%%{poly1[%%%{-4,[1]%%%}
,0]:[1,0,%%%{-1,[1]%%%}]%%},[1,0,1,2]%%%}+%%%{%%%{1,[1]%%%},[0,0,2,2]%%%} E
rror: Bad Argument Value
```

maple [B] time = 0.06, size = 1323, normalized size = 7.97

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x)^(5/2)/(c+d/x)^2,x)
```

```
[Out] -1/2*(2*a^(5/2)*((a*x+b)*x)^(1/2)*((a*d-b*c)/c^2*d)^(1/2)*x*b*c^4*d-5*a^3*l
n(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))*((a*d-b*c)/c^2*d)^(1/2
)*x*b*c^3*d^2-5*a^3*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))*(
(a*d-b*c)/c^2*d)^(1/2)*b*c^2*d^3+a*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1
/2))/a^(1/2))*((a*d-b*c)/c^2*d)^(1/2)*b^3*c^4*d-a*((a*d-b*c)/c^2*d)^(1/2)*l
n(1/2*(2*a*x+b+2*(a*x^2+b*x)^(1/2)*a^(1/2))/a^(1/2))*b^3*c^4*d-2*a^(3/2)*((
a*d-b*c)/c^2*d)^(1/2)*(a*x^2+b*x)^(1/2)*x*b^2*c^5+a^(3/2)*ln((-2*a*d*x+b*c*
x-b*d+2*((a*d-b*c)/c^2*d)^(1/2)*((a*x+b)*x)^(1/2)*c)/(c*x+d))*x*b^3*c^4*d+a
*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))*((a*d-b*c)/c^2*d)^(1
/2)*x*b^3*c^5-a*((a*d-b*c)/c^2*d)^(1/2)*ln(1/2*(2*a*x+b+2*(a*x^2+b*x)^(1/2)
*a^(1/2))/a^(1/2))*x*b^3*c^5-2*a^(3/2)*((a*d-b*c)/c^2*d)^(1/2)*(a*x^2+b*x)^
(1/2)*b^2*c^4*d-2*a^(5/2)*((a*x+b)*x)^(1/2)*((a*d-b*c)/c^2*d)^(1/2)*x^2*b*c
^5-2*a^(7/2)*((a*x+b)*x)^(1/2)*((a*d-b*c)/c^2*d)^(1/2)*x*c^3*d^2-7*a^(7/2)*
ln((-2*a*d*x+b*c*x-b*d+2*((a*d-b*c)/c^2*d)^(1/2)*((a*x+b)*x)^(1/2)*c)/(c*x+
d))*x*b*c^2*d^3+2*a^(5/2)*ln((-2*a*d*x+b*c*x-b*d+2*((a*d-b*c)/c^2*d)^(1/2)*
((a*x+b)*x)^(1/2)*c)/(c*x+d))*x*b^2*c^3*d^2+4*a^4*ln(1/2*(2*a*x+b+2*((a*x+b
)*x)^(1/2)*a^(1/2))/a^(1/2))*((a*d-b*c)/c^2*d)^(1/2)*x*c^2*d^3+4*a^(5/2)*((
a*x+b)*x)^(1/2)*((a*d-b*c)/c^2*d)^(1/2)*b*c^3*d^2+2*a^(7/2)*((a*x+b)*x)^(1/
2)*((a*d-b*c)/c^2*d)^(1/2)*x^2*c^4*d+4*a^(9/2)*ln((-2*a*d*x+b*c*x-b*d+2*((a
*d-b*c)/c^2*d)^(1/2)*((a*x+b)*x)^(1/2)*c)/(c*x+d))*d^5-2*a^(5/2)*((a*x+b)*x
)^(3/2)*((a*d-b*c)/c^2*d)^(1/2)*c^4*d+4*a^(9/2)*ln((-2*a*d*x+b*c*x-b*d+2*((
a*d-b*c)/c^2*d)^(1/2)*((a*x+b)*x)^(1/2)*c)/(c*x+d))*x*c*d^4-4*a^(7/2)*((a*x
+b)*x)^(1/2)*((a*d-b*c)/c^2*d)^(1/2)*c^2*d^3-7*a^(7/2)*ln((-2*a*d*x+b*c*x-b
*d+2*((a*d-b*c)/c^2*d)^(1/2)*((a*x+b)*x)^(1/2)*c)/(c*x+d))*b*c*d^4+2*a^(5/2
)*ln((-2*a*d*x+b*c*x-b*d+2*((a*d-b*c)/c^2*d)^(1/2)*((a*x+b)*x)^(1/2)*c)/(c*
x+d))*b^2*c^2*d^3+4*a^4*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2
))*((a*d-b*c)/c^2*d)^(1/2)*c*d^4+2*a^(3/2)*((a*x+b)*x)^(3/2)*((a*d-b*c)/c^2
*d)^(1/2)*b*c^5+a^(3/2)*ln((-2*a*d*x+b*c*x-b*d+2*((a*d-b*c)/c^2*d)^(1/2)*((
a*x+b)*x)^(1/2)*c)/(c*x+d))*b^3*c^3*d^2)*x*((a*x+b)/x)^(1/2)/c^4/((a*d-b*c)
/c^2*d)^(1/2)/a^(3/2)/(c*x+d)/d^2/((a*x+b)*x)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + \frac{b}{x}\right)^{\frac{5}{2}}}{\left(c + \frac{d}{x}\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(5/2)/(c+d/x)^2,x, algorithm="maxima")
```

```
[Out] integrate((a + b/x)^(5/2)/(c + d/x)^2, x)
```

mupad [B]   time = 2.31, size = 1153, normalized size = 6.95

$$\frac{\frac{\sqrt{a+\frac{b}{x}}\,\left(2a^3bd^2-3a^2b^2cd+ab^3c^2\right)}{c^2d}-\frac{b\left(a+\frac{b}{x}\right)^{3/2}\left(2a^2d^2-2abcd+b^2c^2\right)}{c^2d}}{\left(a+\frac{b}{x}\right)(2ad-bc)-d\left(a+\frac{b}{x}\right)^2-a^2d+abc}-\frac{\operatorname{atanh}\left(\frac{10b^9\sqrt{a+\frac{b}{x}}\,\sqrt{a^3}}{10a^2b^9+\frac{32a^3b^8d}{c}-\frac{132a^4b^7d^2}{c^2}+\frac{130a^5b^6d^3}{c^3}-\frac{40a^6b^5d^4}{c^4}}+\frac{32ab^8\sqrt{a+\frac{b}{x}}\,\sqrt{a^3}}{32a^3b^8+\frac{10a^2b^9c}{d}-\frac{132a^4b^7d}{c}+\frac{130a^5b^6d^2}{c^2}-\frac{40a^6b^5d^3}{c^3}}-\frac{132a^2b^7d\sqrt{a+\frac{b}{x}}\,\sqrt{a^3}}{32a^3b^8c-132a^4b^7d+\frac{10a^2b^9c^2}{d}+\frac{130a^5b^6d^2}{c}-\frac{40a^6b^5d^3}{c^2}}+\frac{130a^3b^6d^2\sqrt{a+\frac{b}{x}}\,\sqrt{a^3}}{32a^3b^8c^2+130a^5b^6d^2+\frac{10a^2b^9c^3}{d}-\frac{40a^6b^5d^3}{c}-132a^4b^7cd}-\frac{40a^4b^5d^3\sqrt{a+\frac{b}{x}}\,\sqrt{a^3}}{32a^3b^8c^3-40a^6b^5d^3-132a^4b^7c^2d+130a^5b^6cd^2+\frac{10a^2b^9c^4}{d}}\right)(4ad-5bc)\sqrt{a^3}}{c^3}+\frac{\operatorname{atanh}\left(\frac{30a^3b^6\sqrt{a+\frac{b}{x}}\,\sqrt{a^3d^6-3a^2bcd^5+3ab^2c^2d^4-b^3c^3d^3}}{14a^2b^9c^3+110a^5b^6d^3-4a^3b^8c^2d-82a^4b^7cd^2+\frac{2ab^{10}c^4}{d}-\frac{40a^6b^5d^4}{c}}+\frac{18a^2b^7\sqrt{a+\frac{b}{x}}\,\sqrt{a^3d^6-3a^2bcd^5+3ab^2c^2d^4-b^3c^3d^3}}{2ab^{10}c^3-82a^4b^7d^3+14a^2b^9c^2d-4a^3b^8cd^2+\frac{110a^5b^6d^4}{c}-\frac{40a^6b^5d^5}{c^2}}+\frac{40a^4b^5\sqrt{a+\frac{b}{x}}\,\sqrt{a^3d^6-3a^2bcd^5+3ab^2c^2d^4-b^3c^3d^3}}{4a^3b^8c^3+40a^6b^5d^3+82a^4b^7c^2d-110a^5b^6cd^2-\frac{2ab^{10}c^5}{d^2}-\frac{14a^2b^9c^4}{d}}-\frac{2ab^8\sqrt{a+\frac{b}{x}}\,\sqrt{a^3d^6-3a^2bcd^5+3ab^2c^2d^4-b^3c^3d^3}}{4a^3b^8d^3-14a^2b^9cd^2+\frac{82a^4b^7d^4}{c}-\frac{110a^5b^6d^5}{c^2}+\frac{40a^6b^5d^6}{c^3}-2ab^{10}c^2d}\right)\sqrt{d^3(ad-bc)^3}\,(4ad+bc)}{c^3d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x)^(5/2)/(c + d/x)^2,x)
```

```
[Out] (((a + b/x)^(1/2)*(a*b^3*c^2 + 2*a^3*b*d^2 - 3*a^2*b^2*c*d))/(c^2*d) - (b*(
a + b/x)^(3/2)*(2*a^2*d^2 + b^2*c^2 - 2*a*b*c*d))/(c^2*d))/((a + b/x)*(2*a*
d - b*c) - d*(a + b/x)^2 - a^2*d + a*b*c) - (atanh((10*b^9*(a + b/x)^(1/2)*
(a^3)^(1/2))/(10*a^2*b^9 + (32*a^3*b^8*d)/c - (132*a^4*b^7*d^2)/c^2 + (130*
a^5*b^6*d^3)/c^3 - (40*a^6*b^5*d^4)/c^4) + (32*a*b^8*(a + b/x)^(1/2)*(a^3)^
(1/2))/(32*a^3*b^8 + (10*a^2*b^9*c)/d - (132*a^4*b^7*d)/c + (130*a^5*b^6*d^
2)/c^2 - (40*a^6*b^5*d^3)/c^3) - (132*a^2*b^7*d*(a + b/x)^(1/2)*(a^3)^(1/2)
)/(32*a^3*b^8*c - 132*a^4*b^7*d + (10*a^2*b^9*c^2)/d + (130*a^5*b^6*d^2)/c
- (40*a^6*b^5*d^3)/c^2) + (130*a^3*b^6*d^2*(a + b/x)^(1/2)*(a^3)^(1/2))/(32
*a^3*b^8*c^2 + 130*a^5*b^6*d^2 + (10*a^2*b^9*c^3)/d - (40*a^6*b^5*d^3)/c -
132*a^4*b^7*c*d) - (40*a^4*b^5*d^3*(a + b/x)^(1/2)*(a^3)^(1/2))/(32*a^3*b^8
*c^3 - 40*a^6*b^5*d^3 - 132*a^4*b^7*c^2*d + 130*a^5*b^6*c*d^2 + (10*a^2*b^9
*c^4)/d))*(4*a*d - 5*b*c)*(a^3)^(1/2))/c^3 + (atanh((30*a^3*b^6*(a + b/x)^(
1/2)*(a^3*d^6 - b^3*c^3*d^3 + 3*a*b^2*c^2*d^4 - 3*a^2*b*c*d^5)^(1/2))/(14*a
^2*b^9*c^3 + 110*a^5*b^6*d^3 - 4*a^3*b^8*c^2*d - 82*a^4*b^7*c*d^2 + (2*a*b^
10*c^4)/d - (40*a^6*b^5*d^4)/c) + (18*a^2*b^7*(a + b/x)^(1/2)*(a^3*d^6 - b^
3*c^3*d^3 + 3*a*b^2*c^2*d^4 - 3*a^2*b*c*d^5)^(1/2))/(2*a*b^10*c^3 - 82*a^4*
b^7*d^3 + 14*a^2*b^9*c^2*d - 4*a^3*b^8*c*d^2 + (110*a^5*b^6*d^4)/c - (40*a^
6*b^5*d^5)/c^2) + (40*a^4*b^5*(a + b/x)^(1/2)*(a^3*d^6 - b^3*c^3*d^3 + 3*a*
b^2*c^2*d^4 - 3*a^2*b*c*d^5)^(1/2))/(4*a^3*b^8*c^3 + 40*a^6*b^5*d^3 + 82*a^
4*b^7*c^2*d - 110*a^5*b^6*c*d^2 - (2*a*b^10*c^5)/d^2 - (14*a^2*b^9*c^4)/d)
- (2*a*b^8*(a + b/x)^(1/2)*(a^3*d^6 - b^3*c^3*d^3 + 3*a*b^2*c^2*d^4 - 3*a^2
*b*c*d^5)^(1/2))/(4*a^3*b^8*d^3 - 14*a^2*b^9*c*d^2 + (82*a^4*b^7*d^4)/c - (
110*a^5*b^6*d^5)/c^2 + (40*a^6*b^5*d^6)/c^3 - 2*a*b^10*c^2*d))*(d^3*(a*d -
b*c)^3)^(1/2)*(4*a*d + b*c))/(c^3*d^3)
```

sympy [F(-1)]   time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)**(5/2)/(c+d/x)**2,x)
```

```
[Out] Timed out
```

3.147 $\int \frac{\left(a+\frac{b}{x}\right)^{5/2}}{\left(c+\frac{d}{x}\right)^3}\,dx$

Optimal. Leaf size=237

$$\frac{a^{3/2}(5bc-6ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{c^4}-\frac{\sqrt{bc-ad}\left(-24a^2d^2+8abcd+b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{4c^4d^{3/2}}-\frac{\sqrt{a+\frac{b}{x}}\left(-12a^2d^2+7abc\right.}{4c^3d\left(c+\frac{d}{x}\right)}$$

Rubi [A] time = 0.37, antiderivative size = 237, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {375, 98, 149, 151, 156, 63, 208, 205}

$$-\frac{\sqrt{a+\frac{b}{x}}\left(-12a^2d^2+7abcd+b^2c^2\right)}{4c^3d\left(c+\frac{d}{x}\right)}-\frac{\sqrt{bc-ad}\left(-24a^2d^2+8abcd+b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{4c^4d^{3/2}}+\frac{a^{3/2}(5bc-6ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{c^4}+\frac{\sqrt{a+\frac{b}{x}}(bc-3ad)(bc-ad)}{2c^2d\left(c+\frac{d}{x}\right)^2}+\frac{ax\left(a+\frac{b}{x}\right)^{3/2}}{c\left(c+\frac{d}{x}\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x)^(5/2)/(c + d/x)^3,x]
```

```
[Out] ((b*c - 3*a*d)*(b*c - a*d)*Sqrt[a + b/x])/(2*c^2*d*(c + d/x)^2) - ((b^2*c^2
 + 7*a*b*c*d - 12*a^2*d^2)*Sqrt[a + b/x])/(4*c^3*d*(c + d/x)) + (a*(a + b/x
)^(3/2)*x)/(c*(c + d/x)^2) - (Sqrt[b*c - a*d]*(b^2*c^2 + 8*a*b*c*d - 24*a^2
*d^2)*ArcTan[(Sqrt[d]*Sqrt[a + b/x])/Sqrt[b*c - a*d]])/(4*c^4*d^(3/2)) + (a
^(3/2)*(5*b*c - 6*a*d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/c^4
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 149

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^n*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] - Dist[1/(b*(
b*e - a*f)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Si
mp[b*c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g
- e*h)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, p}, x] && LtQ[m, -1] && GtQ[n, 0] && IntegerQ[m]
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 375

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[((a + b/x^n)^p*(c + d/x^n)^q)/x^2, x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rubi steps

$$\int \frac{\left(a+\frac{b}{x}\right)^{5/2}}{\left(c+\frac{d}{x}\right)^3}\,dx = -\text{Subst}\left(\int \frac{(a+bx)^{5/2}}{x^2(c+dx)^3}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{a\left(a+\frac{b}{x}\right)^{3/2} x}{c\left(c+\frac{d}{x}\right)^2} + \frac{\text{Subst}\left(\int \frac{\sqrt{a+bx}\left(-\frac{1}{2}a(5bc-6ad)-\frac{1}{2}b(2bc-3ad)x\right)}{x(c+dx)^3}\,dx, x, \frac{1}{x}\right)}{c}$$

$$= \frac{(bc-3ad)(bc-ad)\sqrt{a+\frac{b}{x}}}{2c^2d\left(c+\frac{d}{x}\right)^2} + \frac{a\left(a+\frac{b}{x}\right)^{3/2} x}{c\left(c+\frac{d}{x}\right)^2} - \frac{\text{Subst}\left(\int \frac{a^2d(5bc-6ad)+\frac{1}{2}b\left(b^2c^2+6abcd-9a^2d^2\right)x}{x\sqrt{a+bx}\,(c+dx)^2}\,dx, x, \right.}{2c^2d}$$

$$= \frac{(bc-3ad)(bc-ad)\sqrt{a+\frac{b}{x}}}{2c^2d\left(c+\frac{d}{x}\right)^2} - \frac{\left(b^2c^2+7abcd-12a^2d^2\right)\sqrt{a+\frac{b}{x}}}{4c^3d\left(c+\frac{d}{x}\right)} + \frac{a\left(a+\frac{b}{x}\right)^{3/2} x}{c\left(c+\frac{d}{x}\right)^2} + \frac{\text{Subst}\left(\int \right.}{}$$

$$= \frac{(bc-3ad)(bc-ad)\sqrt{a+\frac{b}{x}}}{2c^2d\left(c+\frac{d}{x}\right)^2} - \frac{\left(b^2c^2+7abcd-12a^2d^2\right)\sqrt{a+\frac{b}{x}}}{4c^3d\left(c+\frac{d}{x}\right)} + \frac{a\left(a+\frac{b}{x}\right)^{3/2} x}{c\left(c+\frac{d}{x}\right)^2} - \frac{\left(a^2(5bc-\right.}{}$$

$$= \frac{(bc-3ad)(bc-ad)\sqrt{a+\frac{b}{x}}}{2c^2d\left(c+\frac{d}{x}\right)^2} - \frac{\left(b^2c^2+7abcd-12a^2d^2\right)\sqrt{a+\frac{b}{x}}}{4c^3d\left(c+\frac{d}{x}\right)} + \frac{a\left(a+\frac{b}{x}\right)^{3/2} x}{c\left(c+\frac{d}{x}\right)^2} - \frac{\left(a^2(5bc-\right.}{}$$

$$= \frac{(bc-3ad)(bc-ad)\sqrt{a+\frac{b}{x}}}{2c^2d\left(c+\frac{d}{x}\right)^2} - \frac{\left(b^2c^2+7abcd-12a^2d^2\right)\sqrt{a+\frac{b}{x}}}{4c^3d\left(c+\frac{d}{x}\right)} + \frac{a\left(a+\frac{b}{x}\right)^{3/2} x}{c\left(c+\frac{d}{x}\right)^2} - \frac{\sqrt{bc-ad}\,\left(\right.}{}$$

Mathematica [A] time = 0.85, size = 191, normalized size = 0.81

$$\frac{-4a^{3/2}(6ad-5bc)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right) + \frac{cx\sqrt{a+\frac{b}{x}}\left(2a^2d\left(2c^2x^2+9cdx+6d^2\right)-abcd(11cx+7d)+b^2c^2(cx-d)\right)}{d(cx+d)^2} - \frac{\sqrt{bc-ad}\left(-24a^2d^2+8abcd+b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{d^{3/2}}}{4c^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x)^(5/2)/(c + d/x)^3,x]
```

```
[Out] ((c*Sqrt[a + b/x]*x*(b^2*c^2*(-d + c*x) - a*b*c*d*(7*d + 11*c*x) + 2*a^2*d*
(6*d^2 + 9*c*d*x + 2*c^2*x^2)))/(d*(d + c*x)^2) - (Sqrt[b*c - a*d]*(b^2*c^2
 + 8*a*b*c*d - 24*a^2*d^2)*ArcTan[(Sqrt[d]*Sqrt[a + b/x])/Sqrt[b*c - a*d]])
/d^(3/2) - 4*a^(3/2)*(-5*b*c + 6*a*d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/(4*c^
4)
```

IntegrateAlgebraic [A] time = 0.52, size = 240, normalized size = 1.01

$$\frac{\left(5a^{3/2}bc-6a^{5/2}d\right)\tanh^{-1}\left(\frac{\sqrt{\frac{ax+b}{x}}}{\sqrt{a}}\right)}{c^4} + \frac{\sqrt{\frac{ax+b}{x}}\left(4a^2c^2dx^3+18a^2cd^2x^2+12a^2d^3x-11abc^2dx^2-7abcd^2x+b^2c^3x^2-b^2c^2dx\right)}{4c^3d(cx+d)^2} + \frac{\left(-24a^3d^3+32a^2bcd^2-7ab^2c^2d-b^3c^3\right)\tan^{-1}\left(\frac{\sqrt{d}\sqrt{\frac{ax+b}{x}}}{\sqrt{bc-ad}}\right)}{4c^4d^{3/2}\sqrt{bc-ad}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b/x)^(5/2)/(c + d/x)^3,x]
```

```
[Out] (Sqrt[(b + a*x)/x]*(-(b^2*c^2*d*x) - 7*a*b*c*d^2*x + 12*a^2*d^3*x + b^2*c^3
*x^2 - 11*a*b*c^2*d*x^2 + 18*a^2*c*d^2*x^2 + 4*a^2*c^2*d*x^3))/(4*c^3*d*(d
+ c*x)^2) + ((-(b^3*c^3) - 7*a*b^2*c^2*d + 32*a^2*b*c*d^2 - 24*a^3*d^3)*Arc
Tan[(Sqrt[d]*Sqrt[(b + a*x)/x])/Sqrt[b*c - a*d]])/(4*c^4*d^(3/2)*Sqrt[b*c -
 a*d]) + ((5*a^(3/2)*b*c - 6*a^(5/2)*d)*ArcTanh[Sqrt[(b + a*x)/x]/Sqrt[a]])
/c^4
```

fricas [A] time = 0.96, size = 1445, normalized size = 6.10

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(5/2)/(c+d/x)^3,x, algorithm="fricas")
```

```
[Out] [-1/8*(4*(5*a*b*c*d^3 - 6*a^2*d^4 + (5*a*b*c^3*d - 6*a^2*c^2*d^2)*x^2 + 2*(
5*a*b*c^2*d^2 - 6*a^2*c*d^3)*x)*sqrt(a)*log(2*a*x - 2*sqrt(a)*x*sqrt((a*x +
 b)/x) + b) + (b^2*c^2*d^2 + 8*a*b*c*d^3 - 24*a^2*d^4 + (b^2*c^4 + 8*a*b*c^
3*d - 24*a^2*c^2*d^2)*x^2 + 2*(b^2*c^3*d + 8*a*b*c^2*d^2 - 24*a^2*c*d^3)*x)
*sqrt(-(b*c - a*d)/d)*log((2*d*x*sqrt(-(b*c - a*d)/d)*sqrt((a*x + b)/x) + b
*d - (b*c - 2*a*d)*x)/(c*x + d)) - 2*(4*a^2*c^3*d*x^3 + (b^2*c^4 - 11*a*b*c
^3*d + 18*a^2*c^2*d^2)*x^2 - (b^2*c^3*d + 7*a*b*c^2*d^2 - 12*a^2*c*d^3)*x)*
sqrt((a*x + b)/x))/(c^6*d*x^2 + 2*c^5*d^2*x + c^4*d^3), 1/4*((b^2*c^2*d^2 +
 8*a*b*c*d^3 - 24*a^2*d^4 + (b^2*c^4 + 8*a*b*c^3*d - 24*a^2*c^2*d^2)*x^2 +
2*(b^2*c^3*d + 8*a*b*c^2*d^2 - 24*a^2*c*d^3)*x)*sqrt((b*c - a*d)/d)*arctan(
-d*sqrt((b*c - a*d)/d)*sqrt((a*x + b)/x)/(b*c - a*d)) - 2*(5*a*b*c*d^3 - 6*
a^2*d^4 + (5*a*b*c^3*d - 6*a^2*c^2*d^2)*x^2 + 2*(5*a*b*c^2*d^2 - 6*a^2*c*d^
3)*x)*sqrt(a)*log(2*a*x - 2*sqrt(a)*x*sqrt((a*x + b)/x) + b) + (4*a^2*c^3*d
*x^3 + (b^2*c^4 - 11*a*b*c^3*d + 18*a^2*c^2*d^2)*x^2 - (b^2*c^3*d + 7*a*b*c
^2*d^2 - 12*a^2*c*d^3)*x)*sqrt((a*x + b)/x))/(c^6*d*x^2 + 2*c^5*d^2*x + c^4
*d^3), -1/8*(8*(5*a*b*c*d^3 - 6*a^2*d^4 + (5*a*b*c^3*d - 6*a^2*c^2*d^2)*x^2
 + 2*(5*a*b*c^2*d^2 - 6*a^2*c*d^3)*x)*sqrt(-a)*arctan(sqrt(-a)*sqrt((a*x +
b)/x)/a) + (b^2*c^2*d^2 + 8*a*b*c*d^3 - 24*a^2*d^4 + (b^2*c^4 + 8*a*b*c^3*d
 - 24*a^2*c^2*d^2)*x^2 + 2*(b^2*c^3*d + 8*a*b*c^2*d^2 - 24*a^2*c*d^3)*x)*sq
rt(-(b*c - a*d)/d)*log((2*d*x*sqrt(-(b*c - a*d)/d)*sqrt((a*x + b)/x) + b*d
- (b*c - 2*a*d)*x)/(c*x + d)) - 2*(4*a^2*c^3*d*x^3 + (b^2*c^4 - 11*a*b*c^3*
d + 18*a^2*c^2*d^2)*x^2 - (b^2*c^3*d + 7*a*b*c^2*d^2 - 12*a^2*c*d^3)*x)*sqr
t((a*x + b)/x))/(c^6*d*x^2 + 2*c^5*d^2*x + c^4*d^3), 1/4*((b^2*c^2*d^2 + 8*
a*b*c*d^3 - 24*a^2*d^4 + (b^2*c^4 + 8*a*b*c^3*d - 24*a^2*c^2*d^2)*x^2 + 2*(
b^2*c^3*d + 8*a*b*c^2*d^2 - 24*a^2*c*d^3)*x)*sqrt((b*c - a*d)/d)*arctan(-d*
sqrt((b*c - a*d)/d)*sqrt((a*x + b)/x)/(b*c - a*d)) - 4*(5*a*b*c*d^3 - 6*a^2
*d^4 + (5*a*b*c^3*d - 6*a^2*c^2*d^2)*x^2 + 2*(5*a*b*c^2*d^2 - 6*a^2*c*d^3)*
x)*sqrt(-a)*arctan(sqrt(-a)*sqrt((a*x + b)/x)/a) + (4*a^2*c^3*d*x^3 + (b^2*
c^4 - 11*a*b*c^3*d + 18*a^2*c^2*d^2)*x^2 - (b^2*c^3*d + 7*a*b*c^2*d^2 - 12*
a^2*c*d^3)*x)*sqrt((a*x + b)/x))/(c^6*d*x^2 + 2*c^5*d^2*x + c^4*d^3)]
```

giac [B] time = 0.49, size = 945, normalized size = 3.99

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(5/2)/(c+d/x)^3,x, algorithm="giac")
```

```
[Out] sqrt(a*x^2 + b*x)*a^2*sgn(x)/c^3 - 1/2*(5*a^2*b*c*sgn(x) - 6*a^3*d*sgn(x))*
log(abs(2*(sqrt(a)*x - sqrt(a*x^2 + b*x))*sqrt(a) + b))/(sqrt(a)*c^4) + 1/4
*(b^3*c^3*sgn(x) + 7*a*b^2*c^2*d*sgn(x) - 32*a^2*b*c*d^2*sgn(x) + 24*a^3*d^
3*sgn(x))*arctan(-((sqrt(a)*x - sqrt(a*x^2 + b*x))*c + sqrt(a)*d)/sqrt(b*c*
d - a*d^2))/(sqrt(b*c*d - a*d^2)*c^4*d) + 1/4*(sqrt(a)*b^3*c^3*arctan(sqrt(
a)*d/sqrt(b*c*d - a*d^2)) + 7*a^(3/2)*b^2*c^2*d*arctan(sqrt(a)*d/sqrt(b*c*d
 - a*d^2)) - 32*a^(5/2)*b*c*d^2*arctan(sqrt(a)*d/sqrt(b*c*d - a*d^2)) + 24*
a^(7/2)*d^3*arctan(sqrt(a)*d/sqrt(b*c*d - a*d^2)) + 10*sqrt(b*c*d - a*d^2)*
```

```
a^2*b*c*d*log(abs(b)) - 12*sqrt(b*c*d - a*d^2)*a^3*d^2*log(abs(b)) - sqrt(b
*c*d - a*d^2)*a*b^2*c^2 + 11*sqrt(b*c*d - a*d^2)*a^2*b*c*d - 10*sqrt(b*c*d
- a*d^2)*a^3*d^2)*sgn(x)/(sqrt(b*c*d - a*d^2)*sqrt(a)*c^4*d) - 1/4*((sqrt(a
)*x - sqrt(a*x^2 + b*x))^3*sqrt(a)*b^3*c^4*sgn(x) - 17*(sqrt(a)*x - sqrt(a*
x^2 + b*x))^3*a^(3/2)*b^2*c^3*d*sgn(x) + 40*(sqrt(a)*x - sqrt(a*x^2 + b*x))
^3*a^(5/2)*b*c^2*d^2*sgn(x) - 24*(sqrt(a)*x - sqrt(a*x^2 + b*x))^3*a^(7/2)*
c*d^3*sgn(x) - 5*(sqrt(a)*x - sqrt(a*x^2 + b*x))^2*a*b^3*c^3*d*sgn(x) - 3*(
sqrt(a)*x - sqrt(a*x^2 + b*x))^2*a^2*b^2*c^2*d^2*sgn(x) + 48*(sqrt(a)*x - s
qrt(a*x^2 + b*x))^2*a^3*b*c*d^3*sgn(x) - 40*(sqrt(a)*x - sqrt(a*x^2 + b*x))
^2*a^4*d^4*sgn(x) - (sqrt(a)*x - sqrt(a*x^2 + b*x))*sqrt(a)*b^4*c^3*d*sgn(x
) - 11*(sqrt(a)*x - sqrt(a*x^2 + b*x))*a^(3/2)*b^3*c^2*d^2*sgn(x) + 52*(sqr
t(a)*x - sqrt(a*x^2 + b*x))*a^(5/2)*b^2*c*d^3*sgn(x) - 40*(sqrt(a)*x - sqrt
(a*x^2 + b*x))*a^(7/2)*b*d^4*sgn(x) - a*b^4*c^2*d^2*sgn(x) + 11*a^2*b^3*c*d
^3*sgn(x) - 10*a^3*b^2*d^4*sgn(x))/(((sqrt(a)*x - sqrt(a*x^2 + b*x))^2*c +
2*(sqrt(a)*x - sqrt(a*x^2 + b*x))*sqrt(a)*d + b*d)^2*sqrt(a)*c^4*d)
```

maple [B] time = 0.07, size = 1638, normalized size = 6.91

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x)^(5/2)/(c+d/x)^3,x)
```

```
[Out] -1/8*(7*a^(5/2)*b^2*c^4*d^2*x^2*ln((-2*a*d*x+b*c*x-b*d+2*((a*d-b*c)/c^2*d)^
(1/2)*((a*x+b)*x)^(1/2)*c)/(c*x+d))+24*((a*d-b*c)/c^2*d)^(1/2)*a^4*c^3*d^3*
x^2*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))-36*((a*x+b)*x)^(1
/2)*((a*d-b*c)/c^2*d)^(1/2)*a^(7/2)*c^3*d^3*x-64*a^(7/2)*b*c^2*d^4*x*ln((-2
*a*d*x+b*c*x-b*d+2*((a*d-b*c)/c^2*d)^(1/2)*((a*x+b)*x)^(1/2)*c)/(c*x+d))+14
*a^(5/2)*b^2*c^3*d^3*x*ln((-2*a*d*x+b*c*x-b*d+2*((a*d-b*c)/c^2*d)^(1/2)*((a
*x+b)*x)^(1/2)*c)/(c*x+d))+48*((a*d-b*c)/c^2*d)^(1/2)*a^4*c^2*d^4*x*ln(1/2*
(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))+14*((a*x+b)*x)^(1/2)*((a*d-b
*c)/c^2*d)^(1/2)*a^(5/2)*b*c^3*d^3-20*((a*d-b*c)/c^2*d)^(1/2)*a^3*b*c^2*d^4
*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))+2*((a*x+b)*x)^(1/2)*
((a*d-b*c)/c^2*d)^(1/2)*a^(3/2)*b^2*c^4*d^2-2*((a*x+b)*x)^(3/2)*((a*d-b*c)/
c^2*d)^(1/2)*a^(3/2)*b*c^6*x+2*((a*x+b)*x)^(1/2)*((a*d-b*c)/c^2*d)^(1/2)*a^
(3/2)*b^2*c^6*x^2+a^(3/2)*b^3*c^5*d*x^2*ln((-2*a*d*x+b*c*x-b*d+2*((a*d-b*c)
/c^2*d)^(1/2)*((a*x+b)*x)^(1/2)*c)/(c*x+d))-6*((a*x+b)*x)^(3/2)*((a*d-b*c)/
c^2*d)^(1/2)*a^(3/2)*b*c^5*d+12*((a*x+b)*x)^(1/2)*((a*d-b*c)/c^2*d)^(1/2)*a
^(7/2)*c^5*d*x^3+2*((a*x+b)*x)^(1/2)*((a*d-b*c)/c^2*d)^(1/2)*a^(5/2)*b*c^6*
x^3-12*((a*x+b)*x)^(3/2)*((a*d-b*c)/c^2*d)^(1/2)*a^(5/2)*c^5*d*x-32*a^(7/2)
*b*c^3*d^3*x^2*ln((-2*a*d*x+b*c*x-b*d+2*((a*d-b*c)/c^2*d)^(1/2)*((a*x+b)*x)
^(1/2)*c)/(c*x+d))+2*a^(3/2)*b^3*c^4*d^2*x*ln((-2*a*d*x+b*c*x-b*d+2*((a*d-b
*c)/c^2*d)^(1/2)*((a*x+b)*x)^(1/2)*c)/(c*x+d))+24*a^(9/2)*d^6*ln((-2*a*d*x+
b*c*x-b*d+2*((a*d-b*c)/c^2*d)^(1/2)*((a*x+b)*x)^(1/2)*c)/(c*x+d))+30*((a*x+
b)*x)^(1/2)*((a*d-b*c)/c^2*d)^(1/2)*a^(5/2)*b*c^4*d^2*x-40*((a*d-b*c)/c^2*d
)^(1/2)*a^3*b*c^3*d^3*x*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2
))+18*((a*x+b)*x)^(1/2)*((a*d-b*c)/c^2*d)^(1/2)*a^(5/2)*b*c^5*d*x^2-20*((a*
d-b*c)/c^2*d)^(1/2)*a^3*b*c^4*d^2*x^2*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a
^(1/2))/a^(1/2))+4*((a*x+b)*x)^(1/2)*((a*d-b*c)/c^2*d)^(1/2)*a^(3/2)*b^2*c^
5*d*x+48*a^(9/2)*c*d^5*x*ln((-2*a*d*x+b*c*x-b*d+2*((a*d-b*c)/c^2*d)^(1/2)*(
(a*x+b)*x)^(1/2)*c)/(c*x+d))-24*((a*x+b)*x)^(1/2)*((a*d-b*c)/c^2*d)^(1/2)*a
^(7/2)*c^2*d^4-32*a^(7/2)*b*c*d^5*ln((-2*a*d*x+b*c*x-b*d+2*((a*d-b*c)/c^2*d
)^(1/2)*((a*x+b)*x)^(1/2)*c)/(c*x+d))+7*a^(5/2)*b^2*c^2*d^4*ln((-2*a*d*x+b*
c*x-b*d+2*((a*d-b*c)/c^2*d)^(1/2)*((a*x+b)*x)^(1/2)*c)/(c*x+d))+24*((a*d-b*
c)/c^2*d)^(1/2)*a^4*c*d^5*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1
/2))+24*a^(9/2)*c^2*d^4*x^2*ln((-2*a*d*x+b*c*x-b*d+2*((a*d-b*c)/c^2*d)^(1/2
)*((a*x+b)*x)^(1/2)*c)/(c*x+d))-8*((a*x+b)*x)^(3/2)*((a*d-b*c)/c^2*d)^(1/2)
*a^(5/2)*c^4*d^2+a^(3/2)*b^3*c^3*d^3*ln((-2*a*d*x+b*c*x-b*d+2*((a*d-b*c)/c^
2*d)^(1/2)*((a*x+b)*x)^(1/2)*c)/(c*x+d)))*x*((a*x+b)/x)^(1/2)/c^5/((a*d-b*c
)/c^2*d)^(1/2)/a^(3/2)/(c*x+d)^2/d^2/((a*x+b)*x)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+\frac{b}{x}\right)^{\frac{5}{2}}}{\left(c+\frac{d}{x}\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(5/2)/(c+d/x)^3,x, algorithm="maxima")
```

```
[Out] integrate((a + b/x)^(5/2)/(c + d/x)^3, x)
```

mupad [B] time = 3.44, size = 1476, normalized size = 6.23

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x)^(5/2)/(c + d/x)^3,x)
```

```
[Out] (atan((b^9*(a + b/x)^(1/2)*(a^3)^(1/2)*5i)/(8*((5*a^2*b^9)/8 + (8*a^3*b^8*d
)/c - (159*a^4*b^7*d^2)/(8*c^2) + (45*a^5*b^6*d^3)/(4*c^3))) + (a*b^8*(a +
b/x)^(1/2)*(a^3)^(1/2)*8i)/(8*a^3*b^8 + (5*a^2*b^9*c)/(8*d) - (159*a^4*b^7*
d)/(8*c) + (45*a^5*b^6*d^2)/(4*c^2)) - (a^2*b^7*d*(a + b/x)^(1/2)*(a^3)^(1/
2)*159i)/(8*(8*a^3*b^8*c - (159*a^4*b^7*d)/8 + (5*a^2*b^9*c^2)/(8*d) + (45*
a^5*b^6*d^2)/(4*c))) + (a^3*b^6*d^2*(a + b/x)^(1/2)*(a^3)^(1/2)*45i)/(4*(8*
a^3*b^8*c^2 + (45*a^5*b^6*d^2)/4 + (5*a^2*b^9*c^3)/(8*d) - (159*a^4*b^7*c*d
)/8)))*(6*a*d - 5*b*c)*(a^3)^(1/2)*1i)/c^4 - (((a + b/x)^(3/2)*(b^4*c^3 - 2
4*a^3*b*d^3 + 32*a^2*b^2*c*d^2 - 9*a*b^3*c^2*d))/(4*c^3*d) - (b*(a + b/x)^(
5/2)*(b^2*c^2 - 12*a^2*d^2 + 7*a*b*c*d))/(4*c^3) + (b*(a + b/x)^(1/2)*(12*a
^4*d^3 - a*b^3*c^3 + 14*a^2*b^2*c^2*d - 25*a^3*b*c*d^2))/(4*c^3*d))/((a + b
/x)^2*(3*a*d^2 - 2*b*c*d) - (a + b/x)*(3*a^2*d^2 + b^2*c^2 - 4*a*b*c*d) - d
^2*(a + b/x)^3 + a^3*d^2 + a*b^2*c^2 - 2*a^2*b*c*d) + (log(- (5*a^2*b^9*c^6
 + 1728*a^8*b^3*d^6 + 64*a^3*b^8*c^5*d - 4752*a^7*b^4*c*d^5 - 59*a^4*b^7*c^
4*d^2 - 1450*a^5*b^6*c^3*d^3 + 4464*a^6*b^5*c^2*d^4)/(16*c^9*d) - (((a + b
/x)^(1/2)*(b^8*c^6 + 1152*a^6*b^2*d^6 - 2496*a^5*b^3*c*d^5 - 15*a^2*b^6*c^4
*d^2 - 400*a^3*b^5*c^3*d^3 + 1760*a^4*b^4*c^2*d^4 + 14*a*b^7*c^5*d))/(8*c^6
*d) - (((16*a*b^5*c^10*d^2 - 208*a^2*b^4*c^9*d^3 + 192*a^3*b^3*c^8*d^4)/(16
*c^9*d) - ((64*b^3*c^9*d^3 - 128*a*b^2*c^8*d^4)*(a + b/x)^(1/2)*(d^3*(a*d -
 b*c))^(1/2)*((b^2*c^2)/8 - 3*a^2*d^2 + a*b*c*d))/(8*c^10*d^4))*(d^3*(a*d -
 b*c))^(1/2)*((b^2*c^2)/8 - 3*a^2*d^2 + a*b*c*d))/(c^4*d^3))*(d^3*(a*d - b*
c))^(1/2)*((b^2*c^2)/8 - 3*a^2*d^2 + a*b*c*d))/(c^4*d^3))*(d^3*(a*d - b*c))
^(1/2)*((b^2*c^2)/8 - 3*a^2*d^2 + a*b*c*d))/(c^4*d^3) - (log(((((a + b/x)^(
1/2)*(b^8*c^6 + 1152*a^6*b^2*d^6 - 2496*a^5*b^3*c*d^5 - 15*a^2*b^6*c^4*d^2
- 400*a^3*b^5*c^3*d^3 + 1760*a^4*b^4*c^2*d^4 + 14*a*b^7*c^5*d))/(8*c^6*d) +
 (((16*a*b^5*c^10*d^2 - 208*a^2*b^4*c^9*d^3 + 192*a^3*b^3*c^8*d^4)/(16*c^9*
d) + ((64*b^3*c^9*d^3 - 128*a*b^2*c^8*d^4)*(a + b/x)^(1/2)*(d^3*(a*d - b*c)
)^(1/2)*(b^2*c^2 - 24*a^2*d^2 + 8*a*b*c*d))/(64*c^10*d^4))*(d^3*(a*d - b*c)
)^(1/2)*(b^2*c^2 - 24*a^2*d^2 + 8*a*b*c*d))/(8*c^4*d^3))*(d^3*(a*d - b*c))^
(1/2)*(b^2*c^2 - 24*a^2*d^2 + 8*a*b*c*d))/(8*c^4*d^3) - (5*a^2*b^9*c^6 + 17
28*a^8*b^3*d^6 + 64*a^3*b^8*c^5*d - 4752*a^7*b^4*c*d^5 - 59*a^4*b^7*c^4*d^2
 - 1450*a^5*b^6*c^3*d^3 + 4464*a^6*b^5*c^2*d^4)/(16*c^9*d))*(d^3*(a*d - b*c
))^(1/2)*(b^2*c^2 - 24*a^2*d^2 + 8*a*b*c*d))/(8*c^4*d^3)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)**(5/2)/(c+d/x)**3,x)
```

```
[Out] Timed out
```

3.148 $\int \frac{\left(c+\frac{d}{x}\right)^3}{\sqrt{a+\frac{b}{x}}}\,dx$

Optimal. Leaf size=126

$$-\frac{c^2(bc-6ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{d\sqrt{a+\frac{b}{x}}\left(2\left(-2a^2d^2+9abcd+3b^2c^2\right)+\frac{bd(2ad+3bc)}{x}\right)}{3ab^2}+\frac{cx\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^2}{a}$$

Rubi [A] time = 0.09, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {375, 98, 147, 63, 208}

$$-\frac{d\sqrt{a+\frac{b}{x}}\left(2\left(-2a^2d^2+9abcd+3b^2c^2\right)+\frac{bd(2ad+3bc)}{x}\right)}{3ab^2}-\frac{c^2(bc-6ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{3/2}}+\frac{cx\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^2}{a}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d/x)^3/Sqrt[a + b/x],x]
```

```
[Out] -(d*Sqrt[a + b/x]*(2*(3*b^2*c^2 + 9*a*b*c*d - 2*a^2*d^2) + (b*d*(3*b*c + 2*
a*d))/x))/(3*a*b^2) + (c*Sqrt[a + b/x]*(c + d/x)^2*x)/a - (c^2*(b*c - 6*a*d
)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/a^(3/2)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 147

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> -Simp[((a*d*f*h*(n + 2) + b*c*f*h*(m
+ 2) - b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1))/(b^2*d^2*(m + n + 2)*(m + n + 3)), x] + Dist[(a^2*d^2
*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m +
n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3
) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3)), In
t[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n},
x] && NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 375

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[((a + b/x^n)^p*(c + d/x^n)^q)/x^2, x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rubi steps

$$\int \frac{\left(c+\frac{d}{x}\right)^3}{\sqrt{a+\frac{b}{x}}}\,dx = -\operatorname{Subst}\left(\int \frac{(c+dx)^3}{x^2\sqrt{a+bx}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{c\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^2 x}{a} + \frac{\operatorname{Subst}\left(\int \frac{(c+dx)\left(\frac{1}{2}c(bc-6ad)-\frac{1}{2}d(3bc+2ad)x\right)}{x\sqrt{a+bx}}\,dx, x, \frac{1}{x}\right)}{a}$$

$$= -\frac{d\sqrt{a+\frac{b}{x}}\left(2\left(3b^2c^2+9abcd-2a^2d^2\right)+\frac{bd(3bc+2ad)}{x}\right)}{3ab^2} + \frac{c\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^2 x}{a} + \frac{\left(c^2(bc-6ad)\right)}{}$$

$$= -\frac{d\sqrt{a+\frac{b}{x}}\left(2\left(3b^2c^2+9abcd-2a^2d^2\right)+\frac{bd(3bc+2ad)}{x}\right)}{3ab^2} + \frac{c\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^2 x}{a} + \frac{\left(c^2(bc-6ad)\right)}{}$$

$$= -\frac{d\sqrt{a+\frac{b}{x}}\left(2\left(3b^2c^2+9abcd-2a^2d^2\right)+\frac{bd(3bc+2ad)}{x}\right)}{3ab^2} + \frac{c\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^2 x}{a} - \frac{c^2(bc-6ad)\,\mathrm{ta}}{a^3}$$

Mathematica [A] time = 0.15, size = 95, normalized size = 0.75

$$\frac{c^2(6ad-bc)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{3/2}} + \frac{\sqrt{a+\frac{b}{x}}\left(4a^2d^3x-2abd^2(9cx+d)+3b^2c^3x^2\right)}{3ab^2x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d/x)^3/Sqrt[a + b/x],x]
```

```
[Out] (Sqrt[a + b/x]*(4*a^2*d^3*x + 3*b^2*c^3*x^2 - 2*a*b*d^2*(d + 9*c*x)))/(3*a*
b^2*x) + (c^2*(-(b*c) + 6*a*d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/a^(3/2)
```

IntegrateAlgebraic [A] time = 0.21, size = 104, normalized size = 0.83

$$\frac{\left(6ac^2d-bc^3\right)\tanh^{-1}\left(\frac{\sqrt{\frac{ax+b}{x}}}{\sqrt{a}}\right)}{a^{3/2}} + \frac{\sqrt{\frac{ax+b}{x}}\left(4a^2d^3x-18abcd^2x-2abd^3+3b^2c^3x^2\right)}{3ab^2x}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c + d/x)^3/Sqrt[a + b/x],x]
```

```
[Out] (Sqrt[(b + a*x)/x]*(-2*a*b*d^3 - 18*a*b*c*d^2*x + 4*a^2*d^3*x + 3*b^2*c^3*x
^2))/(3*a*b^2*x) + ((-(b*c^3) + 6*a*c^2*d)*ArcTanh[Sqrt[(b + a*x)/x]/Sqrt[a
]])/a^(3/2)
```

fricas [A] time = 0.56, size = 233, normalized size = 1.85

$$\left[-\frac{3\left(b^3c^3-6ab^2c^2d\right)\sqrt{a}x\log\left(2ax+2\sqrt{a}x\sqrt{\frac{ax+b}{x}}+b\right)-2\left(3ab^2c^3x^2-2a^2bd^3-2\left(9a^2bcd^2-2a^3d^3\right)x\right)\sqrt{\frac{ax+b}{x}}}{6a^2b^2x},\ \frac{3\left(b^3c^3-6ab^2c^2d\right)\sqrt{-a}x\arctan\left(\frac{\sqrt{-a}\sqrt{\frac{ax+b}{x}}}{a}\right)+\left(3ab^2c^3x^2-2a^2bd^3-2\left(9a^2bcd^2-2a^3d^3\right)x\right)\sqrt{\frac{ax+b}{x}}}{3a^2b^2x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d/x)^3/(a+b/x)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/6*(3*(b^3*c^3 - 6*a*b^2*c^2*d)*sqrt(a)*x*log(2*a*x + 2*sqrt(a)*x*sqrt((
a*x + b)/x) + b) - 2*(3*a*b^2*c^3*x^2 - 2*a^2*b*d^3 - 2*(9*a^2*b*c*d^2 - 2*
a^3*d^3)*x)*sqrt((a*x + b)/x))/(a^2*b^2*x), 1/3*(3*(b^3*c^3 - 6*a*b^2*c^2*d
)*sqrt(-a)*x*arctan(sqrt(-a)*sqrt((a*x + b)/x)/a) + (3*a*b^2*c^3*x^2 - 2*a^
2*b*d^3 - 2*(9*a^2*b*c*d^2 - 2*a^3*d^3)*x)*sqrt((a*x + b)/x))/(a^2*b^2*x)]
```

giac [A] time = 0.20, size = 158, normalized size = 1.25

$$-\frac{\frac{3b^2c^3\sqrt{\frac{ax+b}{x}}}{\left(a-\frac{ax+b}{x}\right)a}-\frac{3\left(b^2c^3-6abc^2d\right)\arctan\left(\frac{\sqrt{\frac{ax+b}{x}}}{\sqrt{-a}}\right)}{\sqrt{-a}\,a}+\frac{2\left(9b^3cd^2\sqrt{\frac{ax+b}{x}}-3ab^2d^3\sqrt{\frac{ax+b}{x}}+\frac{(ax+b)b^2d^3\sqrt{\frac{ax+b}{x}}}{x}\right)}{b^3}}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d/x)^3/(a+b/x)^(1/2),x, algorithm="giac")
```

```
[Out] -1/3*(3*b^2*c^3*sqrt((a*x + b)/x)/((a - (a*x + b)/x)*a) - 3*(b^2*c^3 - 6*a*
b*c^2*d)*arctan(sqrt((a*x + b)/x)/sqrt(-a))/(sqrt(-a)*a) + 2*(9*b^3*c*d^2*s
qrt((a*x + b)/x) - 3*a*b^2*d^3*sqrt((a*x + b)/x) + (a*x + b)*b^2*d^3*sqrt((
a*x + b)/x)/x)/b^3)/b
```

maple [B] time = 0.06, size = 535, normalized size = 4.25

$$\frac{\sqrt{\frac{ax+b}{x}}\left(3x^3a^3bd^3\ln\left(\frac{2ax+b+2\sqrt{(ax+b)x}\sqrt{a}}{2\sqrt{a}}\right)-9x^3a^2b^2cd^2\ln\left(\frac{2ax+b+2\sqrt{(ax+b)x}\sqrt{a}}{2\sqrt{a}}\right)+9x^3ab^3c^2d\ln\left(\frac{2ax+b+2\sqrt{(ax+b)x}\sqrt{a}}{2\sqrt{a}}\right)-3x^3b^4c^3\ln\left(\frac{2ax+b+2\sqrt{(ax+b)x}\sqrt{a}}{2\sqrt{a}}\right)-6\sqrt{ax^2+bx}\,a^{\frac{7}{2}}x^3d^3+18\sqrt{ax^2+bx}\,a^{\frac{5}{2}}x^3bcd^2+18\sqrt{ax^2+bx}\,a^{\frac{3}{2}}x^3b^2c^2d-3x^3a^3bd^3\ln\left(\frac{2ax+b+2\sqrt{ax^2+bx}\sqrt{a}}{2\sqrt{a}}\right)+9x^3a^2b^2cd^2\ln\left(\frac{2ax+b+2\sqrt{ax^2+bx}\sqrt{a}}{2\sqrt{a}}\right)+9x^3ab^3c^2d\ln\left(\frac{2ax+b+2\sqrt{ax^2+bx}\sqrt{a}}{2\sqrt{a}}\right)-6a^{\frac{7}{2}}\sqrt{(ax+b)x}\,x^3d^3+18a^{\frac{5}{2}}\sqrt{(ax+b)x}\,x^3bcd^2-18a^{\frac{3}{2}}\sqrt{(ax+b)x}\,x^3b^2c^2d+6\sqrt{a}\sqrt{(ax+b)x}\,x^3b^3c^3+12\left(ax^2+bx\right)^{\frac{3}{2}}a^{\frac{5}{2}}xd^3-36\left(ax^2+bx\right)^{\frac{3}{2}}a^{\frac{3}{2}}xbcd^2-4d^3\left(ax^2+bx\right)^{\frac{3}{2}}ba^{\frac{3}{2}}\right)}{6x^2\sqrt{(ax+b)x}\,b^3a^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d/x)^3/(a+b/x)^(1/2),x)
```

```
[Out] 1/6*((a*x+b)/x)^(1/2)/x^2*(3*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a
^(1/2))*x^3*a^3*b*d^3-9*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2
))*x^3*a^2*b^2*c*d^2+9*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2)
)*x^3*a*b^3*c^2*d-3*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))*x
^3*b^4*c^3-6*(a*x^2+b*x)^(1/2)*a^(7/2)*x^3*d^3+18*(a*x^2+b*x)^(1/2)*a^(5/2)
*x^3*b*c*d^2+18*(a*x^2+b*x)^(1/2)*a^(3/2)*x^3*b^2*c^2*d-3*ln(1/2*(2*a*x+b+2
*(a*x^2+b*x)^(1/2)*a^(1/2))/a^(1/2))*x^3*a^3*b*d^3+9*ln(1/2*(2*a*x+b+2*(a*x
^2+b*x)^(1/2)*a^(1/2))/a^(1/2))*x^3*a^2*b^2*c*d^2+9*ln(1/2*(2*a*x+b+2*(a*x^
2+b*x)^(1/2)*a^(1/2))/a^(1/2))*x^3*a*b^3*c^2*d-6*a^(7/2)*((a*x+b)*x)^(1/2)*
x^3*d^3+18*a^(5/2)*((a*x+b)*x)^(1/2)*x^3*b*c*d^2-18*a^(3/2)*((a*x+b)*x)^(1/
2)*x^3*b^2*c^2*d+6*a^(1/2)*((a*x+b)*x)^(1/2)*x^3*b^3*c^3+12*(a*x^2+b*x)^(3/
2)*a^(5/2)*x*d^3-36*(a*x^2+b*x)^(3/2)*a^(3/2)*x*b*c*d^2-4*d^3*(a*x^2+b*x)^(
3/2)*b*a^(3/2))/((a*x+b)*x)^(1/2)/b^3/a^(3/2)
```

maxima [A] time = 1.33, size = 166, normalized size = 1.32

$$\frac{1}{2}c^3\left(\frac{2\sqrt{a+\frac{b}{x}}\,b}{\left(a+\frac{b}{x}\right)a-a^2}+\frac{b\log\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{a+\frac{b}{x}}+\sqrt{a}}\right)}{a^{\frac{3}{2}}}\right)-\frac{2}{3}d^3\left(\frac{\left(a+\frac{b}{x}\right)^{\frac{3}{2}}}{b^2}-\frac{3\sqrt{a+\frac{b}{x}}\,a}{b^2}\right)-\frac{3c^2d\log\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{a+\frac{b}{x}}+\sqrt{a}}\right)}{\sqrt{a}}-\frac{6\sqrt{a+\frac{b}{x}}\,cd^2}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d/x)^3/(a+b/x)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*c^3*(2*sqrt(a + b/x)*b/((a + b/x)*a - a^2) + b*log((sqrt(a + b/x) - sqr
t(a))/(sqrt(a + b/x) + sqrt(a)))/a^(3/2)) - 2/3*d^3*((a + b/x)^(3/2)/b^2 -
3*sqrt(a + b/x)*a/b^2) - 3*c^2*d*log((sqrt(a + b/x) - sqrt(a))/(sqrt(a + b/
x) + sqrt(a)))/sqrt(a) - 6*sqrt(a + b/x)*c*d^2/b
```

mupad [B] time = 1.73, size = 107, normalized size = 0.85

$$\sqrt{a+\frac{b}{x}}\,\left(\frac{6\,a\,d^3-6\,b\,c\,d^2}{b^2}-\frac{4\,a\,d^3}{b^2}\right)-\frac{2\,d^3\,{\left(a+\frac{b}{x}\right)}^{3/2}}{3\,b^2}+\frac{c^3\,x\,\sqrt{a+\frac{b}{x}}}{a}-\frac{c^2\,\mathrm{atan}\left(\frac{\sqrt{a+\frac{b}{x}}\,1{}\mathrm{i}}{\sqrt{a}}\right)\,\left(6\,a\,d-b\,c\right)\,1{}\mathrm{i}}{a^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d/x)^3/(a + b/x)^(1/2),x)
```

```
[Out] (a + b/x)^(1/2)*((6*a*d^3 - 6*b*c*d^2)/b^2 - (4*a*d^3)/b^2) - (2*d^3*(a + b
/x)^(3/2))/(3*b^2) + (c^3*x*(a + b/x)^(1/2))/a - (c^2*atan(((a + b/x)^(1/2)
*1i)/a^(1/2))*(6*a*d - b*c)*1i)/a^(3/2)
```

sympy [A] time = 90.00, size = 386, normalized size = 3.06

$$\frac{4a^{\frac{7}{2}}b^{\frac{3}{2}}d^3x^2\sqrt{\frac{ax}{b}+1}}{3a^{\frac{5}{2}}b^3x^{\frac{5}{2}}+3a^{\frac{3}{2}}b^4x^{\frac{3}{2}}}+\frac{2a^{\frac{5}{2}}b^{\frac{5}{2}}d^3x\sqrt{\frac{ax}{b}+1}}{3a^{\frac{5}{2}}b^3x^{\frac{5}{2}}+3a^{\frac{3}{2}}b^4x^{\frac{3}{2}}}-\frac{2a^{\frac{3}{2}}b^{\frac{7}{2}}d^3\sqrt{\frac{ax}{b}+1}}{3a^{\frac{5}{2}}b^3x^{\frac{5}{2}}+3a^{\frac{3}{2}}b^4x^{\frac{3}{2}}}-\frac{4a^4bd^3x^{\frac{5}{2}}}{3a^{\frac{5}{2}}b^3x^{\frac{5}{2}}+3a^{\frac{3}{2}}b^4x^{\frac{3}{2}}}-\frac{4a^3b^2d^3x^{\frac{3}{2}}}{3a^{\frac{5}{2}}b^3x^{\frac{5}{2}}+3a^{\frac{3}{2}}b^4x^{\frac{3}{2}}}+3cd^2\left(\begin{cases}-\frac{1}{\sqrt{a}x} & \text{for}\: b=0\\-\frac{2\sqrt{a+\frac{b}{x}}}{b} & \text{otherwise}\end{cases}\right)+\frac{\sqrt{b}c^3\sqrt{x}\sqrt{\frac{ax}{b}+1}}{a}-\frac{6c^2d\,\mathrm{atan}\left(\frac{1}{\sqrt{-\frac{1}{a}}\sqrt{a+\frac{b}{x}}}\right)}{a\sqrt{-\frac{1}{a}}}-\frac{bc^3\,\mathrm{asinh}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)}{a^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d/x)**3/(a+b/x)**(1/2),x)
```

```
[Out] 4*a**(7/2)*b**(3/2)*d**3*x**2*sqrt(a*x/b + 1)/(3*a**(5/2)*b**3*x**(5/2) + 3
*a**(3/2)*b**4*x**(3/2)) + 2*a**(5/2)*b**(5/2)*d**3*x*sqrt(a*x/b + 1)/(3*a*
*(5/2)*b**3*x**(5/2) + 3*a**(3/2)*b**4*x**(3/2)) - 2*a**(3/2)*b**(7/2)*d**3
*sqrt(a*x/b + 1)/(3*a**(5/2)*b**3*x**(5/2) + 3*a**(3/2)*b**4*x**(3/2)) - 4*
a**4*b*d**3*x**(5/2)/(3*a**(5/2)*b**3*x**(5/2) + 3*a**(3/2)*b**4*x**(3/2))
- 4*a**3*b**2*d**3*x**(3/2)/(3*a**(5/2)*b**3*x**(5/2) + 3*a**(3/2)*b**4*x**
(3/2)) + 3*c*d**2*Piecewise((-1/(sqrt(a)*x), Eq(b, 0)), (-2*sqrt(a + b/x)/b
, True)) + sqrt(b)*c**3*sqrt(x)*sqrt(a*x/b + 1)/a - 6*c**2*d*atan(1/(sqrt(-
1/a)*sqrt(a + b/x)))/(a*sqrt(-1/a)) - b*c**3*asinh(sqrt(a)*sqrt(x)/sqrt(b))
/a**(3/2)
```

3.149 $\int \frac{\left(c+\frac{d}{x}\right)^2}{\sqrt{a+\frac{b}{x}}} \, dx$

Optimal. Leaf size=73

$$-\frac{c(bc-4ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{3/2}}+\frac{c^2x\sqrt{a+\frac{b}{x}}}{a}-\frac{2d^2\sqrt{a+\frac{b}{x}}}{b}$$

Rubi [A] time = 0.05, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {375, 89, 80, 63, 208}

$$-\frac{c(bc-4ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{3/2}}+\frac{c^2x\sqrt{a+\frac{b}{x}}}{a}-\frac{2d^2\sqrt{a+\frac{b}{x}}}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d/x)^2/Sqrt[a + b/x],x]
```

```
[Out] (-2*d^2*Sqrt[a + b/x])/b + (c^2*Sqrt[a + b/x]*x)/a - (c*(b*c - 4*a*d)*ArcTa
nh[Sqrt[a + b/x]/Sqrt[a]])/a^(3/2)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 375

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[((a + b/x^n)^p*(c + d/x^n)^q)/x^2, x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(c+\frac{d}{x}\right)^2}{\sqrt{a+\frac{b}{x}}}\,dx &= -\text{Subst}\left(\int \frac{(c+dx)^2}{x^2\sqrt{a+bx}}\,dx, x, \frac{1}{x}\right)\\
&= \frac{c^2\sqrt{a+\frac{b}{x}}\,x}{a} - \frac{\text{Subst}\left(\int \frac{-\frac{1}{2}c(bc-4ad)+ad^2x}{x\sqrt{a+bx}}\,dx, x, \frac{1}{x}\right)}{a}\\
&= -\frac{2d^2\sqrt{a+\frac{b}{x}}}{b} + \frac{c^2\sqrt{a+\frac{b}{x}}\,x}{a} + \frac{(c(bc-4ad))\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \frac{1}{x}\right)}{2a}\\
&= -\frac{2d^2\sqrt{a+\frac{b}{x}}}{b} + \frac{c^2\sqrt{a+\frac{b}{x}}\,x}{a} + \frac{(c(bc-4ad))\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+\frac{b}{x}}\right)}{ab}\\
&= -\frac{2d^2\sqrt{a+\frac{b}{x}}}{b} + \frac{c^2\sqrt{a+\frac{b}{x}}\,x}{a} - \frac{c(bc-4ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{3/2}}
\end{aligned}$$

Mathematica [A] time = 0.09, size = 66, normalized size = 0.90

$$\frac{c(4ad-bc)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{3/2}} + \frac{\sqrt{a+\frac{b}{x}}\left(bc^2x-2ad^2\right)}{ab}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d/x)^2/Sqrt[a + b/x],x]
```

```
[Out] (Sqrt[a + b/x]*(-2*a*d^2 + b*c^2*x))/(a*b) + (c*(-(b*c) + 4*a*d)*ArcTanh[Sq
rt[a + b/x]/Sqrt[a]])/a^(3/2)
```

IntegrateAlgebraic [A] time = 0.16, size = 72, normalized size = 0.99

$$\frac{\left(4acd-bc^2\right)\tanh^{-1}\left(\frac{\sqrt{\frac{ax+b}{x}}}{\sqrt{a}}\right)}{a^{3/2}} + \frac{\sqrt{\frac{ax+b}{x}}\left(bc^2x-2ad^2\right)}{ab}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c + d/x)^2/Sqrt[a + b/x],x]
```

```
[Out] (Sqrt[(b + a*x)/x]*(-2*a*d^2 + b*c^2*x))/(a*b) + ((-(b*c^2) + 4*a*c*d)*ArcT
anh[Sqrt[(b + a*x)/x]/Sqrt[a]])/a^(3/2)
```

fricas [A] time = 0.61, size = 158, normalized size = 2.16

$$\left[-\frac{\left(b^2c^2-4\,abcd\right)\sqrt{a}\log\left(2\,ax+2\,\sqrt{a}\,x\sqrt{\frac{ax+b}{x}}+b\right)-2\left(abc^2x-2\,a^2d^2\right)\sqrt{\frac{ax+b}{x}}}{2\,a^2b}, \frac{\left(b^2c^2-4\,abcd\right)\sqrt{-a}\arctan\left(\frac{\sqrt{-a}\sqrt{\frac{ax+b}{x}}}{a}\right)+\left(abc^2x-2\,a^2d^2\right)\sqrt{\frac{ax+b}{x}}}{a^2b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d/x)^2/(a+b/x)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/2*((b^2*c^2 - 4*a*b*c*d)*sqrt(a)*log(2*a*x + 2*sqrt(a)*x*sqrt((a*x + b)
/x) + b) - 2*(a*b*c^2*x - 2*a^2*d^2)*sqrt((a*x + b)/x))/(a^2*b), ((b^2*c^2
- 4*a*b*c*d)*sqrt(-a)*arctan(sqrt(-a)*sqrt((a*x + b)/x)/a) + (a*b*c^2*x - 2
*a^2*d^2)*sqrt((a*x + b)/x))/(a^2*b)]
```

giac [**A**] time = 0.18, size = 99, normalized size = 1.36

$$-\frac{\frac{b^2c^2\sqrt{\frac{ax+b}{x}}}{\left(a-\frac{ax+b}{x}\right)a}+2\,d^2\sqrt{\frac{ax+b}{x}}-\frac{\left(b^2c^2-4\,abcd\right)\arctan\left(\frac{\sqrt{\frac{ax+b}{x}}}{\sqrt{-a}}\right)}{\sqrt{-a}\,a}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d/x)^2/(a+b/x)^(1/2),x, algorithm="giac")
```

```
[Out] -(b^2*c^2*sqrt((a*x + b)/x)/((a - (a*x + b)/x)*a) + 2*d^2*sqrt((a*x + b)/x)
 - (b^2*c^2 - 4*a*b*c*d)*arctan(sqrt((a*x + b)/x)/sqrt(-a))/(sqrt(-a)*a))/b
```

maple [**B**] time = 0.06, size = 348, normalized size = 4.77

$$-\frac{\sqrt{\frac{ax+b}{x}}\left(a^2b\,d^2x^2\ln\left(\frac{2ax+b+2\sqrt{(ax+b)x}\,\sqrt{a}}{2\sqrt{a}}\right)-a^2b\,d^2x^2\ln\left(\frac{2ax+b+2\sqrt{a\,x^2+bx}\,\sqrt{a}}{2\sqrt{a}}\right)-2a\,b^2cd\,x^2\ln\left(\frac{2ax+b+2\sqrt{(ax+b)x}\,\sqrt{a}}{2\sqrt{a}}\right)-2a\,b^2cd\,x^2\ln\left(\frac{2ax+b+2\sqrt{a\,x^2+bx}\,\sqrt{a}}{2\sqrt{a}}\right)+b^3c^2x^2\ln\left(\frac{2ax+b+2\sqrt{(ax+b)x}\,\sqrt{a}}{2\sqrt{a}}\right)-2\sqrt{a\,x^2+bx}\,a^{\frac{5}{2}}d^2x^2-2\sqrt{(ax+b)\,x}\,a^{\frac{5}{2}}d^2x^2-4\sqrt{a\,x^2+bx}\,a^{\frac{3}{2}}bcd\,x^2+4\sqrt{(ax+b)\,x}\,a^{\frac{3}{2}}bcd\,x^2-2\sqrt{(ax+b)\,x}\,\sqrt{a}\,b^2c^2x^2+4\left(a\,x^2+bx\right)^{\frac{3}{2}}a^{\frac{3}{2}}d^2\right)}{2\sqrt{(ax+b)\,x}\,a^{\frac{3}{2}}b^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d/x)^2/(a+b/x)^(1/2),x)
```

```
[Out] -1/2*((a*x+b)/x)^(1/2)/x*(ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1
/2))*x^2*a^2*b*d^2-2*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))*
x^2*a*b^2*c*d+ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))*x^2*b^3
*c^2-2*(a*x^2+b*x)^(1/2)*a^(5/2)*x^2*d^2-4*(a*x^2+b*x)^(1/2)*a^(3/2)*x^2*b*
c*d-ln(1/2*(2*a*x+b+2*(a*x^2+b*x)^(1/2)*a^(1/2))/a^(1/2))*x^2*a^2*b*d^2-2*l
n(1/2*(2*a*x+b+2*(a*x^2+b*x)^(1/2)*a^(1/2))/a^(1/2))*x^2*a*b^2*c*d-2*a^(5/2
)*((a*x+b)*x)^(1/2)*x^2*d^2+4*a^(3/2)*((a*x+b)*x)^(1/2)*x^2*b*c*d-2*a^(1/2)
*((a*x+b)*x)^(1/2)*x^2*b^2*c^2+4*(a*x^2+b*x)^(3/2)*a^(3/2)*d^2)/((a*x+b)*x)
^(1/2)/b^2/a^(3/2)
```

maxima [**B**] time = 1.24, size = 129, normalized size = 1.77

$$\frac{1}{2}\,c^2\left(\frac{2\,\sqrt{a+\frac{b}{x}}\,b}{\left(a+\frac{b}{x}\right)a-a^2}+\frac{b\log\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{a+\frac{b}{x}}+\sqrt{a}}\right)}{a^{\frac{3}{2}}}\right)-\frac{2\,cd\log\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{a+\frac{b}{x}}+\sqrt{a}}\right)}{\sqrt{a}}-\frac{2\,\sqrt{a+\frac{b}{x}}\,d^2}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d/x)^2/(a+b/x)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*c^2*(2*sqrt(a + b/x)*b/((a + b/x)*a - a^2) + b*log((sqrt(a + b/x) - sqr
t(a))/(sqrt(a + b/x) + sqrt(a)))/a^(3/2)) - 2*c*d*log((sqrt(a + b/x) - sqrt
(a))/(sqrt(a + b/x) + sqrt(a)))/sqrt(a) - 2*sqrt(a + b/x)*d^2/b
```

mupad [**B**] time = 1.62, size = 63, normalized size = 0.86

$$\frac{c^2\,x\,\sqrt{a+\frac{b}{x}}}{a}-\frac{2\,d^2\,\sqrt{a+\frac{b}{x}}}{b}+\frac{c\,\mathrm{atanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)\,\left(4\,a\,d-b\,c\right)}{a^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d/x)^2/(a + b/x)^(1/2),x)
```

```
[Out] (c^2*x*(a + b/x)^(1/2))/a - (2*d^2*(a + b/x)^(1/2))/b + (c*atanh((a + b/x)^
(1/2)/a^(1/2))*(4*a*d - b*c))/a^(3/2)
```

sympy [**A**] time = 83.77, size = 114, normalized size = 1.56

$$d^2\left(\begin{cases} -\frac{1}{\sqrt{a}\,x} & \text{for } b = 0 \\ -\frac{2\sqrt{a+\frac{b}{x}}}{b} & \text{otherwise} \end{cases}\right) + \frac{\sqrt{b}\,c^2\sqrt{x}\,\sqrt{\frac{ax}{b}+1}}{a} - \frac{4cd\,\mathrm{atan}\left(\frac{1}{\sqrt{-\frac{1}{a}}\,\sqrt{a+\frac{b}{x}}}\right)}{a\sqrt{-\frac{1}{a}}} - \frac{bc^2\,\mathrm{asinh}\left(\frac{\sqrt{a}\,\sqrt{x}}{\sqrt{b}}\right)}{a^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d/x)**2/(a+b/x)**(1/2),x)
```

```
[Out] d**2*Piecewise((-1/(sqrt(a)*x), Eq(b, 0)), (-2*sqrt(a + b/x)/b, True)) + sq
rt(b)*c**2*sqrt(x)*sqrt(a*x/b + 1)/a - 4*c*d*atan(1/(sqrt(-1/a)*sqrt(a + b/
x)))/(a*sqrt(-1/a)) - b*c**2*asinh(sqrt(a)*sqrt(x)/sqrt(b))/a**(3/2)
```

3.150 $\int \frac{c+\frac{d}{x}}{\sqrt{a+\frac{b}{x}}} \, dx$

Optimal. Leaf size=51

$$\frac{cx\sqrt{a+\frac{b}{x}}}{a}-\frac{(bc-2ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{3/2}}$$

Rubi [A] time = 0.03, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {375, 78, 63, 208}

$$\frac{cx\sqrt{a+\frac{b}{x}}}{a}-\frac{(bc-2ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d/x)/Sqrt[a + b/x],x]
```

```
[Out] (c*Sqrt[a + b/x]*x)/a - ((b*c - 2*a*d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/a^(3
/2)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 375

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[((a + b/x^n)^p*(c + d/x^n)^q)/x^2, x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rubi steps

$$\int \frac{c+\frac{d}{x}}{\sqrt{a+\frac{b}{x}}}\,dx = -\mathrm{Subst}\left(\int \frac{c+dx}{x^2\sqrt{a+bx}}\,dx, x, \frac{1}{x}\right)$$
$$= \frac{c\sqrt{a+\frac{b}{x}}\,x}{a} - \frac{\left(-\frac{bc}{2}+ad\right)\mathrm{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \frac{1}{x}\right)}{a}$$
$$= \frac{c\sqrt{a+\frac{b}{x}}\,x}{a} - \frac{\left(2\left(-\frac{bc}{2}+ad\right)\right)\mathrm{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+\frac{b}{x}}\right)}{ab}$$
$$= \frac{c\sqrt{a+\frac{b}{x}}\,x}{a} - \frac{(bc-2ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{3/2}}$$

Mathematica [A] time = 0.04, size = 53, normalized size = 1.04

$$\frac{2\left(ad-\frac{bc}{2}\right)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{3/2}} + \frac{cx\sqrt{a+\frac{b}{x}}}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d/x)/Sqrt[a + b/x],x]
```

```
[Out] (c*Sqrt[a + b/x]*x)/a + (2*(-1/2*(b*c) + a*d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]
])/a^(3/2)
```

IntegrateAlgebraic [A] time = 0.11, size = 55, normalized size = 1.08

$$\frac{(2ad-bc)\tanh^{-1}\left(\frac{\sqrt{\frac{ax+b}{x}}}{\sqrt{a}}\right)}{a^{3/2}} + \frac{cx\sqrt{\frac{ax+b}{x}}}{a}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c + d/x)/Sqrt[a + b/x],x]
```

```
[Out] (c*x*Sqrt[(b + a*x)/x])/a + ((-(b*c) + 2*a*d)*ArcTanh[Sqrt[(b + a*x)/x]/Sqr
t[a]])/a^(3/2)
```

fricas [A] time = 0.85, size = 115, normalized size = 2.25

$$\left[\frac{2\,acx\sqrt{\frac{ax+b}{x}} - (bc-2\,ad)\sqrt{a}\,\log\left(2\,ax+2\,\sqrt{a}\,x\sqrt{\frac{ax+b}{x}}+b\right)}{2\,a^2}, \frac{acx\sqrt{\frac{ax+b}{x}} + (bc-2\,ad)\sqrt{-a}\,\arctan\left(\frac{\sqrt{-a}\sqrt{\frac{ax+b}{x}}}{a}\right)}{a^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d/x)/(a+b/x)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*(2*a*c*x*sqrt((a*x + b)/x) - (b*c - 2*a*d)*sqrt(a)*log(2*a*x + 2*sqrt(
a)*x*sqrt((a*x + b)/x) + b))/a^2, (a*c*x*sqrt((a*x + b)/x) + (b*c - 2*a*d)*
sqrt(-a)*arctan(sqrt(-a)*sqrt((a*x + b)/x)/a))/a^2]
```

giac [A] time = 0.20, size = 78, normalized size = 1.53

$$-\frac{\frac{b^{2} c \sqrt{\frac{a x+b}{x}}}{\left(a-\frac{a x+b}{x}\right) a}-\frac{\left(b^{2} c-2 \, a b d\right) \arctan\left(\frac{\sqrt{\frac{a x+b}{x}}}{\sqrt{-a}}\right)}{\sqrt{-a} a}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d/x)/(a+b/x)^(1/2),x, algorithm="giac")
```

```
[Out] -(b^2*c*sqrt((a*x + b)/x)/((a - (a*x + b)/x)*a) - (b^2*c - 2*a*b*d)*arctan(
sqrt((a*x + b)/x)/sqrt(-a))/(sqrt(-a)*a))/b
```

maple [B] time = 0.06, size = 173, normalized size = 3.39

$$\frac{\sqrt{\frac{a x+b}{x}}\, \left(a b d \ln\left(\frac{2 a x+b+2 \sqrt{\left(a x+b\right) x}\, \sqrt{a}}{2 \sqrt{a}}\right)+a b d \ln\left(\frac{2 a x+b+2 \sqrt{a \,x^{2}+b x}\, \sqrt{a}}{2 \sqrt{a}}\right)-b^{2} c \ln\left(\frac{2 a x+b+2 \sqrt{\left(a x+b\right) x}\, \sqrt{a}}{2 \sqrt{a}}\right)+2 \sqrt{a \,x^{2}+b x}\, a^{\frac{3}{2}} d-2 \sqrt{\left(a x+b\right) x}\, a^{\frac{3}{2}} d+2 \sqrt{\left(a x+b\right) x}\, \sqrt{a}\, b c\right) x}{2 \sqrt{\left(a x+b\right) x}\, a^{\frac{3}{2}} b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d/x)/(a+b/x)^(1/2),x)
```

```
[Out] 1/2*((a*x+b)/x)^(1/2)*x*(2*a^(3/2)*(a*x^2+b*x)^(1/2)*d-2*a^(3/2)*((a*x+b)*x
)^(1/2)*d+2*a^(1/2)*((a*x+b)*x)^(1/2)*b*c+ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/
2)*a^(1/2))/a^(1/2))*a*b*d-ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(
1/2))*b^2*c+ln(1/2*(2*a*x+b+2*(a*x^2+b*x)^(1/2)*a^(1/2))/a^(1/2))*a*b*d)/((
a*x+b)*x)^(1/2)/b/a^(3/2)
```

maxima [B] time = 1.30, size = 109, normalized size = 2.14

$$\frac{1}{2} \, c {\left(\frac{2 \, \sqrt{a + \frac{b}{x}} b}{{\left(a + \frac{b}{x}\right)} a - a^{2}} + \frac{b \log\left(\frac{\sqrt{a + \frac{b}{x}} - \sqrt{a}}{\sqrt{a + \frac{b}{x}} + \sqrt{a}}\right)}{a^{\frac{3}{2}}}\right)} - \frac{d \log\left(\frac{\sqrt{a + \frac{b}{x}} - \sqrt{a}}{\sqrt{a + \frac{b}{x}} + \sqrt{a}}\right)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d/x)/(a+b/x)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*c*(2*sqrt(a + b/x)*b/((a + b/x)*a - a^2) + b*log((sqrt(a + b/x) - sqrt(
a))/(sqrt(a + b/x) + sqrt(a)))/a^(3/2)) - d*log((sqrt(a + b/x) - sqrt(a))/(
sqrt(a + b/x) + sqrt(a)))/sqrt(a)
```

mupad [B] time = 1.98, size = 88, normalized size = 1.73

$$\frac{2\,d\,\mathrm{atanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{\sqrt{a}}+\frac{2\,c\,x\,\left(\frac{3\,\sqrt{b}\,\sqrt{b+a\,x}}{2\,a\,x}+\frac{b^{3/2}\,\mathrm{asin}\left(\frac{\sqrt{a}\,\sqrt{x}\,1{}\mathrm{i}}{\sqrt{b}}\right)\,3{}\mathrm{i}}{2\,a^{3/2}\,x^{3/2}}\right)\,\sqrt{\frac{a\,x}{b}+1}}{3\,\sqrt{a+\frac{b}{x}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d/x)/(a + b/x)^(1/2),x)
```

```
[Out] (2*d*atanh((a + b/x)^(1/2)/a^(1/2)))/a^(1/2) + (2*c*x*((3*b^(1/2)*(b + a*x)
^(1/2))/(2*a*x) + (b^(3/2)*asin((a^(1/2)*x^(1/2)*1i)/b^(1/2))*3i)/(2*a^(3/2
)*x^(3/2)))*((a*x)/b + 1)^(1/2))/(3*(a + b/x)^(1/2))
```

sympy [A] time = 60.25, size = 82, normalized size = 1.61

$$\frac{\sqrt{b}\,c\sqrt{x}\,\sqrt{\frac{ax}{b}+1}}{a}-\frac{2d\,\operatorname{atan}\left(\frac{1}{\sqrt{-\frac{1}{a}}\,\sqrt{a+\frac{b}{x}}}\right)}{a\sqrt{-\frac{1}{a}}}-\frac{bc\,\operatorname{asinh}\left(\frac{\sqrt{a}\,\sqrt{x}}{\sqrt{b}}\right)}{a^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d/x)/(a+b/x)**(1/2),x)
```

```
[Out] sqrt(b)*c*sqrt(x)*sqrt(a*x/b + 1)/a - 2*d*atan(1/(sqrt(-1/a)*sqrt(a + b/x))
)/(a*sqrt(-1/a)) - b*c*asinh(sqrt(a)*sqrt(x)/sqrt(b))/a**(3/2)
```

3.151 $\int \frac{1}{\sqrt{a+\frac{b}{x}}}\,dx$

Optimal. Leaf size=43

$$\frac{x\sqrt{a+\frac{b}{x}}}{a}-\frac{b\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{3/2}}$$

Rubi [A] time = 0.02, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {242, 51, 63, 208}

$$\frac{x\sqrt{a+\frac{b}{x}}}{a}-\frac{b\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a + b/x],x]
```

```
[Out] (Sqrt[a + b/x]*x)/a - (b*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/a^(3/2)
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 242

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a + b/x^n)^p/x^
2, x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+\frac{b}{x}}}\,dx = -\operatorname{Subst}\left(\int \frac{1}{x^2\sqrt{a+bx}}\,dx, x, \frac{1}{x}\right)$$
$$= \frac{\sqrt{a+\frac{b}{x}}\,x}{a} + \frac{b\operatorname{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \frac{1}{x}\right)}{2a}$$
$$= \frac{\sqrt{a+\frac{b}{x}}\,x}{a} + \frac{\operatorname{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+\frac{b}{x}}\right)}{a}$$
$$= \frac{\sqrt{a+\frac{b}{x}}\,x}{a} - \frac{b\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{3/2}}$$

Mathematica [A] time = 0.02, size = 43, normalized size = 1.00

$$\frac{x\sqrt{a+\frac{b}{x}}}{a} - \frac{b\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a + b/x],x]
[Out] (Sqrt[a + b/x]*x)/a - (b*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/a^(3/2)
```

IntegrateAlgebraic [A] time = 0.00, size = 47, normalized size = 1.09

$$\frac{x\sqrt{\frac{ax+b}{x}}}{a} - \frac{b\tanh^{-1}\left(\frac{\sqrt{\frac{ax+b}{x}}}{\sqrt{a}}\right)}{a^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/Sqrt[a + b/x],x]
[Out] (x*Sqrt[(b + a*x)/x])/a - (b*ArcTanh[Sqrt[(b + a*x)/x]/Sqrt[a]])/a^(3/2)
```

fricas [A] time = 0.78, size = 98, normalized size = 2.28

$$\left[\frac{2\,ax\sqrt{\frac{ax+b}{x}} + \sqrt{a}\,b\log\left(2\,ax - 2\,\sqrt{a}\,x\sqrt{\frac{ax+b}{x}} + b\right)}{2\,a^2}, \frac{ax\sqrt{\frac{ax+b}{x}} + \sqrt{-a}\,b\arctan\left(\frac{\sqrt{-a}\sqrt{\frac{ax+b}{x}}}{a}\right)}{a^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)^(1/2),x, algorithm="fricas")
[Out] [1/2*(2*a*x*sqrt((a*x + b)/x) + sqrt(a)*b*log(2*a*x - 2*sqrt(a)*x*sqrt((a*x
 + b)/x) + b))/a^2, (a*x*sqrt((a*x + b)/x) + sqrt(-a)*b*arctan(sqrt(-a)*sqr
t((a*x + b)/x)/a))/a^2]
```

giac [B] time = 0.24, size = 71, normalized size = 1.65

$$-\frac{b\log\left(|b|\right)\operatorname{sgn}(x)}{2\,a^{\frac{3}{2}}} + \frac{b\log\left(\left|-2\left(\sqrt{a}\,x - \sqrt{ax^2+bx}\right)\sqrt{a} - b\right|\right)}{2\,a^{\frac{3}{2}}\operatorname{sgn}(x)} + \frac{\sqrt{ax^2+bx}}{a\operatorname{sgn}(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*b*log(abs(b))*sgn(x)/a^(3/2) + 1/2*b*log(abs(-2*(sqrt(a)*x - sqrt(a*x^
2 + b*x))*sqrt(a) - b))/(a^(3/2)*sgn(x)) + sqrt(a*x^2 + b*x)/(a*sgn(x))
```

maple [A] time = 0.05, size = 71, normalized size = 1.65

$$\frac{\sqrt{\frac{ax+b}{x}}\left(-b\ln\left(\frac{2ax+b+2\sqrt{(ax+b)x}\sqrt{a}}{2\sqrt{a}}\right)+2\sqrt{(ax+b)\,x}\,\sqrt{a}\right)x}{2\sqrt{(ax+b)\,x}\,a^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b/x)^(1/2),x)
```

```
[Out] 1/2*((a*x+b)/x)^(1/2)*x*(2*((a*x+b)*x)^(1/2)*a^(1/2)-b*ln(1/2*(2*a*x+b+2*((
a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2)))/((a*x+b)*x)^(1/2)/a^(3/2)
```

maxima [A] time = 1.21, size = 67, normalized size = 1.56

$$\frac{\sqrt{a+\frac{b}{x}}\,b}{\left(a+\frac{b}{x}\right)a-a^2}+\frac{b\log\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{a+\frac{b}{x}}+\sqrt{a}}\right)}{2\,a^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)^(1/2),x, algorithm="maxima")
```

```
[Out] sqrt(a + b/x)*b/((a + b/x)*a - a^2) + 1/2*b*log((sqrt(a + b/x) - sqrt(a))/(
sqrt(a + b/x) + sqrt(a)))/a^(3/2)
```

mupad [B] time = 1.44, size = 66, normalized size = 1.53

$$\frac{2\,x\left(\frac{3\,\sqrt{b}\,\sqrt{b+a\,x}}{2\,a\,x}+\frac{b^{3/2}\,\mathrm{asin}\left(\frac{\sqrt{a}\,\sqrt{x}\,1\mathrm{i}}{\sqrt{b}}\right)3\mathrm{i}}{2\,a^{3/2}\,x^{3/2}}\right)\sqrt{\frac{a\,x}{b}+1}}{3\,\sqrt{a+\frac{b}{x}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b/x)^(1/2),x)
```

```
[Out] (2*x*((3*b^(1/2)*(b + a*x)^(1/2))/(2*a*x) + (b^(3/2)*asin((a^(1/2)*x^(1/2)*
1i)/b^(1/2))*3i)/(2*a^(3/2)*x^(3/2)))*((a*x)/b + 1)^(1/2))/(3*(a + b/x)^(1/
2))
```

sympy [A] time = 3.09, size = 44, normalized size = 1.02

$$\frac{\sqrt{b}\,\sqrt{x}\,\sqrt{\frac{ax}{b}+1}}{a}-\frac{b\,\mathrm{asinh}\left(\frac{\sqrt{a}\,\sqrt{x}}{\sqrt{b}}\right)}{a^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)**(1/2),x)
```

```
[Out] sqrt(b)*sqrt(x)*sqrt(a*x/b + 1)/a - b*asinh(sqrt(a)*sqrt(x)/sqrt(b))/a**(3/
2)
```

3.152 $\int \frac{1}{\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)}\,dx$

Optimal. Leaf size=108

$$-\frac{(2ad+bc)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{3/2}c^2}-\frac{2d^{3/2}\tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c^2\sqrt{bc-ad}}+\frac{x\sqrt{a+\frac{b}{x}}}{ac}$$

Rubi [A] time = 0.10, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {375, 103, 156, 63, 208, 205}

$$-\frac{(2ad+bc)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{3/2}c^2}-\frac{2d^{3/2}\tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c^2\sqrt{bc-ad}}+\frac{x\sqrt{a+\frac{b}{x}}}{ac}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a + b/x]*(c + d/x)),x]
```

```
[Out] (Sqrt[a + b/x]*x)/(a*c) - (2*d^(3/2)*ArcTan[(Sqrt[d]*Sqrt[a + b/x])/Sqrt[b*
c - a*d]])/(c^2*Sqrt[b*c - a*d]) - ((b*c + 2*a*d)*ArcTanh[Sqrt[a + b/x]/Sqr
t[a]])/(a^(3/2)*c^2)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 375

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[((a + b/x^n)^p*(c + d/x^n)^q)/x^2, x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)}\,dx &= -\operatorname{Subst}\left(\int \frac{1}{x^2\sqrt{a+bx}\,(c+dx)}\,dx, x, \frac{1}{x}\right)\\
&= \frac{\sqrt{a+\frac{b}{x}}\,x}{ac} + \frac{\operatorname{Subst}\left(\int \frac{\frac{1}{2}(bc+2ad)+\frac{bdx}{2}}{x\sqrt{a+bx}\,(c+dx)}\,dx, x, \frac{1}{x}\right)}{ac}\\
&= \frac{\sqrt{a+\frac{b}{x}}\,x}{ac} - \frac{d^2\operatorname{Subst}\left(\int \frac{1}{\sqrt{a+bx}\,(c+dx)}\,dx, x, \frac{1}{x}\right)}{c^2} + \frac{(bc+2ad)\operatorname{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \frac{1}{x}\right)}{2ac^2}\\
&= \frac{\sqrt{a+\frac{b}{x}}\,x}{ac} - \frac{\left(2d^2\right)\operatorname{Subst}\left(\int \frac{1}{c-\frac{ad}{b}+\frac{dx^2}{b}}\,dx, x, \sqrt{a+\frac{b}{x}}\right)}{bc^2} + \frac{(bc+2ad)\operatorname{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x,\right.}{abc^2}\\
&= \frac{\sqrt{a+\frac{b}{x}}\,x}{ac} - \frac{2d^{3/2}\tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c^2\sqrt{bc-ad}} - \frac{(bc+2ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{3/2}c^2}
\end{aligned}
$$

Mathematica [A] time = 0.25, size = 104, normalized size = 0.96

$$
\frac{-\frac{(2ad+bc)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{3/2}} - \frac{2d^{3/2}\tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{\sqrt{bc-ad}} + \frac{cx\sqrt{a+\frac{b}{x}}}{a}}{c^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[a + b/x]*(c + d/x)),x]
```

```
[Out] ((c*Sqrt[a + b/x]*x)/a - (2*d^(3/2)*ArcTan[(Sqrt[d]*Sqrt[a + b/x])/Sqrt[b*c
 - a*d]])/Sqrt[b*c - a*d] - ((b*c + 2*a*d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/
a^(3/2))/c^2
```

IntegrateAlgebraic [A] time = 0.22, size = 114, normalized size = 1.06

$$
\frac{(-2ad-bc)\tanh^{-1}\left(\frac{\sqrt{\frac{ax+b}{x}}}{\sqrt{a}}\right)}{a^{3/2}c^2} - \frac{2d^{3/2}\tan^{-1}\left(\frac{\sqrt{d}\sqrt{\frac{ax+b}{x}}}{\sqrt{bc-ad}}\right)}{c^2\sqrt{bc-ad}} + \frac{x\sqrt{\frac{ax+b}{x}}}{ac}
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(Sqrt[a + b/x]*(c + d/x)),x]
```

```
[Out] (x*Sqrt[(b + a*x)/x])/(a*c) - (2*d^(3/2)*ArcTan[(Sqrt[d]*Sqrt[(b + a*x)/x])
/Sqrt[b*c - a*d]])/(c^2*Sqrt[b*c - a*d]) + ((-(b*c) - 2*a*d)*ArcTanh[Sqrt[(
b + a*x)/x]/Sqrt[a]])/(a^(3/2)*c^2)
```

fricas [A] time = 0.77, size = 542, normalized size = 5.02

$$
\left[\frac{2a^2d\sqrt{-\frac{d}{bc-ad}}\log\left(-\frac{2(bc-ad)x\sqrt{-\frac{d}{bc-ad}}\sqrt{\frac{ax+b}{x}}-bd+(bc-2ad)x}{cx+d}\right)+2acx\sqrt{\frac{ax+b}{x}}+(bc+2ad)\sqrt{a}\log\left(2ax-2\sqrt{a}x\sqrt{\frac{ax+b}{x}}+b\right)}{2a^2c^2}, \frac{a^2d\sqrt{-\frac{d}{bc-ad}}\log\left(-\frac{2(bc-ad)x\sqrt{-\frac{d}{bc-ad}}\sqrt{\frac{ax+b}{x}}-bd+(bc-2ad)x}{cx+d}\right)+acx\sqrt{\frac{ax+b}{x}}+(bc+2ad)\sqrt{-a}\arctan\left(\frac{\sqrt{-a}\sqrt{\frac{ax+b}{x}}}{a}\right)}{a^2c^2}, -\frac{4a^2d\sqrt{\frac{d}{bc-ad}}\arctan\left(-\frac{(bc-ad)x\sqrt{\frac{d}{bc-ad}}\sqrt{\frac{ax+b}{x}}}{adx+bd}\right)-2acx\sqrt{\frac{ax+b}{x}}-(bc+2ad)\sqrt{a}\log\left(2ax-2\sqrt{a}x\sqrt{\frac{ax+b}{x}}+b\right)}{2a^2c^2}, -\frac{2a^2d\sqrt{\frac{d}{bc-ad}}\arctan\left(-\frac{(bc-ad)x\sqrt{\frac{d}{bc-ad}}\sqrt{\frac{ax+b}{x}}}{adx+bd}\right)-acx\sqrt{\frac{ax+b}{x}}-(bc+2ad)\sqrt{-a}\arctan\left(\frac{\sqrt{-a}\sqrt{\frac{ax+b}{x}}}{a}\right)}{a^2c^2}\right]
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+d/x)/(a+b/x)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*(2*a^2*d*sqrt(-d/(b*c - a*d))*log(-(2*(b*c - a*d)*x*sqrt(-d/(b*c - a*d
))*sqrt((a*x + b)/x) - b*d + (b*c - 2*a*d)*x)/(c*x + d)) + 2*a*c*x*sqrt((a*
x + b)/x) + (b*c + 2*a*d)*sqrt(a)*log(2*a*x - 2*sqrt(a)*x*sqrt((a*x + b)/x)
 + b))/(a^2*c^2), (a^2*d*sqrt(-d/(b*c - a*d))*log(-(2*(b*c - a*d)*x*sqrt(-d
/(b*c - a*d))*sqrt((a*x + b)/x) - b*d + (b*c - 2*a*d)*x)/(c*x + d)) + a*c*x
*sqrt((a*x + b)/x) + (b*c + 2*a*d)*sqrt(-a)*arctan(sqrt(-a)*sqrt((a*x + b)/
x)/a))/(a^2*c^2), -1/2*(4*a^2*d*sqrt(d/(b*c - a*d))*arctan(-(b*c - a*d)*x*s
qrt(d/(b*c - a*d))*sqrt((a*x + b)/x)/(a*d*x + b*d)) - 2*a*c*x*sqrt((a*x + b
)/x) - (b*c + 2*a*d)*sqrt(a)*log(2*a*x - 2*sqrt(a)*x*sqrt((a*x + b)/x) + b)
)/(a^2*c^2), -(2*a^2*d*sqrt(d/(b*c - a*d))*arctan(-(b*c - a*d)*x*sqrt(d/(b*
c - a*d))*sqrt((a*x + b)/x)/(a*d*x + b*d)) - a*c*x*sqrt((a*x + b)/x) - (b*c
 + 2*a*d)*sqrt(-a)*arctan(sqrt(-a)*sqrt((a*x + b)/x)/a))/(a^2*c^2)]
```

giac [A] time = 0.18, size = 134, normalized size = 1.24

$$-b^{2}\left(\frac{2\,d^{2}\arctan\left(\frac{d\sqrt{\frac{ax+b}{x}}}{\sqrt{bcd-ad^{2}}}\right)}{\sqrt{bcd-ad^{2}}\,b^{2}c^{2}}+\frac{\sqrt{\frac{ax+b}{x}}}{\left(a-\frac{ax+b}{x}\right)abc}-\frac{(bc+2\,ad)\arctan\left(\frac{\sqrt{\frac{ax+b}{x}}}{\sqrt{-a}}\right)}{\sqrt{-a}\,ab^{2}c^{2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+d/x)/(a+b/x)^(1/2),x, algorithm="giac")
```

```
[Out] -b^2*(2*d^2*arctan(d*sqrt((a*x + b)/x)/sqrt(b*c*d - a*d^2))/(sqrt(b*c*d - a
*d^2)*b^2*c^2) + sqrt((a*x + b)/x)/((a - (a*x + b)/x)*a*b*c) - (b*c + 2*a*d
)*arctan(sqrt((a*x + b)/x)/sqrt(-a))/(sqrt(-a)*a*b^2*c^2))
```

maple [B] time = 0.06, size = 228, normalized size = 2.11

$$-\frac{\left(2a^{\frac{3}{2}}d^{2}\ln\left(\frac{-2adx+bcx-bd+2\sqrt{\frac{(ad-bc)d}{c^{2}}}\,\sqrt{(ax+b)x}\,c}{cx+d}\right)+2\sqrt{\frac{(ad-bc)d}{c^{2}}}\,acd\ln\left(\frac{2ax+b+2\sqrt{(ax+b)x}\,\sqrt{a}}{2\sqrt{a}}\right)+\sqrt{\frac{(ad-bc)d}{c^{2}}}\,bc^{2}\ln\left(\frac{2ax+b+2\sqrt{(ax+b)x}\,\sqrt{a}}{2\sqrt{a}}\right)-2\sqrt{(ax+b)x}\,\sqrt{\frac{(ad-bc)d}{c^{2}}}\,\sqrt{a}\,c^{2}\right)\sqrt{\frac{ax+b}{x}}\,x}{2\sqrt{\frac{(ad-bc)d}{c^{2}}}\,\sqrt{(ax+b)x}\,a^{\frac{3}{2}}c^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c+d/x)/(a+b/x)^(1/2),x)
```

```
[Out] -1/2*(2*a^(3/2)*d^2*ln((-2*a*d*x+b*c*x-b*d+2*((a*d-b*c)/c^2*d)^(1/2)*((a*x+
b)*x)^(1/2)*c)/(c*x+d))-2*((a*x+b)*x)^(1/2)*((a*d-b*c)/c^2*d)^(1/2)*a^(1/2)
*c^2+2*((a*d-b*c)/c^2*d)^(1/2)*a*c*d*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^
(1/2))/a^(1/2))+((a*d-b*c)/c^2*d)^(1/2)*b*c^2*ln(1/2*(2*a*x+b+2*((a*x+b)*x)
^(1/2)*a^(1/2))/a^(1/2)))*x*((a*x+b)/x)^(1/2)/((a*d-b*c)/c^2*d)^(1/2)/c^3/a
^(3/2)/((a*x+b)*x)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+d/x)/(a+b/x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(a + b/x)*(c + d/x)), x)
```

mupad [B] time = 1.98, size = 1183, normalized size = 10.95

$$\frac{x\sqrt{a+\frac{b}{x}}}{a\,c}-\frac{\mathrm{atanh}\left(\frac{12\,b^4\,d^4\,\sqrt{a+\frac{b}{x}}}{\sqrt{a^3}\,\left(\frac{12\,b^4\,d^4}{a}+\frac{10\,b^5\,c\,d^3}{a^2}+\frac{2\,b^6\,c^2\,d^2}{a^3}\right)}+\frac{10\,b^5\,d^3\,\sqrt{a+\frac{b}{x}}}{\sqrt{a^3}\,\left(\frac{10\,b^5\,d^3}{a}+\frac{12\,b^4\,d^4}{c}+\frac{2\,b^6\,c\,d^2}{a^2}\right)}+\frac{2\,b^6\,d^2\,\sqrt{a+\frac{b}{x}}}{\sqrt{a^3}\,\left(\frac{2\,b^6\,d^2}{a}+\frac{10\,b^5\,d^3}{c}+\frac{12\,a\,b^4\,d^4}{c^2}\right)}\right)\,\left(2\,a\,d+b\,c\right)}{c^2\,\sqrt{a^3}}-\frac{\mathrm{atan}\left(\frac{\frac{\left(\frac{\left(\frac{2\,\left(2\,a^2\,b^3\,c^4\,d^3+2\,a\,b^4\,c^5\,d^2\right)}{a^2\,c^3}-\frac{2\,\left(4\,a^2\,b^3\,c^5\,d^2-8\,a^3\,b^2\,c^4\,d^3\right)\,\sqrt{a+\frac{b}{x}}\,\sqrt{a\,d^4-b\,c\,d^3}}{a^2\,c^2\,\left(b\,c^3-a\,c^2\,d\right)}\right)\,\sqrt{a\,d^4-b\,c\,d^3}}{b\,c^3-a\,c^2\,d}-\frac{2\,\sqrt{a+\frac{b}{x}}\,\left(8\,a^2\,b^2\,d^5+4\,a\,b^3\,c\,d^4+b^4\,c^2\,d^3\right)}{a^2\,c^2}\right)\,\sqrt{a\,d^4-b\,c\,d^3}\,1{}\mathrm{i}}{b\,c^3-a\,c^2\,d}-\frac{\left(\frac{\left(\frac{2\,\left(2\,a^2\,b^3\,c^4\,d^3+2\,a\,b^4\,c^5\,d^2\right)}{a^2\,c^3}+\frac{2\,\left(4\,a^2\,b^3\,c^5\,d^2-8\,a^3\,b^2\,c^4\,d^3\right)\,\sqrt{a+\frac{b}{x}}\,\sqrt{a\,d^4-b\,c\,d^3}}{a^2\,c^2\,\left(b\,c^3-a\,c^2\,d\right)}\right)\,\sqrt{a\,d^4-b\,c\,d^3}}{b\,c^3-a\,c^2\,d}+\frac{2\,\sqrt{a+\frac{b}{x}}\,\left(8\,a^2\,b^2\,d^5+4\,a\,b^3\,c\,d^4+b^4\,c^2\,d^3\right)}{a^2\,c^2}\right)\,\sqrt{a\,d^4-b\,c\,d^3}\,1{}\mathrm{i}}{b\,c^3-a\,c^2\,d}}{\frac{\left(\frac{\left(\frac{2\,\left(2\,a^2\,b^3\,c^4\,d^3+2\,a\,b^4\,c^5\,d^2\right)}{a^2\,c^3}-\frac{2\,\left(4\,a^2\,b^3\,c^5\,d^2-8\,a^3\,b^2\,c^4\,d^3\right)\,\sqrt{a+\frac{b}{x}}\,\sqrt{a\,d^4-b\,c\,d^3}}{a^2\,c^2\,\left(b\,c^3-a\,c^2\,d\right)}\right)\,\sqrt{a\,d^4-b\,c\,d^3}}{b\,c^3-a\,c^2\,d}-\frac{2\,\sqrt{a+\frac{b}{x}}\,\left(8\,a^2\,b^2\,d^5+4\,a\,b^3\,c\,d^4+b^4\,c^2\,d^3\right)}{a^2\,c^2}\right)\,\sqrt{a\,d^4-b\,c\,d^3}}{b\,c^3-a\,c^2\,d}-\frac{4\,\left(2\,a\,b^3\,d^5+b^4\,c\,d^4\right)}{a^2\,c^3}+\frac{\left(\frac{\left(\frac{2\,\left(2\,a^2\,b^3\,c^4\,d^3+2\,a\,b^4\,c^5\,d^2\right)}{a^2\,c^3}+\frac{2\,\left(4\,a^2\,b^3\,c^5\,d^2-8\,a^3\,b^2\,c^4\,d^3\right)\,\sqrt{a+\frac{b}{x}}\,\sqrt{a\,d^4-b\,c\,d^3}}{a^2\,c^2\,\left(b\,c^3-a\,c^2\,d\right)}\right)\,\sqrt{a\,d^4-b\,c\,d^3}}{b\,c^3-a\,c^2\,d}+\frac{2\,\sqrt{a+\frac{b}{x}}\,\left(8\,a^2\,b^2\,d^5+4\,a\,b^3\,c\,d^4+b^4\,c^2\,d^3\right)}{a^2\,c^2}\right)\,\sqrt{a\,d^4-b\,c\,d^3}}{b\,c^3-a\,c^2\,d}}\right)\,\sqrt{a\,d^4-b\,c\,d^3}\,2{}\mathrm{i}}{b\,c^3-a\,c^2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b/x)^(1/2)*(c + d/x)),x)
```

```
[Out] (x*(a + b/x)^(1/2))/(a*c) - (atan((((((((2*(2*a*b^4*c^5*d^2 + 2*a^2*b^3*c^4*
d^3))/(a^2*c^3) - (2*(4*a^2*b^3*c^5*d^2 - 8*a^3*b^2*c^4*d^3)*(a + b/x)^(1/2
)*(a*d^4 - b*c*d^3)^(1/2))/(a^2*c^2*(b*c^3 - a*c^2*d)))*(a*d^4 - b*c*d^3)^(
1/2))/(b*c^3 - a*c^2*d) - (2*(a + b/x)^(1/2)*(8*a^2*b^2*d^5 + b^4*c^2*d^3 +
 4*a*b^3*c*d^4))/(a^2*c^2))*(a*d^4 - b*c*d^3)^(1/2)*1i)/(b*c^3 - a*c^2*d) -
 (((((2*(2*a*b^4*c^5*d^2 + 2*a^2*b^3*c^4*d^3))/(a^2*c^3) + (2*(4*a^2*b^3*c^
5*d^2 - 8*a^3*b^2*c^4*d^3)*(a + b/x)^(1/2)*(a*d^4 - b*c*d^3)^(1/2))/(a^2*c^
2*(b*c^3 - a*c^2*d)))*(a*d^4 - b*c*d^3)^(1/2))/(b*c^3 - a*c^2*d) + (2*(a +
b/x)^(1/2)*(8*a^2*b^2*d^5 + b^4*c^2*d^3 + 4*a*b^3*c*d^4))/(a^2*c^2))*(a*d^4
 - b*c*d^3)^(1/2)*1i)/(b*c^3 - a*c^2*d))/((((((2*(2*a*b^4*c^5*d^2 + 2*a^2*b
^3*c^4*d^3))/(a^2*c^3) - (2*(4*a^2*b^3*c^5*d^2 - 8*a^3*b^2*c^4*d^3)*(a + b/
x)^(1/2)*(a*d^4 - b*c*d^3)^(1/2))/(a^2*c^2*(b*c^3 - a*c^2*d)))*(a*d^4 - b*c
*d^3)^(1/2))/(b*c^3 - a*c^2*d) - (2*(a + b/x)^(1/2)*(8*a^2*b^2*d^5 + b^4*c^
2*d^3 + 4*a*b^3*c*d^4))/(a^2*c^2))*(a*d^4 - b*c*d^3)^(1/2))/(b*c^3 - a*c^2*
d) - (4*(2*a*b^3*d^5 + b^4*c*d^4))/(a^2*c^3) + (((((2*(2*a*b^4*c^5*d^2 + 2*
a^2*b^3*c^4*d^3))/(a^2*c^3) + (2*(4*a^2*b^3*c^5*d^2 - 8*a^3*b^2*c^4*d^3)*(a
 + b/x)^(1/2)*(a*d^4 - b*c*d^3)^(1/2))/(a^2*c^2*(b*c^3 - a*c^2*d)))*(a*d^4
- b*c*d^3)^(1/2))/(b*c^3 - a*c^2*d) + (2*(a + b/x)^(1/2)*(8*a^2*b^2*d^5 + b
^4*c^2*d^3 + 4*a*b^3*c*d^4))/(a^2*c^2))*(a*d^4 - b*c*d^3)^(1/2))/(b*c^3 - a
*c^2*d)))*(a*d^4 - b*c*d^3)^(1/2)*2i)/(b*c^3 - a*c^2*d) - (atanh((12*b^4*d^
4*(a + b/x)^(1/2))/((a^3)^(1/2)*((12*b^4*d^4)/a + (10*b^5*c*d^3)/a^2 + (2*b
^6*c^2*d^2)/a^3)) + (10*b^5*d^3*(a + b/x)^(1/2))/((a^3)^(1/2)*((10*b^5*d^3)
/a + (12*b^4*d^4)/c + (2*b^6*c*d^2)/a^2)) + (2*b^6*d^2*(a + b/x)^(1/2))/((a
^3)^(1/2)*((2*b^6*d^2)/a + (10*b^5*d^3)/c + (12*a*b^4*d^4)/c^2)))*(2*a*d +
b*c))/(c^2*(a^3)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{a+\frac{b}{x}}\,\left(cx+d\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+d/x)/(a+b/x)**(1/2),x)
```

```
[Out] Integral(x/(sqrt(a + b/x)*(c*x + d)), x)
```

3.153 $\int \frac{1}{\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^2}\,dx$

Optimal. Leaf size=172

$$-\frac{(4ad+bc)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{3/2}c^3}-\frac{d^{3/2}(5bc-4ad)\tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c^3(bc-ad)^{3/2}}+\frac{d\sqrt{a+\frac{b}{x}}\,(bc-2ad)}{ac^2\left(c+\frac{d}{x}\right)(bc-ad)}+\frac{x\sqrt{a+\frac{b}{x}}}{ac\left(c+\frac{d}{x}\right)}$$

Rubi [A] time = 0.22, antiderivative size = 172, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {375, 103, 151, 156, 63, 208, 205}

$$-\frac{(4ad+bc)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{3/2}c^3}-\frac{d^{3/2}(5bc-4ad)\tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c^3(bc-ad)^{3/2}}+\frac{d\sqrt{a+\frac{b}{x}}\,(bc-2ad)}{ac^2\left(c+\frac{d}{x}\right)(bc-ad)}+\frac{x\sqrt{a+\frac{b}{x}}}{ac\left(c+\frac{d}{x}\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a + b/x]*(c + d/x)^2),x]
```

```
[Out] (d*(b*c - 2*a*d)*Sqrt[a + b/x])/(a*c^2*(b*c - a*d)*(c + d/x)) + (Sqrt[a + b
/x]*x)/(a*c*(c + d/x)) - (d^(3/2)*(5*b*c - 4*a*d)*ArcTan[(Sqrt[d]*Sqrt[a +
b/x])/Sqrt[b*c - a*d]])/(c^3*(b*c - a*d)^(3/2)) - ((b*c + 4*a*d)*ArcTanh[Sq
rt[a + b/x]/Sqrt[a]])/(a^(3/2)*c^3)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 375

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[((a + b/x^n)^p*(c + d/x^n)^q)/x^2, x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^2}\,dx = -\operatorname{Subst}\left(\int \frac{1}{x^2\sqrt{a+bx}\,(c+dx)^2}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{\sqrt{a+\frac{b}{x}}\,x}{ac\left(c+\frac{d}{x}\right)} + \frac{\operatorname{Subst}\left(\int \frac{\frac{1}{2}(bc+4ad)+\frac{3bdx}{2}}{x\sqrt{a+bx}\,(c+dx)^2}\,dx, x, \frac{1}{x}\right)}{ac}$$

$$= \frac{d(bc-2ad)\sqrt{a+\frac{b}{x}}}{ac^2(bc-ad)\left(c+\frac{d}{x}\right)} + \frac{\sqrt{a+\frac{b}{x}}\,x}{ac\left(c+\frac{d}{x}\right)} - \frac{\operatorname{Subst}\left(\int \frac{-\frac{1}{2}(bc-ad)(bc+4ad)-\frac{1}{2}bd(bc-2ad)x}{x\sqrt{a+bx}\,(c+dx)}\,dx, x, \frac{1}{x}\right)}{ac^2(bc-ad)}$$

$$= \frac{d(bc-2ad)\sqrt{a+\frac{b}{x}}}{ac^2(bc-ad)\left(c+\frac{d}{x}\right)} + \frac{\sqrt{a+\frac{b}{x}}\,x}{ac\left(c+\frac{d}{x}\right)} - \frac{\left(d^2(5bc-4ad)\right)\operatorname{Subst}\left(\int \frac{1}{\sqrt{a+bx}\,(c+dx)}\,dx, x, \frac{1}{x}\right)}{2c^3(bc-ad)} + \ldots$$

$$= \frac{d(bc-2ad)\sqrt{a+\frac{b}{x}}}{ac^2(bc-ad)\left(c+\frac{d}{x}\right)} + \frac{\sqrt{a+\frac{b}{x}}\,x}{ac\left(c+\frac{d}{x}\right)} - \frac{\left(d^2(5bc-4ad)\right)\operatorname{Subst}\left(\int \frac{1}{c-\frac{ad}{b}+\frac{dx^2}{b}}\,dx, x, \sqrt{a+\frac{b}{x}}\right)}{bc^3(bc-ad)} \ldots$$

$$= \frac{d(bc-2ad)\sqrt{a+\frac{b}{x}}}{ac^2(bc-ad)\left(c+\frac{d}{x}\right)} + \frac{\sqrt{a+\frac{b}{x}}\,x}{ac\left(c+\frac{d}{x}\right)} - \frac{d^{3/2}(5bc-4ad)\tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c^3(bc-ad)^{3/2}} - \frac{(bc+4ad)\tanh\ldots}{a^{3/2}c\ldots}$$

Mathematica [A] time = 0.78, size = 150, normalized size = 0.87

$$\frac{\frac{ad^{3/2}(4ad-5bc)\tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{(bc-ad)^{3/2}} + \frac{cx\sqrt{a+\frac{b}{x}}\,(bc(cx+d)-ad(cx+2d))}{(cx+d)(bc-ad)} - \frac{(4ad+bc)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{\sqrt{a}}}{ac^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[a + b/x]*(c + d/x)^2),x]
```

```
[Out] ((c*Sqrt[a + b/x]*x*(b*c*(d + c*x) - a*d*(2*d + c*x)))/((b*c - a*d)*(d + c*
x)) + (a*d^(3/2)*(-5*b*c + 4*a*d)*ArcTan[(Sqrt[d]*Sqrt[a + b/x])/Sqrt[b*c -
 a*d]])/(b*c - a*d)^(3/2) - ((b*c + 4*a*d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/
Sqrt[a])/(a*c^3)
```

IntegrateAlgebraic [A] time = 0.63, size = 172, normalized size = 1.00

$$\frac{(-4ad-bc)\tanh^{-1}\left(\frac{\sqrt{\frac{ax+b}{x}}}{\sqrt{a}}\right)}{a^{3/2}c^3}+\frac{\left(4ad^{5/2}-5bcd^{3/2}\right)\tan^{-1}\left(\frac{\sqrt{d}\sqrt{\frac{ax+b}{x}}}{\sqrt{bc-ad}}\right)}{c^3(bc-ad)^{3/2}}+\frac{\sqrt{\frac{ax+b}{x}}\left(acdx^2+2ad^2x-bc^2x^2-bcdx\right)}{ac^2(cx+d)(ad-bc)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(Sqrt[a + b/x]*(c + d/x)^2),x]
```

```
[Out] (Sqrt[(b + a*x)/x]*(-(b*c*d*x) + 2*a*d^2*x - b*c^2*x^2 + a*c*d*x^2))/(a*c^2
*(-(b*c) + a*d)*(d + c*x)) + ((-5*b*c*d^(3/2) + 4*a*d^(5/2))*ArcTan[(Sqrt[d
]*Sqrt[(b + a*x)/x])/Sqrt[b*c - a*d]])/(c^3*(b*c - a*d)^(3/2)) + ((-(b*c) -
 4*a*d)*ArcTanh[Sqrt[(b + a*x)/x]/Sqrt[a]])/(a^(3/2)*c^3)
```

fricas [A] time = 1.06, size = 1163, normalized size = 6.76

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+d/x)^2/(a+b/x)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*((b^2*c^2*d + 3*a*b*c*d^2 - 4*a^2*d^3 + (b^2*c^3 + 3*a*b*c^2*d - 4*a^2
*c*d^2)*x)*sqrt(a)*log(2*a*x - 2*sqrt(a)*x*sqrt((a*x + b)/x) + b) + (5*a^2*
b*c*d^2 - 4*a^3*d^3 + (5*a^2*b*c^2*d - 4*a^3*c*d^2)*x)*sqrt(-d/(b*c - a*d))
*log(-(2*(b*c - a*d)*x*sqrt(-d/(b*c - a*d))*sqrt((a*x + b)/x) - b*d + (b*c
- 2*a*d)*x)/(c*x + d)) + 2*((a*b*c^3 - a^2*c^2*d)*x^2 + (a*b*c^2*d - 2*a^2*
c*d^2)*x)*sqrt((a*x + b)/x))/(a^2*b*c^4*d - a^3*c^3*d^2 + (a^2*b*c^5 - a^3*
c^4*d)*x), -1/2*(2*(5*a^2*b*c*d^2 - 4*a^3*d^3 + (5*a^2*b*c^2*d - 4*a^3*c*d^
2)*x)*sqrt(d/(b*c - a*d))*arctan(-(b*c - a*d)*x*sqrt(d/(b*c - a*d))*sqrt((a
*x + b)/x)/(a*d*x + b*d)) - (b^2*c^2*d + 3*a*b*c*d^2 - 4*a^2*d^3 + (b^2*c^3
 + 3*a*b*c^2*d - 4*a^2*c*d^2)*x)*sqrt(a)*log(2*a*x - 2*sqrt(a)*x*sqrt((a*x
+ b)/x) + b) - 2*((a*b*c^3 - a^2*c^2*d)*x^2 + (a*b*c^2*d - 2*a^2*c*d^2)*x)*
sqrt((a*x + b)/x))/(a^2*b*c^4*d - a^3*c^3*d^2 + (a^2*b*c^5 - a^3*c^4*d)*x),
 1/2*(2*(b^2*c^2*d + 3*a*b*c*d^2 - 4*a^2*d^3 + (b^2*c^3 + 3*a*b*c^2*d - 4*a
^2*c*d^2)*x)*sqrt(-a)*arctan(sqrt(-a)*sqrt((a*x + b)/x)/a) + (5*a^2*b*c*d^2
 - 4*a^3*d^3 + (5*a^2*b*c^2*d - 4*a^3*c*d^2)*x)*sqrt(-d/(b*c - a*d))*log(-(
2*(b*c - a*d)*x*sqrt(-d/(b*c - a*d))*sqrt((a*x + b)/x) - b*d + (b*c - 2*a*d
)*x)/(c*x + d)) + 2*((a*b*c^3 - a^2*c^2*d)*x^2 + (a*b*c^2*d - 2*a^2*c*d^2)*
x)*sqrt((a*x + b)/x))/(a^2*b*c^4*d - a^3*c^3*d^2 + (a^2*b*c^5 - a^3*c^4*d)*
x), -((5*a^2*b*c*d^2 - 4*a^3*d^3 + (5*a^2*b*c^2*d - 4*a^3*c*d^2)*x)*sqrt(d/
(b*c - a*d))*arctan(-(b*c - a*d)*x*sqrt(d/(b*c - a*d))*sqrt((a*x + b)/x)/(a
*d*x + b*d)) - (b^2*c^2*d + 3*a*b*c*d^2 - 4*a^2*d^3 + (b^2*c^3 + 3*a*b*c^2*
d - 4*a^2*c*d^2)*x)*sqrt(-a)*arctan(sqrt(-a)*sqrt((a*x + b)/x)/a) - ((a*b*c
^3 - a^2*c^2*d)*x^2 + (a*b*c^2*d - 2*a^2*c*d^2)*x)*sqrt((a*x + b)/x))/(a^2*
b*c^4*d - a^3*c^3*d^2 + (a^2*b*c^5 - a^3*c^4*d)*x)]
```

giac [A] time = 0.22, size = 300, normalized size = 1.74

$$-b^3\left(\frac{\left(5\,bcd^2-4\,ad^3\right)\arctan\left(\frac{d\sqrt{\frac{ax+b}{x}}}{\sqrt{bcd-ad^2}}\right)}{\left(b^4c^4-ab^3c^3d\right)\sqrt{bcd-ad^2}}+\frac{b^2c^2\sqrt{\frac{ax+b}{x}}-2\,abcd\sqrt{\frac{ax+b}{x}}+2\,a^2d^2\sqrt{\frac{ax+b}{x}}+\frac{(ax+b)bcd\sqrt{\frac{ax+b}{x}}}{x}-\frac{2\,(ax+b)ad^2\sqrt{\frac{ax+b}{x}}}{x}}{\left(ab^3c^3-a^2b^2c^2d\right)\left(abc-a^2d-\frac{(ax+b)bc}{x}+\frac{2\,(ax+b)ad}{x}-\frac{(ax+b)^2d}{x^2}\right)}-\frac{(bc+4\,ad)\arctan\left(\frac{\sqrt{\frac{ax+b}{x}}}{\sqrt{-a}}\right)}{\sqrt{-a}\,ab^3c^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+d/x)^2/(a+b/x)^(1/2),x, algorithm="giac")
```

```
[Out] -b^3*((5*b*c*d^2 - 4*a*d^3)*arctan(d*sqrt((a*x + b)/x)/sqrt(b*c*d - a*d^2))
/((b^4*c^4 - a*b^3*c^3*d)*sqrt(b*c*d - a*d^2)) + (b^2*c^2*sqrt((a*x + b)/x)
 - 2*a*b*c*d*sqrt((a*x + b)/x) + 2*a^2*d^2*sqrt((a*x + b)/x) + (a*x + b)*b*
c*d*sqrt((a*x + b)/x)/x - 2*(a*x + b)*a*d^2*sqrt((a*x + b)/x)/x)/((a*b^3*c^
3 - a^2*b^2*c^2*d)*(a*b*c - a^2*d - (a*x + b)*b*c/x + 2*(a*x + b)*a*d/x - (
a*x + b)^2*d/x^2)) - (b*c + 4*a*d)*arctan(sqrt((a*x + b)/x)/sqrt(-a))/(sqrt
(-a)*a*b^3*c^3))
```

maple [B] time = 0.07, size = 1135, normalized size = 6.60

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c+d/x)^2/(a+b/x)^(1/2),x)
```

```
[Out] -1/2*((a*x+b)/x)^(1/2)*x*(4*a^(9/2)*c*d^4*x*ln((-2*a*d*x+b*c*x-b*d+2*((a*d-
b*c)/c^2*d)^(1/2)*((a*x+b)*x)^(1/2)*c)/(c*x+d))+2*((a*x+b)*x)^(1/2)*((a*d-b
*c)/c^2*d)^(1/2)*a^(7/2)*c^4*d*x^2+4*a^(9/2)*d^5*ln((-2*a*d*x+b*c*x-b*d+2*(
(a*d-b*c)/c^2*d)^(1/2)*((a*x+b)*x)^(1/2)*c)/(c*x+d))-2*((a*x+b)*x)^(1/2)*((
a*d-b*c)/c^2*d)^(1/2)*a^(7/2)*c^3*d^2*x-9*a^(7/2)*b*c^2*d^3*x*ln((-2*a*d*x+
b*c*x-b*d+2*((a*d-b*c)/c^2*d)^(1/2)*((a*x+b)*x)^(1/2)*c)/(c*x+d))-4*((a*x+b
)*x)^(1/2)*((a*d-b*c)/c^2*d)^(1/2)*a^(7/2)*c^2*d^3-9*a^(7/2)*b*c*d^4*ln((-2
*a*d*x+b*c*x-b*d+2*((a*d-b*c)/c^2*d)^(1/2)*((a*x+b)*x)^(1/2)*c)/(c*x+d))-2*
((a*x+b)*x)^(3/2)*((a*d-b*c)/c^2*d)^(1/2)*a^(5/2)*c^4*d+6*((a*x+b)*x)^(1/2)
*((a*d-b*c)/c^2*d)^(1/2)*a^(5/2)*b*c^4*d*x+5*a^(5/2)*b^2*c^3*d^2*x*ln((-2*a
*d*x+b*c*x-b*d+2*((a*d-b*c)/c^2*d)^(1/2)*((a*x+b)*x)^(1/2)*c)/(c*x+d))+6*((
a*x+b)*x)^(1/2)*((a*d-b*c)/c^2*d)^(1/2)*a^(5/2)*b*c^3*d^2+5*a^(5/2)*b^2*c^2
*d^3*ln((-2*a*d*x+b*c*x-b*d+2*((a*d-b*c)/c^2*d)^(1/2)*((a*x+b)*x)^(1/2)*c)/
(c*x+d))-2*a^(3/2)*((a*d-b*c)/c^2*d)^(1/2)*((a*x+b)*x)^(1/2)*x*b^2*c^5-2*a^
(3/2)*((a*d-b*c)/c^2*d)^(1/2)*((a*x+b)*x)^(1/2)*b^2*c^4*d+4*((a*d-b*c)/c^2*
d)^(1/2)*a^4*c^2*d^3*x*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2)
)-7*((a*d-b*c)/c^2*d)^(1/2)*a^3*b*c^3*d^2*x*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(
1/2)*a^(1/2))/a^(1/2))+2*a^2*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a
^(1/2))*((a*d-b*c)/c^2*d)^(1/2)*x*b^2*c^4*d+((a*d-b*c)/c^2*d)^(1/2)*a*b^3*c
^5*x*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))+4*((a*d-b*c)/c^2
*d)^(1/2)*a^4*c*d^4*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))-7
*((a*d-b*c)/c^2*d)^(1/2)*a^3*b*c^2*d^3*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*
a^(1/2))/a^(1/2))+2*a^2*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2
))*((a*d-b*c)/c^2*d)^(1/2)*b^2*c^3*d^2+((a*d-b*c)/c^2*d)^(1/2)*a*b^3*c^4*d*
ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2)))/c^4/((a*x+b)*x)^(1/2
)/(a*d-b*c)^2/(c*x+d)/a^(5/2)/((a*d-b*c)/c^2*d)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a + \frac{b}{x}}\left(c + \frac{d}{x}\right)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+d/x)^2/(a+b/x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(a + b/x)*(c + d/x)^2), x)
```

mupad [B] time = 3.54, size = 3813, normalized size = 22.17

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b/x)^(1/2)*(c + d/x)^2),x)

[Out] (((a + b/x)^(1/2)*(b^3*c^2 + 2*a^2*b*d^2 - 2*a*b^2*c*d))/(c^2*(a^2*d - a*b*
c)) + (d*(a + b/x)^(3/2)*(b^2*c - 2*a*b*d))/(c^2*(a^2*d - a*b*c)))/((a + b/
x)*(2*a*d - b*c) - d*(a + b/x)^2 - a^2*d + a*b*c) - (atan(((((2*(a + b/x)^(
1/2)*(32*a^4*b^2*d^7 + b^6*c^4*d^3 + 6*a*b^5*c^3*d^4 - 64*a^3*b^3*c*d^6 + 2
6*a^2*b^4*c^2*d^5))/(a^2*b^2*c^6 + a^4*c^4*d^2 - 2*a^3*b*c^5*d) + (((4*a*b^
6*c^9*d^2 + 4*a^2*b^5*c^8*d^3 - 16*a^3*b^4*c^7*d^4 + 8*a^4*b^3*c^6*d^5)/(a^
2*b^2*c^8 + a^4*c^6*d^2 - 2*a^3*b*c^7*d) + ((a + b/x)^(1/2)*(4*a*d + b*c)*(
4*a^2*b^5*c^9*d^2 - 16*a^3*b^4*c^8*d^3 + 20*a^4*b^3*c^7*d^4 - 8*a^5*b^2*c^6
*d^5))/(c^3*(a^3)^(1/2)*(a^2*b^2*c^6 + a^4*c^4*d^2 - 2*a^3*b*c^5*d)))*(4*a*
d + b*c))/(2*c^3*(a^3)^(1/2)))*(4*a*d + b*c)*1i)/(2*c^3*(a^3)^(1/2)) + (((2
*(a + b/x)^(1/2)*(32*a^4*b^2*d^7 + b^6*c^4*d^3 + 6*a*b^5*c^3*d^4 - 64*a^3*b
^3*c*d^6 + 26*a^2*b^4*c^2*d^5))/(a^2*b^2*c^6 + a^4*c^4*d^2 - 2*a^3*b*c^5*d)
 - (((4*a*b^6*c^9*d^2 + 4*a^2*b^5*c^8*d^3 - 16*a^3*b^4*c^7*d^4 + 8*a^4*b^3*
c^6*d^5)/(a^2*b^2*c^8 + a^4*c^6*d^2 - 2*a^3*b*c^7*d) - ((a + b/x)^(1/2)*(4*
a*d + b*c)*(4*a^2*b^5*c^9*d^2 - 16*a^3*b^4*c^8*d^3 + 20*a^4*b^3*c^7*d^4 - 8
*a^5*b^2*c^6*d^5))/(c^3*(a^3)^(1/2)*(a^2*b^2*c^6 + a^4*c^4*d^2 - 2*a^3*b*c^
5*d)))*(4*a*d + b*c))/(2*c^3*(a^3)^(1/2)))*(4*a*d + b*c)*1i)/(2*c^3*(a^3)^(
1/2)))/((2*(32*a^3*b^3*d^7 + 5*b^6*c^3*d^4 + 6*a*b^5*c^2*d^5 - 48*a^2*b^4*c
*d^6))/(a^2*b^2*c^8 + a^4*c^6*d^2 - 2*a^3*b*c^7*d) - (((2*(a + b/x)^(1/2)*(
32*a^4*b^2*d^7 + b^6*c^4*d^3 + 6*a*b^5*c^3*d^4 - 64*a^3*b^3*c*d^6 + 26*a^2*
b^4*c^2*d^5))/(a^2*b^2*c^6 + a^4*c^4*d^2 - 2*a^3*b*c^5*d) + (((4*a*b^6*c^9*
d^2 + 4*a^2*b^5*c^8*d^3 - 16*a^3*b^4*c^7*d^4 + 8*a^4*b^3*c^6*d^5)/(a^2*b^2*
c^8 + a^4*c^6*d^2 - 2*a^3*b*c^7*d) + ((a + b/x)^(1/2)*(4*a*d + b*c)*(4*a^2*
b^5*c^9*d^2 - 16*a^3*b^4*c^8*d^3 + 20*a^4*b^3*c^7*d^4 - 8*a^5*b^2*c^6*d^5))
/(c^3*(a^3)^(1/2)*(a^2*b^2*c^6 + a^4*c^4*d^2 - 2*a^3*b*c^5*d)))*(4*a*d + b*
c))/(2*c^3*(a^3)^(1/2)))*(4*a*d + b*c))/(2*c^3*(a^3)^(1/2)) + (((2*(a + b/x
)^(1/2)*(32*a^4*b^2*d^7 + b^6*c^4*d^3 + 6*a*b^5*c^3*d^4 - 64*a^3*b^3*c*d^6
+ 26*a^2*b^4*c^2*d^5))/(a^2*b^2*c^6 + a^4*c^4*d^2 - 2*a^3*b*c^5*d) - (((4*a
*b^6*c^9*d^2 + 4*a^2*b^5*c^8*d^3 - 16*a^3*b^4*c^7*d^4 + 8*a^4*b^3*c^6*d^5)/
(a^2*b^2*c^8 + a^4*c^6*d^2 - 2*a^3*b*c^7*d) - ((a + b/x)^(1/2)*(4*a*d + b*c
)*(4*a^2*b^5*c^9*d^2 - 16*a^3*b^4*c^8*d^3 + 20*a^4*b^3*c^7*d^4 - 8*a^5*b^2*
c^6*d^5))/(c^3*(a^3)^(1/2)*(a^2*b^2*c^6 + a^4*c^4*d^2 - 2*a^3*b*c^5*d)))*(4
*a*d + b*c))/(2*c^3*(a^3)^(1/2)))*(4*a*d + b*c))/(2*c^3*(a^3)^(1/2))))*(4*a
*d + b*c)*1i)/(c^3*(a^3)^(1/2)) - (atan((((d^3*(a*d - b*c)^3)^(1/2)*(4*a*d
- 5*b*c)*((2*(a + b/x)^(1/2)*(32*a^4*b^2*d^7 + b^6*c^4*d^3 + 6*a*b^5*c^3*d^
4 - 64*a^3*b^3*c*d^6 + 26*a^2*b^4*c^2*d^5))/(a^2*b^2*c^6 + a^4*c^4*d^2 - 2*
a^3*b*c^5*d) + ((d^3*(a*d - b*c)^3)^(1/2)*(4*a*d - 5*b*c)*((4*a*b^6*c^9*d^2
 + 4*a^2*b^5*c^8*d^3 - 16*a^3*b^4*c^7*d^4 + 8*a^4*b^3*c^6*d^5)/(a^2*b^2*c^8
 + a^4*c^6*d^2 - 2*a^3*b*c^7*d) + ((d^3*(a*d - b*c)^3)^(1/2)*(a + b/x)^(1/2
)*(4*a*d - 5*b*c)*(4*a^2*b^5*c^9*d^2 - 16*a^3*b^4*c^8*d^3 + 20*a^4*b^3*c^7*
d^4 - 8*a^5*b^2*c^6*d^5))/((a^2*b^2*c^6 + a^4*c^4*d^2 - 2*a^3*b*c^5*d)*(b^3
*c^6 - a^3*c^3*d^3 + 3*a^2*b*c^4*d^2 - 3*a*b^2*c^5*d))))/(2*(b^3*c^6 - a^3*
c^3*d^3 + 3*a^2*b*c^4*d^2 - 3*a*b^2*c^5*d)))*1i)/(2*(b^3*c^6 - a^3*c^3*d^3
+ 3*a^2*b*c^4*d^2 - 3*a*b^2*c^5*d)) + ((d^3*(a*d - b*c)^3)^(1/2)*(4*a*d - 5
*b*c)*((2*(a + b/x)^(1/2)*(32*a^4*b^2*d^7 + b^6*c^4*d^3 + 6*a*b^5*c^3*d^4 -
 64*a^3*b^3*c*d^6 + 26*a^2*b^4*c^2*d^5))/(a^2*b^2*c^6 + a^4*c^4*d^2 - 2*a^3
*b*c^5*d) - ((d^3*(a*d - b*c)^3)^(1/2)*(4*a*d - 5*b*c)*((4*a*b^6*c^9*d^2 +
4*a^2*b^5*c^8*d^3 - 16*a^3*b^4*c^7*d^4 + 8*a^4*b^3*c^6*d^5)/(a^2*b^2*c^8 +
a^4*c^6*d^2 - 2*a^3*b*c^7*d) - ((d^3*(a*d - b*c)^3)^(1/2)*(a + b/x)^(1/2)*(
4*a*d - 5*b*c)*(4*a^2*b^5*c^9*d^2 - 16*a^3*b^4*c^8*d^3 + 20*a^4*b^3*c^7*d^4
 - 8*a^5*b^2*c^6*d^5))/((a^2*b^2*c^6 + a^4*c^4*d^2 - 2*a^3*b*c^5*d)*(b^3*c^
6 - a^3*c^3*d^3 + 3*a^2*b*c^4*d^2 - 3*a*b^2*c^5*d))))/(2*(b^3*c^6 - a^3*c^3
*d^3 + 3*a^2*b*c^4*d^2 - 3*a*b^2*c^5*d)))*1i)/(2*(b^3*c^6 - a^3*c^3*d^3 + 3
*a^2*b*c^4*d^2 - 3*a*b^2*c^5*d)))/((2*(32*a^3*b^3*d^7 + 5*b^6*c^3*d^4 + 6*a
*b^5*c^2*d^5 - 48*a^2*b^4*c*d^6))/(a^2*b^2*c^8 + a^4*c^6*d^2 - 2*a^3*b*c^7*
d) - ((d^3*(a*d - b*c)^3)^(1/2)*(4*a*d - 5*b*c)*((2*(a + b/x)^(1/2)*(32*a^4
*b^2*d^7 + b^6*c^4*d^3 + 6*a*b^5*c^3*d^4 - 64*a^3*b^3*c*d^6 + 26*a^2*b^4*c^
2*d^5))/(a^2*b^2*c^6 + a^4*c^4*d^2 - 2*a^3*b*c^5*d) + ((d^3*(a*d - b*c)^3)^
```

```
(1/2)*(4*a*d - 5*b*c)*((4*a*b^6*c^9*d^2 + 4*a^2*b^5*c^8*d^3 - 16*a^3*b^4*c^
7*d^4 + 8*a^4*b^3*c^6*d^5)/(a^2*b^2*c^8 + a^4*c^6*d^2 - 2*a^3*b*c^7*d) + ((
d^3*(a*d - b*c)^3)^(1/2)*(a + b/x)^(1/2)*(4*a*d - 5*b*c)*(4*a^2*b^5*c^9*d^2
 - 16*a^3*b^4*c^8*d^3 + 20*a^4*b^3*c^7*d^4 - 8*a^5*b^2*c^6*d^5))/((a^2*b^2*
c^6 + a^4*c^4*d^2 - 2*a^3*b*c^5*d)*(b^3*c^6 - a^3*c^3*d^3 + 3*a^2*b*c^4*d^2
 - 3*a*b^2*c^5*d))))/(2*(b^3*c^6 - a^3*c^3*d^3 + 3*a^2*b*c^4*d^2 - 3*a*b^2*
c^5*d))))/(2*(b^3*c^6 - a^3*c^3*d^3 + 3*a^2*b*c^4*d^2 - 3*a*b^2*c^5*d)) + (
(d^3*(a*d - b*c)^3)^(1/2)*(4*a*d - 5*b*c)*((2*(a + b/x)^(1/2)*(32*a^4*b^2*d
^7 + b^6*c^4*d^3 + 6*a*b^5*c^3*d^4 - 64*a^3*b^3*c*d^6 + 26*a^2*b^4*c^2*d^5)
)/(a^2*b^2*c^6 + a^4*c^4*d^2 - 2*a^3*b*c^5*d) - ((d^3*(a*d - b*c)^3)^(1/2)*
(4*a*d - 5*b*c)*((4*a*b^6*c^9*d^2 + 4*a^2*b^5*c^8*d^3 - 16*a^3*b^4*c^7*d^4
+ 8*a^4*b^3*c^6*d^5)/(a^2*b^2*c^8 + a^4*c^6*d^2 - 2*a^3*b*c^7*d) - ((d^3*(a
*d - b*c)^3)^(1/2)*(a + b/x)^(1/2)*(4*a*d - 5*b*c)*(4*a^2*b^5*c^9*d^2 - 16*
a^3*b^4*c^8*d^3 + 20*a^4*b^3*c^7*d^4 - 8*a^5*b^2*c^6*d^5))/((a^2*b^2*c^6 +
a^4*c^4*d^2 - 2*a^3*b*c^5*d)*(b^3*c^6 - a^3*c^3*d^3 + 3*a^2*b*c^4*d^2 - 3*a
*b^2*c^5*d))))/(2*(b^3*c^6 - a^3*c^3*d^3 + 3*a^2*b*c^4*d^2 - 3*a*b^2*c^5*d)
)))/(2*(b^3*c^6 - a^3*c^3*d^3 + 3*a^2*b*c^4*d^2 - 3*a*b^2*c^5*d))))*(d^3*(a
*d - b*c)^3)^(1/2)*(4*a*d - 5*b*c)*1i)/(b^3*c^6 - a^3*c^3*d^3 + 3*a^2*b*c^4
*d^2 - 3*a*b^2*c^5*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+d/x)**2/(a+b/x)**(1/2),x)
```

```
[Out] Timed out
```

3.154 $\int \frac{1}{\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^3}\,dx$

Optimal. Leaf size=250

$$-\frac{(6ad+bc)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{3/2}c^4}-\frac{d^{3/2}\left(24a^2d^2-56abcd+35b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{4c^4(bc-ad)^{5/2}}+\frac{d\sqrt{a+\frac{b}{x}}\,(bc-4ad)(4bc-3a}{4ac^3\left(c+\frac{d}{x}\right)(bc-ad)^2}$$

Rubi [A] time = 0.40, antiderivative size = 250, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {375, 103, 151, 156, 63, 208, 205}

$$-\frac{d^{3/2}\left(24a^2d^2-56abcd+35b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{4c^4(bc-ad)^{5/2}}-\frac{(6ad+bc)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{3/2}c^4}+\frac{d\sqrt{a+\frac{b}{x}}\,(bc-4ad)(4bc-3ad)}{4ac^3\left(c+\frac{d}{x}\right)(bc-ad)^2}+\frac{d\sqrt{a+\frac{b}{x}}\,(2bc-3ad)}{2ac^2\left(c+\frac{d}{x}\right)^2(bc-ad)}+\frac{x\sqrt{a+\frac{b}{x}}}{ac\left(c+\frac{d}{x}\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a + b/x]*(c + d/x)^3),x]
```

```
[Out] (d*(2*b*c - 3*a*d)*Sqrt[a + b/x])/(2*a*c^2*(b*c - a*d)*(c + d/x)^2) + (d*(b
*c - 4*a*d)*(4*b*c - 3*a*d)*Sqrt[a + b/x])/(4*a*c^3*(b*c - a*d)^2*(c + d/x)
) + (Sqrt[a + b/x]*x)/(a*c*(c + d/x)^2) - (d^(3/2)*(35*b^2*c^2 - 56*a*b*c*d
 + 24*a^2*d^2)*ArcTan[(Sqrt[d]*Sqrt[a + b/x])/Sqrt[b*c - a*d]])/(4*c^4*(b*c
 - a*d)^(5/2)) - ((b*c + 6*a*d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/(a^(3/2)*c^
4)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 375

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[((a + b/x^n)^p*(c + d/x^n)^q)/x^2, x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^3}\,dx = -\text{Subst}\left(\int \frac{1}{x^2\sqrt{a+bx}\,(c+dx)^3}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{\sqrt{a+\frac{b}{x}}\,x}{ac\left(c+\frac{d}{x}\right)^2} + \frac{\text{Subst}\left(\int \frac{\frac{1}{2}(bc+6ad)+\frac{5bdx}{2}}{x\sqrt{a+bx}\,(c+dx)^3}\,dx, x, \frac{1}{x}\right)}{ac}$$

$$= \frac{d(2bc-3ad)\sqrt{a+\frac{b}{x}}}{2ac^2(bc-ad)\left(c+\frac{d}{x}\right)^2} + \frac{\sqrt{a+\frac{b}{x}}\,x}{ac\left(c+\frac{d}{x}\right)^2} - \frac{\text{Subst}\left(\int \frac{-((bc-ad)(bc+6ad))-\frac{3}{2}bd(2bc-3ad)x}{x\sqrt{a+bx}\,(c+dx)^2}\,dx, x, \frac{1}{x}\right)}{2ac^2(bc-ad)}$$

$$= \frac{d(2bc-3ad)\sqrt{a+\frac{b}{x}}}{2ac^2(bc-ad)\left(c+\frac{d}{x}\right)^2} + \frac{d(bc-4ad)(4bc-3ad)\sqrt{a+\frac{b}{x}}}{4ac^3(bc-ad)^2\left(c+\frac{d}{x}\right)} + \frac{\sqrt{a+\frac{b}{x}}\,x}{ac\left(c+\frac{d}{x}\right)^2} + \frac{\text{Subst}\left(\int \frac{(bc-\cdots}{\cdots}\right.}{\cdots}$$

$$= \frac{d(2bc-3ad)\sqrt{a+\frac{b}{x}}}{2ac^2(bc-ad)\left(c+\frac{d}{x}\right)^2} + \frac{d(bc-4ad)(4bc-3ad)\sqrt{a+\frac{b}{x}}}{4ac^3(bc-ad)^2\left(c+\frac{d}{x}\right)} + \frac{\sqrt{a+\frac{b}{x}}\,x}{ac\left(c+\frac{d}{x}\right)^2} + \frac{(bc+6ad)\,\text{Su}\cdots}{\cdots}$$

$$= \frac{d(2bc-3ad)\sqrt{a+\frac{b}{x}}}{2ac^2(bc-ad)\left(c+\frac{d}{x}\right)^2} + \frac{d(bc-4ad)(4bc-3ad)\sqrt{a+\frac{b}{x}}}{4ac^3(bc-ad)^2\left(c+\frac{d}{x}\right)} + \frac{\sqrt{a+\frac{b}{x}}\,x}{ac\left(c+\frac{d}{x}\right)^2} + \frac{(bc+6ad)\,\text{Su}\cdots}{\cdots}$$

$$= \frac{d(2bc-3ad)\sqrt{a+\frac{b}{x}}}{2ac^2(bc-ad)\left(c+\frac{d}{x}\right)^2} + \frac{d(bc-4ad)(4bc-3ad)\sqrt{a+\frac{b}{x}}}{4ac^3(bc-ad)^2\left(c+\frac{d}{x}\right)} + \frac{\sqrt{a+\frac{b}{x}}\,x}{ac\left(c+\frac{d}{x}\right)^2} - \frac{d^{3/2}\left(35b^2c^2-\cdots\right.}{\cdots}$$

Mathematica [A] time = 1.75, size = 216, normalized size = 0.86

$$\frac{\frac{cx\sqrt{a+\frac{b}{x}}\left(2a^2d^2\left(2c^2x^2+9cdx+6d^2\right)-abcd\left(8c^2x^2+29cdx+19d^2\right)+4b^2c^2(cx+d)^2\right)}{(cx+d)^2(bc-ad)^2}-\frac{ad^{3/2}\left(24a^2d^2-56abcd+35b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{(bc-ad)^{5/2}}-\frac{4(6ad+bc)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{\sqrt{a}}}{4ac^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[a + b/x]*(c + d/x)^3),x]
```

```
[Out] ((c*Sqrt[a + b/x]*x*(4*b^2*c^2*(d + c*x)^2 + 2*a^2*d^2*(6*d^2 + 9*c*d*x + 2
*c^2*x^2) - a*b*c*d*(19*d^2 + 29*c*d*x + 8*c^2*x^2)))/((b*c - a*d)^2*(d + c
*x)^2) - (a*d^(3/2)*(35*b^2*c^2 - 56*a*b*c*d + 24*a^2*d^2)*ArcTan[(Sqrt[d]*
Sqrt[a + b/x])/Sqrt[b*c - a*d]])/(b*c - a*d)^(5/2) - (4*(b*c + 6*a*d)*ArcTa
nh[Sqrt[a + b/x]/Sqrt[a]])/Sqrt[a])/(4*a*c^4)
```

IntegrateAlgebraic [A] time = 1.65, size = 268, normalized size = 1.07

$$\frac{(-6ad-bc)\tanh^{-1}\left(\frac{\sqrt{\frac{ax+b}{x}}}{\sqrt{a}}\right)}{a^{3/2}c^4}+\frac{\left(-24a^2d^{7/2}+56abcd^{5/2}-35b^2c^2d^{3/2}\right)\tan^{-1}\left(\frac{\sqrt{d}\sqrt{\frac{ax+b}{x}}}{\sqrt{bc-ad}}\right)}{4c^4(bc-ad)^{5/2}}+\frac{\sqrt{\frac{ax+b}{x}}\left(4a^2c^2d^2x^3+18a^2cd^3x^2+12a^2d^4x-8abc^3dx^3-29abc^2d^2x^2-19abcd^3x+4b^2c^4x^3+8b^2c^3dx^2+4b^2c^2d^2x\right)}{4ac^3(cx+d)^2(ad-bc)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(Sqrt[a + b/x]*(c + d/x)^3),x]
```

```
[Out] (Sqrt[(b + a*x)/x]*(4*b^2*c^2*d^2*x - 19*a*b*c*d^3*x + 12*a^2*d^4*x + 8*b^2
*c^3*d*x^2 - 29*a*b*c^2*d^2*x^2 + 18*a^2*c*d^3*x^2 + 4*b^2*c^4*x^3 - 8*a*b*
c^3*d*x^3 + 4*a^2*c^2*d^2*x^3))/(4*a*c^3*(-(b*c) + a*d)^2*(d + c*x)^2) + ((
-35*b^2*c^2*d^(3/2) + 56*a*b*c*d^(5/2) - 24*a^2*d^(7/2))*ArcTan[(Sqrt[d]*Sq
rt[(b + a*x)/x])/Sqrt[b*c - a*d]])/(4*c^4*(b*c - a*d)^(5/2)) + ((-(b*c) - 6
*a*d)*ArcTanh[Sqrt[(b + a*x)/x]/Sqrt[a]])/(a^(3/2)*c^4)
```

fricas [B] time = 2.22, size = 2307, normalized size = 9.23

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+d/x)^3/(a+b/x)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/8*(4*(b^3*c^3*d^2 + 4*a*b^2*c^2*d^3 - 11*a^2*b*c*d^4 + 6*a^3*d^5 + (b^3*
c^5 + 4*a*b^2*c^4*d - 11*a^2*b*c^3*d^2 + 6*a^3*c^2*d^3)*x^2 + 2*(b^3*c^4*d
+ 4*a*b^2*c^3*d^2 - 11*a^2*b*c^2*d^3 + 6*a^3*c*d^4)*x)*sqrt(a)*log(2*a*x -
2*sqrt(a)*x*sqrt((a*x + b)/x) + b) + (35*a^2*b^2*c^2*d^3 - 56*a^3*b*c*d^4 +
 24*a^4*d^5 + (35*a^2*b^2*c^4*d - 56*a^3*b*c^3*d^2 + 24*a^4*c^2*d^3)*x^2 +
2*(35*a^2*b^2*c^3*d^2 - 56*a^3*b*c^2*d^3 + 24*a^4*c*d^4)*x)*sqrt(-d/(b*c -
a*d))*log(-(2*(b*c - a*d)*x*sqrt(-d/(b*c - a*d))*sqrt((a*x + b)/x) - b*d +
(b*c - 2*a*d)*x)/(c*x + d)) + 2*(4*(a*b^2*c^5 - 2*a^2*b*c^4*d + a^3*c^3*d^2
)*x^3 + (8*a*b^2*c^4*d - 29*a^2*b*c^3*d^2 + 18*a^3*c^2*d^3)*x^2 + (4*a*b^2*
c^3*d^2 - 19*a^2*b*c^2*d^3 + 12*a^3*c*d^4)*x)*sqrt((a*x + b)/x))/(a^2*b^2*c
^6*d^2 - 2*a^3*b*c^5*d^3 + a^4*c^4*d^4 + (a^2*b^2*c^8 - 2*a^3*b*c^7*d + a^4
*c^6*d^2)*x^2 + 2*(a^2*b^2*c^7*d - 2*a^3*b*c^6*d^2 + a^4*c^5*d^3)*x), 1/8*(
8*(b^3*c^3*d^2 + 4*a*b^2*c^2*d^3 - 11*a^2*b*c*d^4 + 6*a^3*d^5 + (b^3*c^5 +
4*a*b^2*c^4*d - 11*a^2*b*c^3*d^2 + 6*a^3*c^2*d^3)*x^2 + 2*(b^3*c^4*d + 4*a*
b^2*c^3*d^2 - 11*a^2*b*c^2*d^3 + 6*a^3*c*d^4)*x)*sqrt(-a)*arctan(sqrt(-a)*s
qrt((a*x + b)/x)/a) + (35*a^2*b^2*c^2*d^3 - 56*a^3*b*c*d^4 + 24*a^4*d^5 + (
35*a^2*b^2*c^4*d - 56*a^3*b*c^3*d^2 + 24*a^4*c^2*d^3)*x^2 + 2*(35*a^2*b^2*c
^3*d^2 - 56*a^3*b*c^2*d^3 + 24*a^4*c*d^4)*x)*sqrt(-d/(b*c - a*d))*log(-(2*(
b*c - a*d)*x*sqrt(-d/(b*c - a*d))*sqrt((a*x + b)/x) - b*d + (b*c - 2*a*d)*x
)/(c*x + d)) + 2*(4*(a*b^2*c^5 - 2*a^2*b*c^4*d + a^3*c^3*d^2)*x^3 + (8*a*b^
2*c^4*d - 29*a^2*b*c^3*d^2 + 18*a^3*c^2*d^3)*x^2 + (4*a*b^2*c^3*d^2 - 19*a^
2*b*c^2*d^3 + 12*a^3*c*d^4)*x)*sqrt((a*x + b)/x))/(a^2*b^2*c^6*d^2 - 2*a^3*
b*c^5*d^3 + a^4*c^4*d^4 + (a^2*b^2*c^8 - 2*a^3*b*c^7*d + a^4*c^6*d^2)*x^2 +
```

```
2*(a^2*b^2*c^7*d - 2*a^3*b*c^6*d^2 + a^4*c^5*d^3)*x), -1/4*((35*a^2*b^2*c^
2*d^3 - 56*a^3*b*c*d^4 + 24*a^4*d^5 + (35*a^2*b^2*c^4*d - 56*a^3*b*c^3*d^2
+ 24*a^4*c^2*d^3)*x^2 + 2*(35*a^2*b^2*c^3*d^2 - 56*a^3*b*c^2*d^3 + 24*a^4*c
*d^4)*x)*sqrt(d/(b*c - a*d))*arctan(-(b*c - a*d)*x*sqrt(d/(b*c - a*d))*sqrt
((a*x + b)/x)/(a*d*x + b*d)) - 2*(b^3*c^3*d^2 + 4*a*b^2*c^2*d^3 - 11*a^2*b*
c*d^4 + 6*a^3*d^5 + (b^3*c^5 + 4*a*b^2*c^4*d - 11*a^2*b*c^3*d^2 + 6*a^3*c^2
*d^3)*x^2 + 2*(b^3*c^4*d + 4*a*b^2*c^3*d^2 - 11*a^2*b*c^2*d^3 + 6*a^3*c*d^4
)*x)*sqrt(a)*log(2*a*x - 2*sqrt(a)*x*sqrt((a*x + b)/x) + b) - (4*(a*b^2*c^5
 - 2*a^2*b*c^4*d + a^3*c^3*d^2)*x^3 + (8*a*b^2*c^4*d - 29*a^2*b*c^3*d^2 + 1
8*a^3*c^2*d^3)*x^2 + (4*a*b^2*c^3*d^2 - 19*a^2*b*c^2*d^3 + 12*a^3*c*d^4)*x)
*sqrt((a*x + b)/x))/(a^2*b^2*c^6*d^2 - 2*a^3*b*c^5*d^3 + a^4*c^4*d^4 + (a^2
*b^2*c^8 - 2*a^3*b*c^7*d + a^4*c^6*d^2)*x^2 + 2*(a^2*b^2*c^7*d - 2*a^3*b*c^
6*d^2 + a^4*c^5*d^3)*x), -1/4*((35*a^2*b^2*c^2*d^3 - 56*a^3*b*c*d^4 + 24*a^
4*d^5 + (35*a^2*b^2*c^4*d - 56*a^3*b*c^3*d^2 + 24*a^4*c^2*d^3)*x^2 + 2*(35*
a^2*b^2*c^3*d^2 - 56*a^3*b*c^2*d^3 + 24*a^4*c*d^4)*x)*sqrt(d/(b*c - a*d))*a
rctan(-(b*c - a*d)*x*sqrt(d/(b*c - a*d))*sqrt((a*x + b)/x)/(a*d*x + b*d)) -
 4*(b^3*c^3*d^2 + 4*a*b^2*c^2*d^3 - 11*a^2*b*c*d^4 + 6*a^3*d^5 + (b^3*c^5 +
 4*a*b^2*c^4*d - 11*a^2*b*c^3*d^2 + 6*a^3*c^2*d^3)*x^2 + 2*(b^3*c^4*d + 4*a
*b^2*c^3*d^2 - 11*a^2*b*c^2*d^3 + 6*a^3*c*d^4)*x)*sqrt(-a)*arctan(sqrt(-a)*
sqrt((a*x + b)/x)/a) - (4*(a*b^2*c^5 - 2*a^2*b*c^4*d + a^3*c^3*d^2)*x^3 + (
8*a*b^2*c^4*d - 29*a^2*b*c^3*d^2 + 18*a^3*c^2*d^3)*x^2 + (4*a*b^2*c^3*d^2 -
 19*a^2*b*c^2*d^3 + 12*a^3*c*d^4)*x)*sqrt((a*x + b)/x))/(a^2*b^2*c^6*d^2 -
2*a^3*b*c^5*d^3 + a^4*c^4*d^4 + (a^2*b^2*c^8 - 2*a^3*b*c^7*d + a^4*c^6*d^2)
*x^2 + 2*(a^2*b^2*c^7*d - 2*a^3*b*c^6*d^2 + a^4*c^5*d^3)*x)]
```

giac [A] time = 0.29, size = 352, normalized size = 1.41

$$-\frac{1}{4}\,b^{4}\left(\frac{\left(35\,b^{2}c^{2}d^{2}-56\,abcd^{3}+24\,a^{2}d^{4}\right)\arctan\left(\frac{d\sqrt{\frac{ax+b}{x}}}{\sqrt{bcd-ad^{2}}}\right)}{\left(b^{6}c^{6}-2\,ab^{5}c^{5}d+a^{2}b^{4}c^{4}d^{2}\right)\sqrt{bcd-ad^{2}}}+\frac{13\,b^{2}c^{2}d^{2}\sqrt{\frac{ax+b}{x}}-21\,abcd^{3}\sqrt{\frac{ax+b}{x}}+8\,a^{2}d^{4}\sqrt{\frac{ax+b}{x}}+\frac{11\,(ax+b)bcd^{3}\sqrt{\frac{ax+b}{x}}}{x}-\frac{8\,(ax+b)ad^{4}\sqrt{\frac{ax+b}{x}}}{x}}{\left(b^{5}c^{5}-2\,ab^{4}c^{4}d+a^{2}b^{3}c^{3}d^{2}\right)\left(bc-ad+\frac{(ax+b)d}{x}\right)^{2}}+\frac{4\sqrt{\frac{ax+b}{x}}}{\left(a-\frac{ax+b}{x}\right)ab^{3}c^{3}}-\frac{4\,(bc+6\,ad)\arctan\left(\frac{\sqrt{\frac{ax+b}{x}}}{\sqrt{-a}}\right)}{\sqrt{-a}\,ab^{4}c^{4}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+d/x)^3/(a+b/x)^(1/2),x, algorithm="giac")
```

```
[Out] -1/4*b^4*((35*b^2*c^2*d^2 - 56*a*b*c*d^3 + 24*a^2*d^4)*arctan(d*sqrt((a*x +
 b)/x)/sqrt(b*c*d - a*d^2))/((b^6*c^6 - 2*a*b^5*c^5*d + a^2*b^4*c^4*d^2)*sq
rt(b*c*d - a*d^2)) + (13*b^2*c^2*d^2*sqrt((a*x + b)/x) - 21*a*b*c*d^3*sqrt(
(a*x + b)/x) + 8*a^2*d^4*sqrt((a*x + b)/x) + 11*(a*x + b)*b*c*d^3*sqrt((a*x
 + b)/x)/x - 8*(a*x + b)*a*d^4*sqrt((a*x + b)/x)/x)/((b^5*c^5 - 2*a*b^4*c^4
*d + a^2*b^3*c^3*d^2)*(b*c - a*d + (a*x + b)*d/x)^2) + 4*sqrt((a*x + b)/x)/
((a - (a*x + b)/x)*a*b^3*c^3) - 4*(b*c + 6*a*d)*arctan(sqrt((a*x + b)/x)/sq
rt(-a))/(sqrt(-a)*a*b^4*c^4))
```

maple [B] time = 0.07, size = 2269, normalized size = 9.08

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c+d/x)^3/(a+b/x)^(1/2),x)
```

```
[Out] -1/8*((a*x+b)/x)^(1/2)*x*(60*a^3*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2
))/a^(1/2))*((a*d-b*c)/c^2*d)^(1/2)*b^2*c^3*d^4-12*a^2*ln(1/2*(2*a*x+b+2*((
a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))*((a*d-b*c)/c^2*d)^(1/2)*b^3*c^4*d^3+12*a^
(9/2)*((a*x+b)*x)^(1/2)*((a*d-b*c)/c^2*d)^(1/2)*x^3*c^5*d^2-12*a^(7/2)*((a*
x+b)*x)^(3/2)*((a*d-b*c)/c^2*d)^(1/2)*x*c^5*d^2-80*a^(9/2)*ln((-2*a*d*x+b*c
*x-b*d+2*((a*d-b*c)/c^2*d)^(1/2)*((a*x+b)*x)^(1/2)*c)/(c*x+d))*x^2*b*c^3*d^
4+91*a^(7/2)*ln((-2*a*d*x+b*c*x-b*d+2*((a*d-b*c)/c^2*d)^(1/2)*((a*x+b)*x)^(
1/2)*c)/(c*x+d))*x^2*b^2*c^4*d^3-35*a^(5/2)*ln((-2*a*d*x+b*c*x-b*d+2*((a*d-
b*c)/c^2*d)^(1/2)*((a*x+b)*x)^(1/2)*c)/(c*x+d))*x^2*b^3*c^5*d^2+24*a^5*ln(1
/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))*((a*d-b*c)/c^2*d)^(1/2)*x
^2*c^3*d^4+18*a^(5/2)*((a*x+b)*x)^(3/2)*((a*d-b*c)/c^2*d)^(1/2)*b*c^5*d^2-3
```

```
6*a^(9/2)*((a*x+b)*x)^(1/2)*((a*d-b*c)/c^2*d)^(1/2)*x*c^3*d^4-160*a^(9/2)*l
n((-2*a*d*x+b*c*x-b*d+2*((a*d-b*c)/c^2*d)^(1/2)*((a*x+b)*x)^(1/2)*c)/(c*x+d
))*x*b*c^2*d^5-4*a*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))*((
a*d-b*c)/c^2*d)^(1/2)*x^2*b^4*c^7+8*a^(3/2)*((a*x+b)*x)^(1/2)*((a*d-b*c)/c^
2*d)^(1/2)*b^3*c^5*d^2-4*a*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(
1/2))*((a*d-b*c)/c^2*d)^(1/2)*b^4*c^5*d^2+182*a^(7/2)*ln((-2*a*d*x+b*c*x-b*
d+2*((a*d-b*c)/c^2*d)^(1/2)*((a*x+b)*x)^(1/2)*c)/(c*x+d))*x*b^2*c^3*d^4-70*
a^(5/2)*ln((-2*a*d*x+b*c*x-b*d+2*((a*d-b*c)/c^2*d)^(1/2)*((a*x+b)*x)^(1/2)*
c)/(c*x+d))*x*b^3*c^4*d^3+48*a^5*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2
))/a^(1/2))*((a*d-b*c)/c^2*d)^(1/2)*x*c^2*d^5+62*a^(7/2)*((a*x+b)*x)^(1/2)*
((a*d-b*c)/c^2*d)^(1/2)*b*c^3*d^4-46*a^(5/2)*((a*x+b)*x)^(1/2)*((a*d-b*c)/c
^2*d)^(1/2)*b^2*c^4*d^3-68*a^4*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))
/a^(1/2))*((a*d-b*c)/c^2*d)^(1/2)*b*c^2*d^5+8*a^(3/2)*((a*x+b)*x)^(1/2)*((a
*d-b*c)/c^2*d)^(1/2)*x^2*b^3*c^7+24*a^(11/2)*ln((-2*a*d*x+b*c*x-b*d+2*((a*d
-b*c)/c^2*d)^(1/2)*((a*x+b)*x)^(1/2)*c)/(c*x+d))*d^7-92*a^(5/2)*((a*x+b)*x)
^(1/2)*((a*d-b*c)/c^2*d)^(1/2)*x*b^2*c^5*d^2-136*a^4*ln(1/2*(2*a*x+b+2*((a*
x+b)*x)^(1/2)*a^(1/2))/a^(1/2))*((a*d-b*c)/c^2*d)^(1/2)*x*b*c^3*d^4+120*a^3
*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))*((a*d-b*c)/c^2*d)^(1
/2)*x*b^2*c^4*d^3-24*a^2*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/
2))*((a*d-b*c)/c^2*d)^(1/2)*x*b^3*c^5*d^2+60*a^3*ln(1/2*(2*a*x+b+2*((a*x+b)
*x)^(1/2)*a^(1/2))/a^(1/2))*((a*d-b*c)/c^2*d)^(1/2)*x^2*b^2*c^5*d^2-12*a^2*
ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))*((a*d-b*c)/c^2*d)^(1/
2)*x^2*b^3*c^6*d+102*a^(7/2)*((a*x+b)*x)^(1/2)*((a*d-b*c)/c^2*d)^(1/2)*x*b*
c^4*d^3+22*a^(5/2)*((a*x+b)*x)^(3/2)*((a*d-b*c)/c^2*d)^(1/2)*x*b*c^6*d+18*a
^(7/2)*((a*x+b)*x)^(1/2)*((a*d-b*c)/c^2*d)^(1/2)*x^2*b*c^5*d^2-46*a^(5/2)*(
(a*x+b)*x)^(1/2)*((a*d-b*c)/c^2*d)^(1/2)*x^2*b^2*c^6*d-68*a^4*ln(1/2*(2*a*x
+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))*((a*d-b*c)/c^2*d)^(1/2)*x^2*b*c^4*
d^3-22*a^(7/2)*((a*x+b)*x)^(1/2)*((a*d-b*c)/c^2*d)^(1/2)*x^3*b*c^6*d-8*a*ln
(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))*((a*d-b*c)/c^2*d)^(1/2)
*x*b^4*c^6*d+16*a^(3/2)*((a*x+b)*x)^(1/2)*((a*d-b*c)/c^2*d)^(1/2)*x*b^3*c^6
*d-24*a^(9/2)*((a*x+b)*x)^(1/2)*((a*d-b*c)/c^2*d)^(1/2)*c^2*d^5-80*a^(9/2)*
ln((-2*a*d*x+b*c*x-b*d+2*((a*d-b*c)/c^2*d)^(1/2)*((a*x+b)*x)^(1/2)*c)/(c*x+
d))*b*c*d^6+91*a^(7/2)*ln((-2*a*d*x+b*c*x-b*d+2*((a*d-b*c)/c^2*d)^(1/2)*((a
*x+b)*x)^(1/2)*c)/(c*x+d))*b^2*c^2*d^5-35*a^(5/2)*ln((-2*a*d*x+b*c*x-b*d+2*
((a*d-b*c)/c^2*d)^(1/2)*((a*x+b)*x)^(1/2)*c)/(c*x+d))*b^3*c^3*d^4+24*a^5*ln
(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))*((a*d-b*c)/c^2*d)^(1/2)
*c*d^6+24*a^(11/2)*ln((-2*a*d*x+b*c*x-b*d+2*((a*d-b*c)/c^2*d)^(1/2)*((a*x+b
)*x)^(1/2)*c)/(c*x+d))*x^2*c^2*d^5-8*a^(7/2)*((a*x+b)*x)^(3/2)*((a*d-b*c)/c
^2*d)^(1/2)*c^4*d^3+48*a^(11/2)*ln((-2*a*d*x+b*c*x-b*d+2*((a*d-b*c)/c^2*d)^
(1/2)*((a*x+b)*x)^(1/2)*c)/(c*x+d))*x*c*d^6)/c^5/((a*x+b)*x)^(1/2)/(a*d-b*c
)^3/(c*x+d)^2/a^(5/2)/((a*d-b*c)/c^2*d)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+d/x)^3/(a+b/x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(a + b/x)*(c + d/x)^3), x)
```

mupad [B] time = 5.48, size = 2890, normalized size = 11.56

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b/x)^(1/2)*(c + d/x)^3),x)
```

```
[Out] (log((d^3*(a*d - b*c)^5)^(1/2)*(a + b/x)^(1/2) - a^3*d^4 + b^3*c^3*d - 3*a*
b^2*c^2*d^2 + 3*a^2*b*c*d^3)*(d^3*(a*d - b*c)^5)^(1/2)*(3*a^2*d^2 + (35*b^2
*c^2)/8 - 7*a*b*c*d))/(b^5*c^9 - a^5*c^4*d^5 + 5*a^4*b*c^5*d^4 + 10*a^2*b^3
*c^7*d^2 - 10*a^3*b^2*c^6*d^3 - 5*a*b^4*c^8*d) - ((b*(a + b/x)^(5/2)*(12*a^
2*d^4 + 4*b^2*c^2*d^2 - 19*a*b*c*d^3))/(4*a*c^3*(a*d - b*c)^2) - ((a + b/x)
^(1/2)*(4*b^4*c^3 - 12*a^3*b*d^3 + 25*a^2*b^2*c*d^2 - 12*a*b^3*c^2*d))/(4*a
*c^3*(a*d - b*c)) + (d*(a + b/x)^(3/2)*(8*b^4*c^3 - 24*a^3*b*d^3 + 56*a^2*b
^2*c*d^2 - 37*a*b^3*c^2*d))/(4*c^3*(a^2*d - a*b*c)*(a*d - b*c)))/((a + b/x)
^2*(3*a*d^2 - 2*b*c*d) - (a + b/x)*(3*a^2*d^2 + b^2*c^2 - 4*a*b*c*d) - d^2*
(a + b/x)^3 + a^3*d^2 + a*b^2*c^2 - 2*a^2*b*c*d) - (log((d^3*(a*d - b*c)^5)
^(1/2)*(a + b/x)^(1/2) + a^3*d^4 - b^3*c^3*d + 3*a*b^2*c^2*d^2 - 3*a^2*b*c*
d^3)*(d^3*(a*d - b*c)^5)^(1/2)*(24*a^2*d^2 + 35*b^2*c^2 - 56*a*b*c*d))/(8*(
b^5*c^9 - a^5*c^4*d^5 + 5*a^4*b*c^5*d^4 + 10*a^2*b^3*c^7*d^2 - 10*a^3*b^2*c
^6*d^3 - 5*a*b^4*c^8*d)) - (atan((((((a + b/x)^(1/2)*(1152*a^6*b^2*d^9 + 16
*b^8*c^6*d^3 + 128*a*b^7*c^5*d^4 - 4800*a^5*b^3*c*d^8 + 1129*a^2*b^6*c^4*d^
5 - 5136*a^3*b^5*c^3*d^6 + 7520*a^4*b^4*c^2*d^7))/(8*(a^2*b^4*c^10 + a^6*c^
6*d^4 - 4*a^3*b^3*c^9*d - 4*a^5*b*c^7*d^3 + 6*a^4*b^2*c^8*d^2)) - (((4*a*b^
8*c^13*d^2 + 4*a^2*b^7*c^12*d^3 - 45*a^3*b^6*c^11*d^4 + 74*a^4*b^5*c^10*d^5
 - 49*a^5*b^4*c^9*d^6 + 12*a^6*b^3*c^8*d^7)/(a^2*b^4*c^13 + a^6*c^9*d^4 - 4
*a^3*b^3*c^12*d - 4*a^5*b*c^10*d^3 + 6*a^4*b^2*c^11*d^2) - ((a + b/x)^(1/2)
*(6*a*d + b*c)*(64*a^2*b^7*c^13*d^2 - 384*a^3*b^6*c^12*d^3 + 896*a^4*b^5*c^
11*d^4 - 1024*a^5*b^4*c^10*d^5 + 576*a^6*b^3*c^9*d^6 - 128*a^7*b^2*c^8*d^7)
)/(16*c^4*(a^3)^(1/2)*(a^2*b^4*c^10 + a^6*c^6*d^4 - 4*a^3*b^3*c^9*d - 4*a^5
*b*c^7*d^3 + 6*a^4*b^2*c^8*d^2)))*(6*a*d + b*c))/(2*c^4*(a^3)^(1/2)))*(6*a*
d + b*c)*1i)/(2*c^4*(a^3)^(1/2)) + ((((a + b/x)^(1/2)*(1152*a^6*b^2*d^9 + 1
6*b^8*c^6*d^3 + 128*a*b^7*c^5*d^4 - 4800*a^5*b^3*c*d^8 + 1129*a^2*b^6*c^4*d
^5 - 5136*a^3*b^5*c^3*d^6 + 7520*a^4*b^4*c^2*d^7))/(8*(a^2*b^4*c^10 + a^6*c
^6*d^4 - 4*a^3*b^3*c^9*d - 4*a^5*b*c^7*d^3 + 6*a^4*b^2*c^8*d^2)) + (((4*a*b
^8*c^13*d^2 + 4*a^2*b^7*c^12*d^3 - 45*a^3*b^6*c^11*d^4 + 74*a^4*b^5*c^10*d^
5 - 49*a^5*b^4*c^9*d^6 + 12*a^6*b^3*c^8*d^7)/(a^2*b^4*c^13 + a^6*c^9*d^4 -
4*a^3*b^3*c^12*d - 4*a^5*b*c^10*d^3 + 6*a^4*b^2*c^11*d^2) + ((a + b/x)^(1/2
)*(6*a*d + b*c)*(64*a^2*b^7*c^13*d^2 - 384*a^3*b^6*c^12*d^3 + 896*a^4*b^5*c
^11*d^4 - 1024*a^5*b^4*c^10*d^5 + 576*a^6*b^3*c^9*d^6 - 128*a^7*b^2*c^8*d^7
))/(16*c^4*(a^3)^(1/2)*(a^2*b^4*c^10 + a^6*c^6*d^4 - 4*a^3*b^3*c^9*d - 4*a^
5*b*c^7*d^3 + 6*a^4*b^2*c^8*d^2)))*(6*a*d + b*c))/(2*c^4*(a^3)^(1/2)))*(6*a
*d + b*c)*1i)/(2*c^4*(a^3)^(1/2)))/((216*a^5*b^3*d^9 + (35*b^8*c^5*d^4)/2 -
 (49*a*b^7*c^4*d^5)/8 - 810*a^4*b^4*c*d^8 - (1877*a^2*b^6*c^3*d^6)/4 + 1044
*a^3*b^5*c^2*d^7)/(a^2*b^4*c^13 + a^6*c^9*d^4 - 4*a^3*b^3*c^12*d - 4*a^5*b*
c^10*d^3 + 6*a^4*b^2*c^11*d^2) + ((((a + b/x)^(1/2)*(1152*a^6*b^2*d^9 + 16*
b^8*c^6*d^3 + 128*a*b^7*c^5*d^4 - 4800*a^5*b^3*c*d^8 + 1129*a^2*b^6*c^4*d^5
 - 5136*a^3*b^5*c^3*d^6 + 7520*a^4*b^4*c^2*d^7))/(8*(a^2*b^4*c^10 + a^6*c^6
*d^4 - 4*a^3*b^3*c^9*d - 4*a^5*b*c^7*d^3 + 6*a^4*b^2*c^8*d^2)) - (((4*a*b^8
*c^13*d^2 + 4*a^2*b^7*c^12*d^3 - 45*a^3*b^6*c^11*d^4 + 74*a^4*b^5*c^10*d^5
- 49*a^5*b^4*c^9*d^6 + 12*a^6*b^3*c^8*d^7)/(a^2*b^4*c^13 + a^6*c^9*d^4 - 4*
a^3*b^3*c^12*d - 4*a^5*b*c^10*d^3 + 6*a^4*b^2*c^11*d^2) - ((a + b/x)^(1/2)*
(6*a*d + b*c)*(64*a^2*b^7*c^13*d^2 - 384*a^3*b^6*c^12*d^3 + 896*a^4*b^5*c^1
1*d^4 - 1024*a^5*b^4*c^10*d^5 + 576*a^6*b^3*c^9*d^6 - 128*a^7*b^2*c^8*d^7))
/(16*c^4*(a^3)^(1/2)*(a^2*b^4*c^10 + a^6*c^6*d^4 - 4*a^3*b^3*c^9*d - 4*a^5*
b*c^7*d^3 + 6*a^4*b^2*c^8*d^2)))*(6*a*d + b*c))/(2*c^4*(a^3)^(1/2)))*(6*a*d
 + b*c))/(2*c^4*(a^3)^(1/2)) - ((((a + b/x)^(1/2)*(1152*a^6*b^2*d^9 + 16*b^
8*c^6*d^3 + 128*a*b^7*c^5*d^4 - 4800*a^5*b^3*c*d^8 + 1129*a^2*b^6*c^4*d^5 -
 5136*a^3*b^5*c^3*d^6 + 7520*a^4*b^4*c^2*d^7))/(8*(a^2*b^4*c^10 + a^6*c^6*d
^4 - 4*a^3*b^3*c^9*d - 4*a^5*b*c^7*d^3 + 6*a^4*b^2*c^8*d^2)) + (((4*a*b^8*c
^13*d^2 + 4*a^2*b^7*c^12*d^3 - 45*a^3*b^6*c^11*d^4 + 74*a^4*b^5*c^10*d^5 -
49*a^5*b^4*c^9*d^6 + 12*a^6*b^3*c^8*d^7)/(a^2*b^4*c^13 + a^6*c^9*d^4 - 4*a^
3*b^3*c^12*d - 4*a^5*b*c^10*d^3 + 6*a^4*b^2*c^11*d^2) + ((a + b/x)^(1/2)*(6
*a*d + b*c)*(64*a^2*b^7*c^13*d^2 - 384*a^3*b^6*c^12*d^3 + 896*a^4*b^5*c^11*
d^4 - 1024*a^5*b^4*c^10*d^5 + 576*a^6*b^3*c^9*d^6 - 128*a^7*b^2*c^8*d^7))/(
16*c^4*(a^3)^(1/2)*(a^2*b^4*c^10 + a^6*c^6*d^4 - 4*a^3*b^3*c^9*d - 4*a^5*b*
```

```
c^7*d^3 + 6*a^4*b^2*c^8*d^2)))*(6*a*d + b*c))/(2*c^4*(a^3)^(1/2)))*(6*a*d +
 b*c))/(2*c^4*(a^3)^(1/2))))*(6*a*d + b*c)*1i)/(c^4*(a^3)^(1/2))
```

sympy **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+d/x)**3/(a+b/x)**(1/2),x)
```

```
[Out] Timed out
```

3.155 $\int \frac{\left(c+\frac{d}{x}\right)^3}{\left(a+\frac{b}{x}\right)^{3/2}} dx$

Optimal. Leaf size=132

$$-\frac{3c^2(bc-2ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{5/2}}+\frac{(bc-2ad)\left(2a^2d^2-2abcd+3b^2c^2\right)-\frac{abd^2(2ad+bc)}{x}}{a^2b^2\sqrt{a+\frac{b}{x}}}+\frac{cx\left(c+\frac{d}{x}\right)^2}{a\sqrt{a+\frac{b}{x}}}$$

Rubi [A] time = 0.10, antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {375, 98, 146, 63, 208}

$$\frac{(bc-2ad)\left(2a^2d^2-2abcd+3b^2c^2\right)-\frac{abd^2(2ad+bc)}{x}}{a^2b^2\sqrt{a+\frac{b}{x}}}-\frac{3c^2(bc-2ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{5/2}}+\frac{cx\left(c+\frac{d}{x}\right)^2}{a\sqrt{a+\frac{b}{x}}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d/x)^3/(a + b/x)^(3/2),x]
```

```
[Out] ((b*c - 2*a*d)*(3*b^2*c^2 - 2*a*b*c*d + 2*a^2*d^2) - (a*b*d^2*(b*c + 2*a*d)
)/x)/(a^2*b^2*Sqrt[a + b/x]) + (c*(c + d/x)^2*x)/(a*Sqrt[a + b/x]) - (3*c^2
*(b*c - 2*a*d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/a^(5/2)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 146

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_)
)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((a^2*d*f*h*(n + 2) + b^2*d*e*g*(
m + n + 3) + a*b*(c*f*h*(m + 1) - d*(f*g + e*h)*(m + n + 3)) + b*f*h*(b*c -
 a*d)*(m + 1)*x)*(a + b*x)^(m + 1)*(c + d*x)^(n + 1))/(b^2*d*(b*c - a*d)*(m
 + 1)*(m + n + 3)), x] - Dist[(a^2*d^2*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*
(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m + n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2
) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3) + d^2*e*g*(m + n + 2)*(m + n + 3)))
/(b^2*d*(b*c - a*d)*(m + 1)*(m + n + 3)), Int[(a + b*x)^(m + 1)*(c + d*x)^n
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x] && ((GeQ[m, -2] && LtQ
```

```
[m, -1]) || SumSimplerQ[m, 1]) && NeQ[m, -1] && NeQ[m + n + 3, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 375

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[((a + b/x^n)^p*(c + d/x^n)^q)/x^2, x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rubi steps

$$\int \frac{\left(c+\frac{d}{x}\right)^3}{\left(a+\frac{b}{x}\right)^{3/2}}\,dx = -\text{Subst}\left(\int \frac{(c+dx)^3}{x^2(a+bx)^{3/2}}\,dx,x,\frac{1}{x}\right)$$

$$= \frac{c\left(c+\frac{d}{x}\right)^2 x}{a\sqrt{a+\frac{b}{x}}} + \frac{\text{Subst}\left(\int \frac{(c+dx)\left(\frac{3}{2}c(bc-2ad)-\frac{1}{2}d(bc+2ad)x\right)}{x(a+bx)^{3/2}}\,dx,x,\frac{1}{x}\right)}{a}$$

$$= \frac{(bc-2ad)\left(3b^2c^2-2abcd+2a^2d^2\right)-\frac{abd^2(bc+2ad)}{x}}{a^2b^2\sqrt{a+\frac{b}{x}}} + \frac{c\left(c+\frac{d}{x}\right)^2 x}{a\sqrt{a+\frac{b}{x}}} + \frac{\left(3c^2(bc-2ad)\right)\text{Subst}\left(\int\right.}{2a^2}$$

$$= \frac{(bc-2ad)\left(3b^2c^2-2abcd+2a^2d^2\right)-\frac{abd^2(bc+2ad)}{x}}{a^2b^2\sqrt{a+\frac{b}{x}}} + \frac{c\left(c+\frac{d}{x}\right)^2 x}{a\sqrt{a+\frac{b}{x}}} + \frac{\left(3c^2(bc-2ad)\right)\text{Subst}\left(\int\right.}{a^2}$$

$$= \frac{(bc-2ad)\left(3b^2c^2-2abcd+2a^2d^2\right)-\frac{abd^2(bc+2ad)}{x}}{a^2b^2\sqrt{a+\frac{b}{x}}} + \frac{c\left(c+\frac{d}{x}\right)^2 x}{a\sqrt{a+\frac{b}{x}}} - \frac{3c^2(bc-2ad)\tanh^{-1}\left(\sqrt{\right.}{a^{5/2}}$$

Mathematica [C] time = 0.07, size = 92, normalized size = 0.70

$$\frac{a\left(-4a^2d^3x-2abd^2(d-3cx)+b^2c^3x^2\right)+3b^2c^2x(bc-2ad)\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{b}{ax}+1\right)}{a^2b^2x\sqrt{a+\frac{b}{x}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d/x)^3/(a + b/x)^(3/2),x]
```

```
[Out] (a*(-4*a^2*d^3*x + b^2*c^3*x^2 - 2*a*b*d^2*(d - 3*c*x)) + 3*b^2*c^2*(b*c -
2*a*d)*x*Hypergeometric2F1[-1/2, 1, 1/2, 1 + b/(a*x)])/(a^2*b^2*Sqrt[a + b/
x]*x)
```

IntegrateAlgebraic [A] time = 0.28, size = 130, normalized size = 0.98

$$\frac{3\left(2ac^2d-bc^3\right)\tanh^{-1}\left(\frac{\sqrt{\frac{ax+b}{x}}}{\sqrt{a}}\right)}{a^{5/2}} + \frac{\sqrt{\frac{ax+b}{x}}\left(-4a^3d^3x+6a^2bcd^2x-2a^2bd^3+ab^2c^3x^2-6ab^2c^2dx+3b^3c^3x\right)}{a^2b^2(ax+b)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c + d/x)^3/(a + b/x)^(3/2),x]
```

```
[Out] (Sqrt[(b + a*x)/x]*(-2*a^2*b*d^3 + 3*b^3*c^3*x - 6*a*b^2*c^2*d*x + 6*a^2*b*
c*d^2*x - 4*a^3*d^3*x + a*b^2*c^3*x^2))/(a^2*b^2*(b + a*x)) + (3*(-(b*c^3)
+ 2*a*c^2*d)*ArcTanh[Sqrt[(b + a*x)/x]/Sqrt[a]])/a^(5/2)
```

fricas [A] time = 1.54, size = 336, normalized size = 2.55

$$\left[-\frac{3\left(b^4c^3-2ab^3c^2d+\left(ab^3c^3-2a^2b^2c^2d\right)x\right)\sqrt{a}\log\left(2ax+2\sqrt{a}x\sqrt{\frac{ax+b}{x}}+b\right)-2\left(a^2b^2c^3x^2-2a^3bd^3+\left(3ab^3c^3-6a^2b^2c^2d+6a^3bcd^2-4a^4d^3\right)x\right)\sqrt{\frac{ax+b}{x}}}{2\left(a^4b^2x+a^3b^3\right)}, \frac{3\left(b^4c^3-2ab^3c^2d+\left(ab^3c^3-2a^2b^2c^2d\right)x\right)\sqrt{-a}\arctan\left(\frac{\sqrt{-a}\sqrt{\frac{ax+b}{x}}}{a}\right)+\left(a^2b^2c^3x^2-2a^3bd^3+\left(3ab^3c^3-6a^2b^2c^2d+6a^3bcd^2-4a^4d^3\right)x\right)\sqrt{\frac{ax+b}{x}}}{a^4b^2x+a^3b^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d/x)^3/(a+b/x)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/2*(3*(b^4*c^3 - 2*a*b^3*c^2*d + (a*b^3*c^3 - 2*a^2*b^2*c^2*d)*x)*sqrt(a
)*log(2*a*x + 2*sqrt(a)*x*sqrt((a*x + b)/x) + b) - 2*(a^2*b^2*c^3*x^2 - 2*a
^3*b*d^3 + (3*a*b^3*c^3 - 6*a^2*b^2*c^2*d + 6*a^3*b*c*d^2 - 4*a^4*d^3)*x)*s
qrt((a*x + b)/x))/(a^4*b^2*x + a^3*b^3), (3*(b^4*c^3 - 2*a*b^3*c^2*d + (a*b
^3*c^3 - 2*a^2*b^2*c^2*d)*x)*sqrt(-a)*arctan(sqrt(-a)*sqrt((a*x + b)/x)/a)
+ (a^2*b^2*c^3*x^2 - 2*a^3*b*d^3 + (3*a*b^3*c^3 - 6*a^2*b^2*c^2*d + 6*a^3*b
*c*d^2 - 4*a^4*d^3)*x)*sqrt((a*x + b)/x))/(a^4*b^2*x + a^3*b^3)]
```

giac [A] time = 0.21, size = 222, normalized size = 1.68

$$-\frac{\frac{2d^3\sqrt{\frac{ax+b}{x}}}{b}-\frac{3\left(b^2c^3-2abc^2d\right)\arctan\left(\frac{\sqrt{\frac{ax+b}{x}}}{\sqrt{-a}}\right)}{\sqrt{-a}a^2}-\frac{2ab^3c^3-6a^2b^2c^2d+6a^3bcd^2-2a^4d^3-\frac{3(ax+b)b^3c^3}{x}+\frac{6(ax+b)ab^2c^2d}{x}-\frac{6(ax+b)a^2bcd^2}{x}+\frac{2(ax+b)a^3d^3}{x}}{\left(a\sqrt{\frac{ax+b}{x}}-\frac{(ax+b)\sqrt{\frac{ax+b}{x}}}{x}\right)a^2b}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d/x)^3/(a+b/x)^(3/2),x, algorithm="giac")
```

```
[Out] -(2*d^3*sqrt((a*x + b)/x)/b - 3*(b^2*c^3 - 2*a*b*c^2*d)*arctan(sqrt((a*x +
b)/x)/sqrt(-a))/(sqrt(-a)*a^2) - (2*a*b^3*c^3 - 6*a^2*b^2*c^2*d + 6*a^3*b*c
*d^2 - 2*a^4*d^3 - 3*(a*x + b)*b^3*c^3/x + 6*(a*x + b)*a*b^2*c^2*d/x - 6*(a
*x + b)*a^2*b*c*d^2/x + 2*(a*x + b)*a^3*d^3/x)/((a*sqrt((a*x + b)/x) - (a*x
 + b)*sqrt((a*x + b)/x)/x)*a^2*b))/b
```

maple [B] time = 0.07, size = 969, normalized size = 7.34

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d/x)^3/(a+b/x)^(3/2),x)
```

```
[Out] -1/2*((a*x+b)/x)^(1/2)/x/a^(5/2)*(3*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(
1/2))/a^(1/2))*x^2*b^6*c^3+4*(a*x^2+b*x)^(3/2)*a^(9/2)*x^2*d^3-4*a^(9/2)*((
a*x+b)*x)^(3/2)*x^2*d^3+4*(a*x^2+b*x)^(3/2)*a^(5/2)*b^2*d^3+6*ln(1/2*(2*a*x
+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))*x^3*a^3*b^3*c*d^2-12*ln(1/2*(2*a*x
+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))*x^3*a^2*b^4*c^2*d-6*ln(1/2*(2*a*x+
b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))*x^4*a^3*b^3*c^2*d-3*ln(1/2*(2*a*x+b
+2*(a*x^2+b*x)^(1/2)*a^(1/2))/a^(1/2))*x^4*a^4*b^2*c*d^2+3*ln(1/2*(2*a*x+b+
2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))*x^4*a^4*b^2*c*d^2-6*ln(1/2*(2*a*x+b+2
*(a*x^2+b*x)^(1/2)*a^(1/2))/a^(1/2))*x^3*a^3*b^3*c*d^2+12*a^(3/2)*((a*x+b)*
x)^(1/2)*x^2*b^4*c^2*d-6*(a*x^2+b*x)^(1/2)*a^(9/2)*x^4*b*c*d^2-6*a^(9/2)*((
a*x+b)*x)^(1/2)*x^4*b*c*d^2+12*a^(7/2)*((a*x+b)*x)^(1/2)*x^4*b^2*c^2*d-12*(
```

```
a*x^2+b*x)^(1/2)*a^(7/2)*x^3*b^2*c*d^2+12*a^(7/2)*((a*x+b)*x)^(3/2)*x^2*b*c
*d^2-12*a^(5/2)*((a*x+b)*x)^(3/2)*x^2*b^2*c^2*d-12*a^(7/2)*((a*x+b)*x)^(1/2
)*x^3*b^2*c*d^2+24*a^(5/2)*((a*x+b)*x)^(1/2)*x^3*b^3*c^2*d-6*(a*x^2+b*x)^(1
/2)*a^(5/2)*x^2*b^3*c*d^2-6*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^
(1/2))*x^2*a*b^5*c^2*d-3*ln(1/2*(2*a*x+b+2*(a*x^2+b*x)^(1/2)*a^(1/2))/a^(1/
2))*x^2*a^2*b^4*c*d^2+3*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2
))*x^2*a^2*b^4*c*d^2-6*a^(5/2)*((a*x+b)*x)^(1/2)*x^2*b^3*c*d^2+8*(a*x^2+b*x
)^(3/2)*a^(7/2)*x*b*d^3-12*a^(3/2)*((a*x+b)*x)^(1/2)*x^3*b^4*c^3+4*a^(3/2)*
((a*x+b)*x)^(3/2)*x^2*b^3*c^3-6*a^(5/2)*((a*x+b)*x)^(1/2)*x^4*b^3*c^3+3*ln(
1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))*x^4*a^2*b^4*c^3+6*ln(1/2
*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))*x^3*a*b^5*c^3-6*a^(1/2)*((a
*x+b)*x)^(1/2)*x^2*b^5*c^3)/((a*x+b)*x)^(1/2)/b^3/(a*x+b)^2
```

maxima [A] time = 1.26, size = 200, normalized size = 1.52

$$\frac{1}{2}\,c^{3}\left(\frac{2\left(3\left(a+\frac{b}{x}\right)b-2\,ab\right)}{\left(a+\frac{b}{x}\right)^{\frac{3}{2}}a^{2}-\sqrt{a+\frac{b}{x}}\,a^{3}}+\frac{3\,b\log\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{a+\frac{b}{x}}+\sqrt{a}}\right)}{a^{\frac{5}{2}}}\right)-3\,c^{2}d\left(\frac{\log\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{a+\frac{b}{x}}+\sqrt{a}}\right)}{a^{\frac{3}{2}}}+\frac{2}{\sqrt{a+\frac{b}{x}}\,a}\right)-2\,d^{3}\left(\frac{\sqrt{a+\frac{b}{x}}}{b^{2}}+\frac{a}{\sqrt{a+\frac{b}{x}}\,b^{2}}\right)+\frac{6\,cd^{2}}{\sqrt{a+\frac{b}{x}}\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d/x)^3/(a+b/x)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/2*c^3*(2*(3*(a + b/x)*b - 2*a*b)/((a + b/x)^(3/2)*a^2 - sqrt(a + b/x)*a^3
) + 3*b*log((sqrt(a + b/x) - sqrt(a))/(sqrt(a + b/x) + sqrt(a)))/a^(5/2)) -
 3*c^2*d*(log((sqrt(a + b/x) - sqrt(a))/(sqrt(a + b/x) + sqrt(a)))/a^(3/2)
+ 2/(sqrt(a + b/x)*a)) - 2*d^3*(sqrt(a + b/x)/b^2 + a/(sqrt(a + b/x)*b^2))
+ 6*c*d^2/(sqrt(a + b/x)*b)
```

mupad [B] time = 1.91, size = 172, normalized size = 1.30

$$\frac{\frac{2\left(a^{3}\,d^{3}-3\,a^{2}\,b\,c\,d^{2}+3\,a\,b^{2}\,c^{2}\,d-b^{3}\,c^{3}\right)}{a}-\frac{\left(a+\frac{b}{x}\right)\left(2\,a^{3}\,d^{3}-6\,a^{2}\,b\,c\,d^{2}+6\,a\,b^{2}\,c^{2}\,d-3\,b^{3}\,c^{3}\right)}{a^{2}}}{b^{2}\left(a+\frac{b}{x}\right)^{3/2}-a\,b^{2}\,\sqrt{a+\frac{b}{x}}}-\frac{2\,d^{3}\,\sqrt{a+\frac{b}{x}}}{b^{2}}+\frac{3\,c^{2}\,\mathrm{atanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)(2\,a\,d-b\,c)}{a^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d/x)^3/(a + b/x)^(3/2),x)
```

```
[Out] ((2*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2))/a - ((a + b/x)*(2*
a^3*d^3 - 3*b^3*c^3 + 6*a*b^2*c^2*d - 6*a^2*b*c*d^2))/a^2)/(b^2*(a + b/x)^(
3/2) - a*b^2*(a + b/x)^(1/2)) - (2*d^3*(a + b/x)^(1/2))/b^2 + (3*c^2*atanh(
(a + b/x)^(1/2)/a^(1/2))*(2*a*d - b*c))/a^(5/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(cx+d)^{3}}{x^{3}\left(a+\frac{b}{x}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d/x)**3/(a+b/x)**(3/2),x)
```

```
[Out] Integral((c*x + d)**3/(x**3*(a + b/x)**(3/2)), x)
```

3.156 $\int \frac{\left(c+\frac{d}{x}\right)^2}{\left(a+\frac{b}{x}\right)^{3/2}} dx$

Optimal. Leaf size=94

$$-\frac{c(3bc-4ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{5/2}}+\frac{2a^2d^2+bc(3bc-4ad)}{a^2b\sqrt{a+\frac{b}{x}}}+\frac{c^2x}{a\sqrt{a+\frac{b}{x}}}$$

Rubi [A] time = 0.08, antiderivative size = 90, normalized size of antiderivative = 0.96, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {375, 89, 78, 63, 208}

$$\frac{\frac{c(3bc-4ad)}{a^2}+\frac{2d^2}{b}}{\sqrt{a+\frac{b}{x}}}-\frac{c(3bc-4ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{5/2}}+\frac{c^2x}{a\sqrt{a+\frac{b}{x}}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d/x)^2/(a + b/x)^(3/2),x]
```

```
[Out] ((2*d^2)/b + (c*(3*b*c - 4*a*d))/a^2)/Sqrt[a + b/x] + (c^2*x)/(a*Sqrt[a + b
/x]) - (c*(3*b*c - 4*a*d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/a^(5/2)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 375

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[((a + b/x^n)^p*(c + d/x^n)^q)/x^2, x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rubi steps

$$\int \frac{\left(c+\frac{d}{x}\right)^2}{\left(a+\frac{b}{x}\right)^{3/2}}\,dx = -\mathrm{Subst}\left(\int \frac{(c+dx)^2}{x^2(a+bx)^{3/2}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{c^2x}{a\sqrt{a+\frac{b}{x}}} - \frac{\mathrm{Subst}\left(\int \frac{-\frac{1}{2}c(3bc-4ad)+ad^2x}{x(a+bx)^{3/2}}\,dx, x, \frac{1}{x}\right)}{a}$$

$$= \frac{\frac{3bc^2}{a}-4cd+\frac{2ad^2}{b}}{a\sqrt{a+\frac{b}{x}}} + \frac{c^2x}{a\sqrt{a+\frac{b}{x}}} + \frac{(c(3bc-4ad))\,\mathrm{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \frac{1}{x}\right)}{2a^2}$$

$$= \frac{\frac{3bc^2}{a}-4cd+\frac{2ad^2}{b}}{a\sqrt{a+\frac{b}{x}}} + \frac{c^2x}{a\sqrt{a+\frac{b}{x}}} + \frac{(c(3bc-4ad))\,\mathrm{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+\frac{b}{x}}\right)}{a^2b}$$

$$= \frac{\frac{3bc^2}{a}-4cd+\frac{2ad^2}{b}}{a\sqrt{a+\frac{b}{x}}} + \frac{c^2x}{a\sqrt{a+\frac{b}{x}}} - \frac{c(3bc-4ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{5/2}}$$

Mathematica [A] time = 0.10, size = 81, normalized size = 0.86

$$\frac{c(4ad-3bc)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{5/2}} + \frac{2a^2d^2+abc(cx-4d)+3b^2c^2}{a^2b\sqrt{a+\frac{b}{x}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d/x)^2/(a + b/x)^(3/2),x]
```

```
[Out] (3*b^2*c^2 + 2*a^2*d^2 + a*b*c*(-4*d + c*x))/(a^2*b*Sqrt[a + b/x]) + (c*(-3
*b*c + 4*a*d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/a^(5/2)
```

IntegrateAlgebraic [A] time = 0.28, size = 101, normalized size = 1.07

$$\frac{\left(4acd-3bc^2\right)\tanh^{-1}\left(\frac{\sqrt{\frac{ax+b}{x}}}{\sqrt{a}}\right)}{a^{5/2}} + \frac{\sqrt{\frac{ax+b}{x}}\left(2a^2d^2x+abc^2x^2-4abcdx+3b^2c^2x\right)}{a^2b(ax+b)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c + d/x)^2/(a + b/x)^(3/2),x]
```

```
[Out] (Sqrt[(b + a*x)/x]*(3*b^2*c^2*x - 4*a*b*c*d*x + 2*a^2*d^2*x + a*b*c^2*x^2))
/(a^2*b*(b + a*x)) + ((-3*b*c^2 + 4*a*c*d)*ArcTanh[Sqrt[(b + a*x)/x]/Sqrt[a
]])/a^(5/2)
```

fricas [A] time = 0.62, size = 272, normalized size = 2.89

$$\left[-\frac{\left(3\,b^{3}c^{2}-4\,ab^{2}cd+\left(3\,ab^{2}c^{2}-4\,a^{2}bcd\right)x\right)\sqrt{a}\,\log\left(2\,ax+2\,\sqrt{a}\,x\sqrt{\frac{ax+b}{x}}+b\right)-2\left(a^{2}bc^{2}x^{2}+\left(3\,ab^{2}c^{2}-4\,a^{2}bcd+2\,a^{3}d^{2}\right)x\right)\sqrt{\frac{ax+b}{x}}}{2\left(a^{4}bx+a^{3}b^{2}\right)},\ \frac{\left(3\,b^{3}c^{2}-4\,ab^{2}cd+\left(3\,ab^{2}c^{2}-4\,a^{2}bcd\right)x\right)\sqrt{-a}\,\arctan\left(\frac{\sqrt{-a}\sqrt{\frac{ax+b}{x}}}{a}\right)+\left(a^{2}bc^{2}x^{2}+\left(3\,ab^{2}c^{2}-4\,a^{2}bcd+2\,a^{3}d^{2}\right)x\right)\sqrt{\frac{ax+b}{x}}}{a^{4}bx+a^{3}b^{2}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d/x)^2/(a+b/x)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/2*((3*b^3*c^2 - 4*a*b^2*c*d + (3*a*b^2*c^2 - 4*a^2*b*c*d)*x)*sqrt(a)*lo
g(2*a*x + 2*sqrt(a)*x*sqrt((a*x + b)/x) + b) - 2*(a^2*b*c^2*x^2 + (3*a*b^2*
c^2 - 4*a^2*b*c*d + 2*a^3*d^2)*x)*sqrt((a*x + b)/x))/(a^4*b*x + a^3*b^2), (
(3*b^3*c^2 - 4*a*b^2*c*d + (3*a*b^2*c^2 - 4*a^2*b*c*d)*x)*sqrt(-a)*arctan(s
qrt(-a)*sqrt((a*x + b)/x)/a) + (a^2*b*c^2*x^2 + (3*a*b^2*c^2 - 4*a^2*b*c*d
+ 2*a^3*d^2)*x)*sqrt((a*x + b)/x))/(a^4*b*x + a^3*b^2)]
```

giac [A] time = 0.23, size = 160, normalized size = 1.70

$$\frac{\frac{\left(3\,b^{2}c^{2}-4\,abcd\right)\arctan\left(\frac{\sqrt{\frac{ax+b}{x}}}{\sqrt{-a}}\right)}{\sqrt{-a}\,a^{2}}+\frac{2\,ab^{2}c^{2}-4\,a^{2}bcd+2\,a^{3}d^{2}-\frac{3\,(ax+b)b^{2}c^{2}}{x}+\frac{4\,(ax+b)abcd}{x}-\frac{2\,(ax+b)a^{2}d^{2}}{x}}{\left(a\sqrt{\frac{ax+b}{x}}-\frac{(ax+b)\sqrt{\frac{ax+b}{x}}}{x}\right)a^{2}}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d/x)^2/(a+b/x)^(3/2),x, algorithm="giac")
```

```
[Out] ((3*b^2*c^2 - 4*a*b*c*d)*arctan(sqrt((a*x + b)/x)/sqrt(-a))/(sqrt(-a)*a^2)
+ (2*a*b^2*c^2 - 4*a^2*b*c*d + 2*a^3*d^2 - 3*(a*x + b)*b^2*c^2/x + 4*(a*x +
 b)*a*b*c*d/x - 2*(a*x + b)*a^2*d^2/x)/((a*sqrt((a*x + b)/x) - (a*x + b)*sq
rt((a*x + b)/x)/x)*a^2))/b
```

maple [B] time = 0.06, size = 789, normalized size = 8.39

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d/x)^2/(a+b/x)^(3/2),x)
```

```
[Out] 1/2*((a*x+b)/x)^(1/2)*x*(ln(1/2*(2*a*x+b+2*(a*x^2+b*x)^(1/2)*a^(1/2))/a^(1/
2))*a^2*b^3*d^2+2*a^(9/2)*((a*x+b)*x)^(1/2)*x^2*d^2+2*a^(9/2)*(a*x^2+b*x)^(
1/2)*x^2*d^2-4*a^(3/2)*((a*x+b)*x)^(3/2)*b^2*c^2+2*a^(5/2)*((a*x+b)*x)^(1/2
)*b^2*d^2+2*a^(5/2)*(a*x^2+b*x)^(1/2)*b^2*d^2-ln(1/2*(2*a*x+b+2*((a*x+b)*x)
^(1/2)*a^(1/2))/a^(1/2))*a^2*b^3*d^2+6*a^(1/2)*((a*x+b)*x)^(1/2)*b^4*c^2-16
*a^(5/2)*((a*x+b)*x)^(1/2)*x*b^2*c*d+8*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*
a^(1/2))/a^(1/2))*x*a^2*b^3*c*d+4*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/
2))/a^(1/2))*x^2*a^3*b^2*c*d-8*a^(7/2)*((a*x+b)*x)^(1/2)*x^2*b*c*d-3*ln(1/2
*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))*b^5*c^2-4*a^(7/2)*((a*x+b)*
x)^(3/2)*d^2-2*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))*x*a^3*
b^2*d^2-6*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))*x*a*b^4*c^2
+2*ln(1/2*(2*a*x+b+2*(a*x^2+b*x)^(1/2)*a^(1/2))/a^(1/2))*x*a^3*b^2*d^2+4*ln
(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))*a*b^4*c*d+4*a^(7/2)*((a
*x+b)*x)^(1/2)*x*b*d^2+12*a^(3/2)*((a*x+b)*x)^(1/2)*x*b^3*c^2+4*a^(7/2)*(a*
x^2+b*x)^(1/2)*x*b*d^2-8*a^(3/2)*((a*x+b)*x)^(1/2)*b^3*c*d+6*a^(5/2)*((a*x+
b)*x)^(1/2)*x^2*b^2*c^2+8*a^(5/2)*((a*x+b)*x)^(3/2)*b*c*d-ln(1/2*(2*a*x+b+2
*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))*x^2*a^4*b*d^2-3*ln(1/2*(2*a*x+b+2*((a*
x+b)*x)^(1/2)*a^(1/2))/a^(1/2))*x^2*a^2*b^3*c^2+ln(1/2*(2*a*x+b+2*(a*x^2+b*
x)^(1/2)*a^(1/2))/a^(1/2))*x^2*a^4*b*d^2)/a^(5/2)/((a*x+b)*x)^(1/2)/b^2/(a*
x+b)^2
```

maxima [A] time = 1.16, size = 164, normalized size = 1.74

$$\frac{1}{2}\,c^{2}\left(\frac{2\left(3\left(a+\frac{b}{x}\right)b-2\,ab\right)}{\left(a+\frac{b}{x}\right)^{\frac{3}{2}}a^{2}-\sqrt{a+\frac{b}{x}}\,a^{3}}+\frac{3\,b\log\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{a+\frac{b}{x}}+\sqrt{a}}\right)}{a^{\frac{5}{2}}}\right)-2\,cd\left(\frac{\log\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{a+\frac{b}{x}}+\sqrt{a}}\right)}{a^{\frac{3}{2}}}+\frac{2}{\sqrt{a+\frac{b}{x}}\,a}\right)+\frac{2\,d^{2}}{\sqrt{a+\frac{b}{x}}\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d/x)^2/(a+b/x)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/2*c^2*(2*(3*(a + b/x)*b - 2*a*b)/((a + b/x)^(3/2)*a^2 - sqrt(a + b/x)*a^3
) + 3*b*log((sqrt(a + b/x) - sqrt(a))/(sqrt(a + b/x) + sqrt(a)))/a^(5/2)) -
 2*c*d*(log((sqrt(a + b/x) - sqrt(a))/(sqrt(a + b/x) + sqrt(a)))/a^(3/2) +
2/(sqrt(a + b/x)*a)) + 2*d^2/(sqrt(a + b/x)*b)
```

mupad [B] time = 1.83, size = 120, normalized size = 1.28

$$\frac{c\,\mathrm{atanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)(4\,a\,d-3\,b\,c)}{a^{5/2}}-\frac{\frac{2\left(a^{2}\,d^{2}-2\,a\,b\,c\,d+b^{2}\,c^{2}\right)}{a}-\frac{\left(a+\frac{b}{x}\right)\left(2\,a^{2}\,d^{2}-4\,a\,b\,c\,d+3\,b^{2}\,c^{2}\right)}{a^{2}}}{b\left(a+\frac{b}{x}\right)^{3/2}-a\,b\,\sqrt{a+\frac{b}{x}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d/x)^2/(a + b/x)^(3/2),x)
```

```
[Out] (c*atanh((a + b/x)^(1/2)/a^(1/2))*(4*a*d - 3*b*c))/a^(5/2) - ((2*(a^2*d^2 +
 b^2*c^2 - 2*a*b*c*d))/a - ((a + b/x)*(2*a^2*d^2 + 3*b^2*c^2 - 4*a*b*c*d))/
a^2)/(b*(a + b/x)^(3/2) - a*b*(a + b/x)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(cx+d)^{2}}{x^{2}\left(a+\frac{b}{x}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d/x)**2/(a+b/x)**(3/2),x)
```

```
[Out] Integral((c*x + d)**2/(x**2*(a + b/x)**(3/2)), x)
```

3.157 $\int \frac{c+\frac{d}{x}}{\left(a+\frac{b}{x}\right)^{3/2}} \, dx$

Optimal. Leaf size=76

$$-\frac{(3bc-2ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{5/2}}+\frac{3bc-2ad}{a^2\sqrt{a+\frac{b}{x}}}+\frac{cx}{a\sqrt{a+\frac{b}{x}}}$$

Rubi [A] time = 0.05, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {375, 78, 51, 63, 208}

$$\frac{3bc-2ad}{a^2\sqrt{a+\frac{b}{x}}}-\frac{(3bc-2ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{5/2}}+\frac{cx}{a\sqrt{a+\frac{b}{x}}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d/x)/(a + b/x)^(3/2),x]
```

```
[Out] (3*b*c - 2*a*d)/(a^2*Sqrt[a + b/x]) + (c*x)/(a*Sqrt[a + b/x]) - ((3*b*c - 2
*a*d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/a^(5/2)
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 375

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[((a + b/x^n)^p*(c + d/x^n)^q)/x^2, x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{c+\frac{d}{x}}{\left(a+\frac{b}{x}\right)^{3/2}}\,dx &= -\text{Subst}\left(\int \frac{c+dx}{x^2(a+bx)^{3/2}}\,dx, x, \frac{1}{x}\right)\\
&= \frac{cx}{a\sqrt{a+\frac{b}{x}}} - \frac{\left(-\frac{3bc}{2}+ad\right)\text{Subst}\left(\int \frac{1}{x(a+bx)^{3/2}}\,dx, x, \frac{1}{x}\right)}{a}\\
&= \frac{3bc-2ad}{a^2\sqrt{a+\frac{b}{x}}} + \frac{cx}{a\sqrt{a+\frac{b}{x}}} + \frac{(3bc-2ad)\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \frac{1}{x}\right)}{2a^2}\\
&= \frac{3bc-2ad}{a^2\sqrt{a+\frac{b}{x}}} + \frac{cx}{a\sqrt{a+\frac{b}{x}}} + \frac{(3bc-2ad)\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+\frac{b}{x}}\right)}{a^2b}\\
&= \frac{3bc-2ad}{a^2\sqrt{a+\frac{b}{x}}} + \frac{cx}{a\sqrt{a+\frac{b}{x}}} - \frac{(3bc-2ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{5/2}}
\end{aligned}$$

Mathematica [C] time = 0.02, size = 48, normalized size = 0.63

$$\frac{(3bc-2ad)\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{b}{ax}+1\right)+acx}{a^2\sqrt{a+\frac{b}{x}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d/x)/(a + b/x)^(3/2),x]
```

```
[Out] (a*c*x + (3*b*c - 2*a*d)*Hypergeometric2F1[-1/2, 1, 1/2, 1 + b/(a*x)])/(a^2
*Sqrt[a + b/x])
```

IntegrateAlgebraic [A] time = 0.18, size = 77, normalized size = 1.01

$$\frac{(2ad-3bc)\tanh^{-1}\left(\frac{\sqrt{\frac{ax+b}{x}}}{\sqrt{a}}\right)}{a^{5/2}}+\frac{\sqrt{\frac{ax+b}{x}}\left(acx^2-2adx+3bcx\right)}{a^2(ax+b)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c + d/x)/(a + b/x)^(3/2),x]
```

```
[Out] (Sqrt[(b + a*x)/x]*(3*b*c*x - 2*a*d*x + a*c*x^2))/(a^2*(b + a*x)) + ((-3*b*
c + 2*a*d)*ArcTanh[Sqrt[(b + a*x)/x]/Sqrt[a]])/a^(5/2)
```

fricas [A] time = 0.89, size = 210, normalized size = 2.76

$$\left[-\frac{\left(3b^2c-2abd+\left(3abc-2a^2d\right)x\right)\sqrt{a}\log\left(2ax+2\sqrt{a}x\sqrt{\frac{ax+b}{x}}+b\right)-2\left(a^2cx^2+\left(3abc-2a^2d\right)x\right)\sqrt{\frac{ax+b}{x}}}{2\left(a^4x+a^3b\right)}, \frac{\left(3b^2c-2abd+\left(3abc-2a^2d\right)x\right)\sqrt{-a}\arctan\left(\frac{\sqrt{-a}\sqrt{\frac{ax+b}{x}}}{a}\right)+\left(a^2cx^2+\left(3abc-2a^2d\right)x\right)\sqrt{\frac{ax+b}{x}}}{a^4x+a^3b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d/x)/(a+b/x)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/2*((3*b^2*c - 2*a*b*d + (3*a*b*c - 2*a^2*d)*x)*sqrt(a)*log(2*a*x + 2*sq
rt(a)*x*sqrt((a*x + b)/x) + b) - 2*(a^2*c*x^2 + (3*a*b*c - 2*a^2*d)*x)*sqrt
((a*x + b)/x))/(a^4*x + a^3*b), ((3*b^2*c - 2*a*b*d + (3*a*b*c - 2*a^2*d)*x
)*sqrt(-a)*arctan(sqrt(-a)*sqrt((a*x + b)/x)/a) + (a^2*c*x^2 + (3*a*b*c - 2
*a^2*d)*x)*sqrt((a*x + b)/x))/(a^4*x + a^3*b)]
```

giac [A] time = 0.20, size = 127, normalized size = 1.67

$$\frac{\frac{\left(3\,b^{2}c-2\,abd\right)\arctan\left(\frac{\sqrt{\frac{ax+b}{x}}}{\sqrt{-a}}\right)}{\sqrt{-a}\,a^{2}}+\frac{2\,ab^{2}c-2\,a^{2}bd-\frac{3\,(ax+b)b^{2}c}{x}+\frac{2\,(ax+b)abd}{x}}{\left(a\sqrt{\frac{ax+b}{x}}-\frac{(ax+b)\sqrt{\frac{ax+b}{x}}}{x}\right)a^{2}}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d/x)/(a+b/x)^(3/2),x, algorithm="giac")
```

```
[Out] ((3*b^2*c - 2*a*b*d)*arctan(sqrt((a*x + b)/x)/sqrt(-a))/(sqrt(-a)*a^2) + (2
*a*b^2*c - 2*a^2*b*d - 3*(a*x + b)*b^2*c/x + 2*(a*x + b)*a*b*d/x)/((a*sqrt(
(a*x + b)/x) - (a*x + b)*sqrt((a*x + b)/x)/x)*a^2))/b
```

maple [B] time = 0.06, size = 387, normalized size = 5.09

$$\frac{\sqrt{\frac{ax+b}{x}}\left(2a^{3}bd\,x^{2}\ln\left(\frac{2ax+b+2\sqrt{(ax+b)x}\,\sqrt{a}}{2\sqrt{a}}\right)-3a^{2}b^{2}c\,x^{2}\ln\left(\frac{2ax+b+2\sqrt{(ax+b)x}\,\sqrt{a}}{2\sqrt{a}}\right)+4a^{2}b^{2}dx\ln\left(\frac{2ax+b+2\sqrt{(ax+b)x}\,\sqrt{a}}{2\sqrt{a}}\right)-6a\,b^{3}cx\ln\left(\frac{2ax+b+2\sqrt{(ax+b)x}\,\sqrt{a}}{2\sqrt{a}}\right)-4\sqrt{(ax+b)\,x}\,a^{\frac{7}{2}}d\,x^{2}+6\sqrt{(ax+b)\,x}\,a^{\frac{5}{2}}bc\,x^{2}+2a\,b^{3}d\ln\left(\frac{2ax+b+2\sqrt{(ax+b)x}\,\sqrt{a}}{2\sqrt{a}}\right)-3b^{4}c\ln\left(\frac{2ax+b+2\sqrt{(ax+b)x}\,\sqrt{a}}{2\sqrt{a}}\right)-8\sqrt{(ax+b)\,x}\,a^{\frac{5}{2}}bdx+12\sqrt{(ax+b)\,x}\,a^{\frac{3}{2}}b^{2}cx-4\sqrt{(ax+b)\,x}\,a^{\frac{3}{2}}b^{2}d+6\sqrt{(ax+b)\,x}\,\sqrt{a}\,b^{3}c+4\left((ax+b)\,x\right)^{\frac{3}{2}}a^{\frac{5}{2}}d-4\left((ax+b)\,x\right)^{\frac{3}{2}}a^{\frac{3}{2}}bc\right)x}{2\sqrt{(ax+b)\,x}\,\left(ax+b\right)^{2}a^{\frac{5}{2}}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d/x)/(a+b/x)^(3/2),x)
```

```
[Out] 1/2*((a*x+b)/x)^(1/2)*x/a^(5/2)*(2*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1
/2))/a^(1/2))*x^2*a^3*b*d-3*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^
(1/2))*x^2*a^2*b^2*c-4*a^(7/2)*((a*x+b)*x)^(1/2)*x^2*d+6*a^(5/2)*((a*x+b)*x
)^(1/2)*x^2*b*c+4*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))*x*a
^2*b^2*d-6*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))*x*a*b^3*c+
4*a^(5/2)*((a*x+b)*x)^(3/2)*d-4*a^(3/2)*((a*x+b)*x)^(3/2)*b*c-8*a^(5/2)*((a
*x+b)*x)^(1/2)*x*b*d+12*a^(3/2)*((a*x+b)*x)^(1/2)*x*b^2*c+2*ln(1/2*(2*a*x+b
+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))*a*b^3*d-3*ln(1/2*(2*a*x+b+2*((a*x+b)
*x)^(1/2)*a^(1/2))/a^(1/2))*b^4*c-4*a^(3/2)*((a*x+b)*x)^(1/2)*b^2*d+6*a^(1/
2)*((a*x+b)*x)^(1/2)*b^3*c)/((a*x+b)*x)^(1/2)/b/(a*x+b)^2
```

maxima [B] time = 1.37, size = 144, normalized size = 1.89

$$\frac{1}{2}\,c\left(\frac{2\left(3\left(a+\frac{b}{x}\right)b-2\,ab\right)}{\left(a+\frac{b}{x}\right)^{\frac{3}{2}}a^{2}-\sqrt{a+\frac{b}{x}}\,a^{3}}+\frac{3\,b\log\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{a+\frac{b}{x}}+\sqrt{a}}\right)}{a^{\frac{5}{2}}}\right)-d\left(\frac{\log\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{a+\frac{b}{x}}+\sqrt{a}}\right)}{a^{\frac{3}{2}}}+\frac{2}{\sqrt{a+\frac{b}{x}}\,a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d/x)/(a+b/x)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/2*c*(2*(3*(a + b/x)*b - 2*a*b)/((a + b/x)^(3/2)*a^2 - sqrt(a + b/x)*a^3)
+ 3*b*log((sqrt(a + b/x) - sqrt(a))/(sqrt(a + b/x) + sqrt(a)))/a^(5/2)) - d
*(log((sqrt(a + b/x) - sqrt(a))/(sqrt(a + b/x) + sqrt(a)))/a^(3/2) + 2/(sqr
t(a + b/x)*a))
```

mupad [B] time = 2.44, size = 71, normalized size = 0.93

$$\frac{2\,d\,\mathrm{atanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{2\,d}{a\,\sqrt{a+\frac{b}{x}}}+\frac{2\,c\,x\,{\left(\frac{a\,x}{b}+1\right)}^{3/2}\,{}_2\mathrm{F}_1\left(\frac{3}{2},\frac{5}{2};\ \frac{7}{2};\ -\frac{a\,x}{b}\right)}{5\,{\left(a+\frac{b}{x}\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d/x)/(a + b/x)^(3/2),x)
```

```
[Out] (2*d*atanh((a + b/x)^(1/2)/a^(1/2)))/a^(3/2) - (2*d)/(a*(a + b/x)^(1/2)) +
(2*c*x*((a*x)/b + 1)^(3/2)*hypergeom([3/2, 5/2], 7/2, -(a*x)/b))/(5*(a + b/
x)^(3/2))
```

sympy [B] time = 81.34, size = 224, normalized size = 2.95

$$c\left(\frac{x^{\frac{3}{2}}}{a\sqrt{b}\sqrt{\frac{ax}{b}+1}}+\frac{3\sqrt{b}\sqrt{x}}{a^{2}\sqrt{\frac{ax}{b}+1}}-\frac{3b\,\mathrm{asinh}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)}{a^{\frac{5}{2}}}\right)+d\left(-\frac{2a^{3}x\sqrt{1+\frac{b}{ax}}}{a^{\frac{9}{2}}x+a^{\frac{7}{2}}b}-\frac{a^{3}x\log\left(\frac{b}{ax}\right)}{a^{\frac{9}{2}}x+a^{\frac{7}{2}}b}+\frac{2a^{3}x\log\left(\sqrt{1+\frac{b}{ax}}+1\right)}{a^{\frac{9}{2}}x+a^{\frac{7}{2}}b}-\frac{a^{2}b\log\left(\frac{b}{ax}\right)}{a^{\frac{9}{2}}x+a^{\frac{7}{2}}b}+\frac{2a^{2}b\log\left(\sqrt{1+\frac{b}{ax}}+1\right)}{a^{\frac{9}{2}}x+a^{\frac{7}{2}}b}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d/x)/(a+b/x)**(3/2),x)
```

```
[Out] c*(x**(3/2)/(a*sqrt(b)*sqrt(a*x/b + 1)) + 3*sqrt(b)*sqrt(x)/(a**2*sqrt(a*x/
b + 1)) - 3*b*asinh(sqrt(a)*sqrt(x)/sqrt(b))/a**(5/2)) + d*(-2*a**3*x*sqrt(
1 + b/(a*x))/(a**(9/2)*x + a**(7/2)*b) - a**3*x*log(b/(a*x))/(a**(9/2)*x +
a**(7/2)*b) + 2*a**3*x*log(sqrt(1 + b/(a*x)) + 1)/(a**(9/2)*x + a**(7/2)*b)
 - a**2*b*log(b/(a*x))/(a**(9/2)*x + a**(7/2)*b) + 2*a**2*b*log(sqrt(1 + b/
(a*x)) + 1)/(a**(9/2)*x + a**(7/2)*b))
```

3.158 $\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2}}\,dx$

Optimal. Leaf size=60

$$-\frac{3b\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{5/2}}+\frac{3b}{a^2\sqrt{a+\frac{b}{x}}}+\frac{x}{a\sqrt{a+\frac{b}{x}}}$$

Rubi [A] time = 0.03, antiderivative size = 61, normalized size of antiderivative = 1.02, number of steps used = 5, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {242, 51, 63, 208}

$$\frac{3x\sqrt{a+\frac{b}{x}}}{a^2}-\frac{3b\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{5/2}}-\frac{2x}{a\sqrt{a+\frac{b}{x}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x)^(-3/2),x]
```

```
[Out] (-2*x)/(a*Sqrt[a + b/x]) + (3*Sqrt[a + b/x]*x)/a^2 - (3*b*ArcTanh[Sqrt[a +
b/x]/Sqrt[a]])/a^(5/2)
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 242

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a + b/x^n)^p/x^
2, x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2}}\,dx &= -\operatorname{Subst}\left(\int \frac{1}{x^2(a+bx)^{3/2}}\,dx, x, \frac{1}{x}\right)\\
&= -\frac{2x}{a\sqrt{a+\frac{b}{x}}} - \frac{3\operatorname{Subst}\left(\int \frac{1}{x^2\sqrt{a+bx}}\,dx, x, \frac{1}{x}\right)}{a}\\
&= -\frac{2x}{a\sqrt{a+\frac{b}{x}}} + \frac{3\sqrt{a+\frac{b}{x}}\,x}{a^2} + \frac{(3b)\operatorname{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \frac{1}{x}\right)}{2a^2}\\
&= -\frac{2x}{a\sqrt{a+\frac{b}{x}}} + \frac{3\sqrt{a+\frac{b}{x}}\,x}{a^2} + \frac{3\operatorname{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+\frac{b}{x}}\right)}{a^2}\\
&= -\frac{2x}{a\sqrt{a+\frac{b}{x}}} + \frac{3\sqrt{a+\frac{b}{x}}\,x}{a^2} - \frac{3b\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{5/2}}
\end{aligned}
$$

Mathematica [C] time = 0.01, size = 36, normalized size = 0.60

$$
\frac{2b\,{}_2F_1\left(-\frac{1}{2},2;\frac{1}{2};\frac{a+\frac{b}{x}}{a}\right)}{a^2\sqrt{a+\frac{b}{x}}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x)^(-3/2),x]
[Out] (2*b*Hypergeometric2F1[-1/2, 2, 1/2, (a + b/x)/a])/(a^2*Sqrt[a + b/x])
```

IntegrateAlgebraic [A] time = 0.00, size = 63, normalized size = 1.05

$$
\frac{\sqrt{\frac{ax+b}{x}}\left(ax^2+3bx\right)}{a^2(ax+b)} - \frac{3b\tanh^{-1}\left(\frac{\sqrt{\frac{ax+b}{x}}}{\sqrt{a}}\right)}{a^{5/2}}
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b/x)^(-3/2),x]
[Out] (Sqrt[(b + a*x)/x]*(3*b*x + a*x^2))/(a^2*(b + a*x)) - (3*b*ArcTanh[Sqrt[(b
+ a*x)/x]/Sqrt[a]])/a^(5/2)
```

fricas [A] time = 0.83, size = 156, normalized size = 2.60

$$
\left[\frac{3\left(abx+b^2\right)\sqrt{a}\log\left(2\,ax-2\,\sqrt{a}\,x\sqrt{\frac{ax+b}{x}}+b\right)+2\left(a^2x^2+3\,abx\right)\sqrt{\frac{ax+b}{x}}}{2\left(a^4x+a^3b\right)}, \frac{3\left(abx+b^2\right)\sqrt{-a}\arctan\left(\frac{\sqrt{-a}\sqrt{\frac{ax+b}{x}}}{a}\right)+\left(a^2x^2+3\,abx\right)\sqrt{\frac{ax+b}{x}}}{a^4x+a^3b}\right]
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*(3*(a*b*x + b^2)*sqrt(a)*log(2*a*x - 2*sqrt(a)*x*sqrt((a*x + b)/x) + b
) + 2*(a^2*x^2 + 3*a*b*x)*sqrt((a*x + b)/x))/(a^4*x + a^3*b), (3*(a*b*x + b
^2)*sqrt(-a)*arctan(sqrt(-a)*sqrt((a*x + b)/x)/a) + (a^2*x^2 + 3*a*b*x)*sqr
t((a*x + b)/x))/(a^4*x + a^3*b)]
```

giac [A] time = 0.16, size = 86, normalized size = 1.43

$$b\left(\frac{3\,\arctan\left(\frac{\sqrt{\frac{ax+b}{x}}}{\sqrt{-a}}\right)}{\sqrt{-a}\,a^2}+\frac{2\,a-\frac{3\,(ax+b)}{x}}{\left(a\sqrt{\frac{ax+b}{x}}-\frac{(ax+b)\sqrt{\frac{ax+b}{x}}}{x}\right)a^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)^(3/2),x, algorithm="giac")
```

```
[Out] b*(3*arctan(sqrt((a*x + b)/x)/sqrt(-a))/(sqrt(-a)*a^2) + (2*a - 3*(a*x + b)
/x)/((a*sqrt((a*x + b)/x) - (a*x + b)*sqrt((a*x + b)/x)/x)*a^2))
```

maple [B] time = 0.06, size = 198, normalized size = 3.30

$$\frac{\sqrt{\frac{ax+b}{x}}\left(-3a^2b\,x^2\ln\left(\frac{2ax+b+2\sqrt{(ax+b)x}\,\sqrt{a}}{2\sqrt{a}}\right)-6a\,b^2x\ln\left(\frac{2ax+b+2\sqrt{(ax+b)x}\,\sqrt{a}}{2\sqrt{a}}\right)+6\sqrt{(ax+b)\,x}\,a^{\frac{5}{2}}x^2-3b^3\ln\left(\frac{2ax+b+2\sqrt{(ax+b)x}\,\sqrt{a}}{2\sqrt{a}}\right)+12\sqrt{(ax+b)\,x}\,a^{\frac{3}{2}}bx+6\sqrt{(ax+b)\,x}\,\sqrt{a}\,b^2-4\left((ax+b)\,x\right)^{\frac{3}{2}}a^{\frac{3}{2}}\right)x}{2\sqrt{(ax+b)\,x}\,\left(ax+b\right)^2a^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b/x)^(3/2),x)
```

```
[Out] 1/2*((a*x+b)/x)^(1/2)*x*(6*a^(5/2)*((a*x+b)*x)^(1/2)*x^2-4*a^(3/2)*((a*x+b)
*x)^(3/2)+12*a^(3/2)*((a*x+b)*x)^(1/2)*x*b-3*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^
(1/2)*a^(1/2))/a^(1/2))*x^2*a^2*b-6*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(
1/2))/a^(1/2))*x*a*b^2+6*a^(1/2)*((a*x+b)*x)^(1/2)*b^2-3*ln(1/2*(2*a*x+b+2*
((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))*b^3)/a^(5/2)/((a*x+b)*x)^(1/2)/(a*x+b)^
2
```

maxima [A] time = 1.18, size = 85, normalized size = 1.42

$$\frac{3\left(a+\frac{b}{x}\right)b-2\,ab}{\left(a+\frac{b}{x}\right)^{\frac{3}{2}}a^2-\sqrt{a+\frac{b}{x}}\,a^3}+\frac{3\,b\log\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{a+\frac{b}{x}}+\sqrt{a}}\right)}{2\,a^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)^(3/2),x, algorithm="maxima")
```

```
[Out] (3*(a + b/x)*b - 2*a*b)/((a + b/x)^(3/2)*a^2 - sqrt(a + b/x)*a^3) + 3/2*b*l
og((sqrt(a + b/x) - sqrt(a))/(sqrt(a + b/x) + sqrt(a)))/a^(5/2)
```

mupad [B] time = 1.87, size = 34, normalized size = 0.57

$$\frac{2\,x\,{\left(\frac{a\,x}{b}+1\right)}^{3/2}\,{{}}_2{\mathrm{F}}_1\left(\frac{3}{2},\frac{5}{2};\ \frac{7}{2};\ -\frac{a\,x}{b}\right)}{5\,{\left(a+\frac{b}{x}\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b/x)^(3/2),x)
```

```
[Out] (2*x*((a*x)/b + 1)^(3/2)*hypergeom([3/2, 5/2], 7/2, -(a*x)/b))/(5*(a + b/x)
^(3/2))
```

sympy [A] time = 4.79, size = 71, normalized size = 1.18

$$\frac{x^{\frac{3}{2}}}{a\sqrt{b}\sqrt{\frac{ax}{b}+1}}+\frac{3\sqrt{b}\sqrt{x}}{a^{2}\sqrt{\frac{ax}{b}+1}}-\frac{3b\operatorname{asinh}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)}{a^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)**(3/2),x)
```

```
[Out] x**(3/2)/(a*sqrt(b)*sqrt(a*x/b + 1)) + 3*sqrt(b)*sqrt(x)/(a**2*sqrt(a*x/b +
 1)) - 3*b*asinh(sqrt(a)*sqrt(x)/sqrt(b))/a**(5/2)
```

3.159 $\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)}\,dx$

Optimal. Leaf size=147

$$-\frac{(2ad+3bc)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{5/2}c^2}+\frac{b(3bc-ad)}{a^2c\sqrt{a+\frac{b}{x}}\,(bc-ad)}+\frac{2d^{5/2}\tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c^2(bc-ad)^{3/2}}+\frac{x}{ac\sqrt{a+\frac{b}{x}}}$$

Rubi [A] time = 0.19, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {375, 103, 152, 156, 63, 208, 205}

$$-\frac{(2ad+3bc)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{5/2}c^2}+\frac{b(3bc-ad)}{a^2c\sqrt{a+\frac{b}{x}}\,(bc-ad)}+\frac{2d^{5/2}\tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c^2(bc-ad)^{3/2}}+\frac{x}{ac\sqrt{a+\frac{b}{x}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b/x)^(3/2)*(c + d/x)),x]
```

```
[Out] (b*(3*b*c - a*d))/(a^2*c*(b*c - a*d)*Sqrt[a + b/x]) + x/(a*c*Sqrt[a + b/x])
 + (2*d^(5/2)*ArcTan[(Sqrt[d]*Sqrt[a + b/x])/Sqrt[b*c - a*d]])/(c^2*(b*c -
a*d)^(3/2)) - ((3*b*c + 2*a*d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/(a^(5/2)*c^2
)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 375

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[((a + b/x^n)^p*(c + d/x^n)^q)/x^2, x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rubi steps

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)}\,dx = -\text{Subst}\left(\int \frac{1}{x^2(a+bx)^{3/2}(c+dx)}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{x}{ac\sqrt{a+\frac{b}{x}}} + \frac{\text{Subst}\left(\int \frac{\frac{1}{2}(3bc+2ad)+\frac{3bdx}{2}}{x(a+bx)^{3/2}(c+dx)}\,dx, x, \frac{1}{x}\right)}{ac}$$

$$= \frac{b(3bc-ad)}{a^2c(bc-ad)\sqrt{a+\frac{b}{x}}} + \frac{x}{ac\sqrt{a+\frac{b}{x}}} + \frac{2\,\text{Subst}\left(\int \frac{\frac{1}{4}(bc-ad)(3bc+2ad)+\frac{1}{4}bd(3bc-ad)x}{x\sqrt{a+bx}\,(c+dx)}\,dx, x, \frac{1}{x}\right)}{a^2c(bc-ad)}$$

$$= \frac{b(3bc-ad)}{a^2c(bc-ad)\sqrt{a+\frac{b}{x}}} + \frac{x}{ac\sqrt{a+\frac{b}{x}}} + \frac{d^3\,\text{Subst}\left(\int \frac{1}{\sqrt{a+bx}\,(c+dx)}\,dx, x, \frac{1}{x}\right)}{c^2(bc-ad)} + \frac{(3bc+2ad}{}$$

$$= \frac{b(3bc-ad)}{a^2c(bc-ad)\sqrt{a+\frac{b}{x}}} + \frac{x}{ac\sqrt{a+\frac{b}{x}}} + \frac{\left(2d^3\right)\text{Subst}\left(\int \frac{1}{c-\frac{ad}{b}+\frac{dx^2}{b}}\,dx, x, \sqrt{a+\frac{b}{x}}\right)}{bc^2(bc-ad)} + \frac{(3b}{}$$

$$= \frac{b(3bc-ad)}{a^2c(bc-ad)\sqrt{a+\frac{b}{x}}} + \frac{x}{ac\sqrt{a+\frac{b}{x}}} + \frac{2d^{5/2}\tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c^2(bc-ad)^{3/2}} - \frac{(3bc+2ad)\tanh^{-1}\left(\right.}{a^{5/2}c^2}$$

Mathematica [C] time = 0.07, size = 106, normalized size = 0.72

$$\frac{(ad-bc)\left((2ad+3bc)\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{b}{ax}+1\right)+acx\right)-2a^2d^2\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{d\left(a+\frac{b}{x}\right)}{ad-bc}\right)}{a^2c^2\sqrt{a+\frac{b}{x}}\,(ad-bc)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b/x)^(3/2)*(c + d/x)),x]
```

```
[Out] (-2*a^2*d^2*Hypergeometric2F1[-1/2, 1, 1/2, (d*(a + b/x))/(-(b*c) + a*d)] +
 (-(b*c) + a*d)*(a*c*x + (3*b*c + 2*a*d)*Hypergeometric2F1[-1/2, 1, 1/2, 1
+ b/(a*x)]))/(a^2*c^2*(-(b*c) + a*d)*Sqrt[a + b/x])
```

IntegrateAlgebraic [A] time = 0.34, size = 159, normalized size = 1.08

$$\frac{(-2ad-3bc)\tanh^{-1}\left(\frac{\sqrt{\frac{ax+b}{x}}}{\sqrt{a}}\right)}{a^{5/2}c^2}+\frac{\sqrt{\frac{ax+b}{x}}\left(a^2dx^2-abcx^2+abdx-3b^2cx\right)}{a^2c(ax+b)(ad-bc)}+\frac{2d^{5/2}\tan^{-1}\left(\frac{\sqrt{d}\sqrt{\frac{ax+b}{x}}}{\sqrt{bc-ad}}\right)}{c^2(bc-ad)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((a + b/x)^(3/2)*(c + d/x)),x]
```

```
[Out] (Sqrt[(b + a*x)/x]*(-3*b^2*c*x + a*b*d*x - a*b*c*x^2 + a^2*d*x^2))/(a^2*c*(
-(b*c) + a*d)*(b + a*x)) + (2*d^(5/2)*ArcTan[(Sqrt[d]*Sqrt[(b + a*x)/x])/Sq
rt[b*c - a*d]])/(c^2*(b*c - a*d)^(3/2)) + ((-3*b*c - 2*a*d)*ArcTanh[Sqrt[(b
 + a*x)/x]/Sqrt[a]])/(a^(5/2)*c^2)
```

fricas [B] time = 1.32, size = 1075, normalized size = 7.31

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)^(3/2)/(c+d/x),x, algorithm="fricas")
```

```
[Out] [1/2*((3*b^3*c^2 - a*b^2*c*d - 2*a^2*b*d^2 + (3*a*b^2*c^2 - a^2*b*c*d - 2*a
^3*d^2)*x)*sqrt(a)*log(2*a*x - 2*sqrt(a)*x*sqrt((a*x + b)/x) + b) - 2*(a^4*
d^2*x + a^3*b*d^2)*sqrt(-d/(b*c - a*d))*log(-(2*(b*c - a*d)*x*sqrt(-d/(b*c
- a*d))*sqrt((a*x + b)/x) - b*d + (b*c - 2*a*d)*x)/(c*x + d)) + 2*((a^2*b*c
^2 - a^3*c*d)*x^2 + (3*a*b^2*c^2 - a^2*b*c*d)*x)*sqrt((a*x + b)/x))/(a^3*b^
2*c^3 - a^4*b*c^2*d + (a^4*b*c^3 - a^5*c^2*d)*x), ((3*b^3*c^2 - a*b^2*c*d -
 2*a^2*b*d^2 + (3*a*b^2*c^2 - a^2*b*c*d - 2*a^3*d^2)*x)*sqrt(-a)*arctan(sqr
t(-a)*sqrt((a*x + b)/x)/a) - (a^4*d^2*x + a^3*b*d^2)*sqrt(-d/(b*c - a*d))*l
og(-(2*(b*c - a*d)*x*sqrt(-d/(b*c - a*d))*sqrt((a*x + b)/x) - b*d + (b*c -
2*a*d)*x)/(c*x + d)) + ((a^2*b*c^2 - a^3*c*d)*x^2 + (3*a*b^2*c^2 - a^2*b*c*
d)*x)*sqrt((a*x + b)/x))/(a^3*b^2*c^3 - a^4*b*c^2*d + (a^4*b*c^3 - a^5*c^2*
d)*x), 1/2*(4*(a^4*d^2*x + a^3*b*d^2)*sqrt(d/(b*c - a*d))*arctan(-(b*c - a*
d)*x*sqrt(d/(b*c - a*d))*sqrt((a*x + b)/x)/(a*d*x + b*d)) + (3*b^3*c^2 - a*
b^2*c*d - 2*a^2*b*d^2 + (3*a*b^2*c^2 - a^2*b*c*d - 2*a^3*d^2)*x)*sqrt(a)*lo
g(2*a*x - 2*sqrt(a)*x*sqrt((a*x + b)/x) + b) + 2*((a^2*b*c^2 - a^3*c*d)*x^2
 + (3*a*b^2*c^2 - a^2*b*c*d)*x)*sqrt((a*x + b)/x))/(a^3*b^2*c^3 - a^4*b*c^2
*d + (a^4*b*c^3 - a^5*c^2*d)*x), (2*(a^4*d^2*x + a^3*b*d^2)*sqrt(d/(b*c - a
*d))*arctan(-(b*c - a*d)*x*sqrt(d/(b*c - a*d))*sqrt((a*x + b)/x)/(a*d*x + b
*d)) + (3*b^3*c^2 - a*b^2*c*d - 2*a^2*b*d^2 + (3*a*b^2*c^2 - a^2*b*c*d - 2*
a^3*d^2)*x)*sqrt(-a)*arctan(sqrt(-a)*sqrt((a*x + b)/x)/a) + ((a^2*b*c^2 - a
^3*c*d)*x^2 + (3*a*b^2*c^2 - a^2*b*c*d)*x)*sqrt((a*x + b)/x))/(a^3*b^2*c^3
- a^4*b*c^2*d + (a^4*b*c^3 - a^5*c^2*d)*x)]
```

giac [A] time = 0.21, size = 200, normalized size = 1.36

$$\left(\frac{2\,d^3\arctan\left(\frac{d\sqrt{\frac{ax+b}{x}}}{\sqrt{bcd-ad^2}}\right)}{\left(b^3c^3-ab^2c^2d\right)\sqrt{bcd-ad^2}}+\frac{2\,abc-\frac{3\,(ax+b)bc}{x}+\frac{(ax+b)ad}{x}}{\left(a^2b^2c^2-a^3bcd\right)\left(a\sqrt{\frac{ax+b}{x}}-\frac{(ax+b)\sqrt{\frac{ax+b}{x}}}{x}\right)}+\frac{(3\,bc+2\,ad)\arctan\left(\frac{\sqrt{\frac{ax+b}{x}}}{\sqrt{-a}}\right)}{\sqrt{-a}\,a^2b^2c^2}\right)b^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)^(3/2)/(c+d/x),x, algorithm="giac")
```

```
[Out] (2*d^3*arctan(d*sqrt((a*x + b)/x)/sqrt(b*c*d - a*d^2))/((b^3*c^3 - a*b^2*c^
2*d)*sqrt(b*c*d - a*d^2)) + (2*a*b*c - 3*(a*x + b)*b*c/x + (a*x + b)*a*d/x)
/((a^2*b^2*c^2 - a^3*b*c*d)*(a*sqrt((a*x + b)/x) - (a*x + b)*sqrt((a*x + b)
/x)/x)) + (3*b*c + 2*a*d)*arctan(sqrt((a*x + b)/x)/sqrt(-a))/(sqrt(-a)*a^2*
b^2*c^2))*b^2
```

maple [B] time = 0.07, size = 962, normalized size = 6.54

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b/x)^(3/2)/(c+d/x),x)
```

```
[Out] -1/2*(2*a^(9/2)*ln((-2*a*d*x+b*c*x-b*d+2*((a*d-b*c)/c^2*d)^(1/2)*((a*x+b)*x
)^(1/2)*c)/(c*x+d))*x^2*d^3-2*a^(7/2)*((a*d-b*c)/c^2*d)^(1/2)*((a*x+b)*x)^(
1/2)*x^2*c^2*d+4*a^(7/2)*ln((-2*a*d*x+b*c*x-b*d+2*((a*d-b*c)/c^2*d)^(1/2)*(
(a*x+b)*x)^(1/2)*c)/(c*x+d))*x*b*d^3+6*a^(5/2)*((a*d-b*c)/c^2*d)^(1/2)*((a*
x+b)*x)^(1/2)*x^2*b*c^3-4*a^(5/2)*((a*d-b*c)/c^2*d)^(1/2)*((a*x+b)*x)^(1/2)
*x*b*c^2*d+2*a^(5/2)*ln((-2*a*d*x+b*c*x-b*d+2*((a*d-b*c)/c^2*d)^(1/2)*((a*x
+b)*x)^(1/2)*c)/(c*x+d))*b^2*d^3-4*a^(3/2)*((a*d-b*c)/c^2*d)^(1/2)*((a*x+b)
*x)^(3/2)*b*c^3+12*a^(3/2)*((a*d-b*c)/c^2*d)^(1/2)*((a*x+b)*x)^(1/2)*x*b^2*
c^3+2*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))*((a*d-b*c)/c^2*
d)^(1/2)*x^2*a^4*c*d^2+ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2)
)*((a*d-b*c)/c^2*d)^(1/2)*x^2*a^3*b*c^2*d-3*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(
1/2)*a^(1/2))/a^(1/2))*((a*d-b*c)/c^2*d)^(1/2)*x^2*a^2*b^2*c^3-2*a^(3/2)*((
a*d-b*c)/c^2*d)^(1/2)*((a*x+b)*x)^(1/2)*b^2*c^2*d+4*ln(1/2*(2*a*x+b+2*((a*x
+b)*x)^(1/2)*a^(1/2))/a^(1/2))*((a*d-b*c)/c^2*d)^(1/2)*x*a^3*b*c*d^2+2*ln(1
/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))*((a*d-b*c)/c^2*d)^(1/2)*x
*a^2*b^2*c^2*d-6*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))*((a*
d-b*c)/c^2*d)^(1/2)*x*a*b^3*c^3+6*a^(1/2)*((a*d-b*c)/c^2*d)^(1/2)*((a*x+b)*
x)^(1/2)*b^3*c^3+2*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))*((
a*d-b*c)/c^2*d)^(1/2)*a^2*b^2*c*d^2+ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(
1/2))/a^(1/2))*((a*d-b*c)/c^2*d)^(1/2)*a*b^3*c^2*d-3*ln(1/2*(2*a*x+b+2*((a*
x+b)*x)^(1/2)*a^(1/2))/a^(1/2))*((a*d-b*c)/c^2*d)^(1/2)*b^4*c^3)*x*((a*x+b)
/x)^(1/2)/a^(5/2)/(a*x+b)^2/((a*d-b*c)/c^2*d)^(1/2)/c^3/(a*d-b*c)/((a*x+b)*
x)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a + \frac{b}{x}\right)^{\frac{3}{2}} \left(c + \frac{d}{x}\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)^(3/2)/(c+d/x),x, algorithm="maxima")
```

```
[Out] integrate(1/((a + b/x)^(3/2)*(c + d/x)), x)
```

mupad [B] time = 2.68, size = 3000, normalized size = 20.41

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b/x)^(3/2)*(c + d/x)),x)
```

```
[Out] (atan((((d^5*(a*d - b*c)^3)^(1/2)*((a + b/x)^(1/2)*(18*a^6*b^9*c^10*d^3 - 6
6*a^7*b^8*c^9*d^4 + 68*a^8*b^7*c^8*d^5 + 20*a^9*b^6*c^7*d^6 - 62*a^10*b^5*c
^6*d^7 - 2*a^11*b^4*c^5*d^8 + 40*a^12*b^3*c^4*d^9 - 16*a^13*b^2*c^3*d^10) +
```

((d^5*(a*d - b*c)^3)^(1/2)*(64*a^9*b^8*c^11*d^3 - 12*a^8*b^9*c^12*d^2 - 13
2*a^10*b^7*c^10*d^4 + 128*a^11*b^6*c^9*d^5 - 52*a^12*b^5*c^8*d^6 + 4*a^14*b
^3*c^6*d^8 + ((d^5*(a*d - b*c)^3)^(1/2)*(a + b/x)^(1/2)*(8*a^10*b^8*c^13*d^
2 - 56*a^11*b^7*c^12*d^3 + 160*a^12*b^6*c^11*d^4 - 240*a^13*b^5*c^10*d^5 +
200*a^14*b^4*c^9*d^6 - 88*a^15*b^3*c^8*d^7 + 16*a^16*b^2*c^7*d^8))/(c^2*(a*
d - b*c)^3)))/(c^2*(a*d - b*c)^3))*1i)/(c^2*(a*d - b*c)^3) + ((d^5*(a*d - b
c)^3)^(1/2)((a + b/x)^(1/2)*(18*a^6*b^9*c^10*d^3 - 66*a^7*b^8*c^9*d^4 + 6
8*a^8*b^7*c^8*d^5 + 20*a^9*b^6*c^7*d^6 - 62*a^10*b^5*c^6*d^7 - 2*a^11*b^4*c
^5*d^8 + 40*a^12*b^3*c^4*d^9 - 16*a^13*b^2*c^3*d^10) + ((d^5*(a*d - b*c)^3)
^(1/2)*(12*a^8*b^9*c^12*d^2 - 64*a^9*b^8*c^11*d^3 + 132*a^10*b^7*c^10*d^4 -
128*a^11*b^6*c^9*d^5 + 52*a^12*b^5*c^8*d^6 - 4*a^14*b^3*c^6*d^8 + ((d^5*(a
*d - b*c)^3)^(1/2)*(a + b/x)^(1/2)*(8*a^10*b^8*c^13*d^2 - 56*a^11*b^7*c^12*
d^3 + 160*a^12*b^6*c^11*d^4 - 240*a^13*b^5*c^10*d^5 + 200*a^14*b^4*c^9*d^6
- 88*a^15*b^3*c^8*d^7 + 16*a^16*b^2*c^7*d^8))/(c^2*(a*d - b*c)^3)))/(c^2*(a
*d - b*c)^3))*1i)/(c^2*(a*d - b*c)^3))/(36*a^6*b^8*c^7*d^5 - 96*a^7*b^7*c^6
*d^6 + 64*a^8*b^6*c^5*d^7 + 24*a^9*b^5*c^4*d^8 - 36*a^10*b^4*c^3*d^9 + 8*a^
11*b^3*c^2*d^10 - ((d^5*(a*d - b*c)^3)^(1/2)*((a + b/x)^(1/2)*(18*a^6*b^9*c
^10*d^3 - 66*a^7*b^8*c^9*d^4 + 68*a^8*b^7*c^8*d^5 + 20*a^9*b^6*c^7*d^6 - 62
*a^10*b^5*c^6*d^7 - 2*a^11*b^4*c^5*d^8 + 40*a^12*b^3*c^4*d^9 - 16*a^13*b^2*
c^3*d^10) + ((d^5*(a*d - b*c)^3)^(1/2)*(64*a^9*b^8*c^11*d^3 - 12*a^8*b^9*c^
12*d^2 - 132*a^10*b^7*c^10*d^4 + 128*a^11*b^6*c^9*d^5 - 52*a^12*b^5*c^8*d^6
+ 4*a^14*b^3*c^6*d^8 + ((d^5*(a*d - b*c)^3)^(1/2)*(a + b/x)^(1/2)*(8*a^10*
b^8*c^13*d^2 - 56*a^11*b^7*c^12*d^3 + 160*a^12*b^6*c^11*d^4 - 240*a^13*b^5*
c^10*d^5 + 200*a^14*b^4*c^9*d^6 - 88*a^15*b^3*c^8*d^7 + 16*a^16*b^2*c^7*d^8
))/(c^2*(a*d - b*c)^3)))/(c^2*(a*d - b*c)^3)))/(c^2*(a*d - b*c)^3) + ((d^5*
(a*d - b*c)^3)^(1/2)*((a + b/x)^(1/2)*(18*a^6*b^9*c^10*d^3 - 66*a^7*b^8*c^9
*d^4 + 68*a^8*b^7*c^8*d^5 + 20*a^9*b^6*c^7*d^6 - 62*a^10*b^5*c^6*d^7 - 2*a^
11*b^4*c^5*d^8 + 40*a^12*b^3*c^4*d^9 - 16*a^13*b^2*c^3*d^10) + ((d^5*(a*d -
b*c)^3)^(1/2)*(12*a^8*b^9*c^12*d^2 - 64*a^9*b^8*c^11*d^3 + 132*a^10*b^7*c^
10*d^4 - 128*a^11*b^6*c^9*d^5 + 52*a^12*b^5*c^8*d^6 - 4*a^14*b^3*c^6*d^8 +
((d^5*(a*d - b*c)^3)^(1/2)*(a + b/x)^(1/2)*(8*a^10*b^8*c^13*d^2 - 56*a^11*b
^7*c^12*d^3 + 160*a^12*b^6*c^11*d^4 - 240*a^13*b^5*c^10*d^5 + 200*a^14*b^4*
c^9*d^6 - 88*a^15*b^3*c^8*d^7 + 16*a^16*b^2*c^7*d^8))/(c^2*(a*d - b*c)^3)))
/(c^2*(a*d - b*c)^3)))/(c^2*(a*d - b*c)^3)))*(d^5*(a*d - b*c)^3)^(1/2)*2i)/
(c^2*(a*d - b*c)^3) - (atanh((54*a^5*b^11*c^10*d^2*(a + b/x)^(1/2))/((a^5)^
(1/2)*(54*a^3*b^11*c^10*d^2 - 216*a^4*b^10*c^9*d^3 + 234*a^5*b^9*c^8*d^4 +
124*a^6*b^8*c^7*d^5 - 366*a^7*b^7*c^6*d^6 + 120*a^8*b^6*c^5*d^7 + 110*a^9*b
^5*c^4*d^8 - 60*a^10*b^4*c^3*d^9)) - (216*a^6*b^10*c^9*d^3*(a + b/x)^(1/2))
/((a^5)^(1/2)*(54*a^3*b^11*c^10*d^2 - 216*a^4*b^10*c^9*d^3 + 234*a^5*b^9*c^
8*d^4 + 124*a^6*b^8*c^7*d^5 - 366*a^7*b^7*c^6*d^6 + 120*a^8*b^6*c^5*d^7 + 1
10*a^9*b^5*c^4*d^8 - 60*a^10*b^4*c^3*d^9)) + (234*a^7*b^9*c^8*d^4*(a + b/x)
^(1/2))/((a^5)^(1/2)*(54*a^3*b^11*c^10*d^2 - 216*a^4*b^10*c^9*d^3 + 234*a^5
*b^9*c^8*d^4 + 124*a^6*b^8*c^7*d^5 - 366*a^7*b^7*c^6*d^6 + 120*a^8*b^6*c^5*
d^7 + 110*a^9*b^5*c^4*d^8 - 60*a^10*b^4*c^3*d^9)) + (124*a^8*b^8*c^7*d^5*(a
+ b/x)^(1/2))/((a^5)^(1/2)*(54*a^3*b^11*c^10*d^2 - 216*a^4*b^10*c^9*d^3 +
234*a^5*b^9*c^8*d^4 + 124*a^6*b^8*c^7*d^5 - 366*a^7*b^7*c^6*d^6 + 120*a^8*b
^6*c^5*d^7 + 110*a^9*b^5*c^4*d^8 - 60*a^10*b^4*c^3*d^9)) - (366*a^9*b^7*c^6
d^6(a + b/x)^(1/2))/((a^5)^(1/2)*(54*a^3*b^11*c^10*d^2 - 216*a^4*b^10*c^9
*d^3 + 234*a^5*b^9*c^8*d^4 + 124*a^6*b^8*c^7*d^5 - 366*a^7*b^7*c^6*d^6 + 12
0*a^8*b^6*c^5*d^7 + 110*a^9*b^5*c^4*d^8 - 60*a^10*b^4*c^3*d^9)) + (120*a^10
*b^6*c^5*d^7*(a + b/x)^(1/2))/((a^5)^(1/2)*(54*a^3*b^11*c^10*d^2 - 216*a^4*
b^10*c^9*d^3 + 234*a^5*b^9*c^8*d^4 + 124*a^6*b^8*c^7*d^5 - 366*a^7*b^7*c^6*
d^6 + 120*a^8*b^6*c^5*d^7 + 110*a^9*b^5*c^4*d^8 - 60*a^10*b^4*c^3*d^9)) + (
110*a^11*b^5*c^4*d^8*(a + b/x)^(1/2))/((a^5)^(1/2)*(54*a^3*b^11*c^10*d^2 -
216*a^4*b^10*c^9*d^3 + 234*a^5*b^9*c^8*d^4 + 124*a^6*b^8*c^7*d^5 - 366*a^7*
b^7*c^6*d^6 + 120*a^8*b^6*c^5*d^7 + 110*a^9*b^5*c^4*d^8 - 60*a^10*b^4*c^3*d
^9)) - (60*a^12*b^4*c^3*d^9*(a + b/x)^(1/2))/((a^5)^(1/2)*(54*a^3*b^11*c^10
*d^2 - 216*a^4*b^10*c^9*d^3 + 234*a^5*b^9*c^8*d^4 + 124*a^6*b^8*c^7*d^5 - 3
66*a^7*b^7*c^6*d^6 + 120*a^8*b^6*c^5*d^7 + 110*a^9*b^5*c^4*d^8 - 60*a^10*b^

```
4*c^3*d^9)))*(2*a*d + 3*b*c))/(c^2*(a^5)^(1/2)) - ((2*b^2)/(a^2*d - a*b*c)
+ (b*(a + b/x)*(a*d - 3*b*c))/(a^2*c*(a*d - b*c)))/(a*(a + b/x)^(1/2) - (a
+ b/x)^(3/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(a + \frac{b}{x}\right)^{\frac{3}{2}} (cx + d)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)**(3/2)/(c+d/x),x)
```

```
[Out] Integral(x/((a + b/x)**(3/2)*(c*x + d)), x)
```

3.160 $\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)^2}\,dx$

Optimal. Leaf size=224

$$-\frac{(4ad+3bc)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{5/2}c^3}+\frac{b\left(2a^2d^2-2abcd+3b^2c^2\right)}{a^2c^2\sqrt{a+\frac{b}{x}}\,(bc-ad)^2}+\frac{d^{5/2}(7bc-4ad)\tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c^3(bc-ad)^{5/2}}+\frac{d(bc-2ad}{ac^2\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)}$$

Rubi [A] time = 0.32, antiderivative size = 224, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {375, 103, 151, 152, 156, 63, 208, 205}

$$\frac{b\left(2a^2d^2-2abcd+3b^2c^2\right)}{a^2c^2\sqrt{a+\frac{b}{x}}\,(bc-ad)^2}-\frac{(4ad+3bc)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{5/2}c^3}+\frac{d^{5/2}(7bc-4ad)\tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c^3(bc-ad)^{5/2}}+\frac{d(bc-2ad)}{ac^2\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)(bc-ad)}+\frac{x}{ac\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b/x)^(3/2)*(c + d/x)^2),x]
```

```
[Out] (b*(3*b^2*c^2 - 2*a*b*c*d + 2*a^2*d^2))/(a^2*c^2*(b*c - a*d)^2*Sqrt[a + b/x
]) + (d*(b*c - 2*a*d))/(a*c^2*(b*c - a*d)*Sqrt[a + b/x]*(c + d/x)) + x/(a*c
*Sqrt[a + b/x]*(c + d/x)) + (d^(5/2)*(7*b*c - 4*a*d)*ArcTan[(Sqrt[d]*Sqrt[a
 + b/x])/Sqrt[b*c - a*d]])/(c^3*(b*c - a*d)^(5/2)) - ((3*b*c + 4*a*d)*ArcTa
nh[Sqrt[a + b/x]/Sqrt[a]])/(a^(5/2)*c^3)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 375

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[((a + b/x^n)^p*(c + d/x^n)^q)/x^2, x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rubi steps

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)^2}\,dx = -\text{Subst}\left(\int \frac{1}{x^2(a+bx)^{3/2}(c+dx)^2}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{x}{ac\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)} + \frac{\text{Subst}\left(\int \frac{\frac{1}{2}(3bc+4ad)+\frac{5bdx}{2}}{x(a+bx)^{3/2}(c+dx)^2}\,dx, x, \frac{1}{x}\right)}{ac}$$

$$= \frac{d(bc-2ad)}{ac^2(bc-ad)\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)} + \frac{x}{ac\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)} - \frac{\text{Subst}\left(\int \frac{-\frac{1}{2}(bc-ad)(3bc+4ad)-\frac{3}{2}bd(bc-}{x(a+bx)^{3/2}(c+dx)}\right.}{ac^2(bc-ad)}$$

$$= \frac{b\left(3b^2c^2-2abcd+2a^2d^2\right)}{a^2c^2(bc-ad)^2\sqrt{a+\frac{b}{x}}} + \frac{d(bc-2ad)}{ac^2(bc-ad)\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)} + \frac{x}{ac\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)} - \frac{2\text{S}}{}$$

$$= \frac{b\left(3b^2c^2-2abcd+2a^2d^2\right)}{a^2c^2(bc-ad)^2\sqrt{a+\frac{b}{x}}} + \frac{d(bc-2ad)}{ac^2(bc-ad)\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)} + \frac{x}{ac\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)} + \frac{\left(d^3\right.}{}$$

$$= \frac{b\left(3b^2c^2-2abcd+2a^2d^2\right)}{a^2c^2(bc-ad)^2\sqrt{a+\frac{b}{x}}} + \frac{d(bc-2ad)}{ac^2(bc-ad)\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)} + \frac{x}{ac\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)} + \frac{\left(d^3\right.}{}$$

$$= \frac{b\left(3b^2c^2-2abcd+2a^2d^2\right)}{a^2c^2(bc-ad)^2\sqrt{a+\frac{b}{x}}} + \frac{d(bc-2ad)}{ac^2(bc-ad)\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)} + \frac{x}{ac\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)} + \frac{d^{5/}}{}$$

Mathematica [C] time = 0.14, size = 164, normalized size = 0.73

$$\frac{(bc-ad)\left((cx+d)\left(-4a^2d^2+abcd+3b^2c^2\right)\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{b}{ax}+1\right)+acx(bc(cx+d)-ad(cx+2d))\right)+a^2d^2(cx+d)(7bc-4ad)\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{d\left(a+\frac{b}{x}\right)}{ad-bc}\right)}{a^2c^3\sqrt{a+\frac{b}{x}}(cx+d)(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b/x)^(3/2)*(c + d/x)^2),x]
```

```
[Out] (a^2*d^2*(7*b*c - 4*a*d)*(d + c*x)*Hypergeometric2F1[-1/2, 1, 1/2, (d*(a +
b/x))/(-(b*c) + a*d)] + (b*c - a*d)*(a*c*x*(b*c*(d + c*x) - a*d*(2*d + c*x)
) + (3*b^2*c^2 + a*b*c*d - 4*a^2*d^2)*(d + c*x)*Hypergeometric2F1[-1/2, 1,
1/2, 1 + b/(a*x)]))/(a^2*c^3*(b*c - a*d)^2*Sqrt[a + b/x]*(d + c*x))
```

IntegrateAlgebraic [A] time = 0.86, size = 264, normalized size = 1.18

$$\frac{(-4ad-3bc)\tanh^{-1}\left(\frac{\sqrt{\frac{ax+b}{x}}}{\sqrt{a}}\right)}{a^{5/2}c^3}+\frac{\sqrt{\frac{ax+b}{x}}\left(a^3cd^2x^3+2a^3d^3x^2-2a^2bc^2dx^3-a^2bcd^2x^2+2a^2bd^3x+ab^2c^3x^3-ab^2c^2dx^2-2ab^2cd^2x+3b^3c^3x^2+3b^3c^2dx\right)}{a^2c^2(ax+b)(cx+d)(ad-bc)^2}+\frac{\left(7bcd^{5/2}-4ad^{7/2}\right)\tan^{-1}\left(\frac{\sqrt{d}\sqrt{\frac{ax+b}{x}}}{\sqrt{bc-ad}}\right)}{c^3(bc-ad)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((a + b/x)^(3/2)*(c + d/x)^2),x]
```

```
[Out] (Sqrt[(b + a*x)/x]*(3*b^3*c^2*d*x - 2*a*b^2*c*d^2*x + 2*a^2*b*d^3*x + 3*b^3
*c^3*x^2 - a*b^2*c^2*d*x^2 - a^2*b*c*d^2*x^2 + 2*a^3*d^3*x^2 + a*b^2*c^3*x^
3 - 2*a^2*b*c^2*d*x^3 + a^3*c*d^2*x^3))/(a^2*c^2*(-(b*c) + a*d)^2*(b + a*x)
```

```
*(d + c*x)) + ((7*b*c*d^(5/2) - 4*a*d^(7/2))*ArcTan[(Sqrt[d]*Sqrt[(b + a*x)
/x])/Sqrt[b*c - a*d]])/(c^3*(b*c - a*d)^(5/2)) + ((-3*b*c - 4*a*d)*ArcTanh[
Sqrt[(b + a*x)/x]/Sqrt[a]])/(a^(5/2)*c^3)
```

fricas [B] time = 2.36, size = 2321, normalized size = 10.36

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)^(3/2)/(c+d/x)^2,x, algorithm="fricas")

[Out] [1/2*((3*b^4*c^3*d - 2*a*b^3*c^2*d^2 - 5*a^2*b^2*c*d^3 + 4*a^3*b*d^4 + (3*a
*b^3*c^4 - 2*a^2*b^2*c^3*d - 5*a^3*b*c^2*d^2 + 4*a^4*c*d^3)*x^2 + (3*b^4*c^
4 + a*b^3*c^3*d - 7*a^2*b^2*c^2*d^2 - a^3*b*c*d^3 + 4*a^4*d^4)*x)*sqrt(a)*l
og(2*a*x - 2*sqrt(a)*x*sqrt((a*x + b)/x) + b) - (7*a^3*b^2*c*d^3 - 4*a^4*b*
d^4 + (7*a^4*b*c^2*d^2 - 4*a^5*c*d^3)*x^2 + (7*a^3*b^2*c^2*d^2 + 3*a^4*b*c*
d^3 - 4*a^5*d^4)*x)*sqrt(-d/(b*c - a*d))*log(-(2*(b*c - a*d)*x*sqrt(-d/(b*c
 - a*d))*sqrt((a*x + b)/x) - b*d + (b*c - 2*a*d)*x)/(c*x + d)) + 2*((a^2*b^
2*c^4 - 2*a^3*b*c^3*d + a^4*c^2*d^2)*x^3 + (3*a*b^3*c^4 - a^2*b^2*c^3*d - a
^3*b*c^2*d^2 + 2*a^4*c*d^3)*x^2 + (3*a*b^3*c^3*d - 2*a^2*b^2*c^2*d^2 + 2*a^
3*b*c*d^3)*x)*sqrt((a*x + b)/x))/(a^3*b^3*c^5*d - 2*a^4*b^2*c^4*d^2 + a^5*b
*c^3*d^3 + (a^4*b^2*c^6 - 2*a^5*b*c^5*d + a^6*c^4*d^2)*x^2 + (a^3*b^3*c^6 -
 a^4*b^2*c^5*d - a^5*b*c^4*d^2 + a^6*c^3*d^3)*x), 1/2*(2*(7*a^3*b^2*c*d^3 -
 4*a^4*b*d^4 + (7*a^4*b*c^2*d^2 - 4*a^5*c*d^3)*x^2 + (7*a^3*b^2*c^2*d^2 + 3
*a^4*b*c*d^3 - 4*a^5*d^4)*x)*sqrt(d/(b*c - a*d))*arctan(-(b*c - a*d)*x*sqrt
(d/(b*c - a*d))*sqrt((a*x + b)/x)/(a*d*x + b*d)) + (3*b^4*c^3*d - 2*a*b^3*c
^2*d^2 - 5*a^2*b^2*c*d^3 + 4*a^3*b*d^4 + (3*a*b^3*c^4 - 2*a^2*b^2*c^3*d - 5
*a^3*b*c^2*d^2 + 4*a^4*c*d^3)*x^2 + (3*b^4*c^4 + a*b^3*c^3*d - 7*a^2*b^2*c^
2*d^2 - a^3*b*c*d^3 + 4*a^4*d^4)*x)*sqrt(a)*log(2*a*x - 2*sqrt(a)*x*sqrt((a
*x + b)/x) + b) + 2*((a^2*b^2*c^4 - 2*a^3*b*c^3*d + a^4*c^2*d^2)*x^3 + (3*a
*b^3*c^4 - a^2*b^2*c^3*d - a^3*b*c^2*d^2 + 2*a^4*c*d^3)*x^2 + (3*a*b^3*c^3*
d - 2*a^2*b^2*c^2*d^2 + 2*a^3*b*c*d^3)*x)*sqrt((a*x + b)/x))/(a^3*b^3*c^5*d
 - 2*a^4*b^2*c^4*d^2 + a^5*b*c^3*d^3 + (a^4*b^2*c^6 - 2*a^5*b*c^5*d + a^6*c
^4*d^2)*x^2 + (a^3*b^3*c^6 - a^4*b^2*c^5*d - a^5*b*c^4*d^2 + a^6*c^3*d^3)*x
), 1/2*(2*(3*b^4*c^3*d - 2*a*b^3*c^2*d^2 - 5*a^2*b^2*c*d^3 + 4*a^3*b*d^4 +
(3*a*b^3*c^4 - 2*a^2*b^2*c^3*d - 5*a^3*b*c^2*d^2 + 4*a^4*c*d^3)*x^2 + (3*b^
4*c^4 + a*b^3*c^3*d - 7*a^2*b^2*c^2*d^2 - a^3*b*c*d^3 + 4*a^4*d^4)*x)*sqrt(
-a)*arctan(sqrt(-a)*sqrt((a*x + b)/x)/a) - (7*a^3*b^2*c*d^3 - 4*a^4*b*d^4 +
 (7*a^4*b*c^2*d^2 - 4*a^5*c*d^3)*x^2 + (7*a^3*b^2*c^2*d^2 + 3*a^4*b*c*d^3 -
 4*a^5*d^4)*x)*sqrt(-d/(b*c - a*d))*log(-(2*(b*c - a*d)*x*sqrt(-d/(b*c - a*
d))*sqrt((a*x + b)/x) - b*d + (b*c - 2*a*d)*x)/(c*x + d)) + 2*((a^2*b^2*c^4
 - 2*a^3*b*c^3*d + a^4*c^2*d^2)*x^3 + (3*a*b^3*c^4 - a^2*b^2*c^3*d - a^3*b*
c^2*d^2 + 2*a^4*c*d^3)*x^2 + (3*a*b^3*c^3*d - 2*a^2*b^2*c^2*d^2 + 2*a^3*b*c
*d^3)*x)*sqrt((a*x + b)/x))/(a^3*b^3*c^5*d - 2*a^4*b^2*c^4*d^2 + a^5*b*c^3*
d^3 + (a^4*b^2*c^6 - 2*a^5*b*c^5*d + a^6*c^4*d^2)*x^2 + (a^3*b^3*c^6 - a^4*
b^2*c^5*d - a^5*b*c^4*d^2 + a^6*c^3*d^3)*x), ((7*a^3*b^2*c*d^3 - 4*a^4*b*d^
4 + (7*a^4*b*c^2*d^2 - 4*a^5*c*d^3)*x^2 + (7*a^3*b^2*c^2*d^2 + 3*a^4*b*c*d^
3 - 4*a^5*d^4)*x)*sqrt(d/(b*c - a*d))*arctan(-(b*c - a*d)*x*sqrt(d/(b*c - a
*d))*sqrt((a*x + b)/x)/(a*d*x + b*d)) + (3*b^4*c^3*d - 2*a*b^3*c^2*d^2 - 5*
a^2*b^2*c*d^3 + 4*a^3*b*d^4 + (3*a*b^3*c^4 - 2*a^2*b^2*c^3*d - 5*a^3*b*c^2*
d^2 + 4*a^4*c*d^3)*x^2 + (3*b^4*c^4 + a*b^3*c^3*d - 7*a^2*b^2*c^2*d^2 - a^3
*b*c*d^3 + 4*a^4*d^4)*x)*sqrt(-a)*arctan(sqrt(-a)*sqrt((a*x + b)/x)/a) + ((
a^2*b^2*c^4 - 2*a^3*b*c^3*d + a^4*c^2*d^2)*x^3 + (3*a*b^3*c^4 - a^2*b^2*c^3
*d - a^3*b*c^2*d^2 + 2*a^4*c*d^3)*x^2 + (3*a*b^3*c^3*d - 2*a^2*b^2*c^2*d^2
+ 2*a^3*b*c*d^3)*x)*sqrt((a*x + b)/x))/(a^3*b^3*c^5*d - 2*a^4*b^2*c^4*d^2 +
 a^5*b*c^3*d^3 + (a^4*b^2*c^6 - 2*a^5*b*c^5*d + a^6*c^4*d^2)*x^2 + (a^3*b^3
*c^6 - a^4*b^2*c^5*d - a^5*b*c^4*d^2 + a^6*c^3*d^3)*x)]
```

giac [B] time = 0.26, size = 424, normalized size = 1.89

$$b^{3}\left(\frac{\left(7\,bcd^{3}-4\,ad^{4}\right)\arctan\left(\frac{d\sqrt{\frac{ax+b}{x}}}{\sqrt{bcd-ad^{2}}}\right)}{\left(b^{5}c^{5}-2\,ab^{4}c^{4}d+a^{2}b^{3}c^{3}d^{2}\right)\sqrt{bcd-ad^{2}}}+\frac{2\,ab^{3}c^{3}-2\,a^{2}b^{2}c^{2}d-\frac{3\,(ax+b)b^{3}c^{3}}{x}+\frac{7\,(ax+b)ab^{2}c^{2}d}{x}-\frac{3\,(ax+b)a^{2}bcd^{2}}{x}+\frac{2\,(ax+b)a^{3}d^{3}}{x}-\frac{3\,(ax+b)^{2}b^{2}c^{2}d}{x^{2}}+\frac{2\,(ax+b)^{2}abcd^{2}}{x^{2}}-\frac{2\,(ax+b)^{2}a^{2}d^{3}}{x^{2}}}{\left(a^{2}b^{4}c^{4}-2\,a^{3}b^{3}c^{3}d+a^{4}b^{2}c^{2}d^{2}\right)\left(abc\sqrt{\frac{ax+b}{x}}-a^{2}d\sqrt{\frac{ax+b}{x}}-\frac{(ax+b)bc\sqrt{\frac{ax+b}{x}}}{x}+\frac{2\,(ax+b)ad\sqrt{\frac{ax+b}{x}}}{x}-\frac{(ax+b)^{2}d\sqrt{\frac{ax+b}{x}}}{x^{2}}\right)}+\frac{(3\,bc+4\,ad)\arctan\left(\frac{\sqrt{\frac{ax+b}{x}}}{\sqrt{-a}}\right)}{\sqrt{-a}\,a^{2}b^{3}c^{3}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)^(3/2)/(c+d/x)^2,x, algorithm="giac")
```

```
[Out] b^3*((7*b*c*d^3 - 4*a*d^4)*arctan(d*sqrt((a*x + b)/x)/sqrt(b*c*d - a*d^2))/
((b^5*c^5 - 2*a*b^4*c^4*d + a^2*b^3*c^3*d^2)*sqrt(b*c*d - a*d^2)) + (2*a*b^
3*c^3 - 2*a^2*b^2*c^2*d - 3*(a*x + b)*b^3*c^3/x + 7*(a*x + b)*a*b^2*c^2*d/x
 - 3*(a*x + b)*a^2*b*c*d^2/x + 2*(a*x + b)*a^3*d^3/x - 3*(a*x + b)^2*b^2*c^
2*d/x^2 + 2*(a*x + b)^2*a*b*c*d^2/x^2 - 2*(a*x + b)^2*a^2*d^3/x^2)/((a^2*b^
4*c^4 - 2*a^3*b^3*c^3*d + a^4*b^2*c^2*d^2)*(a*b*c*sqrt((a*x + b)/x) - a^2*d
*sqrt((a*x + b)/x) - (a*x + b)*b*c*sqrt((a*x + b)/x)/x + 2*(a*x + b)*a*d*sq
rt((a*x + b)/x)/x - (a*x + b)^2*d*sqrt((a*x + b)/x)/x^2)) + (3*b*c + 4*a*d)
*arctan(sqrt((a*x + b)/x)/sqrt(-a))/(sqrt(-a)*a^2*b^3*c^3))
```

maple [B] time = 0.07, size = 3119, normalized size = 13.92

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b/x)^(3/2)/(c+d/x)^2,x)
```

```
[Out] -1/2*((a*x+b)/x)^(1/2)*x*(4*a^(15/2)*ln((-2*a*d*x+b*c*x-b*d+2*((a*d-b*c)/c^
2*d)^(1/2)*((a*x+b)*x)^(1/2)*c)/(c*x+d))*x^2*d^6+4*a^(11/2)*ln((-2*a*d*x+b*
c*x-b*d+2*((a*d-b*c)/c^2*d)^(1/2)*((a*x+b)*x)^(1/2)*c)/(c*x+d))*b^2*d^6-6*a
^2*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))*((a*d-b*c)/c^2*d)^
(1/2)*x^2*b^5*c^6+2*a^(7/2)*((a*x+b)*x)^(3/2)*((a*d-b*c)/c^2*d)^(1/2)*b^2*c
^4*d^2-4*a^(5/2)*((a*x+b)*x)^(3/2)*((a*d-b*c)/c^2*d)^(1/2)*b^3*c^5*d-3*a*ln
(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))*((a*d-b*c)/c^2*d)^(1/2)
*x*b^6*c^6+6*a^(3/2)*((a*x+b)*x)^(1/2)*((a*d-b*c)/c^2*d)^(1/2)*b^5*c^5*d-3*
a*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))*((a*d-b*c)/c^2*d)^(
1/2)*b^6*c^5*d+2*a^(13/2)*((a*x+b)*x)^(1/2)*((a*d-b*c)/c^2*d)^(1/2)*x^4*c^4
*d^2-2*a^(11/2)*((a*x+b)*x)^(3/2)*((a*d-b*c)/c^2*d)^(1/2)*x^2*c^4*d^2-2*a^(
13/2)*((a*x+b)*x)^(1/2)*((a*d-b*c)/c^2*d)^(1/2)*x^3*c^3*d^3+6*a^(7/2)*((a*x
+b)*x)^(1/2)*((a*d-b*c)/c^2*d)^(1/2)*x^3*b^3*c^6-11*a^(13/2)*ln((-2*a*d*x+b
*c*x-b*d+2*((a*d-b*c)/c^2*d)^(1/2)*((a*x+b)*x)^(1/2)*c)/(c*x+d))*x^3*b*c^2*
d^4+7*a^(11/2)*ln((-2*a*d*x+b*c*x-b*d+2*((a*d-b*c)/c^2*d)^(1/2)*((a*x+b)*x)
^(1/2)*c)/(c*x+d))*x^3*b^2*c^3*d^3+4*a^7*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2
)*a^(1/2))/a^(1/2))*((a*d-b*c)/c^2*d)^(1/2)*x^3*c^2*d^4-3*a^3*ln(1/2*(2*a*x
+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))*((a*d-b*c)/c^2*d)^(1/2)*x^3*b^4*c^
6-4*a^(5/2)*((a*x+b)*x)^(3/2)*((a*d-b*c)/c^2*d)^(1/2)*x*b^3*c^6-4*a^(13/2)*
((a*x+b)*x)^(1/2)*((a*d-b*c)/c^2*d)^(1/2)*x^2*c^2*d^4+12*a^(5/2)*((a*x+b)*x
)^(1/2)*((a*d-b*c)/c^2*d)^(1/2)*x^2*b^4*c^6-3*a^(13/2)*ln((-2*a*d*x+b*c*x-b
*d+2*((a*d-b*c)/c^2*d)^(1/2)*((a*x+b)*x)^(1/2)*c)/(c*x+d))*x^2*b*c*d^5-18*a
^(11/2)*ln((-2*a*d*x+b*c*x-b*d+2*((a*d-b*c)/c^2*d)^(1/2)*((a*x+b)*x)^(1/2)*
c)/(c*x+d))*x*b^2*c*d^5+3*a^(9/2)*ln((-2*a*d*x+b*c*x-b*d+2*((a*d-b*c)/c^2*d
)^(1/2)*((a*x+b)*x)^(1/2)*c)/(c*x+d))*x*b^3*c^2*d^4+7*a^(7/2)*ln((-2*a*d*x+
b*c*x-b*d+2*((a*d-b*c)/c^2*d)^(1/2)*((a*x+b)*x)^(1/2)*c)/(c*x+d))*x*b^4*c^3
*d^3-4*a^(9/2)*((a*x+b)*x)^(1/2)*((a*d-b*c)/c^2*d)^(1/2)*b^2*c^2*d^4+8*a^(7
/2)*((a*x+b)*x)^(1/2)*((a*d-b*c)/c^2*d)^(1/2)*b^3*c^3*d^3-10*a^(5/2)*((a*x+
b)*x)^(1/2)*((a*d-b*c)/c^2*d)^(1/2)*b^4*c^4*d^2+4*a^5*ln(1/2*(2*a*x+b+2*((a
*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))*((a*d-b*c)/c^2*d)^(1/2)*b^2*c*d^5-9*a^4*ln
(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))*((a*d-b*c)/c^2*d)^(1/2)
*b^3*c^2*d^4+3*a^3*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))*((
a*d-b*c)/c^2*d)^(1/2)*b^4*c^3*d^3+5*a^2*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)
```

```
*a^(1/2))/a^(1/2))*((a*d-b*c)/c^2*d)^(1/2)*b^5*c^4*d^2-15*a^(11/2)*ln((-2*a
*d*x+b*c*x-b*d+2*((a*d-b*c)/c^2*d)^(1/2)*((a*x+b)*x)^(1/2)*c)/(c*x+d))*x^2*
b^2*c^2*d^4+14*a^(9/2)*ln((-2*a*d*x+b*c*x-b*d+2*((a*d-b*c)/c^2*d)^(1/2)*((a
*x+b)*x)^(1/2)*c)/(c*x+d))*x^2*b^3*c^3*d^3+4*a^7*ln(1/2*(2*a*x+b+2*((a*x+b)
*x)^(1/2)*a^(1/2))/a^(1/2))*((a*d-b*c)/c^2*d)^(1/2)*x^2*c*d^5+6*a^(3/2)*((a
*x+b)*x)^(1/2)*((a*d-b*c)/c^2*d)^(1/2)*x*b^5*c^6+13*a^3*ln(1/2*(2*a*x+b+2*(
(a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))*((a*d-b*c)/c^2*d)^(1/2)*x*b^4*c^4*d^2-8*
a^(11/2)*((a*x+b)*x)^(1/2)*((a*d-b*c)/c^2*d)^(1/2)*x*b*c^2*d^4+14*a^(9/2)*(
(a*x+b)*x)^(1/2)*((a*d-b*c)/c^2*d)^(1/2)*x*b^2*c^3*d^3-12*a^(7/2)*((a*x+b)*
x)^(1/2)*((a*d-b*c)/c^2*d)^(1/2)*x*b^3*c^4*d^2+2*a^(5/2)*((a*x+b)*x)^(1/2)*
((a*d-b*c)/c^2*d)^(1/2)*x*b^4*c^5*d+8*a^6*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/
2)*a^(1/2))/a^(1/2))*((a*d-b*c)/c^2*d)^(1/2)*x*b*c*d^5-14*a^5*ln(1/2*(2*a*x
+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))*((a*d-b*c)/c^2*d)^(1/2)*x*b^2*c^2*
d^4+11*a^4*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))*((a*d-b*c)
/c^2*d)^(1/2)*x^2*b^3*c^4*d^2+7*a^3*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(
1/2))/a^(1/2))*((a*d-b*c)/c^2*d)^(1/2)*x^2*b^4*c^5*d-a^6*ln(1/2*(2*a*x+b+2*
((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))*((a*d-b*c)/c^2*d)^(1/2)*x^2*b*c^2*d^4-1
5*a^5*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))*((a*d-b*c)/c^2*
d)^(1/2)*x^2*b^2*c^3*d^3+8*a^(9/2)*((a*x+b)*x)^(1/2)*((a*d-b*c)/c^2*d)^(1/2
)*x^2*b^2*c^4*d^2-14*a^(7/2)*((a*x+b)*x)^(1/2)*((a*d-b*c)/c^2*d)^(1/2)*x^2*
b^3*c^5*d-10*a^(9/2)*((a*x+b)*x)^(1/2)*((a*d-b*c)/c^2*d)^(1/2)*x^3*b^2*c^5*
d-9*a^6*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))*((a*d-b*c)/c^
2*d)^(1/2)*x^3*b*c^3*d^3+3*a^5*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))
/a^(1/2))*((a*d-b*c)/c^2*d)^(1/2)*x^3*b^2*c^4*d^2+5*a^4*ln(1/2*(2*a*x+b+2*(
(a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))*((a*d-b*c)/c^2*d)^(1/2)*x^3*b^3*c^5*d-4*
a^(9/2)*((a*x+b)*x)^(3/2)*((a*d-b*c)/c^2*d)^(1/2)*x*b*c^4*d^2+4*a^(7/2)*((a
*x+b)*x)^(3/2)*((a*d-b*c)/c^2*d)^(1/2)*x*b^2*c^5*d+4*a^(11/2)*((a*x+b)*x)^(
1/2)*((a*d-b*c)/c^2*d)^(1/2)*x^2*b*c^3*d^3+12*a^(11/2)*((a*x+b)*x)^(1/2)*((
a*d-b*c)/c^2*d)^(1/2)*x^3*b*c^4*d^2-a^2*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)
*a^(1/2))/a^(1/2))*((a*d-b*c)/c^2*d)^(1/2)*x*b^5*c^5*d-3*a^4*ln(1/2*(2*a*x+
b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))*((a*d-b*c)/c^2*d)^(1/2)*x*b^3*c^3*d
^3+4*a^(15/2)*ln((-2*a*d*x+b*c*x-b*d+2*((a*d-b*c)/c^2*d)^(1/2)*((a*x+b)*x)^
(1/2)*c)/(c*x+d))*x^3*c*d^5+8*a^(13/2)*ln((-2*a*d*x+b*c*x-b*d+2*((a*d-b*c)/
c^2*d)^(1/2)*((a*x+b)*x)^(1/2)*c)/(c*x+d))*x*b*d^6-11*a^(9/2)*ln((-2*a*d*x+
b*c*x-b*d+2*((a*d-b*c)/c^2*d)^(1/2)*((a*x+b)*x)^(1/2)*c)/(c*x+d))*b^3*c*d^5
+7*a^(7/2)*ln((-2*a*d*x+b*c*x-b*d+2*((a*d-b*c)/c^2*d)^(1/2)*((a*x+b)*x)^(1/
2)*c)/(c*x+d))*b^4*c^2*d^4)/a^(7/2)/c^4/((a*x+b)*x)^(1/2)/(a*d-b*c)^3/(c*x+
d)/((a*d-b*c)/c^2*d)^(1/2)/(a*x+b)^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{\frac{3}{2}}\left(c+\frac{d}{x}\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)^(3/2)/(c+d/x)^2,x, algorithm="maxima")
```

```
[Out] integrate(1/((a + b/x)^(3/2)*(c + d/x)^2), x)
```

mupad [B] time = 6.20, size = 4274, normalized size = 19.08

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b/x)^(3/2)*(c + d/x)^2),x)
```

```
[Out] ((2*b^3)/(a^2*d - a*b*c) + (b*(a + b/x)^2*(2*a^2*d^3 + 3*b^2*c^2*d - 2*a*b*
c*d^2))/(c^2*(a^2*d - a*b*c)^2) - (b*(a + b/x)*(2*a*d - b*c)*(a^2*d^2 + 3*b
^2*c^2 - a*b*c*d))/(c^2*(a^2*d - a*b*c)^2))/(d*(a + b/x)^(5/2) + (a + b/x)^
```

(1/2)*(a^2*d - a*b*c) - (a + b/x)^(3/2)*(2*a*d - b*c)) + (atan((a^13*b^11*c^11*d^3*(a + b/x)^(1/2)*35i - a^12*b^12*c^12*d^2*(a + b/x)^(1/2)*441i - a^10*b^14*c^14*(a + b/x)^(1/2)*27i + a^14*b^10*c^10*d^4*(a + b/x)^(1/2)*1694i - a^15*b^9*c^9*d^5*(a + b/x)^(1/2)*3073i + a^16*b^8*c^8*d^6*(a + b/x)^(1/2)*1316i + a^17*b^7*c^7*d^7*(a + b/x)^(1/2)*2561i - a^18*b^6*c^6*d^8*(a + b/x)^(1/2)*4375i + a^19*b^5*c^5*d^9*(a + b/x)^(1/2)*2996i - a^20*b^4*c^4*d^10*(a + b/x)^(1/2)*1015i + a^21*b^3*c^3*d^11*(a + b/x)^(1/2)*140i + a^11*b^13*c^13*d*(a + b/x)^(1/2)*189i)/(a^5*(a^5)^(1/2)*(a^5*(a^5*(2561*b^7*c^7*d^7 - 4375*a*b^6*c^6*d^8 + 2996*a^2*b^5*c^5*d^9 - 1015*a^3*b^4*c^4*d^10 + 140*a^4*b^3*c^3*d^11) - 441*b^12*c^12*d^2 + 35*a*b^11*c^11*d^3 + 1694*a^2*b^10*c^10*d^4 - 3073*a^3*b^9*c^9*d^5 + 1316*a^4*b^8*c^8*d^6) - 27*a^3*b^14*c^14 + 189*a^4*b^13*c^13*d)))*(4*a*d + 3*b*c)*1i)/(c^3*(a^5)^(1/2)) - (atan((((d^5*(a*d - b*c)^5)^(1/2)*(4*a*d - 7*b*c)*((a + b/x)^(1/2)*(18*a^6*b^14*c^18*d^3 - 132*a^7*b^13*c^17*d^4 + 362*a^8*b^12*c^16*d^5 - 320*a^9*b^11*c^15*d^6 - 442*a^10*b^10*c^14*d^7 + 1004*a^11*b^9*c^13*d^8 + 578*a^12*b^8*c^12*d^9 - 3976*a^13*b^7*c^11*d^10 + 5960*a^14*b^6*c^10*d^11 - 4768*a^15*b^5*c^9*d^12 + 2228*a^16*b^4*c^8*d^13 - 576*a^17*b^3*c^7*d^14 + 64*a^18*b^2*c^6*d^15) - ((d^5*(a*d - b*c)^5)^(1/2)*(4*a*d - 7*b*c)*(12*a^8*b^14*c^21*d^2 - 116*a^9*b^13*c^20*d^3 + 484*a^10*b^12*c^19*d^4 - 1128*a^11*b^11*c^18*d^5 + 1560*a^12*b^10*c^17*d^6 - 1176*a^13*b^9*c^16*d^7 + 168*a^14*b^8*c^15*d^8 + 576*a^15*b^7*c^14*d^9 - 612*a^16*b^6*c^13*d^10 + 300*a^17*b^5*c^12*d^11 - 76*a^18*b^4*c^11*d^12 + 8*a^19*b^3*c^10*d^13 - ((d^5*(a*d - b*c)^5)^(1/2)*(a + b/x)^(1/2)*(4*a*d - 7*b*c)*(8*a^10*b^13*c^23*d^2 - 96*a^11*b^12*c^22*d^3 + 520*a^12*b^11*c^21*d^4 - 1680*a^13*b^10*c^20*d^5 + 3600*a^14*b^9*c^19*d^6 - 5376*a^15*b^8*c^18*d^7 + 5712*a^16*b^7*c^17*d^8 - 4320*a^17*b^6*c^16*d^9 + 2280*a^18*b^5*c^15*d^10 - 800*a^19*b^4*c^14*d^11 + 168*a^20*b^3*c^13*d^12 - 16*a^21*b^2*c^12*d^13))/(2*(b^5*c^8 - a^5*c^3*d^5 + 5*a^4*b*c^4*d^4 + 10*a^2*b^3*c^6*d^2 - 10*a^3*b^2*c^5*d^3 - 5*a*b^4*c^7*d))))/(2*(b^5*c^8 - a^5*c^3*d^5 + 5*a^4*b*c^4*d^4 + 10*a^2*b^3*c^6*d^2 - 10*a^3*b^2*c^5*d^3 - 5*a*b^4*c^7*d)))*1i)/(2*(b^5*c^8 - a^5*c^3*d^5 + 5*a^4*b*c^4*d^4 + 10*a^2*b^3*c^6*d^2 - 10*a^3*b^2*c^5*d^3 - 5*a*b^4*c^7*d)) + ((d^5*(a*d - b*c)^5)^(1/2)*(4*a*d - 7*b*c)*((a + b/x)^(1/2)*(18*a^6*b^14*c^18*d^3 - 132*a^7*b^13*c^17*d^4 + 362*a^8*b^12*c^16*d^5 - 320*a^9*b^11*c^15*d^6 - 442*a^10*b^10*c^14*d^7 + 1004*a^11*b^9*c^13*d^8 + 578*a^12*b^8*c^12*d^9 - 3976*a^13*b^7*c^11*d^10 + 5960*a^14*b^6*c^10*d^11 - 4768*a^15*b^5*c^9*d^12 + 2228*a^16*b^4*c^8*d^13 - 576*a^17*b^3*c^7*d^14 + 64*a^18*b^2*c^6*d^15) + ((d^5*(a*d - b*c)^5)^(1/2)*(4*a*d - 7*b*c)*(12*a^8*b^14*c^21*d^2 - 116*a^9*b^13*c^20*d^3 + 484*a^10*b^12*c^19*d^4 - 1128*a^11*b^11*c^18*d^5 + 1560*a^12*b^10*c^17*d^6 - 1176*a^13*b^9*c^16*d^7 + 168*a^14*b^8*c^15*d^8 + 576*a^15*b^7*c^14*d^9 - 612*a^16*b^6*c^13*d^10 + 300*a^17*b^5*c^12*d^11 - 76*a^18*b^4*c^11*d^12 + 8*a^19*b^3*c^10*d^13 + ((d^5*(a*d - b*c)^5)^(1/2)*(a + b/x)^(1/2)*(4*a*d - 7*b*c)*(8*a^10*b^13*c^23*d^2 - 96*a^11*b^12*c^22*d^3 + 520*a^12*b^11*c^21*d^4 - 1680*a^13*b^10*c^20*d^5 + 3600*a^14*b^9*c^19*d^6 - 5376*a^15*b^8*c^18*d^7 + 5712*a^16*b^7*c^17*d^8 - 4320*a^17*b^6*c^16*d^9 + 2280*a^18*b^5*c^15*d^10 - 800*a^19*b^4*c^14*d^11 + 168*a^20*b^3*c^13*d^12 - 16*a^21*b^2*c^12*d^13))/(2*(b^5*c^8 - a^5*c^3*d^5 + 5*a^4*b*c^4*d^4 + 10*a^2*b^3*c^6*d^2 - 10*a^3*b^2*c^5*d^3 - 5*a*b^4*c^7*d))))/(2*(b^5*c^8 - a^5*c^3*d^5 + 5*a^4*b*c^4*d^4 + 10*a^2*b^3*c^6*d^2 - 10*a^3*b^2*c^5*d^3 - 5*a*b^4*c^7*d)))*1i)/(2*(b^5*c^8 - a^5*c^3*d^5 + 5*a^4*b*c^4*d^4 + 10*a^2*b^3*c^6*d^2 - 10*a^3*b^2*c^5*d^3 - 5*a*b^4*c^7*d)))/(((d^5*(a*d - b*c)^5)^(1/2)*(4*a*d - 7*b*c)*((a + b/x)^(1/2)*(18*a^6*b^14*c^18*d^3 - 132*a^7*b^13*c^17*d^4 + 362*a^8*b^12*c^16*d^5 - 320*a^9*b^11*c^15*d^6 - 442*a^10*b^10*c^14*d^7 + 1004*a^11*b^9*c^13*d^8 + 578*a^12*b^8*c^12*d^9 - 3976*a^13*b^7*c^11*d^10 + 5960*a^14*b^6*c^10*d^11 - 4768*a^15*b^5*c^9*d^12 + 2228*a^16*b^4*c^8*d^13 - 576*a^17*b^3*c^7*d^14 + 64*a^18*b^2*c^6*d^15) - ((d^5*(a*d - b*c)^5)^(1/2)*(4*a*d - 7*b*c)*(12*a^8*b^14*c^21*d^2 - 116*a^9*b^13*c^20*d^3 + 484*a^10*b^12*c^19*d^4 - 1128*a^11*b^11*c^18*d^5 + 1560*a^12*b^10*c^17*d^6 - 1176*a^13*b^9*c^16*d^7 + 168*a^14*b^8*c^15*d^8 + 576*a^15*b^7*c^14*d^9 - 612*a^16*b^6*c^13*d^10 + 300*a^17*b^5*c^12*d^11 - 76*a^18*b^4*c^11*d^12 + 8*a^19*b^3*c^10*d^13 - ((d^5*(a*d - b*c)^

```
5)^(1/2)*(a + b/x)^(1/2)*(4*a*d - 7*b*c)*(8*a^10*b^13*c^23*d^2 - 96*a^11*b^
12*c^22*d^3 + 520*a^12*b^11*c^21*d^4 - 1680*a^13*b^10*c^20*d^5 + 3600*a^14*
b^9*c^19*d^6 - 5376*a^15*b^8*c^18*d^7 + 5712*a^16*b^7*c^17*d^8 - 4320*a^17*
b^6*c^16*d^9 + 2280*a^18*b^5*c^15*d^10 - 800*a^19*b^4*c^14*d^11 + 168*a^20*
b^3*c^13*d^12 - 16*a^21*b^2*c^12*d^13))/(2*(b^5*c^8 - a^5*c^3*d^5 + 5*a^4*b
*c^4*d^4 + 10*a^2*b^3*c^6*d^2 - 10*a^3*b^2*c^5*d^3 - 5*a*b^4*c^7*d))))/(2*(
b^5*c^8 - a^5*c^3*d^5 + 5*a^4*b*c^4*d^4 + 10*a^2*b^3*c^6*d^2 - 10*a^3*b^2*c
^5*d^3 - 5*a*b^4*c^7*d))))/(2*(b^5*c^8 - a^5*c^3*d^5 + 5*a^4*b*c^4*d^4 + 10
*a^2*b^3*c^6*d^2 - 10*a^3*b^2*c^5*d^3 - 5*a*b^4*c^7*d)) - ((d^5*(a*d - b*c)
^5)^(1/2)*(4*a*d - 7*b*c)*((a + b/x)^(1/2)*(18*a^6*b^14*c^18*d^3 - 132*a^7*
b^13*c^17*d^4 + 362*a^8*b^12*c^16*d^5 - 320*a^9*b^11*c^15*d^6 - 442*a^10*b^
10*c^14*d^7 + 1004*a^11*b^9*c^13*d^8 + 578*a^12*b^8*c^12*d^9 - 3976*a^13*b^
7*c^11*d^10 + 5960*a^14*b^6*c^10*d^11 - 4768*a^15*b^5*c^9*d^12 + 2228*a^16*
b^4*c^8*d^13 - 576*a^17*b^3*c^7*d^14 + 64*a^18*b^2*c^6*d^15) + ((d^5*(a*d -
 b*c)^5)^(1/2)*(4*a*d - 7*b*c)*(12*a^8*b^14*c^21*d^2 - 116*a^9*b^13*c^20*d^
3 + 484*a^10*b^12*c^19*d^4 - 1128*a^11*b^11*c^18*d^5 + 1560*a^12*b^10*c^17*
d^6 - 1176*a^13*b^9*c^16*d^7 + 168*a^14*b^8*c^15*d^8 + 576*a^15*b^7*c^14*d^
9 - 612*a^16*b^6*c^13*d^10 + 300*a^17*b^5*c^12*d^11 - 76*a^18*b^4*c^11*d^12
 + 8*a^19*b^3*c^10*d^13 + ((d^5*(a*d - b*c)^5)^(1/2)*(a + b/x)^(1/2)*(4*a*d
 - 7*b*c)*(8*a^10*b^13*c^23*d^2 - 96*a^11*b^12*c^22*d^3 + 520*a^12*b^11*c^2
1*d^4 - 1680*a^13*b^10*c^20*d^5 + 3600*a^14*b^9*c^19*d^6 - 5376*a^15*b^8*c^
18*d^7 + 5712*a^16*b^7*c^17*d^8 - 4320*a^17*b^6*c^16*d^9 + 2280*a^18*b^5*c^
15*d^10 - 800*a^19*b^4*c^14*d^11 + 168*a^20*b^3*c^13*d^12 - 16*a^21*b^2*c^1
2*d^13))/(2*(b^5*c^8 - a^5*c^3*d^5 + 5*a^4*b*c^4*d^4 + 10*a^2*b^3*c^6*d^2 -
 10*a^3*b^2*c^5*d^3 - 5*a*b^4*c^7*d))))/(2*(b^5*c^8 - a^5*c^3*d^5 + 5*a^4*b
*c^4*d^4 + 10*a^2*b^3*c^6*d^2 - 10*a^3*b^2*c^5*d^3 - 5*a*b^4*c^7*d))))/(2*(
b^5*c^8 - a^5*c^3*d^5 + 5*a^4*b*c^4*d^4 + 10*a^2*b^3*c^6*d^2 - 10*a^3*b^2*c
^5*d^3 - 5*a*b^4*c^7*d)) - 126*a^6*b^13*c^14*d^5 + 744*a^7*b^12*c^13*d^6 -
1742*a^8*b^11*c^12*d^7 + 1756*a^9*b^10*c^11*d^8 + 322*a^10*b^9*c^10*d^9 - 3
248*a^11*b^8*c^9*d^10 + 4606*a^12*b^7*c^8*d^11 - 3668*a^13*b^6*c^7*d^12 + 1
804*a^14*b^5*c^6*d^13 - 512*a^15*b^4*c^5*d^14 + 64*a^16*b^3*c^4*d^15))*(d^5
*(a*d - b*c)^5)^(1/2)*(4*a*d - 7*b*c)*1i)/(b^5*c^8 - a^5*c^3*d^5 + 5*a^4*b*
c^4*d^4 + 10*a^2*b^3*c^6*d^2 - 10*a^3*b^2*c^5*d^3 - 5*a*b^4*c^7*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)**(3/2)/(c+d/x)**2,x)
```

```
[Out] Timed out
```

3.161 $\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)^3}\,dx$

Optimal. Leaf size=320

$$-\frac{3(2ad+bc)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{5/2}c^4}+\frac{3d^{5/2}\left(8a^2d^2-24abcd+21b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{4c^4(bc-ad)^{7/2}}+\frac{d\left(12a^2d^2-21abcd+4b^2c^2\right)}{4ac^3\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)(bc-ad)^2}$$

Rubi [A] time = 0.52, antiderivative size = 320, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {375, 103, 151, 152, 156, 63, 208, 205}

$$\frac{d\left(12a^2d^2-21abcd+4b^2c^2\right)}{4ac^3\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)(bc-ad)^2}+\frac{3b(2bc-ad)\left(4a^2d^2-abcd+2b^2c^2\right)}{4a^2c^3\sqrt{a+\frac{b}{x}}(bc-ad)^3}+\frac{3d^{5/2}\left(8a^2d^2-24abcd+21b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{4c^4(bc-ad)^{7/2}}-\frac{3(2ad+bc)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{5/2}c^4}+\frac{d(2bc-3ad)}{2ac^2\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^2(bc-ad)}+\frac{x}{ac\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b/x)^(3/2)*(c + d/x)^3),x]
```

```
[Out] (3*b*(2*b*c - a*d)*(2*b^2*c^2 - a*b*c*d + 4*a^2*d^2))/(4*a^2*c^3*(b*c - a*d
)^3*Sqrt[a + b/x]) + (d*(2*b*c - 3*a*d))/(2*a*c^2*(b*c - a*d)*Sqrt[a + b/x]
*(c + d/x)^2) + (d*(4*b^2*c^2 - 21*a*b*c*d + 12*a^2*d^2))/(4*a*c^3*(b*c - a
*d)^2*Sqrt[a + b/x]*(c + d/x)) + x/(a*c*Sqrt[a + b/x]*(c + d/x)^2) + (3*d^(
5/2)*(21*b^2*c^2 - 24*a*b*c*d + 8*a^2*d^2)*ArcTan[(Sqrt[d]*Sqrt[a + b/x])/S
qrt[b*c - a*d]])/(4*c^4*(b*c - a*d)^(7/2)) - (3*(b*c + 2*a*d)*ArcTanh[Sqrt[
a + b/x]/Sqrt[a]])/(a^(5/2)*c^4)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 375

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[((a + b/x^n)^p*(c + d/x^n)^q)/x^2, x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rubi steps

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)^3}\,dx = -\text{Subst}\left(\int \frac{1}{x^2(a+bx)^{3/2}(c+dx)^3}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{x}{ac\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^2} + \frac{\text{Subst}\left(\int \frac{\frac{3}{2}(bc+2ad)+\frac{7bdx}{2}}{x(a+bx)^{3/2}(c+dx)^3}\,dx, x, \frac{1}{x}\right)}{ac}$$

$$= \frac{d(2bc-3ad)}{2ac^2(bc-ad)\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^2} + \frac{x}{ac\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^2} - \frac{\text{Subst}\left(\int \frac{-3(bc-ad)(bc+2ad)-\frac{5}{2}bd(}{x(a+bx)^{3/2}(c+dx}\right.}{2ac^2(bc-ad}$$

$$= \frac{d(2bc-3ad)}{2ac^2(bc-ad)\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^2} + \frac{d\left(4b^2c^2-21abcd+12a^2d^2\right)}{4ac^3(bc-ad)^2\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)} + \frac{x}{ac\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)}$$

$$= \frac{3b(2bc-ad)\left(2b^2c^2-abcd+4a^2d^2\right)}{4a^2c^3(bc-ad)^3\sqrt{a+\frac{b}{x}}} + \frac{d(2bc-3ad)}{2ac^2(bc-ad)\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^2} + \frac{d\left(4b^2c^2-\right.}{4ac^3(bc-ac}$$

$$= \frac{3b(2bc-ad)\left(2b^2c^2-abcd+4a^2d^2\right)}{4a^2c^3(bc-ad)^3\sqrt{a+\frac{b}{x}}} + \frac{d(2bc-3ad)}{2ac^2(bc-ad)\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^2} + \frac{d\left(4b^2c^2-\right.}{4ac^3(bc-ac}$$

$$= \frac{3b(2bc-ad)\left(2b^2c^2-abcd+4a^2d^2\right)}{4a^2c^3(bc-ad)^3\sqrt{a+\frac{b}{x}}} + \frac{d(2bc-3ad)}{2ac^2(bc-ad)\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^2} + \frac{d\left(4b^2c^2-\right.}{4ac^3(bc-ac}$$

$$= \frac{3b(2bc-ad)\left(2b^2c^2-abcd+4a^2d^2\right)}{4a^2c^3(bc-ad)^3\sqrt{a+\frac{b}{x}}} + \frac{d(2bc-3ad)}{2ac^2(bc-ad)\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^2} + \frac{d\left(4b^2c^2-\right.}{4ac^3(bc-ac}$$

Mathematica [C] time = 0.32, size = 239, normalized size = 0.75

$$\frac{(cx+d)\left(2(cx+d)\left(\frac{3}{4}a^2d^2\left(8a^2d^2-24abcd+21b^2c^2\right){}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{d\left(a+\frac{b}{x}\right)}{ad-bc}\right)+3(2ad+bc)(bc-ad)^3{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{b}{ax}+1\right)\right)-\frac{1}{2}acdx(ad-bc)\left(12a^2d^2-21abcd+4b^2c^2\right)\right)+2ac^3x^3(bc-ad)^3-ac^2dx^2(bc-ad)^2(3ad-2bc)}{2a^2c^4\sqrt{a+\frac{b}{x}}(cx+d)^2(bc-ad)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b/x)^(3/2)*(c + d/x)^3),x]
```

```
[Out] (-(a*c^2*d*(b*c - a*d)^2*(-2*b*c + 3*a*d)*x^2) + 2*a*c^3*(b*c - a*d)^3*x^3
+ (d + c*x)*(-1/2*(a*c*d*(-(b*c) + a*d)*(4*b^2*c^2 - 21*a*b*c*d + 12*a^2*d^
2)*x) + 2*(d + c*x)*((3*a^2*d^2*(21*b^2*c^2 - 24*a*b*c*d + 8*a^2*d^2)*Hyper
geometric2F1[-1/2, 1, 1/2, (d*(a + b/x))/(-(b*c) + a*d)])/4 + 3*(b*c - a*d)
^3*(b*c + 2*a*d)*Hypergeometric2F1[-1/2, 1, 1/2, 1 + b/(a*x)])))/(2*a^2*c^4
*(b*c - a*d)^3*Sqrt[a + b/x]*(d + c*x)^2)
```

IntegrateAlgebraic [A] time = 1.34, size = 412, normalized size = 1.29

$$-\frac{3(2ad+bc)\tanh^{-1}\left(\frac{\sqrt{\frac{ax+b}{x}}}{\sqrt{a}}\right)}{a^{5/2}c^4}+\frac{3\left(8a^2d^{9/2}-24abcd^{7/2}+21b^2c^2d^{5/2}\right)\tan^{-1}\left(\frac{\sqrt{d}\sqrt{\frac{ax+b}{x}}}{\sqrt{bc-ad}}\right)}{4c^4(bc-ad)^{7/2}}+\frac{\sqrt{\frac{ax+b}{x}}\left(4a^4c^2d^3x^4+18a^4cd^4x^3+12a^4d^5x^2-12a^3bc^3d^2x^4-37a^3bc^2d^3x^3-9a^3bcd^4x^2+12a^3bd^5x+12a^2b^2c^4dx^4+12a^2b^2c^3d^2x^3-29a^2b^2c^2d^3x^2-27a^2b^2cd^4x-4ab^3c^5x^4+4ab^3c^4dx^3+20ab^3c^3d^2x^2+12ab^3c^2d^3x-12b^4c^5x^3-24b^4c^4dx^2-12b^4c^3d^2x\right)}{4a^2c^3(ax+b)(cx+d)^2(ad-bc)^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((a + b/x)^(3/2)*(c + d/x)^3),x]
```

```
[Out] (Sqrt[(b + a*x)/x]*(-12*b^4*c^3*d^2*x + 12*a*b^3*c^2*d^3*x - 27*a^2*b^2*c*d
^4*x + 12*a^3*b*d^5*x - 24*b^4*c^4*d*x^2 + 20*a*b^3*c^3*d^2*x^2 - 29*a^2*b^
2*c^2*d^3*x^2 - 9*a^3*b*c*d^4*x^2 + 12*a^4*d^5*x^2 - 12*b^4*c^5*x^3 + 4*a*b
^3*c^4*d*x^3 + 12*a^2*b^2*c^3*d^2*x^3 - 37*a^3*b*c^2*d^3*x^3 + 18*a^4*c*d^4
*x^3 - 4*a*b^3*c^5*x^4 + 12*a^2*b^2*c^4*d*x^4 - 12*a^3*b*c^3*d^2*x^4 + 4*a^
4*c^2*d^3*x^4))/(4*a^2*c^3*(-(b*c) + a*d)^3*(b + a*x)*(d + c*x)^2) + (3*(21
*b^2*c^2*d^(5/2) - 24*a*b*c*d^(7/2) + 8*a^2*d^(9/2))*ArcTan[(Sqrt[d]*Sqrt[(
b + a*x)/x])/Sqrt[b*c - a*d]])/(4*c^4*(b*c - a*d)^(7/2)) - (3*(b*c + 2*a*d)
*ArcTanh[Sqrt[(b + a*x)/x]/Sqrt[a]])/(a^(5/2)*c^4)
```

fricas [B] time = 4.78, size = 4093, normalized size = 12.79

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)^(3/2)/(c+d/x)^3,x, algorithm="fricas")
```

```
[Out] [1/8*(12*(b^5*c^4*d^2 - a*b^4*c^3*d^3 - 3*a^2*b^3*c^2*d^4 + 5*a^3*b^2*c*d^5
 - 2*a^4*b*d^6 + (a*b^4*c^6 - a^2*b^3*c^5*d - 3*a^3*b^2*c^4*d^2 + 5*a^4*b*c
^3*d^3 - 2*a^5*c^2*d^4)*x^3 + (b^5*c^6 + a*b^4*c^5*d - 5*a^2*b^3*c^4*d^2 -
a^3*b^2*c^3*d^3 + 8*a^4*b*c^2*d^4 - 4*a^5*c*d^5)*x^2 + (2*b^5*c^5*d - a*b^4
*c^4*d^2 - 7*a^2*b^3*c^3*d^3 + 7*a^3*b^2*c^2*d^4 + a^4*b*c*d^5 - 2*a^5*d^6)
*x)*sqrt(a)*log(2*a*x - 2*sqrt(a)*x*sqrt((a*x + b)/x) + b) - 3*(21*a^3*b^3*
c^2*d^4 - 24*a^4*b^2*c*d^5 + 8*a^5*b*d^6 + (21*a^4*b^2*c^4*d^2 - 24*a^5*b*c
^3*d^3 + 8*a^6*c^2*d^4)*x^3 + (21*a^3*b^3*c^4*d^2 + 18*a^4*b^2*c^3*d^3 - 40
*a^5*b*c^2*d^4 + 16*a^6*c*d^5)*x^2 + (42*a^3*b^3*c^3*d^3 - 27*a^4*b^2*c^2*d
^4 - 8*a^5*b*c*d^5 + 8*a^6*d^6)*x)*sqrt(-d/(b*c - a*d))*log(-(2*(b*c - a*d)
*x*sqrt(-d/(b*c - a*d))*sqrt((a*x + b)/x) - b*d + (b*c - 2*a*d)*x)/(c*x + d
)) + 2*(4*(a^2*b^3*c^6 - 3*a^3*b^2*c^5*d + 3*a^4*b*c^4*d^2 - a^5*c^3*d^3)*x
^4 + (12*a*b^4*c^6 - 4*a^2*b^3*c^5*d - 12*a^3*b^2*c^4*d^2 + 37*a^4*b*c^3*d^
3 - 18*a^5*c^2*d^4)*x^3 + (24*a*b^4*c^5*d - 20*a^2*b^3*c^4*d^2 + 29*a^3*b^2
*c^3*d^3 + 9*a^4*b*c^2*d^4 - 12*a^5*c*d^5)*x^2 + 3*(4*a*b^4*c^4*d^2 - 4*a^2
*b^3*c^3*d^3 + 9*a^3*b^2*c^2*d^4 - 4*a^4*b*c*d^5)*x)*sqrt((a*x + b)/x))/(a^
3*b^4*c^7*d^2 - 3*a^4*b^3*c^6*d^3 + 3*a^5*b^2*c^5*d^4 - a^6*b*c^4*d^5 + (a^
4*b^3*c^9 - 3*a^5*b^2*c^8*d + 3*a^6*b*c^7*d^2 - a^7*c^6*d^3)*x^3 + (a^3*b^4
*c^9 - a^4*b^3*c^8*d - 3*a^5*b^2*c^7*d^2 + 5*a^6*b*c^6*d^3 - 2*a^7*c^5*d^4)
*x^2 + (2*a^3*b^4*c^8*d - 5*a^4*b^3*c^7*d^2 + 3*a^5*b^2*c^6*d^3 + a^6*b*c^5
*d^4 - a^7*c^4*d^5)*x), 1/8*(24*(b^5*c^4*d^2 - a*b^4*c^3*d^3 - 3*a^2*b^3*c^
2*d^4 + 5*a^3*b^2*c*d^5 - 2*a^4*b*d^6 + (a*b^4*c^6 - a^2*b^3*c^5*d - 3*a^3*
b^2*c^4*d^2 + 5*a^4*b*c^3*d^3 - 2*a^5*c^2*d^4)*x^3 + (b^5*c^6 + a*b^4*c^5*d
 - 5*a^2*b^3*c^4*d^2 - a^3*b^2*c^3*d^3 + 8*a^4*b*c^2*d^4 - 4*a^5*c*d^5)*x^2
 + (2*b^5*c^5*d - a*b^4*c^4*d^2 - 7*a^2*b^3*c^3*d^3 + 7*a^3*b^2*c^2*d^4 + a
^4*b*c*d^5 - 2*a^5*d^6)*x)*sqrt(-a)*arctan(sqrt(-a)*sqrt((a*x + b)/x)/a) -
3*(21*a^3*b^3*c^2*d^4 - 24*a^4*b^2*c*d^5 + 8*a^5*b*d^6 + (21*a^4*b^2*c^4*d^
2 - 24*a^5*b*c^3*d^3 + 8*a^6*c^2*d^4)*x^3 + (21*a^3*b^3*c^4*d^2 + 18*a^4*b^
2*c^3*d^3 - 40*a^5*b*c^2*d^4 + 16*a^6*c*d^5)*x^2 + (42*a^3*b^3*c^3*d^3 - 27
*a^4*b^2*c^2*d^4 - 8*a^5*b*c*d^5 + 8*a^6*d^6)*x)*sqrt(-d/(b*c - a*d))*log(-
(2*(b*c - a*d)*x*sqrt(-d/(b*c - a*d))*sqrt((a*x + b)/x) - b*d + (b*c - 2*a*
d)*x)/(c*x + d)) + 2*(4*(a^2*b^3*c^6 - 3*a^3*b^2*c^5*d + 3*a^4*b*c^4*d^2 -
a^5*c^3*d^3)*x^4 + (12*a*b^4*c^6 - 4*a^2*b^3*c^5*d - 12*a^3*b^2*c^4*d^2 + 3
7*a^4*b*c^3*d^3 - 18*a^5*c^2*d^4)*x^3 + (24*a*b^4*c^5*d - 20*a^2*b^3*c^4*d^
2 + 29*a^3*b^2*c^3*d^3 + 9*a^4*b*c^2*d^4 - 12*a^5*c*d^5)*x^2 + 3*(4*a*b^4*c
^4*d^2 - 4*a^2*b^3*c^3*d^3 + 9*a^3*b^2*c^2*d^4 - 4*a^4*b*c*d^5)*x)*sqrt((a*
x + b)/x))/(a^3*b^4*c^7*d^2 - 3*a^4*b^3*c^6*d^3 + 3*a^5*b^2*c^5*d^4 - a^6*b
*c^4*d^5 + (a^4*b^3*c^9 - 3*a^5*b^2*c^8*d + 3*a^6*b*c^7*d^2 - a^7*c^6*d^3)*
x^3 + (a^3*b^4*c^9 - a^4*b^3*c^8*d - 3*a^5*b^2*c^7*d^2 + 5*a^6*b*c^6*d^3 -
2*a^7*c^5*d^4)*x^2 + (2*a^3*b^4*c^8*d - 5*a^4*b^3*c^7*d^2 + 3*a^5*b^2*c^6*d
^3 + a^6*b*c^5*d^4 - a^7*c^4*d^5)*x), 1/4*(3*(21*a^3*b^3*c^2*d^4 - 24*a^4*b
^2*c*d^5 + 8*a^5*b*d^6 + (21*a^4*b^2*c^4*d^2 - 24*a^5*b*c^3*d^3 + 8*a^6*c^2
```

```
*d^4)*x^3 + (21*a^3*b^3*c^4*d^2 + 18*a^4*b^2*c^3*d^3 - 40*a^5*b*c^2*d^4 + 1
6*a^6*c*d^5)*x^2 + (42*a^3*b^3*c^3*d^3 - 27*a^4*b^2*c^2*d^4 - 8*a^5*b*c*d^5
 + 8*a^6*d^6)*x)*sqrt(d/(b*c - a*d))*arctan(-(b*c - a*d)*x*sqrt(d/(b*c - a*
d))*sqrt((a*x + b)/x)/(a*d*x + b*d)) + 6*(b^5*c^4*d^2 - a*b^4*c^3*d^3 - 3*a
^2*b^3*c^2*d^4 + 5*a^3*b^2*c*d^5 - 2*a^4*b*d^6 + (a*b^4*c^6 - a^2*b^3*c^5*d
 - 3*a^3*b^2*c^4*d^2 + 5*a^4*b*c^3*d^3 - 2*a^5*c^2*d^4)*x^3 + (b^5*c^6 + a*
b^4*c^5*d - 5*a^2*b^3*c^4*d^2 - a^3*b^2*c^3*d^3 + 8*a^4*b*c^2*d^4 - 4*a^5*c
*d^5)*x^2 + (2*b^5*c^5*d - a*b^4*c^4*d^2 - 7*a^2*b^3*c^3*d^3 + 7*a^3*b^2*c^
2*d^4 + a^4*b*c*d^5 - 2*a^5*d^6)*x)*sqrt(a)*log(2*a*x - 2*sqrt(a)*x*sqrt((a
*x + b)/x) + b) + (4*(a^2*b^3*c^6 - 3*a^3*b^2*c^5*d + 3*a^4*b*c^4*d^2 - a^5
*c^3*d^3)*x^4 + (12*a*b^4*c^6 - 4*a^2*b^3*c^5*d - 12*a^3*b^2*c^4*d^2 + 37*a
^4*b*c^3*d^3 - 18*a^5*c^2*d^4)*x^3 + (24*a*b^4*c^5*d - 20*a^2*b^3*c^4*d^2 +
 29*a^3*b^2*c^3*d^3 + 9*a^4*b*c^2*d^4 - 12*a^5*c*d^5)*x^2 + 3*(4*a*b^4*c^4*
d^2 - 4*a^2*b^3*c^3*d^3 + 9*a^3*b^2*c^2*d^4 - 4*a^4*b*c*d^5)*x)*sqrt((a*x +
 b)/x))/(a^3*b^4*c^7*d^2 - 3*a^4*b^3*c^6*d^3 + 3*a^5*b^2*c^5*d^4 - a^6*b*c^
4*d^5 + (a^4*b^3*c^9 - 3*a^5*b^2*c^8*d + 3*a^6*b*c^7*d^2 - a^7*c^6*d^3)*x^3
 + (a^3*b^4*c^9 - a^4*b^3*c^8*d - 3*a^5*b^2*c^7*d^2 + 5*a^6*b*c^6*d^3 - 2*a
^7*c^5*d^4)*x^2 + (2*a^3*b^4*c^8*d - 5*a^4*b^3*c^7*d^2 + 3*a^5*b^2*c^6*d^3
+ a^6*b*c^5*d^4 - a^7*c^4*d^5)*x), 1/4*(3*(21*a^3*b^3*c^2*d^4 - 24*a^4*b^2*
c*d^5 + 8*a^5*b*d^6 + (21*a^4*b^2*c^4*d^2 - 24*a^5*b*c^3*d^3 + 8*a^6*c^2*d^
4)*x^3 + (21*a^3*b^3*c^4*d^2 + 18*a^4*b^2*c^3*d^3 - 40*a^5*b*c^2*d^4 + 16*a
^6*c*d^5)*x^2 + (42*a^3*b^3*c^3*d^3 - 27*a^4*b^2*c^2*d^4 - 8*a^5*b*c*d^5 +
8*a^6*d^6)*x)*sqrt(d/(b*c - a*d))*arctan(-(b*c - a*d)*x*sqrt(d/(b*c - a*d))
*sqrt((a*x + b)/x)/(a*d*x + b*d)) + 12*(b^5*c^4*d^2 - a*b^4*c^3*d^3 - 3*a^2
*b^3*c^2*d^4 + 5*a^3*b^2*c*d^5 - 2*a^4*b*d^6 + (a*b^4*c^6 - a^2*b^3*c^5*d -
 3*a^3*b^2*c^4*d^2 + 5*a^4*b*c^3*d^3 - 2*a^5*c^2*d^4)*x^3 + (b^5*c^6 + a*b^
4*c^5*d - 5*a^2*b^3*c^4*d^2 - a^3*b^2*c^3*d^3 + 8*a^4*b*c^2*d^4 - 4*a^5*c*d
^5)*x^2 + (2*b^5*c^5*d - a*b^4*c^4*d^2 - 7*a^2*b^3*c^3*d^3 + 7*a^3*b^2*c^2*
d^4 + a^4*b*c*d^5 - 2*a^5*d^6)*x)*sqrt(-a)*arctan(sqrt(-a)*sqrt((a*x + b)/x
)/a) + (4*(a^2*b^3*c^6 - 3*a^3*b^2*c^5*d + 3*a^4*b*c^4*d^2 - a^5*c^3*d^3)*x
^4 + (12*a*b^4*c^6 - 4*a^2*b^3*c^5*d - 12*a^3*b^2*c^4*d^2 + 37*a^4*b*c^3*d^
3 - 18*a^5*c^2*d^4)*x^3 + (24*a*b^4*c^5*d - 20*a^2*b^3*c^4*d^2 + 29*a^3*b^2
*c^3*d^3 + 9*a^4*b*c^2*d^4 - 12*a^5*c*d^5)*x^2 + 3*(4*a*b^4*c^4*d^2 - 4*a^2
*b^3*c^3*d^3 + 9*a^3*b^2*c^2*d^4 - 4*a^4*b*c*d^5)*x)*sqrt((a*x + b)/x))/(a^
3*b^4*c^7*d^2 - 3*a^4*b^3*c^6*d^3 + 3*a^5*b^2*c^5*d^4 - a^6*b*c^4*d^5 + (a^
4*b^3*c^9 - 3*a^5*b^2*c^8*d + 3*a^6*b*c^7*d^2 - a^7*c^6*d^3)*x^3 + (a^3*b^4
*c^9 - a^4*b^3*c^8*d - 3*a^5*b^2*c^7*d^2 + 5*a^6*b*c^6*d^3 - 2*a^7*c^5*d^4)
*x^2 + (2*a^3*b^4*c^8*d - 5*a^4*b^3*c^7*d^2 + 3*a^5*b^2*c^6*d^3 + a^6*b*c^5
*d^4 - a^7*c^4*d^5)*x)]
```

giac [A] time = 0.31, size = 516, normalized size = 1.61

$$\frac{1}{4}\,b^4\left(\frac{3\left(21\,b^2c^2d^3-24\,abcd^4+8\,a^2d^5\right)\arctan\left(\frac{d\sqrt{\frac{ax+b}{x}}}{\sqrt{bcd-ad^2}}\right)}{\left(b^7c^7-3\,ab^6c^6d+3\,a^2b^5c^5d^2-a^3b^4c^4d^3\right)\sqrt{bcd-ad^2}}+\frac{4\left(2\,ab^3c^3-\frac{3\,(ax+b)b^3c^3}{x}+\frac{3\,(ax+b)ab^2c^2d}{x}-\frac{3\,(ax+b)a^2bcd^2}{x}+\frac{(ax+b)a^3d^3}{x}\right)}{\left(a^2b^6c^6-3\,a^3b^5c^5d+3\,a^4b^4c^4d^2-a^5b^3c^3d^3\right)\left(a\sqrt{\frac{ax+b}{x}}-\frac{(ax+b)\sqrt{\frac{ax+b}{x}}}{x}\right)}+\frac{17\,b^2c^2d^3\sqrt{\frac{ax+b}{x}}-25\,abcd^4\sqrt{\frac{ax+b}{x}}+8\,a^2d^5\sqrt{\frac{ax+b}{x}}+\frac{15\,(ax+b)bcd^4\sqrt{\frac{ax+b}{x}}}{x}-\frac{8\,(ax+b)ad^5\sqrt{\frac{ax+b}{x}}}{x}}{\left(b^6c^6-3\,ab^5c^5d+3\,a^2b^4c^4d^2-a^3b^3c^3d^3\right)\left(bc-ad+\frac{(ax+b)d}{x}\right)^2}+\frac{12\,(bc+2\,ad)\arctan\left(\frac{\sqrt{\frac{ax+b}{x}}}{\sqrt{-a}}\right)}{\sqrt{-a}\,a^2b^4c^4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)^(3/2)/(c+d/x)^3,x, algorithm="giac")
```

```
[Out] 1/4*b^4*(3*(21*b^2*c^2*d^3 - 24*a*b*c*d^4 + 8*a^2*d^5)*arctan(d*sqrt((a*x +
 b)/x)/sqrt(b*c*d - a*d^2))/((b^7*c^7 - 3*a*b^6*c^6*d + 3*a^2*b^5*c^5*d^2 -
 a^3*b^4*c^4*d^3)*sqrt(b*c*d - a*d^2)) + 4*(2*a*b^3*c^3 - 3*(a*x + b)*b^3*c
^3/x + 3*(a*x + b)*a*b^2*c^2*d/x - 3*(a*x + b)*a^2*b*c*d^2/x + (a*x + b)*a^
3*d^3/x)/((a^2*b^6*c^6 - 3*a^3*b^5*c^5*d + 3*a^4*b^4*c^4*d^2 - a^5*b^3*c^3*
d^3)*(a*sqrt((a*x + b)/x) - (a*x + b)*sqrt((a*x + b)/x)/x)) + (17*b^2*c^2*d
^3*sqrt((a*x + b)/x) - 25*a*b*c*d^4*sqrt((a*x + b)/x) + 8*a^2*d^5*sqrt((a*x
 + b)/x) + 15*(a*x + b)*b*c*d^4*sqrt((a*x + b)/x)/x - 8*(a*x + b)*a*d^5*sqr
t((a*x + b)/x)/x)/((b^6*c^6 - 3*a*b^5*c^5*d + 3*a^2*b^4*c^4*d^2 - a^3*b^3*c
^3*d^3)*(b*c - a*d + (a*x + b)*d/x)^2) + 12*(b*c + 2*a*d)*arctan(sqrt((a*x
+ b)/x)/sqrt(-a))/(sqrt(-a)*a^2*b^4*c^4))
```

maple [B] time = 0.08, size = 5158, normalized size = 16.12

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b/x)^(3/2)/(c+d/x)^3,x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{\frac{3}{2}}\left(c+\frac{d}{x}\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)^(3/2)/(c+d/x)^3,x, algorithm="maxima")
```

```
[Out] integrate(1/((a + b/x)^(3/2)*(c + d/x)^3), x)
```

mupad [B] time = 9.49, size = 8936, normalized size = 27.92

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b/x)^(3/2)*(c + d/x)^3),x)
```

```
[Out] ((2*b^4)/(a^2*d - a*b*c) + (b*(a + b/x)*(12*a^4*d^4 + 12*b^4*c^4 + 24*a^2*b
^2*c^2*d^2 - 40*a*b^3*c^3*d - 33*a^3*b*c*d^3))/(4*a*c^3*(a^2*d - a*b*c)*(a*
d - b*c)) + (3*b*(a + b/x)^3*(4*a^3*d^5 - 4*b^3*c^3*d^2 + 4*a*b^2*c^2*d^3 -
 9*a^2*b*c*d^4))/(4*a*c^3*(a^2*d - a*b*c)*(a*d - b*c)^2) - (b*(a + b/x)^2*(
24*a^4*d^5 + 24*b^4*c^4*d - 56*a*b^3*c^3*d^2 + 65*a^2*b^2*c^2*d^3 - 72*a^3*
b*c*d^4))/(4*a*c^3*(a^2*d - a*b*c)*(a*d - b*c)^2))/((a + b/x)^(3/2)*(3*a^2*
d^2 + b^2*c^2 - 4*a*b*c*d) - (a + b/x)^(5/2)*(3*a*d^2 - 2*b*c*d) + d^2*(a +
 b/x)^(7/2) - (a + b/x)^(1/2)*(a^3*d^2 + a*b^2*c^2 - 2*a^2*b*c*d)) + (atan(
((((a + b/x)^(1/2)*(18432*a^6*b^19*c^26*d^3 - 202752*a^7*b^18*c^25*d^4 + 90
3168*a^8*b^17*c^24*d^5 - 1751040*a^9*b^16*c^23*d^6 - 137088*a^10*b^15*c^22*
d^7 + 6007680*a^11*b^14*c^21*d^8 + 1276416*a^12*b^13*c^20*d^9 - 65382912*a^
13*b^12*c^19*d^10 + 216610560*a^14*b^11*c^18*d^11 - 407418624*a^15*b^10*c^1
7*d^12 + 521961984*a^16*b^9*c^16*d^13 - 482904576*a^17*b^8*c^15*d^14 + 3288
09600*a^18*b^7*c^14*d^15 - 164257920*a^19*b^6*c^13*d^16 + 58816512*a^20*b^5
*c^12*d^17 - 14340096*a^21*b^4*c^11*d^18 + 2138112*a^22*b^3*c^10*d^19 - 147
456*a^23*b^2*c^9*d^20) - (3*(d^5*(a*d - b*c)^7)^(1/2)*(8*a^2*d^2 + 21*b^2*c
^2 - 24*a*b*c*d)*(12288*a^8*b^19*c^30*d^2 - 172032*a^9*b^18*c^29*d^3 + 1081
344*a^10*b^17*c^28*d^4 - 3996672*a^11*b^16*c^27*d^5 + 9449472*a^12*b^15*c^2
6*d^6 - 14112768*a^13*b^14*c^25*d^7 + 10407936*a^14*b^13*c^24*d^8 + 6454272
*a^15*b^12*c^23*d^9 - 30007296*a^16*b^11*c^22*d^10 + 45551616*a^17*b^10*c^2
1*d^11 - 44064768*a^18*b^9*c^20*d^12 + 30096384*a^19*b^8*c^19*d^13 - 148316
16*a^20*b^7*c^18*d^14 + 5203968*a^21*b^6*c^17*d^15 - 1241088*a^22*b^5*c^16*
d^16 + 181248*a^23*b^4*c^15*d^17 - 12288*a^24*b^3*c^14*d^18 - (3*(d^5*(a*d
- b*c)^7)^(1/2)*(a + b/x)^(1/2)*(8*a^2*d^2 + 21*b^2*c^2 - 24*a*b*c*d)*(8192
*a^10*b^18*c^33*d^2 - 139264*a^11*b^17*c^32*d^3 + 1105920*a^12*b^16*c^31*d^
4 - 5447680*a^13*b^15*c^30*d^5 + 18636800*a^14*b^14*c^29*d^6 - 46964736*a^1
5*b^13*c^28*d^7 + 90202112*a^16*b^12*c^27*d^8 - 134717440*a^17*b^11*c^26*d^
9 + 158146560*a^18*b^10*c^25*d^10 - 146432000*a^19*b^9*c^24*d^11 + 10660249
6*a^20*b^8*c^23*d^12 - 60383232*a^21*b^7*c^22*d^13 + 26091520*a^22*b^6*c^21
*d^14 - 8314880*a^23*b^5*c^20*d^15 + 1843200*a^24*b^4*c^19*d^16 - 253952*a^
25*b^3*c^18*d^17 + 16384*a^26*b^2*c^17*d^18))/(8*(b^7*c^11 - a^7*c^4*d^7 +
7*a^6*b*c^5*d^6 + 21*a^2*b^5*c^9*d^2 - 35*a^3*b^4*c^8*d^3 + 35*a^4*b^3*c^7*
```

d^4 - 21*a^5*b^2*c^6*d^5 - 7*a*b^6*c^10*d))))/(8*(b^7*c^11 - a^7*c^4*d^7 + 7*a^6*b*c^5*d^6 + 21*a^2*b^5*c^9*d^2 - 35*a^3*b^4*c^8*d^3 + 35*a^4*b^3*c^7*d^4 - 21*a^5*b^2*c^6*d^5 - 7*a*b^6*c^10*d)))*(d^5*(a*d - b*c)^7)^(1/2)*(8*a^2*d^2 + 21*b^2*c^2 - 24*a*b*c*d)*3i)/(8*(b^7*c^11 - a^7*c^4*d^7 + 7*a^6*b*c^5*d^6 + 21*a^2*b^5*c^9*d^2 - 35*a^3*b^4*c^8*d^3 + 35*a^4*b^3*c^7*d^4 - 21*a^5*b^2*c^6*d^5 - 7*a*b^6*c^10*d)) + (((a + b/x)^(1/2)*(18432*a^6*b^19*c^26*d^3 - 202752*a^7*b^18*c^25*d^4 + 903168*a^8*b^17*c^24*d^5 - 1751040*a^9*b^16*c^23*d^6 - 137088*a^10*b^15*c^22*d^7 + 6007680*a^11*b^14*c^21*d^8 + 1276416*a^12*b^13*c^20*d^9 - 65382912*a^13*b^12*c^19*d^10 + 216610560*a^14*b^11*c^18*d^11 - 407418624*a^15*b^10*c^17*d^12 + 521961984*a^16*b^9*c^16*d^13 - 482904576*a^17*b^8*c^15*d^14 + 328809600*a^18*b^7*c^14*d^15 - 164257920*a^19*b^6*c^13*d^16 + 58816512*a^20*b^5*c^12*d^17 - 14340096*a^21*b^4*c^11*d^18 + 2138112*a^22*b^3*c^10*d^19 - 147456*a^23*b^2*c^9*d^20) + (3*(d^5*(a*d - b*c)^7)^(1/2)*(8*a^2*d^2 + 21*b^2*c^2 - 24*a*b*c*d)*(12288*a^8*b^19*c^30*d^2 - 172032*a^9*b^18*c^29*d^3 + 1081344*a^10*b^17*c^28*d^4 - 3996672*a^11*b^16*c^27*d^5 + 9449472*a^12*b^15*c^26*d^6 - 14112768*a^13*b^14*c^25*d^7 + 10407936*a^14*b^13*c^24*d^8 + 6454272*a^15*b^12*c^23*d^9 - 30007296*a^16*b^11*c^22*d^10 + 45551616*a^17*b^10*c^21*d^11 - 44064768*a^18*b^9*c^20*d^12 + 30096384*a^19*b^8*c^19*d^13 - 14831616*a^20*b^7*c^18*d^14 + 5203968*a^21*b^6*c^17*d^15 - 1241088*a^22*b^5*c^16*d^16 + 181248*a^23*b^4*c^15*d^17 - 12288*a^24*b^3*c^14*d^18 + (3*(d^5*(a*d - b*c)^7)^(1/2)*(a + b/x)^(1/2)*(8*a^2*d^2 + 21*b^2*c^2 - 24*a*b*c*d)*(8192*a^10*b^18*c^33*d^2 - 139264*a^11*b^17*c^32*d^3 + 1105920*a^12*b^16*c^31*d^4 - 5447680*a^13*b^15*c^30*d^5 + 18636800*a^14*b^14*c^29*d^6 - 46964736*a^15*b^13*c^28*d^7 + 90202112*a^16*b^12*c^27*d^8 - 134717440*a^17*b^11*c^26*d^9 + 158146560*a^18*b^10*c^25*d^10 - 146432000*a^19*b^9*c^24*d^11 + 106602496*a^20*b^8*c^23*d^12 - 60383232*a^21*b^7*c^22*d^13 + 26091520*a^22*b^6*c^21*d^14 - 8314880*a^23*b^5*c^20*d^15 + 1843200*a^24*b^4*c^19*d^16 - 253952*a^25*b^3*c^18*d^17 + 16384*a^26*b^2*c^17*d^18))/(8*(b^7*c^11 - a^7*c^4*d^7 + 7*a^6*b*c^5*d^6 + 21*a^2*b^5*c^9*d^2 - 35*a^3*b^4*c^8*d^3 + 35*a^4*b^3*c^7*d^4 - 21*a^5*b^2*c^6*d^5 - 7*a*b^6*c^10*d))))/(8*(b^7*c^11 - a^7*c^4*d^7 + 7*a^6*b*c^5*d^6 + 21*a^2*b^5*c^9*d^2 - 35*a^3*b^4*c^8*d^3 + 35*a^4*b^3*c^7*d^4 - 21*a^5*b^2*c^6*d^5 - 7*a*b^6*c^10*d)))*(d^5*(a*d - b*c)^7)^(1/2)*(8*a^2*d^2 + 21*b^2*c^2 - 24*a*b*c*d)*3i)/(8*(b^7*c^11 - a^7*c^4*d^7 + 7*a^6*b*c^5*d^6 + 21*a^2*b^5*c^9*d^2 - 35*a^3*b^4*c^8*d^3 + 35*a^4*b^3*c^7*d^4 - 21*a^5*b^2*c^6*d^5 - 7*a*b^6*c^10*d)))/(290304*a^6*b^18*c^21*d^5 - 2654208*a^7*b^17*c^20*d^6 + 10675584*a^8*b^16*c^19*d^7 - 23497344*a^9*b^15*c^18*d^8 + 23604480*a^10*b^14*c^17*d^9 + 24731136*a^11*b^13*c^16*d^10 - 148172544*a^12*b^12*c^15*d^11 + 320101632*a^13*b^11*c^14*d^12 - 452086272*a^14*b^10*c^13*d^13 + 459302400*a^15*b^9*c^12*d^14 - 343108224*a^16*b^8*c^11*d^15 + 187373952*a^17*b^7*c^10*d^16 - 72873216*a^18*b^6*c^9*d^17 + 19132416*a^19*b^5*c^8*d^18 - 3041280*a^20*b^4*c^7*d^19 + 221184*a^21*b^3*c^6*d^20 - (3*((a + b/x)^(1/2)*(18432*a^6*b^19*c^26*d^3 - 202752*a^7*b^18*c^25*d^4 + 903168*a^8*b^17*c^24*d^5 - 1751040*a^9*b^16*c^23*d^6 - 137088*a^10*b^15*c^22*d^7 + 6007680*a^11*b^14*c^21*d^8 + 1276416*a^12*b^13*c^20*d^9 - 65382912*a^13*b^12*c^19*d^10 + 216610560*a^14*b^11*c^18*d^11 - 407418624*a^15*b^10*c^17*d^12 + 521961984*a^16*b^9*c^16*d^13 - 482904576*a^17*b^8*c^15*d^14 + 328809600*a^18*b^7*c^14*d^15 - 164257920*a^19*b^6*c^13*d^16 + 58816512*a^20*b^5*c^12*d^17 - 14340096*a^21*b^4*c^11*d^18 + 2138112*a^22*b^3*c^10*d^19 - 147456*a^23*b^2*c^9*d^20) - (3*(d^5*(a*d - b*c)^7)^(1/2)*(8*a^2*d^2 + 21*b^2*c^2 - 24*a*b*c*d)*(12288*a^8*b^19*c^30*d^2 - 172032*a^9*b^18*c^29*d^3 + 1081344*a^10*b^17*c^28*d^4 - 3996672*a^11*b^16*c^27*d^5 + 9449472*a^12*b^15*c^26*d^6 - 14112768*a^13*b^14*c^25*d^7 + 10407936*a^14*b^13*c^24*d^8 + 6454272*a^15*b^12*c^23*d^9 - 30007296*a^16*b^11*c^22*d^10 + 45551616*a^17*b^10*c^21*d^11 - 44064768*a^18*b^9*c^20*d^12 + 30096384*a^19*b^8*c^19*d^13 - 14831616*a^20*b^7*c^18*d^14 + 5203968*a^21*b^6*c^17*d^15 - 1241088*a^22*b^5*c^16*d^16 + 181248*a^23*b^4*c^15*d^17 - 12288*a^24*b^3*c^14*d^18 - (3*(d^5*(a*d - b*c)^7)^(1/2)*(a + b/x)^(1/2)*(8*a^2*d^2 + 21*b^2*c^2 - 24*a*b*c*d)*(8192*a^10*b^18*c^33*d^2 - 139264*a^11*b^17*c^32*d^3 + 1105920*a^12*b^16*c^31*d^4 - 5447680*a^13*b^15*c^30*d^5 + 18636800*a^14*b

^14*c^29*d^6 - 46964736*a^15*b^13*c^28*d^7 + 90202112*a^16*b^12*c^27*d^8 - 134717440*a^17*b^11*c^26*d^9 + 158146560*a^18*b^10*c^25*d^10 - 146432000*a^19*b^9*c^24*d^11 + 106602496*a^20*b^8*c^23*d^12 - 60383232*a^21*b^7*c^22*d^13 + 26091520*a^22*b^6*c^21*d^14 - 8314880*a^23*b^5*c^20*d^15 + 1843200*a^24*b^4*c^19*d^16 - 253952*a^25*b^3*c^18*d^17 + 16384*a^26*b^2*c^17*d^18))/(8*(b^7*c^11 - a^7*c^4*d^7 + 7*a^6*b*c^5*d^6 + 21*a^2*b^5*c^9*d^2 - 35*a^3*b^4*c^8*d^3 + 35*a^4*b^3*c^7*d^4 - 21*a^5*b^2*c^6*d^5 - 7*a*b^6*c^10*d))))/(8*(b^7*c^11 - a^7*c^4*d^7 + 7*a^6*b*c^5*d^6 + 21*a^2*b^5*c^9*d^2 - 35*a^3*b^4*c^8*d^3 + 35*a^4*b^3*c^7*d^4 - 21*a^5*b^2*c^6*d^5 - 7*a*b^6*c^10*d)))*(d^5*(a*d - b*c)^7)^(1/2)*(8*a^2*d^2 + 21*b^2*c^2 - 24*a*b*c*d))/(8*(b^7*c^11 - a^7*c^4*d^7 + 7*a^6*b*c^5*d^6 + 21*a^2*b^5*c^9*d^2 - 35*a^3*b^4*c^8*d^3 + 35*a^4*b^3*c^7*d^4 - 21*a^5*b^2*c^6*d^5 - 7*a*b^6*c^10*d)) + (3*((a + b/x)^(1/2)*(18432*a^6*b^19*c^26*d^3 - 202752*a^7*b^18*c^25*d^4 + 903168*a^8*b^17*c^24*d^5 - 1751040*a^9*b^16*c^23*d^6 - 137088*a^10*b^15*c^22*d^7 + 6007680*a^11*b^14*c^21*d^8 + 1276416*a^12*b^13*c^20*d^9 - 65382912*a^13*b^12*c^19*d^10 + 216610560*a^14*b^11*c^18*d^11 - 407418624*a^15*b^10*c^17*d^12 + 521961984*a^16*b^9*c^16*d^13 - 482904576*a^17*b^8*c^15*d^14 + 328809600*a^18*b^7*c^14*d^15 - 164257920*a^19*b^6*c^13*d^16 + 58816512*a^20*b^5*c^12*d^17 - 14340096*a^21*b^4*c^11*d^18 + 2138112*a^22*b^3*c^10*d^19 - 147456*a^23*b^2*c^9*d^20) + (3*(d^5*(a*d - b*c)^7)^(1/2)*(8*a^2*d^2 + 21*b^2*c^2 - 24*a*b*c*d)*(12288*a^8*b^19*c^30*d^2 - 172032*a^9*b^18*c^29*d^3 + 1081344*a^10*b^17*c^28*d^4 - 3996672*a^11*b^16*c^27*d^5 + 9449472*a^12*b^15*c^26*d^6 - 14112768*a^13*b^14*c^25*d^7 + 10407936*a^14*b^13*c^24*d^8 + 6454272*a^15*b^12*c^23*d^9 - 30007296*a^16*b^11*c^22*d^10 + 45551616*a^17*b^10*c^21*d^11 - 44064768*a^18*b^9*c^20*d^12 + 30096384*a^19*b^8*c^19*d^13 - 14831616*a^20*b^7*c^18*d^14 + 5203968*a^21*b^6*c^17*d^15 - 1241088*a^22*b^5*c^16*d^16 + 181248*a^23*b^4*c^15*d^17 - 12288*a^24*b^3*c^14*d^18 + (3*(d^5*(a*d - b*c)^7)^(1/2)*(a + b/x)^(1/2)*(8*a^2*d^2 + 21*b^2*c^2 - 24*a*b*c*d)*(8192*a^10*b^18*c^33*d^2 - 139264*a^11*b^17*c^32*d^3 + 1105920*a^12*b^16*c^31*d^4 - 5447680*a^13*b^15*c^30*d^5 + 18636800*a^14*b^14*c^29*d^6 - 46964736*a^15*b^13*c^28*d^7 + 90202112*a^16*b^12*c^27*d^8 - 134717440*a^17*b^11*c^26*d^9 + 158146560*a^18*b^10*c^25*d^10 - 146432000*a^19*b^9*c^24*d^11 + 106602496*a^20*b^8*c^23*d^12 - 60383232*a^21*b^7*c^22*d^13 + 26091520*a^22*b^6*c^21*d^14 - 8314880*a^23*b^5*c^20*d^15 + 1843200*a^24*b^4*c^19*d^16 - 253952*a^25*b^3*c^18*d^17 + 16384*a^26*b^2*c^17*d^18))/(8*(b^7*c^11 - a^7*c^4*d^7 + 7*a^6*b*c^5*d^6 + 21*a^2*b^5*c^9*d^2 - 35*a^3*b^4*c^8*d^3 + 35*a^4*b^3*c^7*d^4 - 21*a^5*b^2*c^6*d^5 - 7*a*b^6*c^10*d))))/(8*(b^7*c^11 - a^7*c^4*d^7 + 7*a^6*b*c^5*d^6 + 21*a^2*b^5*c^9*d^2 - 35*a^3*b^4*c^8*d^3 + 35*a^4*b^3*c^7*d^4 - 21*a^5*b^2*c^6*d^5 - 7*a*b^6*c^10*d)))*(d^5*(a*d - b*c)^7)^(1/2)*(8*a^2*d^2 + 21*b^2*c^2 - 24*a*b*c*d))/(8*(b^7*c^11 - a^7*c^4*d^7 + 7*a^6*b*c^5*d^6 + 21*a^2*b^5*c^9*d^2 - 35*a^3*b^4*c^8*d^3 + 35*a^4*b^3*c^7*d^4 - 21*a^5*b^2*c^6*d^5 - 7*a*b^6*c^10*d))))*(d^5*(a*d - b*c)^7)^(1/2)*(8*a^2*d^2 + 21*b^2*c^2 - 24*a*b*c*d)*3i)/(4*(b^7*c^11 - a^7*c^4*d^7 + 7*a^6*b*c^5*d^6 + 21*a^2*b^5*c^9*d^2 - 35*a^3*b^4*c^8*d^3 + 35*a^4*b^3*c^7*d^4 - 21*a^5*b^2*c^6*d^5 - 7*a*b^6*c^10*d)) + (atan((((2*a*d + b*c)*((a + b/x)^(1/2)*(18432*a^6*b^19*c^26*d^3 - 202752*a^7*b^18*c^25*d^4 + 903168*a^8*b^17*c^24*d^5 - 1751040*a^9*b^16*c^23*d^6 - 137088*a^10*b^15*c^22*d^7 + 6007680*a^11*b^14*c^21*d^8 + 1276416*a^12*b^13*c^20*d^9 - 65382912*a^13*b^12*c^19*d^10 + 216610560*a^14*b^11*c^18*d^11 - 407418624*a^15*b^10*c^17*d^12 + 521961984*a^16*b^9*c^16*d^13 - 482904576*a^17*b^8*c^15*d^14 + 328809600*a^18*b^7*c^14*d^15 - 164257920*a^19*b^6*c^13*d^16 + 58816512*a^20*b^5*c^12*d^17 - 14340096*a^21*b^4*c^11*d^18 + 2138112*a^22*b^3*c^10*d^19 - 147456*a^23*b^2*c^9*d^20) - (3*(2*a*d + b*c)*(12288*a^8*b^19*c^30*d^2 - 172032*a^9*b^18*c^29*d^3 + 1081344*a^10*b^17*c^28*d^4 - 3996672*a^11*b^16*c^27*d^5 + 9449472*a^12*b^15*c^26*d^6 - 14112768*a^13*b^14*c^25*d^7 + 10407936*a^14*b^13*c^24*d^8 + 6454272*a^15*b^12*c^23*d^9 - 30007296*a^16*b^11*c^22*d^10 + 45551616*a^17*b^10*c^21*d^11 - 44064768*a^18*b^9*c^20*d^12 + 30096384*a^19*b^8*c^19*d^13 - 14831616*a^20*b^7*c^18*d^14 + 5203968*a^21*b^6*c^17*d^15 - 1241088*a^22*b^5*c^16*d^16 + 181248*a^23*b^4*c^15*d^17 - 12288*a^24*b^3*c^14*d^18 - (3*(a + b/x)^(1/2)*(2*a*d +

b*c)*(8192*a^10*b^18*c^33*d^2 - 139264*a^11*b^17*c^32*d^3 + 1105920*a^12*b
^16*c^31*d^4 - 5447680*a^13*b^15*c^30*d^5 + 18636800*a^14*b^14*c^29*d^6 - 4
6964736*a^15*b^13*c^28*d^7 + 90202112*a^16*b^12*c^27*d^8 - 134717440*a^17*b
^11*c^26*d^9 + 158146560*a^18*b^10*c^25*d^10 - 146432000*a^19*b^9*c^24*d^11
 + 106602496*a^20*b^8*c^23*d^12 - 60383232*a^21*b^7*c^22*d^13 + 26091520*a^
22*b^6*c^21*d^14 - 8314880*a^23*b^5*c^20*d^15 + 1843200*a^24*b^4*c^19*d^16
 - 253952*a^25*b^3*c^18*d^17 + 16384*a^26*b^2*c^17*d^18))/(2*c^4*(a^5)^(1/2)
)))/(2*c^4*(a^5)^(1/2)))*3i)/(2*c^4*(a^5)^(1/2)) + ((2*a*d + b*c)*((a + b/x
)^(1/2)*(18432*a^6*b^19*c^26*d^3 - 202752*a^7*b^18*c^25*d^4 + 903168*a^8*b^
17*c^24*d^5 - 1751040*a^9*b^16*c^23*d^6 - 137088*a^10*b^15*c^22*d^7 + 60076
80*a^11*b^14*c^21*d^8 + 1276416*a^12*b^13*c^20*d^9 - 65382912*a^13*b^12*c^1
9*d^10 + 216610560*a^14*b^11*c^18*d^11 - 407418624*a^15*b^10*c^17*d^12 + 52
1961984*a^16*b^9*c^16*d^13 - 482904576*a^17*b^8*c^15*d^14 + 328809600*a^18*
b^7*c^14*d^15 - 164257920*a^19*b^6*c^13*d^16 + 58816512*a^20*b^5*c^12*d^17
 - 14340096*a^21*b^4*c^11*d^18 + 2138112*a^22*b^3*c^10*d^19 - 147456*a^23*b^
2*c^9*d^20) + (3*(2*a*d + b*c)*(12288*a^8*b^19*c^30*d^2 - 172032*a^9*b^18*c
^29*d^3 + 1081344*a^10*b^17*c^28*d^4 - 3996672*a^11*b^16*c^27*d^5 + 9449472
*a^12*b^15*c^26*d^6 - 14112768*a^13*b^14*c^25*d^7 + 10407936*a^14*b^13*c^24
*d^8 + 6454272*a^15*b^12*c^23*d^9 - 30007296*a^16*b^11*c^22*d^10 + 45551616
*a^17*b^10*c^21*d^11 - 44064768*a^18*b^9*c^20*d^12 + 30096384*a^19*b^8*c^19
*d^13 - 14831616*a^20*b^7*c^18*d^14 + 5203968*a^21*b^6*c^17*d^15 - 1241088*
a^22*b^5*c^16*d^16 + 181248*a^23*b^4*c^15*d^17 - 12288*a^24*b^3*c^14*d^18 +
 (3*(a + b/x)^(1/2)*(2*a*d + b*c)*(8192*a^10*b^18*c^33*d^2 - 139264*a^11*b^
17*c^32*d^3 + 1105920*a^12*b^16*c^31*d^4 - 5447680*a^13*b^15*c^30*d^5 + 186
36800*a^14*b^14*c^29*d^6 - 46964736*a^15*b^13*c^28*d^7 + 90202112*a^16*b^12
*c^27*d^8 - 134717440*a^17*b^11*c^26*d^9 + 158146560*a^18*b^10*c^25*d^10 -
146432000*a^19*b^9*c^24*d^11 + 106602496*a^20*b^8*c^23*d^12 - 60383232*a^21
*b^7*c^22*d^13 + 26091520*a^22*b^6*c^21*d^14 - 8314880*a^23*b^5*c^20*d^15 +
 1843200*a^24*b^4*c^19*d^16 - 253952*a^25*b^3*c^18*d^17 + 16384*a^26*b^2*c^
17*d^18))/(2*c^4*(a^5)^(1/2))))/(2*c^4*(a^5)^(1/2)))*3i)/(2*c^4*(a^5)^(1/2)
))/(290304*a^6*b^18*c^21*d^5 - 2654208*a^7*b^17*c^20*d^6 + 10675584*a^8*b^1
6*c^19*d^7 - 23497344*a^9*b^15*c^18*d^8 + 23604480*a^10*b^14*c^17*d^9 + 247
31136*a^11*b^13*c^16*d^10 - 148172544*a^12*b^12*c^15*d^11 + 320101632*a^13*
b^11*c^14*d^12 - 452086272*a^14*b^10*c^13*d^13 + 459302400*a^15*b^9*c^12*d^
14 - 343108224*a^16*b^8*c^11*d^15 + 187373952*a^17*b^7*c^10*d^16 - 72873216
*a^18*b^6*c^9*d^17 + 19132416*a^19*b^5*c^8*d^18 - 3041280*a^20*b^4*c^7*d^19
 + 221184*a^21*b^3*c^6*d^20 - (3*(2*a*d + b*c)*((a + b/x)^(1/2)*(18432*a^6*
b^19*c^26*d^3 - 202752*a^7*b^18*c^25*d^4 + 903168*a^8*b^17*c^24*d^5 - 17510
40*a^9*b^16*c^23*d^6 - 137088*a^10*b^15*c^22*d^7 + 6007680*a^11*b^14*c^21*d
^8 + 1276416*a^12*b^13*c^20*d^9 - 65382912*a^13*b^12*c^19*d^10 + 216610560*
a^14*b^11*c^18*d^11 - 407418624*a^15*b^10*c^17*d^12 + 521961984*a^16*b^9*c^
16*d^13 - 482904576*a^17*b^8*c^15*d^14 + 328809600*a^18*b^7*c^14*d^15 - 164
257920*a^19*b^6*c^13*d^16 + 58816512*a^20*b^5*c^12*d^17 - 14340096*a^21*b^4
*c^11*d^18 + 2138112*a^22*b^3*c^10*d^19 - 147456*a^23*b^2*c^9*d^20) - (3*(2
*a*d + b*c)*(12288*a^8*b^19*c^30*d^2 - 172032*a^9*b^18*c^29*d^3 + 1081344*a
^10*b^17*c^28*d^4 - 3996672*a^11*b^16*c^27*d^5 + 9449472*a^12*b^15*c^26*d^6
 - 14112768*a^13*b^14*c^25*d^7 + 10407936*a^14*b^13*c^24*d^8 + 6454272*a^15
*b^12*c^23*d^9 - 30007296*a^16*b^11*c^22*d^10 + 45551616*a^17*b^10*c^21*d^1
1 - 44064768*a^18*b^9*c^20*d^12 + 30096384*a^19*b^8*c^19*d^13 - 14831616*a^
20*b^7*c^18*d^14 + 5203968*a^21*b^6*c^17*d^15 - 1241088*a^22*b^5*c^16*d^16
+ 181248*a^23*b^4*c^15*d^17 - 12288*a^24*b^3*c^14*d^18 - (3*(a + b/x)^(1/2)
*(2*a*d + b*c)*(8192*a^10*b^18*c^33*d^2 - 139264*a^11*b^17*c^32*d^3 + 11059
20*a^12*b^16*c^31*d^4 - 5447680*a^13*b^15*c^30*d^5 + 18636800*a^14*b^14*c^2
9*d^6 - 46964736*a^15*b^13*c^28*d^7 + 90202112*a^16*b^12*c^27*d^8 - 1347174
40*a^17*b^11*c^26*d^9 + 158146560*a^18*b^10*c^25*d^10 - 146432000*a^19*b^9*
c^24*d^11 + 106602496*a^20*b^8*c^23*d^12 - 60383232*a^21*b^7*c^22*d^13 + 26
091520*a^22*b^6*c^21*d^14 - 8314880*a^23*b^5*c^20*d^15 + 1843200*a^24*b^4*c
^19*d^16 - 253952*a^25*b^3*c^18*d^17 + 16384*a^26*b^2*c^17*d^18))/(2*c^4*(a
^5)^(1/2))))/(2*c^4*(a^5)^(1/2))))/(2*c^4*(a^5)^(1/2)) + (3*(2*a*d + b*c)*(

```
(a + b/x)^(1/2)*(18432*a^6*b^19*c^26*d^3 - 202752*a^7*b^18*c^25*d^4 + 90316
8*a^8*b^17*c^24*d^5 - 1751040*a^9*b^16*c^23*d^6 - 137088*a^10*b^15*c^22*d^7
 + 6007680*a^11*b^14*c^21*d^8 + 1276416*a^12*b^13*c^20*d^9 - 65382912*a^13*
b^12*c^19*d^10 + 216610560*a^14*b^11*c^18*d^11 - 407418624*a^15*b^10*c^17*d
^12 + 521961984*a^16*b^9*c^16*d^13 - 482904576*a^17*b^8*c^15*d^14 + 3288096
00*a^18*b^7*c^14*d^15 - 164257920*a^19*b^6*c^13*d^16 + 58816512*a^20*b^5*c^
12*d^17 - 14340096*a^21*b^4*c^11*d^18 + 2138112*a^22*b^3*c^10*d^19 - 147456
*a^23*b^2*c^9*d^20) + (3*(2*a*d + b*c)*(12288*a^8*b^19*c^30*d^2 - 172032*a^
9*b^18*c^29*d^3 + 1081344*a^10*b^17*c^28*d^4 - 3996672*a^11*b^16*c^27*d^5 +
 9449472*a^12*b^15*c^26*d^6 - 14112768*a^13*b^14*c^25*d^7 + 10407936*a^14*b
^13*c^24*d^8 + 6454272*a^15*b^12*c^23*d^9 - 30007296*a^16*b^11*c^22*d^10 +
45551616*a^17*b^10*c^21*d^11 - 44064768*a^18*b^9*c^20*d^12 + 30096384*a^19*
b^8*c^19*d^13 - 14831616*a^20*b^7*c^18*d^14 + 5203968*a^21*b^6*c^17*d^15 -
1241088*a^22*b^5*c^16*d^16 + 181248*a^23*b^4*c^15*d^17 - 12288*a^24*b^3*c^1
4*d^18 + (3*(a + b/x)^(1/2)*(2*a*d + b*c)*(8192*a^10*b^18*c^33*d^2 - 139264
*a^11*b^17*c^32*d^3 + 1105920*a^12*b^16*c^31*d^4 - 5447680*a^13*b^15*c^30*d
^5 + 18636800*a^14*b^14*c^29*d^6 - 46964736*a^15*b^13*c^28*d^7 + 90202112*a
^16*b^12*c^27*d^8 - 134717440*a^17*b^11*c^26*d^9 + 158146560*a^18*b^10*c^25
*d^10 - 146432000*a^19*b^9*c^24*d^11 + 106602496*a^20*b^8*c^23*d^12 - 60383
232*a^21*b^7*c^22*d^13 + 26091520*a^22*b^6*c^21*d^14 - 8314880*a^23*b^5*c^2
0*d^15 + 1843200*a^24*b^4*c^19*d^16 - 253952*a^25*b^3*c^18*d^17 + 16384*a^2
6*b^2*c^17*d^18))/(2*c^4*(a^5)^(1/2))))/(2*c^4*(a^5)^(1/2))))/(2*c^4*(a^5)^
(1/2))))*(2*a*d + b*c)*3i)/(c^4*(a^5)^(1/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)**(3/2)/(c+d/x)**3,x)
```

```
[Out] Timed out
```

3.162 $\int \frac{\left(c+\frac{d}{x}\right)^3}{\left(a+\frac{b}{x}\right)^{5/2}}\,dx$

Optimal. Leaf size=143

$$-\frac{c^2(5bc-6ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{7/2}}+\frac{(bc-ad)\left(-4a^3d^2x-2a^2bd(5cx+3d)+ab^2c(20cx-3d)+15b^3c^2\right)}{3a^3b^2x\left(a+\frac{b}{x}\right)^{3/2}}+\frac{cx\left(c+\frac{d}{x}\right)^2}{a\left(a+\frac{b}{x}\right)^{3/2}}$$

Rubi [A] time = 0.15, antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {375, 98, 145, 63, 208}

$$\frac{(bc-ad)\left(-2a^2bd(5cx+3d)-4a^3d^2x+ab^2c(20cx-3d)+15b^3c^2\right)}{3a^3b^2x\left(a+\frac{b}{x}\right)^{3/2}}-\frac{c^2(5bc-6ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{7/2}}+\frac{cx\left(c+\frac{d}{x}\right)^2}{a\left(a+\frac{b}{x}\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d/x)^3/(a + b/x)^(5/2),x]
```

```
[Out] (c*(c + d/x)^2*x)/(a*(a + b/x)^(3/2)) + ((b*c - a*d)*(15*b^3*c^2 - 4*a^3*d^
2*x - 2*a^2*b*d*(3*d + 5*c*x) + a*b^2*c*(-3*d + 20*c*x)))/(3*a^3*b^2*(a + b
/x)^(3/2)*x) - (c^2*(5*b*c - 6*a*d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/a^(7/2)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 145

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_)
)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b^3*c*e*g*(m + 2) - a^3*d*f*h*(
n + 2) - a^2*b*(c*f*h*m - d*(f*g + e*h)*(m + n + 3)) - a*b^2*(c*(f*g + e*h)
 + d*e*g*(2*m + n + 4)) + b*(a^2*d*f*h*(m - n) - a*b*(2*c*f*h*(m + 1) - d*(
f*g + e*h)*(n + 1)) + b^2*(c*(f*g + e*h)*(m + 1) - d*e*g*(m + n + 2)))*x)*(
a + b*x)^(m + 1)*(c + d*x)^(n + 1))/(b^2*(b*c - a*d)^2*(m + 1)*(m + 2)), x]
 + Dist[(f*h)/b^2 - (d*(m + n + 3)*(a^2*d*f*h*(m - n) - a*b*(2*c*f*h*(m + 1
) - d*(f*g + e*h)*(n + 1)) + b^2*(c*(f*g + e*h)*(m + 1) - d*e*g*(m + n + 2)
)))/(b^2*(b*c - a*d)^2*(m + 1)*(m + 2)), Int[(a + b*x)^(m + 2)*(c + d*x)^n,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x] && (LtQ[m, -2] || (EqQ[
```

```
m + n + 3, 0] &&  !LtQ[n, -2]))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 375

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[((a + b/x^n)^p*(c + d/x^n)^q)/x^2, x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rubi steps

$$\int \frac{\left(c+\frac{d}{x}\right)^3}{\left(a+\frac{b}{x}\right)^{5/2}}\,dx = -\text{Subst}\left(\int \frac{(c+dx)^3}{x^2(a+bx)^{5/2}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{c\left(c+\frac{d}{x}\right)^2 x}{a\left(a+\frac{b}{x}\right)^{3/2}} + \frac{\text{Subst}\left(\int \frac{(c+dx)\left(\frac{1}{2}c(5bc-6ad)+\frac{1}{2}d(bc-2ad)x\right)}{x(a+bx)^{5/2}}\,dx, x, \frac{1}{x}\right)}{a}$$

$$= \frac{c\left(c+\frac{d}{x}\right)^2 x}{a\left(a+\frac{b}{x}\right)^{3/2}} + \frac{(bc-ad)\left(15b^3c^2-4a^3d^2x-ab^2c(3d-20cx)-2a^2bd(3d+5cx)\right)}{3a^3b^2\left(a+\frac{b}{x}\right)^{3/2}x} + \frac{\left(c^2(5bc\right.}{}$$

$$= \frac{c\left(c+\frac{d}{x}\right)^2 x}{a\left(a+\frac{b}{x}\right)^{3/2}} + \frac{(bc-ad)\left(15b^3c^2-4a^3d^2x-ab^2c(3d-20cx)-2a^2bd(3d+5cx)\right)}{3a^3b^2\left(a+\frac{b}{x}\right)^{3/2}x} + \frac{\left(c^2(5bc\right.}{}$$

$$= \frac{c\left(c+\frac{d}{x}\right)^2 x}{a\left(a+\frac{b}{x}\right)^{3/2}} + \frac{(bc-ad)\left(15b^3c^2-4a^3d^2x-ab^2c(3d-20cx)-2a^2bd(3d+5cx)\right)}{3a^3b^2\left(a+\frac{b}{x}\right)^{3/2}x} - \frac{c^2(5bc-}{}$$

Mathematica [A] time = 0.43, size = 145, normalized size = 1.01

$$\frac{\frac{4a^5d^3x}{b^2}+\frac{6a^4d^2(cx+d)}{b}+3a^3c^2x(cx-8d)+2a^2bc^2(10cx-9d)+15ab^2c^3+3ac^2\sqrt{\frac{b}{ax}+1}\,(ax+b)(6ad-5bc)\tanh^{-1}\left(\sqrt{\frac{b}{ax}+1}\right)}{3a^4\sqrt{a+\frac{b}{x}}\,(ax+b)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d/x)^3/(a + b/x)^(5/2),x]
```

```
[Out] (15*a*b^2*c^3 + (4*a^5*d^3*x)/b^2 + 3*a^3*c^2*x*(-8*d + c*x) + (6*a^4*d^2*(
d + c*x))/b + 2*a^2*b*c^2*(-9*d + 10*c*x) + 3*a*c^2*(-5*b*c + 6*a*d)*Sqrt[1
 + b/(a*x)]*(b + a*x)*ArcTanh[Sqrt[1 + b/(a*x)]])/(3*a^4*Sqrt[a + b/x]*(b +
 a*x))
```

IntegrateAlgebraic [A] time = 0.28, size = 167, normalized size = 1.17

$$\frac{\left(6ac^2d-5bc^3\right)\tanh^{-1}\left(\frac{\sqrt{\frac{ax+b}{x}}}{\sqrt{a}}\right)}{a^{7/2}}+\frac{\sqrt{\frac{ax+b}{x}}\left(4a^4d^3x^2+6a^3bcd^2x^2+6a^3bd^3x+3a^2b^2c^3x^3-24a^2b^2c^2dx^2+20ab^3c^3x^2-18ab^3c^2dx+15b^4c^3x\right)}{3a^3b^2(ax+b)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c + d/x)^3/(a + b/x)^(5/2),x]
```

```
[Out] (Sqrt[(b + a*x)/x]*(15*b^4*c^3*x - 18*a*b^3*c^2*d*x + 6*a^3*b*d^3*x + 20*a*
b^3*c^3*x^2 - 24*a^2*b^2*c^2*d*x^2 + 6*a^3*b*c*d^2*x^2 + 4*a^4*d^3*x^2 + 3*
a^2*b^2*c^3*x^3))/(3*a^3*b^2*(b + a*x)^2) + ((-5*b*c^3 + 6*a*c^2*d)*ArcTanh
[Sqrt[(b + a*x)/x]/Sqrt[a]])/a^(7/2)
```

fricas [A] time = 0.82, size = 483, normalized size = 3.38

$$\left[-\frac{3\left(5 b^{5} c^{3}-6 a b^{4} c^{2} d+\left(5 a^{2} b^{3} c^{3}-6 a^{3} b^{2} c^{2} d\right) x^{2}+2\left(5 a b^{4} c^{3}-6 a^{2} b^{3} c^{2} d\right) x\right) \sqrt{a} \log \left(2 a x+2 \sqrt{a} x \sqrt{\frac{a x+b}{x}}+b\right)-2\left(3 a^{3} b^{2} c^{3} x^{3}+2\left(10 a^{2} b^{3} c^{3}-12 a^{3} b^{2} c^{2} d+3 a^{4} b c d^{2}+2 a^{5} d^{3}\right) x^{2}+3\left(5 a b^{4} c^{3}-6 a^{2} b^{3} c^{2} d+2 a^{4} b d^{3}\right) x\right) \sqrt{\frac{a x+b}{x}}}{6\left(a^{6} b^{2} x^{2}+2 a^{5} b^{3} x+a^{4} b^{4}\right)}, \frac{3\left(5 b^{5} c^{3}-6 a b^{4} c^{2} d+\left(5 a^{2} b^{3} c^{3}-6 a^{3} b^{2} c^{2} d\right) x^{2}+2\left(5 a b^{4} c^{3}-6 a^{2} b^{3} c^{2} d\right) x\right) \sqrt{-a} \arctan \left(\frac{\sqrt{-a} \sqrt{\frac{a x+b}{x}}}{a}\right)+\left(3 a^{3} b^{2} c^{3} x^{3}+2\left(10 a^{2} b^{3} c^{3}-12 a^{3} b^{2} c^{2} d+3 a^{4} b c d^{2}+2 a^{5} d^{3}\right) x^{2}+3\left(5 a b^{4} c^{3}-6 a^{2} b^{3} c^{2} d+2 a^{4} b d^{3}\right) x\right) \sqrt{\frac{a x+b}{x}}}{3\left(a^{6} b^{2} x^{2}+2 a^{5} b^{3} x+a^{4} b^{4}\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d/x)^3/(a+b/x)^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/6*(3*(5*b^5*c^3 - 6*a*b^4*c^2*d + (5*a^2*b^3*c^3 - 6*a^3*b^2*c^2*d)*x^2
 + 2*(5*a*b^4*c^3 - 6*a^2*b^3*c^2*d)*x)*sqrt(a)*log(2*a*x + 2*sqrt(a)*x*sqr
t((a*x + b)/x) + b) - 2*(3*a^3*b^2*c^3*x^3 + 2*(10*a^2*b^3*c^3 - 12*a^3*b^2
*c^2*d + 3*a^4*b*c*d^2 + 2*a^5*d^3)*x^2 + 3*(5*a*b^4*c^3 - 6*a^2*b^3*c^2*d
+ 2*a^4*b*d^3)*x)*sqrt((a*x + b)/x))/(a^6*b^2*x^2 + 2*a^5*b^3*x + a^4*b^4),
 1/3*(3*(5*b^5*c^3 - 6*a*b^4*c^2*d + (5*a^2*b^3*c^3 - 6*a^3*b^2*c^2*d)*x^2
+ 2*(5*a*b^4*c^3 - 6*a^2*b^3*c^2*d)*x)*sqrt(-a)*arctan(sqrt(-a)*sqrt((a*x +
 b)/x)/a) + (3*a^3*b^2*c^3*x^3 + 2*(10*a^2*b^3*c^3 - 12*a^3*b^2*c^2*d + 3*a
^4*b*c*d^2 + 2*a^5*d^3)*x^2 + 3*(5*a*b^4*c^3 - 6*a^2*b^3*c^2*d + 2*a^4*b*d^
3)*x)*sqrt((a*x + b)/x))/(a^6*b^2*x^2 + 2*a^5*b^3*x + a^4*b^4)]
```

giac [A] time = 0.26, size = 203, normalized size = 1.42

$$-\frac{\frac{3 b^{2} c^{3} \sqrt{\frac{a x+b}{x}}}{\left(a-\frac{a x+b}{x}\right) a^{3}}-\frac{3\left(5 b^{2} c^{3}-6 a b c^{2} d\right) \arctan \left(\frac{\sqrt{\frac{a x+b}{x}}}{\sqrt{-a}}\right)}{\sqrt{-a}\, a^{3}}-\frac{2\left(a b^{3} c^{3}-3 a^{2} b^{2} c^{2} d+3 a^{3} b c d^{2}-a^{4} d^{3}+\frac{6(a x+b) b^{3} c^{3}}{x}-\frac{9(a x+b) a b^{2} c^{2} d}{x}+\frac{3(a x+b) a^{3} d^{3}}{x}\right) x}{(a x+b) a^{3} b \sqrt{\frac{a x+b}{x}}}}{3 b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d/x)^3/(a+b/x)^(5/2),x, algorithm="giac")
```

```
[Out] -1/3*(3*b^2*c^3*sqrt((a*x + b)/x)/((a - (a*x + b)/x)*a^3) - 3*(5*b^2*c^3 -
6*a*b*c^2*d)*arctan(sqrt((a*x + b)/x)/sqrt(-a))/(sqrt(-a)*a^3) - 2*(a*b^3*c
^3 - 3*a^2*b^2*c^2*d + 3*a^3*b*c*d^2 - a^4*d^3 + 6*(a*x + b)*b^3*c^3/x - 9*
(a*x + b)*a*b^2*c^2*d/x + 3*(a*x + b)*a^3*d^3/x)*x/((a*x + b)*a^3*b*sqrt((a
*x + b)/x)))/b
```

maple [B] time = 0.06, size = 1150, normalized size = 8.04

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d/x)^3/(a+b/x)^(5/2),x)
```

```
[Out] -1/6*((a*x+b)/x)^(1/2)*x/a^(7/2)*(3*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(
1/2))/a^(1/2))*a^3*b^4*d^3-3*ln(1/2*(2*a*x+b+2*(a*x^2+b*x)^(1/2)*a^(1/2))/a
^(1/2))*a^3*b^4*d^3-30*a^(1/2)*((a*x+b)*x)^(1/2)*b^6*c^3-6*(a*x^2+b*x)^(1/2
)*a^(13/2)*x^3*d^3-6*a^(13/2)*((a*x+b)*x)^(1/2)*x^3*d^3+12*a^(11/2)*((a*x+b
)*x)^(3/2)*x*d^3+16*a^(9/2)*((a*x+b)*x)^(3/2)*b*d^3+20*a^(3/2)*((a*x+b)*x)^
(3/2)*b^4*c^3-6*(a*x^2+b*x)^(1/2)*a^(7/2)*b^3*d^3-6*a^(7/2)*((a*x+b)*x)^(1/
2)*b^3*d^3+36*a^(9/2)*((a*x+b)*x)^(1/2)*x^3*b^2*c^2*d-36*a^(7/2)*((a*x+b)*x
)^(3/2)*x*b^2*c^2*d+108*a^(7/2)*((a*x+b)*x)^(1/2)*x^2*b^3*c^2*d-54*ln(1/2*(
2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))*x*a^2*b^5*c^2*d-54*ln(1/2*(2*
a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))*x^2*a^3*b^4*c^2*d-18*ln(1/2*(2*
```

```
a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))*x^3*a^4*b^3*c^2*d+108*a^(5/2)*(
(a*x+b)*x)^(1/2)*x*b^4*c^2*d+15*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2)
)/a^(1/2))*b^7*c^3+9*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))*
x*a^4*b^3*d^3+45*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))*x*a*
b^6*c^3-9*ln(1/2*(2*a*x+b+2*(a*x^2+b*x)^(1/2)*a^(1/2))/a^(1/2))*x*a^4*b^3*d
^3-18*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))*a*b^6*c^2*d-18*
(a*x^2+b*x)^(1/2)*a^(9/2)*x*b^2*d^3-12*a^(7/2)*((a*x+b)*x)^(3/2)*b^2*c*d^2-
24*a^(5/2)*((a*x+b)*x)^(3/2)*b^3*c^2*d-18*a^(9/2)*((a*x+b)*x)^(1/2)*x*b^2*d
^3-90*a^(3/2)*((a*x+b)*x)^(1/2)*x*b^5*c^3+36*a^(3/2)*((a*x+b)*x)^(1/2)*b^5*
c^2*d-90*a^(5/2)*((a*x+b)*x)^(1/2)*x^2*b^4*c^3+24*a^(5/2)*((a*x+b)*x)^(3/2)
*x*b^3*c^3-18*a^(11/2)*((a*x+b)*x)^(1/2)*x^2*b*d^3-18*(a*x^2+b*x)^(1/2)*a^(
11/2)*x^2*b*d^3-30*a^(7/2)*((a*x+b)*x)^(1/2)*x^3*b^3*c^3+3*ln(1/2*(2*a*x+b+
2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))*x^3*a^6*b*d^3+15*ln(1/2*(2*a*x+b+2*((
a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))*x^3*a^3*b^4*c^3-3*ln(1/2*(2*a*x+b+2*(a*x^
2+b*x)^(1/2)*a^(1/2))/a^(1/2))*x^3*a^6*b*d^3+9*ln(1/2*(2*a*x+b+2*((a*x+b)*x
)^(1/2)*a^(1/2))/a^(1/2))*x^2*a^5*b^2*d^3+45*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^
(1/2)*a^(1/2))/a^(1/2))*x^2*a^2*b^5*c^3-9*ln(1/2*(2*a*x+b+2*(a*x^2+b*x)^(1/
2)*a^(1/2))/a^(1/2))*x^2*a^5*b^2*d^3)/((a*x+b)*x)^(1/2)/b^3/(a*x+b)^3
```

maxima [A] time = 1.34, size = 228, normalized size = 1.59

$$\frac{1}{6}c^3\left(\frac{2\left(15\left(a+\frac{b}{x}\right)^2 b-10\left(a+\frac{b}{x}\right)ab-2a^2b\right)}{\left(a+\frac{b}{x}\right)^{\frac{5}{2}}a^3-\left(a+\frac{b}{x}\right)^{\frac{3}{2}}a^4}+\frac{15b\log\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{a+\frac{b}{x}}+\sqrt{a}}\right)}{a^{\frac{7}{2}}}\right)-c^2d\left(\frac{3\log\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{a+\frac{b}{x}}+\sqrt{a}}\right)}{a^{\frac{5}{2}}}+\frac{2\left(4a+\frac{3b}{x}\right)}{\left(a+\frac{b}{x}\right)^{\frac{3}{2}}a^2}\right)+\frac{2}{3}d^3\left(\frac{3}{\sqrt{a+\frac{b}{x}}b^2}-\frac{a}{\left(a+\frac{b}{x}\right)^{\frac{3}{2}}b^2}\right)+\frac{2cd^2}{\left(a+\frac{b}{x}\right)^{\frac{3}{2}}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d/x)^3/(a+b/x)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/6*c^3*(2*(15*(a + b/x)^2*b - 10*(a + b/x)*a*b - 2*a^2*b)/((a + b/x)^(5/2)
*a^3 - (a + b/x)^(3/2)*a^4) + 15*b*log((sqrt(a + b/x) - sqrt(a))/(sqrt(a +
b/x) + sqrt(a)))/a^(7/2)) - c^2*d*(3*log((sqrt(a + b/x) - sqrt(a))/(sqrt(a
+ b/x) + sqrt(a)))/a^(5/2) + 2*(4*a + 3*b/x)/((a + b/x)^(3/2)*a^2)) + 2/3*d
^3*(3/(sqrt(a + b/x)*b^2) - a/((a + b/x)^(3/2)*b^2)) + 2*c*d^2/((a + b/x)^(
3/2)*b)
```

mupad [B] time = 2.05, size = 194, normalized size = 1.36

$$\frac{\frac{2\left(a^3d^3-3a^2bcd^2+3ab^2c^2d-b^3c^3\right)}{3a}+\frac{\left(a+\frac{b}{x}\right)^2\left(2a^3d^3-6ab^2c^2d+5b^3c^3\right)}{a^3}-\frac{2\left(a+\frac{b}{x}\right)\left(4a^3d^3-3a^2bcd^2-6ab^2c^2d+5b^3c^3\right)}{3a^2}}{b^2\left(a+\frac{b}{x}\right)^{5/2}-ab^2\left(a+\frac{b}{x}\right)^{3/2}}+\frac{c^2\,\mathrm{atanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)(6ad-5bc)}{a^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d/x)^3/(a + b/x)^(5/2),x)
```

```
[Out] ((2*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2))/(3*a) + ((a + b/x)
^2*(2*a^3*d^3 + 5*b^3*c^3 - 6*a*b^2*c^2*d))/a^3 - (2*(a + b/x)*(4*a^3*d^3 +
 5*b^3*c^3 - 6*a*b^2*c^2*d - 3*a^2*b*c*d^2))/(3*a^2))/(b^2*(a + b/x)^(5/2)
- a*b^2*(a + b/x)^(3/2)) + (c^2*atanh((a + b/x)^(1/2)/a^(1/2))*(6*a*d - 5*b
*c))/a^(7/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx+d)^3}{x^3\left(a+\frac{b}{x}\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d/x)**3/(a+b/x)**(5/2),x)
```

```
[Out] Integral((c*x + d)**3/(x**3*(a + b/x)**(5/2)), x)
```

3.163 $\int \frac{\left(c+\frac{d}{x}\right)^2}{\left(a+\frac{b}{x}\right)^{5/2}}\,dx$

Optimal. Leaf size=122

$$-\frac{c(5bc-4ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{7/2}}+\frac{c(5bc-4ad)}{a^3\sqrt{a+\frac{b}{x}}}+\frac{2a^2d^2+bc(5bc-4ad)}{3a^2b\left(a+\frac{b}{x}\right)^{3/2}}+\frac{c^2x}{a\left(a+\frac{b}{x}\right)^{3/2}}$$

Rubi [A] time = 0.10, antiderivative size = 118, normalized size of antiderivative = 0.97, number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {375, 89, 78, 51, 63, 208}

$$\frac{\frac{c(5bc-4ad)}{a^2}+\frac{2d^2}{b}}{3\left(a+\frac{b}{x}\right)^{3/2}}+\frac{c(5bc-4ad)}{a^3\sqrt{a+\frac{b}{x}}}-\frac{c(5bc-4ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{7/2}}+\frac{c^2x}{a\left(a+\frac{b}{x}\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d/x)^2/(a + b/x)^(5/2),x]
```

```
[Out] ((2*d^2)/b + (c*(5*b*c - 4*a*d))/a^2)/(3*(a + b/x)^(3/2)) + (c*(5*b*c - 4*a
*d))/(a^3*Sqrt[a + b/x]) + (c^2*x)/(a*(a + b/x)^(3/2)) - (c*(5*b*c - 4*a*d)
*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/a^(7/2)
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
```

```
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 375

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[((a + b/x^n)^p*(c + d/x^n)^q)/x^2, x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(c+\frac{d}{x}\right)^2}{\left(a+\frac{b}{x}\right)^{5/2}}\,dx &= -\text{Subst}\left(\int \frac{(c+dx)^2}{x^2(a+bx)^{5/2}}\,dx, x, \frac{1}{x}\right)\\
&= \frac{c^2x}{a\left(a+\frac{b}{x}\right)^{3/2}} - \frac{\text{Subst}\left(\int \frac{-\frac{1}{2}c(5bc-4ad)+ad^2x}{x(a+bx)^{5/2}}\,dx, x, \frac{1}{x}\right)}{a}\\
&= \frac{\frac{5bc^2}{a}-4cd+\frac{2ad^2}{b}}{3a\left(a+\frac{b}{x}\right)^{3/2}} + \frac{c^2x}{a\left(a+\frac{b}{x}\right)^{3/2}} + \frac{(c(5bc-4ad))\,\text{Subst}\left(\int \frac{1}{x(a+bx)^{3/2}}\,dx, x, \frac{1}{x}\right)}{2a^2}\\
&= \frac{\frac{5bc^2}{a}-4cd+\frac{2ad^2}{b}}{3a\left(a+\frac{b}{x}\right)^{3/2}} + \frac{c(5bc-4ad)}{a^3\sqrt{a+\frac{b}{x}}} + \frac{c^2x}{a\left(a+\frac{b}{x}\right)^{3/2}} + \frac{(c(5bc-4ad))\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \frac{1}{x}\right)}{2a^3}\\
&= \frac{\frac{5bc^2}{a}-4cd+\frac{2ad^2}{b}}{3a\left(a+\frac{b}{x}\right)^{3/2}} + \frac{c(5bc-4ad)}{a^3\sqrt{a+\frac{b}{x}}} + \frac{c^2x}{a\left(a+\frac{b}{x}\right)^{3/2}} + \frac{(c(5bc-4ad))\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a}\right.}{a^3b}\\
&= \frac{\frac{5bc^2}{a}-4cd+\frac{2ad^2}{b}}{3a\left(a+\frac{b}{x}\right)^{3/2}} + \frac{c(5bc-4ad)}{a^3\sqrt{a+\frac{b}{x}}} + \frac{c^2x}{a\left(a+\frac{b}{x}\right)^{3/2}} - \frac{c(5bc-4ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{7/2}}
\end{aligned}$$

Mathematica [C] time = 0.08, size = 97, normalized size = 0.80

$$\frac{ax\left(2a^2d^2+abc(3cx-4d)+5b^2c^2\right)+3bc(ax+b)(5bc-4ad)\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{b}{ax}+1\right)}{3a^3b\sqrt{a+\frac{b}{x}}\,(ax+b)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d/x)^2/(a + b/x)^(5/2),x]
```

```
[Out] (a*x*(5*b^2*c^2 + 2*a^2*d^2 + a*b*c*(-4*d + 3*c*x)) + 3*b*c*(5*b*c - 4*a*d)
*(b + a*x)*Hypergeometric2F1[-1/2, 1, 1/2, 1 + b/(a*x)])/(3*a^3*b*Sqrt[a +
b/x]*(b + a*x))
```

IntegrateAlgebraic [A] time = 0.25, size = 134, normalized size = 1.10

$$\frac{\left(4acd-5bc^2\right)\tanh^{-1}\left(\frac{\sqrt{\frac{ax+b}{x}}}{\sqrt{a}}\right)}{a^{7/2}}+\frac{\sqrt{\frac{ax+b}{x}}\left(2a^3d^2x^2+3a^2bc^2x^3-16a^2bcdx^2+20ab^2c^2x^2-12ab^2cdx+15b^3c^2x\right)}{3a^3b(ax+b)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c + d/x)^2/(a + b/x)^(5/2),x]
```

```
[Out] (Sqrt[(b + a*x)/x]*(15*b^3*c^2*x - 12*a*b^2*c*d*x + 20*a*b^2*c^2*x^2 - 16*a
^2*b*c*d*x^2 + 2*a^3*d^2*x^2 + 3*a^2*b*c^2*x^3))/(3*a^3*b*(b + a*x)^2) + ((
-5*b*c^2 + 4*a*c*d)*ArcTanh[Sqrt[(b + a*x)/x]/Sqrt[a]])/a^(7/2)
```

fricas [A] time = 0.99, size = 407, normalized size = 3.34

$$\left[-\frac{3\left(5b^4c^2-4ab^3cd+\left(5a^2b^2c^2-4a^3bcd\right)x^2+2\left(5ab^3c^2-4a^2b^2cd\right)x\right)\sqrt{a}\log\left(2ax+2\sqrt{a}x\sqrt{\frac{ax+b}{x}}+b\right)-2\left(3a^3bc^2x^3+2\left(10a^2b^2c^2-8a^3bcd+a^4d^2\right)x^2+3\left(5ab^3c^2-4a^2b^2cd\right)x\right)\sqrt{\frac{ax+b}{x}}}{6\left(a^6bx^2+2a^5b^2x+a^4b^3\right)},\frac{3\left(5b^4c^2-4ab^3cd+\left(5a^2b^2c^2-4a^3bcd\right)x^2+2\left(5ab^3c^2-4a^2b^2cd\right)x\right)\sqrt{-a}\arctan\left(\frac{\sqrt{-a}\sqrt{\frac{ax+b}{x}}}{a}\right)+\left(3a^3bc^2x^3+2\left(10a^2b^2c^2-8a^3bcd+a^4d^2\right)x^2+3\left(5ab^3c^2-4a^2b^2cd\right)x\right)\sqrt{\frac{ax+b}{x}}}{3\left(a^6bx^2+2a^5b^2x+a^4b^3\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d/x)^2/(a+b/x)^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/6*(3*(5*b^4*c^2 - 4*a*b^3*c*d + (5*a^2*b^2*c^2 - 4*a^3*b*c*d)*x^2 + 2*(
5*a*b^3*c^2 - 4*a^2*b^2*c*d)*x)*sqrt(a)*log(2*a*x + 2*sqrt(a)*x*sqrt((a*x +
 b)/x) + b) - 2*(3*a^3*b*c^2*x^3 + 2*(10*a^2*b^2*c^2 - 8*a^3*b*c*d + a^4*d^
2)*x^2 + 3*(5*a*b^3*c^2 - 4*a^2*b^2*c*d)*x)*sqrt((a*x + b)/x))/(a^6*b*x^2 +
 2*a^5*b^2*x + a^4*b^3), 1/3*(3*(5*b^4*c^2 - 4*a*b^3*c*d + (5*a^2*b^2*c^2 -
 4*a^3*b*c*d)*x^2 + 2*(5*a*b^3*c^2 - 4*a^2*b^2*c*d)*x)*sqrt(-a)*arctan(sqrt
(-a)*sqrt((a*x + b)/x)/a) + (3*a^3*b*c^2*x^3 + 2*(10*a^2*b^2*c^2 - 8*a^3*b*
c*d + a^4*d^2)*x^2 + 3*(5*a*b^3*c^2 - 4*a^2*b^2*c*d)*x)*sqrt((a*x + b)/x))/
(a^6*b*x^2 + 2*a^5*b^2*x + a^4*b^3)]
```

giac [A] time = 0.21, size = 163, normalized size = 1.34

$$-\frac{\frac{3\,b^2c^2\sqrt{\frac{ax+b}{x}}}{\left(a-\frac{ax+b}{x}\right)a^3}-\frac{3\left(5\,b^2c^2-4\,abcd\right)\arctan\left(\frac{\sqrt{\frac{ax+b}{x}}}{\sqrt{-a}}\right)}{\sqrt{-a}\,a^3}-\frac{2\left(ab^2c^2-2\,a^2bcd+a^3d^2+\frac{6\,(ax+b)b^2c^2}{x}-\frac{6\,(ax+b)abcd}{x}\right)x}{(ax+b)a^3\sqrt{\frac{ax+b}{x}}}}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d/x)^2/(a+b/x)^(5/2),x, algorithm="giac")
```

```
[Out] -1/3*(3*b^2*c^2*sqrt((a*x + b)/x)/((a - (a*x + b)/x)*a^3) - 3*(5*b^2*c^2 -
4*a*b*c*d)*arctan(sqrt((a*x + b)/x)/sqrt(-a))/(sqrt(-a)*a^3) - 2*(a*b^2*c^2
 - 2*a^2*b*c*d + a^3*d^2 + 6*(a*x + b)*b^2*c^2/x - 6*(a*x + b)*a*b*c*d/x)*x
/((a*x + b)*a^3*sqrt((a*x + b)/x)))/b
```

maple [B] time = 0.06, size = 588, normalized size = 4.82

$$-\frac{\sqrt{\frac{ax+b}{x}}\left(-12a^4bcd\,x^3\ln\left(\frac{2ax+b+2\sqrt{(ax+b)x}\,\sqrt{a}}{2\sqrt{a}}\right)+15a^3b^2c^2x^3\ln\left(\frac{2ax+b+2\sqrt{(ax+b)x}\,\sqrt{a}}{2\sqrt{a}}\right)-36a^3b^2cd\,x^2\ln\left(\frac{2ax+b+2\sqrt{(ax+b)x}\,\sqrt{a}}{2\sqrt{a}}\right)+45a^2b^3c^2x^2\ln\left(\frac{2ax+b+2\sqrt{(ax+b)x}\,\sqrt{a}}{2\sqrt{a}}\right)+24\sqrt{(ax+b)x}\,a^{\frac{9}{2}}cd\,x^3-30\sqrt{(ax+b)x}\,a^{\frac{7}{2}}bc^2x^3-36a^2b^3cdx\ln\left(\frac{2ax+b+2\sqrt{(ax+b)x}\,\sqrt{a}}{2\sqrt{a}}\right)+45a\,b^4c^2x\ln\left(\frac{2ax+b+2\sqrt{(ax+b)x}\,\sqrt{a}}{2\sqrt{a}}\right)+72\sqrt{(ax+b)x}\,a^{\frac{7}{2}}bcd\,x^2-90\sqrt{(ax+b)x}\,a^{\frac{5}{2}}b^2c^2x^2-12a\,b^4cd\ln\left(\frac{2ax+b+2\sqrt{(ax+b)x}\,\sqrt{a}}{2\sqrt{a}}\right)+15b^5c^2\ln\left(\frac{2ax+b+2\sqrt{(ax+b)x}\,\sqrt{a}}{2\sqrt{a}}\right)+72\sqrt{(ax+b)x}\,a^{\frac{5}{2}}b^2cdx-90\sqrt{(ax+b)x}\,a^{\frac{3}{2}}b^3c^2x-24\left((ax+b)x\right)^{\frac{3}{2}}a^{\frac{7}{2}}cdx+24\left((ax+b)x\right)^{\frac{3}{2}}a^{\frac{5}{2}}b\,c^2x+24\sqrt{(ax+b)x}\,a^{\frac{3}{2}}b^3cd-30\sqrt{(ax+b)x}\,\sqrt{a}\,b^4c^2-4\left((ax+b)x\right)^{\frac{3}{2}}a^{\frac{7}{2}}d^2-16\left((ax+b)x\right)^{\frac{3}{2}}a^{\frac{5}{2}}bcd+20\left((ax+b)x\right)^{\frac{3}{2}}a^{\frac{3}{2}}b^2c^2\right)x}{6\sqrt{(ax+b)x}\,(ax+b)^3a^{\frac{7}{2}}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d/x)^2/(a+b/x)^(5/2),x)
```

```
[Out] -1/6*((a*x+b)/x)^(1/2)*x*(24*a^(9/2)*((a*x+b)*x)^(1/2)*x^3*c*d-30*a^(7/2)*(
(a*x+b)*x)^(1/2)*x^3*b*c^2-24*a^(7/2)*((a*x+b)*x)^(3/2)*x*c*d+72*((a*x+b)*x
)^(1/2)*a^(7/2)*b*c*d*x^2-4*((a*x+b)*x)^(3/2)*a^(7/2)*d^2+24*a^(5/2)*((a*x+
b)*x)^(3/2)*x*b*c^2-90*((a*x+b)*x)^(1/2)*a^(5/2)*b^2*c^2*x^2-16*((a*x+b)*x)
^(3/2)*a^(5/2)*b*c*d+72*((a*x+b)*x)^(1/2)*a^(5/2)*b^2*c*d*x-12*ln(1/2*(2*a*
x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))*x^3*a^4*b*c*d+15*ln(1/2*(2*a*x+b+
2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))*x^3*a^3*b^2*c^2+20*((a*x+b)*x)^(3/2)*
a^(3/2)*b^2*c^2-90*((a*x+b)*x)^(1/2)*a^(3/2)*b^3*c^2*x-36*a^3*b^2*c*d*x^2*l
n(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))+45*a^2*b^3*c^2*x^2*ln(
1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))+24*((a*x+b)*x)^(1/2)*a^(
3/2)*b^3*c*d-36*a^2*b^3*c*d*x*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/
a^(1/2))+45*a*b^4*c^2*x*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2
))-30*((a*x+b)*x)^(1/2)*a^(1/2)*b^4*c^2-12*a*b^4*c*d*ln(1/2*(2*a*x+b+2*((a*
x+b)*x)^(1/2)*a^(1/2))/a^(1/2))+15*b^5*c^2*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1
/2)*a^(1/2))/a^(1/2)))/a^(7/2)/((a*x+b)*x)^(1/2)/b/(a*x+b)^3
```

maxima [A] time = 1.31, size = 190, normalized size = 1.56

$$\frac{1}{6}c^2\left(\frac{2\left(15\left(a+\frac{b}{x}\right)^2b-10\left(a+\frac{b}{x}\right)ab-2a^2b\right)}{\left(a+\frac{b}{x}\right)^{\frac{5}{2}}a^3-\left(a+\frac{b}{x}\right)^{\frac{3}{2}}a^4}+\frac{15b\log\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{a+\frac{b}{x}}+\sqrt{a}}\right)}{a^{\frac{7}{2}}}\right)-\frac{2}{3}cd\left(\frac{3\log\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{a+\frac{b}{x}}+\sqrt{a}}\right)}{a^{\frac{5}{2}}}+\frac{2\left(4a+\frac{3b}{x}\right)}{\left(a+\frac{b}{x}\right)^{\frac{3}{2}}a^2}\right)+\frac{2d^2}{3\left(a+\frac{b}{x}\right)^{\frac{3}{2}}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d/x)^2/(a+b/x)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/6*c^2*(2*(15*(a + b/x)^2*b - 10*(a + b/x)*a*b - 2*a^2*b)/((a + b/x)^(5/2)
*a^3 - (a + b/x)^(3/2)*a^4) + 15*b*log((sqrt(a + b/x) - sqrt(a))/(sqrt(a +
b/x) + sqrt(a)))/a^(7/2)) - 2/3*c*d*(3*log((sqrt(a + b/x) - sqrt(a))/(sqrt(
a + b/x) + sqrt(a)))/a^(5/2) + 2*(4*a + 3*b/x)/((a + b/x)^(3/2)*a^2)) + 2/3
*d^2/((a + b/x)^(3/2)*b)
```

mupad [B] time = 2.22, size = 144, normalized size = 1.18

$$\frac{\frac{2\left(a+\frac{b}{x}\right)\left(a^2d^2+4abcd-5b^2c^2\right)}{3a^2}-\frac{2\left(a^2d^2-2abcd+b^2c^2\right)}{3a}+\frac{b\left(a+\frac{b}{x}\right)^2\left(5bc^2-4acd\right)}{a^3}}{b\left(a+\frac{b}{x}\right)^{5/2}-ab\left(a+\frac{b}{x}\right)^{3/2}}+\frac{c\,\mathrm{atanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)(4ad-5bc)}{a^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d/x)^2/(a + b/x)^(5/2),x)
```

```
[Out] ((2*(a + b/x)*(a^2*d^2 - 5*b^2*c^2 + 4*a*b*c*d))/(3*a^2) - (2*(a^2*d^2 + b^
2*c^2 - 2*a*b*c*d))/(3*a) + (b*(a + b/x)^2*(5*b*c^2 - 4*a*c*d))/a^3)/(b*(a
+ b/x)^(5/2) - a*b*(a + b/x)^(3/2)) + (c*atanh((a + b/x)^(1/2)/a^(1/2))*(4*
a*d - 5*b*c))/a^(7/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(cx+d)^2}{x^2\left(a+\frac{b}{x}\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d/x)**2/(a+b/x)**(5/2),x)
```

```
[Out] Integral((c*x + d)**2/(x**2*(a + b/x)**(5/2)), x)
```

3.164 $\int \frac{c+\frac{d}{x}}{\left(a+\frac{b}{x}\right)^{5/2}}\,dx$

Optimal. Leaf size=103

$$-\frac{(5bc-2ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{7/2}}+\frac{5bc-2ad}{a^3\sqrt{a+\frac{b}{x}}}+\frac{5bc-2ad}{3a^2\left(a+\frac{b}{x}\right)^{3/2}}+\frac{cx}{a\left(a+\frac{b}{x}\right)^{3/2}}$$

Rubi [A] time = 0.07, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {375, 78, 51, 63, 208}

$$\frac{5bc-2ad}{a^3\sqrt{a+\frac{b}{x}}}+\frac{5bc-2ad}{3a^2\left(a+\frac{b}{x}\right)^{3/2}}-\frac{(5bc-2ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{7/2}}+\frac{cx}{a\left(a+\frac{b}{x}\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d/x)/(a + b/x)^(5/2),x]
```

```
[Out] (5*b*c - 2*a*d)/(3*a^2*(a + b/x)^(3/2)) + (5*b*c - 2*a*d)/(a^3*Sqrt[a + b/x
]) + (c*x)/(a*(a + b/x)^(3/2)) - ((5*b*c - 2*a*d)*ArcTanh[Sqrt[a + b/x]/Sqr
t[a]])/a^(7/2)
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 375

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[((a + b/x^n)^p*(c + d/x^n)^q)/x^2, x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rubi steps

$$\int \frac{c+\frac{d}{x}}{\left(a+\frac{b}{x}\right)^{5/2}}\,dx = -\text{Subst}\left(\int \frac{c+dx}{x^2(a+bx)^{5/2}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{cx}{a\left(a+\frac{b}{x}\right)^{3/2}} - \frac{\left(-\frac{5bc}{2}+ad\right)\text{Subst}\left(\int \frac{1}{x(a+bx)^{5/2}}\,dx, x, \frac{1}{x}\right)}{a}$$

$$= \frac{5bc-2ad}{3a^2\left(a+\frac{b}{x}\right)^{3/2}} + \frac{cx}{a\left(a+\frac{b}{x}\right)^{3/2}} + \frac{(5bc-2ad)\,\text{Subst}\left(\int \frac{1}{x(a+bx)^{3/2}}\,dx, x, \frac{1}{x}\right)}{2a^2}$$

$$= \frac{5bc-2ad}{3a^2\left(a+\frac{b}{x}\right)^{3/2}} + \frac{5bc-2ad}{a^3\sqrt{a+\frac{b}{x}}} + \frac{cx}{a\left(a+\frac{b}{x}\right)^{3/2}} + \frac{(5bc-2ad)\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \frac{1}{x}\right)}{2a^3}$$

$$= \frac{5bc-2ad}{3a^2\left(a+\frac{b}{x}\right)^{3/2}} + \frac{5bc-2ad}{a^3\sqrt{a+\frac{b}{x}}} + \frac{cx}{a\left(a+\frac{b}{x}\right)^{3/2}} + \frac{(5bc-2ad)\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+\frac{b}{x}}\right)}{a^3b}$$

$$= \frac{5bc-2ad}{3a^2\left(a+\frac{b}{x}\right)^{3/2}} + \frac{5bc-2ad}{a^3\sqrt{a+\frac{b}{x}}} + \frac{cx}{a\left(a+\frac{b}{x}\right)^{3/2}} - \frac{(5bc-2ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{7/2}}$$

Mathematica [C] time = 0.04, size = 60, normalized size = 0.58

$$\frac{x\left((5bc-2ad)\,{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};\frac{b}{ax}+1\right)+3acx\right)}{3a^2\sqrt{a+\frac{b}{x}}\,(ax+b)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d/x)/(a + b/x)^(5/2),x]
```

```
[Out] (x*(3*a*c*x + (5*b*c - 2*a*d)*Hypergeometric2F1[-3/2, 1, -1/2, 1 + b/(a*x)]
))/(3*a^2*Sqrt[a + b/x]*(b + a*x))
```

IntegrateAlgebraic [A] time = 0.20, size = 103, normalized size = 1.00

$$\frac{(2ad-5bc)\tanh^{-1}\left(\frac{\sqrt{\frac{ax+b}{x}}}{\sqrt{a}}\right)}{a^{7/2}} + \frac{\sqrt{\frac{ax+b}{x}}\left(3a^2cx^3-8a^2dx^2+20abcx^2-6abdx+15b^2cx\right)}{3a^3(ax+b)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c + d/x)/(a + b/x)^(5/2),x]
```

```
[Out] (Sqrt[(b + a*x)/x]*(15*b^2*c*x - 6*a*b*d*x + 20*a*b*c*x^2 - 8*a^2*d*x^2 + 3
*a^2*c*x^3))/(3*a^3*(b + a*x)^2) + ((-5*b*c + 2*a*d)*ArcTanh[Sqrt[(b + a*x)
/x]/Sqrt[a]])/a^(7/2)
```

fricas [A] time = 0.63, size = 331, normalized size = 3.21

$$\left[-\frac{3\left(5b^3c-2ab^2d+\left(5a^2bc-2a^3d\right)x^2+2\left(5ab^2c-2a^2bd\right)x\right)\sqrt{a}\log\left(2ax+2\sqrt{a}x\sqrt{\frac{ax+b}{x}}+b\right)-2\left(3a^3cx^3+4\left(5a^2bc-2a^3d\right)x^2+3\left(5ab^2c-2a^2bd\right)x\right)\sqrt{\frac{ax+b}{x}}}{6\left(a^6x^2+2a^5bx+a^4b^2\right)},\ \frac{3\left(5b^3c-2ab^2d+\left(5a^2bc-2a^3d\right)x^2+2\left(5ab^2c-2a^2bd\right)x\right)\sqrt{-a}\arctan\left(\frac{\sqrt{-a}\sqrt{\frac{ax+b}{x}}}{a}\right)+\left(3a^3cx^3+4\left(5a^2bc-2a^3d\right)x^2+3\left(5ab^2c-2a^2bd\right)x\right)\sqrt{\frac{ax+b}{x}}}{3\left(a^6x^2+2a^5bx+a^4b^2\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d/x)/(a+b/x)^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/6*(3*(5*b^3*c - 2*a*b^2*d + (5*a^2*b*c - 2*a^3*d)*x^2 + 2*(5*a*b^2*c -
2*a^2*b*d)*x)*sqrt(a)*log(2*a*x + 2*sqrt(a)*x*sqrt((a*x + b)/x) + b) - 2*(3
*a^3*c*x^3 + 4*(5*a^2*b*c - 2*a^3*d)*x^2 + 3*(5*a*b^2*c - 2*a^2*b*d)*x)*sqr
t((a*x + b)/x))/(a^6*x^2 + 2*a^5*b*x + a^4*b^2), 1/3*(3*(5*b^3*c - 2*a*b^2*
d + (5*a^2*b*c - 2*a^3*d)*x^2 + 2*(5*a*b^2*c - 2*a^2*b*d)*x)*sqrt(-a)*arcta
n(sqrt(-a)*sqrt((a*x + b)/x)/a) + (3*a^3*c*x^3 + 4*(5*a^2*b*c - 2*a^3*d)*x^
2 + 3*(5*a*b^2*c - 2*a^2*b*d)*x)*sqrt((a*x + b)/x))/(a^6*x^2 + 2*a^5*b*x +
a^4*b^2)]
```

giac [A] time = 0.22, size = 145, normalized size = 1.41

$$-\frac{\frac{3\,b^2c\sqrt{\frac{ax+b}{x}}}{\left(a-\frac{ax+b}{x}\right)a^3}-\frac{3\left(5\,b^2c-2\,abd\right)\arctan\left(\frac{\sqrt{\frac{ax+b}{x}}}{\sqrt{-a}}\right)}{\sqrt{-a}\,a^3}-\frac{2\left(ab^2c-a^2bd+\frac{6\,(ax+b)b^2c}{x}-\frac{3\,(ax+b)abd}{x}\right)x}{(ax+b)a^3\sqrt{\frac{ax+b}{x}}}}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d/x)/(a+b/x)^(5/2),x, algorithm="giac")
```

```
[Out] -1/3*(3*b^2*c*sqrt((a*x + b)/x)/((a - (a*x + b)/x)*a^3) - 3*(5*b^2*c - 2*a*
b*d)*arctan(sqrt((a*x + b)/x)/sqrt(-a))/(sqrt(-a)*a^3) - 2*(a*b^2*c - a^2*b
*d + 6*(a*x + b)*b^2*c/x - 3*(a*x + b)*a*b*d/x)*x/((a*x + b)*a^3*sqrt((a*x
+ b)/x)))/b
```

maple [B] time = 0.06, size = 541, normalized size = 5.25

$$\frac{\sqrt{\frac{ax+b}{x}}\left(6a^4bd\,x^3\ln\left(\frac{2ax+b+2\sqrt{(ax+b)x}\,\sqrt{a}}{2\sqrt{a}}\right)-15a^3b^2c\,x^3\ln\left(\frac{2ax+b+2\sqrt{(ax+b)x}\,\sqrt{a}}{2\sqrt{a}}\right)+18a^3b^2d\,x^2\ln\left(\frac{2ax+b+2\sqrt{(ax+b)x}\,\sqrt{a}}{2\sqrt{a}}\right)-45a^2b^3c\,x^2\ln\left(\frac{2ax+b+2\sqrt{(ax+b)x}\,\sqrt{a}}{2\sqrt{a}}\right)-12\sqrt{(ax+b)x}\,a^{\frac{9}{2}}d\,x^3+30\sqrt{(ax+b)x}\,a^{\frac{7}{2}}bc\,x^3+18a^2b^3dx\ln\left(\frac{2ax+b+2\sqrt{(ax+b)x}\,\sqrt{a}}{2\sqrt{a}}\right)-45ab^4cx\ln\left(\frac{2ax+b+2\sqrt{(ax+b)x}\,\sqrt{a}}{2\sqrt{a}}\right)+12\left((ax+b)x\right)^{\frac{3}{2}}a^{\frac{7}{2}}dx-24\left((ax+b)x\right)^{\frac{3}{2}}a^{\frac{5}{2}}bcx-36\sqrt{(ax+b)x}\,a^{\frac{7}{2}}bd\,x^2+90\sqrt{(ax+b)x}\,a^{\frac{5}{2}}b^2c\,x^2+6ab^4d\ln\left(\frac{2ax+b+2\sqrt{(ax+b)x}\,\sqrt{a}}{2\sqrt{a}}\right)-15b^5c\ln\left(\frac{2ax+b+2\sqrt{(ax+b)x}\,\sqrt{a}}{2\sqrt{a}}\right)+8\left((ax+b)x\right)^{\frac{3}{2}}a^{\frac{5}{2}}bd-20\left((ax+b)x\right)^{\frac{3}{2}}a^{\frac{3}{2}}b^2c-36\sqrt{(ax+b)x}\,a^{\frac{5}{2}}b^2dx+90\sqrt{(ax+b)x}\,a^{\frac{3}{2}}b^3cx-12\sqrt{(ax+b)x}\,a^{\frac{3}{2}}b^3d+30\sqrt{(ax+b)x}\,\sqrt{a}\,b^4c\right)x}{6\sqrt{(ax+b)x}\,(ax+b)^3a^{\frac{7}{2}}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d/x)/(a+b/x)^(5/2),x)
```

```
[Out] 1/6*((a*x+b)/x)^(1/2)*x/a^(7/2)*(6*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1
/2))/a^(1/2))*x^3*a^4*b*d-15*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a
^(1/2))*x^3*a^3*b^2*c-12*a^(9/2)*((a*x+b)*x)^(1/2)*x^3*d+30*a^(7/2)*((a*x+b
)*x)^(1/2)*x^3*b*c+18*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))
*x^2*a^3*b^2*d-45*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))*x^2
*a^2*b^3*c+12*a^(7/2)*((a*x+b)*x)^(3/2)*x*d-24*a^(5/2)*((a*x+b)*x)^(3/2)*x*
b*c-36*a^(7/2)*((a*x+b)*x)^(1/2)*x^2*b*d+90*a^(5/2)*((a*x+b)*x)^(1/2)*x^2*b
^2*c+18*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))*x*a^2*b^3*d-4
5*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))*x*a*b^4*c+8*a^(5/2)
*((a*x+b)*x)^(3/2)*b*d-20*a^(3/2)*((a*x+b)*x)^(3/2)*b^2*c-36*a^(5/2)*((a*x+
b)*x)^(1/2)*x*b^2*d+90*a^(3/2)*((a*x+b)*x)^(1/2)*x*b^3*c+6*ln(1/2*(2*a*x+b+
2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))*a*b^4*d-15*ln(1/2*(2*a*x+b+2*((a*x+b)
*x)^(1/2)*a^(1/2))/a^(1/2))*b^5*c-12*a^(3/2)*((a*x+b)*x)^(1/2)*b^3*d+30*a^(
1/2)*((a*x+b)*x)^(1/2)*b^4*c)/((a*x+b)*x)^(1/2)/b/(a*x+b)^3
```

maxima [A] time = 1.25, size = 170, normalized size = 1.65

$$\frac{1}{6}\,c\left(\frac{2\left(15\left(a+\frac{b}{x}\right)^{2}b-10\left(a+\frac{b}{x}\right)ab-2\,a^{2}b\right)}{\left(a+\frac{b}{x}\right)^{\frac{5}{2}}a^{3}-\left(a+\frac{b}{x}\right)^{\frac{3}{2}}a^{4}}+\frac{15\,b\log\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{a+\frac{b}{x}}+\sqrt{a}}\right)}{a^{\frac{7}{2}}}\right)-\frac{1}{3}\,d\left(\frac{3\,\log\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{a+\frac{b}{x}}+\sqrt{a}}\right)}{a^{\frac{5}{2}}}+\frac{2\left(4\,a+\frac{3\,b}{x}\right)}{\left(a+\frac{b}{x}\right)^{\frac{3}{2}}a^{2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d/x)/(a+b/x)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/6*c*(2*(15*(a + b/x)^2*b - 10*(a + b/x)*a*b - 2*a^2*b)/((a + b/x)^(5/2)*a
^3 - (a + b/x)^(3/2)*a^4) + 15*b*log((sqrt(a + b/x) - sqrt(a))/(sqrt(a + b/
x) + sqrt(a)))/a^(7/2)) - 1/3*d*(3*log((sqrt(a + b/x) - sqrt(a))/(sqrt(a +
b/x) + sqrt(a)))/a^(5/2) + 2*(4*a + 3*b/x)/((a + b/x)^(3/2)*a^2))
```

mupad [B] time = 2.91, size = 87, normalized size = 0.84

$$\frac{2\,d\,\mathrm{atanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{5/2}}-\frac{\frac{2\,d}{3\,a}+\frac{2\,d\left(a+\frac{b}{x}\right)}{a^{2}}}{\left(a+\frac{b}{x}\right)^{3/2}}+\frac{2\,c\,x\left(\frac{a\,x}{b}+1\right)^{5/2}{}_2\mathrm{F}_1\left(\frac{5}{2},\frac{7}{2};\ \frac{9}{2};\ -\frac{a\,x}{b}\right)}{7\left(a+\frac{b}{x}\right)^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d/x)/(a + b/x)^(5/2),x)
```

```
[Out] (2*d*atanh((a + b/x)^(1/2)/a^(1/2)))/a^(5/2) - ((2*d)/(3*a) + (2*d*(a + b/x
))/a^2)/(a + b/x)^(3/2) + (2*c*x*((a*x)/b + 1)^(5/2)*hypergeom([5/2, 7/2],
9/2, -(a*x)/b))/(7*(a + b/x)^(5/2))
```

sympy [B] time = 155.82, size = 1479, normalized size = 14.36

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d/x)/(a+b/x)**(5/2),x)
```

```
[Out] c*(6*a**17*x**4*sqrt(1 + b/(a*x))/(6*a**(39/2)*x**3 + 18*a**(37/2)*b*x**2 +
 18*a**(35/2)*b**2*x + 6*a**(33/2)*b**3) + 46*a**16*b*x**3*sqrt(1 + b/(a*x)
)/(6*a**(39/2)*x**3 + 18*a**(37/2)*b*x**2 + 18*a**(35/2)*b**2*x + 6*a**(33/
2)*b**3) + 15*a**16*b*x**3*log(b/(a*x))/(6*a**(39/2)*x**3 + 18*a**(37/2)*b*
x**2 + 18*a**(35/2)*b**2*x + 6*a**(33/2)*b**3) - 30*a**16*b*x**3*log(sqrt(1
 + b/(a*x)) + 1)/(6*a**(39/2)*x**3 + 18*a**(37/2)*b*x**2 + 18*a**(35/2)*b**
2*x + 6*a**(33/2)*b**3) + 70*a**15*b**2*x**2*sqrt(1 + b/(a*x))/(6*a**(39/2)
*x**3 + 18*a**(37/2)*b*x**2 + 18*a**(35/2)*b**2*x + 6*a**(33/2)*b**3) + 45*
a**15*b**2*x**2*log(b/(a*x))/(6*a**(39/2)*x**3 + 18*a**(37/2)*b*x**2 + 18*a
**(35/2)*b**2*x + 6*a**(33/2)*b**3) - 90*a**15*b**2*x**2*log(sqrt(1 + b/(a*
x)) + 1)/(6*a**(39/2)*x**3 + 18*a**(37/2)*b*x**2 + 18*a**(35/2)*b**2*x + 6*
a**(33/2)*b**3) + 30*a**14*b**3*x*sqrt(1 + b/(a*x))/(6*a**(39/2)*x**3 + 18*
a**(37/2)*b*x**2 + 18*a**(35/2)*b**2*x + 6*a**(33/2)*b**3) + 45*a**14*b**3*
x*log(b/(a*x))/(6*a**(39/2)*x**3 + 18*a**(37/2)*b*x**2 + 18*a**(35/2)*b**2*
x + 6*a**(33/2)*b**3) - 90*a**14*b**3*x*log(sqrt(1 + b/(a*x)) + 1)/(6*a**(3
9/2)*x**3 + 18*a**(37/2)*b*x**2 + 18*a**(35/2)*b**2*x + 6*a**(33/2)*b**3) +
 15*a**13*b**4*log(b/(a*x))/(6*a**(39/2)*x**3 + 18*a**(37/2)*b*x**2 + 18*a*
*(35/2)*b**2*x + 6*a**(33/2)*b**3) - 30*a**13*b**4*log(sqrt(1 + b/(a*x)) +
1)/(6*a**(39/2)*x**3 + 18*a**(37/2)*b*x**2 + 18*a**(35/2)*b**2*x + 6*a**(33
/2)*b**3)) + d*(-8*a**7*x**3*sqrt(1 + b/(a*x))/(3*a**(19/2)*x**3 + 9*a**(17
/2)*b*x**2 + 9*a**(15/2)*b**2*x + 3*a**(13/2)*b**3) - 3*a**7*x**3*log(b/(a*
x))/(3*a**(19/2)*x**3 + 9*a**(17/2)*b*x**2 + 9*a**(15/2)*b**2*x + 3*a**(13/
```

```
2)*b**3) + 6*a**7*x**3*log(sqrt(1 + b/(a*x)) + 1)/(3*a**(19/2)*x**3 + 9*a**
(17/2)*b*x**2 + 9*a**(15/2)*b**2*x + 3*a**(13/2)*b**3) - 14*a**6*b*x**2*sqr
t(1 + b/(a*x))/(3*a**(19/2)*x**3 + 9*a**(17/2)*b*x**2 + 9*a**(15/2)*b**2*x
+ 3*a**(13/2)*b**3) - 9*a**6*b*x**2*log(b/(a*x))/(3*a**(19/2)*x**3 + 9*a**(
17/2)*b*x**2 + 9*a**(15/2)*b**2*x + 3*a**(13/2)*b**3) + 18*a**6*b*x**2*log(
sqrt(1 + b/(a*x)) + 1)/(3*a**(19/2)*x**3 + 9*a**(17/2)*b*x**2 + 9*a**(15/2)
*b**2*x + 3*a**(13/2)*b**3) - 6*a**5*b**2*x*sqrt(1 + b/(a*x))/(3*a**(19/2)*
x**3 + 9*a**(17/2)*b*x**2 + 9*a**(15/2)*b**2*x + 3*a**(13/2)*b**3) - 9*a**5
*b**2*x*log(b/(a*x))/(3*a**(19/2)*x**3 + 9*a**(17/2)*b*x**2 + 9*a**(15/2)*b
**2*x + 3*a**(13/2)*b**3) + 18*a**5*b**2*x*log(sqrt(1 + b/(a*x)) + 1)/(3*a*
*(19/2)*x**3 + 9*a**(17/2)*b*x**2 + 9*a**(15/2)*b**2*x + 3*a**(13/2)*b**3)
- 3*a**4*b**3*log(b/(a*x))/(3*a**(19/2)*x**3 + 9*a**(17/2)*b*x**2 + 9*a**(1
5/2)*b**2*x + 3*a**(13/2)*b**3) + 6*a**4*b**3*log(sqrt(1 + b/(a*x)) + 1)/(3
*a**(19/2)*x**3 + 9*a**(17/2)*b*x**2 + 9*a**(15/2)*b**2*x + 3*a**(13/2)*b**
3))
```

3.165 $\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2}} dx$

Optimal. Leaf size=79

$$-\frac{5b\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{7/2}}+\frac{5b}{a^3\sqrt{a+\frac{b}{x}}}+\frac{5b}{3a^2\left(a+\frac{b}{x}\right)^{3/2}}+\frac{x}{a\left(a+\frac{b}{x}\right)^{3/2}}$$

Rubi [A] time = 0.04, antiderivative size = 82, normalized size of antiderivative = 1.04, number of steps used = 6, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {242, 51, 63, 208}

$$\frac{5x\sqrt{a+\frac{b}{x}}}{a^3}-\frac{10x}{3a^2\sqrt{a+\frac{b}{x}}}-\frac{5b\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{7/2}}-\frac{2x}{3a\left(a+\frac{b}{x}\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x)^(-5/2),x]
```

```
[Out] (-2*x)/(3*a*(a + b/x)^(3/2)) - (10*x)/(3*a^2*Sqrt[a + b/x]) + (5*Sqrt[a + b
/x]*x)/a^3 - (5*b*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/a^(7/2)
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 242

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a + b/x^n)^p/x^
2, x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2}}\,dx &= -\text{Subst}\left(\int \frac{1}{x^2(a+bx)^{5/2}}\,dx, x, \frac{1}{x}\right)\\
&= -\frac{2x}{3a\left(a+\frac{b}{x}\right)^{3/2}} - \frac{5\,\text{Subst}\left(\int \frac{1}{x^2(a+bx)^{3/2}}\,dx, x, \frac{1}{x}\right)}{3a}\\
&= -\frac{2x}{3a\left(a+\frac{b}{x}\right)^{3/2}} - \frac{10x}{3a^2\sqrt{a+\frac{b}{x}}} - \frac{5\,\text{Subst}\left(\int \frac{1}{x^2\sqrt{a+bx}}\,dx, x, \frac{1}{x}\right)}{a^2}\\
&= -\frac{2x}{3a\left(a+\frac{b}{x}\right)^{3/2}} - \frac{10x}{3a^2\sqrt{a+\frac{b}{x}}} + \frac{5\sqrt{a+\frac{b}{x}}\,x}{a^3} + \frac{(5b)\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \frac{1}{x}\right)}{2a^3}\\
&= -\frac{2x}{3a\left(a+\frac{b}{x}\right)^{3/2}} - \frac{10x}{3a^2\sqrt{a+\frac{b}{x}}} + \frac{5\sqrt{a+\frac{b}{x}}\,x}{a^3} + \frac{5\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+\frac{b}{x}}\right)}{a^3}\\
&= -\frac{2x}{3a\left(a+\frac{b}{x}\right)^{3/2}} - \frac{10x}{3a^2\sqrt{a+\frac{b}{x}}} + \frac{5\sqrt{a+\frac{b}{x}}\,x}{a^3} - \frac{5b\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{7/2}}
\end{aligned}
$$

Mathematica [C] time = 0.02, size = 38, normalized size = 0.48

$$
\frac{2b\,{}_2F_1\left(-\frac{3}{2},2;-\frac{1}{2};\frac{a+\frac{b}{x}}{a}\right)}{3a^2\left(a+\frac{b}{x}\right)^{3/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x)^(-5/2),x]
[Out] (2*b*Hypergeometric2F1[-3/2, 2, -1/2, (a + b/x)/a])/(3*a^2*(a + b/x)^(3/2))
```

IntegrateAlgebraic [A] time = 0.00, size = 78, normalized size = 0.99

$$
\frac{\sqrt{\frac{ax+b}{x}}\left(3a^2x^3+20abx^2+15b^2x\right)}{3a^3(ax+b)^2} - \frac{5b\tanh^{-1}\left(\frac{\sqrt{\frac{ax+b}{x}}}{\sqrt{a}}\right)}{a^{7/2}}
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b/x)^(-5/2),x]
[Out] (Sqrt[(b + a*x)/x]*(15*b^2*x + 20*a*b*x^2 + 3*a^2*x^3))/(3*a^3*(b + a*x)^2)
 - (5*b*ArcTanh[Sqrt[(b + a*x)/x]/Sqrt[a]])/a^(7/2)
```

fricas [A] time = 0.83, size = 225, normalized size = 2.85

$$
\left[\frac{15\left(a^2bx^2+2\,ab^2x+b^3\right)\sqrt{a}\,\log\left(2\,ax-2\,\sqrt{a}\,x\sqrt{\frac{ax+b}{x}}+b\right)+2\left(3\,a^3x^3+20\,a^2bx^2+15\,ab^2x\right)\sqrt{\frac{ax+b}{x}}}{6\left(a^6x^2+2\,a^5bx+a^4b^2\right)}, \frac{15\left(a^2bx^2+2\,ab^2x+b^3\right)\sqrt{-a}\,\arctan\left(\frac{\sqrt{-a}\sqrt{\frac{ax+b}{x}}}{a}\right)+\left(3\,a^3x^3+20\,a^2bx^2+15\,ab^2x\right)\sqrt{\frac{ax+b}{x}}}{3\left(a^6x^2+2\,a^5bx+a^4b^2\right)}\right]
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/6*(15*(a^2*b*x^2 + 2*a*b^2*x + b^3)*sqrt(a)*log(2*a*x - 2*sqrt(a)*x*sqrt
((a*x + b)/x) + b) + 2*(3*a^3*x^3 + 20*a^2*b*x^2 + 15*a*b^2*x)*sqrt((a*x +
b)/x))/(a^6*x^2 + 2*a^5*b*x + a^4*b^2), 1/3*(15*(a^2*b*x^2 + 2*a*b^2*x + b^
3)*sqrt(-a)*arctan(sqrt(-a)*sqrt((a*x + b)/x)/a) + (3*a^3*x^3 + 20*a^2*b*x^
2 + 15*a*b^2*x)*sqrt((a*x + b)/x))/(a^6*x^2 + 2*a^5*b*x + a^4*b^2)]
```

giac [A] time = 0.24, size = 98, normalized size = 1.24

$$\frac{1}{3} \, b {\left(\frac{2 \, {\left(a + \frac{6 \, {\left(a x + b\right)}}{x}\right)} x}{{\left(a x + b\right)} a^{3} \sqrt{\frac{a x + b}{x}}} + \frac{15 \, \arctan\left(\frac{\sqrt{\frac{a x + b}{x}}}{\sqrt{-a}}\right)}{\sqrt{-a} a^{3}} - \frac{3 \, \sqrt{\frac{a x + b}{x}}}{{\left(a - \frac{a x + b}{x}\right)} a^{3}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)^(5/2),x, algorithm="giac")
```

```
[Out] 1/3*b*(2*(a + 6*(a*x + b)/x)*x/((a*x + b)*a^3*sqrt((a*x + b)/x)) + 15*arcta
n(sqrt((a*x + b)/x)/sqrt(-a))/(sqrt(-a)*a^3) - 3*sqrt((a*x + b)/x)/((a - (a
*x + b)/x)*a^3))
```

maple [B] time = 0.07, size = 271, normalized size = 3.43

$$\frac{\sqrt{\frac{a x+b}{x}}\, \left(-15 a^{3} b\, x^{3} \ln \left(\frac{2 a x+b+2 \sqrt{\left(a x+b\right) x}\, \sqrt{a}}{2 \sqrt{a}}\right)-45 a^{2} b^{2} x^{2} \ln \left(\frac{2 a x+b+2 \sqrt{\left(a x+b\right) x}\, \sqrt{a}}{2 \sqrt{a}}\right)+30 \sqrt{\left(a x+b\right) x}\, a^{\frac{7}{2}} x^{3}-45 a\, b^{3} x \ln \left(\frac{2 a x+b+2 \sqrt{\left(a x+b\right) x}\, \sqrt{a}}{2 \sqrt{a}}\right)+90 \sqrt{\left(a x+b\right) x}\, a^{\frac{5}{2}} b\, x^{2}-15 b^{4} \ln \left(\frac{2 a x+b+2 \sqrt{\left(a x+b\right) x}\, \sqrt{a}}{2 \sqrt{a}}\right)+90 \sqrt{\left(a x+b\right) x}\, a^{\frac{3}{2}} b^{2} x-24 \left(\left(a x+b\right) x\right)^{\frac{3}{2}} a^{\frac{5}{2}} x+30 \sqrt{\left(a x+b\right) x}\, \sqrt{a}\, b^{3}-20 \left(\left(a x+b\right) x\right)^{\frac{3}{2}} a^{\frac{3}{2}} b\right) x}{6 \sqrt{\left(a x+b\right) x}\, \left(a x+b\right)^{3} a^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b/x)^(5/2),x)
```

```
[Out] 1/6*((a*x+b)/x)^(1/2)*x*(30*a^(7/2)*((a*x+b)*x)^(1/2)*x^3-24*a^(5/2)*((a*x+
b)*x)^(3/2)*x+90*a^(5/2)*((a*x+b)*x)^(1/2)*x^2*b-15*ln(1/2*(2*a*x+b+2*((a*x
+b)*x)^(1/2)*a^(1/2))/a^(1/2))*x^3*a^3*b-20*b*a^(3/2)*((a*x+b)*x)^(3/2)+90*
a^(3/2)*((a*x+b)*x)^(1/2)*x*b^2-45*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1
/2))/a^(1/2))*x^2*a^2*b^2-45*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a
^(1/2))*x*a*b^3+30*a^(1/2)*((a*x+b)*x)^(1/2)*b^3-15*ln(1/2*(2*a*x+b+2*((a*x
+b)*x)^(1/2)*a^(1/2))/a^(1/2))*b^4)/a^(7/2)/((a*x+b)*x)^(1/2)/(a*x+b)^3
```

maxima [A] time = 1.29, size = 101, normalized size = 1.28

$$\frac{15 \, {\left(a + \frac{b}{x}\right)}^{2} b - 10 \, {\left(a + \frac{b}{x}\right)} a b - 2 \, a^{2} b}{3 \, {\left({\left(a + \frac{b}{x}\right)}^{\frac{5}{2}} a^{3} - {\left(a + \frac{b}{x}\right)}^{\frac{3}{2}} a^{4}\right)}} + \frac{5 \, b \log\left(\frac{\sqrt{a + \frac{b}{x}} - \sqrt{a}}{\sqrt{a + \frac{b}{x}} + \sqrt{a}}\right)}{2 \, a^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/3*(15*(a + b/x)^2*b - 10*(a + b/x)*a*b - 2*a^2*b)/((a + b/x)^(5/2)*a^3 -
(a + b/x)^(3/2)*a^4) + 5/2*b*log((sqrt(a + b/x) - sqrt(a))/(sqrt(a + b/x) +
 sqrt(a)))/a^(7/2)
```

mupad [B] time = 1.72, size = 34, normalized size = 0.43

$$\frac{2\,x\,{\left(\frac{a\,x}{b}+1\right)}^{5/2}\,{}_2\mathrm{F}_1\left(\frac{5}{2},\frac{7}{2};\ \frac{9}{2};\ -\frac{a\,x}{b}\right)}{7\,{\left(a+\frac{b}{x}\right)}^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b/x)^(5/2),x)
```

```
[Out] (2*x*((a*x)/b + 1)^(5/2)*hypergeom([5/2, 7/2], 9/2, -(a*x)/b))/(7*(a + b/x)
^(5/2))
```

sympy [B] time = 7.92, size = 774, normalized size = 9.80

$$\frac{6a^{17}x^{4}\sqrt{1+\frac{b}{ax}}}{6a^{\frac{39}{2}}x^{3}+18a^{\frac{37}{2}}bx^{2}+18a^{\frac{35}{2}}b^{2}x+6a^{\frac{33}{2}}b^{3}}+\frac{46a^{16}bx^{3}\sqrt{1+\frac{b}{ax}}}{6a^{\frac{39}{2}}x^{3}+18a^{\frac{37}{2}}bx^{2}+18a^{\frac{35}{2}}b^{2}x+6a^{\frac{33}{2}}b^{3}}+\frac{15a^{16}bx^{3}\log\left(\frac{b}{ax}\right)}{6a^{\frac{39}{2}}x^{3}+18a^{\frac{37}{2}}bx^{2}+18a^{\frac{35}{2}}b^{2}x+6a^{\frac{33}{2}}b^{3}}-\frac{30a^{16}bx^{3}\log\left(\sqrt{1+\frac{b}{ax}}+1\right)}{6a^{\frac{39}{2}}x^{3}+18a^{\frac{37}{2}}bx^{2}+18a^{\frac{35}{2}}b^{2}x+6a^{\frac{33}{2}}b^{3}}+\frac{70a^{15}b^{2}x^{2}\sqrt{1+\frac{b}{ax}}}{6a^{\frac{39}{2}}x^{3}+18a^{\frac{37}{2}}bx^{2}+18a^{\frac{35}{2}}b^{2}x+6a^{\frac{33}{2}}b^{3}}+\frac{45a^{15}b^{2}x^{2}\log\left(\frac{b}{ax}\right)}{6a^{\frac{39}{2}}x^{3}+18a^{\frac{37}{2}}bx^{2}+18a^{\frac{35}{2}}b^{2}x+6a^{\frac{33}{2}}b^{3}}-\frac{90a^{15}b^{2}x^{2}\log\left(\sqrt{1+\frac{b}{ax}}+1\right)}{6a^{\frac{39}{2}}x^{3}+18a^{\frac{37}{2}}bx^{2}+18a^{\frac{35}{2}}b^{2}x+6a^{\frac{33}{2}}b^{3}}+\frac{30a^{14}b^{3}x\sqrt{1+\frac{b}{ax}}}{6a^{\frac{39}{2}}x^{3}+18a^{\frac{37}{2}}bx^{2}+18a^{\frac{35}{2}}b^{2}x+6a^{\frac{33}{2}}b^{3}}+\frac{45a^{14}b^{3}x\log\left(\frac{b}{ax}\right)}{6a^{\frac{39}{2}}x^{3}+18a^{\frac{37}{2}}bx^{2}+18a^{\frac{35}{2}}b^{2}x+6a^{\frac{33}{2}}b^{3}}-\frac{90a^{14}b^{3}x\log\left(\sqrt{1+\frac{b}{ax}}+1\right)}{6a^{\frac{39}{2}}x^{3}+18a^{\frac{37}{2}}bx^{2}+18a^{\frac{35}{2}}b^{2}x+6a^{\frac{33}{2}}b^{3}}+\frac{15a^{13}b^{4}\log\left(\frac{b}{ax}\right)}{6a^{\frac{39}{2}}x^{3}+18a^{\frac{37}{2}}bx^{2}+18a^{\frac{35}{2}}b^{2}x+6a^{\frac{33}{2}}b^{3}}-\frac{30a^{13}b^{4}\log\left(\sqrt{1+\frac{b}{ax}}+1\right)}{6a^{\frac{39}{2}}x^{3}+18a^{\frac{37}{2}}bx^{2}+18a^{\frac{35}{2}}b^{2}x+6a^{\frac{33}{2}}b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)**(5/2),x)
```

```
[Out] 6*a**17*x**4*sqrt(1 + b/(a*x))/(6*a**(39/2)*x**3 + 18*a**(37/2)*b*x**2 + 18
*a**(35/2)*b**2*x + 6*a**(33/2)*b**3) + 46*a**16*b*x**3*sqrt(1 + b/(a*x))/(
6*a**(39/2)*x**3 + 18*a**(37/2)*b*x**2 + 18*a**(35/2)*b**2*x + 6*a**(33/2)*
b**3) + 15*a**16*b*x**3*log(b/(a*x))/(6*a**(39/2)*x**3 + 18*a**(37/2)*b*x**
2 + 18*a**(35/2)*b**2*x + 6*a**(33/2)*b**3) - 30*a**16*b*x**3*log(sqrt(1 +
b/(a*x)) + 1)/(6*a**(39/2)*x**3 + 18*a**(37/2)*b*x**2 + 18*a**(35/2)*b**2*x
 + 6*a**(33/2)*b**3) + 70*a**15*b**2*x**2*sqrt(1 + b/(a*x))/(6*a**(39/2)*x*
*3 + 18*a**(37/2)*b*x**2 + 18*a**(35/2)*b**2*x + 6*a**(33/2)*b**3) + 45*a**
15*b**2*x**2*log(b/(a*x))/(6*a**(39/2)*x**3 + 18*a**(37/2)*b*x**2 + 18*a**(
35/2)*b**2*x + 6*a**(33/2)*b**3) - 90*a**15*b**2*x**2*log(sqrt(1 + b/(a*x))
 + 1)/(6*a**(39/2)*x**3 + 18*a**(37/2)*b*x**2 + 18*a**(35/2)*b**2*x + 6*a**
(33/2)*b**3) + 30*a**14*b**3*x*sqrt(1 + b/(a*x))/(6*a**(39/2)*x**3 + 18*a**
(37/2)*b*x**2 + 18*a**(35/2)*b**2*x + 6*a**(33/2)*b**3) + 45*a**14*b**3*x*l
og(b/(a*x))/(6*a**(39/2)*x**3 + 18*a**(37/2)*b*x**2 + 18*a**(35/2)*b**2*x +
 6*a**(33/2)*b**3) - 90*a**14*b**3*x*log(sqrt(1 + b/(a*x)) + 1)/(6*a**(39/2
)*x**3 + 18*a**(37/2)*b*x**2 + 18*a**(35/2)*b**2*x + 6*a**(33/2)*b**3) + 15
*a**13*b**4*log(b/(a*x))/(6*a**(39/2)*x**3 + 18*a**(37/2)*b*x**2 + 18*a**(3
5/2)*b**2*x + 6*a**(33/2)*b**3) - 30*a**13*b**4*log(sqrt(1 + b/(a*x)) + 1)/
(6*a**(39/2)*x**3 + 18*a**(37/2)*b*x**2 + 18*a**(35/2)*b**2*x + 6*a**(33/2)
*b**3)
```

3.166 $\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2}\left(c+\frac{d}{x}\right)}\,dx$

Optimal. Leaf size=201

$$-\frac{(2ad+5bc)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{7/2}c^2}+\frac{b(5bc-3ad)}{3a^2c\left(a+\frac{b}{x}\right)^{3/2}(bc-ad)}+\frac{b\left(a^2d^2-8abcd+5b^2c^2\right)}{a^3c\sqrt{a+\frac{b}{x}}\,(bc-ad)^2}-\frac{2d^{7/2}\tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c^2(bc-ad)^{5/2}}+\frac{}{a}$$

Rubi [A] time = 0.32, antiderivative size = 201, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {375, 103, 152, 156, 63, 208, 205}

$$\frac{b\left(a^2d^2-8abcd+5b^2c^2\right)}{a^3c\sqrt{a+\frac{b}{x}}\,(bc-ad)^2}-\frac{(2ad+5bc)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{7/2}c^2}+\frac{b(5bc-3ad)}{3a^2c\left(a+\frac{b}{x}\right)^{3/2}(bc-ad)}-\frac{2d^{7/2}\tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c^2(bc-ad)^{5/2}}+\frac{x}{ac\left(a+\frac{b}{x}\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b/x)^(5/2)*(c + d/x)),x]
```

```
[Out] (b*(5*b*c - 3*a*d))/(3*a^2*c*(b*c - a*d)*(a + b/x)^(3/2)) + (b*(5*b^2*c^2 -
 8*a*b*c*d + a^2*d^2))/(a^3*c*(b*c - a*d)^2*Sqrt[a + b/x]) + x/(a*c*(a + b/
x)^(3/2)) - (2*d^(7/2)*ArcTan[(Sqrt[d]*Sqrt[a + b/x])/Sqrt[b*c - a*d]])/(c^
2*(b*c - a*d)^(5/2)) - ((5*b*c + 2*a*d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/(a^
(7/2)*c^2)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 375

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[((a + b/x^n)^p*(c + d/x^n)^q)/x^2, x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rubi steps

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2}\left(c+\frac{d}{x}\right)}\,dx = -\operatorname{Subst}\left(\int \frac{1}{x^2(a+bx)^{5/2}(c+dx)}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{x}{ac\left(a+\frac{b}{x}\right)^{3/2}} + \frac{\operatorname{Subst}\left(\int \frac{\frac{1}{2}(5bc+2ad)+\frac{5bdx}{2}}{x(a+bx)^{5/2}(c+dx)}\,dx, x, \frac{1}{x}\right)}{ac}$$

$$= \frac{b(5bc-3ad)}{3a^2c(bc-ad)\left(a+\frac{b}{x}\right)^{3/2}} + \frac{x}{ac\left(a+\frac{b}{x}\right)^{3/2}} + \frac{2\operatorname{Subst}\left(\int \frac{\frac{3}{4}(bc-ad)(5bc+2ad)+\frac{3}{4}bd(5bc-3ad)x}{x(a+bx)^{3/2}(c+dx)}\,dx, \ldots\right.}{3a^2c(bc-ad)}$$

$$= \frac{b(5bc-3ad)}{3a^2c(bc-ad)\left(a+\frac{b}{x}\right)^{3/2}} + \frac{b\left(5b^2c^2-8abcd+a^2d^2\right)}{a^3c(bc-ad)^2\sqrt{a+\frac{b}{x}}} + \frac{x}{ac\left(a+\frac{b}{x}\right)^{3/2}} + \frac{4\operatorname{Subst}\left(\int \frac{\frac{3}{8}(bc \ldots}{\ldots}\right.}{\ldots}$$

$$= \frac{b(5bc-3ad)}{3a^2c(bc-ad)\left(a+\frac{b}{x}\right)^{3/2}} + \frac{b\left(5b^2c^2-8abcd+a^2d^2\right)}{a^3c(bc-ad)^2\sqrt{a+\frac{b}{x}}} + \frac{x}{ac\left(a+\frac{b}{x}\right)^{3/2}} - \frac{d^4\operatorname{Subst}\left(\int \frac{\ldots}{\sqrt{a \ldots}}\right.}{c^2(bc \ldots}$$

$$= \frac{b(5bc-3ad)}{3a^2c(bc-ad)\left(a+\frac{b}{x}\right)^{3/2}} + \frac{b\left(5b^2c^2-8abcd+a^2d^2\right)}{a^3c(bc-ad)^2\sqrt{a+\frac{b}{x}}} + \frac{x}{ac\left(a+\frac{b}{x}\right)^{3/2}} - \frac{\left(2d^4\right)\operatorname{Subst}\left(\int \ldots\right.}{bc \ldots}$$

$$= \frac{b(5bc-3ad)}{3a^2c(bc-ad)\left(a+\frac{b}{x}\right)^{3/2}} + \frac{b\left(5b^2c^2-8abcd+a^2d^2\right)}{a^3c(bc-ad)^2\sqrt{a+\frac{b}{x}}} + \frac{x}{ac\left(a+\frac{b}{x}\right)^{3/2}} - \frac{2d^{7/2}\tan^{-1}\left(\frac{\sqrt{a \ldots}}{\sqrt{\ldots}}\right.}{c^2(bc-ad \ldots}$$

Mathematica [C] time = 0.08, size = 118, normalized size = 0.59

$$\frac{x\left((ad-bc)\left((2ad+5bc)\,_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};\frac{b}{ax}+1\right)+3acx\right)-2a^2d^2\,_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};\frac{d\left(a+\frac{b}{x}\right)}{ad-bc}\right)\right)}{3a^2c^2\sqrt{a+\frac{b}{x}}\,(ax+b)(ad-bc)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b/x)^(5/2)*(c + d/x)),x]
```

```
[Out] (x*(-2*a^2*d^2*Hypergeometric2F1[-3/2, 1, -1/2, (d*(a + b/x))/(-(b*c) + a*d
)] + (-(b*c) + a*d)*(3*a*c*x + (5*b*c + 2*a*d)*Hypergeometric2F1[-3/2, 1, -
1/2, 1 + b/(a*x)])))/(3*a^2*c^2*(-(b*c) + a*d)*Sqrt[a + b/x]*(b + a*x))
```

IntegrateAlgebraic [A] time = 0.46, size = 237, normalized size = 1.18

$$\frac{(-2ad-5bc)\tanh^{-1}\left(\frac{\sqrt{\frac{ax+b}{x}}}{\sqrt{a}}\right)}{a^{7/2}c^2}+\frac{\sqrt{\frac{ax+b}{x}}\left(3a^4d^2x^3-6a^3bcdx^3+6a^3bd^2x^2+3a^2b^2c^2x^3-32a^2b^2cdx^2+3a^2b^2d^2x+20ab^3c^2x^2-24ab^3cdx+15b^4c^2x\right)}{3a^3c(ax+b)^2(ad-bc)^2}-\frac{2d^{7/2}\tan^{-1}\left(\frac{\sqrt{d}\sqrt{\frac{ax+b}{x}}}{\sqrt{bc-ad}}\right)}{c^2(bc-ad)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((a + b/x)^(5/2)*(c + d/x)),x]
```

```
[Out] (Sqrt[(b + a*x)/x]*(15*b^4*c^2*x - 24*a*b^3*c*d*x + 3*a^2*b^2*d^2*x + 20*a*
b^3*c^2*x^2 - 32*a^2*b^2*c*d*x^2 + 6*a^3*b*d^2*x^2 + 3*a^2*b^2*c^2*x^3 - 6*
a^3*b*c*d*x^3 + 3*a^4*d^2*x^3))/(3*a^3*c*(-(b*c) + a*d)^2*(b + a*x)^2) - (2
*d^(7/2)*ArcTan[(Sqrt[d]*Sqrt[(b + a*x)/x])/Sqrt[b*c - a*d]])/(c^2*(b*c - a
*d)^(5/2)) + ((-5*b*c - 2*a*d)*ArcTanh[Sqrt[(b + a*x)/x]/Sqrt[a]])/(a^(7/2)
*c^2)
```

fricas [B] time = 4.97, size = 1990, normalized size = 9.90

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)^(5/2)/(c+d/x),x, algorithm="fricas")
```

```
[Out] [1/6*(3*(5*b^5*c^3 - 8*a*b^4*c^2*d + a^2*b^3*c*d^2 + 2*a^3*b^2*d^3 + (5*a^2
*b^3*c^3 - 8*a^3*b^2*c^2*d + a^4*b*c*d^2 + 2*a^5*d^3)*x^2 + 2*(5*a*b^4*c^3
- 8*a^2*b^3*c^2*d + a^3*b^2*c*d^2 + 2*a^4*b*d^3)*x)*sqrt(a)*log(2*a*x - 2*s
qrt(a)*x*sqrt((a*x + b)/x) + b) + 6*(a^6*d^3*x^2 + 2*a^5*b*d^3*x + a^4*b^2*
d^3)*sqrt(-d/(b*c - a*d))*log(-(2*(b*c - a*d)*x*sqrt(-d/(b*c - a*d))*sqrt((
a*x + b)/x) - b*d + (b*c - 2*a*d)*x)/(c*x + d)) + 2*(3*(a^3*b^2*c^3 - 2*a^4
*b*c^2*d + a^5*c*d^2)*x^3 + 2*(10*a^2*b^3*c^3 - 16*a^3*b^2*c^2*d + 3*a^4*b*
c*d^2)*x^2 + 3*(5*a*b^4*c^3 - 8*a^2*b^3*c^2*d + a^3*b^2*c*d^2)*x)*sqrt((a*x
 + b)/x))/(a^4*b^4*c^4 - 2*a^5*b^3*c^3*d + a^6*b^2*c^2*d^2 + (a^6*b^2*c^4 -
 2*a^7*b*c^3*d + a^8*c^2*d^2)*x^2 + 2*(a^5*b^3*c^4 - 2*a^6*b^2*c^3*d + a^7*
b*c^2*d^2)*x), 1/3*(3*(5*b^5*c^3 - 8*a*b^4*c^2*d + a^2*b^3*c*d^2 + 2*a^3*b^
2*d^3 + (5*a^2*b^3*c^3 - 8*a^3*b^2*c^2*d + a^4*b*c*d^2 + 2*a^5*d^3)*x^2 + 2
*(5*a*b^4*c^3 - 8*a^2*b^3*c^2*d + a^3*b^2*c*d^2 + 2*a^4*b*d^3)*x)*sqrt(-a)*
arctan(sqrt(-a)*sqrt((a*x + b)/x)/a) + 3*(a^6*d^3*x^2 + 2*a^5*b*d^3*x + a^4
*b^2*d^3)*sqrt(-d/(b*c - a*d))*log(-(2*(b*c - a*d)*x*sqrt(-d/(b*c - a*d))*s
qrt((a*x + b)/x) - b*d + (b*c - 2*a*d)*x)/(c*x + d)) + (3*(a^3*b^2*c^3 - 2*
a^4*b*c^2*d + a^5*c*d^2)*x^3 + 2*(10*a^2*b^3*c^3 - 16*a^3*b^2*c^2*d + 3*a^4
*b*c*d^2)*x^2 + 3*(5*a*b^4*c^3 - 8*a^2*b^3*c^2*d + a^3*b^2*c*d^2)*x)*sqrt((
a*x + b)/x))/(a^4*b^4*c^4 - 2*a^5*b^3*c^3*d + a^6*b^2*c^2*d^2 + (a^6*b^2*c^
4 - 2*a^7*b*c^3*d + a^8*c^2*d^2)*x^2 + 2*(a^5*b^3*c^4 - 2*a^6*b^2*c^3*d + a
^7*b*c^2*d^2)*x), -1/6*(12*(a^6*d^3*x^2 + 2*a^5*b*d^3*x + a^4*b^2*d^3)*sqrt
(d/(b*c - a*d))*arctan(-(b*c - a*d)*x*sqrt(d/(b*c - a*d))*sqrt((a*x + b)/x)
/(a*d*x + b*d)) - 3*(5*b^5*c^3 - 8*a*b^4*c^2*d + a^2*b^3*c*d^2 + 2*a^3*b^2*
```

```
d^3 + (5*a^2*b^3*c^3 - 8*a^3*b^2*c^2*d + a^4*b*c*d^2 + 2*a^5*d^3)*x^2 + 2*(
5*a*b^4*c^3 - 8*a^2*b^3*c^2*d + a^3*b^2*c*d^2 + 2*a^4*b*d^3)*x)*sqrt(a)*log
(2*a*x - 2*sqrt(a)*x*sqrt((a*x + b)/x) + b) - 2*(3*(a^3*b^2*c^3 - 2*a^4*b*c
^2*d + a^5*c*d^2)*x^3 + 2*(10*a^2*b^3*c^3 - 16*a^3*b^2*c^2*d + 3*a^4*b*c*d^
2)*x^2 + 3*(5*a*b^4*c^3 - 8*a^2*b^3*c^2*d + a^3*b^2*c*d^2)*x)*sqrt((a*x + b
)/x))/(a^4*b^4*c^4 - 2*a^5*b^3*c^3*d + a^6*b^2*c^2*d^2 + (a^6*b^2*c^4 - 2*a
^7*b*c^3*d + a^8*c^2*d^2)*x^2 + 2*(a^5*b^3*c^4 - 2*a^6*b^2*c^3*d + a^7*b*c^
2*d^2)*x), -1/3*(6*(a^6*d^3*x^2 + 2*a^5*b*d^3*x + a^4*b^2*d^3)*sqrt(d/(b*c
- a*d))*arctan(-(b*c - a*d)*x*sqrt(d/(b*c - a*d))*sqrt((a*x + b)/x)/(a*d*x
+ b*d)) - 3*(5*b^5*c^3 - 8*a*b^4*c^2*d + a^2*b^3*c*d^2 + 2*a^3*b^2*d^3 + (5
*a^2*b^3*c^3 - 8*a^3*b^2*c^2*d + a^4*b*c*d^2 + 2*a^5*d^3)*x^2 + 2*(5*a*b^4*
c^3 - 8*a^2*b^3*c^2*d + a^3*b^2*c*d^2 + 2*a^4*b*d^3)*x)*sqrt(-a)*arctan(sqr
t(-a)*sqrt((a*x + b)/x)/a) - (3*(a^3*b^2*c^3 - 2*a^4*b*c^2*d + a^5*c*d^2)*x
^3 + 2*(10*a^2*b^3*c^3 - 16*a^3*b^2*c^2*d + 3*a^4*b*c*d^2)*x^2 + 3*(5*a*b^4
*c^3 - 8*a^2*b^3*c^2*d + a^3*b^2*c*d^2)*x)*sqrt((a*x + b)/x))/(a^4*b^4*c^4
- 2*a^5*b^3*c^3*d + a^6*b^2*c^2*d^2 + (a^6*b^2*c^4 - 2*a^7*b*c^3*d + a^8*c^
2*d^2)*x^2 + 2*(a^5*b^3*c^4 - 2*a^6*b^2*c^3*d + a^7*b*c^2*d^2)*x)]
```

giac [**A**] time = 0.22, size = 247, normalized size = 1.23

$$-\frac{1}{3}\left(\frac{6\,d^4\arctan\left(\frac{d\sqrt{\frac{ax+b}{x}}}{\sqrt{bcd-ad^2}}\right)}{\left(b^4c^4-2\,ab^3c^3d+a^2b^2c^2d^2\right)\sqrt{bcd-ad^2}}-\frac{2\left(abc-a^2d+\frac{6\,(ax+b)bc}{x}-\frac{9\,(ax+b)ad}{x}\right)x}{\left(a^3b^2c^2-2\,a^4bcd+a^5d^2\right)(ax+b)\sqrt{\frac{ax+b}{x}}}+\frac{3\sqrt{\frac{ax+b}{x}}}{\left(a-\frac{ax+b}{x}\right)a^3bc}-\frac{3\,(5\,bc+2\,ad)\arctan\left(\frac{\sqrt{\frac{ax+b}{x}}}{\sqrt{-a}}\right)}{\sqrt{-a}\,a^3b^2c^2}\right)b^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)^(5/2)/(c+d/x),x, algorithm="giac")
```

```
[Out] -1/3*(6*d^4*arctan(d*sqrt((a*x + b)/x)/sqrt(b*c*d - a*d^2))/((b^4*c^4 - 2*a
*b^3*c^3*d + a^2*b^2*c^2*d^2)*sqrt(b*c*d - a*d^2)) - 2*(a*b*c - a^2*d + 6*(
a*x + b)*b*c/x - 9*(a*x + b)*a*d/x)*x/((a^3*b^2*c^2 - 2*a^4*b*c*d + a^5*d^2
)*(a*x + b)*sqrt((a*x + b)/x)) + 3*sqrt((a*x + b)/x)/((a - (a*x + b)/x)*a^3
*b*c) - 3*(5*b*c + 2*a*d)*arctan(sqrt((a*x + b)/x)/sqrt(-a))/(sqrt(-a)*a^3*
b^2*c^2))*b^2
```

maple [**B**] time = 0.07, size = 1767, normalized size = 8.79

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b/x)^(5/2)/(c+d/x),x)
```

```
[Out] -1/6*(15*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))*((a*d-b*c)/c
^2*d)^(1/2)*b^6*c^4+6*a^(13/2)*ln((-2*a*d*x+b*c*x-b*d+2*((a*d-b*c)/c^2*d)^(
1/2)*((a*x+b)*x)^(1/2)*c)/(c*x+d))*x^3*d^4+6*a^(7/2)*ln((-2*a*d*x+b*c*x-b*d
+2*((a*d-b*c)/c^2*d)^(1/2)*((a*x+b)*x)^(1/2)*c)/(c*x+d))*b^3*d^4+3*ln(1/2*(
2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))*((a*d-b*c)/c^2*d)^(1/2)*a^2*b
^4*c^2*d^2+45*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))*((a*d-b
*c)/c^2*d)^(1/2)*x^2*a^2*b^4*c^4+6*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1
/2))/a^(1/2))*((a*d-b*c)/c^2*d)^(1/2)*x^3*a^6*c*d^3+15*ln(1/2*(2*a*x+b+2*((
a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))*((a*d-b*c)/c^2*d)^(1/2)*x^3*a^3*b^3*c^4-3
0*a^(7/2)*((a*x+b)*x)^(1/2)*((a*d-b*c)/c^2*d)^(1/2)*x^3*b^2*c^4+24*a^(5/2)*
((a*x+b)*x)^(3/2)*((a*d-b*c)/c^2*d)^(1/2)*x*b^2*c^4-90*a^(5/2)*((a*x+b)*x)^
(1/2)*((a*d-b*c)/c^2*d)^(1/2)*x^2*b^3*c^4-32*a^(5/2)*((a*x+b)*x)^(3/2)*((a*
d-b*c)/c^2*d)^(1/2)*b^2*c^3*d-90*a^(3/2)*((a*x+b)*x)^(1/2)*((a*d-b*c)/c^2*d
)^(1/2)*x*b^4*c^4-6*a^(5/2)*((a*x+b)*x)^(1/2)*((a*d-b*c)/c^2*d)^(1/2)*b^3*c
^2*d^2+48*a^(3/2)*((a*x+b)*x)^(1/2)*((a*d-b*c)/c^2*d)^(1/2)*b^4*c^3*d-6*a^(
11/2)*((a*x+b)*x)^(1/2)*((a*d-b*c)/c^2*d)^(1/2)*x^3*c^2*d^2+45*ln(1/2*(2*a*
x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))*((a*d-b*c)/c^2*d)^(1/2)*x*a*b^5*c
^4+6*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))*((a*d-b*c)/c^2*d
)^(1/2)*a^3*b^3*c*d^3-24*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/
```

```
2))*((a*d-b*c)/c^2*d)^(1/2)*a*b^5*c^3*d+144*a^(7/2)*((a*x+b)*x)^(1/2)*((a*d
-b*c)/c^2*d)^(1/2)*x^2*b^2*c^3*d-18*a^(7/2)*((a*x+b)*x)^(1/2)*((a*d-b*c)/c^
2*d)^(1/2)*x*b^2*c^2*d^2+144*a^(5/2)*((a*x+b)*x)^(1/2)*((a*d-b*c)/c^2*d)^(1
/2)*x*b^3*c^3*d-18*a^(9/2)*((a*x+b)*x)^(1/2)*((a*d-b*c)/c^2*d)^(1/2)*x^2*b*
c^2*d^2+48*a^(9/2)*((a*x+b)*x)^(1/2)*((a*d-b*c)/c^2*d)^(1/2)*x^3*b*c^3*d-36
*a^(7/2)*((a*x+b)*x)^(3/2)*((a*d-b*c)/c^2*d)^(1/2)*x*b*c^3*d+3*ln(1/2*(2*a*
x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))*((a*d-b*c)/c^2*d)^(1/2)*x^3*a^5*b
*c^2*d^2-24*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))*((a*d-b*c
)/c^2*d)^(1/2)*x^3*a^4*b^2*c^3*d+18*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(
1/2))/a^(1/2))*((a*d-b*c)/c^2*d)^(1/2)*x^2*a^5*b*c*d^3+9*ln(1/2*(2*a*x+b+2*
((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))*((a*d-b*c)/c^2*d)^(1/2)*x^2*a^4*b^2*c^2
*d^2-72*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))*((a*d-b*c)/c^
2*d)^(1/2)*x^2*a^3*b^3*c^3*d+18*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2)
)/a^(1/2))*((a*d-b*c)/c^2*d)^(1/2)*x*a^4*b^2*c*d^3+9*ln(1/2*(2*a*x+b+2*((a*
x+b)*x)^(1/2)*a^(1/2))/a^(1/2))*((a*d-b*c)/c^2*d)^(1/2)*x*a^3*b^3*c^2*d^2-7
2*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))*((a*d-b*c)/c^2*d)^(
1/2)*x*a^2*b^4*c^3*d+18*a^(11/2)*ln((-2*a*d*x+b*c*x-b*d+2*((a*d-b*c)/c^2*d)
^(1/2)*((a*x+b)*x)^(1/2)*c)/(c*x+d))*x^2*b*d^4-30*a^(1/2)*((a*x+b)*x)^(1/2)
*((a*d-b*c)/c^2*d)^(1/2)*b^5*c^4+20*a^(3/2)*((a*x+b)*x)^(3/2)*((a*d-b*c)/c^
2*d)^(1/2)*b^3*c^4+18*a^(9/2)*ln((-2*a*d*x+b*c*x-b*d+2*((a*d-b*c)/c^2*d)^(1
/2)*((a*x+b)*x)^(1/2)*c)/(c*x+d))*x*b^2*d^4)*x*((a*x+b)/x)^(1/2)/a^(7/2)/(a
*x+b)^3/((a*d-b*c)/c^2*d)^(1/2)/c^3/(a*d-b*c)^2/((a*x+b)*x)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{\frac{5}{2}}\left(c+\frac{d}{x}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)^(5/2)/(c+d/x),x, algorithm="maxima")
```

```
[Out] integrate(1/((a + b/x)^(5/2)*(c + d/x)), x)
```

mupad [B] time = 4.62, size = 5387, normalized size = 26.80

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b/x)^(5/2)*(c + d/x)),x)
```

```
[Out] - ((2*b^2)/(3*(a^2*d - a*b*c)) + (2*b^2*(a + b/x)*(8*a*d - 5*b*c))/(3*(a^2*
d - a*b*c)^2) + (b*(a + b/x)^2*(a^2*d^2 + 5*b^2*c^2 - 8*a*b*c*d))/(a^2*c*(a
^2*d - a*b*c)*(a*d - b*c)))/(a*(a + b/x)^(3/2) - (a + b/x)^(5/2)) - (atan((
(((a + b/x)^(1/2)*(50*a^9*b^14*c^15*d^3 - 460*a^10*b^13*c^14*d^4 + 1858*a^1
1*b^12*c^13*d^5 - 4280*a^12*b^11*c^12*d^6 + 6060*a^13*b^10*c^11*d^7 - 5160*
a^14*b^9*c^10*d^8 + 2108*a^15*b^8*c^9*d^9 + 336*a^16*b^7*c^8*d^10 - 750*a^1
7*b^6*c^7*d^11 + 180*a^18*b^5*c^6*d^12 + 130*a^19*b^4*c^5*d^13 - 88*a^20*b^
3*c^4*d^14 + 16*a^21*b^2*c^3*d^15) - ((2*a*d + 5*b*c)*(20*a^12*b^14*c^17*d^
2 - 212*a^13*b^13*c^16*d^3 + 1012*a^14*b^12*c^15*d^4 - 2860*a^15*b^11*c^14*
d^5 + 5288*a^16*b^10*c^13*d^6 - 6664*a^17*b^9*c^12*d^7 + 5768*a^18*b^8*c^11
*d^8 - 3352*a^19*b^7*c^10*d^9 + 1220*a^20*b^6*c^9*d^10 - 228*a^21*b^5*c^8*d
^11 + 4*a^22*b^4*c^7*d^12 + 4*a^23*b^3*c^6*d^13 - ((a + b/x)^(1/2)*(2*a*d +
 5*b*c)*(8*a^15*b^13*c^18*d^2 - 96*a^16*b^12*c^17*d^3 + 520*a^17*b^11*c^16*
d^4 - 1680*a^18*b^10*c^15*d^5 + 3600*a^19*b^9*c^14*d^6 - 5376*a^20*b^8*c^13
*d^7 + 5712*a^21*b^7*c^12*d^8 - 4320*a^22*b^6*c^11*d^9 + 2280*a^23*b^5*c^10
*d^10 - 800*a^24*b^4*c^9*d^11 + 168*a^25*b^3*c^8*d^12 - 16*a^26*b^2*c^7*d^1
3))/(2*c^2*(a^7)^(1/2))))/(2*c^2*(a^7)^(1/2)))*(2*a*d + 5*b*c)*1i)/(2*c^2*(
a^7)^(1/2)) + (((a + b/x)^(1/2)*(50*a^9*b^14*c^15*d^3 - 460*a^10*b^13*c^14*
d^4 + 1858*a^11*b^12*c^13*d^5 - 4280*a^12*b^11*c^12*d^6 + 6060*a^13*b^10*c^
```

11*d^7 - 5160*a^14*b^9*c^10*d^8 + 2108*a^15*b^8*c^9*d^9 + 336*a^16*b^7*c^8*
d^10 - 750*a^17*b^6*c^7*d^11 + 180*a^18*b^5*c^6*d^12 + 130*a^19*b^4*c^5*d^1
3 - 88*a^20*b^3*c^4*d^14 + 16*a^21*b^2*c^3*d^15) + ((2*a*d + 5*b*c)*(20*a^1
2*b^14*c^17*d^2 - 212*a^13*b^13*c^16*d^3 + 1012*a^14*b^12*c^15*d^4 - 2860*a
^15*b^11*c^14*d^5 + 5288*a^16*b^10*c^13*d^6 - 6664*a^17*b^9*c^12*d^7 + 5768
*a^18*b^8*c^11*d^8 - 3352*a^19*b^7*c^10*d^9 + 1220*a^20*b^6*c^9*d^10 - 228*
a^21*b^5*c^8*d^11 + 4*a^22*b^4*c^7*d^12 + 4*a^23*b^3*c^6*d^13 + ((a + b/x)^
(1/2)*(2*a*d + 5*b*c)*(8*a^15*b^13*c^18*d^2 - 96*a^16*b^12*c^17*d^3 + 520*a
^17*b^11*c^16*d^4 - 1680*a^18*b^10*c^15*d^5 + 3600*a^19*b^9*c^14*d^6 - 5376
*a^20*b^8*c^13*d^7 + 5712*a^21*b^7*c^12*d^8 - 4320*a^22*b^6*c^11*d^9 + 2280
*a^23*b^5*c^10*d^10 - 800*a^24*b^4*c^9*d^11 + 168*a^25*b^3*c^8*d^12 - 16*a^
26*b^2*c^7*d^13))/(2*c^2*(a^7)^(1/2))))/(2*c^2*(a^7)^(1/2)))*(2*a*d + 5*b*c
)*1i)/(2*c^2*(a^7)^(1/2)))/(100*a^9*b^12*c^11*d^6 - 720*a^10*b^11*c^10*d^7
+ 2176*a^11*b^10*c^9*d^8 - 3528*a^12*b^9*c^8*d^9 + 3192*a^13*b^8*c^7*d^10 -
1400*a^14*b^7*c^6*d^11 + 264*a^16*b^5*c^4*d^13 - 92*a^17*b^4*c^3*d^14 + 8*
a^18*b^3*c^2*d^15 + (((a + b/x)^(1/2)*(50*a^9*b^14*c^15*d^3 - 460*a^10*b^13
*c^14*d^4 + 1858*a^11*b^12*c^13*d^5 - 4280*a^12*b^11*c^12*d^6 + 6060*a^13*b
^10*c^11*d^7 - 5160*a^14*b^9*c^10*d^8 + 2108*a^15*b^8*c^9*d^9 + 336*a^16*b^
7*c^8*d^10 - 750*a^17*b^6*c^7*d^11 + 180*a^18*b^5*c^6*d^12 + 130*a^19*b^4*c
^5*d^13 - 88*a^20*b^3*c^4*d^14 + 16*a^21*b^2*c^3*d^15) - ((2*a*d + 5*b*c)*(
20*a^12*b^14*c^17*d^2 - 212*a^13*b^13*c^16*d^3 + 1012*a^14*b^12*c^15*d^4 -
2860*a^15*b^11*c^14*d^5 + 5288*a^16*b^10*c^13*d^6 - 6664*a^17*b^9*c^12*d^7
+ 5768*a^18*b^8*c^11*d^8 - 3352*a^19*b^7*c^10*d^9 + 1220*a^20*b^6*c^9*d^10
- 228*a^21*b^5*c^8*d^11 + 4*a^22*b^4*c^7*d^12 + 4*a^23*b^3*c^6*d^13 - ((a +
b/x)^(1/2)*(2*a*d + 5*b*c)*(8*a^15*b^13*c^18*d^2 - 96*a^16*b^12*c^17*d^3 +
520*a^17*b^11*c^16*d^4 - 1680*a^18*b^10*c^15*d^5 + 3600*a^19*b^9*c^14*d^6
- 5376*a^20*b^8*c^13*d^7 + 5712*a^21*b^7*c^12*d^8 - 4320*a^22*b^6*c^11*d^9
+ 2280*a^23*b^5*c^10*d^10 - 800*a^24*b^4*c^9*d^11 + 168*a^25*b^3*c^8*d^12 -
16*a^26*b^2*c^7*d^13))/(2*c^2*(a^7)^(1/2))))/(2*c^2*(a^7)^(1/2)))*(2*a*d +
5*b*c))/(2*c^2*(a^7)^(1/2)) - (((a + b/x)^(1/2)*(50*a^9*b^14*c^15*d^3 - 46
0*a^10*b^13*c^14*d^4 + 1858*a^11*b^12*c^13*d^5 - 4280*a^12*b^11*c^12*d^6 +
6060*a^13*b^10*c^11*d^7 - 5160*a^14*b^9*c^10*d^8 + 2108*a^15*b^8*c^9*d^9 +
336*a^16*b^7*c^8*d^10 - 750*a^17*b^6*c^7*d^11 + 180*a^18*b^5*c^6*d^12 + 130
*a^19*b^4*c^5*d^13 - 88*a^20*b^3*c^4*d^14 + 16*a^21*b^2*c^3*d^15) + ((2*a*d
+ 5*b*c)*(20*a^12*b^14*c^17*d^2 - 212*a^13*b^13*c^16*d^3 + 1012*a^14*b^12*
c^15*d^4 - 2860*a^15*b^11*c^14*d^5 + 5288*a^16*b^10*c^13*d^6 - 6664*a^17*b^
9*c^12*d^7 + 5768*a^18*b^8*c^11*d^8 - 3352*a^19*b^7*c^10*d^9 + 1220*a^20*b^
6*c^9*d^10 - 228*a^21*b^5*c^8*d^11 + 4*a^22*b^4*c^7*d^12 + 4*a^23*b^3*c^6*d
^13 + ((a + b/x)^(1/2)*(2*a*d + 5*b*c)*(8*a^15*b^13*c^18*d^2 - 96*a^16*b^12
*c^17*d^3 + 520*a^17*b^11*c^16*d^4 - 1680*a^18*b^10*c^15*d^5 + 3600*a^19*b^
9*c^14*d^6 - 5376*a^20*b^8*c^13*d^7 + 5712*a^21*b^7*c^12*d^8 - 4320*a^22*b^
6*c^11*d^9 + 2280*a^23*b^5*c^10*d^10 - 800*a^24*b^4*c^9*d^11 + 168*a^25*b^3
*c^8*d^12 - 16*a^26*b^2*c^7*d^13))/(2*c^2*(a^7)^(1/2))))/(2*c^2*(a^7)^(1/2)
))*(2*a*d + 5*b*c))/(2*c^2*(a^7)^(1/2))))*(2*a*d + 5*b*c)*1i)/(c^2*(a^7)^(1
/2)) - (atan((((d^7*(a*d - b*c)^5)^(1/2)*((a + b/x)^(1/2)*(50*a^9*b^14*c^15
*d^3 - 460*a^10*b^13*c^14*d^4 + 1858*a^11*b^12*c^13*d^5 - 4280*a^12*b^11*c^
12*d^6 + 6060*a^13*b^10*c^11*d^7 - 5160*a^14*b^9*c^10*d^8 + 2108*a^15*b^8*c
^9*d^9 + 336*a^16*b^7*c^8*d^10 - 750*a^17*b^6*c^7*d^11 + 180*a^18*b^5*c^6*d
^12 + 130*a^19*b^4*c^5*d^13 - 88*a^20*b^3*c^4*d^14 + 16*a^21*b^2*c^3*d^15)
+ ((d^7*(a*d - b*c)^5)^(1/2)*(20*a^12*b^14*c^17*d^2 - 212*a^13*b^13*c^16*d^
3 + 1012*a^14*b^12*c^15*d^4 - 2860*a^15*b^11*c^14*d^5 + 5288*a^16*b^10*c^13
*d^6 - 6664*a^17*b^9*c^12*d^7 + 5768*a^18*b^8*c^11*d^8 - 3352*a^19*b^7*c^10
*d^9 + 1220*a^20*b^6*c^9*d^10 - 228*a^21*b^5*c^8*d^11 + 4*a^22*b^4*c^7*d^12
+ 4*a^23*b^3*c^6*d^13 + ((d^7*(a*d - b*c)^5)^(1/2)*(a + b/x)^(1/2)*(8*a^15
*b^13*c^18*d^2 - 96*a^16*b^12*c^17*d^3 + 520*a^17*b^11*c^16*d^4 - 1680*a^18
*b^10*c^15*d^5 + 3600*a^19*b^9*c^14*d^6 - 5376*a^20*b^8*c^13*d^7 + 5712*a^2
1*b^7*c^12*d^8 - 4320*a^22*b^6*c^11*d^9 + 2280*a^23*b^5*c^10*d^10 - 800*a^2
4*b^4*c^9*d^11 + 168*a^25*b^3*c^8*d^12 - 16*a^26*b^2*c^7*d^13))/(c^2*(a*d -
b*c)^5)))/(c^2*(a*d - b*c)^5))*1i)/(c^2*(a*d - b*c)^5) + ((d^7*(a*d - b*c)

```
^5)^(1/2)*((a + b/x)^(1/2)*(50*a^9*b^14*c^15*d^3 - 460*a^10*b^13*c^14*d^4 +
 1858*a^11*b^12*c^13*d^5 - 4280*a^12*b^11*c^12*d^6 + 6060*a^13*b^10*c^11*d^
7 - 5160*a^14*b^9*c^10*d^8 + 2108*a^15*b^8*c^9*d^9 + 336*a^16*b^7*c^8*d^10
- 750*a^17*b^6*c^7*d^11 + 180*a^18*b^5*c^6*d^12 + 130*a^19*b^4*c^5*d^13 - 8
8*a^20*b^3*c^4*d^14 + 16*a^21*b^2*c^3*d^15) - ((d^7*(a*d - b*c)^5)^(1/2)*(2
0*a^12*b^14*c^17*d^2 - 212*a^13*b^13*c^16*d^3 + 1012*a^14*b^12*c^15*d^4 - 2
860*a^15*b^11*c^14*d^5 + 5288*a^16*b^10*c^13*d^6 - 6664*a^17*b^9*c^12*d^7 +
 5768*a^18*b^8*c^11*d^8 - 3352*a^19*b^7*c^10*d^9 + 1220*a^20*b^6*c^9*d^10 -
 228*a^21*b^5*c^8*d^11 + 4*a^22*b^4*c^7*d^12 + 4*a^23*b^3*c^6*d^13 - ((d^7*
(a*d - b*c)^5)^(1/2)*(a + b/x)^(1/2)*(8*a^15*b^13*c^18*d^2 - 96*a^16*b^12*c
^17*d^3 + 520*a^17*b^11*c^16*d^4 - 1680*a^18*b^10*c^15*d^5 + 3600*a^19*b^9*
c^14*d^6 - 5376*a^20*b^8*c^13*d^7 + 5712*a^21*b^7*c^12*d^8 - 4320*a^22*b^6*
c^11*d^9 + 2280*a^23*b^5*c^10*d^10 - 800*a^24*b^4*c^9*d^11 + 168*a^25*b^3*c
^8*d^12 - 16*a^26*b^2*c^7*d^13))/(c^2*(a*d - b*c)^5)))/(c^2*(a*d - b*c)^5))
*1i)/(c^2*(a*d - b*c)^5))/(((d^7*(a*d - b*c)^5)^(1/2)*((a + b/x)^(1/2)*(50*
a^9*b^14*c^15*d^3 - 460*a^10*b^13*c^14*d^4 + 1858*a^11*b^12*c^13*d^5 - 4280
*a^12*b^11*c^12*d^6 + 6060*a^13*b^10*c^11*d^7 - 5160*a^14*b^9*c^10*d^8 + 21
08*a^15*b^8*c^9*d^9 + 336*a^16*b^7*c^8*d^10 - 750*a^17*b^6*c^7*d^11 + 180*a
^18*b^5*c^6*d^12 + 130*a^19*b^4*c^5*d^13 - 88*a^20*b^3*c^4*d^14 + 16*a^21*b
^2*c^3*d^15) - ((d^7*(a*d - b*c)^5)^(1/2)*(20*a^12*b^14*c^17*d^2 - 212*a^13
*b^13*c^16*d^3 + 1012*a^14*b^12*c^15*d^4 - 2860*a^15*b^11*c^14*d^5 + 5288*a
^16*b^10*c^13*d^6 - 6664*a^17*b^9*c^12*d^7 + 5768*a^18*b^8*c^11*d^8 - 3352*
a^19*b^7*c^10*d^9 + 1220*a^20*b^6*c^9*d^10 - 228*a^21*b^5*c^8*d^11 + 4*a^22
*b^4*c^7*d^12 + 4*a^23*b^3*c^6*d^13 - ((d^7*(a*d - b*c)^5)^(1/2)*(a + b/x)^
(1/2)*(8*a^15*b^13*c^18*d^2 - 96*a^16*b^12*c^17*d^3 + 520*a^17*b^11*c^16*d^
4 - 1680*a^18*b^10*c^15*d^5 + 3600*a^19*b^9*c^14*d^6 - 5376*a^20*b^8*c^13*d
^7 + 5712*a^21*b^7*c^12*d^8 - 4320*a^22*b^6*c^11*d^9 + 2280*a^23*b^5*c^10*d
^10 - 800*a^24*b^4*c^9*d^11 + 168*a^25*b^3*c^8*d^12 - 16*a^26*b^2*c^7*d^13)
)/(c^2*(a*d - b*c)^5)))/(c^2*(a*d - b*c)^5)))/(c^2*(a*d - b*c)^5) - ((d^7*(
a*d - b*c)^5)^(1/2)*((a + b/x)^(1/2)*(50*a^9*b^14*c^15*d^3 - 460*a^10*b^13*
c^14*d^4 + 1858*a^11*b^12*c^13*d^5 - 4280*a^12*b^11*c^12*d^6 + 6060*a^13*b^
10*c^11*d^7 - 5160*a^14*b^9*c^10*d^8 + 2108*a^15*b^8*c^9*d^9 + 336*a^16*b^7
*c^8*d^10 - 750*a^17*b^6*c^7*d^11 + 180*a^18*b^5*c^6*d^12 + 130*a^19*b^4*c^
5*d^13 - 88*a^20*b^3*c^4*d^14 + 16*a^21*b^2*c^3*d^15) + ((d^7*(a*d - b*c)^5
)^(1/2)*(20*a^12*b^14*c^17*d^2 - 212*a^13*b^13*c^16*d^3 + 1012*a^14*b^12*c^
15*d^4 - 2860*a^15*b^11*c^14*d^5 + 5288*a^16*b^10*c^13*d^6 - 6664*a^17*b^9*
c^12*d^7 + 5768*a^18*b^8*c^11*d^8 - 3352*a^19*b^7*c^10*d^9 + 1220*a^20*b^6*
c^9*d^10 - 228*a^21*b^5*c^8*d^11 + 4*a^22*b^4*c^7*d^12 + 4*a^23*b^3*c^6*d^1
3 + ((d^7*(a*d - b*c)^5)^(1/2)*(a + b/x)^(1/2)*(8*a^15*b^13*c^18*d^2 - 96*a
^16*b^12*c^17*d^3 + 520*a^17*b^11*c^16*d^4 - 1680*a^18*b^10*c^15*d^5 + 3600
*a^19*b^9*c^14*d^6 - 5376*a^20*b^8*c^13*d^7 + 5712*a^21*b^7*c^12*d^8 - 4320
*a^22*b^6*c^11*d^9 + 2280*a^23*b^5*c^10*d^10 - 800*a^24*b^4*c^9*d^11 + 168*
a^25*b^3*c^8*d^12 - 16*a^26*b^2*c^7*d^13))/(c^2*(a*d - b*c)^5)))/(c^2*(a*d
- b*c)^5)))/(c^2*(a*d - b*c)^5) + 100*a^9*b^12*c^11*d^6 - 720*a^10*b^11*c^1
0*d^7 + 2176*a^11*b^10*c^9*d^8 - 3528*a^12*b^9*c^8*d^9 + 3192*a^13*b^8*c^7*
d^10 - 1400*a^14*b^7*c^6*d^11 + 264*a^16*b^5*c^4*d^13 - 92*a^17*b^4*c^3*d^1
4 + 8*a^18*b^3*c^2*d^15))*(d^7*(a*d - b*c)^5)^(1/2)*2i)/(c^2*(a*d - b*c)^5)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(a+\frac{b}{x}\right)^{\frac{5}{2}}(cx+d)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)**(5/2)/(c+d/x),x)
```

```
[Out] Integral(x/((a + b/x)**(5/2)*(c*x + d)), x)
```

3.167 $\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2}\left(c+\frac{d}{x}\right)^2}\,dx$

Optimal. Leaf size=287

$$-\frac{(4ad+5bc)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{7/2}c^3}+\frac{b\left(6a^2d^2-6abcd+5b^2c^2\right)}{3a^2c^2\left(a+\frac{b}{x}\right)^{3/2}(bc-ad)^2}+\frac{b(bc-2ad)\left(a^2d^2-abcd+5b^2c^2\right)}{a^3c^2\sqrt{a+\frac{b}{x}}\,(bc-ad)^3}-\frac{d^{7/2}(9bc-4ad)\,\mathrm{ta}}{c^3(bc-}$$

Rubi [A] time = 0.45, antiderivative size = 287, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {375, 103, 151, 152, 156, 63, 208, 205}

$$\frac{b(bc-2ad)\left(a^2d^2-abcd+5b^2c^2\right)}{a^3c^2\sqrt{a+\frac{b}{x}}\,(bc-ad)^3}+\frac{b\left(6a^2d^2-6abcd+5b^2c^2\right)}{3a^2c^2\left(a+\frac{b}{x}\right)^{3/2}(bc-ad)^2}-\frac{(4ad+5bc)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{7/2}c^3}-\frac{d^{7/2}(9bc-4ad)\tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c^3(bc-ad)^{7/2}}+\frac{d(bc-2ad)}{ac^2\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)(bc-ad)}+\frac{x}{ac\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b/x)^(5/2)*(c + d/x)^2),x]
```

```
[Out] (b*(5*b^2*c^2 - 6*a*b*c*d + 6*a^2*d^2))/(3*a^2*c^2*(b*c - a*d)^2*(a + b/x)^
(3/2)) + (b*(b*c - 2*a*d)*(5*b^2*c^2 - a*b*c*d + a^2*d^2))/(a^3*c^2*(b*c -
a*d)^3*Sqrt[a + b/x]) + (d*(b*c - 2*a*d))/(a*c^2*(b*c - a*d)*(a + b/x)^(3/2
)*(c + d/x)) + x/(a*c*(a + b/x)^(3/2)*(c + d/x)) - (d^(7/2)*(9*b*c - 4*a*d)
*ArcTan[(Sqrt[d]*Sqrt[a + b/x])/Sqrt[b*c - a*d]])/(c^3*(b*c - a*d)^(7/2)) -
 ((5*b*c + 4*a*d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/(a^(7/2)*c^3)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 375

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[((a + b/x^n)^p*(c + d/x^n)^q)/x^2, x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rubi steps

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2}\left(c+\frac{d}{x}\right)^2}\,dx = -\operatorname{Subst}\left(\int \frac{1}{x^2(a+bx)^{5/2}(c+dx)^2}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{x}{ac\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)} + \frac{\operatorname{Subst}\left(\int \frac{\frac{1}{2}(5bc+4ad)+\frac{7bdx}{2}}{x(a+bx)^{5/2}(c+dx)^2}\,dx, x, \frac{1}{x}\right)}{ac}$$

$$= \frac{d(bc-2ad)}{ac^2(bc-ad)\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)} + \frac{x}{ac\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)} - \frac{\operatorname{Subst}\left(\int \frac{-\frac{1}{2}(bc-ad)(5bc+4ad)-\cdots}{x(a+bx)^{5/2}(c+\cdots}\cdots\right)}{ac^2(bc-a\cdots}$$

$$= \frac{b\left(5b^2c^2-6abcd+6a^2d^2\right)}{3a^2c^2(bc-ad)^2\left(a+\frac{b}{x}\right)^{3/2}} + \frac{d(bc-2ad)}{ac^2(bc-ad)\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)} + \frac{x}{ac\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)} - \cdots$$

$$= \frac{b\left(5b^2c^2-6abcd+6a^2d^2\right)}{3a^2c^2(bc-ad)^2\left(a+\frac{b}{x}\right)^{3/2}} + \frac{b(bc-2ad)\left(5b^2c^2-abcd+a^2d^2\right)}{a^3c^2(bc-ad)^3\sqrt{a+\frac{b}{x}}} + \frac{d(bc-2a\cdots}{ac^2(bc-ad)\left(a+\cdots\right.}$$

$$= \frac{b\left(5b^2c^2-6abcd+6a^2d^2\right)}{3a^2c^2(bc-ad)^2\left(a+\frac{b}{x}\right)^{3/2}} + \frac{b(bc-2ad)\left(5b^2c^2-abcd+a^2d^2\right)}{a^3c^2(bc-ad)^3\sqrt{a+\frac{b}{x}}} + \frac{d(bc-2a\cdots}{ac^2(bc-ad)\left(a+\cdots\right.}$$

$$= \frac{b\left(5b^2c^2-6abcd+6a^2d^2\right)}{3a^2c^2(bc-ad)^2\left(a+\frac{b}{x}\right)^{3/2}} + \frac{b(bc-2ad)\left(5b^2c^2-abcd+a^2d^2\right)}{a^3c^2(bc-ad)^3\sqrt{a+\frac{b}{x}}} + \frac{d(bc-2a\cdots}{ac^2(bc-ad)\left(a+\cdots\right.}$$

$$= \frac{b\left(5b^2c^2-6abcd+6a^2d^2\right)}{3a^2c^2(bc-ad)^2\left(a+\frac{b}{x}\right)^{3/2}} + \frac{b(bc-2ad)\left(5b^2c^2-abcd+a^2d^2\right)}{a^3c^2(bc-ad)^3\sqrt{a+\frac{b}{x}}} + \frac{d(bc-2a\cdots}{ac^2(bc-ad)\left(a+\cdots\right.}$$

Mathematica [C] time = 0.19, size = 178, normalized size = 0.62

$$\frac{x\left((ad-bc)\left(3acx(ad(cx+2d)-bc(cx+d))-(cx+d)\left(-4a^2d^2-abcd+5b^2c^2\right){}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};\frac{b}{ax}+1\right)\right)+a^2d^2(cx+d)(9bc-4ad)\,{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};\frac{d\left(a+\frac{b}{x}\right)}{ad-bc}\right)\right)}{3a^2c^3\sqrt{a+\frac{b}{x}}\,(ax+b)(cx+d)(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b/x)^(5/2)*(c + d/x)^2),x]
```

```
[Out] (x*(a^2*d^2*(9*b*c - 4*a*d)*(d + c*x)*Hypergeometric2F1[-3/2, 1, -1/2, (d*(
a + b/x))/(-(b*c) + a*d)] + (-(b*c) + a*d)*(3*a*c*x*(-(b*c*(d + c*x)) + a*d
*(2*d + c*x)) - (5*b^2*c^2 - a*b*c*d - 4*a^2*d^2)*(d + c*x)*Hypergeometric2
F1[-3/2, 1, -1/2, 1 + b/(a*x)])))/(3*a^2*c^3*(b*c - a*d)^2*Sqrt[a + b/x]*(b
 + a*x)*(d + c*x))
```

IntegrateAlgebraic [A] time = 1.04, size = 393, normalized size = 1.37

$$\frac{(-4ad-5bc)\tanh^{-1}\left(\frac{\sqrt{\frac{ax+b}{x}}}{\sqrt{a}}\right)}{a^{7/2}c^3}+\frac{\sqrt{\frac{ax+b}{x}}\left(3a^5cd^3x^4+6a^5d^4x^3-9a^4bc^2d^2x^4-3a^4bcd^3x^3+12a^4bd^4x^2+9a^3b^2c^3dx^4-9a^3b^2c^2d^2x^3-15a^3b^2cd^3x^2+6a^3b^2d^4x-3a^2b^3c^4x^4+41a^2b^3c^3dx^3+35a^2b^3c^2d^2x^2-9a^2b^3cd^3x-20ab^4c^4x^3+13ab^4c^3dx^2+33ab^4c^2d^2x-15b^5c^4x^2-15b^5c^3dx\right)}{3a^3c^2(ax+b)^2(cx+d)(ad-bc)^3}+\frac{\left(4ad^{9/2}-9bcd^{7/2}\right)\tan^{-1}\left(\frac{\sqrt{d}\sqrt{\frac{ax+b}{x}}}{\sqrt{bc-ad}}\right)}{c^3(bc-ad)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((a + b/x)^(5/2)*(c + d/x)^2),x]
```

```
[Out] (Sqrt[(b + a*x)/x]*(-15*b^5*c^3*d*x + 33*a*b^4*c^2*d^2*x - 9*a^2*b^3*c*d^3*
x + 6*a^3*b^2*d^4*x - 15*b^5*c^4*x^2 + 13*a*b^4*c^3*d*x^2 + 35*a^2*b^3*c^2*
d^2*x^2 - 15*a^3*b^2*c*d^3*x^2 + 12*a^4*b*d^4*x^2 - 20*a*b^4*c^4*x^3 + 41*a
^2*b^3*c^3*d*x^3 - 9*a^3*b^2*c^2*d^2*x^3 - 3*a^4*b*c*d^3*x^3 + 6*a^5*d^4*x^
3 - 3*a^2*b^3*c^4*x^4 + 9*a^3*b^2*c^3*d*x^4 - 9*a^4*b*c^2*d^2*x^4 + 3*a^5*c
*d^3*x^4))/(3*a^3*c^2*(-(b*c) + a*d)^3*(b + a*x)^2*(d + c*x)) + ((-9*b*c*d^
(7/2) + 4*a*d^(9/2))*ArcTan[(Sqrt[d]*Sqrt[(b + a*x)/x])/Sqrt[b*c - a*d]])/(
c^3*(b*c - a*d)^(7/2)) + ((-5*b*c - 4*a*d)*ArcTanh[Sqrt[(b + a*x)/x]/Sqrt[a
]])/(a^(7/2)*c^3)
```

fricas [B] time = 5.15, size = 3887, normalized size = 13.54

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)^(5/2)/(c+d/x)^2,x, algorithm="fricas")
```

```
[Out] [1/6*(3*(5*b^6*c^4*d - 11*a*b^5*c^3*d^2 + 3*a^2*b^4*c^2*d^3 + 7*a^3*b^3*c*d
^4 - 4*a^4*b^2*d^5 + (5*a^2*b^4*c^5 - 11*a^3*b^3*c^4*d + 3*a^4*b^2*c^3*d^2
+ 7*a^5*b*c^2*d^3 - 4*a^6*c*d^4)*x^3 + (10*a*b^5*c^5 - 17*a^2*b^4*c^4*d - 5
*a^3*b^3*c^3*d^2 + 17*a^4*b^2*c^2*d^3 - a^5*b*c*d^4 - 4*a^6*d^5)*x^2 + (5*b
^6*c^5 - a*b^5*c^4*d - 19*a^2*b^4*c^3*d^2 + 13*a^3*b^3*c^2*d^3 + 10*a^4*b^2
*c*d^4 - 8*a^5*b*d^5)*x)*sqrt(a)*log(2*a*x - 2*sqrt(a)*x*sqrt((a*x + b)/x)
+ b) + 3*(9*a^4*b^3*c*d^4 - 4*a^5*b^2*d^5 + (9*a^6*b*c^2*d^3 - 4*a^7*c*d^4)
*x^3 + (18*a^5*b^2*c^2*d^3 + a^6*b*c*d^4 - 4*a^7*d^5)*x^2 + (9*a^4*b^3*c^2*
d^3 + 14*a^5*b^2*c*d^4 - 8*a^6*b*d^5)*x)*sqrt(-d/(b*c - a*d))*log(-(2*(b*c
- a*d)*x*sqrt(-d/(b*c - a*d))*sqrt((a*x + b)/x) - b*d + (b*c - 2*a*d)*x)/(c
*x + d)) + 2*(3*(a^3*b^3*c^5 - 3*a^4*b^2*c^4*d + 3*a^5*b*c^3*d^2 - a^6*c^2*
d^3)*x^4 + (20*a^2*b^4*c^5 - 41*a^3*b^3*c^4*d + 9*a^4*b^2*c^3*d^2 + 3*a^5*b
*c^2*d^3 - 6*a^6*c*d^4)*x^3 + (15*a*b^5*c^5 - 13*a^2*b^4*c^4*d - 35*a^3*b^3
*c^3*d^2 + 15*a^4*b^2*c^2*d^3 - 12*a^5*b*c*d^4)*x^2 + 3*(5*a*b^5*c^4*d - 11
*a^2*b^4*c^3*d^2 + 3*a^3*b^3*c^2*d^3 - 2*a^4*b^2*c*d^4)*x)*sqrt((a*x + b)/x
))/(a^4*b^5*c^6*d - 3*a^5*b^4*c^5*d^2 + 3*a^6*b^3*c^4*d^3 - a^7*b^2*c^3*d^4
 + (a^6*b^3*c^7 - 3*a^7*b^2*c^6*d + 3*a^8*b*c^5*d^2 - a^9*c^4*d^3)*x^3 + (2
*a^5*b^4*c^7 - 5*a^6*b^3*c^6*d + 3*a^7*b^2*c^5*d^2 + a^8*b*c^4*d^3 - a^9*c^
3*d^4)*x^2 + (a^4*b^5*c^7 - a^5*b^4*c^6*d - 3*a^6*b^3*c^5*d^2 + 5*a^7*b^2*c
^4*d^3 - 2*a^8*b*c^3*d^4)*x), -1/6*(6*(9*a^4*b^3*c*d^4 - 4*a^5*b^2*d^5 + (9
*a^6*b*c^2*d^3 - 4*a^7*c*d^4)*x^3 + (18*a^5*b^2*c^2*d^3 + a^6*b*c*d^4 - 4*a
^7*d^5)*x^2 + (9*a^4*b^3*c^2*d^3 + 14*a^5*b^2*c*d^4 - 8*a^6*b*d^5)*x)*sqrt(
d/(b*c - a*d))*arctan(-(b*c - a*d)*x*sqrt(d/(b*c - a*d))*sqrt((a*x + b)/x)/
(a*d*x + b*d)) - 3*(5*b^6*c^4*d - 11*a*b^5*c^3*d^2 + 3*a^2*b^4*c^2*d^3 + 7*
a^3*b^3*c*d^4 - 4*a^4*b^2*d^5 + (5*a^2*b^4*c^5 - 11*a^3*b^3*c^4*d + 3*a^4*b
^2*c^3*d^2 + 7*a^5*b*c^2*d^3 - 4*a^6*c*d^4)*x^3 + (10*a*b^5*c^5 - 17*a^2*b^
4*c^4*d - 5*a^3*b^3*c^3*d^2 + 17*a^4*b^2*c^2*d^3 - a^5*b*c*d^4 - 4*a^6*d^5)
*x^2 + (5*b^6*c^5 - a*b^5*c^4*d - 19*a^2*b^4*c^3*d^2 + 13*a^3*b^3*c^2*d^3 +
 10*a^4*b^2*c*d^4 - 8*a^5*b*d^5)*x)*sqrt(a)*log(2*a*x - 2*sqrt(a)*x*sqrt((a
*x + b)/x) + b) - 2*(3*(a^3*b^3*c^5 - 3*a^4*b^2*c^4*d + 3*a^5*b*c^3*d^2 - a
^6*c^2*d^3)*x^4 + (20*a^2*b^4*c^5 - 41*a^3*b^3*c^4*d + 9*a^4*b^2*c^3*d^2 +
3*a^5*b*c^2*d^3 - 6*a^6*c*d^4)*x^3 + (15*a*b^5*c^5 - 13*a^2*b^4*c^4*d - 35*
a^3*b^3*c^3*d^2 + 15*a^4*b^2*c^2*d^3 - 12*a^5*b*c*d^4)*x^2 + 3*(5*a*b^5*c^4
*d - 11*a^2*b^4*c^3*d^2 + 3*a^3*b^3*c^2*d^3 - 2*a^4*b^2*c*d^4)*x)*sqrt((a*x
 + b)/x))/(a^4*b^5*c^6*d - 3*a^5*b^4*c^5*d^2 + 3*a^6*b^3*c^4*d^3 - a^7*b^2*
c^3*d^4 + (a^6*b^3*c^7 - 3*a^7*b^2*c^6*d + 3*a^8*b*c^5*d^2 - a^9*c^4*d^3)*x
^3 + (2*a^5*b^4*c^7 - 5*a^6*b^3*c^6*d + 3*a^7*b^2*c^5*d^2 + a^8*b*c^4*d^3 -
 a^9*c^3*d^4)*x^2 + (a^4*b^5*c^7 - a^5*b^4*c^6*d - 3*a^6*b^3*c^5*d^2 + 5*a^
7*b^2*c^4*d^3 - 2*a^8*b*c^3*d^4)*x), 1/6*(6*(5*b^6*c^4*d - 11*a*b^5*c^3*d^2
 + 3*a^2*b^4*c^2*d^3 + 7*a^3*b^3*c*d^4 - 4*a^4*b^2*d^5 + (5*a^2*b^4*c^5 - 1
1*a^3*b^3*c^4*d + 3*a^4*b^2*c^3*d^2 + 7*a^5*b*c^2*d^3 - 4*a^6*c*d^4)*x^3 +
(10*a*b^5*c^5 - 17*a^2*b^4*c^4*d - 5*a^3*b^3*c^3*d^2 + 17*a^4*b^2*c^2*d^3 -
```

```
 a^5*b*c*d^4 - 4*a^6*d^5)*x^2 + (5*b^6*c^5 - a*b^5*c^4*d - 19*a^2*b^4*c^3*d
^2 + 13*a^3*b^3*c^2*d^3 + 10*a^4*b^2*c*d^4 - 8*a^5*b*d^5)*x)*sqrt(-a)*arcta
n(sqrt(-a)*sqrt((a*x + b)/x)/a) + 3*(9*a^4*b^3*c*d^4 - 4*a^5*b^2*d^5 + (9*a
^6*b*c^2*d^3 - 4*a^7*c*d^4)*x^3 + (18*a^5*b^2*c^2*d^3 + a^6*b*c*d^4 - 4*a^7
*d^5)*x^2 + (9*a^4*b^3*c^2*d^3 + 14*a^5*b^2*c*d^4 - 8*a^6*b*d^5)*x)*sqrt(-d
/(b*c - a*d))*log(-(2*(b*c - a*d)*x*sqrt(-d/(b*c - a*d))*sqrt((a*x + b)/x)
- b*d + (b*c - 2*a*d)*x)/(c*x + d)) + 2*(3*(a^3*b^3*c^5 - 3*a^4*b^2*c^4*d +
 3*a^5*b*c^3*d^2 - a^6*c^2*d^3)*x^4 + (20*a^2*b^4*c^5 - 41*a^3*b^3*c^4*d +
9*a^4*b^2*c^3*d^2 + 3*a^5*b*c^2*d^3 - 6*a^6*c*d^4)*x^3 + (15*a*b^5*c^5 - 13
*a^2*b^4*c^4*d - 35*a^3*b^3*c^3*d^2 + 15*a^4*b^2*c^2*d^3 - 12*a^5*b*c*d^4)*
x^2 + 3*(5*a*b^5*c^4*d - 11*a^2*b^4*c^3*d^2 + 3*a^3*b^3*c^2*d^3 - 2*a^4*b^2
*c*d^4)*x)*sqrt((a*x + b)/x))/(a^4*b^5*c^6*d - 3*a^5*b^4*c^5*d^2 + 3*a^6*b^
3*c^4*d^3 - a^7*b^2*c^3*d^4 + (a^6*b^3*c^7 - 3*a^7*b^2*c^6*d + 3*a^8*b*c^5*
d^2 - a^9*c^4*d^3)*x^3 + (2*a^5*b^4*c^7 - 5*a^6*b^3*c^6*d + 3*a^7*b^2*c^5*d
^2 + a^8*b*c^4*d^3 - a^9*c^3*d^4)*x^2 + (a^4*b^5*c^7 - a^5*b^4*c^6*d - 3*a^
6*b^3*c^5*d^2 + 5*a^7*b^2*c^4*d^3 - 2*a^8*b*c^3*d^4)*x), -1/3*(3*(9*a^4*b^3
*c*d^4 - 4*a^5*b^2*d^5 + (9*a^6*b*c^2*d^3 - 4*a^7*c*d^4)*x^3 + (18*a^5*b^2*
c^2*d^3 + a^6*b*c*d^4 - 4*a^7*d^5)*x^2 + (9*a^4*b^3*c^2*d^3 + 14*a^5*b^2*c*
d^4 - 8*a^6*b*d^5)*x)*sqrt(d/(b*c - a*d))*arctan(-(b*c - a*d)*x*sqrt(d/(b*c
 - a*d))*sqrt((a*x + b)/x)/(a*d*x + b*d)) - 3*(5*b^6*c^4*d - 11*a*b^5*c^3*d
^2 + 3*a^2*b^4*c^2*d^3 + 7*a^3*b^3*c*d^4 - 4*a^4*b^2*d^5 + (5*a^2*b^4*c^5 -
 11*a^3*b^3*c^4*d + 3*a^4*b^2*c^3*d^2 + 7*a^5*b*c^2*d^3 - 4*a^6*c*d^4)*x^3
+ (10*a*b^5*c^5 - 17*a^2*b^4*c^4*d - 5*a^3*b^3*c^3*d^2 + 17*a^4*b^2*c^2*d^3
 - a^5*b*c*d^4 - 4*a^6*d^5)*x^2 + (5*b^6*c^5 - a*b^5*c^4*d - 19*a^2*b^4*c^3
*d^2 + 13*a^3*b^3*c^2*d^3 + 10*a^4*b^2*c*d^4 - 8*a^5*b*d^5)*x)*sqrt(-a)*arc
tan(sqrt(-a)*sqrt((a*x + b)/x)/a) - (3*(a^3*b^3*c^5 - 3*a^4*b^2*c^4*d + 3*a
^5*b*c^3*d^2 - a^6*c^2*d^3)*x^4 + (20*a^2*b^4*c^5 - 41*a^3*b^3*c^4*d + 9*a^
4*b^2*c^3*d^2 + 3*a^5*b*c^2*d^3 - 6*a^6*c*d^4)*x^3 + (15*a*b^5*c^5 - 13*a^2
*b^4*c^4*d - 35*a^3*b^3*c^3*d^2 + 15*a^4*b^2*c^2*d^3 - 12*a^5*b*c*d^4)*x^2
+ 3*(5*a*b^5*c^4*d - 11*a^2*b^4*c^3*d^2 + 3*a^3*b^3*c^2*d^3 - 2*a^4*b^2*c*d
^4)*x)*sqrt((a*x + b)/x))/(a^4*b^5*c^6*d - 3*a^5*b^4*c^5*d^2 + 3*a^6*b^3*c^
4*d^3 - a^7*b^2*c^3*d^4 + (a^6*b^3*c^7 - 3*a^7*b^2*c^6*d + 3*a^8*b*c^5*d^2
- a^9*c^4*d^3)*x^3 + (2*a^5*b^4*c^7 - 5*a^6*b^3*c^6*d + 3*a^7*b^2*c^5*d^2 +
 a^8*b*c^4*d^3 - a^9*c^3*d^4)*x^2 + (a^4*b^5*c^7 - a^5*b^4*c^6*d - 3*a^6*b^
3*c^5*d^2 + 5*a^7*b^2*c^4*d^3 - 2*a^8*b*c^3*d^4)*x)]
```

giac [B] time = 0.33, size = 576, normalized size = 2.01

$$-\frac{1}{3}\,b^{3}\left(\frac{3\left(9\,bcd^{4}-4\,ad^{5}\right)\arctan\left(\frac{d\sqrt{\frac{ax+b}{x}}}{\sqrt{bcd-ad^{2}}}\right)}{\left(b^{6}c^{6}-3\,ab^{5}c^{5}d+3\,a^{2}b^{4}c^{4}d^{2}-a^{3}b^{3}c^{3}d^{3}\right)\sqrt{bcd-ad^{2}}}-\frac{2\left(abc-a^{2}d+\frac{6\,(ax+b)bc}{x}-\frac{12\,(ax+b)ad}{x}\right)x}{\left(a^{3}b^{3}c^{3}-3\,a^{4}b^{2}c^{2}d+3\,a^{5}bcd^{2}-a^{6}d^{3}\right)(ax+b)\sqrt{\frac{ax+b}{x}}}+\frac{3\left(b^{4}c^{4}\sqrt{\frac{ax+b}{x}}-4\,ab^{3}c^{3}d\sqrt{\frac{ax+b}{x}}+6\,a^{2}b^{2}c^{2}d^{2}\sqrt{\frac{ax+b}{x}}-4\,a^{3}bcd^{3}\sqrt{\frac{ax+b}{x}}+2\,a^{4}d^{4}\sqrt{\frac{ax+b}{x}}+\frac{(ax+b)b^{3}c^{3}d\sqrt{\frac{ax+b}{x}}}{x}-\frac{3\,(ax+b)ab^{2}c^{2}d^{2}\sqrt{\frac{ax+b}{x}}}{x}+\frac{3\,(ax+b)a^{2}bcd^{3}\sqrt{\frac{ax+b}{x}}}{x}-\frac{2\,(ax+b)a^{3}d^{4}\sqrt{\frac{ax+b}{x}}}{x}\right)}{\left(a^{3}b^{5}c^{5}-3\,a^{4}b^{4}c^{4}d+3\,a^{5}b^{3}c^{3}d^{2}-a^{6}b^{2}c^{2}d^{3}\right)\left(abc-a^{2}d-\frac{(ax+b)bc}{x}+\frac{2\,(ax+b)ad}{x}-\frac{(ax+b)^{2}d}{x^{2}}\right)}-\frac{3\,(5\,bc+4\,ad)\arctan\left(\frac{\sqrt{\frac{ax+b}{x}}}{\sqrt{-a}}\right)}{\sqrt{-a}\,a^{3}b^{3}c^{3}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)^(5/2)/(c+d/x)^2,x, algorithm="giac")
```

```
[Out] -1/3*b^3*(3*(9*b*c*d^4 - 4*a*d^5)*arctan(d*sqrt((a*x + b)/x)/sqrt(b*c*d - a
*d^2))/((b^6*c^6 - 3*a*b^5*c^5*d + 3*a^2*b^4*c^4*d^2 - a^3*b^3*c^3*d^3)*sqr
t(b*c*d - a*d^2)) - 2*(a*b*c - a^2*d + 6*(a*x + b)*b*c/x - 12*(a*x + b)*a*d
/x)*x/((a^3*b^3*c^3 - 3*a^4*b^2*c^2*d + 3*a^5*b*c*d^2 - a^6*d^3)*(a*x + b)*
sqrt((a*x + b)/x)) + 3*(b^4*c^4*sqrt((a*x + b)/x) - 4*a*b^3*c^3*d*sqrt((a*x
 + b)/x) + 6*a^2*b^2*c^2*d^2*sqrt((a*x + b)/x) - 4*a^3*b*c*d^3*sqrt((a*x +
b)/x) + 2*a^4*d^4*sqrt((a*x + b)/x) + (a*x + b)*b^3*c^3*d*sqrt((a*x + b)/x)
/x - 3*(a*x + b)*a*b^2*c^2*d^2*sqrt((a*x + b)/x)/x + 3*(a*x + b)*a^2*b*c*d^
3*sqrt((a*x + b)/x)/x - 2*(a*x + b)*a^3*d^4*sqrt((a*x + b)/x)/x)/((a^3*b^5*
c^5 - 3*a^4*b^4*c^4*d + 3*a^5*b^3*c^3*d^2 - a^6*b^2*c^2*d^3)*(a*b*c - a^2*d
 - (a*x + b)*b*c/x + 2*(a*x + b)*a*d/x - (a*x + b)^2*d/x^2)) - 3*(5*b*c + 4
*a*d)*arctan(sqrt((a*x + b)/x)/sqrt(-a))/(sqrt(-a)*a^3*b^3*c^3))
```

maple [B] time = 0.08, size = 4644, normalized size = 16.18

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b/x)^(5/2)/(c+d/x)^2,x)
```

```
[Out] -1/6*((a*x+b)/x)^(1/2)*x*(12*a^(19/2)*ln((-2*a*d*x+b*c*x-b*d+2*((a*d-b*c)/c
^2*d)^(1/2)*((a*x+b)*x)^(1/2)*c)/(c*x+d))*x^3*d^7+12*a^(13/2)*ln((-2*a*d*x+
b*c*x-b*d+2*((a*d-b*c)/c^2*d)^(1/2)*((a*x+b)*x)^(1/2)*c)/(c*x+d))*b^3*d^7+3
0*a^(9/2)*((a*x+b)*x)^(1/2)*((a*d-b*c)/c^2*d)^(1/2)*b^4*c^3*d^4-84*a^(7/2)*
((a*x+b)*x)^(1/2)*((a*d-b*c)/c^2*d)^(1/2)*b^5*c^4*d^3+96*a^(5/2)*((a*x+b)*x
)^(1/2)*((a*d-b*c)/c^2*d)^(1/2)*b^6*c^5*d^2+12*a^6*ln(1/2*(2*a*x+b+2*((a*x+
b)*x)^(1/2)*a^(1/2))/a^(1/2))*((a*d-b*c)/c^2*d)^(1/2)*b^3*c*d^6-33*a^5*ln(1
/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))*((a*d-b*c)/c^2*d)^(1/2)*b
^4*c^2*d^5+12*a^4*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))*((a
*d-b*c)/c^2*d)^(1/2)*b^5*c^3*d^4-6*a^(17/2)*((a*x+b)*x)^(1/2)*((a*d-b*c)/c^
2*d)^(1/2)*x^4*c^3*d^4-30*a^(9/2)*((a*x+b)*x)^(1/2)*((a*d-b*c)/c^2*d)^(1/2)
*x^4*b^4*c^7-39*a^(17/2)*ln((-2*a*d*x+b*c*x-b*d+2*((a*d-b*c)/c^2*d)^(1/2)*(
(a*x+b)*x)^(1/2)*c)/(c*x+d))*x^4*b*c^2*d^5+27*a^(15/2)*ln((-2*a*d*x+b*c*x-b
*d+2*((a*d-b*c)/c^2*d)^(1/2)*((a*x+b)*x)^(1/2)*c)/(c*x+d))*x^4*b^2*c^3*d^4+
12*a^9*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))*((a*d-b*c)/c^2
*d)^(1/2)*x^4*c^2*d^5+15*a^4*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a
^(1/2))*((a*d-b*c)/c^2*d)^(1/2)*x^4*b^5*c^7+24*a^(7/2)*((a*x+b)*x)^(3/2)*((
a*d-b*c)/c^2*d)^(1/2)*x^2*b^4*c^7-12*a^(17/2)*((a*x+b)*x)^(1/2)*((a*d-b*c)/
c^2*d)^(1/2)*x^3*c^2*d^5-90*a^(7/2)*((a*x+b)*x)^(1/2)*((a*d-b*c)/c^2*d)^(1/
2)*x^3*b^5*c^7-3*a^(17/2)*ln((-2*a*d*x+b*c*x-b*d+2*((a*d-b*c)/c^2*d)^(1/2)*
((a*x+b)*x)^(1/2)*c)/(c*x+d))*x^3*b*c*d^6-90*a^(15/2)*ln((-2*a*d*x+b*c*x-b*
d+2*((a*d-b*c)/c^2*d)^(1/2)*((a*x+b)*x)^(1/2)*c)/(c*x+d))*x^3*b^2*c^2*d^5+8
1*a^(13/2)*ln((-2*a*d*x+b*c*x-b*d+2*((a*d-b*c)/c^2*d)^(1/2)*((a*x+b)*x)^(1/
2)*c)/(c*x+d))*x^3*b^3*c^3*d^4+12*a^9*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a
^(1/2))/a^(1/2))*((a*d-b*c)/c^2*d)^(1/2)*x^3*c*d^6+45*a^3*ln(1/2*(2*a*x+b+2
*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))*((a*d-b*c)/c^2*d)^(1/2)*x^3*b^6*c^7+20
*a^(5/2)*((a*x+b)*x)^(3/2)*((a*d-b*c)/c^2*d)^(1/2)*x*b^5*c^7-90*a^(5/2)*((a
*x+b)*x)^(1/2)*((a*d-b*c)/c^2*d)^(1/2)*x^2*b^6*c^7-81*a^(15/2)*ln((-2*a*d*x
+b*c*x-b*d+2*((a*d-b*c)/c^2*d)^(1/2)*((a*x+b)*x)^(1/2)*c)/(c*x+d))*x^2*b^2*
c*d^6-36*a^(13/2)*ln((-2*a*d*x+b*c*x-b*d+2*((a*d-b*c)/c^2*d)^(1/2)*((a*x+b)
*x)^(1/2)*c)/(c*x+d))*x^2*b^3*c^2*d^5+81*a^(11/2)*ln((-2*a*d*x+b*c*x-b*d+2*
((a*d-b*c)/c^2*d)^(1/2)*((a*x+b)*x)^(1/2)*c)/(c*x+d))*x^2*b^4*c^3*d^4+45*a^
2*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))*((a*d-b*c)/c^2*d)^(
1/2)*x^2*b^7*c^7+38*a^(9/2)*((a*x+b)*x)^(3/2)*((a*d-b*c)/c^2*d)^(1/2)*b^3*c
^4*d^3-64*a^(7/2)*((a*x+b)*x)^(3/2)*((a*d-b*c)/c^2*d)^(1/2)*b^4*c^5*d^2+20*
a^(5/2)*((a*x+b)*x)^(3/2)*((a*d-b*c)/c^2*d)^(1/2)*b^5*c^6*d-105*a^(13/2)*ln
((-2*a*d*x+b*c*x-b*d+2*((a*d-b*c)/c^2*d)^(1/2)*((a*x+b)*x)^(1/2)*c)/(c*x+d)
)*x*b^3*c*d^6+42*a^(11/2)*ln((-2*a*d*x+b*c*x-b*d+2*((a*d-b*c)/c^2*d)^(1/2)*
((a*x+b)*x)^(1/2)*c)/(c*x+d))*x*b^4*c^2*d^5+27*a^(9/2)*ln((-2*a*d*x+b*c*x-b
*d+2*((a*d-b*c)/c^2*d)^(1/2)*((a*x+b)*x)^(1/2)*c)/(c*x+d))*x*b^5*c^3*d^4-12
*a^(11/2)*((a*x+b)*x)^(1/2)*((a*d-b*c)/c^2*d)^(1/2)*b^3*c^2*d^5+15*a*ln(1/2
*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))*((a*d-b*c)/c^2*d)^(1/2)*x*b
^8*c^7-30*a^(3/2)*((a*x+b)*x)^(1/2)*((a*d-b*c)/c^2*d)^(1/2)*b^7*c^6*d+15*a*
ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))*((a*d-b*c)/c^2*d)^(1/
2)*b^8*c^6*d+42*a^3*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))*(
(a*d-b*c)/c^2*d)^(1/2)*b^6*c^4*d^3-48*a^2*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/
2)*a^(1/2))/a^(1/2))*((a*d-b*c)/c^2*d)^(1/2)*b^7*c^5*d^2+6*a^(17/2)*((a*x+b
)*x)^(1/2)*((a*d-b*c)/c^2*d)^(1/2)*x^5*c^4*d^3-6*a^(15/2)*((a*x+b)*x)^(3/2)
*((a*d-b*c)/c^2*d)^(1/2)*x^3*c^4*d^3-30*a^(3/2)*((a*x+b)*x)^(1/2)*((a*d-b*c
)/c^2*d)^(1/2)*x*b^7*c^7-102*a^3*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2
))/a^(1/2))*((a*d-b*c)/c^2*d)^(1/2)*x*b^6*c^5*d^2-3*a^2*ln(1/2*(2*a*x+b+2*(
(a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))*((a*d-b*c)/c^2*d)^(1/2)*x*b^7*c^6*d+36*a
^8*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))*((a*d-b*c)/c^2*d)^
(1/2)*x^2*b*c*d^6-63*a^7*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/
2))*((a*d-b*c)/c^2*d)^(1/2)*x^2*b^2*c^2*d^5+12*a^(15/2)*((a*x+b)*x)^(1/2)*(
(a*d-b*c)/c^2*d)^(1/2)*x^3*b*c^3*d^4+24*a^(13/2)*((a*x+b)*x)^(1/2)*((a*d-b*
```

```
c)/c^2*d)^(1/2)*x^3*b^2*c^4*d^3-33*a^8*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*
a^(1/2))/a^(1/2))*((a*d-b*c)/c^2*d)^(1/2)*x^4*b*c^3*d^4+12*a^7*ln(1/2*(2*a*
x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))*((a*d-b*c)/c^2*d)^(1/2)*x^4*b^2*c
^4*d^3+42*a^6*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))*((a*d-b
*c)/c^2*d)^(1/2)*x^4*b^3*c^5*d^2-48*a^5*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)
*a^(1/2))/a^(1/2))*((a*d-b*c)/c^2*d)^(1/2)*x^4*b^4*c^6*d+48*a^(15/2)*((a*x+
b)*x)^(1/2)*((a*d-b*c)/c^2*d)^(1/2)*x^4*b*c^4*d^3-84*a^(13/2)*((a*x+b)*x)^(
1/2)*((a*d-b*c)/c^2*d)^(1/2)*x^4*b^2*c^5*d^2+96*a^(11/2)*((a*x+b)*x)^(1/2)*
((a*d-b*c)/c^2*d)^(1/2)*x^4*b^3*c^6*d-36*a^(13/2)*((a*x+b)*x)^(1/2)*((a*d-b
*c)/c^2*d)^(1/2)*x*b^2*c^2*d^5-63*a^6*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a
^(1/2))/a^(1/2))*((a*d-b*c)/c^2*d)^(1/2)*x^2*b^3*c^3*d^4+162*a^5*ln(1/2*(2*
a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))*((a*d-b*c)/c^2*d)^(1/2)*x^2*b^4
*c^4*d^3-18*a^4*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))*((a*d
-b*c)/c^2*d)^(1/2)*x^2*b^5*c^5*d^2-99*a^3*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/
2)*a^(1/2))/a^(1/2))*((a*d-b*c)/c^2*d)^(1/2)*x^2*b^6*c^6*d+198*a^(7/2)*((a*
x+b)*x)^(1/2)*((a*d-b*c)/c^2*d)^(1/2)*x^2*b^5*c^6*d-28*a^(9/2)*((a*x+b)*x)^
(3/2)*((a*d-b*c)/c^2*d)^(1/2)*x*b^3*c^5*d^2-40*a^(7/2)*((a*x+b)*x)^(3/2)*((
a*d-b*c)/c^2*d)^(1/2)*x*b^4*c^6*d-36*a^(15/2)*((a*x+b)*x)^(1/2)*((a*d-b*c)/
c^2*d)^(1/2)*x^2*b*c^2*d^5+72*a^(13/2)*((a*x+b)*x)^(1/2)*((a*d-b*c)/c^2*d)^
(1/2)*x^2*b^2*c^3*d^4-156*a^(11/2)*((a*x+b)*x)^(1/2)*((a*d-b*c)/c^2*d)^(1/2
)*x^2*b^3*c^4*d^3+36*a^(9/2)*((a*x+b)*x)^(1/2)*((a*d-b*c)/c^2*d)^(1/2)*x^2*
b^4*c^5*d^2+78*a^5*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))*((
a*d-b*c)/c^2*d)^(1/2)*x^3*b^4*c^5*d^2-129*a^4*ln(1/2*(2*a*x+b+2*((a*x+b)*x)
^(1/2)*a^(1/2))/a^(1/2))*((a*d-b*c)/c^2*d)^(1/2)*x^3*b^5*c^6*d+30*a^(11/2)*
((a*x+b)*x)^(3/2)*((a*d-b*c)/c^2*d)^(1/2)*x*b^2*c^4*d^3+3*a^8*ln(1/2*(2*a*x
+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))*((a*d-b*c)/c^2*d)^(1/2)*x^3*b*c^2*
d^5-87*a^7*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))*((a*d-b*c)
/c^2*d)^(1/2)*x^3*b^2*c^3*d^4+78*a^6*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^
(1/2))/a^(1/2))*((a*d-b*c)/c^2*d)^(1/2)*x^3*b^3*c^4*d^3-156*a^(11/2)*((a*x+
b)*x)^(1/2)*((a*d-b*c)/c^2*d)^(1/2)*x^3*b^3*c^5*d^2+258*a^(9/2)*((a*x+b)*x)
^(1/2)*((a*d-b*c)/c^2*d)^(1/2)*x^3*b^4*c^6*d-18*a^(13/2)*((a*x+b)*x)^(3/2)*
((a*d-b*c)/c^2*d)^(1/2)*x^2*b*c^4*d^3+48*a^(11/2)*((a*x+b)*x)^(3/2)*((a*d-b
*c)/c^2*d)^(1/2)*x^2*b^2*c^5*d^2-72*a^(9/2)*((a*x+b)*x)^(3/2)*((a*d-b*c)/c^
2*d)^(1/2)*x^2*b^3*c^6*d+84*a^(11/2)*((a*x+b)*x)^(1/2)*((a*d-b*c)/c^2*d)^(1
/2)*x*b^3*c^3*d^4-222*a^(9/2)*((a*x+b)*x)^(1/2)*((a*d-b*c)/c^2*d)^(1/2)*x*b
^4*c^4*d^3+204*a^(7/2)*((a*x+b)*x)^(1/2)*((a*d-b*c)/c^2*d)^(1/2)*x*b^5*c^5*
d^2+6*a^(5/2)*((a*x+b)*x)^(1/2)*((a*d-b*c)/c^2*d)^(1/2)*x*b^6*c^6*d+36*a^7*
ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))*((a*d-b*c)/c^2*d)^(1/
2)*x*b^2*c*d^6-87*a^6*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))
*((a*d-b*c)/c^2*d)^(1/2)*x*b^3*c^2*d^5+3*a^5*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^
(1/2)*a^(1/2))/a^(1/2))*((a*d-b*c)/c^2*d)^(1/2)*x*b^4*c^3*d^4+138*a^4*ln(1/
2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))*((a*d-b*c)/c^2*d)^(1/2)*x*
b^5*c^4*d^3+12*a^(19/2)*ln((-2*a*d*x+b*c*x-b*d+2*((a*d-b*c)/c^2*d)^(1/2)*((
a*x+b)*x)^(1/2)*c)/(c*x+d))*x^4*c*d^6+36*a^(17/2)*ln((-2*a*d*x+b*c*x-b*d+2*
((a*d-b*c)/c^2*d)^(1/2)*((a*x+b)*x)^(1/2)*c)/(c*x+d))*x^2*b*d^7+36*a^(15/2)
*ln((-2*a*d*x+b*c*x-b*d+2*((a*d-b*c)/c^2*d)^(1/2)*((a*x+b)*x)^(1/2)*c)/(c*x
+d))*x*b^2*d^7-39*a^(11/2)*ln((-2*a*d*x+b*c*x-b*d+2*((a*d-b*c)/c^2*d)^(1/2)
*((a*x+b)*x)^(1/2)*c)/(c*x+d))*b^4*c*d^6+27*a^(9/2)*ln((-2*a*d*x+b*c*x-b*d+
2*((a*d-b*c)/c^2*d)^(1/2)*((a*x+b)*x)^(1/2)*c)/(c*x+d))*b^5*c^2*d^5)/a^(9/2
)/c^4/((a*x+b)*x)^(1/2)/(a*d-b*c)^4/(c*x+d)/((a*d-b*c)/c^2*d)^(1/2)/(a*x+b)
^3
```

<u>maxima</u> [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{\frac{5}{2}}\left(c+\frac{d}{x}\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)^(5/2)/(c+d/x)^2,x, algorithm="maxima")
```

```
[Out] integrate(1/((a + b/x)^(5/2)*(c + d/x)^2), x)
```

mupad [B] time = 8.73, size = 5789, normalized size = 20.17

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b/x)^(5/2)*(c + d/x)^2),x)
```

```
[Out] ((2*b^3)/(3*(a^2*d - a*b*c)) + (10*b^3*(a + b/x)*(2*a*d - b*c))/(3*(a^2*d -
 a*b*c)^2) - (b*(a + b/x)^2*(6*a^4*d^4 + 15*b^4*c^4 + 64*a^2*b^2*c^2*d^2 -
58*a*b^3*c^3*d - 12*a^3*b*c*d^3))/(3*c^2*(a^2*d - a*b*c)^3) + (b*(a + b/x)^
3*(2*a*d - b*c)*(a^2*d^3 + 5*b^2*c^2*d - a*b*c*d^2))/(c^2*(a^2*d - a*b*c)^3
))/(d*(a + b/x)^(7/2) + (a + b/x)^(3/2)*(a^2*d - a*b*c) - (a + b/x)^(5/2)*(
2*a*d - b*c)) + (atan((a^15*b^19*c^19*(a + b/x)^(1/2)*125i + a^17*b^17*c^17
*d^2*(a + b/x)^(1/2)*10440i - a^18*b^16*c^16*d^3*(a + b/x)^(1/2)*37776i + a
^19*b^15*c^15*d^4*(a + b/x)^(1/2)*87276i - a^20*b^14*c^14*d^5*(a + b/x)^(1/
2)*126720i + a^21*b^13*c^13*d^6*(a + b/x)^(1/2)*91560i + a^22*b^12*c^12*d^7
*(a + b/x)^(1/2)*40965i - a^23*b^11*c^11*d^8*(a + b/x)^(1/2)*184563i + a^24
*b^10*c^10*d^9*(a + b/x)^(1/2)*212608i - a^25*b^9*c^9*d^10*(a + b/x)^(1/2)*
107740i - a^26*b^8*c^8*d^11*(a + b/x)^(1/2)*19530i + a^27*b^7*c^7*d^12*(a +
 b/x)^(1/2)*71070i - a^28*b^6*c^6*d^13*(a + b/x)^(1/2)*52836i + a^29*b^5*c^
5*d^14*(a + b/x)^(1/2)*20916i - a^30*b^4*c^4*d^15*(a + b/x)^(1/2)*4515i + a
^31*b^3*c^3*d^16*(a + b/x)^(1/2)*420i - a^16*b^18*c^18*d*(a + b/x)^(1/2)*17
00i)/(a^7*(a^7)^(1/2)*(a^7*(a^7*(212608*b^10*c^10*d^9 - 107740*a*b^9*c^9*d^
10 - 19530*a^2*b^8*c^8*d^11 + 71070*a^3*b^7*c^7*d^12 - 52836*a^4*b^6*c^6*d^
13 + 20916*a^5*b^5*c^5*d^14 - 4515*a^6*b^4*c^4*d^15 + 420*a^7*b^3*c^3*d^16)
 + 10440*b^17*c^17*d^2 - 37776*a*b^16*c^16*d^3 + 87276*a^2*b^15*c^15*d^4 -
126720*a^3*b^14*c^14*d^5 + 91560*a^4*b^13*c^13*d^6 + 40965*a^5*b^12*c^12*d^
7 - 184563*a^6*b^11*c^11*d^8) + 125*a^5*b^19*c^19 - 1700*a^6*b^18*c^18*d)))
*(4*a*d + 5*b*c)*1i)/(c^3*(a^7)^(1/2)) - (atan((((d^7*(a*d - b*c)^7)^(1/2)*
((a + b/x)^(1/2)*(670*a^10*b^18*c^22*d^4 - 50*a^9*b^19*c^23*d^3 - 4082*a^11
*b^17*c^21*d^5 + 14830*a^12*b^16*c^20*d^6 - 35210*a^13*b^15*c^19*d^7 + 5551
0*a^14*b^14*c^18*d^8 - 53852*a^15*b^13*c^17*d^9 + 19048*a^16*b^12*c^16*d^10
 + 25730*a^17*b^11*c^15*d^11 - 39550*a^18*b^10*c^14*d^12 + 10670*a^19*b^9*c
^13*d^13 + 29414*a^20*b^8*c^12*d^14 - 45430*a^21*b^7*c^11*d^15 + 34490*a^22
*b^6*c^10*d^16 - 16240*a^23*b^5*c^9*d^17 + 4820*a^24*b^4*c^8*d^18 - 832*a^2
5*b^3*c^7*d^19 + 64*a^26*b^2*c^6*d^20) - ((d^7*(a*d - b*c)^7)^(1/2)*(4*a*d
- 9*b*c)*(304*a^13*b^18*c^25*d^3 - 20*a^12*b^19*c^26*d^2 - 2144*a^14*b^17*c
^24*d^4 + 9280*a^15*b^16*c^23*d^5 - 27476*a^16*b^15*c^22*d^6 + 58688*a^17*b
^14*c^21*d^7 - 92840*a^18*b^13*c^20*d^8 + 109648*a^19*b^12*c^19*d^9 - 95700
*a^20*b^11*c^18*d^10 + 59312*a^21*b^10*c^17*d^11 - 23056*a^22*b^9*c^16*d^12
 + 2528*a^23*b^8*c^15*d^13 + 2996*a^24*b^7*c^14*d^14 - 2080*a^25*b^6*c^13*d
^15 + 664*a^26*b^5*c^12*d^16 - 112*a^27*b^4*c^11*d^17 + 8*a^28*b^3*c^10*d^1
8 + ((d^7*(a*d - b*c)^7)^(1/2)*(a + b/x)^(1/2)*(4*a*d - 9*b*c)*(8*a^15*b^18
*c^28*d^2 - 136*a^16*b^17*c^27*d^3 + 1080*a^17*b^16*c^26*d^4 - 5320*a^18*b^
15*c^25*d^5 + 18200*a^19*b^14*c^24*d^6 - 45864*a^20*b^13*c^23*d^7 + 88088*a
^21*b^12*c^22*d^8 - 131560*a^22*b^11*c^21*d^9 + 154440*a^23*b^10*c^20*d^10
- 143000*a^24*b^9*c^19*d^11 + 104104*a^25*b^8*c^18*d^12 - 58968*a^26*b^7*c^
17*d^13 + 25480*a^27*b^6*c^16*d^14 - 8120*a^28*b^5*c^15*d^15 + 1800*a^29*b^
4*c^14*d^16 - 248*a^30*b^3*c^13*d^17 + 16*a^31*b^2*c^12*d^18))/(2*(b^7*c^10
 - a^7*c^3*d^7 + 7*a^6*b*c^4*d^6 + 21*a^2*b^5*c^8*d^2 - 35*a^3*b^4*c^7*d^3
+ 35*a^4*b^3*c^6*d^4 - 21*a^5*b^2*c^5*d^5 - 7*a*b^6*c^9*d))))/(2*(b^7*c^10
- a^7*c^3*d^7 + 7*a^6*b*c^4*d^6 + 21*a^2*b^5*c^8*d^2 - 35*a^3*b^4*c^7*d^3 +
 35*a^4*b^3*c^6*d^4 - 21*a^5*b^2*c^5*d^5 - 7*a*b^6*c^9*d)))*(4*a*d - 9*b*c)
*1i)/(2*(b^7*c^10 - a^7*c^3*d^7 + 7*a^6*b*c^4*d^6 + 21*a^2*b^5*c^8*d^2 - 35
*a^3*b^4*c^7*d^3 + 35*a^4*b^3*c^6*d^4 - 21*a^5*b^2*c^5*d^5 - 7*a*b^6*c^9*d)
) + ((d^7*(a*d - b*c)^7)^(1/2)*((a + b/x)^(1/2)*(670*a^10*b^18*c^22*d^4 - 5
0*a^9*b^19*c^23*d^3 - 4082*a^11*b^17*c^21*d^5 + 14830*a^12*b^16*c^20*d^6 -
35210*a^13*b^15*c^19*d^7 + 55510*a^14*b^14*c^18*d^8 - 53852*a^15*b^13*c^17*
```

d^9 + 19048*a^16*b^12*c^16*d^10 + 25730*a^17*b^11*c^15*d^11 - 39550*a^18*b^10*c^14*d^12 + 10670*a^19*b^9*c^13*d^13 + 29414*a^20*b^8*c^12*d^14 - 45430*a^21*b^7*c^11*d^15 + 34490*a^22*b^6*c^10*d^16 - 16240*a^23*b^5*c^9*d^17 + 4820*a^24*b^4*c^8*d^18 - 832*a^25*b^3*c^7*d^19 + 64*a^26*b^2*c^6*d^20) - ((d^7*(a*d - b*c)^7)^(1/2)*(4*a*d - 9*b*c)*(20*a^12*b^19*c^26*d^2 - 304*a^13*b^18*c^25*d^3 + 2144*a^14*b^17*c^24*d^4 - 9280*a^15*b^16*c^23*d^5 + 27476*a^16*b^15*c^22*d^6 - 58688*a^17*b^14*c^21*d^7 + 92840*a^18*b^13*c^20*d^8 - 109648*a^19*b^12*c^19*d^9 + 95700*a^20*b^11*c^18*d^10 - 59312*a^21*b^10*c^17*d^11 + 23056*a^22*b^9*c^16*d^12 - 2528*a^23*b^8*c^15*d^13 - 2996*a^24*b^7*c^14*d^14 + 2080*a^25*b^6*c^13*d^15 - 664*a^26*b^5*c^12*d^16 + 112*a^27*b^4*c^11*d^17 - 8*a^28*b^3*c^10*d^18 + ((d^7*(a*d - b*c)^7)^(1/2)*(a + b/x)^(1/2)*(4*a*d - 9*b*c)*(8*a^15*b^18*c^28*d^2 - 136*a^16*b^17*c^27*d^3 + 1080*a^17*b^16*c^26*d^4 - 5320*a^18*b^15*c^25*d^5 + 18200*a^19*b^14*c^24*d^6 - 45864*a^20*b^13*c^23*d^7 + 88088*a^21*b^12*c^22*d^8 - 131560*a^22*b^11*c^21*d^9 + 154440*a^23*b^10*c^20*d^10 - 143000*a^24*b^9*c^19*d^11 + 104104*a^25*b^8*c^18*d^12 - 58968*a^26*b^7*c^17*d^13 + 25480*a^27*b^6*c^16*d^14 - 8120*a^28*b^5*c^15*d^15 + 1800*a^29*b^4*c^14*d^16 - 248*a^30*b^3*c^13*d^17 + 16*a^31*b^2*c^12*d^18))/(2*(b^7*c^10 - a^7*c^3*d^7 + 7*a^6*b*c^4*d^6 + 21*a^2*b^5*c^8*d^2 - 35*a^3*b^4*c^7*d^3 + 35*a^4*b^3*c^6*d^4 - 21*a^5*b^2*c^5*d^5 - 7*a*b^6*c^9*d))))/(2*(b^7*c^10 - a^7*c^3*d^7 + 7*a^6*b*c^4*d^6 + 21*a^2*b^5*c^8*d^2 - 35*a^3*b^4*c^7*d^3 + 35*a^4*b^3*c^6*d^4 - 21*a^5*b^2*c^5*d^5 - 7*a*b^6*c^9*d)))*(4*a*d - 9*b*c)*1i)/(2*(b^7*c^10 - a^7*c^3*d^7 + 7*a^6*b*c^4*d^6 + 21*a^2*b^5*c^8*d^2 - 35*a^3*b^4*c^7*d^3 + 35*a^4*b^3*c^6*d^4 - 21*a^5*b^2*c^5*d^5 - 7*a*b^6*c^9*d)))/(4880*a^10*b^16*c^17*d^7 - 450*a^9*b^17*c^18*d^6 - 23428*a^11*b^15*c^16*d^8 + 65234*a^12*b^14*c^15*d^9 - 115136*a^13*b^13*c^14*d^10 + 129800*a^14*b^12*c^13*d^11 - 83040*a^15*b^11*c^12*d^12 + 5916*a^16*b^10*c^11*d^13 + 45702*a^17*b^9*c^10*d^14 - 51528*a^18*b^8*c^9*d^15 + 32500*a^19*b^7*c^8*d^16 - 13790*a^20*b^6*c^7*d^17 + 4012*a^21*b^5*c^6*d^18 - 736*a^22*b^4*c^5*d^19 + 64*a^23*b^3*c^4*d^20 + ((d^7*(a*d - b*c)^7)^(1/2)*((a + b/x)^(1/2)*(670*a^10*b^18*c^22*d^4 - 50*a^9*b^19*c^23*d^3 - 4082*a^11*b^17*c^21*d^5 + 14830*a^12*b^16*c^20*d^6 - 35210*a^13*b^15*c^19*d^7 + 55510*a^14*b^14*c^18*d^8 - 53852*a^15*b^13*c^17*d^9 + 19048*a^16*b^12*c^16*d^10 + 25730*a^17*b^11*c^15*d^11 - 39550*a^18*b^10*c^14*d^12 + 10670*a^19*b^9*c^13*d^13 + 29414*a^20*b^8*c^12*d^14 - 45430*a^21*b^7*c^11*d^15 + 34490*a^22*b^6*c^10*d^16 - 16240*a^23*b^5*c^9*d^17 + 4820*a^24*b^4*c^8*d^18 - 832*a^25*b^3*c^7*d^19 + 64*a^26*b^2*c^6*d^20) - ((d^7*(a*d - b*c)^7)^(1/2)*(4*a*d - 9*b*c)*(304*a^13*b^18*c^25*d^3 - 20*a^12*b^19*c^26*d^2 - 2144*a^14*b^17*c^24*d^4 + 9280*a^15*b^16*c^23*d^5 - 27476*a^16*b^15*c^22*d^6 + 58688*a^17*b^14*c^21*d^7 - 92840*a^18*b^13*c^20*d^8 + 109648*a^19*b^12*c^19*d^9 - 95700*a^20*b^11*c^18*d^10 + 59312*a^21*b^10*c^17*d^11 - 23056*a^22*b^9*c^16*d^12 + 2528*a^23*b^8*c^15*d^13 + 2996*a^24*b^7*c^14*d^14 - 2080*a^25*b^6*c^13*d^15 + 664*a^26*b^5*c^12*d^16 - 112*a^27*b^4*c^11*d^17 + 8*a^28*b^3*c^10*d^18 + ((d^7*(a*d - b*c)^7)^(1/2)*(a + b/x)^(1/2)*(4*a*d - 9*b*c)*(8*a^15*b^18*c^28*d^2 - 136*a^16*b^17*c^27*d^3 + 1080*a^17*b^16*c^26*d^4 - 5320*a^18*b^15*c^25*d^5 + 18200*a^19*b^14*c^24*d^6 - 45864*a^20*b^13*c^23*d^7 + 88088*a^21*b^12*c^22*d^8 - 131560*a^22*b^11*c^21*d^9 + 154440*a^23*b^10*c^20*d^10 - 143000*a^24*b^9*c^19*d^11 + 104104*a^25*b^8*c^18*d^12 - 58968*a^26*b^7*c^17*d^13 + 25480*a^27*b^6*c^16*d^14 - 8120*a^28*b^5*c^15*d^15 + 1800*a^29*b^4*c^14*d^16 - 248*a^30*b^3*c^13*d^17 + 16*a^31*b^2*c^12*d^18))/(2*(b^7*c^10 - a^7*c^3*d^7 + 7*a^6*b*c^4*d^6 + 21*a^2*b^5*c^8*d^2 - 35*a^3*b^4*c^7*d^3 + 35*a^4*b^3*c^6*d^4 - 21*a^5*b^2*c^5*d^5 - 7*a*b^6*c^9*d))))/(2*(b^7*c^10 - a^7*c^3*d^7 + 7*a^6*b*c^4*d^6 + 21*a^2*b^5*c^8*d^2 - 35*a^3*b^4*c^7*d^3 + 35*a^4*b^3*c^6*d^4 - 21*a^5*b^2*c^5*d^5 - 7*a*b^6*c^9*d)))*(4*a*d - 9*b*c))/(2*(b^7*c^10 - a^7*c^3*d^7 + 7*a^6*b*c^4*d^6 + 21*a^2*b^5*c^8*d^2 - 35*a^3*b^4*c^7*d^3 + 35*a^4*b^3*c^6*d^4 - 21*a^5*b^2*c^5*d^5 - 7*a*b^6*c^9*d)) - ((d^7*(a*d - b*c)^7)^(1/2)*((a + b/x)^(1/2)*(670*a^10*b^18*c^22*d^4 - 50*a^9*b^19*c^23*d^3 - 4082*a^11*b^17*c^21*d^5 + 14830*a^12*b^16*c^20*d^6 - 35210*a^13*b^15*c^19*d^7 + 55510*a^14*b^14*c^18*d^8 - 53852*a^15*b^13*c^17*d^9 + 19048*a^16*b^12*c^16*d^10 + 25730*a^17*b^11*c^15*d^11 - 39550*a^18

```
*b^10*c^14*d^12 + 10670*a^19*b^9*c^13*d^13 + 29414*a^20*b^8*c^12*d^14 - 454
30*a^21*b^7*c^11*d^15 + 34490*a^22*b^6*c^10*d^16 - 16240*a^23*b^5*c^9*d^17
+ 4820*a^24*b^4*c^8*d^18 - 832*a^25*b^3*c^7*d^19 + 64*a^26*b^2*c^6*d^20) -
((d^7*(a*d - b*c)^7)^(1/2)*(4*a*d - 9*b*c)*(20*a^12*b^19*c^26*d^2 - 304*a^1
3*b^18*c^25*d^3 + 2144*a^14*b^17*c^24*d^4 - 9280*a^15*b^16*c^23*d^5 + 27476
*a^16*b^15*c^22*d^6 - 58688*a^17*b^14*c^21*d^7 + 92840*a^18*b^13*c^20*d^8 -
 109648*a^19*b^12*c^19*d^9 + 95700*a^20*b^11*c^18*d^10 - 59312*a^21*b^10*c^
17*d^11 + 23056*a^22*b^9*c^16*d^12 - 2528*a^23*b^8*c^15*d^13 - 2996*a^24*b^
7*c^14*d^14 + 2080*a^25*b^6*c^13*d^15 - 664*a^26*b^5*c^12*d^16 + 112*a^27*b
^4*c^11*d^17 - 8*a^28*b^3*c^10*d^18 + ((d^7*(a*d - b*c)^7)^(1/2)*(a + b/x)^
(1/2)*(4*a*d - 9*b*c)*(8*a^15*b^18*c^28*d^2 - 136*a^16*b^17*c^27*d^3 + 1080
*a^17*b^16*c^26*d^4 - 5320*a^18*b^15*c^25*d^5 + 18200*a^19*b^14*c^24*d^6 -
45864*a^20*b^13*c^23*d^7 + 88088*a^21*b^12*c^22*d^8 - 131560*a^22*b^11*c^21
*d^9 + 154440*a^23*b^10*c^20*d^10 - 143000*a^24*b^9*c^19*d^11 + 104104*a^25
*b^8*c^18*d^12 - 58968*a^26*b^7*c^17*d^13 + 25480*a^27*b^6*c^16*d^14 - 8120
*a^28*b^5*c^15*d^15 + 1800*a^29*b^4*c^14*d^16 - 248*a^30*b^3*c^13*d^17 + 16
*a^31*b^2*c^12*d^18))/(2*(b^7*c^10 - a^7*c^3*d^7 + 7*a^6*b*c^4*d^6 + 21*a^2
*b^5*c^8*d^2 - 35*a^3*b^4*c^7*d^3 + 35*a^4*b^3*c^6*d^4 - 21*a^5*b^2*c^5*d^5
 - 7*a*b^6*c^9*d))))/(2*(b^7*c^10 - a^7*c^3*d^7 + 7*a^6*b*c^4*d^6 + 21*a^2*
b^5*c^8*d^2 - 35*a^3*b^4*c^7*d^3 + 35*a^4*b^3*c^6*d^4 - 21*a^5*b^2*c^5*d^5
- 7*a*b^6*c^9*d)))*(4*a*d - 9*b*c))/(2*(b^7*c^10 - a^7*c^3*d^7 + 7*a^6*b*c^
4*d^6 + 21*a^2*b^5*c^8*d^2 - 35*a^3*b^4*c^7*d^3 + 35*a^4*b^3*c^6*d^4 - 21*a
^5*b^2*c^5*d^5 - 7*a*b^6*c^9*d))))*(d^7*(a*d - b*c)^7)^(1/2)*(4*a*d - 9*b*c
)*1i)/(b^7*c^10 - a^7*c^3*d^7 + 7*a^6*b*c^4*d^6 + 21*a^2*b^5*c^8*d^2 - 35*a
^3*b^4*c^7*d^3 + 35*a^4*b^3*c^6*d^4 - 21*a^5*b^2*c^5*d^5 - 7*a*b^6*c^9*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)**(5/2)/(c+d/x)**2,x)
```

```
[Out] Timed out
```

3.168 $\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2}\left(c+\frac{d}{x}\right)^3}\,dx$

Optimal. Leaf size=409

$$-\frac{(6ad+5bc)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{7/2}c^4}-\frac{d^{7/2}\left(24a^2d^2-88abcd+99b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{4c^4(bc-ad)^{9/2}}+\frac{d\left(12a^2d^2-23abcd+4b^2c^2\right)}{4ac^3\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)(bc-a}$$

Rubi [A] time = 0.70, antiderivative size = 409, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {375, 103, 151, 152, 156, 63, 208, 205}

$$\frac{d\left(12a^2d^2-23abcd+4b^2c^2\right)}{4ac^3\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)(bc-ad)^2}+\frac{b\left(24a^2b^2c^2d^2-35a^3bcd^3+12a^4d^4-56ab^3c^3d+20b^4c^4\right)}{4a^3c^3\sqrt{a+\frac{b}{x}}(bc-ad)^4}+\frac{b\left(87a^2bcd^2-36a^3d^3-36ab^2c^2d+20b^3c^3\right)}{12a^2c^3\left(a+\frac{b}{x}\right)^{3/2}(bc-ad)^3}-\frac{d^{7/2}\left(24a^2d^2-88abcd+99b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{4c^4(bc-ad)^{9/2}}-\frac{(6ad+5bc)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{7/2}c^4}+\frac{d(2bc-3ad)}{2ac^2\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)^2(bc-ad)}+\frac{x}{ac\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b/x)^(5/2)*(c + d/x)^3),x]
```

```
[Out] (b*(20*b^3*c^3 - 36*a*b^2*c^2*d + 87*a^2*b*c*d^2 - 36*a^3*d^3))/(12*a^2*c^3
*(b*c - a*d)^3*(a + b/x)^(3/2)) + (b*(20*b^4*c^4 - 56*a*b^3*c^3*d + 24*a^2*
b^2*c^2*d^2 - 35*a^3*b*c*d^3 + 12*a^4*d^4))/(4*a^3*c^3*(b*c - a*d)^4*Sqrt[a
 + b/x]) + (d*(2*b*c - 3*a*d))/(2*a*c^2*(b*c - a*d)*(a + b/x)^(3/2)*(c + d/
x)^2) + (d*(4*b^2*c^2 - 23*a*b*c*d + 12*a^2*d^2))/(4*a*c^3*(b*c - a*d)^2*(a
 + b/x)^(3/2)*(c + d/x)) + x/(a*c*(a + b/x)^(3/2)*(c + d/x)^2) - (d^(7/2)*(
99*b^2*c^2 - 88*a*b*c*d + 24*a^2*d^2)*ArcTan[(Sqrt[d]*Sqrt[a + b/x])/Sqrt[b
*c - a*d]])/(4*c^4*(b*c - a*d)^(9/2)) - ((5*b*c + 6*a*d)*ArcTanh[Sqrt[a + b
/x]/Sqrt[a]])/(a^(7/2)*c^4)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 375

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[((a + b/x^n)^p*(c + d/x^n)^q)/x^2, x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rubi steps

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2}\left(c+\frac{d}{x}\right)^3}\,dx = -\mathrm{Subst}\left(\int \frac{1}{x^2(a+bx)^{5/2}(c+dx)^3}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{x}{ac\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)^2} + \frac{\mathrm{Subst}\left(\int \frac{\frac{1}{2}(5bc+6ad)+\frac{9bdx}{2}}{x(a+bx)^{5/2}(c+dx)^3}\,dx, x, \frac{1}{x}\right)}{ac}$$

$$= \frac{d(2bc-3ad)}{2ac^2(bc-ad)\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)^2} + \frac{x}{ac\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)^2} - \frac{\mathrm{Subst}\left(\int \frac{-((bc-ad)(5bc+6ad}{x(a+bx)^{5/}}\right.}{2ac^2(bc}$$

$$= \frac{d(2bc-3ad)}{2ac^2(bc-ad)\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)^2} + \frac{d\left(4b^2c^2-23abcd+12a^2d^2\right)}{4ac^3(bc-ad)^2\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)} + \frac{x}{ac\left(a+\frac{b}{x}\right)^{3/2}}$$

$$= \frac{b\left(20b^3c^3-36ab^2c^2d+87a^2bcd^2-36a^3d^3\right)}{12a^2c^3(bc-ad)^3\left(a+\frac{b}{x}\right)^{3/2}} + \frac{d(2bc-3ad)}{2ac^2(bc-ad)\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)^2} + \frac{}{4a}$$

$$= \frac{b\left(20b^3c^3-36ab^2c^2d+87a^2bcd^2-36a^3d^3\right)}{12a^2c^3(bc-ad)^3\left(a+\frac{b}{x}\right)^{3/2}} + \frac{b\left(20b^4c^4-56ab^3c^3d+24a^2b^2c^2d^2-\right.}{4a^3c^3(bc-ad)^4\sqrt{a+}}$$

$$= \frac{b\left(20b^3c^3-36ab^2c^2d+87a^2bcd^2-36a^3d^3\right)}{12a^2c^3(bc-ad)^3\left(a+\frac{b}{x}\right)^{3/2}} + \frac{b\left(20b^4c^4-56ab^3c^3d+24a^2b^2c^2d^2-\right.}{4a^3c^3(bc-ad)^4\sqrt{a+}}$$

$$= \frac{b\left(20b^3c^3-36ab^2c^2d+87a^2bcd^2-36a^3d^3\right)}{12a^2c^3(bc-ad)^3\left(a+\frac{b}{x}\right)^{3/2}} + \frac{b\left(20b^4c^4-56ab^3c^3d+24a^2b^2c^2d^2-\right.}{4a^3c^3(bc-ad)^4\sqrt{a+}}$$

$$= \frac{b\left(20b^3c^3-36ab^2c^2d+87a^2bcd^2-36a^3d^3\right)}{12a^2c^3(bc-ad)^3\left(a+\frac{b}{x}\right)^{3/2}} + \frac{b\left(20b^4c^4-56ab^3c^3d+24a^2b^2c^2d^2-\right.}{4a^3c^3(bc-ad)^4\sqrt{a+}}$$

Mathematica [C] time = 0.39, size = 239, normalized size = 0.58

$$\frac{(cx+d)\left(2(cx+d)\left(\frac{1}{4}a^2d^2\left(24a^2d^2-88abcd+99b^2c^2\right){}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};\frac{d\left(a+\frac{b}{x}\right)}{ad-bc}\right)+(6ad+5bc)(bc-ad)^3{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};\frac{b}{ax}+1\right)\right)-\frac{3}{2}acdx(ad-bc)\left(12a^2d^2-23abcd+4b^2c^2\right)\right)+6ac^3x^3(bc-ad)^3-3ac^2dx^2(bc-ad)^2(3ad-2bc)}{6a^2c^4\left(a+\frac{b}{x}\right)^{3/2}(cx+d)^2(bc-ad)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b/x)^(5/2)*(c + d/x)^3),x]
```

```
[Out] (-3*a*c^2*d*(b*c - a*d)^2*(-2*b*c + 3*a*d)*x^2 + 6*a*c^3*(b*c - a*d)^3*x^3
+ (d + c*x)*((-3*a*c*d*(-(b*c) + a*d)*(4*b^2*c^2 - 23*a*b*c*d + 12*a^2*d^2)
*x)/2 + 2*(d + c*x)*((a^2*d^2*(99*b^2*c^2 - 88*a*b*c*d + 24*a^2*d^2)*Hyperg
eometric2F1[-3/2, 1, -1/2, (d*(a + b/x))/(-(b*c) + a*d)])/4 + (b*c - a*d)^3
*(5*b*c + 6*a*d)*Hypergeometric2F1[-3/2, 1, -1/2, 1 + b/(a*x)])))/(6*a^2*c^
4*(b*c - a*d)^3*(a + b/x)^(3/2)*(d + c*x)^2)
```

IntegrateAlgebraic [A] time = 1.60, size = 587, normalized size = 1.44

$$\frac{(-6ad-5bc)\tanh^{-1}\left(\frac{\sqrt{\frac{ax+b}{x}}}{\sqrt{a}}\right)}{a^{7/2}c^4}+\frac{\left(-24a^2d^{11/2}+88abcd^{9/2}-99b^2c^2d^{7/2}\right)\tan^{-1}\left(\frac{\sqrt{d}\sqrt{\frac{ax+b}{x}}}{\sqrt{bc-ad}}\right)}{4c^4(bc-ad)^{9/2}}+\frac{\sqrt{\frac{ax+b}{x}}\left(12a^6c^2d^4x^5+54a^6cd^5x^4+36a^6d^6x^3-48a^5bc^3d^3x^5-135a^5bc^2d^4x^4+3a^5bcd^5x^3+72a^5bd^6x^2+72a^4b^2c^4d^2x^5+48a^4b^2c^3d^3x^4-234a^4b^2c^2d^4x^3-156a^4b^2cd^5x^2+36a^4b^2d^6x-48a^3b^3c^5dx^5+48a^3b^3c^4d^2x^4+192a^3b^3c^3d^3x^3-15a^3b^3c^2d^4x^2-105a^3b^3cd^5x+12a^2b^4c^6x^5-200a^2b^4c^5dx^4-364a^2b^4c^4d^2x^3-80a^2b^4c^3d^3x^2+72a^2b^4c^2d^4x+80ab^5c^6x^4-8ab^5c^5dx^3-256ab^5c^4d^2x^2-168ab^5c^3d^3x+60b^6c^6x^3+120b^6c^5dx^2+60b^6c^4d^2x\right)}{12a^3c^3(ax+b)^2(cx+d)^2(ad-bc)^4}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((a + b/x)^(5/2)*(c + d/x)^3),x]
```

```
[Out] (Sqrt[(b + a*x)/x]*(60*b^6*c^4*d^2*x - 168*a*b^5*c^3*d^3*x + 72*a^2*b^4*c^2
*d^4*x - 105*a^3*b^3*c*d^5*x + 36*a^4*b^2*d^6*x + 120*b^6*c^5*d*x^2 - 256*a
*b^5*c^4*d^2*x^2 - 80*a^2*b^4*c^3*d^3*x^2 - 15*a^3*b^3*c^2*d^4*x^2 - 156*a^
4*b^2*c*d^5*x^2 + 72*a^5*b*d^6*x^2 + 60*b^6*c^6*x^3 - 8*a*b^5*c^5*d*x^3 - 3
64*a^2*b^4*c^4*d^2*x^3 + 192*a^3*b^3*c^3*d^3*x^3 - 234*a^4*b^2*c^2*d^4*x^3
+ 3*a^5*b*c*d^5*x^3 + 36*a^6*d^6*x^3 + 80*a*b^5*c^6*x^4 - 200*a^2*b^4*c^5*d
*x^4 + 48*a^3*b^3*c^4*d^2*x^4 + 48*a^4*b^2*c^3*d^3*x^4 - 135*a^5*b*c^2*d^4*
x^4 + 54*a^6*c*d^5*x^4 + 12*a^2*b^4*c^6*x^5 - 48*a^3*b^3*c^5*d*x^5 + 72*a^4
*b^2*c^4*d^2*x^5 - 48*a^5*b*c^3*d^3*x^5 + 12*a^6*c^2*d^4*x^5))/(12*a^3*c^3*
(-(b*c) + a*d)^4*(b + a*x)^2*(d + c*x)^2) + ((-99*b^2*c^2*d^(7/2) + 88*a*b*
c*d^(9/2) - 24*a^2*d^(11/2))*ArcTan[(Sqrt[d]*Sqrt[(b + a*x)/x])/Sqrt[b*c -
a*d]])/(4*c^4*(b*c - a*d)^(9/2)) + ((-5*b*c - 6*a*d)*ArcTanh[Sqrt[(b + a*x)
/x]/Sqrt[a]])/(a^(7/2)*c^4)
```

fricas [B] time = 16.66, size = 6171, normalized size = 15.09

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)^(5/2)/(c+d/x)^3,x, algorithm="fricas")
```

```
[Out] [1/24*(12*(5*b^7*c^5*d^2 - 14*a*b^6*c^4*d^3 + 6*a^2*b^5*c^3*d^4 + 16*a^3*b^
4*c^2*d^5 - 19*a^4*b^3*c*d^6 + 6*a^5*b^2*d^7 + (5*a^2*b^5*c^7 - 14*a^3*b^4*
c^6*d + 6*a^4*b^3*c^5*d^2 + 16*a^5*b^2*c^4*d^3 - 19*a^6*b*c^3*d^4 + 6*a^7*c
^2*d^5)*x^4 + 2*(5*a*b^6*c^7 - 9*a^2*b^5*c^6*d - 8*a^3*b^4*c^5*d^2 + 22*a^4
*b^3*c^4*d^3 - 3*a^5*b^2*c^3*d^4 - 13*a^6*b*c^2*d^5 + 6*a^7*c*d^6)*x^3 + (5
*b^7*c^7 + 6*a*b^6*c^6*d - 45*a^2*b^5*c^5*d^2 + 26*a^3*b^4*c^4*d^3 + 51*a^4
*b^3*c^3*d^4 - 54*a^5*b^2*c^2*d^5 + 5*a^6*b*c*d^6 + 6*a^7*d^7)*x^2 + 2*(5*b
^7*c^6*d - 9*a*b^6*c^5*d^2 - 8*a^2*b^5*c^4*d^3 + 22*a^3*b^4*c^3*d^4 - 3*a^4
*b^3*c^2*d^5 - 13*a^5*b^2*c*d^6 + 6*a^6*b*d^7)*x)*sqrt(a)*log(2*a*x - 2*sqr
t(a)*x*sqrt((a*x + b)/x) + b) + 3*(99*a^4*b^4*c^2*d^5 - 88*a^5*b^3*c*d^6 +
24*a^6*b^2*d^7 + (99*a^6*b^2*c^4*d^3 - 88*a^7*b*c^3*d^4 + 24*a^8*c^2*d^5)*x
^4 + 2*(99*a^5*b^3*c^4*d^3 + 11*a^6*b^2*c^3*d^4 - 64*a^7*b*c^2*d^5 + 24*a^8
*c*d^6)*x^3 + (99*a^4*b^4*c^4*d^3 + 308*a^5*b^3*c^3*d^4 - 229*a^6*b^2*c^2*d
^5 + 8*a^7*b*c*d^6 + 24*a^8*d^7)*x^2 + 2*(99*a^4*b^4*c^3*d^4 + 11*a^5*b^3*c
^2*d^5 - 64*a^6*b^2*c*d^6 + 24*a^7*b*d^7)*x)*sqrt(-d/(b*c - a*d))*log(-(2*(
b*c - a*d)*x*sqrt(-d/(b*c - a*d))*sqrt((a*x + b)/x) - b*d + (b*c - 2*a*d)*x
)/(c*x + d)) + 2*(12*(a^3*b^4*c^7 - 4*a^4*b^3*c^6*d + 6*a^5*b^2*c^5*d^2 - 4
*a^6*b*c^4*d^3 + a^7*c^3*d^4)*x^5 + (80*a^2*b^5*c^7 - 200*a^3*b^4*c^6*d + 4
8*a^4*b^3*c^5*d^2 + 48*a^5*b^2*c^4*d^3 - 135*a^6*b*c^3*d^4 + 54*a^7*c^2*d^5
)*x^4 + (60*a*b^6*c^7 - 8*a^2*b^5*c^6*d - 364*a^3*b^4*c^5*d^2 + 192*a^4*b^3
*c^4*d^3 - 234*a^5*b^2*c^3*d^4 + 3*a^6*b*c^2*d^5 + 36*a^7*c*d^6)*x^3 + (120
*a*b^6*c^6*d - 256*a^2*b^5*c^5*d^2 - 80*a^3*b^4*c^4*d^3 - 15*a^4*b^3*c^3*d^
4 - 156*a^5*b^2*c^2*d^5 + 72*a^6*b*c*d^6)*x^2 + 3*(20*a*b^6*c^5*d^2 - 56*a^
2*b^5*c^4*d^3 + 24*a^3*b^4*c^3*d^4 - 35*a^4*b^3*c^2*d^5 + 12*a^5*b^2*c*d^6)
*x)*sqrt((a*x + b)/x))/(a^4*b^6*c^8*d^2 - 4*a^5*b^5*c^7*d^3 + 6*a^6*b^4*c^6
*d^4 - 4*a^7*b^3*c^5*d^5 + a^8*b^2*c^4*d^6 + (a^6*b^4*c^10 - 4*a^7*b^3*c^9*
d + 6*a^8*b^2*c^8*d^2 - 4*a^9*b*c^7*d^3 + a^10*c^6*d^4)*x^4 + 2*(a^5*b^5*c^
10 - 3*a^6*b^4*c^9*d + 2*a^7*b^3*c^8*d^2 + 2*a^8*b^2*c^7*d^3 - 3*a^9*b*c^6*
d^4 + a^10*c^5*d^5)*x^3 + (a^4*b^6*c^10 - 9*a^6*b^4*c^8*d^2 + 16*a^7*b^3*c^
7*d^3 - 9*a^8*b^2*c^6*d^4 + a^10*c^4*d^6)*x^2 + 2*(a^4*b^6*c^9*d - 3*a^5*b^
5*c^8*d^2 + 2*a^6*b^4*c^7*d^3 + 2*a^7*b^3*c^6*d^4 - 3*a^8*b^2*c^5*d^5 + a^9
*b*c^4*d^6)*x), 1/24*(24*(5*b^7*c^5*d^2 - 14*a*b^6*c^4*d^3 + 6*a^2*b^5*c^3*
```

d^4 + 16*a^3*b^4*c^2*d^5 - 19*a^4*b^3*c*d^6 + 6*a^5*b^2*d^7 + (5*a^2*b^5*c^
7 - 14*a^3*b^4*c^6*d + 6*a^4*b^3*c^5*d^2 + 16*a^5*b^2*c^4*d^3 - 19*a^6*b*c^
3*d^4 + 6*a^7*c^2*d^5)*x^4 + 2*(5*a*b^6*c^7 - 9*a^2*b^5*c^6*d - 8*a^3*b^4*c
^5*d^2 + 22*a^4*b^3*c^4*d^3 - 3*a^5*b^2*c^3*d^4 - 13*a^6*b*c^2*d^5 + 6*a^7*
c*d^6)*x^3 + (5*b^7*c^7 + 6*a*b^6*c^6*d - 45*a^2*b^5*c^5*d^2 + 26*a^3*b^4*c
^4*d^3 + 51*a^4*b^3*c^3*d^4 - 54*a^5*b^2*c^2*d^5 + 5*a^6*b*c*d^6 + 6*a^7*d^
7)*x^2 + 2*(5*b^7*c^6*d - 9*a*b^6*c^5*d^2 - 8*a^2*b^5*c^4*d^3 + 22*a^3*b^4*
c^3*d^4 - 3*a^4*b^3*c^2*d^5 - 13*a^5*b^2*c*d^6 + 6*a^6*b*d^7)*x)*sqrt(-a)*a
rctan(sqrt(-a)*sqrt((a*x + b)/x)/a) + 3*(99*a^4*b^4*c^2*d^5 - 88*a^5*b^3*c*
d^6 + 24*a^6*b^2*d^7 + (99*a^6*b^2*c^4*d^3 - 88*a^7*b*c^3*d^4 + 24*a^8*c^2*
d^5)*x^4 + 2*(99*a^5*b^3*c^4*d^3 + 11*a^6*b^2*c^3*d^4 - 64*a^7*b*c^2*d^5 +
24*a^8*c*d^6)*x^3 + (99*a^4*b^4*c^4*d^3 + 308*a^5*b^3*c^3*d^4 - 229*a^6*b^2
*c^2*d^5 + 8*a^7*b*c*d^6 + 24*a^8*d^7)*x^2 + 2*(99*a^4*b^4*c^3*d^4 + 11*a^5
*b^3*c^2*d^5 - 64*a^6*b^2*c*d^6 + 24*a^7*b*d^7)*x)*sqrt(-d/(b*c - a*d))*log
(-(2*(b*c - a*d)*x*sqrt(-d/(b*c - a*d))*sqrt((a*x + b)/x) - b*d + (b*c - 2*
a*d)*x)/(c*x + d)) + 2*(12*(a^3*b^4*c^7 - 4*a^4*b^3*c^6*d + 6*a^5*b^2*c^5*d
^2 - 4*a^6*b*c^4*d^3 + a^7*c^3*d^4)*x^5 + (80*a^2*b^5*c^7 - 200*a^3*b^4*c^6
*d + 48*a^4*b^3*c^5*d^2 + 48*a^5*b^2*c^4*d^3 - 135*a^6*b*c^3*d^4 + 54*a^7*c
^2*d^5)*x^4 + (60*a*b^6*c^7 - 8*a^2*b^5*c^6*d - 364*a^3*b^4*c^5*d^2 + 192*a
^4*b^3*c^4*d^3 - 234*a^5*b^2*c^3*d^4 + 3*a^6*b*c^2*d^5 + 36*a^7*c*d^6)*x^3
+ (120*a*b^6*c^6*d - 256*a^2*b^5*c^5*d^2 - 80*a^3*b^4*c^4*d^3 - 15*a^4*b^3*
c^3*d^4 - 156*a^5*b^2*c^2*d^5 + 72*a^6*b*c*d^6)*x^2 + 3*(20*a*b^6*c^5*d^2 -
 56*a^2*b^5*c^4*d^3 + 24*a^3*b^4*c^3*d^4 - 35*a^4*b^3*c^2*d^5 + 12*a^5*b^2*
c*d^6)*x)*sqrt((a*x + b)/x))/(a^4*b^6*c^8*d^2 - 4*a^5*b^5*c^7*d^3 + 6*a^6*b
^4*c^6*d^4 - 4*a^7*b^3*c^5*d^5 + a^8*b^2*c^4*d^6 + (a^6*b^4*c^10 - 4*a^7*b^
3*c^9*d + 6*a^8*b^2*c^8*d^2 - 4*a^9*b*c^7*d^3 + a^10*c^6*d^4)*x^4 + 2*(a^5*
b^5*c^10 - 3*a^6*b^4*c^9*d + 2*a^7*b^3*c^8*d^2 + 2*a^8*b^2*c^7*d^3 - 3*a^9*
b*c^6*d^4 + a^10*c^5*d^5)*x^3 + (a^4*b^6*c^10 - 9*a^6*b^4*c^8*d^2 + 16*a^7*
b^3*c^7*d^3 - 9*a^8*b^2*c^6*d^4 + a^10*c^4*d^6)*x^2 + 2*(a^4*b^6*c^9*d - 3*
a^5*b^5*c^8*d^2 + 2*a^6*b^4*c^7*d^3 + 2*a^7*b^3*c^6*d^4 - 3*a^8*b^2*c^5*d^5
 + a^9*b*c^4*d^6)*x), -1/12*(3*(99*a^4*b^4*c^2*d^5 - 88*a^5*b^3*c*d^6 + 24*
a^6*b^2*d^7 + (99*a^6*b^2*c^4*d^3 - 88*a^7*b*c^3*d^4 + 24*a^8*c^2*d^5)*x^4
+ 2*(99*a^5*b^3*c^4*d^3 + 11*a^6*b^2*c^3*d^4 - 64*a^7*b*c^2*d^5 + 24*a^8*c*
d^6)*x^3 + (99*a^4*b^4*c^4*d^3 + 308*a^5*b^3*c^3*d^4 - 229*a^6*b^2*c^2*d^5
+ 8*a^7*b*c*d^6 + 24*a^8*d^7)*x^2 + 2*(99*a^4*b^4*c^3*d^4 + 11*a^5*b^3*c^2*
d^5 - 64*a^6*b^2*c*d^6 + 24*a^7*b*d^7)*x)*sqrt(d/(b*c - a*d))*arctan(-(b*c
- a*d)*x*sqrt(d/(b*c - a*d))*sqrt((a*x + b)/x)/(a*d*x + b*d)) - 6*(5*b^7*c^
5*d^2 - 14*a*b^6*c^4*d^3 + 6*a^2*b^5*c^3*d^4 + 16*a^3*b^4*c^2*d^5 - 19*a^4*
b^3*c*d^6 + 6*a^5*b^2*d^7 + (5*a^2*b^5*c^7 - 14*a^3*b^4*c^6*d + 6*a^4*b^3*c
^5*d^2 + 16*a^5*b^2*c^4*d^3 - 19*a^6*b*c^3*d^4 + 6*a^7*c^2*d^5)*x^4 + 2*(5*
a*b^6*c^7 - 9*a^2*b^5*c^6*d - 8*a^3*b^4*c^5*d^2 + 22*a^4*b^3*c^4*d^3 - 3*a^
5*b^2*c^3*d^4 - 13*a^6*b*c^2*d^5 + 6*a^7*c*d^6)*x^3 + (5*b^7*c^7 + 6*a*b^6*
c^6*d - 45*a^2*b^5*c^5*d^2 + 26*a^3*b^4*c^4*d^3 + 51*a^4*b^3*c^3*d^4 - 54*a
^5*b^2*c^2*d^5 + 5*a^6*b*c*d^6 + 6*a^7*d^7)*x^2 + 2*(5*b^7*c^6*d - 9*a*b^6*
c^5*d^2 - 8*a^2*b^5*c^4*d^3 + 22*a^3*b^4*c^3*d^4 - 3*a^4*b^3*c^2*d^5 - 13*a
^5*b^2*c*d^6 + 6*a^6*b*d^7)*x)*sqrt(a)*log(2*a*x - 2*sqrt(a)*x*sqrt((a*x +
b)/x) + b) - (12*(a^3*b^4*c^7 - 4*a^4*b^3*c^6*d + 6*a^5*b^2*c^5*d^2 - 4*a^6
*b*c^4*d^3 + a^7*c^3*d^4)*x^5 + (80*a^2*b^5*c^7 - 200*a^3*b^4*c^6*d + 48*a^
4*b^3*c^5*d^2 + 48*a^5*b^2*c^4*d^3 - 135*a^6*b*c^3*d^4 + 54*a^7*c^2*d^5)*x^
4 + (60*a*b^6*c^7 - 8*a^2*b^5*c^6*d - 364*a^3*b^4*c^5*d^2 + 192*a^4*b^3*c^4
*d^3 - 234*a^5*b^2*c^3*d^4 + 3*a^6*b*c^2*d^5 + 36*a^7*c*d^6)*x^3 + (120*a*b
^6*c^6*d - 256*a^2*b^5*c^5*d^2 - 80*a^3*b^4*c^4*d^3 - 15*a^4*b^3*c^3*d^4 -
156*a^5*b^2*c^2*d^5 + 72*a^6*b*c*d^6)*x^2 + 3*(20*a*b^6*c^5*d^2 - 56*a^2*b^
5*c^4*d^3 + 24*a^3*b^4*c^3*d^4 - 35*a^4*b^3*c^2*d^5 + 12*a^5*b^2*c*d^6)*x)*
sqrt((a*x + b)/x))/(a^4*b^6*c^8*d^2 - 4*a^5*b^5*c^7*d^3 + 6*a^6*b^4*c^6*d^4
 - 4*a^7*b^3*c^5*d^5 + a^8*b^2*c^4*d^6 + (a^6*b^4*c^10 - 4*a^7*b^3*c^9*d +
6*a^8*b^2*c^8*d^2 - 4*a^9*b*c^7*d^3 + a^10*c^6*d^4)*x^4 + 2*(a^5*b^5*c^10 -
 3*a^6*b^4*c^9*d + 2*a^7*b^3*c^8*d^2 + 2*a^8*b^2*c^7*d^3 - 3*a^9*b*c^6*d^4
+ a^10*c^5*d^5)*x^3 + (a^4*b^6*c^10 - 9*a^6*b^4*c^8*d^2 + 16*a^7*b^3*c^7*d^

```
3 - 9*a^8*b^2*c^6*d^4 + a^10*c^4*d^6)*x^2 + 2*(a^4*b^6*c^9*d - 3*a^5*b^5*c^
8*d^2 + 2*a^6*b^4*c^7*d^3 + 2*a^7*b^3*c^6*d^4 - 3*a^8*b^2*c^5*d^5 + a^9*b*c
^4*d^6)*x), -1/12*(3*(99*a^4*b^4*c^2*d^5 - 88*a^5*b^3*c*d^6 + 24*a^6*b^2*d^
7 + (99*a^6*b^2*c^4*d^3 - 88*a^7*b*c^3*d^4 + 24*a^8*c^2*d^5)*x^4 + 2*(99*a^
5*b^3*c^4*d^3 + 11*a^6*b^2*c^3*d^4 - 64*a^7*b*c^2*d^5 + 24*a^8*c*d^6)*x^3 +
 (99*a^4*b^4*c^4*d^3 + 308*a^5*b^3*c^3*d^4 - 229*a^6*b^2*c^2*d^5 + 8*a^7*b*
c*d^6 + 24*a^8*d^7)*x^2 + 2*(99*a^4*b^4*c^3*d^4 + 11*a^5*b^3*c^2*d^5 - 64*a
^6*b^2*c*d^6 + 24*a^7*b*d^7)*x)*sqrt(d/(b*c - a*d))*arctan(-(b*c - a*d)*x*s
qrt(d/(b*c - a*d))*sqrt((a*x + b)/x)/(a*d*x + b*d)) - 12*(5*b^7*c^5*d^2 - 1
4*a*b^6*c^4*d^3 + 6*a^2*b^5*c^3*d^4 + 16*a^3*b^4*c^2*d^5 - 19*a^4*b^3*c*d^6
 + 6*a^5*b^2*d^7 + (5*a^2*b^5*c^7 - 14*a^3*b^4*c^6*d + 6*a^4*b^3*c^5*d^2 +
16*a^5*b^2*c^4*d^3 - 19*a^6*b*c^3*d^4 + 6*a^7*c^2*d^5)*x^4 + 2*(5*a*b^6*c^7
 - 9*a^2*b^5*c^6*d - 8*a^3*b^4*c^5*d^2 + 22*a^4*b^3*c^4*d^3 - 3*a^5*b^2*c^3
*d^4 - 13*a^6*b*c^2*d^5 + 6*a^7*c*d^6)*x^3 + (5*b^7*c^7 + 6*a*b^6*c^6*d - 4
5*a^2*b^5*c^5*d^2 + 26*a^3*b^4*c^4*d^3 + 51*a^4*b^3*c^3*d^4 - 54*a^5*b^2*c^
2*d^5 + 5*a^6*b*c*d^6 + 6*a^7*d^7)*x^2 + 2*(5*b^7*c^6*d - 9*a*b^6*c^5*d^2 -
 8*a^2*b^5*c^4*d^3 + 22*a^3*b^4*c^3*d^4 - 3*a^4*b^3*c^2*d^5 - 13*a^5*b^2*c*
d^6 + 6*a^6*b*d^7)*x)*sqrt(-a)*arctan(sqrt(-a)*sqrt((a*x + b)/x)/a) - (12*(
a^3*b^4*c^7 - 4*a^4*b^3*c^6*d + 6*a^5*b^2*c^5*d^2 - 4*a^6*b*c^4*d^3 + a^7*c
^3*d^4)*x^5 + (80*a^2*b^5*c^7 - 200*a^3*b^4*c^6*d + 48*a^4*b^3*c^5*d^2 + 48
*a^5*b^2*c^4*d^3 - 135*a^6*b*c^3*d^4 + 54*a^7*c^2*d^5)*x^4 + (60*a*b^6*c^7
- 8*a^2*b^5*c^6*d - 364*a^3*b^4*c^5*d^2 + 192*a^4*b^3*c^4*d^3 - 234*a^5*b^2
*c^3*d^4 + 3*a^6*b*c^2*d^5 + 36*a^7*c*d^6)*x^3 + (120*a*b^6*c^6*d - 256*a^2
*b^5*c^5*d^2 - 80*a^3*b^4*c^4*d^3 - 15*a^4*b^3*c^3*d^4 - 156*a^5*b^2*c^2*d^
5 + 72*a^6*b*c*d^6)*x^2 + 3*(20*a*b^6*c^5*d^2 - 56*a^2*b^5*c^4*d^3 + 24*a^3
*b^4*c^3*d^4 - 35*a^4*b^3*c^2*d^5 + 12*a^5*b^2*c*d^6)*x)*sqrt((a*x + b)/x))
/(a^4*b^6*c^8*d^2 - 4*a^5*b^5*c^7*d^3 + 6*a^6*b^4*c^6*d^4 - 4*a^7*b^3*c^5*d
^5 + a^8*b^2*c^4*d^6 + (a^6*b^4*c^10 - 4*a^7*b^3*c^9*d + 6*a^8*b^2*c^8*d^2
- 4*a^9*b*c^7*d^3 + a^10*c^6*d^4)*x^4 + 2*(a^5*b^5*c^10 - 3*a^6*b^4*c^9*d +
 2*a^7*b^3*c^8*d^2 + 2*a^8*b^2*c^7*d^3 - 3*a^9*b*c^6*d^4 + a^10*c^5*d^5)*x^
3 + (a^4*b^6*c^10 - 9*a^6*b^4*c^8*d^2 + 16*a^7*b^3*c^7*d^3 - 9*a^8*b^2*c^6*
d^4 + a^10*c^4*d^6)*x^2 + 2*(a^4*b^6*c^9*d - 3*a^5*b^5*c^8*d^2 + 2*a^6*b^4*
c^7*d^3 + 2*a^7*b^3*c^6*d^4 - 3*a^8*b^2*c^5*d^5 + a^9*b*c^4*d^6)*x)]
```

giac [A] time = 0.29, size = 523, normalized size = 1.28

$$-\frac{1}{12}\,b^{4}\left(\frac{3\left(99\,b^{2}c^{2}d^{4}-88\,abcd^{5}+24\,a^{2}d^{6}\right)\arctan\left(\frac{d\sqrt{\frac{ax+b}{x}}}{\sqrt{bcd-ad^{2}}}\right)}{\left(b^{8}c^{8}-4\,ab^{7}c^{7}d+6\,a^{2}b^{6}c^{6}d^{2}-4\,a^{3}b^{5}c^{5}d^{3}+a^{4}b^{4}c^{4}d^{4}\right)\sqrt{bcd-ad^{2}}}-\frac{8\left(abc-a^{2}d+\frac{6\,(ax+b)bc}{x}-\frac{15\,(ax+b)ad}{x}\right)x}{\left(a^{3}b^{4}c^{4}-4\,a^{4}b^{3}c^{3}d+6\,a^{5}b^{2}c^{2}d^{2}-4\,a^{6}bcd^{3}+a^{7}d^{4}\right)(ax+b)\sqrt{\frac{ax+b}{x}}}+\frac{3\left(21\,b^{2}c^{2}d^{4}\sqrt{\frac{ax+b}{x}}-29\,abcd^{5}\sqrt{\frac{ax+b}{x}}+8\,a^{2}d^{6}\sqrt{\frac{ax+b}{x}}+\frac{19\,(ax+b)bcd^{5}\sqrt{\frac{ax+b}{x}}}{x}-\frac{8\,(ax+b)ad^{6}\sqrt{\frac{ax+b}{x}}}{x}\right)}{\left(b^{7}c^{7}-4\,ab^{6}c^{6}d+6\,a^{2}b^{5}c^{5}d^{2}-4\,a^{3}b^{4}c^{4}d^{3}+a^{4}b^{3}c^{3}d^{4}\right)\left(bc-ad+\frac{(ax+b)d}{x}\right)^{2}}+\frac{12\sqrt{\frac{ax+b}{x}}}{\left(a-\frac{ax+b}{x}\right)a^{3}b^{3}c^{3}}-\frac{12\,(5\,bc+6\,ad)\arctan\left(\frac{\sqrt{\frac{ax+b}{x}}}{\sqrt{-a}}\right)}{\sqrt{-a}\,a^{3}b^{4}c^{4}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)^(5/2)/(c+d/x)^3,x, algorithm="giac")
```

```
[Out] -1/12*b^4*(3*(99*b^2*c^2*d^4 - 88*a*b*c*d^5 + 24*a^2*d^6)*arctan(d*sqrt((a*
x + b)/x)/sqrt(b*c*d - a*d^2))/((b^8*c^8 - 4*a*b^7*c^7*d + 6*a^2*b^6*c^6*d^
2 - 4*a^3*b^5*c^5*d^3 + a^4*b^4*c^4*d^4)*sqrt(b*c*d - a*d^2)) - 8*(a*b*c -
a^2*d + 6*(a*x + b)*b*c/x - 15*(a*x + b)*a*d/x)*x/((a^3*b^4*c^4 - 4*a^4*b^3
*c^3*d + 6*a^5*b^2*c^2*d^2 - 4*a^6*b*c*d^3 + a^7*d^4)*(a*x + b)*sqrt((a*x +
 b)/x)) + 3*(21*b^2*c^2*d^4*sqrt((a*x + b)/x) - 29*a*b*c*d^5*sqrt((a*x + b)
/x) + 8*a^2*d^6*sqrt((a*x + b)/x) + 19*(a*x + b)*b*c*d^5*sqrt((a*x + b)/x)/
x - 8*(a*x + b)*a*d^6*sqrt((a*x + b)/x)/x)/((b^7*c^7 - 4*a*b^6*c^6*d + 6*a^
2*b^5*c^5*d^2 - 4*a^3*b^4*c^4*d^3 + a^4*b^3*c^3*d^4)*(b*c - a*d + (a*x + b)
*d/x)^2) + 12*sqrt((a*x + b)/x)/((a - (a*x + b)/x)*a^3*b^3*c^3) - 12*(5*b*c
 + 6*a*d)*arctan(sqrt((a*x + b)/x)/sqrt(-a))/(sqrt(-a)*a^3*b^4*c^4))
```

maple [B] time = 0.08, size = 7300, normalized size = 17.85

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b/x)^(5/2)/(c+d/x)^3,x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{\frac{5}{2}}\left(c+\frac{d}{x}\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)^(5/2)/(c+d/x)^3,x, algorithm="maxima")
```

```
[Out] integrate(1/((a + b/x)^(5/2)*(c + d/x)^3), x)
```

mupad [B] time = 8.23, size = 4284, normalized size = 10.47

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b/x)^(5/2)*(c + d/x)^3),x)
```

```
[Out] ((2*b^4)/(3*(a^2*d - a*b*c)) + (2*b^4*(a + b/x)*(12*a*d - 5*b*c))/(3*(a^2*d
 - a*b*c)^2) + (b*(a + b/x)^2*(36*a^5*d^5 - 60*b^5*c^5 - 456*a^2*b^3*c^3*d^
2 + 120*a^3*b^2*c^2*d^3 + 308*a*b^4*c^4*d - 123*a^4*b*c*d^4))/(12*a^2*c^3*(
a^2*d - a*b*c)*(a*d - b*c)^2) + (b*(a + b/x)^4*(12*a^4*d^6 + 20*b^4*c^4*d^2
 - 56*a*b^3*c^3*d^3 + 24*a^2*b^2*c^2*d^4 - 35*a^3*b*c*d^5))/(4*a^2*c^3*(a^2
*d - a*b*c)*(a*d - b*c)^3) - (b*(a + b/x)^3*(72*a^5*d^6 - 120*b^5*c^5*d + 4
96*a*b^4*c^4*d^2 - 592*a^2*b^3*c^3*d^3 + 303*a^3*b^2*c^2*d^4 - 264*a^4*b*c*
d^5))/(12*a^2*c^3*(a^2*d - a*b*c)*(a*d - b*c)^3))/((a + b/x)^(5/2)*(3*a^2*d
^2 + b^2*c^2 - 4*a*b*c*d) - (a + b/x)^(7/2)*(3*a*d^2 - 2*b*c*d) + d^2*(a +
b/x)^(9/2) - (a + b/x)^(3/2)*(a^3*d^2 + a*b^2*c^2 - 2*a^2*b*c*d)) + (atan((
a^15*b^24*c^24*(a + b/x)^(1/2)*2000i + a^17*b^22*c^22*d^2*(a + b/x)^(1/2)*2
77440i - a^18*b^21*c^21*d^3*(a + b/x)^(1/2)*1325984i + a^19*b^20*c^20*d^4*(
a + b/x)^(1/2)*4135824i - a^20*b^19*c^19*d^5*(a + b/x)^(1/2)*8371440i + a^2
1*b^18*c^18*d^6*(a + b/x)^(1/2)*9129120i + a^22*b^17*c^17*d^7*(a + b/x)^(1/
2)*3058605i - a^23*b^16*c^16*d^8*(a + b/x)^(1/2)*32337558i + a^24*b^15*c^15
*d^9*(a + b/x)^(1/2)*63677218i - a^25*b^14*c^14*d^10*(a + b/x)^(1/2)*666652
80i + a^26*b^13*c^13*d^11*(a + b/x)^(1/2)*24871035i + a^27*b^12*c^12*d^12*(
a + b/x)^(1/2)*40203170i - a^28*b^11*c^11*d^13*(a + b/x)^(1/2)*85652532i +
a^29*b^10*c^10*d^14*(a + b/x)^(1/2)*88170192i - a^30*b^9*c^9*d^15*(a + b/x)
^(1/2)*60362445i + a^31*b^8*c^8*d^16*(a + b/x)^(1/2)*29178270i - a^32*b^7*c
^7*d^17*(a + b/x)^(1/2)*9940590i + a^33*b^6*c^6*d^18*(a + b/x)^(1/2)*228782
4i - a^34*b^5*c^5*d^19*(a + b/x)^(1/2)*320859i + a^35*b^4*c^4*d^20*(a + b/x
)^(1/2)*20790i - a^16*b^23*c^23*d*(a + b/x)^(1/2)*34800i)/(a^7*(a^7)^(1/2)*
(a^7*(a^7*(a^7*(29178270*b^8*c^8*d^16 - 9940590*a*b^7*c^7*d^17 + 2287824*a^
2*b^6*c^6*d^18 - 320859*a^3*b^5*c^5*d^19 + 20790*a^4*b^4*c^4*d^20) + 636772
18*b^15*c^15*d^9 - 66665280*a*b^14*c^14*d^10 + 24871035*a^2*b^13*c^13*d^11
+ 40203170*a^3*b^12*c^12*d^12 - 85652532*a^4*b^11*c^11*d^13 + 88170192*a^5*
b^10*c^10*d^14 - 60362445*a^6*b^9*c^9*d^15) + 277440*b^22*c^22*d^2 - 132598
4*a*b^21*c^21*d^3 + 4135824*a^2*b^20*c^20*d^4 - 8371440*a^3*b^19*c^19*d^5 +
 9129120*a^4*b^18*c^18*d^6 + 3058605*a^5*b^17*c^17*d^7 - 32337558*a^6*b^16*
c^16*d^8) + 2000*a^5*b^24*c^24 - 34800*a^6*b^23*c^23*d)))*(6*a*d + 5*b*c)*1
i)/(c^4*(a^7)^(1/2)) + (log(400*b^25*c^25*d^4 - 8240*a*b^24*c^24*d^5 - 1152
*a^11*d^5*(d^7*(a*d - b*c)^9)^(3/2)*(a + b/x)^(1/2) + 1152*a^20*d^21*(d^7*(
a*d - b*c)^9)^(1/2)*(a + b/x)^(1/2) + 79696*a^2*b^23*c^23*d^6 - 478768*a^3*
b^22*c^22*d^7 + 1987568*a^4*b^21*c^21*d^8 - 5978896*a^5*b^20*c^20*d^9 + 131
76240*a^6*b^19*c^19*d^10 - 20525703*a^7*b^18*c^18*d^11 + 18765714*a^8*b^17*
c^17*d^12 + 3763331*a^9*b^16*c^16*d^13 - 49787452*a^10*b^15*c^15*d^14 + 104
120705*a^11*b^14*c^14*d^15 - 140185682*a^12*b^13*c^13*d^16 + 139985251*a^13
*b^12*c^12*d^17 - 108046616*a^14*b^11*c^11*d^18 + 65184867*a^15*b^10*c^10*d
^19 - 30607170*a^16*b^9*c^9*d^20 + 10996689*a^17*b^8*c^8*d^21 - 2926572*a^1
```

8*b^7*c^7*d^22 + 544467*a^19*b^6*c^6*d^23 - 63294*a^20*b^5*c^5*d^24 + 3465*
a^21*b^4*c^4*d^25 + 400*b^20*c^20*d*(d^7*(a*d - b*c)^9)^(1/2)*(a + b/x)^(1/
2) + 9801*a^6*b^5*c^5*(d^7*(a*d - b*c)^9)^(3/2)*(a + b/x)^(1/2) - 37026*a^7
*b^4*c^4*d*(d^7*(a*d - b*c)^9)^(3/2)*(a + b/x)^(1/2) - 6240*a*b^19*c^19*d^2
(d^7(a*d - b*c)^9)^(1/2)*(a + b/x)^(1/2) + 47344*a^8*b^3*c^3*d^2*(d^7*(a*
d - b*c)^9)^(3/2)*(a + b/x)^(1/2) - 29216*a^9*b^2*c^2*d^3*(d^7*(a*d - b*c)^
9)^(3/2)*(a + b/x)^(1/2) + 44496*a^2*b^18*c^18*d^3*(d^7*(a*d - b*c)^9)^(1/2
)*(a + b/x)^(1/2) - 189888*a^3*b^17*c^17*d^4*(d^7*(a*d - b*c)^9)^(1/2)*(a +
b/x)^(1/2) + 528768*a^4*b^16*c^16*d^5*(d^7*(a*d - b*c)^9)^(1/2)*(a + b/x)^
(1/2) - 959616*a^5*b^15*c^15*d^6*(d^7*(a*d - b*c)^9)^(1/2)*(a + b/x)^(1/2)
+ 972681*a^6*b^14*c^14*d^7*(d^7*(a*d - b*c)^9)^(1/2)*(a + b/x)^(1/2) + 4123
8*a^7*b^13*c^13*d^8*(d^7*(a*d - b*c)^9)^(1/2)*(a + b/x)^(1/2) - 1727195*a^8
*b^12*c^12*d^9*(d^7*(a*d - b*c)^9)^(1/2)*(a + b/x)^(1/2) + 2139672*a^9*b^11
*c^11*d^10*(d^7*(a*d - b*c)^9)^(1/2)*(a + b/x)^(1/2) + 786834*a^10*b^10*c^1
0*d^11*(d^7*(a*d - b*c)^9)^(1/2)*(a + b/x)^(1/2) - 6551292*a^11*b^9*c^9*d^1
2*(d^7*(a*d - b*c)^9)^(1/2)*(a + b/x)^(1/2) + 11685186*a^12*b^8*c^8*d^13*(d
^7*(a*d - b*c)^9)^(1/2)*(a + b/x)^(1/2) - 12876696*a^13*b^7*c^7*d^14*(d^7*(
a*d - b*c)^9)^(1/2)*(a + b/x)^(1/2) + 10033077*a^14*b^6*c^6*d^15*(d^7*(a*d
- b*c)^9)^(1/2)*(a + b/x)^(1/2) - 5737770*a^15*b^5*c^5*d^16*(d^7*(a*d - b*c
)^9)^(1/2)*(a + b/x)^(1/2) + 2414601*a^16*b^4*c^4*d^17*(d^7*(a*d - b*c)^9)^
(1/2)*(a + b/x)^(1/2) - 731920*a^17*b^3*c^3*d^18*(d^7*(a*d - b*c)^9)^(1/2)*
(a + b/x)^(1/2) + 151904*a^18*b^2*c^2*d^19*(d^7*(a*d - b*c)^9)^(1/2)*(a + b
/x)^(1/2) + 9024*a^10*b*c*d^4*(d^7*(a*d - b*c)^9)^(3/2)*(a + b/x)^(1/2) - 1
9392*a^19*b*c*d^20*(d^7*(a*d - b*c)^9)^(1/2)*(a + b/x)^(1/2))*(d^7*(a*d - b
c)^9)^(1/2)(3*a^2*d^2 + (99*b^2*c^2)/8 - 11*a*b*c*d))/(b^9*c^13 - a^9*c^4
*d^9 + 9*a^8*b*c^5*d^8 + 36*a^2*b^7*c^11*d^2 - 84*a^3*b^6*c^10*d^3 + 126*a^
4*b^5*c^9*d^4 - 126*a^5*b^4*c^8*d^5 + 84*a^6*b^3*c^7*d^6 - 36*a^7*b^2*c^6*d
^7 - 9*a*b^8*c^12*d) - (log(8240*a*b^24*c^24*d^5 - 400*b^25*c^25*d^4 - 1152
*a^11*d^5*(d^7*(a*d - b*c)^9)^(3/2)*(a + b/x)^(1/2) + 1152*a^20*d^21*(d^7*(
a*d - b*c)^9)^(1/2)*(a + b/x)^(1/2) - 79696*a^2*b^23*c^23*d^6 + 478768*a^3*
b^22*c^22*d^7 - 1987568*a^4*b^21*c^21*d^8 + 5978896*a^5*b^20*c^20*d^9 - 131
76240*a^6*b^19*c^19*d^10 + 20525703*a^7*b^18*c^18*d^11 - 18765714*a^8*b^17*
c^17*d^12 - 3763331*a^9*b^16*c^16*d^13 + 49787452*a^10*b^15*c^15*d^14 - 104
120705*a^11*b^14*c^14*d^15 + 140185682*a^12*b^13*c^13*d^16 - 139985251*a^13
*b^12*c^12*d^17 + 108046616*a^14*b^11*c^11*d^18 - 65184867*a^15*b^10*c^10*d
^19 + 30607170*a^16*b^9*c^9*d^20 - 10996689*a^17*b^8*c^8*d^21 + 2926572*a^1
8*b^7*c^7*d^22 - 544467*a^19*b^6*c^6*d^23 + 63294*a^20*b^5*c^5*d^24 - 3465*
a^21*b^4*c^4*d^25 + 400*b^20*c^20*d*(d^7*(a*d - b*c)^9)^(1/2)*(a + b/x)^(1/
2) + 9801*a^6*b^5*c^5*(d^7*(a*d - b*c)^9)^(3/2)*(a + b/x)^(1/2) - 37026*a^7
*b^4*c^4*d*(d^7*(a*d - b*c)^9)^(3/2)*(a + b/x)^(1/2) - 6240*a*b^19*c^19*d^2
(d^7(a*d - b*c)^9)^(1/2)*(a + b/x)^(1/2) + 47344*a^8*b^3*c^3*d^2*(d^7*(a*
d - b*c)^9)^(3/2)*(a + b/x)^(1/2) - 29216*a^9*b^2*c^2*d^3*(d^7*(a*d - b*c)^
9)^(3/2)*(a + b/x)^(1/2) + 44496*a^2*b^18*c^18*d^3*(d^7*(a*d - b*c)^9)^(1/2
)*(a + b/x)^(1/2) - 189888*a^3*b^17*c^17*d^4*(d^7*(a*d - b*c)^9)^(1/2)*(a +
b/x)^(1/2) + 528768*a^4*b^16*c^16*d^5*(d^7*(a*d - b*c)^9)^(1/2)*(a + b/x)^
(1/2) - 959616*a^5*b^15*c^15*d^6*(d^7*(a*d - b*c)^9)^(1/2)*(a + b/x)^(1/2)
+ 972681*a^6*b^14*c^14*d^7*(d^7*(a*d - b*c)^9)^(1/2)*(a + b/x)^(1/2) + 4123
8*a^7*b^13*c^13*d^8*(d^7*(a*d - b*c)^9)^(1/2)*(a + b/x)^(1/2) - 1727195*a^8
*b^12*c^12*d^9*(d^7*(a*d - b*c)^9)^(1/2)*(a + b/x)^(1/2) + 2139672*a^9*b^11
*c^11*d^10*(d^7*(a*d - b*c)^9)^(1/2)*(a + b/x)^(1/2) + 786834*a^10*b^10*c^1
0*d^11*(d^7*(a*d - b*c)^9)^(1/2)*(a + b/x)^(1/2) - 6551292*a^11*b^9*c^9*d^1
2*(d^7*(a*d - b*c)^9)^(1/2)*(a + b/x)^(1/2) + 11685186*a^12*b^8*c^8*d^13*(d
^7*(a*d - b*c)^9)^(1/2)*(a + b/x)^(1/2) - 12876696*a^13*b^7*c^7*d^14*(d^7*(
a*d - b*c)^9)^(1/2)*(a + b/x)^(1/2) + 10033077*a^14*b^6*c^6*d^15*(d^7*(a*d
- b*c)^9)^(1/2)*(a + b/x)^(1/2) - 5737770*a^15*b^5*c^5*d^16*(d^7*(a*d - b*c
)^9)^(1/2)*(a + b/x)^(1/2) + 2414601*a^16*b^4*c^4*d^17*(d^7*(a*d - b*c)^9)^
(1/2)*(a + b/x)^(1/2) - 731920*a^17*b^3*c^3*d^18*(d^7*(a*d - b*c)^9)^(1/2)*
(a + b/x)^(1/2) + 151904*a^18*b^2*c^2*d^19*(d^7*(a*d - b*c)^9)^(1/2)*(a + b
/x)^(1/2) + 9024*a^10*b*c*d^4*(d^7*(a*d - b*c)^9)^(3/2)*(a + b/x)^(1/2) - 1

```
9392*a^19*b*c*d^20*(d^7*(a*d - b*c)^9)^(1/2)*(a + b/x)^(1/2))*(d^7*(a*d - b
*c)^9)^(1/2)*(24*a^2*d^2 + 99*b^2*c^2 - 88*a*b*c*d))/(8*(b^9*c^13 - a^9*c^4
*d^9 + 9*a^8*b*c^5*d^8 + 36*a^2*b^7*c^11*d^2 - 84*a^3*b^6*c^10*d^3 + 126*a^
4*b^5*c^9*d^4 - 126*a^5*b^4*c^8*d^5 + 84*a^6*b^3*c^7*d^6 - 36*a^7*b^2*c^6*d
^7 - 9*a*b^8*c^12*d))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)**(5/2)/(c+d/x)**3,x)
```

```
[Out] Timed out
```

3.169 $\int \sqrt{a+\frac{b}{x}}\sqrt{c+\frac{d}{x}}\,dx$

Optimal. Leaf size=123

$$x\sqrt{a+\frac{b}{x}}\sqrt{c+\frac{d}{x}}+\frac{(ad+bc)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+\frac{b}{x}}}{\sqrt{a}\sqrt{c+\frac{d}{x}}}\right)}{\sqrt{a}\sqrt{c}}-2\sqrt{b}\sqrt{d}\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{b}\sqrt{c+\frac{d}{x}}}\right)$$

Rubi [A] time = 0.09, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {375, 97, 157, 63, 217, 206, 93, 208}

$$x\sqrt{a+\frac{b}{x}}\sqrt{c+\frac{d}{x}}+\frac{(ad+bc)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+\frac{b}{x}}}{\sqrt{a}\sqrt{c+\frac{d}{x}}}\right)}{\sqrt{a}\sqrt{c}}-2\sqrt{b}\sqrt{d}\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{b}\sqrt{c+\frac{d}{x}}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b/x]*Sqrt[c + d/x],x]
```

```
[Out] Sqrt[a + b/x]*Sqrt[c + d/x]*x + ((b*c + a*d)*ArcTanh[(Sqrt[c]*Sqrt[a + b/x]
)/(Sqrt[a]*Sqrt[c + d/x])])/(Sqrt[a]*Sqrt[c]) - 2*Sqrt[b]*Sqrt[d]*ArcTanh[(
Sqrt[d]*Sqrt[a + b/x])/(Sqrt[b]*Sqrt[c + d/x])]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 97

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p)/(b*
(m + 1)), x] - Dist[1/(b*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*
(e + f*x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ
[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 157

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[((c + d*x)^n*(e + f*x)^p)/(a + b*x), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 375

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[((a + b/x^n)^p*(c + d/x^n)^q)/x^2, x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rubi steps

$$\int \sqrt{a+\frac{b}{x}}\sqrt{c+\frac{d}{x}}\,dx = -\text{Subst}\left(\int \frac{\sqrt{a+bx}\sqrt{c+dx}}{x^2}\,dx, x, \frac{1}{x}\right)$$
$$= \sqrt{a+\frac{b}{x}}\sqrt{c+\frac{d}{x}}\,x - \text{Subst}\left(\int \frac{\frac{1}{2}(bc+ad)+bdx}{x\sqrt{a+bx}\sqrt{c+dx}}\,dx, x, \frac{1}{x}\right)$$
$$= \sqrt{a+\frac{b}{x}}\sqrt{c+\frac{d}{x}}\,x - (bd)\,\text{Subst}\left(\int \frac{1}{\sqrt{a+bx}\sqrt{c+dx}}\,dx, x, \frac{1}{x}\right) - \frac{1}{2}(bc+ad)\,\text{Subst}\left(\int$$
$$= \sqrt{a+\frac{b}{x}}\sqrt{c+\frac{d}{x}}\,x - (2d)\,\text{Subst}\left(\int \frac{1}{\sqrt{c-\frac{ad}{b}+\frac{dx^2}{b}}}\,dx, x, \sqrt{a+\frac{b}{x}}\right) - (bc+ad)\,\text{Subst}\left(\right.$$
$$= \sqrt{a+\frac{b}{x}}\sqrt{c+\frac{d}{x}}\,x + \frac{(bc+ad)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+\frac{b}{x}}}{\sqrt{a}\sqrt{c+\frac{d}{x}}}\right)}{\sqrt{a}\sqrt{c}} - (2d)\,\text{Subst}\left(\int \frac{1}{1-\frac{dx^2}{b}}\,dx, x, \frac{\sqrt{a+}}{\sqrt{c+}}\right.$$
$$= \sqrt{a+\frac{b}{x}}\sqrt{c+\frac{d}{x}}\,x + \frac{(bc+ad)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+\frac{b}{x}}}{\sqrt{a}\sqrt{c+\frac{d}{x}}}\right)}{\sqrt{a}\sqrt{c}} - 2\sqrt{b}\sqrt{d}\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{b}\sqrt{c+\frac{d}{x}}}\right)$$

Mathematica [A] time = 1.40, size = 167, normalized size = 1.36

$$\frac{\sqrt{a+\frac{b}{x}}(cx+d) - 2\sqrt{d}\sqrt{bc-ad}\sqrt{\frac{bcx+bd}{bcx-adx}}\sinh^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right) + \frac{\sqrt{c+\frac{d}{x}}(ad+bc)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+\frac{b}{x}}}{\sqrt{a}\sqrt{c+\frac{d}{x}}}\right)}{\sqrt{a}\sqrt{c}}}{\sqrt{c+\frac{d}{x}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b/x]*Sqrt[c + d/x],x]
```

```
[Out] (Sqrt[a + b/x]*(d + c*x) - 2*Sqrt[d]*Sqrt[b*c - a*d]*Sqrt[(b*d + b*c*x)/(b*
c*x - a*d*x)]*ArcSinh[(Sqrt[d]*Sqrt[a + b/x])/Sqrt[b*c - a*d]] + ((b*c + a*
d)*Sqrt[c + d/x]*ArcTanh[(Sqrt[c]*Sqrt[a + b/x])/(Sqrt[a]*Sqrt[c + d/x])])/
(Sqrt[a]*Sqrt[c]))/Sqrt[c + d/x]
```

IntegrateAlgebraic [A] time = 0.40, size = 179, normalized size = 1.46

$$\frac{x\sqrt{a+\frac{b}{x}}\sqrt{c+\frac{d}{x}}\left(\frac{\sqrt{cx+d}\,(bc-ad)}{\sqrt{ax+b}\left(c-\frac{a(cx+d)}{ax+b}\right)}+\frac{(ad+bc)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{cx+d}}{\sqrt{c}\sqrt{ax+b}}\right)}{\sqrt{a}\sqrt{c}}-2\sqrt{b}\sqrt{d}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{cx+d}}{\sqrt{d}\sqrt{ax+b}}\right)\right)}{\sqrt{ax+b}\sqrt{cx+d}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[a + b/x]*Sqrt[c + d/x],x]
```

```
[Out] (Sqrt[a + b/x]*Sqrt[c + d/x]*x*(((b*c - a*d)*Sqrt[d + c*x])/(Sqrt[b + a*x]*
(c - (a*(d + c*x))/(b + a*x))) + ((b*c + a*d)*ArcTanh[(Sqrt[a]*Sqrt[d + c*x
])/(Sqrt[c]*Sqrt[b + a*x])])/(Sqrt[a]*Sqrt[c]) - 2*Sqrt[b]*Sqrt[d]*ArcTanh[
(Sqrt[b]*Sqrt[d + c*x])/(Sqrt[d]*Sqrt[b + a*x])]))/(Sqrt[b + a*x]*Sqrt[d +
c*x])
```

fricas [A] time = 1.78, size = 890, normalized size = 7.24

$$\left[\frac{4\,acx\sqrt{\frac{ax+b}{x}}\sqrt{\frac{cx+d}{x}}+2\sqrt{bd}\,ac\log\left(-\frac{8\,b^2d^2+(b^2c^2+6\,abcd+a^2d^2)x^2-4\,(2\,bdx+(bc+ad)x^2)\sqrt{bd}\sqrt{\frac{ax+b}{x}}\sqrt{\frac{cx+d}{x}}+8\,(b^2cd+abd^2)x}{x^2}\right)+\sqrt{ac}\,(bc+ad)\log\left(-8\,a^2c^2x^2-b^2c^2-6\,abcd-a^2d^2-4\,(2\,acx^2+(bc+ad)x)\sqrt{ac}\sqrt{\frac{ax+b}{x}}\sqrt{\frac{cx+d}{x}}-8\,(abc^2+a^2cd)x\right)}{4\,ac},\ \frac{4\,acx\sqrt{\frac{ax+b}{x}}\sqrt{\frac{cx+d}{x}}+4\sqrt{-bd}\,ac\arctan\left(\frac{(2\,bdx+(bc+ad)x^2)\sqrt{-bd}\sqrt{\frac{ax+b}{x}}\sqrt{\frac{cx+d}{x}}}{2\,(abcdx^2+b^2d^2+(b^2cd+abd^2)x)}\right)+\sqrt{ac}\,(bc+ad)\log\left(-8\,a^2c^2x^2-b^2c^2-6\,abcd-a^2d^2-4\,(2\,acx^2+(bc+ad)x)\sqrt{ac}\sqrt{\frac{ax+b}{x}}\sqrt{\frac{cx+d}{x}}-8\,(abc^2+a^2cd)x\right)}{4\,ac},\ \frac{2\,acx\sqrt{\frac{ax+b}{x}}\sqrt{\frac{cx+d}{x}}+\sqrt{bd}\,ac\log\left(-\frac{8\,b^2d^2+(b^2c^2+6\,abcd+a^2d^2)x^2-4\,(2\,bdx+(bc+ad)x^2)\sqrt{bd}\sqrt{\frac{ax+b}{x}}\sqrt{\frac{cx+d}{x}}+8\,(b^2cd+abd^2)x}{x^2}\right)-\sqrt{-ac}\,(bc+ad)\arctan\left(\frac{2\sqrt{-ac}\,x\sqrt{\frac{ax+b}{x}}\sqrt{\frac{cx+d}{x}}}{2\,acx+bc+ad}\right)}{2\,ac},\ \frac{2\,acx\sqrt{\frac{ax+b}{x}}\sqrt{\frac{cx+d}{x}}+2\sqrt{-bd}\,ac\arctan\left(\frac{(2\,bdx+(bc+ad)x^2)\sqrt{-bd}\sqrt{\frac{ax+b}{x}}\sqrt{\frac{cx+d}{x}}}{2\,(abcdx^2+b^2d^2+(b^2cd+abd^2)x)}\right)-\sqrt{-ac}\,(bc+ad)\arctan\left(\frac{2\sqrt{-ac}\,x\sqrt{\frac{ax+b}{x}}\sqrt{\frac{cx+d}{x}}}{2\,acx+bc+ad}\right)}{2\,ac}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d/x)^(1/2)*(a+b/x)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(4*a*c*x*sqrt((a*x + b)/x)*sqrt((c*x + d)/x) + 2*sqrt(b*d)*a*c*log(-(8
*b^2*d^2 + (b^2*c^2 + 6*a*b*c*d + a^2*d^2)*x^2 - 4*(2*b*d*x + (b*c + a*d)*x
^2)*sqrt(b*d)*sqrt((a*x + b)/x)*sqrt((c*x + d)/x) + 8*(b^2*c*d + a*b*d^2)*x
)/x^2) + sqrt(a*c)*(b*c + a*d)*log(-8*a^2*c^2*x^2 - b^2*c^2 - 6*a*b*c*d - a
^2*d^2 - 4*(2*a*c*x^2 + (b*c + a*d)*x)*sqrt(a*c)*sqrt((a*x + b)/x)*sqrt((c*
x + d)/x) - 8*(a*b*c^2 + a^2*c*d)*x))/(a*c), 1/4*(4*a*c*x*sqrt((a*x + b)/x)
*sqrt((c*x + d)/x) + 4*sqrt(-b*d)*a*c*arctan(1/2*(2*b*d*x + (b*c + a*d)*x^2
)*sqrt(-b*d)*sqrt((a*x + b)/x)*sqrt((c*x + d)/x)/(a*b*c*d*x^2 + b^2*d^2 + (
b^2*c*d + a*b*d^2)*x)) + sqrt(a*c)*(b*c + a*d)*log(-8*a^2*c^2*x^2 - b^2*c^2
 - 6*a*b*c*d - a^2*d^2 - 4*(2*a*c*x^2 + (b*c + a*d)*x)*sqrt(a*c)*sqrt((a*x
+ b)/x)*sqrt((c*x + d)/x) - 8*(a*b*c^2 + a^2*c*d)*x))/(a*c), 1/2*(2*a*c*x*s
qrt((a*x + b)/x)*sqrt((c*x + d)/x) + sqrt(b*d)*a*c*log(-(8*b^2*d^2 + (b^2*c
^2 + 6*a*b*c*d + a^2*d^2)*x^2 - 4*(2*b*d*x + (b*c + a*d)*x^2)*sqrt(b*d)*sqr
t((a*x + b)/x)*sqrt((c*x + d)/x) + 8*(b^2*c*d + a*b*d^2)*x)/x^2) - sqrt(-a*
c)*(b*c + a*d)*arctan(2*sqrt(-a*c)*x*sqrt((a*x + b)/x)*sqrt((c*x + d)/x)/(2
*a*c*x + b*c + a*d)))/(a*c), 1/2*(2*a*c*x*sqrt((a*x + b)/x)*sqrt((c*x + d)/
x) + 2*sqrt(-b*d)*a*c*arctan(1/2*(2*b*d*x + (b*c + a*d)*x^2)*sqrt(-b*d)*sqr
t((a*x + b)/x)*sqrt((c*x + d)/x)/(a*b*c*d*x^2 + b^2*d^2 + (b^2*c*d + a*b*d^
2)*x)) - sqrt(-a*c)*(b*c + a*d)*arctan(2*sqrt(-a*c)*x*sqrt((a*x + b)/x)*sqr
t((c*x + d)/x)/(2*a*c*x + b*c + a*d)))/(a*c)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a+\frac{b}{x}}\sqrt{c+\frac{d}{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d/x)^(1/2)*(a+b/x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a + b/x)*sqrt(c + d/x), x)
```

maple [B] time = 0.10, size = 253, normalized size = 2.06

$$\frac{\sqrt{\frac{ax+b}{x}}\,\sqrt{\frac{cx+d}{x}}\left(\sqrt{bd}\,ad\ln\left(\frac{2acx+ad+bc+2\sqrt{ac\,x^{2}+adx+bcx+bd}\,\sqrt{ac}}{2\sqrt{ac}}\right)+\sqrt{bd}\,bc\ln\left(\frac{2acx+ad+bc+2\sqrt{ac\,x^{2}+adx+bcx+bd}\,\sqrt{ac}}{2\sqrt{ac}}\right)-2\sqrt{ac}\,bd\ln\left(\frac{adx+bcx+2bd+2\sqrt{bd}\,\sqrt{ac\,x^{2}+adx+bcx+bd}}{x}\right)+2\sqrt{ac\,x^{2}+adx+bcx+bd}\,\sqrt{ac}\,\sqrt{bd}\right)x}{2\sqrt{ac\,x^{2}+adx+bcx+bd}\,\sqrt{ac}\,\sqrt{bd}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d/x)^(1/2)*(a+b/x)^(1/2),x)
```

```
[Out] 1/2*((a*x+b)/x)^(1/2)*x*((c*x+d)/x)^(1/2)*((b*d)^(1/2)*ln(1/2*(2*a*c*x+2*(a
*c*x^2+a*d*x+b*c*x+b*d)^(1/2)*(a*c)^(1/2)+a*d+b*c)/(a*c)^(1/2))*a*d+(b*d)^(
1/2)*ln(1/2*(2*a*c*x+2*(a*c*x^2+a*d*x+b*c*x+b*d)^(1/2)*(a*c)^(1/2)+a*d+b*c)
/(a*c)^(1/2))*b*c-2*b*d*ln((a*d*x+b*c*x+2*(b*d)^(1/2)*(a*c*x^2+a*d*x+b*c*x+
b*d)^(1/2)+2*b*d)/x)*(a*c)^(1/2)+2*(a*c*x^2+a*d*x+b*c*x+b*d)^(1/2)*(a*c)^(1
/2)*(b*d)^(1/2))/(a*c*x^2+a*d*x+b*c*x+b*d)^(1/2)/(a*c)^(1/2)/(b*d)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + \frac{b}{x}}\,\sqrt{c + \frac{d}{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d/x)^(1/2)*(a+b/x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a + b/x)*sqrt(c + d/x), x)
```

mupad [B] time = 22.22, size = 4674, normalized size = 38.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x)^(1/2)*(c + d/x)^(1/2),x)
```

```
[Out] atan(((b*d)^(1/2)*(2*(b*d)^(1/2)*(2*(b*d)^(1/2)*(2*(b*d)^(1/2)*((2*(4*a^(9/
2)*b^9*c^(19/2) - 4*a^(13/2)*b^7*c^(15/2)*d^2 - 4*a^(15/2)*b^6*c^(13/2)*d^3
 + 4*a^(19/2)*b^4*c^(9/2)*d^5))/(a^7*c^7*d^9) - (((a + b/x)^(1/2) - a^(1/2)
)*(32*a^4*b^9*c^10 - 120*a^5*b^8*c^9*d + 288*a^6*b^7*c^8*d^2 - 400*a^7*b^6*
c^7*d^3 + 288*a^8*b^5*c^6*d^4 - 120*a^9*b^4*c^5*d^5 + 32*a^10*b^3*c^4*d^6))
/(2*a^7*c^7*d^9*((c + d/x)^(1/2) - c^(1/2)))) - (2*(8*a^5*b^9*c^9*d + 16*a^
6*b^8*c^8*d^2 - 48*a^7*b^7*c^7*d^3 + 16*a^8*b^6*c^6*d^4 + 8*a^9*b^5*c^5*d^5
))/(a^7*c^7*d^9) + (((a + b/x)^(1/2) - a^(1/2))*(16*a^(7/2)*b^10*c^(21/2) -
 76*a^(9/2)*b^9*c^(19/2)*d + 228*a^(11/2)*b^8*c^(17/2)*d^2 - 168*a^(13/2)*b
^7*c^(15/2)*d^3 - 168*a^(15/2)*b^6*c^(13/2)*d^4 + 228*a^(17/2)*b^5*c^(11/2)
*d^5 - 76*a^(19/2)*b^4*c^(9/2)*d^6 + 16*a^(21/2)*b^3*c^(7/2)*d^7))/(2*a^7*c
^7*d^9*((c + d/x)^(1/2) - c^(1/2)))) - (2*(a^(7/2)*b^11*c^(21/2) + 16*a^(9/
2)*b^10*c^(19/2)*d - 42*a^(11/2)*b^9*c^(17/2)*d^2 + 25*a^(13/2)*b^8*c^(15/2
)*d^3 + 25*a^(15/2)*b^7*c^(13/2)*d^4 - 42*a^(17/2)*b^6*c^(11/2)*d^5 + 16*a^
(19/2)*b^5*c^(9/2)*d^6 + a^(21/2)*b^4*c^(7/2)*d^7))/(a^7*c^7*d^9) + (((a +
b/x)^(1/2) - a^(1/2))*(146*a^4*b^10*c^10*d - 556*a^5*b^9*c^9*d^2 + 1006*a^6
*b^8*c^8*d^3 - 1192*a^7*b^7*c^7*d^4 + 1006*a^8*b^6*c^6*d^5 - 556*a^9*b^5*c^
5*d^6 + 146*a^10*b^4*c^4*d^7))/(2*a^7*c^7*d^9*((c + d/x)^(1/2) - c^(1/2))))
 + (2*(2*a^4*b^11*c^10*d + 8*a^5*b^10*c^9*d^2 - 2*a^6*b^9*c^8*d^3 - 16*a^7*
b^8*c^7*d^4 - 2*a^8*b^7*c^6*d^5 + 8*a^9*b^6*c^5*d^6 + 2*a^10*b^5*c^4*d^7))/
(a^7*c^7*d^9) - (((a + b/x)^(1/2) - a^(1/2))*(65*a^(7/2)*b^11*c^(21/2)*d -
297*a^(9/2)*b^10*c^(19/2)*d^2 + 597*a^(11/2)*b^9*c^(17/2)*d^3 - 365*a^(13/2
)*b^8*c^(15/2)*d^4 - 365*a^(15/2)*b^7*c^(13/2)*d^5 + 597*a^(17/2)*b^6*c^(11
/2)*d^6 - 297*a^(19/2)*b^5*c^(9/2)*d^7 + 65*a^(21/2)*b^4*c^(7/2)*d^8))/(2*a
^7*c^7*d^9*((c + d/x)^(1/2) - c^(1/2))))*1i - (b*d)^(1/2)*(2*(b*d)^(1/2)*(2
*(b*d)^(1/2)*(2*(b*d)^(1/2)*((2*(4*a^(9/2)*b^9*c^(19/2) - 4*a^(13/2)*b^7*c^
```

(15/2)*d^2 - 4*a^(15/2)*b^6*c^(13/2)*d^3 + 4*a^(19/2)*b^4*c^(9/2)*d^5))/(a^
7*c^7*d^9) - (((a + b/x)^(1/2) - a^(1/2))*(32*a^4*b^9*c^10 - 120*a^5*b^8*c^
9*d + 288*a^6*b^7*c^8*d^2 - 400*a^7*b^6*c^7*d^3 + 288*a^8*b^5*c^6*d^4 - 120
*a^9*b^4*c^5*d^5 + 32*a^10*b^3*c^4*d^6))/(2*a^7*c^7*d^9*((c + d/x)^(1/2) -
c^(1/2)))) + (2*(8*a^5*b^9*c^9*d + 16*a^6*b^8*c^8*d^2 - 48*a^7*b^7*c^7*d^3
+ 16*a^8*b^6*c^6*d^4 + 8*a^9*b^5*c^5*d^5))/(a^7*c^7*d^9) - (((a + b/x)^(1/2
) - a^(1/2))*(16*a^(7/2)*b^10*c^(21/2) - 76*a^(9/2)*b^9*c^(19/2)*d + 228*a^
(11/2)*b^8*c^(17/2)*d^2 - 168*a^(13/2)*b^7*c^(15/2)*d^3 - 168*a^(15/2)*b^6*
c^(13/2)*d^4 + 228*a^(17/2)*b^5*c^(11/2)*d^5 - 76*a^(19/2)*b^4*c^(9/2)*d^6
+ 16*a^(21/2)*b^3*c^(7/2)*d^7))/(2*a^7*c^7*d^9*((c + d/x)^(1/2) - c^(1/2)))
) - (2*(a^(7/2)*b^11*c^(21/2) + 16*a^(9/2)*b^10*c^(19/2)*d - 42*a^(11/2)*b^
9*c^(17/2)*d^2 + 25*a^(13/2)*b^8*c^(15/2)*d^3 + 25*a^(15/2)*b^7*c^(13/2)*d^
4 - 42*a^(17/2)*b^6*c^(11/2)*d^5 + 16*a^(19/2)*b^5*c^(9/2)*d^6 + a^(21/2)*b
^4*c^(7/2)*d^7))/(a^7*c^7*d^9) + (((a + b/x)^(1/2) - a^(1/2))*(146*a^4*b^10
*c^10*d - 556*a^5*b^9*c^9*d^2 + 1006*a^6*b^8*c^8*d^3 - 1192*a^7*b^7*c^7*d^4
+ 1006*a^8*b^6*c^6*d^5 - 556*a^9*b^5*c^5*d^6 + 146*a^10*b^4*c^4*d^7))/(2*a
^7*c^7*d^9*((c + d/x)^(1/2) - c^(1/2)))) - (2*(2*a^4*b^11*c^10*d + 8*a^5*b^
10*c^9*d^2 - 2*a^6*b^9*c^8*d^3 - 16*a^7*b^8*c^7*d^4 - 2*a^8*b^7*c^6*d^5 + 8
*a^9*b^6*c^5*d^6 + 2*a^10*b^5*c^4*d^7))/(a^7*c^7*d^9) + (((a + b/x)^(1/2) -
a^(1/2))*(65*a^(7/2)*b^11*c^(21/2)*d - 297*a^(9/2)*b^10*c^(19/2)*d^2 + 597
*a^(11/2)*b^9*c^(17/2)*d^3 - 365*a^(13/2)*b^8*c^(15/2)*d^4 - 365*a^(15/2)*b
^7*c^(13/2)*d^5 + 597*a^(17/2)*b^6*c^(11/2)*d^6 - 297*a^(19/2)*b^5*c^(9/2)*
d^7 + 65*a^(21/2)*b^4*c^(7/2)*d^8))/(2*a^7*c^7*d^9*((c + d/x)^(1/2) - c^(1/
2))))*1i)/((b*d)^(1/2)*(2*(b*d)^(1/2)*(2*(b*d)^(1/2)*(2*(b*d)^(1/2)*((2*(4*
a^(9/2)*b^9*c^(19/2) - 4*a^(13/2)*b^7*c^(15/2)*d^2 - 4*a^(15/2)*b^6*c^(13/2
)*d^3 + 4*a^(19/2)*b^4*c^(9/2)*d^5))/(a^7*c^7*d^9) - (((a + b/x)^(1/2) - a^
(1/2))*(32*a^4*b^9*c^10 - 120*a^5*b^8*c^9*d + 288*a^6*b^7*c^8*d^2 - 400*a^7
*b^6*c^7*d^3 + 288*a^8*b^5*c^6*d^4 - 120*a^9*b^4*c^5*d^5 + 32*a^10*b^3*c^4*
d^6))/(2*a^7*c^7*d^9*((c + d/x)^(1/2) - c^(1/2)))) - (2*(8*a^5*b^9*c^9*d +
16*a^6*b^8*c^8*d^2 - 48*a^7*b^7*c^7*d^3 + 16*a^8*b^6*c^6*d^4 + 8*a^9*b^5*c^
5*d^5))/(a^7*c^7*d^9) + (((a + b/x)^(1/2) - a^(1/2))*(16*a^(7/2)*b^10*c^(21
/2) - 76*a^(9/2)*b^9*c^(19/2)*d + 228*a^(11/2)*b^8*c^(17/2)*d^2 - 168*a^(13
/2)*b^7*c^(15/2)*d^3 - 168*a^(15/2)*b^6*c^(13/2)*d^4 + 228*a^(17/2)*b^5*c^(
11/2)*d^5 - 76*a^(19/2)*b^4*c^(9/2)*d^6 + 16*a^(21/2)*b^3*c^(7/2)*d^7))/(2*
a^7*c^7*d^9*((c + d/x)^(1/2) - c^(1/2)))) - (2*(a^(7/2)*b^11*c^(21/2) + 16*
a^(9/2)*b^10*c^(19/2)*d - 42*a^(11/2)*b^9*c^(17/2)*d^2 + 25*a^(13/2)*b^8*c^
(15/2)*d^3 + 25*a^(15/2)*b^7*c^(13/2)*d^4 - 42*a^(17/2)*b^6*c^(11/2)*d^5 +
16*a^(19/2)*b^5*c^(9/2)*d^6 + a^(21/2)*b^4*c^(7/2)*d^7))/(a^7*c^7*d^9) + ((
(a + b/x)^(1/2) - a^(1/2))*(146*a^4*b^10*c^10*d - 556*a^5*b^9*c^9*d^2 + 100
6*a^6*b^8*c^8*d^3 - 1192*a^7*b^7*c^7*d^4 + 1006*a^8*b^6*c^6*d^5 - 556*a^9*b
^5*c^5*d^6 + 146*a^10*b^4*c^4*d^7))/(2*a^7*c^7*d^9*((c + d/x)^(1/2) - c^(1/
2)))) + (2*(2*a^4*b^11*c^10*d + 8*a^5*b^10*c^9*d^2 - 2*a^6*b^9*c^8*d^3 - 16
*a^7*b^8*c^7*d^4 - 2*a^8*b^7*c^6*d^5 + 8*a^9*b^6*c^5*d^6 + 2*a^10*b^5*c^4*d
^7))/(a^7*c^7*d^9) - (((a + b/x)^(1/2) - a^(1/2))*(65*a^(7/2)*b^11*c^(21/2)
*d - 297*a^(9/2)*b^10*c^(19/2)*d^2 + 597*a^(11/2)*b^9*c^(17/2)*d^3 - 365*a^
(13/2)*b^8*c^(15/2)*d^4 - 365*a^(15/2)*b^7*c^(13/2)*d^5 + 597*a^(17/2)*b^6*
c^(11/2)*d^6 - 297*a^(19/2)*b^5*c^(9/2)*d^7 + 65*a^(21/2)*b^4*c^(7/2)*d^8))
/(2*a^7*c^7*d^9*((c + d/x)^(1/2) - c^(1/2)))) + (b*d)^(1/2)*(2*(b*d)^(1/2)*
(2*(b*d)^(1/2)*(2*(b*d)^(1/2)*((2*(4*a^(9/2)*b^9*c^(19/2) - 4*a^(13/2)*b^7*
c^(15/2)*d^2 - 4*a^(15/2)*b^6*c^(13/2)*d^3 + 4*a^(19/2)*b^4*c^(9/2)*d^5))/(
a^7*c^7*d^9) - (((a + b/x)^(1/2) - a^(1/2))*(32*a^4*b^9*c^10 - 120*a^5*b^8*
c^9*d + 288*a^6*b^7*c^8*d^2 - 400*a^7*b^6*c^7*d^3 + 288*a^8*b^5*c^6*d^4 - 1
20*a^9*b^4*c^5*d^5 + 32*a^10*b^3*c^4*d^6))/(2*a^7*c^7*d^9*((c + d/x)^(1/2)
- c^(1/2)))) + (2*(8*a^5*b^9*c^9*d + 16*a^6*b^8*c^8*d^2 - 48*a^7*b^7*c^7*d^
3 + 16*a^8*b^6*c^6*d^4 + 8*a^9*b^5*c^5*d^5))/(a^7*c^7*d^9) - (((a + b/x)^(1
/2) - a^(1/2))*(16*a^(7/2)*b^10*c^(21/2) - 76*a^(9/2)*b^9*c^(19/2)*d + 228*
a^(11/2)*b^8*c^(17/2)*d^2 - 168*a^(13/2)*b^7*c^(15/2)*d^3 - 168*a^(15/2)*b^
6*c^(13/2)*d^4 + 228*a^(17/2)*b^5*c^(11/2)*d^5 - 76*a^(19/2)*b^4*c^(9/2)*d^
6 + 16*a^(21/2)*b^3*c^(7/2)*d^7))/(2*a^7*c^7*d^9*((c + d/x)^(1/2) - c^(1/2)

```
))) - (2*(a^(7/2)*b^11*c^(21/2) + 16*a^(9/2)*b^10*c^(19/2)*d - 42*a^(11/2)*
b^9*c^(17/2)*d^2 + 25*a^(13/2)*b^8*c^(15/2)*d^3 + 25*a^(15/2)*b^7*c^(13/2)*
d^4 - 42*a^(17/2)*b^6*c^(11/2)*d^5 + 16*a^(19/2)*b^5*c^(9/2)*d^6 + a^(21/2)
*b^4*c^(7/2)*d^7))/(a^7*c^7*d^9) + (((a + b/x)^(1/2) - a^(1/2))*(146*a^4*b^
10*c^10*d - 556*a^5*b^9*c^9*d^2 + 1006*a^6*b^8*c^8*d^3 - 1192*a^7*b^7*c^7*d
^4 + 1006*a^8*b^6*c^6*d^5 - 556*a^9*b^5*c^5*d^6 + 146*a^10*b^4*c^4*d^7))/(2
*a^7*c^7*d^9*((c + d/x)^(1/2) - c^(1/2)))) - (2*(2*a^4*b^11*c^10*d + 8*a^5*
b^10*c^9*d^2 - 2*a^6*b^9*c^8*d^3 - 16*a^7*b^8*c^7*d^4 - 2*a^8*b^7*c^6*d^5 +
 8*a^9*b^6*c^5*d^6 + 2*a^10*b^5*c^4*d^7))/(a^7*c^7*d^9) + (((a + b/x)^(1/2)
 - a^(1/2))*(65*a^(7/2)*b^11*c^(21/2)*d - 297*a^(9/2)*b^10*c^(19/2)*d^2 + 5
97*a^(11/2)*b^9*c^(17/2)*d^3 - 365*a^(13/2)*b^8*c^(15/2)*d^4 - 365*a^(15/2)
*b^7*c^(13/2)*d^5 + 597*a^(17/2)*b^6*c^(11/2)*d^6 - 297*a^(19/2)*b^5*c^(9/2
)*d^7 + 65*a^(21/2)*b^4*c^(7/2)*d^8))/(2*a^7*c^7*d^9*((c + d/x)^(1/2) - c^(
1/2)))) + (7*a^(7/2)*b^12*c^(21/2)*d - 7*a^(9/2)*b^11*c^(19/2)*d^2 - 21*a^(
11/2)*b^10*c^(17/2)*d^3 + 21*a^(13/2)*b^9*c^(15/2)*d^4 + 21*a^(15/2)*b^8*c^
(13/2)*d^5 - 21*a^(17/2)*b^7*c^(11/2)*d^6 - 7*a^(19/2)*b^6*c^(9/2)*d^7 + 7*
a^(21/2)*b^5*c^(7/2)*d^8)/(a^7*c^7*d^9) + (((a + b/x)^(1/2) - a^(1/2))*(112
*a^5*b^10*c^9*d^3 - 56*a^4*b^11*c^10*d^2 + 56*a^6*b^9*c^8*d^4 - 224*a^7*b^8
*c^7*d^5 + 56*a^8*b^7*c^6*d^6 + 112*a^9*b^6*c^5*d^7 - 56*a^10*b^5*c^4*d^8))
/(2*a^7*c^7*d^9*((c + d/x)^(1/2) - c^(1/2)))))*(b*d)^(1/2)*4i - ((((a + b/x
)^(1/2) - a^(1/2))*((b^2*c)/4 + (a*b*d)/4))/(a^(1/2)*c^(1/2)*d*((c + d/x)^(
1/2) - c^(1/2))) - b^2/(4*d) + (((a + b/x)^(1/2) - a^(1/2))^2*((a^2*d^2)/4
+ (b^2*c^2)/4 - (3*a*b*c*d)/4))/(a*c*d*((c + d/x)^(1/2) - c^(1/2))^2))/(((a
 + b/x)^(1/2) - a^(1/2))^3/((c + d/x)^(1/2) - c^(1/2))^3 + (b*((a + b/x)^(1
/2) - a^(1/2)))/(d*((c + d/x)^(1/2) - c^(1/2))) - (((a + b/x)^(1/2) - a^(1/
2))^2*(a*d + b*c))/(a^(1/2)*c^(1/2)*d*((c + d/x)^(1/2) - c^(1/2))^2)) + (d*
((a + b/x)^(1/2) - a^(1/2)))/(4*((c + d/x)^(1/2) - c^(1/2))) + (log(((a + b
/x)^(1/2) - a^(1/2))/((c + d/x)^(1/2) - c^(1/2)))*(a*d + b*c))/(2*a^(1/2)*c
^(1/2)) - (log(((c^(1/2)*(a + b/x)^(1/2) - a^(1/2)*(c + d/x)^(1/2))*(b*c^(1
/2) - (a^(1/2)*d*((a + b/x)^(1/2) - a^(1/2)))/((c + d/x)^(1/2) - c^(1/2))))
/((c + d/x)^(1/2) - c^(1/2)))*(a^(1/2)*b*c^(3/2) + a^(3/2)*c^(1/2)*d))/(2*a
*c)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a+\frac{b}{x}}\sqrt{c+\frac{d}{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d/x)**(1/2)*(a+b/x)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b/x)*sqrt(c + d/x), x)
```

3.170 $\int \frac{\sqrt{a+\frac{b}{x}}}{\sqrt{c+\frac{d}{x}}}\,dx$

Optimal. Leaf size=81

$$\frac{(bc-ad)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+\frac{b}{x}}}{\sqrt{a}\sqrt{c+\frac{d}{x}}}\right)}{\sqrt{a}\,c^{3/2}}+\frac{x\sqrt{a+\frac{b}{x}}\sqrt{c+\frac{d}{x}}}{c}$$

Rubi [A] time = 0.05, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {375, 94, 93, 208}

$$\frac{(bc-ad)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+\frac{b}{x}}}{\sqrt{a}\sqrt{c+\frac{d}{x}}}\right)}{\sqrt{a}\,c^{3/2}}+\frac{x\sqrt{a+\frac{b}{x}}\sqrt{c+\frac{d}{x}}}{c}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b/x]/Sqrt[c + d/x],x]
```

```
[Out] (Sqrt[a + b/x]*Sqrt[c + d/x]*x)/c + ((b*c - a*d)*ArcTanh[(Sqrt[c]*Sqrt[a +
b/x])/(Sqrt[a]*Sqrt[c + d/x])])/(Sqrt[a]*c^(3/2))
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 375

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[((a + b/x^n)^p*(c + d/x^n)^q)/x^2, x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rubi steps

$$\int \frac{\sqrt{a+\frac{b}{x}}}{\sqrt{c+\frac{d}{x}}}\,dx = -\operatorname{Subst}\left(\int \frac{\sqrt{a+bx}}{x^2\sqrt{c+dx}}\,dx, x, \frac{1}{x}\right)$$
$$= \frac{\sqrt{a+\frac{b}{x}}\sqrt{c+\frac{d}{x}}\,x}{c} - \frac{(bc-ad)\operatorname{Subst}\left(\int \frac{1}{x\sqrt{a+bx}\sqrt{c+dx}}\,dx, x, \frac{1}{x}\right)}{2c}$$
$$= \frac{\sqrt{a+\frac{b}{x}}\sqrt{c+\frac{d}{x}}\,x}{c} - \frac{(bc-ad)\operatorname{Subst}\left(\int \frac{1}{-a+cx^2}\,dx, x, \frac{\sqrt{a+\frac{b}{x}}}{\sqrt{c+\frac{d}{x}}}\right)}{c}$$
$$= \frac{\sqrt{a+\frac{b}{x}}\sqrt{c+\frac{d}{x}}\,x}{c} + \frac{(bc-ad)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+\frac{b}{x}}}{\sqrt{a}\sqrt{c+\frac{d}{x}}}\right)}{\sqrt{a}\,c^{3/2}}$$

Mathematica [A] time = 0.07, size = 81, normalized size = 1.00

$$\frac{(bc-ad)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+\frac{b}{x}}}{\sqrt{a}\sqrt{c+\frac{d}{x}}}\right)}{\sqrt{a}\,c^{3/2}} + \frac{x\sqrt{a+\frac{b}{x}}\sqrt{c+\frac{d}{x}}}{c}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b/x]/Sqrt[c + d/x],x]
```

```
[Out] (Sqrt[a + b/x]*Sqrt[c + d/x]*x)/c + ((b*c - a*d)*ArcTanh[(Sqrt[c]*Sqrt[a + 
b/x])/(Sqrt[a]*Sqrt[c + d/x])])/(Sqrt[a]*c^(3/2))
```

IntegrateAlgebraic [A] time = 0.43, size = 147, normalized size = 1.81

$$\frac{\sqrt{a+\frac{b}{x}}\sqrt{cx+d}\left(\frac{\sqrt{cx+d}\sqrt{\frac{a(cx+d)}{c}-\frac{ad}{c}+b}}{c}+\frac{\sqrt{\frac{a}{c}}\,(ad-bc)\log\left(\sqrt{\frac{a(cx+d)}{c}-\frac{ad}{c}+b}-\sqrt{\frac{a}{c}}\sqrt{cx+d}\right)}{ac}\right)}{\sqrt{ax+b}\sqrt{c+\frac{d}{x}}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[a + b/x]/Sqrt[c + d/x],x]
```

```
[Out] (Sqrt[a + b/x]*Sqrt[d + c*x]*((Sqrt[d + c*x]*Sqrt[b - (a*d)/c + (a*(d + c*x
))/c])/c + (Sqrt[a/c]*(-(b*c) + a*d)*Log[-(Sqrt[a/c]*Sqrt[d + c*x]) + Sqrt[
b - (a*d)/c + (a*(d + c*x))/c]])/(a*c)))/(Sqrt[c + d/x]*Sqrt[b + a*x])
```

fricas [A] time = 1.19, size = 247, normalized size = 3.05

$$\left[\frac{4\,acx\sqrt{\frac{ax+b}{x}}\sqrt{\frac{cx+d}{x}}-\sqrt{ac}\,(bc-ad)\log\left(-8\,a^2c^2x^2-b^2c^2-6\,abcd-a^2d^2+4\left(2\,acx^2+(bc+ad)x\right)\sqrt{ac}\sqrt{\frac{ax+b}{x}}\sqrt{\frac{cx+d}{x}}-8\left(abc^2+a^2cd\right)x\right)}{4\,ac^2}, \frac{2\,acx\sqrt{\frac{ax+b}{x}}\sqrt{\frac{cx+d}{x}}-\sqrt{-ac}\,(bc-ad)\arctan\left(\frac{2\sqrt{-ac}\,x\sqrt{\frac{ax+b}{x}}\sqrt{\frac{cx+d}{x}}}{2\,acx+bc+ad}\right)}{2\,ac^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(1/2)/(c+d/x)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(4*a*c*x*sqrt((a*x + b)/x)*sqrt((c*x + d)/x) - sqrt(a*c)*(b*c - a*d)*l
og(-8*a^2*c^2*x^2 - b^2*c^2 - 6*a*b*c*d - a^2*d^2 + 4*(2*a*c*x^2 + (b*c + a
*d)*x)*sqrt(a*c)*sqrt((a*x + b)/x)*sqrt((c*x + d)/x) - 8*(a*b*c^2 + a^2*c*d
)*x))/(a*c^2), 1/2*(2*a*c*x*sqrt((a*x + b)/x)*sqrt((c*x + d)/x) - sqrt(-a*c
```

```
)*(b*c - a*d)*arctan(2*sqrt(-a*c)*x*sqrt((a*x + b)/x)*sqrt((c*x + d)/x)/(2*
a*c*x + b*c + a*d)))/(a*c^2)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+\frac{b}{x}}}{\sqrt{c+\frac{d}{x}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(1/2)/(c+d/x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a + b/x)/sqrt(c + d/x), x)
```

maple [B] time = 0.10, size = 155, normalized size = 1.91

$$\frac{\sqrt{\frac{ax+b}{x}}\,\sqrt{\frac{cx+d}{x}}\,\left(-ad\ln\left(\frac{2acx+ad+bc+2\sqrt{(ax+b)(cx+d)}\,\sqrt{ac}}{2\sqrt{ac}}\right)+bc\ln\left(\frac{2acx+ad+bc+2\sqrt{(ax+b)(cx+d)}\,\sqrt{ac}}{2\sqrt{ac}}\right)+2\sqrt{(ax+b)(cx+d)}\,\sqrt{ac}\right)x}{2\sqrt{(ax+b)(cx+d)}\,\sqrt{ac}\;c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x)^(1/2)/(c+d/x)^(1/2),x)
```

```
[Out] 1/2*((a*x+b)/x)^(1/2)*x*((c*x+d)/x)^(1/2)*(-ln(1/2*(2*a*c*x+2*((a*x+b)*(c*x
+d))^(1/2)*(a*c)^(1/2)+a*d+b*c)/(a*c)^(1/2))*a*d+ln(1/2*(2*a*c*x+2*((a*x+b)
*(c*x+d))^(1/2)*(a*c)^(1/2)+a*d+b*c)/(a*c)^(1/2))*b*c+2*((a*x+b)*(c*x+d))^(
1/2)*(a*c)^(1/2))/((a*x+b)*(c*x+d))^(1/2)/c/(a*c)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+\frac{b}{x}}}{\sqrt{c+\frac{d}{x}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(1/2)/(c+d/x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a + b/x)/sqrt(c + d/x), x)
```

mupad [B] time = 6.58, size = 478, normalized size = 5.90

$$\frac{d\left(\sqrt{a+\frac{b}{x}}-\sqrt{a}\right)}{4c\left(\sqrt{c+\frac{d}{x}}-\sqrt{c}\right)}-\frac{\frac{\left(\sqrt{a+\frac{b}{x}}-\sqrt{a}\right)\left(\frac{c\,b^2}{4}+\frac{a\,d\,b}{4}\right)}{\sqrt{a}\,c^{3/2}\,d\left(\sqrt{c+\frac{d}{x}}-\sqrt{c}\right)}-\frac{b^2}{4\,c\,d}+\frac{\left(\sqrt{a+\frac{b}{x}}-\sqrt{a}\right)^2\left(\frac{a^2\,d^2}{4}-\frac{3\,a\,b\,c\,d}{4}+\frac{b^2\,c^2}{4}\right)}{a\,c^2\,d\left(\sqrt{c+\frac{d}{x}}-\sqrt{c}\right)^2}}{\frac{\left(\sqrt{a+\frac{b}{x}}-\sqrt{a}\right)^3}{\left(\sqrt{c+\frac{d}{x}}-\sqrt{c}\right)^3}+\frac{b\left(\sqrt{a+\frac{b}{x}}-\sqrt{a}\right)}{d\left(\sqrt{c+\frac{d}{x}}-\sqrt{c}\right)}-\frac{\left(\sqrt{a+\frac{b}{x}}-\sqrt{a}\right)^2(a\,d+b\,c)}{\sqrt{a}\,\sqrt{c}\,d\left(\sqrt{c+\frac{d}{x}}-\sqrt{c}\right)^2}}+\frac{\ln\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{c+\frac{d}{x}}-\sqrt{c}}\right)\left(\sqrt{a}\,b\,c^{3/2}-a^{3/2}\,\sqrt{c}\,d\right)}{2\,a\,c^2}-\frac{\ln\left(\frac{\left(\sqrt{c}\,\sqrt{a+\frac{b}{x}}-\sqrt{a}\,\sqrt{c+\frac{d}{x}}\right)\left(b\,\sqrt{c}-\frac{\sqrt{a}\,d\left(\sqrt{a+\frac{b}{x}}-\sqrt{a}\right)}{\sqrt{c+\frac{d}{x}}-\sqrt{c}}\right)}{\sqrt{c+\frac{d}{x}}-\sqrt{c}}\right)\left(\sqrt{a}\,b\,c^{3/2}-a^{3/2}\,\sqrt{c}\,d\right)}{2\,a\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x)^(1/2)/(c + d/x)^(1/2),x)
```

```
[Out] (d*((a + b/x)^(1/2) - a^(1/2)))/(4*c*((c + d/x)^(1/2) - c^(1/2))) - (((a +
 b/x)^(1/2) - a^(1/2))*((b^2*c)/4 + (a*b*d)/4))/(a^(1/2)*c^(3/2)*d*((c + d/
x)^(1/2) - c^(1/2))) - b^2/(4*c*d) + (((a + b/x)^(1/2) - a^(1/2))^2*((a^2*d
^2)/4 + (b^2*c^2)/4 - (3*a*b*c*d)/4))/(a*c^2*d*((c + d/x)^(1/2) - c^(1/2))^
2))/(((a + b/x)^(1/2) - a^(1/2))^3/((c + d/x)^(1/2) - c^(1/2))^3 + (b*((a +
 b/x)^(1/2) - a^(1/2)))/(d*((c + d/x)^(1/2) - c^(1/2))) - (((a + b/x)^(1/2)
 - a^(1/2))^2*(a*d + b*c))/(a^(1/2)*c^(1/2)*d*((c + d/x)^(1/2) - c^(1/2))^2
)) + (log(((a + b/x)^(1/2) - a^(1/2))/((c + d/x)^(1/2) - c^(1/2)))*(a^(1/2)
```

```
*b*c^(3/2) - a^(3/2)*c^(1/2)*d))/(2*a*c^2) - (log(((c^(1/2)*(a + b/x)^(1/2)
 - a^(1/2)*(c + d/x)^(1/2))*(b*c^(1/2) - (a^(1/2)*d*((a + b/x)^(1/2) - a^(1
/2)))/((c + d/x)^(1/2) - c^(1/2))))/((c + d/x)^(1/2) - c^(1/2)))*(a^(1/2)*b
*c^(3/2) - a^(3/2)*c^(1/2)*d))/(2*a*c^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a + \frac{b}{x}}}{\sqrt{c + \frac{d}{x}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)**(1/2)/(c+d/x)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b/x)/sqrt(c + d/x), x)
```

3.171 $\int \frac{\sqrt{a+\frac{b}{x}}}{\left(c+\frac{d}{x}\right)^{3/2}} \, dx$

Optimal. Leaf size=122

$$\frac{(bc-3ad)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+\frac{b}{x}}}{\sqrt{a}\sqrt{c+\frac{d}{x}}}\right)}{\sqrt{a}\,c^{5/2}}-\frac{\sqrt{a+\frac{b}{x}}\,(bc-3ad)}{ac^2\sqrt{c+\frac{d}{x}}}+\frac{x\left(a+\frac{b}{x}\right)^{3/2}}{ac\sqrt{c+\frac{d}{x}}}$$

Rubi [A] time = 0.08, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {375, 96, 94, 93, 208}

$$-\frac{\sqrt{a+\frac{b}{x}}\,(bc-3ad)}{ac^2\sqrt{c+\frac{d}{x}}}+\frac{(bc-3ad)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+\frac{b}{x}}}{\sqrt{a}\sqrt{c+\frac{d}{x}}}\right)}{\sqrt{a}\,c^{5/2}}+\frac{x\left(a+\frac{b}{x}\right)^{3/2}}{ac\sqrt{c+\frac{d}{x}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b/x]/(c + d/x)^(3/2),x]
```

```
[Out] -(((b*c - 3*a*d)*Sqrt[a + b/x])/(a*c^2*Sqrt[c + d/x])) + ((a + b/x)^(3/2)*x
)/(a*c*Sqrt[c + d/x]) + ((b*c - 3*a*d)*ArcTanh[(Sqrt[c]*Sqrt[a + b/x])/(Sqr
t[a]*Sqrt[c + d/x])])/(Sqrt[a]*c^(5/2))
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[(a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*
x)^(m + 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m
, 1])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 375

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[((a + b/x^n)^p*(c + d/x^n)^q)/x^2, x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rubi steps

$$\int \frac{\sqrt{a+\frac{b}{x}}}{\left(c+\frac{d}{x}\right)^{3/2}}\,dx = -\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x^2(c+dx)^{3/2}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{\left(a+\frac{b}{x}\right)^{3/2} x}{ac\sqrt{c+\frac{d}{x}}} + \frac{\left(-\frac{bc}{2}+\frac{3ad}{2}\right)\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x(c+dx)^{3/2}}\,dx, x, \frac{1}{x}\right)}{ac}$$

$$= -\frac{(bc-3ad)\sqrt{a+\frac{b}{x}}}{ac^2\sqrt{c+\frac{d}{x}}} + \frac{\left(a+\frac{b}{x}\right)^{3/2} x}{ac\sqrt{c+\frac{d}{x}}} - \frac{(bc-3ad)\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}\sqrt{c+dx}}\,dx, x, \frac{1}{x}\right)}{2c^2}$$

$$= -\frac{(bc-3ad)\sqrt{a+\frac{b}{x}}}{ac^2\sqrt{c+\frac{d}{x}}} + \frac{\left(a+\frac{b}{x}\right)^{3/2} x}{ac\sqrt{c+\frac{d}{x}}} - \frac{(bc-3ad)\,\text{Subst}\left(\int \frac{1}{-a+cx^2}\,dx, x, \frac{\sqrt{a+\frac{b}{x}}}{\sqrt{c+\frac{d}{x}}}\right)}{c^2}$$

$$= -\frac{(bc-3ad)\sqrt{a+\frac{b}{x}}}{ac^2\sqrt{c+\frac{d}{x}}} + \frac{\left(a+\frac{b}{x}\right)^{3/2} x}{ac\sqrt{c+\frac{d}{x}}} + \frac{(bc-3ad)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+\frac{b}{x}}}{\sqrt{a}\sqrt{c+\frac{d}{x}}}\right)}{\sqrt{a}\,c^{5/2}}$$

Mathematica [A] time = 0.11, size = 87, normalized size = 0.71

$$\frac{(bc-3ad)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+\frac{b}{x}}}{\sqrt{a}\sqrt{c+\frac{d}{x}}}\right)}{\sqrt{a}\,c^{5/2}} + \frac{\sqrt{a+\frac{b}{x}}\,(cx+3d)}{c^2\sqrt{c+\frac{d}{x}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b/x]/(c + d/x)^(3/2),x]
```

```
[Out] (Sqrt[a + b/x]*(3*d + c*x))/(c^2*Sqrt[c + d/x]) + ((b*c - 3*a*d)*ArcTanh[(S
qrt[c]*Sqrt[a + b/x])/(Sqrt[a]*Sqrt[c + d/x])])/(Sqrt[a]*c^(5/2))
```

IntegrateAlgebraic [A] time = 0.43, size = 158, normalized size = 1.30

$$\frac{x\sqrt{a+\frac{b}{x}}\sqrt{c+\frac{d}{x}}\left(\frac{(bc-3ad)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{ax+b}}{\sqrt{a}\sqrt{cx+d}}\right)}{\sqrt{a}\,c^{5/2}} + \frac{\sqrt{ax+b}\left(\frac{2cd(ax+b)}{cx+d}-3ad+bc\right)}{c^2\sqrt{cx+d}\left(\frac{c(ax+b)}{cx+d}-a\right)}\right)}{\sqrt{ax+b}\sqrt{cx+d}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[a + b/x]/(c + d/x)^(3/2),x]
```

```
[Out] (Sqrt[a + b/x]*Sqrt[c + d/x]*x*((Sqrt[b + a*x]*(b*c - 3*a*d + (2*c*d*(b + a
*x))/(d + c*x)))/(c^2*Sqrt[d + c*x]*(-a + (c*(b + a*x))/(d + c*x))) + ((b*c
```

```
 - 3*a*d)*ArcTanh[(Sqrt[c]*Sqrt[b + a*x])/(Sqrt[a]*Sqrt[d + c*x])])/(Sqrt[a
]*c^(5/2))))/(Sqrt[b + a*x]*Sqrt[d + c*x])
```

fricas [A] time = 1.20, size = 319, normalized size = 2.61

$$\left[-\frac{\left(bcd - 3\,ad^2 + \left(bc^2 - 3\,acd\right)x\right)\sqrt{ac}\,\log\left(-8\,a^2c^2x^2 - b^2c^2 - 6\,abcd - a^2d^2 + 4\left(2\,acx^2 + (bc + ad)x\right)\sqrt{ac}\,\sqrt{\frac{ax+b}{x}}\,\sqrt{\frac{cx+d}{x}} - 8\left(abc^2 + a^2cd\right)x\right) - 4\left(ac^2x^2 + 3\,acdx\right)\sqrt{\frac{ax+b}{x}}\,\sqrt{\frac{cx+d}{x}}}{4\left(ac^4x + ac^3d\right)}, -\frac{\left(bcd - 3\,ad^2 + \left(bc^2 - 3\,acd\right)x\right)\sqrt{-ac}\,\arctan\left(\frac{2\,\sqrt{-ac}\,x\sqrt{\frac{ax+b}{x}}\,\sqrt{\frac{cx+d}{x}}}{2\,acx+bc+ad}\right) - 2\left(ac^2x^2 + 3\,acdx\right)\sqrt{\frac{ax+b}{x}}\,\sqrt{\frac{cx+d}{x}}}{2\left(ac^4x + ac^3d\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(1/2)/(c+d/x)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/4*((b*c*d - 3*a*d^2 + (b*c^2 - 3*a*c*d)*x)*sqrt(a*c)*log(-8*a^2*c^2*x^2
 - b^2*c^2 - 6*a*b*c*d - a^2*d^2 + 4*(2*a*c*x^2 + (b*c + a*d)*x)*sqrt(a*c)*
sqrt((a*x + b)/x)*sqrt((c*x + d)/x) - 8*(a*b*c^2 + a^2*c*d)*x) - 4*(a*c^2*x
^2 + 3*a*c*d*x)*sqrt((a*x + b)/x)*sqrt((c*x + d)/x))/(a*c^4*x + a*c^3*d), -
1/2*((b*c*d - 3*a*d^2 + (b*c^2 - 3*a*c*d)*x)*sqrt(-a*c)*arctan(2*sqrt(-a*c)
*x*sqrt((a*x + b)/x)*sqrt((c*x + d)/x)/(2*a*c*x + b*c + a*d)) - 2*(a*c^2*x^
2 + 3*a*c*d*x)*sqrt((a*x + b)/x)*sqrt((c*x + d)/x))/(a*c^4*x + a*c^3*d)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(1/2)/(c+d/x)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(x)]
Warning, integration of abs or sign assumes constant sign by intervals (cor
rect if the argument is real):Check [abs(t_nostep)]Unable to divide, perhap
s due to rounding error%%%{%%%{1,[1]%%%},[2,1,2]%%%}+%%%{%%{[-2,0]:[1,0,%%%
{-1,[1]%%%}]%%},[1,1,3]%%%}+%%%{1,[0,1,4]%%%} / %%%{%%%{1,[2]%%%},[2,0,0]%%
%}+%%%{%%{[%%%{-2,[1]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[1,0,1]%%%}+%%%{%%%{1,
[1]%%%},[0,0,2]%%%} Error: Bad Argument Value
```

maple [B] time = 0.08, size = 280, normalized size = 2.30

$$\frac{\sqrt{\frac{ax+b}{x}}\,\sqrt{\frac{cx+d}{x}}\,\left(-3acdx\ln\left(\frac{2acx+ad+bc+2\sqrt{(ax+b)(cx+d)}\,\sqrt{ac}}{2\sqrt{ac}}\right) + b\,c^2x\ln\left(\frac{2acx+ad+bc+2\sqrt{(ax+b)(cx+d)}\,\sqrt{ac}}{2\sqrt{ac}}\right) - 3a\,d^2\ln\left(\frac{2acx+ad+bc+2\sqrt{(ax+b)(cx+d)}\,\sqrt{ac}}{2\sqrt{ac}}\right) + bcd\ln\left(\frac{2acx+ad+bc+2\sqrt{(ax+b)(cx+d)}\,\sqrt{ac}}{2\sqrt{ac}}\right) + 2\sqrt{(ax+b)(cx+d)}\,\sqrt{ac}\,cx + 6\sqrt{(ax+b)(cx+d)}\,\sqrt{ac}\,d\right)x}{2\sqrt{ac}\,(cx+d)\sqrt{(ax+b)(cx+d)}\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x)^(1/2)/(c+d/x)^(3/2),x)
```

```
[Out] 1/2*((a*x+b)/x)^(1/2)*x*((c*x+d)/x)^(1/2)*(-3*ln(1/2*(2*a*c*x+a*d+b*c+2*((a
*x+b)*(c*x+d))^(1/2)*(a*c)^(1/2))/(a*c)^(1/2))*x*a*c*d+ln(1/2*(2*a*c*x+a*d+
b*c+2*((a*x+b)*(c*x+d))^(1/2)*(a*c)^(1/2))/(a*c)^(1/2))*x*b*c^2+2*x*c*((a*x
+b)*(c*x+d))^(1/2)*(a*c)^(1/2)-3*ln(1/2*(2*a*c*x+a*d+b*c+2*((a*x+b)*(c*x+d)
)^(1/2)*(a*c)^(1/2))/(a*c)^(1/2))*a*d^2+ln(1/2*(2*a*c*x+a*d+b*c+2*((a*x+b)*
(c*x+d))^(1/2)*(a*c)^(1/2))/(a*c)^(1/2))*b*c*d+6*d*((a*x+b)*(c*x+d))^(1/2)*
(a*c)^(1/2))/(a*c)^(1/2)/(c*x+d)/((a*x+b)*(c*x+d))^(1/2)/c^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a + \frac{b}{x}}}{\left(c + \frac{d}{x}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(1/2)/(c+d/x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a + b/x)/(c + d/x)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{a+\frac{b}{x}}}{\left(c+\frac{d}{x}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x)^(1/2)/(c + d/x)^(3/2),x)
```

```
[Out] int((a + b/x)^(1/2)/(c + d/x)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+\frac{b}{x}}}{\left(c+\frac{d}{x}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)**(1/2)/(c+d/x)**(3/2),x)
```

```
[Out] Integral(sqrt(a + b/x)/(c + d/x)**(3/2), x)
```

3.172 $\int \frac{a+\frac{b}{x^2}}{c+\frac{d}{x^2}}\,dx$

Optimal. Leaf size=39

$$\frac{(bc-ad)\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{d}}\right)}{c^{3/2}\sqrt{d}}+\frac{ax}{c}$$

Rubi [A] time = 0.02, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {374, 388, 205}

$$\frac{(bc-ad)\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{d}}\right)}{c^{3/2}\sqrt{d}}+\frac{ax}{c}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x^2)/(c + d/x^2),x]
```

```
[Out] (a*x)/c + ((b*c - a*d)*ArcTan[(Sqrt[c]*x)/Sqrt[d]])/(c^(3/2)*Sqrt[d])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 374

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[x^(n*(p + q))*(b + a/x^n)^p*(d + c/x^n)^q, x] /; FreeQ[{a, b, c, d
, n}, x] && NeQ[b*c - a*d, 0] && IntegersQ[p, q] && NegQ[n]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{a+\frac{b}{x^2}}{c+\frac{d}{x^2}}\,dx &= \int \frac{b+ax^2}{d+cx^2}\,dx\\ &= \frac{ax}{c}-\frac{(-bc+ad)\int \frac{1}{d+cx^2}\,dx}{c}\\ &= \frac{ax}{c}+\frac{(bc-ad)\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{d}}\right)}{c^{3/2}\sqrt{d}}\end{aligned}$$

Mathematica [A] time = 0.03, size = 40, normalized size = 1.03

$$\frac{ax}{c}-\frac{(ad-bc)\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{d}}\right)}{c^{3/2}\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x^2)/(c + d/x^2),x]
```

```
[Out] (a*x)/c - ((-(b*c) + a*d)*ArcTan[(Sqrt[c]*x)/Sqrt[d]])/(c^(3/2)*Sqrt[d])
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a+\frac{b}{x^2}}{c+\frac{d}{x^2}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b/x^2)/(c + d/x^2),x]
```

```
[Out] IntegrateAlgebraic[(a + b/x^2)/(c + d/x^2), x]
```

fricas [A] time = 0.74, size = 98, normalized size = 2.51

$$\left[\frac{2\,acdx+(bc-ad)\sqrt{-cd}\,\log\left(\frac{cx^2+2\,\sqrt{-cd}\,x-d}{cx^2+d}\right)}{2\,c^2d},\,\frac{acdx+(bc-ad)\sqrt{cd}\,\arctan\left(\frac{\sqrt{cd}\,x}{d}\right)}{c^2d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)/(c+d/x^2),x, algorithm="fricas")
```

```
[Out] [1/2*(2*a*c*d*x + (b*c - a*d)*sqrt(-c*d)*log((c*x^2 + 2*sqrt(-c*d)*x - d)/(
c*x^2 + d)))/(c^2*d), (a*c*d*x + (b*c - a*d)*sqrt(c*d)*arctan(sqrt(c*d)*x/d
))/(c^2*d)]
```

giac [A] time = 0.15, size = 33, normalized size = 0.85

$$\frac{ax}{c}+\frac{(bc-ad)\arctan\left(\frac{cx}{\sqrt{cd}}\right)}{\sqrt{cd}\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)/(c+d/x^2),x, algorithm="giac")
```

```
[Out] a*x/c + (b*c - a*d)*arctan(c*x/sqrt(c*d))/(sqrt(c*d)*c)
```

maple [A] time = 0.05, size = 45, normalized size = 1.15

$$-\frac{ad\arctan\left(\frac{cx}{\sqrt{cd}}\right)}{\sqrt{cd}\;c}+\frac{b\arctan\left(\frac{cx}{\sqrt{cd}}\right)}{\sqrt{cd}}+\frac{ax}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^2)/(c+d/x^2),x)
```

```
[Out] a*x/c-1/c/(c*d)^(1/2)*arctan(c*x/(c*d)^(1/2))*a*d+1/(c*d)^(1/2)*arctan(c*x/
(c*d)^(1/2))*b
```

maxima [A] time = 1.30, size = 33, normalized size = 0.85

$$\frac{ax}{c}+\frac{(bc-ad)\arctan\left(\frac{cx}{\sqrt{cd}}\right)}{\sqrt{cd}\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)/(c+d/x^2),x, algorithm="maxima")
```

```
[Out] a*x/c + (b*c - a*d)*arctan(c*x/sqrt(c*d))/(sqrt(c*d)*c)
```

mupad [B] time = 0.07, size = 32, normalized size = 0.82

$$\frac{a\,x}{c}-\frac{\operatorname{atan}\left(\frac{\sqrt{c}\,x}{\sqrt{d}}\right)\,\left(a\,d-b\,c\right)}{c^{3/2}\,\sqrt{d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x^2)/(c + d/x^2),x)
```

```
[Out] (a*x)/c - (atan((c^(1/2)*x)/d^(1/2))*(a*d - b*c))/(c^(3/2)*d^(1/2))
```

sympy [B] time = 0.33, size = 82, normalized size = 2.10

$$\frac{ax}{c}+\frac{\sqrt{-\frac{1}{c^3d}}\,(ad-bc)\log\left(-cd\sqrt{-\frac{1}{c^3d}}+x\right)}{2}-\frac{\sqrt{-\frac{1}{c^3d}}\,(ad-bc)\log\left(cd\sqrt{-\frac{1}{c^3d}}+x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)/(c+d/x**2),x)
```

```
[Out] a*x/c + sqrt(-1/(c**3*d))*(a*d - b*c)*log(-c*d*sqrt(-1/(c**3*d)) + x)/2 - s
qrt(-1/(c**3*d))*(a*d - b*c)*log(c*d*sqrt(-1/(c**3*d)) + x)/2
```

3.173 $\int \frac{a+\frac{b}{x^3}}{c+\frac{d}{x^3}} \, dx$

Optimal. Leaf size=145

$$-\frac{(bc-ad)\log\left(c^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}\right)}{6c^{4/3}d^{2/3}}+\frac{(bc-ad)\log\left(\sqrt[3]{c}x+\sqrt[3]{d}\right)}{3c^{4/3}d^{2/3}}-\frac{(bc-ad)\tan^{-1}\left(\frac{\sqrt[3]{d}-2\sqrt[3]{c}x}{\sqrt{3}\sqrt[3]{d}}\right)}{\sqrt{3}c^{4/3}d^{2/3}}+\frac{ax}{c}$$

Rubi [A] time = 0.11, antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.471, Rules used = {374, 388, 200, 31, 634, 617, 204, 628}

$$-\frac{(bc-ad)\log\left(c^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}\right)}{6c^{4/3}d^{2/3}}+\frac{(bc-ad)\log\left(\sqrt[3]{c}x+\sqrt[3]{d}\right)}{3c^{4/3}d^{2/3}}-\frac{(bc-ad)\tan^{-1}\left(\frac{\sqrt[3]{d}-2\sqrt[3]{c}x}{\sqrt{3}\sqrt[3]{d}}\right)}{\sqrt{3}c^{4/3}d^{2/3}}+\frac{ax}{c}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x^3)/(c + d/x^3),x]
```

```
[Out] (a*x)/c - ((b*c - a*d)*ArcTan[(d^(1/3) - 2*c^(1/3)*x)/(Sqrt[3]*d^(1/3))])/(
Sqrt[3]*c^(4/3)*d^(2/3)) + ((b*c - a*d)*Log[d^(1/3) + c^(1/3)*x])/(3*c^(4/3
)*d^(2/3)) - ((b*c - a*d)*Log[d^(2/3) - c^(1/3)*d^(1/3)*x + c^(2/3)*x^2])/(
6*c^(4/3)*d^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 374

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[x^(n*(p + q))*(b + a/x^n)^p*(d + c/x^n)^q, x] /; FreeQ[{a, b, c, d
, n}, x] && NeQ[b*c - a*d, 0] && IntegersQ[p, q] && NegQ[n]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{a+\frac{b}{x^3}}{c+\frac{d}{x^3}}\,dx = \int \frac{b+ax^3}{d+cx^3}\,dx$$

$$= \frac{ax}{c} - \frac{(-bc+ad)\int \frac{1}{d+cx^3}\,dx}{c}$$

$$= \frac{ax}{c} + \frac{(bc-ad)\int \frac{1}{\sqrt[3]{d}+\sqrt[3]{c}x}\,dx}{3cd^{2/3}} + \frac{(bc-ad)\int \frac{2\sqrt[3]{d}-\sqrt[3]{c}x}{d^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}x^2}\,dx}{3cd^{2/3}}$$

$$= \frac{ax}{c} + \frac{(bc-ad)\log\left(\sqrt[3]{d}+\sqrt[3]{c}x\right)}{3c^{4/3}d^{2/3}} - \frac{(bc-ad)\int \frac{-\sqrt[3]{c}\sqrt[3]{d}+2c^{2/3}x}{d^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}x^2}\,dx}{6c^{4/3}d^{2/3}} + \frac{(bc-ad)\int \frac{1}{d^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+c^2 \ldots}}{2c\sqrt[3]{d}}$$

$$= \frac{ax}{c} + \frac{(bc-ad)\log\left(\sqrt[3]{d}+\sqrt[3]{c}x\right)}{3c^{4/3}d^{2/3}} - \frac{(bc-ad)\log\left(d^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}x^2\right)}{6c^{4/3}d^{2/3}} + \frac{(bc-ad)\,\mathrm{Subst}\Big(\ldots}{c \ldots}$$

$$= \frac{ax}{c} - \frac{(bc-ad)\tan^{-1}\left(\frac{\sqrt[3]{d}-2\sqrt[3]{c}x}{\sqrt{3}\sqrt[3]{d}}\right)}{\sqrt{3}\,c^{4/3}d^{2/3}} + \frac{(bc-ad)\log\left(\sqrt[3]{d}+\sqrt[3]{c}x\right)}{3c^{4/3}d^{2/3}} - \frac{(bc-ad)\log\left(d^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x \ldots\right.}{6c^{4/3}d^{2/3}}$$

Mathematica [A] time = 0.10, size = 129, normalized size = 0.89

$$\frac{-(bc-ad)\log\left(c^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}\right)+2(bc-ad)\log\left(\sqrt[3]{c}x+\sqrt[3]{d}\right)-2\sqrt{3}(bc-ad)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{c}x}{\sqrt[3]{d}}}{\sqrt{3}}\right)+6a\sqrt[3]{c}d^{2/3}x}{6c^{4/3}d^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x^3)/(c + d/x^3),x]
```

```
[Out] (6*a*c^(1/3)*d^(2/3)*x - 2*Sqrt[3]*(b*c - a*d)*ArcTan[(1 - (2*c^(1/3)*x)/d^
(1/3))/Sqrt[3]] + 2*(b*c - a*d)*Log[d^(1/3) + c^(1/3)*x] - (b*c - a*d)*Log[
d^(2/3) - c^(1/3)*d^(1/3)*x + c^(2/3)*x^2])/(6*c^(4/3)*d^(2/3))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a+\frac{b}{x^3}}{c+\frac{d}{x^3}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b/x^3)/(c + d/x^3),x]
```

```
[Out] IntegrateAlgebraic[(a + b/x^3)/(c + d/x^3), x]
```

fricas [A] time = 0.92, size = 390, normalized size = 2.69

$$\left[\frac{6\,acd^2x-3\,\sqrt{\frac{1}{3}}\left(bc^2d-acd^2\right)\sqrt{\frac{\left(-cd^2\right)^{\frac{1}{3}}}{c}}\,\log\left(\frac{2\,cdx^3+3\left(-cd^2\right)^{\frac{1}{3}}dx-d^2-3\,\sqrt{\frac{1}{3}}\left(2\,cdx^2+\left(-cd^2\right)^{\frac{2}{3}}x+\left(-cd^2\right)^{\frac{1}{3}}d\right)\sqrt{\frac{\left(-cd^2\right)^{\frac{1}{3}}}{c}}}{cx^3+d}\right)-\left(-cd^2\right)^{\frac{2}{3}}(bc-ad)\log\left(cdx^2-\left(-cd^2\right)^{\frac{2}{3}}x-\left(-cd^2\right)^{\frac{1}{3}}d\right)+2\left(-cd^2\right)^{\frac{2}{3}}(bc-ad)\log\left(cdx+\left(-cd^2\right)^{\frac{2}{3}}\right)}{6\,c^2d^2},\ \frac{6\,acd^2x+6\,\sqrt{\frac{1}{3}}\left(bc^2d-acd^2\right)\sqrt{-\frac{\left(-cd^2\right)^{\frac{1}{3}}}{c}}\,\arctan\left(\frac{\sqrt{\frac{1}{3}}\left(2\left(-cd^2\right)^{\frac{2}{3}}x+\left(-cd^2\right)^{\frac{1}{3}}d\right)\sqrt{-\frac{\left(-cd^2\right)^{\frac{1}{3}}}{c}}}{d^2}\right)-\left(-cd^2\right)^{\frac{2}{3}}(bc-ad)\log\left(cdx^2-\left(-cd^2\right)^{\frac{2}{3}}x-\left(-cd^2\right)^{\frac{1}{3}}d\right)+2\left(-cd^2\right)^{\frac{2}{3}}(bc-ad)\log\left(cdx+\left(-cd^2\right)^{\frac{2}{3}}\right)}{6\,c^2d^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^3)/(c+d/x^3),x, algorithm="fricas")
```

```
[Out] [1/6*(6*a*c*d^2*x - 3*sqrt(1/3)*(b*c^2*d - a*c*d^2)*sqrt((-c*d^2)^(1/3)/c)*
log((2*c*d*x^3 + 3*(-c*d^2)^(1/3)*d*x - d^2 - 3*sqrt(1/3)*(2*c*d*x^2 + (-c*
d^2)^(2/3)*x + (-c*d^2)^(1/3)*d)*sqrt((-c*d^2)^(1/3)/c))/(c*x^3 + d)) - (-c
*d^2)^(2/3)*(b*c - a*d)*log(c*d*x^2 - (-c*d^2)^(2/3)*x - (-c*d^2)^(1/3)*d)
+ 2*(-c*d^2)^(2/3)*(b*c - a*d)*log(c*d*x + (-c*d^2)^(2/3)))/(c^2*d^2), 1/6*
(6*a*c*d^2*x + 6*sqrt(1/3)*(b*c^2*d - a*c*d^2)*sqrt(-(-c*d^2)^(1/3)/c)*arct
an(sqrt(1/3)*(2*(-c*d^2)^(2/3)*x + (-c*d^2)^(1/3)*d)*sqrt(-(-c*d^2)^(1/3)/c
)/d^2) - (-c*d^2)^(2/3)*(b*c - a*d)*log(c*d*x^2 - (-c*d^2)^(2/3)*x - (-c*d^
2)^(1/3)*d) + 2*(-c*d^2)^(2/3)*(b*c - a*d)*log(c*d*x + (-c*d^2)^(2/3)))/(c^
2*d^2)]
```

giac [A] time = 0.22, size = 133, normalized size = 0.92

$$-\frac{\sqrt{3}\,(bc-ad)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{d}{c}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{d}{c}\right)^{\frac{1}{3}}}\right)}{3\left(-c^2d\right)^{\frac{2}{3}}}-\frac{(bc-ad)\log\left(x^2+x\left(-\frac{d}{c}\right)^{\frac{1}{3}}+\left(-\frac{d}{c}\right)^{\frac{2}{3}}\right)}{6\left(-c^2d\right)^{\frac{2}{3}}}+\frac{ax}{c}-\frac{(bc-ad)\left(-\frac{d}{c}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{d}{c}\right)^{\frac{1}{3}}\right|\right)}{3\,cd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^3)/(c+d/x^3),x, algorithm="giac")
```

```
[Out] -1/3*sqrt(3)*(b*c - a*d)*arctan(1/3*sqrt(3)*(2*x + (-d/c)^(1/3))/(-d/c)^(1/
3))/(-c^2*d)^(2/3) - 1/6*(b*c - a*d)*log(x^2 + x*(-d/c)^(1/3) + (-d/c)^(2/3
))/(-c^2*d)^(2/3) + a*x/c - 1/3*(b*c - a*d)*(-d/c)^(1/3)*log(abs(x - (-d/c)
^(1/3)))/(c*d)
```

maple [A] time = 0.05, size = 195, normalized size = 1.34

$$\frac{ax}{c}-\frac{\sqrt{3}\,ad\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{d}{c}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3\left(\frac{d}{c}\right)^{\frac{2}{3}}c^2}-\frac{ad\ln\left(x+\left(\frac{d}{c}\right)^{\frac{1}{3}}\right)}{3\left(\frac{d}{c}\right)^{\frac{2}{3}}c^2}+\frac{ad\ln\left(x^2-\left(\frac{d}{c}\right)^{\frac{1}{3}}x+\left(\frac{d}{c}\right)^{\frac{2}{3}}\right)}{6\left(\frac{d}{c}\right)^{\frac{2}{3}}c^2}+\frac{\sqrt{3}\,b\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{d}{c}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3\left(\frac{d}{c}\right)^{\frac{2}{3}}c}+\frac{b\ln\left(x+\left(\frac{d}{c}\right)^{\frac{1}{3}}\right)}{3\left(\frac{d}{c}\right)^{\frac{2}{3}}c}-\frac{b\ln\left(x^2-\left(\frac{d}{c}\right)^{\frac{1}{3}}x+\left(\frac{d}{c}\right)^{\frac{2}{3}}\right)}{6\left(\frac{d}{c}\right)^{\frac{2}{3}}c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^3)/(c+d/x^3),x)
```

```
[Out] a/c*x-1/3/c^2/(1/c*d)^(2/3)*ln(x+(1/c*d)^(1/3))*a*d+1/3/c/(1/c*d)^(2/3)*ln(
x+(1/c*d)^(1/3))*b+1/6/c^2/(1/c*d)^(2/3)*ln(x^2-(1/c*d)^(1/3)*x+(1/c*d)^(2/
3))*a*d-1/6/c/(1/c*d)^(2/3)*ln(x^2-(1/c*d)^(1/3)*x+(1/c*d)^(2/3))*b-1/3/c^2
/(1/c*d)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/c*d)^(1/3)*x-1))*a*d+1/3/c/
(1/c*d)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/c*d)^(1/3)*x-1))*b
```

maxima [A] time = 1.29, size = 128, normalized size = 0.88

$$\frac{ax}{c}+\frac{\sqrt{3}\,(bc-ad)\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{d}{c}\right)^{\frac{1}{3}}\right)}{3\left(\frac{d}{c}\right)^{\frac{1}{3}}}\right)}{3\,c^2\left(\frac{d}{c}\right)^{\frac{2}{3}}}-\frac{(bc-ad)\log\left(x^2-x\left(\frac{d}{c}\right)^{\frac{1}{3}}+\left(\frac{d}{c}\right)^{\frac{2}{3}}\right)}{6\,c^2\left(\frac{d}{c}\right)^{\frac{2}{3}}}+\frac{(bc-ad)\log\left(x+\left(\frac{d}{c}\right)^{\frac{1}{3}}\right)}{3\,c^2\left(\frac{d}{c}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^3)/(c+d/x^3),x, algorithm="maxima")
```

```
[Out] a*x/c + 1/3*sqrt(3)*(b*c - a*d)*arctan(1/3*sqrt(3)*(2*x - (d/c)^(1/3))/(d/c
)^(1/3))/(c^2*(d/c)^(2/3)) - 1/6*(b*c - a*d)*log(x^2 - x*(d/c)^(1/3) + (d/c
)^(2/3))/(c^2*(d/c)^(2/3)) + 1/3*(b*c - a*d)*log(x + (d/c)^(1/3))/(c^2*(d/c
)^(2/3))
```

mupad [B] time = 0.27, size = 123, normalized size = 0.85

$$\frac{a\,x}{c}-\frac{\ln\left(c^{1/3}\,x+d^{1/3}\right)\,\left(a\,d-b\,c\right)}{3\,c^{4/3}\,d^{2/3}}+\frac{\ln\left(d^{1/3}-2\,c^{1/3}\,x+\sqrt{3}\,d^{1/3}\,1\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(a\,d-b\,c\right)}{3\,c^{4/3}\,d^{2/3}}-\frac{\ln\left(2\,c^{1/3}\,x-d^{1/3}+\sqrt{3}\,d^{1/3}\,1\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(a\,d-b\,c\right)}{3\,c^{4/3}\,d^{2/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x^3)/(c + d/x^3),x)
```

```
[Out] (a*x)/c - (log(c^(1/3)*x + d^(1/3))*(a*d - b*c))/(3*c^(4/3)*d^(2/3)) + (log
(3^(1/2)*d^(1/3)*1i - 2*c^(1/3)*x + d^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(a*d -
b*c))/(3*c^(4/3)*d^(2/3)) - (log(3^(1/2)*d^(1/3)*1i + 2*c^(1/3)*x - d^(1/3)
)*((3^(1/2)*1i)/2 - 1/2)*(a*d - b*c))/(3*c^(4/3)*d^(2/3))
```

sympy [A] time = 0.45, size = 71, normalized size = 0.49

$$\frac{ax}{c}+\operatorname{RootSum}\left(27t^3c^4d^2+a^3d^3-3a^2bcd^2+3ab^2c^2d-b^3c^3,\left(t\mapsto t\log\left(-\frac{3tcd}{ad-bc}+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**3)/(c+d/x**3),x)
```

```
[Out] a*x/c + RootSum(27*_t**3*c**4*d**2 + a**3*d**3 - 3*a**2*b*c*d**2 + 3*a*b**2
*c**2*d - b**3*c**3, Lambda(_t, _t*log(-3*_t*c*d/(a*d - b*c) + x)))
```

3.174 $\int \frac{a+b\sqrt{x}}{c+d\sqrt{x}}\,dx$

Optimal. Leaf size=49

$$\frac{2c(bc-ad)\log\left(c+d\sqrt{x}\right)}{d^3}-\frac{2\sqrt{x}\,(bc-ad)}{d^2}+\frac{bx}{d}$$

Rubi [A] time = 0.05, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {376, 77}

$$-\frac{2\sqrt{x}\,(bc-ad)}{d^2}+\frac{2c(bc-ad)\log\left(c+d\sqrt{x}\right)}{d^3}+\frac{bx}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sqrt[x])/(c + d*Sqrt[x]),x]
```

```
[Out] (-2*(b*c - a*d)*Sqrt[x])/d^2 + (b*x)/d + (2*c*(b*c - a*d)*Log[c + d*Sqrt[x]
])/d^3
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 376

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(a + b*x^(g*n))
^p*(c + d*x^(g*n))^q, x], x, x^(1/g)], x]] /; FreeQ[{a, b, c, d, p, q}, x]
&& NeQ[b*c - a*d, 0] && FractionQ[n]
```

Rubi steps

$$\begin{aligned}\int \frac{a+b\sqrt{x}}{c+d\sqrt{x}}\,dx &= 2\,\text{Subst}\left(\int \frac{x(a+bx)}{c+dx}\,dx,x,\sqrt{x}\right)\\ &= 2\,\text{Subst}\left(\int \left(\frac{-bc+ad}{d^2}+\frac{bx}{d}+\frac{c(bc-ad)}{d^2(c+dx)}\right)dx,x,\sqrt{x}\right)\\ &= -\frac{2(bc-ad)\sqrt{x}}{d^2}+\frac{bx}{d}+\frac{2c(bc-ad)\log\left(c+d\sqrt{x}\right)}{d^3}\end{aligned}$$

Mathematica [A] time = 0.07, size = 41, normalized size = 0.84

$$\frac{2(ad-bc)\left(d\sqrt{x}-c\log\left(c+d\sqrt{x}\right)\right)}{d^3}+\frac{bx}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sqrt[x])/(c + d*Sqrt[x]),x]
```

```
[Out] (b*x)/d + (2*(-(b*c) + a*d)*(d*Sqrt[x] - c*Log[c + d*Sqrt[x]]))/d^3
```

IntegrateAlgebraic [A] time = 0.03, size = 53, normalized size = 1.08

$$\frac{2\left(bc^2-acd\right)\log\left(c+d\sqrt{x}\right)}{d^3}+\frac{\sqrt{x}\left(2ad-2bc+bd\sqrt{x}\right)}{d^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*Sqrt[x])/(c + d*Sqrt[x]),x]
```

```
[Out] ((-2*b*c + 2*a*d + b*d*Sqrt[x])*Sqrt[x])/d^2 + (2*(b*c^2 - a*c*d)*Log[c + d
*Sqrt[x]])/d^3
```

fricas [A] time = 0.82, size = 48, normalized size = 0.98

$$\frac{bd^2x+2\left(bc^2-acd\right)\log\left(d\sqrt{x}+c\right)-2\left(bcd-ad^2\right)\sqrt{x}}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/2))/(c+d*x^(1/2)),x, algorithm="fricas")
```

```
[Out] (b*d^2*x + 2*(b*c^2 - a*c*d)*log(d*sqrt(x) + c) - 2*(b*c*d - a*d^2)*sqrt(x)
)/d^3
```

giac [A] time = 0.19, size = 49, normalized size = 1.00

$$\frac{bdx-2\,bc\sqrt{x}+2\,ad\sqrt{x}}{d^2}+\frac{2\left(bc^2-acd\right)\log\left(\left|d\sqrt{x}+c\right|\right)}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/2))/(c+d*x^(1/2)),x, algorithm="giac")
```

```
[Out] (b*d*x - 2*b*c*sqrt(x) + 2*a*d*sqrt(x))/d^2 + 2*(b*c^2 - a*c*d)*log(abs(d*s
qrt(x) + c))/d^3
```

maple [A] time = 0.05, size = 59, normalized size = 1.20

$$-\frac{2ac\ln\left(d\sqrt{x}+c\right)}{d^2}+\frac{2b\,c^2\ln\left(d\sqrt{x}+c\right)}{d^3}+\frac{bx}{d}+\frac{2a\sqrt{x}}{d}-\frac{2bc\sqrt{x}}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^(1/2)+a)/(c+d*x^(1/2)),x)
```

```
[Out] b/d*x+2/d*a*x^(1/2)-2/d^2*b*c*x^(1/2)-2*c/d^2*ln(c+d*x^(1/2))*a+2*c^2/d^3*l
n(c+d*x^(1/2))*b
```

maxima [A] time = 0.64, size = 47, normalized size = 0.96

$$\frac{bdx-2\,(bc-ad)\sqrt{x}}{d^2}+\frac{2\left(bc^2-acd\right)\log\left(d\sqrt{x}+c\right)}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/2))/(c+d*x^(1/2)),x, algorithm="maxima")
```

```
[Out] (b*d*x - 2*(b*c - a*d)*sqrt(x))/d^2 + 2*(b*c^2 - a*c*d)*log(d*sqrt(x) + c)/
d^3
```

mupad [B] time = 0.07, size = 49, normalized size = 1.00

$$\sqrt{x}\,\left(\frac{2\,a}{d}-\frac{2\,b\,c}{d^2}\right)+\frac{\ln\left(c+d\,\sqrt{x}\right)\,\left(2\,b\,c^2-2\,a\,c\,d\right)}{d^3}+\frac{b\,x}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^(1/2))/(c + d*x^(1/2)),x)
```

```
[Out] x^(1/2)*((2*a)/d - (2*b*c)/d^2) + (log(c + d*x^(1/2))*(2*b*c^2 - 2*a*c*d))/
d^3 + (b*x)/d
```

sympy [A] time = 0.30, size = 82, normalized size = 1.67

$$\begin{cases} -\frac{2ac\log\left(\frac{c}{d}+\sqrt{x}\right)}{d^2} + \frac{2a\sqrt{x}}{d} + \frac{2bc^2\log\left(\frac{c}{d}+\sqrt{x}\right)}{d^3} - \frac{2bc\sqrt{x}}{d^2} + \frac{bx}{d} & \text{for } d \neq 0 \\ \frac{ax+\frac{2bx^{\frac{3}{2}}}{3}}{c} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**(1/2))/(c+d*x**(1/2)),x)
```

```
[Out] Piecewise((-2*a*c*log(c/d + sqrt(x))/d**2 + 2*a*sqrt(x)/d + 2*b*c**2*log(c/
d + sqrt(x))/d**3 - 2*b*c*sqrt(x)/d**2 + b*x/d, Ne(d, 0)), ((a*x + 2*b*x**(
3/2)/3)/c, True))
```

3.175 $\int \frac{-1+\sqrt[3]{x}}{1+\sqrt[3]{x}}\,dx$

Optimal. Leaf size=26

$$-3x^{2/3}+x+6\sqrt[3]{x}-6\log\left(\sqrt[3]{x}+1\right)$$

Rubi [A] time = 0.02, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {376, 77}

$$-3x^{2/3}+x+6\sqrt[3]{x}-6\log\left(\sqrt[3]{x}+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + x^(1/3))/(1 + x^(1/3)),x]
```

```
[Out] 6*x^(1/3) - 3*x^(2/3) + x - 6*Log[1 + x^(1/3)]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 376

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(a + b*x^(g*n))
^p*(c + d*x^(g*n))^q, x], x, x^(1/g)], x]] /; FreeQ[{a, b, c, d, p, q}, x]
&& NeQ[b*c - a*d, 0] && FractionQ[n]
```

Rubi steps

$$\begin{aligned}\int \frac{-1+\sqrt[3]{x}}{1+\sqrt[3]{x}}\,dx &= 3\,\text{Subst}\left(\int \frac{(-1+x)x^2}{1+x}\,dx,x,\sqrt[3]{x}\right)\\ &= 3\,\text{Subst}\left(\int \left(2-2x+x^2-\frac{2}{1+x}\right)dx,x,\sqrt[3]{x}\right)\\ &= 6\sqrt[3]{x}-3x^{2/3}+x-6\log\left(1+\sqrt[3]{x}\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 26, normalized size = 1.00

$$-3x^{2/3}+x+6\sqrt[3]{x}-6\log\left(\sqrt[3]{x}+1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + x^(1/3))/(1 + x^(1/3)),x]
```

```
[Out] 6*x^(1/3) - 3*x^(2/3) + x - 6*Log[1 + x^(1/3)]
```

IntegrateAlgebraic [A] time = 0.01, size = 31, normalized size = 1.19

$$\left(x^{2/3}-3\sqrt[3]{x}+6\right)\sqrt[3]{x}-6\log\left(\sqrt[3]{x}+1\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-1 + x^(1/3))/(1 + x^(1/3)),x]
```

```
[Out] (6 - 3*x^(1/3) + x^(2/3))*x^(1/3) - 6*Log[1 + x^(1/3)]
```

fricas [A] time = 0.81, size = 20, normalized size = 0.77

$$x - 3 x^{\frac{2}{3}} + 6 x^{\frac{1}{3}} - 6 \log\left(x^{\frac{1}{3}} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x^(1/3))/(1+x^(1/3)),x, algorithm="fricas")
```

```
[Out] x - 3*x^(2/3) + 6*x^(1/3) - 6*log(x^(1/3) + 1)
```

giac [A] time = 0.20, size = 20, normalized size = 0.77

$$x - 3 x^{\frac{2}{3}} + 6 x^{\frac{1}{3}} - 6 \log\left(x^{\frac{1}{3}} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x^(1/3))/(1+x^(1/3)),x, algorithm="giac")
```

```
[Out] x - 3*x^(2/3) + 6*x^(1/3) - 6*log(x^(1/3) + 1)
```

maple [A] time = 0.04, size = 21, normalized size = 0.81

$$x - 6 \ln\left(x^{\frac{1}{3}} + 1\right) - 3 x^{\frac{2}{3}} + 6 x^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(1/3)-1)/(x^(1/3)+1),x)
```

```
[Out] 6*x^(1/3)-3*x^(2/3)+x-6*ln(x^(1/3)+1)
```

maxima [A] time = 0.55, size = 20, normalized size = 0.77

$$x - 3 x^{\frac{2}{3}} + 6 x^{\frac{1}{3}} - 6 \log\left(x^{\frac{1}{3}} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x^(1/3))/(1+x^(1/3)),x, algorithm="maxima")
```

```
[Out] x - 3*x^(2/3) + 6*x^(1/3) - 6*log(x^(1/3) + 1)
```

mupad [B] time = 0.03, size = 20, normalized size = 0.77

$$x - 6 \ln\left(x^{1/3} + 1\right) + 6 x^{1/3} - 3 x^{2/3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(1/3) - 1)/(x^(1/3) + 1),x)
```

```
[Out] x - 6*log(x^(1/3) + 1) + 6*x^(1/3) - 3*x^(2/3)
```

sympy [A] time = 0.20, size = 24, normalized size = 0.92

$$-3 x^{\frac{2}{3}} + 6 \sqrt[3]{x} + x - 6 \log\left(\sqrt[3]{x} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x**(1/3))/(1+x**(1/3)),x)
```

```
[Out] -3*x**(2/3) + 6*x**(1/3) + x - 6*log(x**(1/3) + 1)
```

3.176 $\int \frac{1+x^{2/3}}{-1+x^{2/3}} \, dx$

Optimal. Leaf size=17

$$x+6\sqrt[3]{x}-6\tanh^{-1}\left(\sqrt[3]{x}\right)$$

Rubi [A] time = 0.01, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {376, 459, 321, 207}

$$x+6\sqrt[3]{x}-6\tanh^{-1}\left(\sqrt[3]{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^(2/3))/(-1 + x^(2/3)),x]
```

```
[Out] 6*x^(1/3) + x - 6*ArcTanh[x^(1/3)]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 376

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(a + b*x^(g*n))
^p*(c + d*x^(g*n))^q, x], x, x^(1/g)], x]] /; FreeQ[{a, b, c, d, p, q}, x]
&& NeQ[b*c - a*d, 0] && FractionQ[n]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1+x^{2/3}}{-1+x^{2/3}} \, dx &= 3\,\text{Subst}\left(\int \frac{x^2\left(1+x^2\right)}{-1+x^2} \, dx, x, \sqrt[3]{x}\right)\\
&= x+6\,\text{Subst}\left(\int \frac{x^2}{-1+x^2} \, dx, x, \sqrt[3]{x}\right)\\
&= 6\sqrt[3]{x}+x+6\,\text{Subst}\left(\int \frac{1}{-1+x^2} \, dx, x, \sqrt[3]{x}\right)\\
&= 6\sqrt[3]{x}+x-6\tanh^{-1}\left(\sqrt[3]{x}\right)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 17, normalized size = 1.00

$$x+6\sqrt[3]{x}-6\tanh^{-1}\left(\sqrt[3]{x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^(2/3))/(-1 + x^(2/3)),x]
[Out] 6*x^(1/3) + x - 6*ArcTanh[x^(1/3)]
```

IntegrateAlgebraic [A] time = 0.01, size = 17, normalized size = 1.00

$$x+6\sqrt[3]{x}-6\tanh^{-1}\left(\sqrt[3]{x}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 + x^(2/3))/(-1 + x^(2/3)),x]
[Out] 6*x^(1/3) + x - 6*ArcTanh[x^(1/3)]
```

fricas [A] time = 0.62, size = 23, normalized size = 1.35

$$x+6\,x^{\frac{1}{3}}-3\,\log\left(x^{\frac{1}{3}}+1\right)+3\,\log\left(x^{\frac{1}{3}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x^(2/3))/(-1+x^(2/3)),x, algorithm="fricas")
[Out] x + 6*x^(1/3) - 3*log(x^(1/3) + 1) + 3*log(x^(1/3) - 1)
```

giac [A] time = 0.18, size = 24, normalized size = 1.41

$$x+6\,x^{\frac{1}{3}}-3\,\log\left(x^{\frac{1}{3}}+1\right)+3\,\log\left(\left|x^{\frac{1}{3}}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x^(2/3))/(-1+x^(2/3)),x, algorithm="giac")
[Out] x + 6*x^(1/3) - 3*log(x^(1/3) + 1) + 3*log(abs(x^(1/3) - 1))
```

maple [A] time = 0.05, size = 24, normalized size = 1.41

$$x-3\ln\left(x^{\frac{1}{3}}+1\right)+3\ln\left(x^{\frac{1}{3}}-1\right)+6x^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(2/3)+1)/(x^(2/3)-1),x)
[Out] x+6*x^(1/3)+3*ln(x^(1/3)-1)-3*ln(x^(1/3)+1)
```

maxima [A] time = 0.56, size = 23, normalized size = 1.35

$$x+6\,x^{\frac{1}{3}}-3\,\log\left(x^{\frac{1}{3}}+1\right)+3\,\log\left(x^{\frac{1}{3}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x^(2/3))/(-1+x^(2/3)),x, algorithm="maxima")
[Out] x + 6*x^(1/3) - 3*log(x^(1/3) + 1) + 3*log(x^(1/3) - 1)
```

mupad [B] time = 1.46, size = 13, normalized size = 0.76

$$x - 6\,\mathrm{atanh}\left(x^{1/3}\right) + 6\,x^{1/3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(2/3) + 1)/(x^(2/3) - 1),x)
[Out] x - 6*atanh(x^(1/3)) + 6*x^(1/3)
```

sympy [A] time = 0.27, size = 27, normalized size = 1.59

$$6\sqrt[3]{x} + x + 3\log\left(\sqrt[3]{x} - 1\right) - 3\log\left(\sqrt[3]{x} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x**(2/3))/(-1+x**(2/3)),x)
[Out] 6*x**(1/3) + x + 3*log(x**(1/3) - 1) - 3*log(x**(1/3) + 1)
```

3.177 $\int \frac{-16+x^{3/4}}{16+x^{3/4}} \, dx$

Optimal. Leaf size=104

$$x-128\sqrt[4]{x}+\frac{256}{3}\sqrt[3]{2}\log\left(\sqrt[4]{x}+2\sqrt[3]{2}\right)-\frac{128}{3}\sqrt[3]{2}\log\left(\sqrt{x}-2\sqrt[3]{2}\sqrt[4]{x}+4\,2^{2/3}\right)-\frac{256\sqrt[3]{2}\tan^{-1}\left(\frac{\sqrt[3]{2}-\sqrt[4]{x}}{\sqrt[3]{2}\sqrt{3}}\right)}{\sqrt{3}}$$

Rubi [A] time = 0.09, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.529, Rules used = {376, 459, 321, 200, 31, 634, 617, 204, 628}

$$x-128\sqrt[4]{x}+\frac{256}{3}\sqrt[3]{2}\log\left(\sqrt[4]{x}+2\sqrt[3]{2}\right)-\frac{128}{3}\sqrt[3]{2}\log\left(\sqrt{x}-2\sqrt[3]{2}\sqrt[4]{x}+4\,2^{2/3}\right)-\frac{256\sqrt[3]{2}\tan^{-1}\left(\frac{\sqrt[3]{2}-\sqrt[4]{x}}{\sqrt[3]{2}\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(-16 + x^(3/4))/(16 + x^(3/4)),x]
```

```
[Out] -128*x^(1/4) + x - (256*2^(1/3)*ArcTan[(2^(1/3) - x^(1/4))/(2^(1/3)*Sqrt[3]
)])/Sqrt[3] + (256*2^(1/3)*Log[2*2^(1/3) + x^(1/4)])/3 - (128*2^(1/3)*Log[4
*2^(2/3) - 2*2^(1/3)*x^(1/4) + Sqrt[x]])/3
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 376

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(a + b*x^(g*n))
^p*(c + d*x^(g*n))^q, x], x, x^(1/g)], x]] /; FreeQ[{a, b, c, d, p, q}, x]
&& NeQ[b*c - a*d, 0] && FractionQ[n]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}
\int \frac{-16+x^{3/4}}{16+x^{3/4}}\,dx &= 4\,\text{Subst}\left(\int \frac{x^3\left(-16+x^3\right)}{16+x^3}\,dx,x,\sqrt[4]{x}\right)\\
&= x-128\,\text{Subst}\left(\int \frac{x^3}{16+x^3}\,dx,x,\sqrt[4]{x}\right)\\
&= -128\sqrt[4]{x}+x+2048\,\text{Subst}\left(\int \frac{1}{16+x^3}\,dx,x,\sqrt[4]{x}\right)\\
&= -128\sqrt[4]{x}+x+\frac{1}{3}\left(256\sqrt[3]{2}\right)\text{Subst}\left(\int \frac{1}{2\sqrt[3]{2}+x}\,dx,x,\sqrt[4]{x}\right)+\frac{1}{3}\left(256\sqrt[3]{2}\right)\text{Subst}\left(\int \frac{4}{4\,2^{2/3}\right.\\
&= -128\sqrt[4]{x}+x+\frac{256}{3}\sqrt[3]{2}\,\log\left(2\sqrt[3]{2}+\sqrt[4]{x}\right)-\frac{1}{3}\left(128\sqrt[3]{2}\right)\text{Subst}\left(\int \frac{-2\sqrt[3]{2}+2x}{4\,2^{2/3}-2\sqrt[3]{2}\,x+x^2}\,dx,\right.\\
&= -128\sqrt[4]{x}+x+\frac{256}{3}\sqrt[3]{2}\,\log\left(2\sqrt[3]{2}+\sqrt[4]{x}\right)-\frac{128}{3}\sqrt[3]{2}\,\log\left(4\,2^{2/3}-2\sqrt[3]{2}\sqrt[4]{x}+\sqrt{x}\right)+\left(256\sqrt[3]{\right.\\
&= -128\sqrt[4]{x}+x-\frac{256\sqrt[3]{2}\,\tan^{-1}\left(\frac{2-2^{2/3}\sqrt[4]{x}}{2\sqrt{3}}\right)}{\sqrt{3}}+\frac{256}{3}\sqrt[3]{2}\,\log\left(2\sqrt[3]{2}+\sqrt[4]{x}\right)-\frac{128}{3}\sqrt[3]{2}\,\log\left(4\,2^{2/3}\right.
\end{aligned}$$

Mathematica [C] time = 0.01, size = 22, normalized size = 0.21

$$x-2x\,{}_2F_1\left(1,\frac{4}{3};\frac{7}{3};-\frac{x^{3/4}}{16}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-16 + x^(3/4))/(16 + x^(3/4)),x]
```

```
[Out] x - 2*x*Hypergeometric2F1[1, 4/3, 7/3, -1/16*x^(3/4)]
```

IntegrateAlgebraic [A] time = 0.13, size = 104, normalized size = 1.00

$$x-128\sqrt[4]{x}+\frac{256}{3}\sqrt[3]{2}\log\left(2^{2/3}\sqrt[4]{x}+4\right)-\frac{128}{3}\sqrt[3]{2}\log\left(-\sqrt[3]{2}\sqrt{x}+2\,2^{2/3}\sqrt[4]{x}-8\right)-\frac{256\sqrt[3]{2}\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{\sqrt[4]{x}}{\sqrt[3]{2}\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-16 + x^(3/4))/(16 + x^(3/4)),x]
```

```
[Out] -128*x^(1/4) + x - (256*2^(1/3)*ArcTan[1/Sqrt[3] - x^(1/4)/(2^(1/3)*Sqrt[3]
)])/Sqrt[3] + (256*2^(1/3)*Log[4 + 2^(2/3)*x^(1/4)])/3 - (128*2^(1/3)*Log[-
8 + 2*2^(2/3)*x^(1/4) - 2^(1/3)*Sqrt[x]])/3
```

fricas [A] time = 0.82, size = 71, normalized size = 0.68

$$\frac{256}{3}\sqrt{3}\,2^{\frac{1}{3}}\arctan\left(\frac{1}{6}\sqrt{3}\,2^{\frac{2}{3}}x^{\frac{1}{4}}-\frac{1}{3}\sqrt{3}\right)-\frac{128}{3}\cdot 2^{\frac{1}{3}}\log\left(4\cdot 2^{\frac{2}{3}}-2\cdot 2^{\frac{1}{3}}x^{\frac{1}{4}}+\sqrt{x}\right)+\frac{256}{3}\cdot 2^{\frac{1}{3}}\log\left(2\cdot 2^{\frac{1}{3}}+x^{\frac{1}{4}}\right)+x-128\,x^{\frac{1}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-16+x^(3/4))/(16+x^(3/4)),x, algorithm="fricas")
```

```
[Out] 256/3*sqrt(3)*2^(1/3)*arctan(1/6*sqrt(3)*2^(2/3)*x^(1/4) - 1/3*sqrt(3)) - 1
28/3*2^(1/3)*log(4*2^(2/3) - 2*2^(1/3)*x^(1/4) + sqrt(x)) + 256/3*2^(1/3)*l
og(2*2^(1/3) + x^(1/4)) + x - 128*x^(1/4)
```

giac [A] time = 0.17, size = 71, normalized size = 0.68

$$\frac{256}{3}\sqrt{3}\,2^{\frac{1}{3}}\arctan\left(-\frac{1}{6}\sqrt{3}\,2^{\frac{2}{3}}\left(2^{\frac{1}{3}}-x^{\frac{1}{4}}\right)\right)-\frac{128}{3}\cdot 2^{\frac{1}{3}}\log\left(4\cdot 2^{\frac{2}{3}}-2\cdot 2^{\frac{1}{3}}x^{\frac{1}{4}}+\sqrt{x}\right)+\frac{256}{3}\cdot 2^{\frac{1}{3}}\log\left(2\cdot 2^{\frac{1}{3}}+x^{\frac{1}{4}}\right)+x-128\,x^{\frac{1}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-16+x^(3/4))/(16+x^(3/4)),x, algorithm="giac")
```

```
[Out] 256/3*sqrt(3)*2^(1/3)*arctan(-1/6*sqrt(3)*2^(2/3)*(2^(1/3) - x^(1/4))) - 12
8/3*2^(1/3)*log(4*2^(2/3) - 2*2^(1/3)*x^(1/4) + sqrt(x)) + 256/3*2^(1/3)*lo
g(2*2^(1/3) + x^(1/4)) + x - 128*x^(1/4)
```

maple [A] time = 0.04, size = 66, normalized size = 0.63

$$x+\frac{128\,16^{\frac{1}{3}}\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{16^{\frac{2}{3}}x^{\frac{1}{4}}}{8}-1\right)}{3}\right)}{3}+\frac{128\,16^{\frac{1}{3}}\ln\left(x^{\frac{1}{4}}+16^{\frac{1}{3}}\right)}{3}-\frac{64\,16^{\frac{1}{3}}\ln\left(\sqrt{x}-16^{\frac{1}{3}}x^{\frac{1}{4}}+16^{\frac{2}{3}}\right)}{3}-128x^{\frac{1}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-16+x^(3/4))/(16+x^(3/4)),x)
```

```
[Out] x-128*x^(1/4)+128/3*16^(1/3)*ln(x^(1/4)+16^(1/3))-64/3*16^(1/3)*ln(x^(1/2)-
16^(1/3)*x^(1/4)+16^(2/3))+128/3*16^(1/3)*3^(1/2)*arctan(1/3*3^(1/2)*(1/8*1
6^(2/3)*x^(1/4)-1))
```

maxima [A] time = 1.13, size = 71, normalized size = 0.68

$$\frac{256}{3}\sqrt{3}\,2^{\frac{1}{3}}\arctan\left(-\frac{1}{6}\sqrt{3}\,2^{\frac{2}{3}}\left(2^{\frac{1}{3}}-x^{\frac{1}{4}}\right)\right)-\frac{128}{3}\cdot 2^{\frac{1}{3}}\log\left(4\cdot 2^{\frac{2}{3}}-2\cdot 2^{\frac{1}{3}}x^{\frac{1}{4}}+\sqrt{x}\right)+\frac{256}{3}\cdot 2^{\frac{1}{3}}\log\left(2\cdot 2^{\frac{1}{3}}+x^{\frac{1}{4}}\right)+x-128\,x^{\frac{1}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-16+x^(3/4))/(16+x^(3/4)),x, algorithm="maxima")

[Out] 256/3*sqrt(3)*2^(1/3)*arctan(-1/6*sqrt(3)*2^(2/3)*(2^(1/3) - x^(1/4))) - 128/3*2^(1/3)*log(4*2^(2/3) - 2*2^(1/3)*x^(1/4) + sqrt(x)) + 256/3*2^(1/3)*log(2*2^(1/3) + x^(1/4)) + x - 128*x^(1/4)

mupad [B] time = 1.50, size = 90, normalized size = 0.87

$$x+\frac{256\,2^{1/3}\,\ln\left(12288\,2^{1/3}+6144\,x^{1/4}\right)}{3}-128\,x^{1/4}+\frac{128\,2^{1/3}\,\ln\left(6144\,x^{1/4}+6144\,2^{1/3}\,\left(-1+\sqrt{3}\,1\mathrm{i}\right)\right)\,\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{3}-\frac{128\,2^{1/3}\,\ln\left(6144\,x^{1/4}-6144\,2^{1/3}\,\left(1+\sqrt{3}\,1\mathrm{i}\right)\right)\,\left(1+\sqrt{3}\,1\mathrm{i}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^(3/4) - 16)/(x^(3/4) + 16),x)

[Out] x + (256*2^(1/3)*log(12288*2^(1/3) + 6144*x^(1/4)))/3 - 128*x^(1/4) + (128*2^(1/3)*log(6144*x^(1/4) + 6144*2^(1/3)*(3^(1/2)*1i - 1))*(3^(1/2)*1i - 1))/3 - (128*2^(1/3)*log(6144*x^(1/4) - 6144*2^(1/3)*(3^(1/2)*1i + 1))*(3^(1/2)*1i + 1))/3

sympy [A] time = 5.72, size = 102, normalized size = 0.98

$$-128\sqrt[4]{x}+x+\frac{256\sqrt[3]{2}\log\left(\sqrt[4]{x}+2\sqrt[3]{2}\right)}{3}-\frac{128\sqrt[3]{2}\log\left(-2\sqrt[3]{2}\sqrt[4]{x}+\sqrt{x}+4\cdot 2^{\frac{2}{3}}\right)}{3}+\frac{256\sqrt[3]{2}\sqrt{3}\operatorname{atan}\left(\frac{2^{\frac{2}{3}}\sqrt{3}\sqrt[4]{x}}{6}-\frac{\sqrt{3}}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-16+x**(3/4))/(16+x**(3/4)),x)

[Out] -128*x**(1/4) + x + 256*2**(1/3)*log(x**(1/4) + 2*2**(1/3))/3 - 128*2**(1/3)*log(-2*2**(1/3)*x**(1/4) + sqrt(x) + 4*2**(2/3))/3 + 256*2**(1/3)*sqrt(3)*atan(2**(2/3)*sqrt(3)*x**(1/4)/6 - sqrt(3)/3)/3

3.178 $\int \frac{1+\frac{1}{\sqrt[3]{x}}}{-1+\frac{1}{\sqrt[3]{x}}} \, dx$

Optimal. Leaf size=30

$$-3x^{2/3}-x-6\sqrt[3]{x}-6\log\left(1-\sqrt[3]{x}\right)$$

Rubi [A] time = 0.02, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {374, 376, 77}

$$-3x^{2/3}-x-6\sqrt[3]{x}-6\log\left(1-\sqrt[3]{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^(-1/3))/(-1 + x^(-1/3)),x]
```

```
[Out] -6*x^(1/3) - 3*x^(2/3) - x - 6*Log[1 - x^(1/3)]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 374

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[x^(n*(p + q))*(b + a/x^n)^p*(d + c/x^n)^q, x] /; FreeQ[{a, b, c, d
, n}, x] && NeQ[b*c - a*d, 0] && IntegersQ[p, q] && NegQ[n]
```

Rule 376

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(a + b*x^(g*n))
^p*(c + d*x^(g*n))^q, x], x, x^(1/g)], x]] /; FreeQ[{a, b, c, d, p, q}, x]
&& NeQ[b*c - a*d, 0] && FractionQ[n]
```

Rubi steps

$$\begin{aligned}
\int \frac{1+\frac{1}{\sqrt[3]{x}}}{-1+\frac{1}{\sqrt[3]{x}}} \, dx &= \int \frac{1+\sqrt[3]{x}}{1-\sqrt[3]{x}} \, dx\\
&= 3\,\text{Subst}\left(\int \frac{x^2(1+x)}{1-x} \, dx, x, \sqrt[3]{x}\right)\\
&= 3\,\text{Subst}\left(\int \left(-2-\frac{2}{-1+x}-2x-x^2\right) dx, x, \sqrt[3]{x}\right)\\
&= -6\sqrt[3]{x}-3x^{2/3}-x-6\log\left(1-\sqrt[3]{x}\right)
\end{aligned}$$

Mathematica [A] time = 0.03, size = 30, normalized size = 1.00

$$-3x^{2/3}-x-6\sqrt[3]{x}-6\log\left(1-\sqrt[3]{x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^(-1/3))/(-1 + x^(-1/3)),x]
[Out] -6*x^(1/3) - 3*x^(2/3) - x - 6*Log[1 - x^(1/3)]
```

IntegrateAlgebraic [A] time = 0.02, size = 32, normalized size = 1.07

$$-\sqrt[3]{x}\left(x^{2/3}+3\sqrt[3]{x}+6\right)-6\log\left(\sqrt[3]{x}-1\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 + x^(-1/3))/(-1 + x^(-1/3)),x]
[Out] -((6 + 3*x^(1/3) + x^(2/3))*x^(1/3)) - 6*Log[-1 + x^(1/3)]
```

fricas [A] time = 0.56, size = 22, normalized size = 0.73

$$-x-3\,x^{\frac{2}{3}}-6\,x^{\frac{1}{3}}-6\,\log\left(x^{\frac{1}{3}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+1/x^(1/3))/(-1+1/x^(1/3)),x, algorithm="fricas")
[Out] -x - 3*x^(2/3) - 6*x^(1/3) - 6*log(x^(1/3) - 1)
```

giac [A] time = 0.15, size = 23, normalized size = 0.77

$$-x-3\,x^{\frac{2}{3}}-6\,x^{\frac{1}{3}}-6\,\log\left(\left|x^{\frac{1}{3}}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+1/x^(1/3))/(-1+1/x^(1/3)),x, algorithm="giac")
[Out] -x - 3*x^(2/3) - 6*x^(1/3) - 6*log(abs(x^(1/3) - 1))
```

maple [A] time = 0.04, size = 23, normalized size = 0.77

$$-x-6\ln\left(x^{\frac{1}{3}}-1\right)-3x^{\frac{2}{3}}-6x^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+1/x^(1/3))/(-1+1/x^(1/3)),x)
[Out] -x-3*x^(2/3)-6*x^(1/3)-6*ln(x^(1/3)-1)
```

maxima [A] time = 0.56, size = 22, normalized size = 0.73

$$-x-3\,x^{\frac{2}{3}}-6\,x^{\frac{1}{3}}-6\,\log\left(x^{\frac{1}{3}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+1/x^(1/3))/(-1+1/x^(1/3)),x, algorithm="maxima")
[Out] -x - 3*x^(2/3) - 6*x^(1/3) - 6*log(x^(1/3) - 1)
```

mupad [B] time = 0.04, size = 22, normalized size = 0.73

$$-x-6\,\ln\left(x^{1/3}-1\right)-6\,x^{1/3}-3\,x^{2/3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/x^(1/3) + 1)/(1/x^(1/3) - 1),x)
```

```
[Out] - x - 6*log(x^(1/3) - 1) - 6*x^(1/3) - 3*x^(2/3)
```

sympy [**A**] time = 0.18, size = 26, normalized size = 0.87

$$-3x^{\frac{2}{3}} - 6\sqrt[3]{x} - x - 6\log\left(\sqrt[3]{x} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+1/x**(1/3))/(-1+1/x**(1/3)),x)
```

```
[Out] -3*x**(2/3) - 6*x**(1/3) - x - 6*log(x**(1/3) - 1)
```

3.179 $\int (a+bx^n)(c+dx^n)^4 \, dx$

Optimal. Leaf size=132

$$\frac{c^3x^{n+1}(4ad+bc)}{n+1}+\frac{2c^2dx^{2n+1}(3ad+2bc)}{2n+1}+\frac{d^3x^{4n+1}(ad+4bc)}{4n+1}+\frac{2cd^2x^{3n+1}(2ad+3bc)}{3n+1}+ac^4x+\frac{bd^4x^{5n+1}}{5n+1}$$

Rubi [A] time = 0.11, antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {373}

$$\frac{c^3x^{n+1}(4ad+bc)}{n+1}+\frac{2c^2dx^{2n+1}(3ad+2bc)}{2n+1}+\frac{2cd^2x^{3n+1}(2ad+3bc)}{3n+1}+\frac{d^3x^{4n+1}(ad+4bc)}{4n+1}+ac^4x+\frac{bd^4x^{5n+1}}{5n+1}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^n)*(c + d*x^n)^4,x]
```

```
[Out] a*c^4*x + (c^3*(b*c + 4*a*d)*x^(1 + n))/(1 + n) + (2*c^2*d*(2*b*c + 3*a*d)*
x^(1 + 2*n))/(1 + 2*n) + (2*c*d^2*(3*b*c + 2*a*d)*x^(1 + 3*n))/(1 + 3*n) +
(d^3*(4*b*c + a*d)*x^(1 + 4*n))/(1 + 4*n) + (b*d^4*x^(1 + 5*n))/(1 + 5*n)
```

Rule 373

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\int (a+bx^n)(c+dx^n)^4 \, dx = \int \left(ac^4+c^3(bc+4ad)x^n+2c^2d(2bc+3ad)x^{2n}+2cd^2(3bc+2ad)x^{3n}+d^3(4bc+\right.$$

$$= ac^4x+\frac{c^3(bc+4ad)x^{1+n}}{1+n}+\frac{2c^2d(2bc+3ad)x^{1+2n}}{1+2n}+\frac{2cd^2(3bc+2ad)x^{1+3n}}{1+3n}+\frac{d^3(4}{}$$

Mathematica [A] time = 0.28, size = 110, normalized size = 0.83

$$\frac{bx(c+dx^n)^5-x\left(c^4+\frac{4c^3dx^n}{n+1}+\frac{6c^2d^2x^{2n}}{2n+1}+\frac{4cd^3x^{3n}}{3n+1}+\frac{d^4x^{4n}}{4n+1}\right)(bc-ad(5n+1))}{5dn+d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^n)*(c + d*x^n)^4,x]
```

```
[Out] (b*x*(c + d*x^n)^5 - (b*c - a*d*(1 + 5*n))*x*(c^4 + (4*c^3*d*x^n)/(1 + n) +
 (6*c^2*d^2*x^(2*n))/(1 + 2*n) + (4*c*d^3*x^(3*n))/(1 + 3*n) + (d^4*x^(4*n)
)/(1 + 4*n)))/(d + 5*d*n)
```

IntegrateAlgebraic [F] time = 0.50, size = 0, normalized size = 0.00

$$\int (a+bx^n)(c+dx^n)^4 \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x^n)*(c + d*x^n)^4,x]
```

```
[Out] a*c^4*x + Defer[IntegrateAlgebraic][x^n*(b*c^4 + 4*a*c^3*d + 4*b*c^3*d*x^n
+ 6*a*c^2*d^2*x^n + 6*b*c^2*d^2*x^(2*n) + 4*a*c*d^3*x^(2*n) + 4*b*c*d^3*x^(
3*n) + a*d^4*x^(3*n) + b*d^4*x^(4*n)), x]
```

fricas [B] time = 0.86, size = 527, normalized size = 3.99

$$\frac{\left(24\,bd^4n^4+50\,bd^4n^3+35\,bd^4n^2+10\,bd^4n+bd^4\right)xx^{5\,n}+\left(4\,bcd^3+ad^4+30\left(4\,bcd^3+ad^4\right)n^4+61\left(4\,bcd^3+ad^4\right)n^3+41\left(4\,bcd^3+ad^4\right)n^2+11\left(4\,bcd^3+ad^4\right)n\right)xx^{4\,n}+2\left(3\,bc^2d^2+2\,acd^3+40\left(3\,bc^2d^2+2\,acd^3\right)n^4+78\left(3\,bc^2d^2+2\,acd^3\right)n^3+49\left(3\,bc^2d^2+2\,acd^3\right)n^2+12\left(3\,bc^2d^2+2\,acd^3\right)n\right)xx^{3\,n}+2\left(2\,bc^3d+3\,ac^2d^2+60\left(2\,bc^3d+3\,ac^2d^2\right)n^4+107\left(2\,bc^3d+3\,ac^2d^2\right)n^3+59\left(2\,bc^3d+3\,ac^2d^2\right)n^2+13\left(2\,bc^3d+3\,ac^2d^2\right)n\right)xx^{2\,n}+\left(bc^4+4\,ac^3d+120\left(bc^4+4\,ac^3d\right)n^4+154\left(bc^4+4\,ac^3d\right)n^3+71\left(bc^4+4\,ac^3d\right)n^2+14\left(bc^4+4\,ac^3d\right)n\right)xx^n+\left(120\,ac^4n^5+274\,ac^4n^4+225\,ac^4n^3+85\,ac^4n^2+15\,ac^4n+ac^4\right)x}{120\,n^5+274\,n^4+225\,n^3+85\,n^2+15\,n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)*(c+d*x^n)^4,x, algorithm="fricas")
```

```
[Out] ((24*b*d^4*n^4 + 50*b*d^4*n^3 + 35*b*d^4*n^2 + 10*b*d^4*n + b*d^4)*x*x^(5*n
) + (4*b*c*d^3 + a*d^4 + 30*(4*b*c*d^3 + a*d^4)*n^4 + 61*(4*b*c*d^3 + a*d^4
)*n^3 + 41*(4*b*c*d^3 + a*d^4)*n^2 + 11*(4*b*c*d^3 + a*d^4)*n)*x*x^(4*n) +
2*(3*b*c^2*d^2 + 2*a*c*d^3 + 40*(3*b*c^2*d^2 + 2*a*c*d^3)*n^4 + 78*(3*b*c^2
*d^2 + 2*a*c*d^3)*n^3 + 49*(3*b*c^2*d^2 + 2*a*c*d^3)*n^2 + 12*(3*b*c^2*d^2
+ 2*a*c*d^3)*n)*x*x^(3*n) + 2*(2*b*c^3*d + 3*a*c^2*d^2 + 60*(2*b*c^3*d + 3*
a*c^2*d^2)*n^4 + 107*(2*b*c^3*d + 3*a*c^2*d^2)*n^3 + 59*(2*b*c^3*d + 3*a*c^
2*d^2)*n^2 + 13*(2*b*c^3*d + 3*a*c^2*d^2)*n)*x*x^(2*n) + (b*c^4 + 4*a*c^3*d
 + 120*(b*c^4 + 4*a*c^3*d)*n^4 + 154*(b*c^4 + 4*a*c^3*d)*n^3 + 71*(b*c^4 +
4*a*c^3*d)*n^2 + 14*(b*c^4 + 4*a*c^3*d)*n)*x*x^n + (120*a*c^4*n^5 + 274*a*c
^4*n^4 + 225*a*c^4*n^3 + 85*a*c^4*n^2 + 15*a*c^4*n + a*c^4)*x)/(120*n^5 + 2
74*n^4 + 225*n^3 + 85*n^2 + 15*n + 1)
```

giac [B] time = 0.22, size = 740, normalized size = 5.61

$$\frac{120\,ac^4n^5x+24\,bd^4n^4xx^{5\,n}+120\,bcd^3n^4xx^{4\,n}+30\,ad^4n^4xx^{4\,n}+240\,bc^2d^2n^4xx^{3\,n}+160\,acd^3n^4xx^{3\,n}+240\,bc^3dn^4xx^{2\,n}+360\,ac^2d^2n^4xx^{2\,n}+120\,bc^4n^4xx^n+480\,ac^3dn^4xx^n+274\,ac^4n^4x+50\,bd^4n^3xx^{5\,n}+244\,bcd^3n^3xx^{4\,n}+61\,ad^4n^3xx^{4\,n}+468\,bc^2d^2n^3xx^{3\,n}+312\,acd^3n^3xx^{3\,n}+428\,bc^3dn^3xx^{2\,n}+642\,ac^2d^2n^3xx^{2\,n}+154\,bc^4n^3xx^n+616\,ac^3dn^3xx^n+225\,ac^4n^3x+35\,bd^4n^2xx^{5\,n}+164\,bcd^3n^2xx^{4\,n}+41\,ad^4n^2xx^{4\,n}+294\,bc^2d^2n^2xx^{3\,n}+196\,acd^3n^2xx^{3\,n}+236\,bc^3dn^2xx^{2\,n}+354\,ac^2d^2n^2xx^{2\,n}+71\,bc^4n^2xx^n+284\,ac^3dn^2xx^n+85\,ac^4n^2x+10\,bd^4nxx^{5\,n}+44\,bcd^3nxx^{4\,n}+11\,ad^4nxx^{4\,n}+72\,bc^2d^2nxx^{3\,n}+48\,acd^3nxx^{3\,n}+52\,bc^3dnxx^{2\,n}+78\,ac^2d^2nxx^{2\,n}+14\,bc^4nxx^n+56\,ac^3dnxx^n+15\,ac^4nx+bd^4xx^{5\,n}+4\,bcd^3xx^{4\,n}+ad^4xx^{4\,n}+6\,bc^2d^2xx^{3\,n}+4\,acd^3xx^{3\,n}+4\,bc^3dxx^{2\,n}+6\,ac^2d^2xx^{2\,n}+bc^4xx^n+4\,ac^3dxx^n+ac^4x}{120\,n^5+274\,n^4+225\,n^3+85\,n^2+15\,n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)*(c+d*x^n)^4,x, algorithm="giac")
```

```
[Out] (120*a*c^4*n^5*x + 24*b*d^4*n^4*x*x^(5*n) + 120*b*c*d^3*n^4*x*x^(4*n) + 30*
a*d^4*n^4*x*x^(4*n) + 240*b*c^2*d^2*n^4*x*x^(3*n) + 160*a*c*d^3*n^4*x*x^(3*
n) + 240*b*c^3*d*n^4*x*x^(2*n) + 360*a*c^2*d^2*n^4*x*x^(2*n) + 120*b*c^4*n^
4*x*x^n + 480*a*c^3*d*n^4*x*x^n + 274*a*c^4*n^4*x + 50*b*d^4*n^3*x*x^(5*n)
+ 244*b*c*d^3*n^3*x*x^(4*n) + 61*a*d^4*n^3*x*x^(4*n) + 468*b*c^2*d^2*n^3*x*
x^(3*n) + 312*a*c*d^3*n^3*x*x^(3*n) + 428*b*c^3*d*n^3*x*x^(2*n) + 642*a*c^2
*d^2*n^3*x*x^(2*n) + 154*b*c^4*n^3*x*x^n + 616*a*c^3*d*n^3*x*x^n + 225*a*c^
4*n^3*x + 35*b*d^4*n^2*x*x^(5*n) + 164*b*c*d^3*n^2*x*x^(4*n) + 41*a*d^4*n^2
*x*x^(4*n) + 294*b*c^2*d^2*n^2*x*x^(3*n) + 196*a*c*d^3*n^2*x*x^(3*n) + 236*
b*c^3*d*n^2*x*x^(2*n) + 354*a*c^2*d^2*n^2*x*x^(2*n) + 71*b*c^4*n^2*x*x^n +
284*a*c^3*d*n^2*x*x^n + 85*a*c^4*n^2*x + 10*b*d^4*n*x*x^(5*n) + 44*b*c*d^3*
n*x*x^(4*n) + 11*a*d^4*n*x*x^(4*n) + 72*b*c^2*d^2*n*x*x^(3*n) + 48*a*c*d^3*
n*x*x^(3*n) + 52*b*c^3*d*n*x*x^(2*n) + 78*a*c^2*d^2*n*x*x^(2*n) + 14*b*c^4*
n*x*x^n + 56*a*c^3*d*n*x*x^n + 15*a*c^4*n*x + b*d^4*x*x^(5*n) + 4*b*c*d^3*x
*x^(4*n) + a*d^4*x*x^(4*n) + 6*b*c^2*d^2*x*x^(3*n) + 4*a*c*d^3*x*x^(3*n) +
4*b*c^3*d*x*x^(2*n) + 6*a*c^2*d^2*x*x^(2*n) + b*c^4*x*x^n + 4*a*c^3*d*x*x^n
 + a*c^4*x)/(120*n^5 + 274*n^4 + 225*n^3 + 85*n^2 + 15*n + 1)
```

maple [A] time = 0.05, size = 138, normalized size = 1.05

$$\frac{b\,d^4x\,\mathrm{e}^{5n\ln(x)}}{5n+1}+a\,c^4x+\frac{(4ad+bc)\,c^3x\,\mathrm{e}^{n\ln(x)}}{n+1}+\frac{2\,(3ad+2bc)\,c^2dx\,\mathrm{e}^{2n\ln(x)}}{2n+1}+\frac{2\,(2ad+3bc)\,c\,d^2x\,\mathrm{e}^{3n\ln(x)}}{3n+1}+\frac{(ad+4bc)\,d^3x\,\mathrm{e}^{4n\ln(x)}}{4n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^n+a)*(c+d*x^n)^4,x)
```

```
[Out] a*c^4*x+b*d^4/(1+5*n)*x*exp(n*ln(x))^5+c^3*(4*a*d+b*c)/(n+1)*x*exp(n*ln(x))
+d^3*(a*d+4*b*c)/(4*n+1)*x*exp(n*ln(x))^4+2*c*d^2*(2*a*d+3*b*c)/(3*n+1)*x*e
xp(n*ln(x))^3+2*c^2*d*(3*a*d+2*b*c)/(2*n+1)*x*exp(n*ln(x))^2
```

maxima [A] time = 0.61, size = 186, normalized size = 1.41

$$ac^4x+\frac{bd^4x^{5\,n+1}}{5\,n+1}+\frac{4\,bcd^3x^{4\,n+1}}{4\,n+1}+\frac{ad^4x^{4\,n+1}}{4\,n+1}+\frac{6\,bc^2d^2x^{3\,n+1}}{3\,n+1}+\frac{4\,acd^3x^{3\,n+1}}{3\,n+1}+\frac{4\,bc^3dx^{2\,n+1}}{2\,n+1}+\frac{6\,ac^2d^2x^{2\,n+1}}{2\,n+1}+\frac{bc^4x^{n+1}}{n+1}+\frac{4\,ac^3dx^{n+1}}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)*(c+d*x^n)^4,x, algorithm="maxima")
```

```
[Out] a*c^4*x + b*d^4*x^(5*n + 1)/(5*n + 1) + 4*b*c*d^3*x^(4*n + 1)/(4*n + 1) + a
*d^4*x^(4*n + 1)/(4*n + 1) + 6*b*c^2*d^2*x^(3*n + 1)/(3*n + 1) + 4*a*c*d^3*
x^(3*n + 1)/(3*n + 1) + 4*b*c^3*d*x^(2*n + 1)/(2*n + 1) + 6*a*c^2*d^2*x^(2*
n + 1)/(2*n + 1) + b*c^4*x^(n + 1)/(n + 1) + 4*a*c^3*d*x^(n + 1)/(n + 1)
```

mupad [B] time = 1.64, size = 131, normalized size = 0.99

$$a\,c^4\,x+\frac{x\,x^n\,\left(b\,c^4+4\,a\,d\,c^3\right)}{n+1}+\frac{x\,x^{4\,n}\,\left(a\,d^4+4\,b\,c\,d^3\right)}{4\,n+1}+\frac{b\,d^4\,x\,x^{5\,n}}{5\,n+1}+\frac{2\,c^2\,d\,x\,x^{2\,n}\,\left(3\,a\,d+2\,b\,c\right)}{2\,n+1}+\frac{2\,c\,d^2\,x\,x^{3\,n}\,\left(2\,a\,d+3\,b\,c\right)}{3\,n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^n)*(c + d*x^n)^4,x)
```

```
[Out] a*c^4*x + (x*x^n*(b*c^4 + 4*a*c^3*d))/(n + 1) + (x*x^(4*n)*(a*d^4 + 4*b*c*d
^3))/(4*n + 1) + (b*d^4*x*x^(5*n))/(5*n + 1) + (2*c^2*d*x*x^(2*n)*(3*a*d +
2*b*c))/(2*n + 1) + (2*c*d^2*x*x^(3*n)*(2*a*d + 3*b*c))/(3*n + 1)
```

sympy [A] time = 3.67, size = 2744, normalized size = 20.79

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**n)*(c+d*x**n)**4,x)
```

```
[Out] Piecewise((a*c**4*x + 4*a*c**3*d*log(x) - 6*a*c**2*d**2/x - 2*a*c*d**3/x**2
 - a*d**4/(3*x**3) + b*c**4*log(x) - 4*b*c**3*d/x - 3*b*c**2*d**2/x**2 - 4*
b*c*d**3/(3*x**3) - b*d**4/(4*x**4), Eq(n, -1)), (a*c**4*x + 8*a*c**3*d*sqr
t(x) + 6*a*c**2*d**2*log(x) - 8*a*c*d**3/sqrt(x) - a*d**4/x + 2*b*c**4*sqrt
(x) + 4*b*c**3*d*log(x) - 12*b*c**2*d**2/sqrt(x) - 4*b*c*d**3/x - 2*b*d**4/
(3*x**(3/2)), Eq(n, -1/2)), (a*c**4*x + 6*a*c**3*d*x**(2/3) + 18*a*c**2*d**
2*x**(1/3) + 4*a*c*d**3*log(x) - 3*a*d**4/x**(1/3) + 3*b*c**4*x**(2/3)/2 +
12*b*c**3*d*x**(1/3) + 6*b*c**2*d**2*log(x) - 12*b*c*d**3/x**(1/3) - 3*b*d*
*4/(2*x**(2/3)), Eq(n, -1/3)), (a*c**4*x + 16*a*c**3*d*x**(3/4)/3 + 12*a*c*
*2*d**2*sqrt(x) + 16*a*c*d**3*x**(1/4) + a*d**4*log(x) + 4*b*c**4*x**(3/4)/
3 + 8*b*c**3*d*sqrt(x) + 24*b*c**2*d**2*x**(1/4) + 4*b*c*d**3*log(x) - 4*b*
d**4/x**(1/4), Eq(n, -1/4)), (a*c**4*x + 5*a*c**3*d*x**(4/5) + 10*a*c**2*d*
*2*x**(3/5) + 10*a*c*d**3*x**(2/5) + 5*a*d**4*x**(1/5) + 5*b*c**4*x**(4/5)/
4 + 20*b*c**3*d*x**(3/5)/3 + 15*b*c**2*d**2*x**(2/5) + 20*b*c*d**3*x**(1/5)
 + b*d**4*log(x), Eq(n, -1/5)), (120*a*c**4*n**5*x/(120*n**5 + 274*n**4 + 2
25*n**3 + 85*n**2 + 15*n + 1) + 274*a*c**4*n**4*x/(120*n**5 + 274*n**4 + 22
5*n**3 + 85*n**2 + 15*n + 1) + 225*a*c**4*n**3*x/(120*n**5 + 274*n**4 + 225
*n**3 + 85*n**2 + 15*n + 1) + 85*a*c**4*n**2*x/(120*n**5 + 274*n**4 + 225*n
**3 + 85*n**2 + 15*n + 1) + 15*a*c**4*n*x/(120*n**5 + 274*n**4 + 225*n**3 +
 85*n**2 + 15*n + 1) + a*c**4*x/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 +
 15*n + 1) + 480*a*c**3*d*n**4*x*x**n/(120*n**5 + 274*n**4 + 225*n**3 + 85*
n**2 + 15*n + 1) + 616*a*c**3*d*n**3*x*x**n/(120*n**5 + 274*n**4 + 225*n**3
 + 85*n**2 + 15*n + 1) + 284*a*c**3*d*n**2*x*x**n/(120*n**5 + 274*n**4 + 22
5*n**3 + 85*n**2 + 15*n + 1) + 56*a*c**3*d*n*x*x**n/(120*n**5 + 274*n**4 +
225*n**3 + 85*n**2 + 15*n + 1) + 4*a*c**3*d*x*x**n/(120*n**5 + 274*n**4 + 2
25*n**3 + 85*n**2 + 15*n + 1) + 360*a*c**2*d**2*n**4*x*x**(2*n)/(120*n**5 +
 274*n**4 + 225*n**3 + 85*n**2 + 15*n + 1) + 642*a*c**2*d**2*n**3*x*x**(2*n
)/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n + 1) + 354*a*c**2*d**2*n
**2*x*x**(2*n)/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n + 1) + 78*a
*c**2*d**2*n*x*x**(2*n)/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n +
1) + 6*a*c**2*d**2*x*x**(2*n)/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 1
5*n + 1) + 160*a*c*d**3*n**4*x*x**(3*n)/(120*n**5 + 274*n**4 + 225*n**3 + 8
5*n**2 + 15*n + 1) + 312*a*c*d**3*n**3*x*x**(3*n)/(120*n**5 + 274*n**4 + 22
```

```
5*n**3 + 85*n**2 + 15*n + 1) + 196*a*c*d**3*n**2*x*x**(3*n)/(120*n**5 + 274
*n**4 + 225*n**3 + 85*n**2 + 15*n + 1) + 48*a*c*d**3*n*x*x**(3*n)/(120*n**5
 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n + 1) + 4*a*c*d**3*x*x**(3*n)/(120*n
**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n + 1) + 30*a*d**4*n**4*x*x**(4*n)
/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n + 1) + 61*a*d**4*n**3*x*x
**(4*n)/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n + 1) + 41*a*d**4*n
**2*x*x**(4*n)/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n + 1) + 11*a
*d**4*n*x*x**(4*n)/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n + 1) +
a*d**4*x*x**(4*n)/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n + 1) + 1
20*b*c**4*n**4*x*x**n/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n + 1)
 + 154*b*c**4*n**3*x*x**n/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n
+ 1) + 71*b*c**4*n**2*x*x**n/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15
*n + 1) + 14*b*c**4*n*x*x**n/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15
*n + 1) + b*c**4*x*x**n/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n +
1) + 240*b*c**3*d*n**4*x*x**(2*n)/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2
 + 15*n + 1) + 428*b*c**3*d*n**3*x*x**(2*n)/(120*n**5 + 274*n**4 + 225*n**3
 + 85*n**2 + 15*n + 1) + 236*b*c**3*d*n**2*x*x**(2*n)/(120*n**5 + 274*n**4
+ 225*n**3 + 85*n**2 + 15*n + 1) + 52*b*c**3*d*n*x*x**(2*n)/(120*n**5 + 274
*n**4 + 225*n**3 + 85*n**2 + 15*n + 1) + 4*b*c**3*d*x*x**(2*n)/(120*n**5 +
274*n**4 + 225*n**3 + 85*n**2 + 15*n + 1) + 240*b*c**2*d**2*n**4*x*x**(3*n)
/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n + 1) + 468*b*c**2*d**2*n*
*3*x*x**(3*n)/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n + 1) + 294*b
*c**2*d**2*n**2*x*x**(3*n)/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n
 + 1) + 72*b*c**2*d**2*n*x*x**(3*n)/(120*n**5 + 274*n**4 + 225*n**3 + 85*n*
*2 + 15*n + 1) + 6*b*c**2*d**2*x*x**(3*n)/(120*n**5 + 274*n**4 + 225*n**3 +
 85*n**2 + 15*n + 1) + 120*b*c*d**3*n**4*x*x**(4*n)/(120*n**5 + 274*n**4 +
225*n**3 + 85*n**2 + 15*n + 1) + 244*b*c*d**3*n**3*x*x**(4*n)/(120*n**5 + 2
74*n**4 + 225*n**3 + 85*n**2 + 15*n + 1) + 164*b*c*d**3*n**2*x*x**(4*n)/(12
0*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n + 1) + 44*b*c*d**3*n*x*x**(4*
n)/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n + 1) + 4*b*c*d**3*x*x**
(4*n)/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n + 1) + 24*b*d**4*n**
4*x*x**(5*n)/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n + 1) + 50*b*d
**4*n**3*x*x**(5*n)/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n + 1) +
 35*b*d**4*n**2*x*x**(5*n)/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n
 + 1) + 10*b*d**4*n*x*x**(5*n)/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 +
15*n + 1) + b*d**4*x*x**(5*n)/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 1
5*n + 1), True))
```

3.180 $\int (a+bx^n)(c+dx^n)^3 \, dx$

Optimal. Leaf size=99

$$\frac{c^2x^{n+1}(3ad+bc)}{n+1}+\frac{d^2x^{3n+1}(ad+3bc)}{3n+1}+\frac{3cdx^{2n+1}(ad+bc)}{2n+1}+ac^3x+\frac{bd^3x^{4n+1}}{4n+1}$$

Rubi [A] time = 0.07, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {373}

$$\frac{c^2x^{n+1}(3ad+bc)}{n+1}+\frac{d^2x^{3n+1}(ad+3bc)}{3n+1}+\frac{3cdx^{2n+1}(ad+bc)}{2n+1}+ac^3x+\frac{bd^3x^{4n+1}}{4n+1}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^n)*(c + d*x^n)^3,x]
```

```
[Out] a*c^3*x + (c^2*(b*c + 3*a*d)*x^(1 + n))/(1 + n) + (3*c*d*(b*c + a*d)*x^(1 +
 2*n))/(1 + 2*n) + (d^2*(3*b*c + a*d)*x^(1 + 3*n))/(1 + 3*n) + (b*d^3*x^(1
+ 4*n))/(1 + 4*n)
```

Rule 373

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int (a+bx^n)(c+dx^n)^3 \, dx &= \int \left(ac^3+c^2(bc+3ad)x^n+3cd(bc+ad)x^{2n}+d^2(3bc+ad)x^{3n}+bd^3x^{4n}\right) dx\\ &= ac^3x+\frac{c^2(bc+3ad)x^{1+n}}{1+n}+\frac{3cd(bc+ad)x^{1+2n}}{1+2n}+\frac{d^2(3bc+ad)x^{1+3n}}{1+3n}+\frac{bd^3x^{1+4n}}{1+4n}\end{aligned}$$

Mathematica [A] time = 0.13, size = 90, normalized size = 0.91

$$\frac{bx(c+dx^n)^4-x\left(c^3+\frac{3c^2dx^n}{n+1}+\frac{3cd^2x^{2n}}{2n+1}+\frac{d^3x^{3n}}{3n+1}\right)(bc-ad(4n+1))}{4dn+d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^n)*(c + d*x^n)^3,x]
```

```
[Out] (b*x*(c + d*x^n)^4 - (b*c - a*d*(1 + 4*n))*x*(c^3 + (3*c^2*d*x^n)/(1 + n) +
 (3*c*d^2*x^(2*n))/(1 + 2*n) + (d^3*x^(3*n))/(1 + 3*n)))/(d + 4*d*n)
```

IntegrateAlgebraic [F] time = 0.11, size = 0, normalized size = 0.00

$$\int (a+bx^n)(c+dx^n)^3 \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x^n)*(c + d*x^n)^3,x]
```

```
[Out] a*c^3*x + Defer[IntegrateAlgebraic][x^n*(b*c^3 + 3*a*c^2*d + 3*b*c^2*d*x^n
+ 3*a*c*d^2*x^n + 3*b*c*d^2*x^(2*n) + a*d^3*x^(2*n) + b*d^3*x^(3*n)), x]
```

fricas [B] time = 0.74, size = 319, normalized size = 3.22

$$\frac{\left(6\,bd^{3}n^{3}+11\,bd^{3}n^{2}+6\,bd^{3}n+bd^{3}\right)xx^{4\,n}+\left(3\,bcd^{2}+ad^{3}+8\left(3\,bcd^{2}+ad^{3}\right)n^{3}+14\left(3\,bcd^{2}+ad^{3}\right)n^{2}+7\left(3\,bcd^{2}+ad^{3}\right)n\right)xx^{3\,n}+3\left(bc^{2}d+acd^{2}+12\left(bc^{2}d+acd^{2}\right)n^{3}+19\left(bc^{2}d+acd^{2}\right)n^{2}+8\left(bc^{2}d+acd^{2}\right)n\right)xx^{2\,n}+\left(bc^{3}+3\,ac^{2}d+24\left(bc^{3}+3\,ac^{2}d\right)n^{3}+26\left(bc^{3}+3\,ac^{2}d\right)n^{2}+9\left(bc^{3}+3\,ac^{2}d\right)n\right)xx^{n}+\left(24\,ac^{3}n^{4}+50\,ac^{3}n^{3}+35\,ac^{3}n^{2}+10\,ac^{3}n+ac^{3}\right)x}{24\,n^{4}+50\,n^{3}+35\,n^{2}+10\,n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)*(c+d*x^n)^3,x, algorithm="fricas")
```

```
[Out] ((6*b*d^3*n^3 + 11*b*d^3*n^2 + 6*b*d^3*n + b*d^3)*x*x^(4*n) + (3*b*c*d^2 +
a*d^3 + 8*(3*b*c*d^2 + a*d^3)*n^3 + 14*(3*b*c*d^2 + a*d^3)*n^2 + 7*(3*b*c*d
^2 + a*d^3)*n)*x*x^(3*n) + 3*(b*c^2*d + a*c*d^2 + 12*(b*c^2*d + a*c*d^2)*n^
3 + 19*(b*c^2*d + a*c*d^2)*n^2 + 8*(b*c^2*d + a*c*d^2)*n)*x*x^(2*n) + (b*c^
3 + 3*a*c^2*d + 24*(b*c^3 + 3*a*c^2*d)*n^3 + 26*(b*c^3 + 3*a*c^2*d)*n^2 + 9
*(b*c^3 + 3*a*c^2*d)*n)*x*x^n + (24*a*c^3*n^4 + 50*a*c^3*n^3 + 35*a*c^3*n^2
 + 10*a*c^3*n + a*c^3)*x)/(24*n^4 + 50*n^3 + 35*n^2 + 10*n + 1)
```

giac [B] time = 0.19, size = 450, normalized size = 4.55

$$\frac{24\,ac^{3}n^{4}x+6\,bd^{3}n^{3}xx^{4\,n}+24\,bcd^{2}n^{3}xx^{3\,n}+8\,ad^{3}n^{3}xx^{3\,n}+36\,bc^{2}dn^{3}xx^{2\,n}+36\,acd^{2}n^{3}xx^{2\,n}+24\,bc^{3}n^{3}xx^{n}+72\,ac^{2}dn^{3}xx^{n}+50\,ac^{3}n^{3}x+11\,bd^{3}n^{2}xx^{4\,n}+42\,bcd^{2}n^{2}xx^{3\,n}+14\,ad^{3}n^{2}xx^{3\,n}+57\,bc^{2}dn^{2}xx^{2\,n}+57\,acd^{2}n^{2}xx^{2\,n}+26\,bc^{3}n^{2}xx^{n}+78\,ac^{2}dn^{2}xx^{n}+35\,ac^{3}n^{2}x+6\,bd^{3}nxx^{4\,n}+21\,bcd^{2}nxx^{3\,n}+7\,ad^{3}nxx^{3\,n}+24\,bc^{2}dnxx^{2\,n}+24\,acd^{2}nxx^{2\,n}+9\,bc^{3}nxx^{n}+27\,ac^{2}dnxx^{n}+10\,ac^{3}nx+bd^{3}xx^{4\,n}+3\,bcd^{2}xx^{3\,n}+ad^{3}xx^{3\,n}+3\,bc^{2}dxx^{2\,n}+3\,acd^{2}xx^{2\,n}+bc^{3}xx^{n}+3\,ac^{2}dxx^{n}+ac^{3}x}{24\,n^{4}+50\,n^{3}+35\,n^{2}+10\,n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)*(c+d*x^n)^3,x, algorithm="giac")
```

```
[Out] (24*a*c^3*n^4*x + 6*b*d^3*n^3*x*x^(4*n) + 24*b*c*d^2*n^3*x*x^(3*n) + 8*a*d^
3*n^3*x*x^(3*n) + 36*b*c^2*d*n^3*x*x^(2*n) + 36*a*c*d^2*n^3*x*x^(2*n) + 24*
b*c^3*n^3*x*x^n + 72*a*c^2*d*n^3*x*x^n + 50*a*c^3*n^3*x + 11*b*d^3*n^2*x*x^
(4*n) + 42*b*c*d^2*n^2*x*x^(3*n) + 14*a*d^3*n^2*x*x^(3*n) + 57*b*c^2*d*n^2*
x*x^(2*n) + 57*a*c*d^2*n^2*x*x^(2*n) + 26*b*c^3*n^2*x*x^n + 78*a*c^2*d*n^2*
x*x^n + 35*a*c^3*n^2*x + 6*b*d^3*n*x*x^(4*n) + 21*b*c*d^2*n*x*x^(3*n) + 7*a
*d^3*n*x*x^(3*n) + 24*b*c^2*d*n*x*x^(2*n) + 24*a*c*d^2*n*x*x^(2*n) + 9*b*c^
3*n*x*x^n + 27*a*c^2*d*n*x*x^n + 10*a*c^3*n*x + b*d^3*x*x^(4*n) + 3*b*c*d^2
*x*x^(3*n) + a*d^3*x*x^(3*n) + 3*b*c^2*d*x*x^(2*n) + 3*a*c*d^2*x*x^(2*n) +
b*c^3*x*x^n + 3*a*c^2*d*x*x^n + a*c^3*x)/(24*n^4 + 50*n^3 + 35*n^2 + 10*n +
 1)
```

maple [A] time = 0.06, size = 104, normalized size = 1.05

$$\frac{b\,d^{3}x\,{\rm e}^{4n\ln(x)}}{4n+1}+a\,c^{3}x+\frac{(3ad+bc)\,c^{2}x\,{\rm e}^{n\ln(x)}}{n+1}+\frac{3\,(ad+bc)\,cdx\,{\rm e}^{2n\ln(x)}}{2n+1}+\frac{(ad+3bc)\,d^{2}x\,{\rm e}^{3n\ln(x)}}{3n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^n+a)*(c+d*x^n)^3,x)
```

```
[Out] a*c^3*x+b*d^3/(4*n+1)*x*exp(n*ln(x))^4+c^2*(3*a*d+b*c)/(n+1)*x*exp(n*ln(x))
+d^2*(a*d+3*b*c)/(3*n+1)*x*exp(n*ln(x))^3+3*c*d*(a*d+b*c)/(2*n+1)*x*exp(n*l
n(x))^2
```

maxima [A] time = 0.50, size = 140, normalized size = 1.41

$$ac^{3}x+\frac{bd^{3}x^{4\,n+1}}{4\,n+1}+\frac{3\,bcd^{2}x^{3\,n+1}}{3\,n+1}+\frac{ad^{3}x^{3\,n+1}}{3\,n+1}+\frac{3\,bc^{2}dx^{2\,n+1}}{2\,n+1}+\frac{3\,acd^{2}x^{2\,n+1}}{2\,n+1}+\frac{bc^{3}x^{n+1}}{n+1}+\frac{3\,ac^{2}dx^{n+1}}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)*(c+d*x^n)^3,x, algorithm="maxima")
```

```
[Out] a*c^3*x + b*d^3*x^(4*n + 1)/(4*n + 1) + 3*b*c*d^2*x^(3*n + 1)/(3*n + 1) + a
*d^3*x^(3*n + 1)/(3*n + 1) + 3*b*c^2*d*x^(2*n + 1)/(2*n + 1) + 3*a*c*d^2*x^
(2*n + 1)/(2*n + 1) + b*c^3*x^(n + 1)/(n + 1) + 3*a*c^2*d*x^(n + 1)/(n + 1)
```

mupad [B] time = 1.56, size = 99, normalized size = 1.00

$$a\,c^3\,x+\frac{x\,x^n\,\left(b\,c^3+3\,a\,d\,c^2\right)}{n+1}+\frac{x\,x^{3\,n}\,\left(a\,d^3+3\,b\,c\,d^2\right)}{3\,n+1}+\frac{b\,d^3\,x\,x^{4\,n}}{4\,n+1}+\frac{3\,c\,d\,x\,x^{2\,n}\,\left(a\,d+b\,c\right)}{2\,n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^n)*(c + d*x^n)^3,x)
```

```
[Out] a*c^3*x + (x*x^n*(b*c^3 + 3*a*c^2*d))/(n + 1) + (x*x^(3*n)*(a*d^3 + 3*b*c*d
^2))/(3*n + 1) + (b*d^3*x*x^(4*n))/(4*n + 1) + (3*c*d*x*x^(2*n)*(a*d + b*c)
)/(2*n + 1)
```

sympy [A] time = 3.35, size = 1540, normalized size = 15.56

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**n)*(c+d*x**n)**3,x)
```

```
[Out] Piecewise((a*c**3*x + 3*a*c**2*d*log(x) - 3*a*c*d**2/x - a*d**3/(2*x**2) +
b*c**3*log(x) - 3*b*c**2*d/x - 3*b*c*d**2/(2*x**2) - b*d**3/(3*x**3), Eq(n,
 -1)), (a*c**3*x + 6*a*c**2*d*sqrt(x) + 3*a*c*d**2*log(x) - 2*a*d**3/sqrt(x
) + 2*b*c**3*sqrt(x) + 3*b*c**2*d*log(x) - 6*b*c*d**2/sqrt(x) - b*d**3/x, E
q(n, -1/2)), (a*c**3*x + 9*a*c**2*d*x**(2/3)/2 + 9*a*c*d**2*x**(1/3) + a*d*
*3*log(x) + 3*b*c**3*x**(2/3)/2 + 9*b*c**2*d*x**(1/3) + 3*b*c*d**2*log(x) -
 3*b*d**3/x**(1/3), Eq(n, -1/3)), (a*c**3*x + 4*a*c**2*d*x**(3/4) + 6*a*c*d
**2*sqrt(x) + 4*a*d**3*x**(1/4) + 4*b*c**3*x**(3/4)/3 + 6*b*c**2*d*sqrt(x)
+ 12*b*c*d**2*x**(1/4) + b*d**3*log(x), Eq(n, -1/4)), (24*a*c**3*n**4*x/(24
*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 50*a*c**3*n**3*x/(24*n**4 + 50*n**3
 + 35*n**2 + 10*n + 1) + 35*a*c**3*n**2*x/(24*n**4 + 50*n**3 + 35*n**2 + 10
*n + 1) + 10*a*c**3*n*x/(24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + a*c**3*x
/(24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 72*a*c**2*d*n**3*x*x**n/(24*n**
4 + 50*n**3 + 35*n**2 + 10*n + 1) + 78*a*c**2*d*n**2*x*x**n/(24*n**4 + 50*n
**3 + 35*n**2 + 10*n + 1) + 27*a*c**2*d*n*x*x**n/(24*n**4 + 50*n**3 + 35*n*
*2 + 10*n + 1) + 3*a*c**2*d*x*x**n/(24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1)
 + 36*a*c*d**2*n**3*x*x**(2*n)/(24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 5
7*a*c*d**2*n**2*x*x**(2*n)/(24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 24*a*
c*d**2*n*x*x**(2*n)/(24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 3*a*c*d**2*x
*x**(2*n)/(24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 8*a*d**3*n**3*x*x**(3*
n)/(24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 14*a*d**3*n**2*x*x**(3*n)/(24
*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 7*a*d**3*n*x*x**(3*n)/(24*n**4 + 50
*n**3 + 35*n**2 + 10*n + 1) + a*d**3*x*x**(3*n)/(24*n**4 + 50*n**3 + 35*n**
2 + 10*n + 1) + 24*b*c**3*n**3*x*x**n/(24*n**4 + 50*n**3 + 35*n**2 + 10*n +
 1) + 26*b*c**3*n**2*x*x**n/(24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 9*b*
c**3*n*x*x**n/(24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + b*c**3*x*x**n/(24*
n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 36*b*c**2*d*n**3*x*x**(2*n)/(24*n**4
 + 50*n**3 + 35*n**2 + 10*n + 1) + 57*b*c**2*d*n**2*x*x**(2*n)/(24*n**4 + 5
0*n**3 + 35*n**2 + 10*n + 1) + 24*b*c**2*d*n*x*x**(2*n)/(24*n**4 + 50*n**3
+ 35*n**2 + 10*n + 1) + 3*b*c**2*d*x*x**(2*n)/(24*n**4 + 50*n**3 + 35*n**2
+ 10*n + 1) + 24*b*c*d**2*n**3*x*x**(3*n)/(24*n**4 + 50*n**3 + 35*n**2 + 10
*n + 1) + 42*b*c*d**2*n**2*x*x**(3*n)/(24*n**4 + 50*n**3 + 35*n**2 + 10*n +
 1) + 21*b*c*d**2*n*x*x**(3*n)/(24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 3
*b*c*d**2*x*x**(3*n)/(24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 6*b*d**3*n*
*3*x*x**(4*n)/(24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 11*b*d**3*n**2*x*x
**(4*n)/(24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 6*b*d**3*n*x*x**(4*n)/(2
4*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + b*d**3*x*x**(4*n)/(24*n**4 + 50*n*
*3 + 35*n**2 + 10*n + 1), True))
```

3.181 $\int (a+bx^n)(c+dx^n)^2 \, dx$

Optimal. Leaf size=70

$$\frac{cx^{n+1}(2ad+bc)}{n+1}+\frac{dx^{2n+1}(ad+2bc)}{2n+1}+ac^2x+\frac{bd^2x^{3n+1}}{3n+1}$$

Rubi [A] time = 0.05, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {373}

$$\frac{cx^{n+1}(2ad+bc)}{n+1}+\frac{dx^{2n+1}(ad+2bc)}{2n+1}+ac^2x+\frac{bd^2x^{3n+1}}{3n+1}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^n)*(c + d*x^n)^2,x]
```

```
[Out] a*c^2*x + (c*(b*c + 2*a*d)*x^(1 + n))/(1 + n) + (d*(2*b*c + a*d)*x^(1 + 2*n
))/(1 + 2*n) + (b*d^2*x^(1 + 3*n))/(1 + 3*n)
```

Rule 373

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int (a+bx^n)(c+dx^n)^2 \, dx &= \int \left(ac^2+c(bc+2ad)x^n+d(2bc+ad)x^{2n}+bd^2x^{3n}\right) dx\\ &= ac^2x+\frac{c(bc+2ad)x^{1+n}}{1+n}+\frac{d(2bc+ad)x^{1+2n}}{1+2n}+\frac{bd^2x^{1+3n}}{1+3n}\end{aligned}$$

Mathematica [A] time = 0.10, size = 70, normalized size = 1.00

$$\frac{bx(c+dx^n)^3-x\left(c^2+\frac{2cdx^n}{n+1}+\frac{d^2x^{2n}}{2n+1}\right)(bc-ad(3n+1))}{3dn+d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^n)*(c + d*x^n)^2,x]
```

```
[Out] (b*x*(c + d*x^n)^3 - (b*c - a*d*(1 + 3*n))*x*(c^2 + (2*c*d*x^n)/(1 + n) + (
d^2*x^(2*n))/(1 + 2*n)))/(d + 3*d*n)
```

IntegrateAlgebraic [F] time = 0.06, size = 0, normalized size = 0.00

$$\int (a+bx^n)(c+dx^n)^2 \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x^n)*(c + d*x^n)^2,x]
```

```
[Out] a*c^2*x + Defer[IntegrateAlgebraic][x^n*(b*c^2 + 2*a*c*d + 2*b*c*d*x^n + a*
d^2*x^n + b*d^2*x^(2*n)), x]
```

fricas [B] time = 0.91, size = 175, normalized size = 2.50

$$\frac{\left(2\,bd^2n^2+3\,bd^2n+bd^2\right)xx^{3\,n}+\left(2\,bcd+ad^2+3\left(2\,bcd+ad^2\right)n^2+4\left(2\,bcd+ad^2\right)n\right)xx^{2\,n}+\left(bc^2+2\,acd+6\left(bc^2+2\,acd\right)n^2+5\left(bc^2+2\,acd\right)n\right)xx^n+\left(6\,ac^2n^3+11\,ac^2n^2+6\,ac^2n+ac^2\right)x}{6\,n^3+11\,n^2+6\,n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)*(c+d*x^n)^2,x, algorithm="fricas")
```

```
[Out] ((2*b*d^2*n^2 + 3*b*d^2*n + b*d^2)*x*x^(3*n) + (2*b*c*d + a*d^2 + 3*(2*b*c*
d + a*d^2)*n^2 + 4*(2*b*c*d + a*d^2)*n)*x*x^(2*n) + (b*c^2 + 2*a*c*d + 6*(b
*c^2 + 2*a*c*d)*n^2 + 5*(b*c^2 + 2*a*c*d)*n)*x*x^n + (6*a*c^2*n^3 + 11*a*c^
2*n^2 + 6*a*c^2*n + a*c^2)*x)/(6*n^3 + 11*n^2 + 6*n + 1)
```

giac [B] time = 0.19, size = 232, normalized size = 3.31

$$\frac{6\,ac^2n^3x+2\,bd^2n^2xx^{3\,n}+6\,bcdn^2xx^{2\,n}+3\,ad^2n^2xx^{2\,n}+6\,bc^2n^2xx^n+12\,acdn^2xx^n+11\,ac^2n^2x+3\,bd^2nxx^{3\,n}+8\,bcdnxx^{2\,n}+4\,ad^2nxx^{2\,n}+5\,bc^2nxx^n+10\,acdnxx^n+6\,ac^2nx+bd^2xx^{3\,n}+2\,bcdxx^{2\,n}+ad^2xx^{2\,n}+bc^2xx^n+2\,acdxx^n+ac^2x}{6\,n^3+11\,n^2+6\,n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)*(c+d*x^n)^2,x, algorithm="giac")
```

```
[Out] (6*a*c^2*n^3*x + 2*b*d^2*n^2*x*x^(3*n) + 6*b*c*d*n^2*x*x^(2*n) + 3*a*d^2*n^
2*x*x^(2*n) + 6*b*c^2*n^2*x*x^n + 12*a*c*d*n^2*x*x^n + 11*a*c^2*n^2*x + 3*b
*d^2*n*x*x^(3*n) + 8*b*c*d*n*x*x^(2*n) + 4*a*d^2*n*x*x^(2*n) + 5*b*c^2*n*x*
x^n + 10*a*c*d*n*x*x^n + 6*a*c^2*n*x + b*d^2*x*x^(3*n) + 2*b*c*d*x*x^(2*n)
+ a*d^2*x*x^(2*n) + b*c^2*x*x^n + 2*a*c*d*x*x^n + a*c^2*x)/(6*n^3 + 11*n^2
+ 6*n + 1)
```

maple [A] time = 0.05, size = 74, normalized size = 1.06

$$\frac{b\,d^2x\,{\rm e}^{3n\ln(x)}}{3n+1}+a\,c^2x+\frac{(2ad+bc)\,cx\,{\rm e}^{n\ln(x)}}{n+1}+\frac{(ad+2bc)\,dx\,{\rm e}^{2n\ln(x)}}{2n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^n+a)*(c+d*x^n)^2,x)
```

```
[Out] a*c^2*x+b*d^2/(3*n+1)*x*exp(n*ln(x))^3+c*(2*a*d+b*c)/(n+1)*x*exp(n*ln(x))+d
*(a*d+2*b*c)/(2*n+1)*x*exp(n*ln(x))^2
```

maxima [A] time = 0.44, size = 94, normalized size = 1.34

$$ac^2x+\frac{bd^2x^{3\,n+1}}{3\,n+1}+\frac{2\,bcdx^{2\,n+1}}{2\,n+1}+\frac{ad^2x^{2\,n+1}}{2\,n+1}+\frac{bc^2x^{n+1}}{n+1}+\frac{2\,acdx^{n+1}}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)*(c+d*x^n)^2,x, algorithm="maxima")
```

```
[Out] a*c^2*x + b*d^2*x^(3*n + 1)/(3*n + 1) + 2*b*c*d*x^(2*n + 1)/(2*n + 1) + a*d
^2*x^(2*n + 1)/(2*n + 1) + b*c^2*x^(n + 1)/(n + 1) + 2*a*c*d*x^(n + 1)/(n +
 1)
```

mupad [B] time = 1.53, size = 71, normalized size = 1.01

$$a\,c^2\,x+\frac{x\,x^{2\,n}\,\left(a\,d^2+2\,b\,c\,d\right)}{2\,n+1}+\frac{x\,x^n\,\left(b\,c^2+2\,a\,d\,c\right)}{n+1}+\frac{b\,d^2\,x\,x^{3\,n}}{3\,n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^n)*(c + d*x^n)^2,x)
```

```
[Out] a*c^2*x + (x*x^(2*n)*(a*d^2 + 2*b*c*d))/(2*n + 1) + (x*x^n*(b*c^2 + 2*a*c*d
))/(n + 1) + (b*d^2*x*x^(3*n))/(3*n + 1)
```

sympy [**A**]   time = 1.96, size = 726, normalized size = 10.37

$$\begin{cases} ac^2x + 2acd\log(x) - \frac{ad^2}{x} + bc^2\log(x) - \frac{2bcd}{x} - \frac{bd^2}{2x^2} & \text{for } n = -1 \\ ac^2x + 4acd\sqrt{x} + ad^2\log(x) + 2bc^2\sqrt{x} + 2bcd\log(x) - \frac{2bd^2}{\sqrt{x}} & \text{for } n = -\frac{1}{2} \\ ac^2x + 3acdx^{\frac{2}{3}} + 3ad^2\sqrt[3]{x} + \frac{3bc^2x^{\frac{2}{3}}}{2} + 6bcd\sqrt[3]{x} + bd^2\log(x) & \text{for } n = -\frac{1}{3} \\ \frac{6ac^2n^3x}{6n^3+11n^2+6n+1} + \frac{11ac^2n^2x}{6n^3+11n^2+6n+1} + \frac{6ac^2nx}{6n^3+11n^2+6n+1} + \frac{ac^2x}{6n^3+11n^2+6n+1} + \frac{12acdn^2xx^n}{6n^3+11n^2+6n+1} + \frac{10acdnxx^n}{6n^3+11n^2+6n+1} + \frac{2acdxx^n}{6n^3+11n^2+6n+1} + \frac{3ad^2n^2xx^{2n}}{6n^3+11n^2+6n+1} + \frac{4ad^2nxx^{2n}}{6n^3+11n^2+6n+1} + \frac{ad^2xx^{2n}}{6n^3+11n^2+6n+1} + \frac{6bc^2n^2xx^n}{6n^3+11n^2+6n+1} + \frac{5bc^2nxx^n}{6n^3+11n^2+6n+1} + \frac{bc^2xx^n}{6n^3+11n^2+6n+1} + \frac{6bcdn^2xx^{2n}}{6n^3+11n^2+6n+1} + \frac{8bcdnxx^{2n}}{6n^3+11n^2+6n+1} + \frac{2bcdxx^{2n}}{6n^3+11n^2+6n+1} + \frac{2bd^2n^2xx^{3n}}{6n^3+11n^2+6n+1} + \frac{3bd^2nxx^{3n}}{6n^3+11n^2+6n+1} + \frac{bd^2xx^{3n}}{6n^3+11n^2+6n+1} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**n)*(c+d*x**n)**2,x)
```

```
[Out] Piecewise((a*c**2*x + 2*a*c*d*log(x) - a*d**2/x + b*c**2*log(x) - 2*b*c*d/x
 - b*d**2/(2*x**2), Eq(n, -1)), (a*c**2*x + 4*a*c*d*sqrt(x) + a*d**2*log(x)
 + 2*b*c**2*sqrt(x) + 2*b*c*d*log(x) - 2*b*d**2/sqrt(x), Eq(n, -1/2)), (a*c
**2*x + 3*a*c*d*x**(2/3) + 3*a*d**2*x**(1/3) + 3*b*c**2*x**(2/3)/2 + 6*b*c*
d*x**(1/3) + b*d**2*log(x), Eq(n, -1/3)), (6*a*c**2*n**3*x/(6*n**3 + 11*n**
2 + 6*n + 1) + 11*a*c**2*n**2*x/(6*n**3 + 11*n**2 + 6*n + 1) + 6*a*c**2*n*x
/(6*n**3 + 11*n**2 + 6*n + 1) + a*c**2*x/(6*n**3 + 11*n**2 + 6*n + 1) + 12*
a*c*d*n**2*x*x**n/(6*n**3 + 11*n**2 + 6*n + 1) + 10*a*c*d*n*x*x**n/(6*n**3
+ 11*n**2 + 6*n + 1) + 2*a*c*d*x*x**n/(6*n**3 + 11*n**2 + 6*n + 1) + 3*a*d*
*2*n**2*x*x**(2*n)/(6*n**3 + 11*n**2 + 6*n + 1) + 4*a*d**2*n*x*x**(2*n)/(6*
n**3 + 11*n**2 + 6*n + 1) + a*d**2*x*x**(2*n)/(6*n**3 + 11*n**2 + 6*n + 1)
+ 6*b*c**2*n**2*x*x**n/(6*n**3 + 11*n**2 + 6*n + 1) + 5*b*c**2*n*x*x**n/(6*
n**3 + 11*n**2 + 6*n + 1) + b*c**2*x*x**n/(6*n**3 + 11*n**2 + 6*n + 1) + 6*
b*c*d*n**2*x*x**(2*n)/(6*n**3 + 11*n**2 + 6*n + 1) + 8*b*c*d*n*x*x**(2*n)/(
6*n**3 + 11*n**2 + 6*n + 1) + 2*b*c*d*x*x**(2*n)/(6*n**3 + 11*n**2 + 6*n +
1) + 2*b*d**2*n**2*x*x**(3*n)/(6*n**3 + 11*n**2 + 6*n + 1) + 3*b*d**2*n*x*x
**(3*n)/(6*n**3 + 11*n**2 + 6*n + 1) + b*d**2*x*x**(3*n)/(6*n**3 + 11*n**2
+ 6*n + 1), True))
```

3.182 $\int (a+bx^n)(c+dx^n)\,dx$

Optimal. Leaf size=40

$$\frac{x^{n+1}(ad+bc)}{n+1}+acx+\frac{bdx^{2n+1}}{2n+1}$$

Rubi [A] time = 0.02, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {373}

$$\frac{x^{n+1}(ad+bc)}{n+1}+acx+\frac{bdx^{2n+1}}{2n+1}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^n)*(c + d*x^n),x]
```

```
[Out] a*c*x + ((b*c + a*d)*x^(1 + n))/(1 + n) + (b*d*x^(1 + 2*n))/(1 + 2*n)
```

Rule 373

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int (a+bx^n)(c+dx^n)\,dx &= \int \left(ac+(bc+ad)x^n+bdx^{2n}\right)dx\\ &= acx+\frac{(bc+ad)x^{1+n}}{1+n}+\frac{bdx^{1+2n}}{1+2n}\end{aligned}$$

Mathematica [A] time = 0.08, size = 37, normalized size = 0.92

$$x\left(\frac{x^n(ad+bc)}{n+1}+ac+\frac{bdx^{2n}}{2n+1}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^n)*(c + d*x^n),x]
```

```
[Out] x*(a*c + ((b*c + a*d)*x^n)/(1 + n) + (b*d*x^(2*n))/(1 + 2*n))
```

IntegrateAlgebraic [F] time = 0.04, size = 0, normalized size = 0.00

$$\int (a+bx^n)(c+dx^n)\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x^n)*(c + d*x^n),x]
```

```
[Out] a*c*x + Defer[IntegrateAlgebraic][x^n*(b*c + a*d + b*d*x^n), x]
```

fricas [A] time = 0.94, size = 69, normalized size = 1.72

$$\frac{(bdn+bd)xx^{2n}+(bc+ad+2(bc+ad)n)xx^n+\left(2acn^2+3acn+ac\right)x}{2n^2+3n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)*(c+d*x^n),x, algorithm="fricas")
```

```
[Out] ((b*d*n + b*d)*x*x^(2*n) + (b*c + a*d + 2*(b*c + a*d)*n)*x*x^n + (2*a*c*n^2
 + 3*a*c*n + a*c)*x)/(2*n^2 + 3*n + 1)
```

giac [B] time = 0.17, size = 83, normalized size = 2.08

$$\frac{2\,acn^2x + bdnxx^{2\,n} + 2\,bcnxx^n + 2\,adnxx^n + 3\,acnx + bdxx^{2\,n} + bcxx^n + adxx^n + acx}{2\,n^2 + 3\,n + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)*(c+d*x^n),x, algorithm="giac")
```

```
[Out] (2*a*c*n^2*x + b*d*n*x*x^(2*n) + 2*b*c*n*x*x^n + 2*a*d*n*x*x^n + 3*a*c*n*x
+ b*d*x*x^(2*n) + b*c*x*x^n + a*d*x*x^n + a*c*x)/(2*n^2 + 3*n + 1)
```

maple [A] time = 0.04, size = 43, normalized size = 1.08

$$\frac{bdx\,\mathrm{e}^{2n\ln(x)}}{2n+1} + acx + \frac{(ad+bc)\,x\,\mathrm{e}^{n\ln(x)}}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^n+a)*(c+d*x^n),x)
```

```
[Out] a*c*x+(a*d+b*c)/(n+1)*x*exp(n*ln(x))+b*d/(2*n+1)*x*exp(n*ln(x))^2
```

maxima [A] time = 0.51, size = 48, normalized size = 1.20

$$acx + \frac{bdx^{2\,n+1}}{2\,n+1} + \frac{bcx^{n+1}}{n+1} + \frac{adx^{n+1}}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)*(c+d*x^n),x, algorithm="maxima")
```

```
[Out] a*c*x + b*d*x^(2*n + 1)/(2*n + 1) + b*c*x^(n + 1)/(n + 1) + a*d*x^(n + 1)/(
n + 1)
```

mupad [B] time = 1.48, size = 38, normalized size = 0.95

$$a\,c\,x + \frac{x\,x^n\,\left(a\,d+b\,c\right)}{n+1} + \frac{b\,d\,x\,x^{2\,n}}{2\,n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^n)*(c + d*x^n),x)
```

```
[Out] a*c*x + (x*x^n*(a*d + b*c))/(n + 1) + (b*d*x*x^(2*n))/(2*n + 1)
```

sympy [A] time = 0.65, size = 236, normalized size = 5.90

$$\begin{cases} acx + ad\log{(x)} + bc\log{(x)} - \frac{bd}{x} & \text{for}\: n = -1 \\ acx + 2ad\sqrt{x} + 2bc\sqrt{x} + bd\log{(x)} & \text{for}\: n = -\frac{1}{2} \\ \frac{2acn^2x}{2n^2+3n+1} + \frac{3acnx}{2n^2+3n+1} + \frac{acx}{2n^2+3n+1} + \frac{2adnxx^n}{2n^2+3n+1} + \frac{adxx^n}{2n^2+3n+1} + \frac{2bcnxx^n}{2n^2+3n+1} + \frac{bcxx^n}{2n^2+3n+1} + \frac{bdnxx^{2n}}{2n^2+3n+1} + \frac{bdxx^{2n}}{2n^2+3n+1} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**n)*(c+d*x**n),x)
```

```
[Out] Piecewise((a*c*x + a*d*log(x) + b*c*log(x) - b*d/x, Eq(n, -1)), (a*c*x + 2*
a*d*sqrt(x) + 2*b*c*sqrt(x) + b*d*log(x), Eq(n, -1/2)), (2*a*c*n**2*x/(2*n*
*2 + 3*n + 1) + 3*a*c*n*x/(2*n**2 + 3*n + 1) + a*c*x/(2*n**2 + 3*n + 1) + 2
*a*d*n*x*x**n/(2*n**2 + 3*n + 1) + a*d*x*x**n/(2*n**2 + 3*n + 1) + 2*b*c*n*
x*x**n/(2*n**2 + 3*n + 1) + b*c*x*x**n/(2*n**2 + 3*n + 1) + b*d*n*x*x**(2*n
)/(2*n**2 + 3*n + 1) + b*d*x*x**(2*n)/(2*n**2 + 3*n + 1), True))
```

3.183 $\int (a+bx^n)^2 (d+ex^n)^3\, dx$

Optimal. Leaf size=158

$$\frac{dx^{2n+1}\left(3a^2e^2+6abde+b^2d^2\right)}{2n+1}+\frac{ex^{3n+1}\left(a^2e^2+6abde+3b^2d^2\right)}{3n+1}+a^2d^3x+\frac{ad^2x^{n+1}(3ae+2bd)}{n+1}+\frac{be^2x^{4n+1}(2ae+3bd)}{4n+1}$$

Rubi [A] time = 0.13, antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {373}

$$\frac{dx^{2n+1}\left(3a^2e^2+6abde+b^2d^2\right)}{2n+1}+\frac{ex^{3n+1}\left(a^2e^2+6abde+3b^2d^2\right)}{3n+1}+a^2d^3x+\frac{ad^2x^{n+1}(3ae+2bd)}{n+1}+\frac{be^2x^{4n+1}(2ae+3bd)}{4n+1}+\frac{b^2e^3x^{5n+1}}{5n+1}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^n)^2*(d + e*x^n)^3,x]
```

```
[Out] a^2*d^3*x + (a*d^2*(2*b*d + 3*a*e)*x^(1 + n))/(1 + n) + (d*(b^2*d^2 + 6*a*b
*d*e + 3*a^2*e^2)*x^(1 + 2*n))/(1 + 2*n) + (e*(3*b^2*d^2 + 6*a*b*d*e + a^2*
e^2)*x^(1 + 3*n))/(1 + 3*n) + (b*e^2*(3*b*d + 2*a*e)*x^(1 + 4*n))/(1 + 4*n)
 + (b^2*e^3*x^(1 + 5*n))/(1 + 5*n)
```

Rule 373

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\int (a+bx^n)^2 (d+ex^n)^3\, dx = \int \left(a^2d^3+ad^2(2bd+3ae)x^n+d\left(b^2d^2+6abde+3a^2e^2\right)x^{2n}+e\left(3b^2d^2+6abde\right.\right.$$

$$= a^2d^3x+\frac{ad^2(2bd+3ae)x^{1+n}}{1+n}+\frac{d\left(b^2d^2+6abde+3a^2e^2\right)x^{1+2n}}{1+2n}+\frac{e\left(3b^2d^2+6ab\right.}{1+}$$

Mathematica [A] time = 0.21, size = 149, normalized size = 0.94

$$x\left(\frac{dx^{2n}\left(3a^2e^2+6abde+b^2d^2\right)}{2n+1}+\frac{ex^{3n}\left(a^2e^2+6abde+3b^2d^2\right)}{3n+1}+a^2d^3+\frac{ad^2x^n(3ae+2bd)}{n+1}+\frac{be^2x^{4n}(2ae+3bd)}{4n+1}+\frac{b^2e^3x^{5n}}{5n+1}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^n)^2*(d + e*x^n)^3,x]
```

```
[Out] x*(a^2*d^3 + (a*d^2*(2*b*d + 3*a*e)*x^n)/(1 + n) + (d*(b^2*d^2 + 6*a*b*d*e
+ 3*a^2*e^2)*x^(2*n))/(1 + 2*n) + (e*(3*b^2*d^2 + 6*a*b*d*e + a^2*e^2)*x^(3
*n))/(1 + 3*n) + (b*e^2*(3*b*d + 2*a*e)*x^(4*n))/(1 + 4*n) + (b^2*e^3*x^(5*
n))/(1 + 5*n))
```

IntegrateAlgebraic [F] time = 0.60, size = 0, normalized size = 0.00

$$\int (a+bx^n)^2 (d+ex^n)^3\, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x^n)^2*(d + e*x^n)^3,x]
```

[Out] a^2*d^3*x + Defer[IntegrateAlgebraic][x^n*(2*a*b*d^3 + 3*a^2*d^2*e + b^2*d^3*x^n + 6*a*b*d^2*e*x^n + 3*a^2*d*e^2*x^n + 3*b^2*d^2*e*x^(2*n) + 6*a*b*d*e^2*x^(2*n) + a^2*e^3*x^(2*n) + 3*b^2*d*e^2*x^(3*n) + 2*a*b*e^3*x^(3*n) + b^2*e^3*x^(4*n)), x]

fricas [B] time = 0.88, size = 667, normalized size = 4.22

$$\frac{\left(24 b^2e^3n^4 + 50 b^2e^3n^3 + 35 b^2e^3n^2 + 10 b^2e^3n + b^2e^3\right)xx^{5n} + \left(3b^2de^2 + 2abe^3 + 30\left(3b^2de^2 + 2abe^3\right)n^4 + 61\left(3b^2de^2 + 2abe^3\right)n^3 + 41\left(3b^2de^2 + 2abe^3\right)n^2 + 11\left(3b^2de^2 + 2abe^3\right)n\right)xx^{4n} + \left(3b^2d^2e + 6abde^2 + a^2e^3 + 40\left(3b^2d^2e + 6abde^2 + a^2e^3\right)n^4 + 78\left(3b^2d^2e + 6abde^2 + a^2e^3\right)n^3 + 49\left(3b^2d^2e + 6abde^2 + a^2e^3\right)n^2 + 12\left(3b^2d^2e + 6abde^2 + a^2e^3\right)n\right)xx^{3n} + \left(b^2d^3 + 6abd^2e + 3a^2de^2 + 60\left(b^2d^3 + 6abd^2e + 3a^2de^2\right)n^4 + 107\left(b^2d^3 + 6abd^2e + 3a^2de^2\right)n^3 + 59\left(b^2d^3 + 6abd^2e + 3a^2de^2\right)n^2 + 13\left(b^2d^3 + 6abd^2e + 3a^2de^2\right)n\right)xx^{2n} + \left(2abd^3 + 3a^2d^2e + 120\left(2abd^3 + 3a^2d^2e\right)n^4 + 154\left(2abd^3 + 3a^2d^2e\right)n^3 + 71\left(2abd^3 + 3a^2d^2e\right)n^2 + 14\left(2abd^3 + 3a^2d^2e\right)n\right)xx^n + \left(120a^2d^3n^5 + 274a^2d^3n^4 + 225a^2d^3n^3 + 85a^2d^3n^2 + 15a^2d^3n + a^2d^3\right)x}{120n^5 + 274n^4 + 225n^3 + 85n^2 + 15n + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*x^n)^2*(d+e*x^n)^3,x, algorithm="fricas")

[Out] ((24*b^2*e^3*n^4 + 50*b^2*e^3*n^3 + 35*b^2*e^3*n^2 + 10*b^2*e^3*n + b^2*e^3)*x*x^(5*n) + (3*b^2*d*e^2 + 2*a*b*e^3 + 30*(3*b^2*d*e^2 + 2*a*b*e^3)*n^4 + 61*(3*b^2*d*e^2 + 2*a*b*e^3)*n^3 + 41*(3*b^2*d*e^2 + 2*a*b*e^3)*n^2 + 11*(3*b^2*d*e^2 + 2*a*b*e^3)*n)*x*x^(4*n) + (3*b^2*d^2*e + 6*a*b*d*e^2 + a^2*e^3 + 40*(3*b^2*d^2*e + 6*a*b*d*e^2 + a^2*e^3)*n^4 + 78*(3*b^2*d^2*e + 6*a*b*d*e^2 + a^2*e^3)*n^3 + 49*(3*b^2*d^2*e + 6*a*b*d*e^2 + a^2*e^3)*n^2 + 12*(3*b^2*d^2*e + 6*a*b*d*e^2 + a^2*e^3)*n)*x*x^(3*n) + (b^2*d^3 + 6*a*b*d^2*e + 3*a^2*d*e^2 + 60*(b^2*d^3 + 6*a*b*d^2*e + 3*a^2*d*e^2)*n^4 + 107*(b^2*d^3 + 6*a*b*d^2*e + 3*a^2*d*e^2)*n^3 + 59*(b^2*d^3 + 6*a*b*d^2*e + 3*a^2*d*e^2)*n^2 + 13*(b^2*d^3 + 6*a*b*d^2*e + 3*a^2*d*e^2)*n)*x*x^(2*n) + (2*a*b*d^3 + 3*a^2*d^2*e + 120*(2*a*b*d^3 + 3*a^2*d^2*e)*n^4 + 154*(2*a*b*d^3 + 3*a^2*d^2*e)*n^3 + 71*(2*a*b*d^3 + 3*a^2*d^2*e)*n^2 + 14*(2*a*b*d^3 + 3*a^2*d^2*e)*n)*x*x^n + (120*a^2*d^3*n^5 + 274*a^2*d^3*n^4 + 225*a^2*d^3*n^3 + 85*a^2*d^3*n^2 + 15*a^2*d^3*n + a^2*d^3)*x)/(120*n^5 + 274*n^4 + 225*n^3 + 85*n^2 + 15*n + 1)

giac [B] time = 0.24, size = 947, normalized size = 5.99

$$\frac{120a^2d^3n^5x + 60b^2d^3n^4xx^{2n} + 240abd^3n^4xx^n + 120b^2d^2n^4xx^{3n}e + 360abd^2n^4xx^{2n}e + 360a^2d^2n^4xx^ne + 274a^2d^3n^4x + 107b^2d^3n^3xx^{2n} + 308abd^3n^3xx^n + 90b^2dn^4xx^{4n}e^2 + 240abdn^4xx^{3n}e^2 + 180a^2dn^4xx^{2n}e^2 + 234b^2d^2n^3xx^{3n}e + 642abd^2n^3xx^{2n}e + 462a^2d^2n^3xx^ne + 225a^2d^3n^3x + 59b^2d^3n^2xx^{2n} + 142abd^3n^2xx^n + 24b^2n^4xx^{5n}e^3 + 60abn^4xx^{4n}e^3 + 40a^2n^4xx^{3n}e^3 + 183b^2dn^3xx^{4n}e^2 + 468abdn^3xx^{3n}e^2 + 321a^2dn^3xx^{2n}e^2 + 147b^2d^2n^2xx^{3n}e + 354abd^2n^2xx^{2n}e + 213a^2d^2n^2xx^ne + 85a^2d^3n^2x + 13b^2d^3nxx^{2n} + 28abd^3nxx^n + 50b^2n^3xx^{5n}e^3 + 122abn^3xx^{4n}e^3 + 78a^2n^3xx^{3n}e^3 + 123b^2dn^2xx^{4n}e^2 + 294abdn^2xx^{3n}e^2 + 177a^2dn^2xx^{2n}e^2 + 36b^2d^2nxx^{3n}e + 78abd^2nxx^{2n}e + 42a^2d^2nxx^ne + 15a^2d^3nx + b^2d^3xx^{2n} + 2abd^3xx^n + 35b^2n^2xx^{5n}e^3 + 82abn^2xx^{4n}e^3 + 49a^2n^2xx^{3n}e^3 + 33b^2dnxx^{4n}e^2 + 72abdnxx^{3n}e^2 + 39a^2dnxx^{2n}e^2 + 3b^2d^2xx^{3n}e + 6abd^2xx^{2n}e + 3a^2d^2xx^ne + a^2d^3x + 10b^2nxx^{5n}e^3 + 22abnxx^{4n}e^3 + 12a^2nxx^{3n}e^3 + 3b^2dxx^{4n}e^2 + 6abdxx^{3n}e^2 + 3a^2dxx^{2n}e^2 + b^2xx^{5n}e^3 + 2abxx^{4n}e^3 + a^2xx^{3n}e^3}{120n^5 + 274n^4 + 225n^3 + 85n^2 + 15n + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*x^n)^2*(d+e*x^n)^3,x, algorithm="giac")

[Out] (120*a^2*d^3*n^5*x + 60*b^2*d^3*n^4*x*x^(2*n) + 240*a*b*d^3*n^4*x*x^n + 120*b^2*d^2*n^4*x*x^(3*n)*e + 360*a*b*d^2*n^4*x*x^(2*n)*e + 360*a^2*d^2*n^4*x*x^n*e + 274*a^2*d^3*n^4*x + 107*b^2*d^3*n^3*x*x^(2*n) + 308*a*b*d^3*n^3*x*x^n + 90*b^2*d*n^4*x*x^(4*n)*e^2 + 240*a*b*d*n^4*x*x^(3*n)*e^2 + 180*a^2*d*n^4*x*x^(2*n)*e^2 + 234*b^2*d^2*n^3*x*x^(3*n)*e + 642*a*b*d^2*n^3*x*x^(2*n)*e + 462*a^2*d^2*n^3*x*x^n*e + 225*a^2*d^3*n^3*x + 59*b^2*d^3*n^2*x*x^(2*n) + 142*a*b*d^3*n^2*x*x^n + 24*b^2*n^4*x*x^(5*n)*e^3 + 60*a*b*n^4*x*x^(4*n)*e^3 + 40*a^2*n^4*x*x^(3*n)*e^3 + 183*b^2*d*n^3*x*x^(4*n)*e^2 + 468*a*b*d*n^3*x*x^(3*n)*e^2 + 321*a^2*d*n^3*x*x^(2*n)*e^2 + 147*b^2*d^2*n^2*x*x^(3*n)*e + 354*a*b*d^2*n^2*x*x^(2*n)*e + 213*a^2*d^2*n^2*x*x^n*e + 85*a^2*d^3*n^2*x + 13*b^2*d^3*n*x*x^(2*n) + 28*a*b*d^3*n*x*x^n + 50*b^2*n^3*x*x^(5*n)*e^3 + 122*a*b*n^3*x*x^(4*n)*e^3 + 78*a^2*n^3*x*x^(3*n)*e^3 + 123*b^2*d*n^2*x*x^(4*n)*e^2 + 294*a*b*d*n^2*x*x^(3*n)*e^2 + 177*a^2*d*n^2*x*x^(2*n)*e^2 + 36*b^2*d^2*n*x*x^(3*n)*e + 78*a*b*d^2*n*x*x^(2*n)*e + 42*a^2*d^2*n*x*x^n*e + 15*a^2*d^3*n*x + b^2*d^3*x*x^(2*n) + 2*a*b*d^3*x*x^n + 35*b^2*n^2*x*x^(5*n)*e^3 + 82*a*b*n^2*x*x^(4*n)*e^3 + 49*a^2*n^2*x*x^(3*n)*e^3 + 33*b^2*d*n*x*x^(4*n)*e^2 + 72*a*b*d*n*x*x^(3*n)*e^2 + 39*a^2*d*n*x*x^(2*n)*e^2 + 3*b^2*d^2*x*x^(3*n)*e + 6*a*b*d^2*x*x^(2*n)*e + 3*a^2*d^2*x*x^n*e + a^2*d^3*x + 10*b^2*n*x*x^(5*n)*e^3 + 22*a*b*n*x*x^(4*n)*e^3 + 12*a^2*n*x*x^(3*n)*e^3 + 3*b^2*d*x*x^(4*n)*e^2 + 6*a*b*d*x*x^(3*n)*e^2 + 3*a^2*d*x*x^(2*n)*e^2 + b^2*x*x^(5*n)*e^3 + 2*a*b*x*x^(4*n)*e^3 + a^2*x*x^(3*n)*e^3)/(120*n^5 + 274*n^4 + 225*n^3 + 85*n^2 + 15*n + 1)

maple [A] time = 0.06, size = 164, normalized size = 1.04

$$\frac{b^2e^3x\,\mathrm{e}^{5n\ln(x)}}{5n+1} + a^2d^3x + \frac{(3ae+2bd)\,a\,d^2x\,\mathrm{e}^{n\ln(x)}}{n+1} + \frac{(2ae+3bd)\,b\,e^2x\,\mathrm{e}^{4n\ln(x)}}{4n+1} + \frac{\left(3a^2e^2+6abde+b^2d^2\right)dx\,\mathrm{e}^{2n\ln(x)}}{2n+1} + \frac{\left(a^2e^2+6abde+3b^2d^2\right)ex\,\mathrm{e}^{3n\ln(x)}}{3n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^n+a)^2*(d+e*x^n)^3,x)
```

```
[Out] a^2*d^3*x+b^2*e^3/(5*n+1)*x*exp(n*ln(x))^5+d*(3*a^2*e^2+6*a*b*d*e+b^2*d^2)/
(2*n+1)*x*exp(n*ln(x))^2+e*(a^2*e^2+6*a*b*d*e+3*b^2*d^2)/(3*n+1)*x*exp(n*ln
(x))^3+a*d^2*(3*a*e+2*b*d)/(n+1)*x*exp(n*ln(x))+b*e^2*(2*a*e+3*b*d)/(4*n+1)
*x*exp(n*ln(x))^4
```

maxima [A] time = 0.62, size = 242, normalized size = 1.53

$$a^2d^3x + \frac{b^2e^3x^{5n+1}}{5n+1} + \frac{3b^2de^2x^{4n+1}}{4n+1} + \frac{2abe^3x^{4n+1}}{4n+1} + \frac{3b^2d^2ex^{3n+1}}{3n+1} + \frac{6abde^2x^{3n+1}}{3n+1} + \frac{a^2e^3x^{3n+1}}{3n+1} + \frac{b^2d^3x^{2n+1}}{2n+1} + \frac{6abd^2ex^{2n+1}}{2n+1} + \frac{3a^2de^2x^{2n+1}}{2n+1} + \frac{2abd^3x^{n+1}}{n+1} + \frac{3a^2d^2ex^{n+1}}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^2*(d+e*x^n)^3,x, algorithm="maxima")
```

```
[Out] a^2*d^3*x + b^2*e^3*x^(5*n + 1)/(5*n + 1) + 3*b^2*d*e^2*x^(4*n + 1)/(4*n +
1) + 2*a*b*e^3*x^(4*n + 1)/(4*n + 1) + 3*b^2*d^2*e*x^(3*n + 1)/(3*n + 1) +
6*a*b*d*e^2*x^(3*n + 1)/(3*n + 1) + a^2*e^3*x^(3*n + 1)/(3*n + 1) + b^2*d^3
*x^(2*n + 1)/(2*n + 1) + 6*a*b*d^2*e*x^(2*n + 1)/(2*n + 1) + 3*a^2*d*e^2*x^
(2*n + 1)/(2*n + 1) + 2*a*b*d^3*x^(n + 1)/(n + 1) + 3*a^2*d^2*e*x^(n + 1)/(
n + 1)
```

mupad [B] time = 1.71, size = 157, normalized size = 0.99

$$a^2\,d^3\,x + \frac{x\,x^{2\,n}\,\left(3\,a^2\,d\,e^2 + 6\,a\,b\,d^2\,e + b^2\,d^3\right)}{2\,n+1} + \frac{x\,x^{3\,n}\,\left(a^2\,e^3 + 6\,a\,b\,d\,e^2 + 3\,b^2\,d^2\,e\right)}{3\,n+1} + \frac{b^2\,e^3\,x\,x^{5\,n}}{5\,n+1} + \frac{a\,d^2\,x\,x^n\,\left(3\,a\,e + 2\,b\,d\right)}{n+1} + \frac{b\,e^2\,x\,x^{4\,n}\,\left(2\,a\,e + 3\,b\,d\right)}{4\,n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^n)^2*(d + e*x^n)^3,x)
```

```
[Out] a^2*d^3*x + (x*x^(2*n)*(b^2*d^3 + 3*a^2*d*e^2 + 6*a*b*d^2*e))/(2*n + 1) + (
x*x^(3*n)*(a^2*e^3 + 3*b^2*d^2*e + 6*a*b*d*e^2))/(3*n + 1) + (b^2*e^3*x*x^(
5*n))/(5*n + 1) + (a*d^2*x*x^n*(3*a*e + 2*b*d))/(n + 1) + (b*e^2*x*x^(4*n)*
(2*a*e + 3*b*d))/(4*n + 1)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**n)**2*(d+e*x**n)**3,x)
```

```
[Out] Timed out
```

3.184 $\int (a+bx^n)^2 (d+ex^n)^2 \, dx$

Optimal. Leaf size=112

$$\frac{x^{2n+1}\left(a^2e^2+4abde+b^2d^2\right)}{2n+1}+a^2d^2x+\frac{2adx^{n+1}(ae+bd)}{n+1}+\frac{2bex^{3n+1}(ae+bd)}{3n+1}+\frac{b^2e^2x^{4n+1}}{4n+1}$$

Rubi [A] time = 0.08, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = {373}

$$\frac{x^{2n+1}\left(a^2e^2+4abde+b^2d^2\right)}{2n+1}+a^2d^2x+\frac{2adx^{n+1}(ae+bd)}{n+1}+\frac{2bex^{3n+1}(ae+bd)}{3n+1}+\frac{b^2e^2x^{4n+1}}{4n+1}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^n)^2*(d + e*x^n)^2,x]
```

```
[Out] a^2*d^2*x + (2*a*d*(b*d + a*e)*x^(1 + n))/(1 + n) + ((b^2*d^2 + 4*a*b*d*e +
 a^2*e^2)*x^(1 + 2*n))/(1 + 2*n) + (2*b*e*(b*d + a*e)*x^(1 + 3*n))/(1 + 3*n
) + (b^2*e^2*x^(1 + 4*n))/(1 + 4*n)
```

Rule 373

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int (a+bx^n)^2 (d+ex^n)^2 \, dx &= \int \left(a^2d^2+2ad(bd+ae)x^n+\left(b^2d^2+4abde+a^2e^2\right)x^{2n}+2be(bd+ae)x^{3n}+b^2e^2x^{4n}\right) dx\\ &= a^2d^2x+\frac{2ad(bd+ae)x^{1+n}}{1+n}+\frac{\left(b^2d^2+4abde+a^2e^2\right)x^{1+2n}}{1+2n}+\frac{2be(bd+ae)x^{1+3n}}{1+3n}+\frac{b^2}{}\end{aligned}$$

Mathematica [A] time = 0.20, size = 105, normalized size = 0.94

$$x\left(\frac{x^{2n}\left(a^2e^2+4abde+b^2d^2\right)}{2n+1}+a^2d^2+\frac{2bex^{3n}(ae+bd)}{3n+1}+\frac{2adx^n(ae+bd)}{n+1}+\frac{b^2e^2x^{4n}}{4n+1}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^n)^2*(d + e*x^n)^2,x]
```

```
[Out] x*(a^2*d^2 + (2*a*d*(b*d + a*e)*x^n)/(1 + n) + ((b^2*d^2 + 4*a*b*d*e + a^2*
e^2)*x^(2*n))/(1 + 2*n) + (2*b*e*(b*d + a*e)*x^(3*n))/(1 + 3*n) + (b^2*e^2*
x^(4*n))/(1 + 4*n))
```

IntegrateAlgebraic [F] time = 0.62, size = 0, normalized size = 0.00

$$\int (a+bx^n)^2 (d+ex^n)^2 \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x^n)^2*(d + e*x^n)^2,x]
```

```
[Out] a^2*d^2*x + Defer[IntegrateAlgebraic][x^n*(2*a*b*d^2 + 2*a^2*d*e + b^2*d^2*
x^n + 4*a*b*d*e*x^n + a^2*e^2*x^n + 2*b^2*d*e*x^(2*n) + 2*a*b*e^2*x^(2*n) +
 b^2*e^2*x^(3*n)), x]
```

fricas [B] time = 0.84, size = 370, normalized size = 3.30

$$\frac{\left(6\,b^2e^2n^3+11\,b^2e^2n^2+6\,b^2e^2n+b^2e^2\right)xx^{4\,n}+2\left(b^2de+abe^2+8\left(b^2de+abe^2\right)n^3+14\left(b^2de+abe^2\right)n^2+7\left(b^2de+abe^2\right)n\right)xx^{3\,n}+\left(b^2d^2+4\,abde+a^2e^2+12\left(b^2d^2+4\,abde+a^2e^2\right)n^3+19\left(b^2d^2+4\,abde+a^2e^2\right)n^2+8\left(b^2d^2+4\,abde+a^2e^2\right)n\right)xx^{2\,n}+2\left(abd^2+a^2de+24\left(abd^2+a^2de\right)n^3+26\left(abd^2+a^2de\right)n^2+9\left(abd^2+a^2de\right)n\right)xx^{n}+\left(24\,a^2d^2n^4+50\,a^2d^2n^3+35\,a^2d^2n^2+10\,a^2d^2n+a^2d^2\right)x}{24\,n^4+50\,n^3+35\,n^2+10\,n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^2*(d+e*x^n)^2,x, algorithm="fricas")
```

```
[Out] ((6*b^2*e^2*n^3 + 11*b^2*e^2*n^2 + 6*b^2*e^2*n + b^2*e^2)*x*x^(4*n) + 2*(b^
2*d*e + a*b*e^2 + 8*(b^2*d*e + a*b*e^2)*n^3 + 14*(b^2*d*e + a*b*e^2)*n^2 +
7*(b^2*d*e + a*b*e^2)*n)*x*x^(3*n) + (b^2*d^2 + 4*a*b*d*e + a^2*e^2 + 12*(b
^2*d^2 + 4*a*b*d*e + a^2*e^2)*n^3 + 19*(b^2*d^2 + 4*a*b*d*e + a^2*e^2)*n^2
+ 8*(b^2*d^2 + 4*a*b*d*e + a^2*e^2)*n)*x*x^(2*n) + 2*(a*b*d^2 + a^2*d*e + 2
4*(a*b*d^2 + a^2*d*e)*n^3 + 26*(a*b*d^2 + a^2*d*e)*n^2 + 9*(a*b*d^2 + a^2*d
*e)*n)*x*x^n + (24*a^2*d^2*n^4 + 50*a^2*d^2*n^3 + 35*a^2*d^2*n^2 + 10*a^2*d
^2*n + a^2*d^2)*x)/(24*n^4 + 50*n^3 + 35*n^2 + 10*n + 1)
```

giac [B] time = 0.22, size = 539, normalized size = 4.81

$$\frac{24\,a^2d^2n^4x+12\,b^2d^2n^3xx^{2\,n}+48\,abd^2n^3xx^{n}+16\,b^2dn^3xx^{3\,n}e+48\,abdn^3xx^{2\,n}e+48\,a^2dn^3xx^{n}e+50\,a^2d^2n^3x+19\,b^2d^2n^2xx^{2\,n}+52\,abd^2n^2xx^{n}+6\,b^2n^3xx^{4\,n}e^2+16\,abn^3xx^{3\,n}e^2+12\,a^2n^3xx^{2\,n}e^2+28\,b^2dn^2xx^{3\,n}e+76\,abdn^2xx^{2\,n}e+52\,a^2dn^2xx^{n}e+35\,a^2d^2n^2x+8\,b^2d^2nxx^{2\,n}+18\,abd^2nxx^{n}+11\,b^2n^2xx^{4\,n}e^2+28\,abn^2xx^{3\,n}e^2+19\,a^2n^2xx^{2\,n}e^2+14\,b^2dnxx^{3\,n}e+32\,abdnxx^{2\,n}e+18\,a^2dnxx^{n}e+10\,a^2d^2nx+b^2d^2xx^{2\,n}+2\,abd^2xx^{n}+6\,b^2nxx^{4\,n}e^2+14\,abnxx^{3\,n}e^2+8\,a^2nxx^{2\,n}e^2+2\,b^2dxx^{3\,n}e+4\,abdxx^{2\,n}e+2\,a^2dxx^{n}e+a^2d^2x+b^2xx^{4\,n}e^2+2\,abxx^{3\,n}e^2+a^2xx^{2\,n}e^2}{24\,n^4+50\,n^3+35\,n^2+10\,n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^2*(d+e*x^n)^2,x, algorithm="giac")
```

```
[Out] (24*a^2*d^2*n^4*x + 12*b^2*d^2*n^3*x*x^(2*n) + 48*a*b*d^2*n^3*x*x^n + 16*b^
2*d*n^3*x*x^(3*n)*e + 48*a*b*d*n^3*x*x^(2*n)*e + 48*a^2*d*n^3*x*x^n*e + 50*
a^2*d^2*n^3*x + 19*b^2*d^2*n^2*x*x^(2*n) + 52*a*b*d^2*n^2*x*x^n + 6*b^2*n^3
*x*x^(4*n)*e^2 + 16*a*b*n^3*x*x^(3*n)*e^2 + 12*a^2*n^3*x*x^(2*n)*e^2 + 28*b
^2*d*n^2*x*x^(3*n)*e + 76*a*b*d*n^2*x*x^(2*n)*e + 52*a^2*d*n^2*x*x^n*e + 35
*a^2*d^2*n^2*x + 8*b^2*d^2*n*x*x^(2*n) + 18*a*b*d^2*n*x*x^n + 11*b^2*n^2*x*
x^(4*n)*e^2 + 28*a*b*n^2*x*x^(3*n)*e^2 + 19*a^2*n^2*x*x^(2*n)*e^2 + 14*b^2*
d*n*x*x^(3*n)*e + 32*a*b*d*n*x*x^(2*n)*e + 18*a^2*d*n*x*x^n*e + 10*a^2*d^2*
n*x + b^2*d^2*x*x^(2*n) + 2*a*b*d^2*x*x^n + 6*b^2*n*x*x^(4*n)*e^2 + 14*a*b*
n*x*x^(3*n)*e^2 + 8*a^2*n*x*x^(2*n)*e^2 + 2*b^2*d*x*x^(3*n)*e + 4*a*b*d*x*x
^(2*n)*e + 2*a^2*d*x*x^n*e + a^2*d^2*x + b^2*x*x^(4*n)*e^2 + 2*a*b*x*x^(3*n
)*e^2 + a^2*x*x^(2*n)*e^2)/(24*n^4 + 50*n^3 + 35*n^2 + 10*n + 1)
```

maple [A] time = 0.06, size = 117, normalized size = 1.04

$$\frac{b^2e^2x\,\mathrm{e}^{4n\ln(x)}}{4n+1}+a^2d^2x+\frac{2\,(ae+bd)\,adx\,\mathrm{e}^{n\ln(x)}}{n+1}+\frac{2\,(ae+bd)\,bex\,\mathrm{e}^{3n\ln(x)}}{3n+1}+\frac{\left(a^2e^2+4abde+b^2d^2\right)x\,\mathrm{e}^{2n\ln(x)}}{2n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^n+a)^2*(d+e*x^n)^2,x)
```

```
[Out] a^2*d^2*x+(a^2*e^2+4*a*b*d*e+b^2*d^2)/(2*n+1)*x*exp(n*ln(x))^2+b^2*e^2/(4*n
+1)*x*exp(n*ln(x))^4+2*a*d*(a*e+b*d)/(n+1)*x*exp(n*ln(x))+2*b*e*(a*e+b*d)/(
3*n+1)*x*exp(n*ln(x))^3
```

maxima [A] time = 0.60, size = 168, normalized size = 1.50

$$a^2d^2x+\frac{b^2e^2x^{4\,n+1}}{4\,n+1}+\frac{2\,b^2dex^{3\,n+1}}{3\,n+1}+\frac{2\,abe^2x^{3\,n+1}}{3\,n+1}+\frac{b^2d^2x^{2\,n+1}}{2\,n+1}+\frac{4\,abdex^{2\,n+1}}{2\,n+1}+\frac{a^2e^2x^{2\,n+1}}{2\,n+1}+\frac{2\,abd^2x^{n+1}}{n+1}+\frac{2\,a^2dex^{n+1}}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^2*(d+e*x^n)^2,x, algorithm="maxima")
```

```
[Out] a^2*d^2*x + b^2*e^2*x^(4*n + 1)/(4*n + 1) + 2*b^2*d*e*x^(3*n + 1)/(3*n + 1)
 + 2*a*b*e^2*x^(3*n + 1)/(3*n + 1) + b^2*d^2*x^(2*n + 1)/(2*n + 1) + 4*a*b*
d*e*x^(2*n + 1)/(2*n + 1) + a^2*e^2*x^(2*n + 1)/(2*n + 1) + 2*a*b*d^2*x^(n
+ 1)/(n + 1) + 2*a^2*d*e*x^(n + 1)/(n + 1)
```

mupad [B] time = 1.57, size = 108, normalized size = 0.96

$$a^2\,d^2\,x+\frac{x\,x^{2\,n}\,\left(a^2\,e^2+4\,a\,b\,d\,e+b^2\,d^2\right)}{2\,n+1}+\frac{b^2\,e^2\,x\,x^{4\,n}}{4\,n+1}+\frac{2\,b\,e\,x\,x^{3\,n}\,\left(a\,e+b\,d\right)}{3\,n+1}+\frac{2\,a\,d\,x\,x^n\,\left(a\,e+b\,d\right)}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^n)^2*(d + e*x^n)^2,x)
```

```
[Out] a^2*d^2*x + (x*x^(2*n)*(a^2*e^2 + b^2*d^2 + 4*a*b*d*e))/(2*n + 1) + (b^2*e^
2*x*x^(4*n))/(4*n + 1) + (2*b*e*x*x^(3*n)*(a*e + b*d))/(3*n + 1) + (2*a*d*x
*x^n*(a*e + b*d))/(n + 1)
```

sympy [A] time = 77.47, size = 1765, normalized size = 15.76

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**n)**2*(d+e*x**n)**2,x)
```

```
[Out] Piecewise((a**2*d**2*x + 2*a**2*d*e*log(x) - a**2*e**2/x + 2*a*b*d**2*log(x
) - 4*a*b*d*e/x - a*b*e**2/x**2 - b**2*d**2/x - b**2*d*e/x**2 - b**2*e**2/(
3*x**3), Eq(n, -1)), (a**2*d**2*x + 4*a**2*d*e*sqrt(x) + a**2*e**2*log(x) +
 4*a*b*d**2*sqrt(x) + 4*a*b*d*e*log(x) - 4*a*b*e**2/sqrt(x) + b**2*d**2*log
(x) - 4*b**2*d*e/sqrt(x) - b**2*e**2/x, Eq(n, -1/2)), (a**2*d**2*x + 3*a**2
*d*e*x**(2/3) + 3*a**2*e**2*x**(1/3) + 3*a*b*d**2*x**(2/3) + 12*a*b*d*e*x**
(1/3) + 2*a*b*e**2*log(x) + 3*b**2*d**2*x**(1/3) + 2*b**2*d*e*log(x) - 3*b*
*2*e**2/x**(1/3), Eq(n, -1/3)), (a**2*d**2*x + 8*a*d*x**(3/4)*(a*e + b*d)/3
 - 4*b**2*e**2*log(x**(-1/4)) + 8*b*e*x**(1/4)*(a*e + b*d) - sqrt(x)*(-4*a*
*2*e**2 - 16*a*b*d*e - 4*b**2*d**2)/2, Eq(n, -1/4)), (24*a**2*d**2*n**4*x/(
24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 50*a**2*d**2*n**3*x/(24*n**4 + 50
*n**3 + 35*n**2 + 10*n + 1) + 35*a**2*d**2*n**2*x/(24*n**4 + 50*n**3 + 35*n
**2 + 10*n + 1) + 10*a**2*d**2*n*x/(24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1)
 + a**2*d**2*x/(24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 48*a**2*d*e*n**3*
x*x**n/(24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 52*a**2*d*e*n**2*x*x**n/(
24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 18*a**2*d*e*n*x*x**n/(24*n**4 + 5
0*n**3 + 35*n**2 + 10*n + 1) + 2*a**2*d*e*x*x**n/(24*n**4 + 50*n**3 + 35*n*
*2 + 10*n + 1) + 12*a**2*e**2*n**3*x*x**(2*n)/(24*n**4 + 50*n**3 + 35*n**2
+ 10*n + 1) + 19*a**2*e**2*n**2*x*x**(2*n)/(24*n**4 + 50*n**3 + 35*n**2 + 1
0*n + 1) + 8*a**2*e**2*n*x*x**(2*n)/(24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1
) + a**2*e**2*x*x**(2*n)/(24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 48*a*b*
d**2*n**3*x*x**n/(24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 52*a*b*d**2*n**
2*x*x**n/(24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 18*a*b*d**2*n*x*x**n/(2
4*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 2*a*b*d**2*x*x**n/(24*n**4 + 50*n*
*3 + 35*n**2 + 10*n + 1) + 48*a*b*d*e*n**3*x*x**(2*n)/(24*n**4 + 50*n**3 +
35*n**2 + 10*n + 1) + 76*a*b*d*e*n**2*x*x**(2*n)/(24*n**4 + 50*n**3 + 35*n*
*2 + 10*n + 1) + 32*a*b*d*e*n*x*x**(2*n)/(24*n**4 + 50*n**3 + 35*n**2 + 10*
n + 1) + 4*a*b*d*e*x*x**(2*n)/(24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 16
*a*b*e**2*n**3*x*x**(3*n)/(24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 28*a*b
*e**2*n**2*x*x**(3*n)/(24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 14*a*b*e**
2*n*x*x**(3*n)/(24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 2*a*b*e**2*x*x**(
3*n)/(24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 12*b**2*d**2*n**3*x*x**(2*n
)/(24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 19*b**2*d**2*n**2*x*x**(2*n)/(
24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 8*b**2*d**2*n*x*x**(2*n)/(24*n**4
 + 50*n**3 + 35*n**2 + 10*n + 1) + b**2*d**2*x*x**(2*n)/(24*n**4 + 50*n**3
+ 35*n**2 + 10*n + 1) + 16*b**2*d*e*n**3*x*x**(3*n)/(24*n**4 + 50*n**3 + 35
```

```
*n**2 + 10*n + 1) + 28*b**2*d*e*n**2*x*x**(3*n)/(24*n**4 + 50*n**3 + 35*n**
2 + 10*n + 1) + 14*b**2*d*e*n*x*x**(3*n)/(24*n**4 + 50*n**3 + 35*n**2 + 10*
n + 1) + 2*b**2*d*e*x*x**(3*n)/(24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 6
*b**2*e**2*n**3*x*x**(4*n)/(24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 11*b*
*2*e**2*n**2*x*x**(4*n)/(24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 6*b**2*e
**2*n*x*x**(4*n)/(24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + b**2*e**2*x*x**
(4*n)/(24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1), True))
```

3.185 $\int (a+bx^n)^2 (c+dx^n) \, dx$

Optimal. Leaf size=70

$$a^2cx+\frac{ax^{n+1}(ad+2bc)}{n+1}+\frac{bx^{2n+1}(2ad+bc)}{2n+1}+\frac{b^2dx^{3n+1}}{3n+1}$$

Rubi [A] time = 0.05, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {373}

$$a^2cx+\frac{ax^{n+1}(ad+2bc)}{n+1}+\frac{bx^{2n+1}(2ad+bc)}{2n+1}+\frac{b^2dx^{3n+1}}{3n+1}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^n)^2*(c + d*x^n),x]
```

```
[Out] a^2*c*x + (a*(2*b*c + a*d)*x^(1 + n))/(1 + n) + (b*(b*c + 2*a*d)*x^(1 + 2*n
))/(1 + 2*n) + (b^2*d*x^(1 + 3*n))/(1 + 3*n)
```

Rule 373

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}
\int (a+bx^n)^2 (c+dx^n) \, dx &= \int \left(a^2c+a(2bc+ad)x^n+b(bc+2ad)x^{2n}+b^2dx^{3n}\right) dx \\
&= a^2cx+\frac{a(2bc+ad)x^{1+n}}{1+n}+\frac{b(bc+2ad)x^{1+2n}}{1+2n}+\frac{b^2dx^{1+3n}}{1+3n}
\end{aligned}$$

Mathematica [A] time = 0.11, size = 70, normalized size = 1.00

$$\frac{dx\,(a+bx^n)^3-x\left(a^2+\frac{2abx^n}{n+1}+\frac{b^2x^{2n}}{2n+1}\right)(ad-b(3cn+c))}{3bn+b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^n)^2*(c + d*x^n),x]
```

```
[Out] (d*x*(a + b*x^n)^3 - (a*d - b*(c + 3*c*n))*x*(a^2 + (2*a*b*x^n)/(1 + n) + (
b^2*x^(2*n))/(1 + 2*n)))/(b + 3*b*n)
```

IntegrateAlgebraic [F] time = 0.08, size = 0, normalized size = 0.00

$$\int (a+bx^n)^2 (c+dx^n) \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x^n)^2*(c + d*x^n),x]
```

```
[Out] a^2*c*x + Defer[IntegrateAlgebraic][x^n*(2*a*b*c + a^2*d + b^2*c*x^n + 2*a*
b*d*x^n + b^2*d*x^(2*n)), x]
```

fricas [B] time = 0.80, size = 175, normalized size = 2.50

$$\frac{\left(2\,b^{2}dn^{2}+3\,b^{2}dn+b^{2}d\right)xx^{3\,n}+\left(b^{2}c+2\,abd+3\,\left(b^{2}c+2\,abd\right)n^{2}+4\,\left(b^{2}c+2\,abd\right)n\right)xx^{2\,n}+\left(2\,abc+a^{2}d+6\,\left(2\,abc+a^{2}d\right)n^{2}+5\,\left(2\,abc+a^{2}d\right)n\right)xx^{n}+\left(6\,a^{2}cn^{3}+11\,a^{2}cn^{2}+6\,a^{2}cn+a^{2}c\right)x}{6\,n^{3}+11\,n^{2}+6\,n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^2*(c+d*x^n),x, algorithm="fricas")
```

```
[Out] ((2*b^2*d*n^2 + 3*b^2*d*n + b^2*d)*x*x^(3*n) + (b^2*c + 2*a*b*d + 3*(b^2*c
+ 2*a*b*d)*n^2 + 4*(b^2*c + 2*a*b*d)*n)*x*x^(2*n) + (2*a*b*c + a^2*d + 6*(2
*a*b*c + a^2*d)*n^2 + 5*(2*a*b*c + a^2*d)*n)*x*x^n + (6*a^2*c*n^3 + 11*a^2*
c*n^2 + 6*a^2*c*n + a^2*c)*x)/(6*n^3 + 11*n^2 + 6*n + 1)
```

giac [B] time = 0.27, size = 232, normalized size = 3.31

$$\frac{6\,a^{2}cn^{3}x+2\,b^{2}dn^{2}xx^{3\,n}+3\,b^{2}cn^{2}xx^{2\,n}+6\,abdn^{2}xx^{2\,n}+12\,abcn^{2}xx^{n}+6\,a^{2}dn^{2}xx^{n}+11\,a^{2}cn^{2}x+3\,b^{2}dnxx^{3\,n}+4\,b^{2}cnxx^{2\,n}+8\,abdnxx^{2\,n}+10\,abcnxx^{n}+5\,a^{2}dnxx^{n}+6\,a^{2}cnx+b^{2}dxx^{3\,n}+b^{2}cxx^{2\,n}+2\,abdxx^{2\,n}+2\,abcxx^{n}+a^{2}dxx^{n}+a^{2}cx}{6\,n^{3}+11\,n^{2}+6\,n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^2*(c+d*x^n),x, algorithm="giac")
```

```
[Out] (6*a^2*c*n^3*x + 2*b^2*d*n^2*x*x^(3*n) + 3*b^2*c*n^2*x*x^(2*n) + 6*a*b*d*n^
2*x*x^(2*n) + 12*a*b*c*n^2*x*x^n + 6*a^2*d*n^2*x*x^n + 11*a^2*c*n^2*x + 3*b
^2*d*n*x*x^(3*n) + 4*b^2*c*n*x*x^(2*n) + 8*a*b*d*n*x*x^(2*n) + 10*a*b*c*n*x
*x^n + 5*a^2*d*n*x*x^n + 6*a^2*c*n*x + b^2*d*x*x^(3*n) + b^2*c*x*x^(2*n) +
2*a*b*d*x*x^(2*n) + 2*a*b*c*x*x^n + a^2*d*x*x^n + a^2*c*x)/(6*n^3 + 11*n^2
+ 6*n + 1)
```

maple [A] time = 0.05, size = 74, normalized size = 1.06

$$\frac{b^{2}dx\,{\rm e}^{3n\ln(x)}}{3n+1}+a^{2}cx+\frac{(ad+2bc)\,ax\,{\rm e}^{n\ln(x)}}{n+1}+\frac{(2ad+bc)\,bx\,{\rm e}^{2n\ln(x)}}{2n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^n+a)^2*(d*x^n+c),x)
```

```
[Out] a^2*c*x+a*(a*d+2*b*c)/(n+1)*x*exp(n*ln(x))+b*(2*a*d+b*c)/(2*n+1)*x*exp(n*ln
(x))^2+b^2*d/(3*n+1)*x*exp(n*ln(x))^3
```

maxima [A] time = 0.63, size = 94, normalized size = 1.34

$$a^{2}cx+\frac{b^{2}dx^{3\,n+1}}{3\,n+1}+\frac{b^{2}cx^{2\,n+1}}{2\,n+1}+\frac{2\,abdx^{2\,n+1}}{2\,n+1}+\frac{2\,abcx^{n+1}}{n+1}+\frac{a^{2}dx^{n+1}}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^2*(c+d*x^n),x, algorithm="maxima")
```

```
[Out] a^2*c*x + b^2*d*x^(3*n + 1)/(3*n + 1) + b^2*c*x^(2*n + 1)/(2*n + 1) + 2*a*b
*d*x^(2*n + 1)/(2*n + 1) + 2*a*b*c*x^(n + 1)/(n + 1) + a^2*d*x^(n + 1)/(n +
 1)
```

mupad [B] time = 1.53, size = 71, normalized size = 1.01

$$a^2\,c\,x+\frac{x\,x^{2\,n}\,\left(c\,b^2+2\,a\,d\,b\right)}{2\,n+1}+\frac{x\,x^n\,\left(d\,a^2+2\,b\,c\,a\right)}{n+1}+\frac{b^2\,d\,x\,x^{3\,n}}{3\,n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^n)^2*(c + d*x^n),x)
```

```
[Out] a^2*c*x + (x*x^(2*n)*(b^2*c + 2*a*b*d))/(2*n + 1) + (x*x^n*(a^2*d + 2*a*b*c
))/(n + 1) + (b^2*d*x*x^(3*n))/(3*n + 1)
```

sympy [A] time = 2.74, size = 726, normalized size = 10.37

$$\begin{cases} a^2cx + a^2d\log(x) + 2abc\log(x) - \frac{2abd}{x} - \frac{b^2c}{x} - \frac{b^2d}{2x^2} & \text{for } n = -1 \\ a^2cx + 2a^2d\sqrt{x} + 4abc\sqrt{x} + 2abd\log(x) + b^2c\log(x) - \frac{2b^2d}{\sqrt{x}} & \text{for } n = -\frac{1}{2} \\ a^2cx + \frac{3a^2dx^{\frac{2}{3}}}{2} + 3abcx^{\frac{2}{3}} + 6abd\sqrt[3]{x} + 3b^2c\sqrt[3]{x} + b^2d\log(x) & \text{for } n = -\frac{1}{3} \\ \frac{6a^2cn^3x}{6n^3+11n^2+6n+1} + \frac{11a^2cn^2x}{6n^3+11n^2+6n+1} + \frac{6a^2cnx}{6n^3+11n^2+6n+1} + \frac{a^2cx}{6n^3+11n^2+6n+1} + \frac{6a^2dn^2xx^n}{6n^3+11n^2+6n+1} + \frac{5a^2dnxx^n}{6n^3+11n^2+6n+1} + \frac{a^2dxx^n}{6n^3+11n^2+6n+1} + \frac{12abcn^2xx^n}{6n^3+11n^2+6n+1} + \frac{10abcnxx^n}{6n^3+11n^2+6n+1} + \frac{2abcxx^n}{6n^3+11n^2+6n+1} + \frac{6abdn^2xx^{2n}}{6n^3+11n^2+6n+1} + \frac{8abdnxx^{2n}}{6n^3+11n^2+6n+1} + \frac{2abdxx^{2n}}{6n^3+11n^2+6n+1} + \frac{3b^2cn^2xx^{2n}}{6n^3+11n^2+6n+1} + \frac{4b^2cnxx^{2n}}{6n^3+11n^2+6n+1} + \frac{b^2cxx^{2n}}{6n^3+11n^2+6n+1} + \frac{2b^2dn^2xx^{3n}}{6n^3+11n^2+6n+1} + \frac{3b^2dnxx^{3n}}{6n^3+11n^2+6n+1} + \frac{b^2dxx^{3n}}{6n^3+11n^2+6n+1} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**n)**2*(c+d*x**n),x)
```

```
[Out] Piecewise((a**2*c*x + a**2*d*log(x) + 2*a*b*c*log(x) - 2*a*b*d/x - b**2*c/x
 - b**2*d/(2*x**2), Eq(n, -1)), (a**2*c*x + 2*a**2*d*sqrt(x) + 4*a*b*c*sqrt
(x) + 2*a*b*d*log(x) + b**2*c*log(x) - 2*b**2*d/sqrt(x), Eq(n, -1/2)), (a**
2*c*x + 3*a**2*d*x**(2/3)/2 + 3*a*b*c*x**(2/3) + 6*a*b*d*x**(1/3) + 3*b**2*
c*x**(1/3) + b**2*d*log(x), Eq(n, -1/3)), (6*a**2*c*n**3*x/(6*n**3 + 11*n**
2 + 6*n + 1) + 11*a**2*c*n**2*x/(6*n**3 + 11*n**2 + 6*n + 1) + 6*a**2*c*n*x
/(6*n**3 + 11*n**2 + 6*n + 1) + a**2*c*x/(6*n**3 + 11*n**2 + 6*n + 1) + 6*a
**2*d*n**2*x*x**n/(6*n**3 + 11*n**2 + 6*n + 1) + 5*a**2*d*n*x*x**n/(6*n**3
+ 11*n**2 + 6*n + 1) + a**2*d*x*x**n/(6*n**3 + 11*n**2 + 6*n + 1) + 12*a*b*
c*n**2*x*x**n/(6*n**3 + 11*n**2 + 6*n + 1) + 10*a*b*c*n*x*x**n/(6*n**3 + 11
*n**2 + 6*n + 1) + 2*a*b*c*x*x**n/(6*n**3 + 11*n**2 + 6*n + 1) + 6*a*b*d*n*
*2*x*x**(2*n)/(6*n**3 + 11*n**2 + 6*n + 1) + 8*a*b*d*n*x*x**(2*n)/(6*n**3 +
 11*n**2 + 6*n + 1) + 2*a*b*d*x*x**(2*n)/(6*n**3 + 11*n**2 + 6*n + 1) + 3*b
**2*c*n**2*x*x**(2*n)/(6*n**3 + 11*n**2 + 6*n + 1) + 4*b**2*c*n*x*x**(2*n)/
(6*n**3 + 11*n**2 + 6*n + 1) + b**2*c*x*x**(2*n)/(6*n**3 + 11*n**2 + 6*n +
1) + 2*b**2*d*n**2*x*x**(3*n)/(6*n**3 + 11*n**2 + 6*n + 1) + 3*b**2*d*n*x*x
**(3*n)/(6*n**3 + 11*n**2 + 6*n + 1) + b**2*d*x*x**(3*n)/(6*n**3 + 11*n**2
+ 6*n + 1), True))
```

3.186 $\int (a+bx^n)^3 (c+dx^n)^{-4-\frac{1}{n}}\, dx$

Optimal. Leaf size=178

$$\frac{6a^3n^3x(c+dx^n)^{-1/n}}{c^4(n+1)(2n+1)(3n+1)}+\frac{6a^2n^2x(a+bx^n)(c+dx^n)^{-\frac{1}{n}-1}}{c^3(n+1)(2n+1)(3n+1)}+\frac{3anx(a+bx^n)^2(c+dx^n)^{-\frac{1}{n}-2}}{c^2\left(6n^2+5n+1\right)}+\frac{x(a+bx^n)^3(c+ \cdots}{c(3n+1)}$$

Rubi [A] time = 0.09, antiderivative size = 178, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {378, 191}

$$\frac{6a^2n^2x(a+bx^n)(c+dx^n)^{-\frac{1}{n}-1}}{c^3(n+1)(2n+1)(3n+1)}+\frac{6a^3n^3x(c+dx^n)^{-1/n}}{c^4(n+1)(2n+1)(3n+1)}+\frac{3anx(a+bx^n)^2(c+dx^n)^{-\frac{1}{n}-2}}{c^2\left(6n^2+5n+1\right)}+\frac{x(a+bx^n)^3(c+dx^n)^{-\frac{1}{n}-3}}{c(3n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^n)^3*(c + d*x^n)^(-4 - n^(-1)),x]
```

```
[Out] (x*(a + b*x^n)^3*(c + d*x^n)^(-3 - n^(-1)))/(c*(1 + 3*n)) + (3*a*n*x*(a + b
*x^n)^2*(c + d*x^n)^(-2 - n^(-1)))/(c^2*(1 + 5*n + 6*n^2)) + (6*a^2*n^2*x*(
a + b*x^n)*(c + d*x^n)^(-1 - n^(-1)))/(c^3*(1 + n)*(1 + 2*n)*(1 + 3*n)) + (
6*a^3*n^3*x)/(c^4*(1 + n)*(1 + 2*n)*(1 + 3*n)*(c + d*x^n)^n^(-1))
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 378

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> -Simp[(x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*n*(p + 1)), x] - Dist[(c
*q)/(a*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1, 0]
 && GtQ[q, 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int (a+bx^n)^3(c+dx^n)^{-4-\frac{1}{n}}\,dx &= \frac{x(a+bx^n)^3(c+dx^n)^{-3-\frac{1}{n}}}{c(1+3n)}+\frac{(3an)\int (a+bx^n)^2(c+dx^n)^{-3-\frac{1}{n}}\,dx}{c(1+3n)}\\ &= \frac{x(a+bx^n)^3(c+dx^n)^{-3-\frac{1}{n}}}{c(1+3n)}+\frac{3anx(a+bx^n)^2(c+dx^n)^{-2-\frac{1}{n}}}{c^2\left(1+5n+6n^2\right)}+\frac{\left(6a^2n^2\right)\int (a+ \cdots}{c^2 \left(1 \cdots\right.}\\ &= \frac{x(a+bx^n)^3(c+dx^n)^{-3-\frac{1}{n}}}{c(1+3n)}+\frac{3anx(a+bx^n)^2(c+dx^n)^{-2-\frac{1}{n}}}{c^2\left(1+5n+6n^2\right)}+\frac{6a^2n^2x(a+b \cdots}{c^3(1+n)\left(1 \cdots\right.}\\ &= \frac{x(a+bx^n)^3(c+dx^n)^{-3-\frac{1}{n}}}{c(1+3n)}+\frac{3anx(a+bx^n)^2(c+dx^n)^{-2-\frac{1}{n}}}{c^2\left(1+5n+6n^2\right)}+\frac{6a^2n^2x(a+b \cdots}{c^3(1+n)\left(1 \cdots\right.}\end{aligned}$$

Mathematica [A] time = 0.14, size = 218, normalized size = 1.22

$$\frac{x(c+dx^n)^{-\frac{1}{n}-3}\left(a^3\left(c^3\left(6n^3+11n^2+6n+1\right)+3c^2dn\left(6n^2+5n+1\right)x^n+6cd^2n^2(3n+1)x^{2n}+6d^3n^3x^{3n}\right)+3a^2bcx^n\left(c^2\left(6n^2+5n+1\right)+2cdn(3n+1)x^n+2d^2n^2x^{2n}\right)+3ab^2c^2(n+1)x^{2n}(3cn+c+dnx^n)+b^3c^3\left(2n^2+3n+1\right)x^{3n}\right)}{c^4(n+1)(2n+1)(3n+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^n)^3*(c + d*x^n)^(-4 - n^(-1)),x]
```

```
[Out] (x*(c + d*x^n)^(-3 - n^(-1))*(b^3*c^3*(1 + 3*n + 2*n^2)*x^(3*n) + 3*a*b^2*c
^2*(1 + n)*x^(2*n)*(c + 3*c*n + d*n*x^n) + 3*a^2*b*c*x^n*(c^2*(1 + 5*n + 6*
n^2) + 2*c*d*n*(1 + 3*n)*x^n + 2*d^2*n^2*x^(2*n)) + a^3*(c^3*(1 + 6*n + 11*
n^2 + 6*n^3) + 3*c^2*d*n*(1 + 5*n + 6*n^2)*x^n + 6*c*d^2*n^2*(1 + 3*n)*x^(2
*n) + 6*d^3*n^3*x^(3*n))))/(c^4*(1 + n)*(1 + 2*n)*(1 + 3*n))
```

IntegrateAlgebraic [F] time = 0.07, size = 0, normalized size = 0.00

$$\int (a + bx^n)^3 (c + dx^n)^{-4-\frac{1}{n}}\, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x^n)^3*(c + d*x^n)^(-4 - n^(-1)),x]
```

```
[Out] Defer[IntegrateAlgebraic][(a + b*x^n)^3*(c + d*x^n)^(-4 - n^(-1)), x]
```

fricas [B] time = 0.91, size = 478, normalized size = 2.69

$$\frac{\left(6\,a^3d^4n^3 + b^3c^3d + \left(2\,b^3c^3d + 3\,ab^2c^2d^2 + 6\,a^2bcd^3\right)n^2 + 3\left(b^3c^3d + ab^2c^2d^2\right)n\right)xx^{4\,n} + \left(24\,a^3cd^3n^3 + b^3c^4 + 3\,ab^2c^3d + 2\left(b^3c^4 + 6\,ab^2c^3d + 12\,a^2bc^2d^2 + 3\,a^3cd^3\right)n^2 + 3\left(b^3c^4 + 5\,ab^2c^3d + 2\,a^2bc^2d^2\right)n\right)xx^{3\,n} + 3\left(12\,a^3c^2d^2n^3 + ab^2c^4 + a^2bc^3d + \left(3\,ab^2c^4 + 12\,a^2bc^3d + 7\,a^3c^2d^2\right)n^2 + \left(4\,ab^2c^4 + 7\,a^2bc^3d + a^3c^2d^2\right)n\right)xx^{2\,n} + \left(24\,a^3c^3dn^3 + 3\,a^2bc^4 + a^3c^3d + 2\left(9\,a^2bc^4 + 13\,a^3c^3d\right)n^2 + 3\left(5\,a^2bc^4 + 3\,a^3c^3d\right)n\right)xx^n + \left(6\,a^3c^4n^3 + 11\,a^3c^4n^2 + 6\,a^3c^4n + a^3c^4\right)x}{\left(6\,c^4n^3 + 11\,c^4n^2 + 6\,c^4n + c^4\right)\left(dx^n + c\right)^{\frac{4n+1}{n}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^3*(c+d*x^n)^(-4-1/n),x, algorithm="fricas")
```

```
[Out] ((6*a^3*d^4*n^3 + b^3*c^3*d + (2*b^3*c^3*d + 3*a*b^2*c^2*d^2 + 6*a^2*b*c*d^
3)*n^2 + 3*(b^3*c^3*d + a*b^2*c^2*d^2)*n)*x*x^(4*n) + (24*a^3*c*d^3*n^3 + b
^3*c^4 + 3*a*b^2*c^3*d + 2*(b^3*c^4 + 6*a*b^2*c^3*d + 12*a^2*b*c^2*d^2 + 3*
a^3*c*d^3)*n^2 + 3*(b^3*c^4 + 5*a*b^2*c^3*d + 2*a^2*b*c^2*d^2)*n)*x*x^(3*n)
 + 3*(12*a^3*c^2*d^2*n^3 + a*b^2*c^4 + a^2*b*c^3*d + (3*a*b^2*c^4 + 12*a^2*
b*c^3*d + 7*a^3*c^2*d^2)*n^2 + (4*a*b^2*c^4 + 7*a^2*b*c^3*d + a^3*c^2*d^2)*
n)*x*x^(2*n) + (24*a^3*c^3*d*n^3 + 3*a^2*b*c^4 + a^3*c^3*d + 2*(9*a^2*b*c^4
 + 13*a^3*c^3*d)*n^2 + 3*(5*a^2*b*c^4 + 3*a^3*c^3*d)*n)*x*x^n + (6*a^3*c^4*
n^3 + 11*a^3*c^4*n^2 + 6*a^3*c^4*n + a^3*c^4)*x)/((6*c^4*n^3 + 11*c^4*n^2 +
 6*c^4*n + c^4)*(d*x^n + c)^((4*n + 1)/n))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^3*(c+d*x^n)^(-4-1/n),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to divide, perhaps due to rounding er
ror%%%{81,[2,0,6,4,2,4,3,0]%%%}+%%%{108,[2,0,6,3,2,4,3,0]%%%}+%%%{54,[2,0,6
,2,2,4,3,0]%%%}+%%%{12,[2,0,6,1,2,4,3,0]%%%}+%%%{1,[2,0,6,0,2,4,3,0]%%%}+%%
%{243,[1,0,6,4,2,4,2,1]%%%}+%%%{-81,[1,0,6,4,1,5,3,0]%%%}+%%%{324,[1,0,6,3,
2,4,2,1]%%%}+%%%{-108,[1,0,6,3,1,5,3,0]%%%}+%%%{162,[1,0,6,2,2,4,2,1]%%%}+%
%%{-54,[1,0,6,2,1,5,3,0]%%%}+%%%{36,[1,0,6,1,2,4,2,1]%%%}+%%%{-12,[1,0,6,1,
1,5,3,0]%%%}+%%%{3,[1,0,6,0,2,4,2,1]%%%}+%%%{-1,[1,0,6,0,1,5,3,0]%%%}+%%%{8
1,[0,0,6,4,3,3,0,3]%%%}+%%%{81,[0,0,6,3,3,3,0,3]%%%}+%%%{81,[0,0,6,3,2,4,1,
2]%%%}+%%%{-81,[0,0,6,3,1,5,2,1]%%%}+%%%{27,[0,0,6,3,0,6,3,0]%%%}+%%%{27,[0
,0,6,2,3,3,0,3]%%%}+%%%{81,[0,0,6,2,2,4,1,2]%%%}+%%%{-81,[0,0,6,2,1,5,2,1]%
%%}+%%%{27,[0,0,6,2,0,6,3,0]%%%}+%%%{3,[0,0,6,1,3,3,0,3]%%%}+%%%{27,[0,0,6,
1,2,4,1,2]%%%}+%%%{-27,[0,0,6,1,1,5,2,1]%%%}+%%%{9,[0,0,6,1,0,6,3,0]%%%}+%%
%{3,[0,0,6,0,2,4,1,2]%%%}+%%%{-3,[0,0,6,0,1,5,2,1]%%%}+%%%{1,[0,0,6,0,0,6,3
,0]%%%} / %%%{81,[0,0,7,4,3,4,0,0]%%%}+%%%{108,[0,0,7,3,3,4,0,0]%%%}+%%%{54
,[0,0,7,2,3,4,0,0]%%%}+%%%{12,[0,0,7,1,3,4,0,0]%%%}+%%%{1,[0,0,7,0,3,4,0,0]
%%%} Error: Bad Argument Value
```

maple [F]   time = 0.83, size = 0, normalized size = 0.00

$$\int (b\,x^{n}+a)^{3} (d\,x^{n}+c)^{-\frac{1}{n}-4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^n+a)^3*(d*x^n+c)^(-1/n-4),x)
[Out] int((b*x^n+a)^3*(d*x^n+c)^(-1/n-4),x)
```

maxima [F]   time = 0.00, size = 0, normalized size = 0.00

$$\int (bx^{n}+a)^{3}(dx^{n}+c)^{-\frac{1}{n}-4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^3*(c+d*x^n)^(-4-1/n),x, algorithm="maxima")
[Out] integrate((b*x^n + a)^3*(d*x^n + c)^(-1/n - 4), x)
```

mupad [F]   time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\,x^{n})^{3}}{(c+d\,x^{n})^{\frac{1}{n}+4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^n)^3/(c + d*x^n)^(1/n + 4),x)
[Out] int((a + b*x^n)^3/(c + d*x^n)^(1/n + 4), x)
```

sympy [F(-1)]   time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**n)**3*(c+d*x**n)**(-4-1/n),x)
[Out] Timed out
```

3.187 $\int (a+bx^n)^2 (c+dx^n)^{-3-\frac{1}{n}}\,dx$

Optimal. Leaf size=116

$$\frac{2a^2n^2x(c+dx^n)^{-1/n}}{c^3(n+1)(2n+1)}+\frac{2anx(a+bx^n)(c+dx^n)^{-\frac{1}{n}-1}}{c^2(n+1)(2n+1)}+\frac{x(a+bx^n)^2(c+dx^n)^{-\frac{1}{n}-2}}{c(2n+1)}$$

Rubi [A] time = 0.04, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {378, 191}

$$\frac{2a^2n^2x(c+dx^n)^{-1/n}}{c^3(n+1)(2n+1)}+\frac{2anx(a+bx^n)(c+dx^n)^{-\frac{1}{n}-1}}{c^2(n+1)(2n+1)}+\frac{x(a+bx^n)^2(c+dx^n)^{-\frac{1}{n}-2}}{c(2n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^n)^2*(c + d*x^n)^(-3 - n^(-1)),x]
```

```
[Out] (x*(a + b*x^n)^2*(c + d*x^n)^(-2 - n^(-1)))/(c*(1 + 2*n)) + (2*a*n*x*(a + b
*x^n)*(c + d*x^n)^(-1 - n^(-1)))/(c^2*(1 + n)*(1 + 2*n)) + (2*a^2*n^2*x)/(c
^3*(1 + n)*(1 + 2*n)*(c + d*x^n)^n^(-1))
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 378

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> -Simp[(x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*n*(p + 1)), x] - Dist[(c
*q)/(a*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1, 0]
 && GtQ[q, 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int (a+bx^n)^2(c+dx^n)^{-3-\frac{1}{n}}\,dx &= \frac{x(a+bx^n)^2(c+dx^n)^{-2-\frac{1}{n}}}{c(1+2n)}+\frac{(2an)\int (a+bx^n)(c+dx^n)^{-2-\frac{1}{n}}\,dx}{c(1+2n)}\\ &= \frac{x(a+bx^n)^2(c+dx^n)^{-2-\frac{1}{n}}}{c(1+2n)}+\frac{2anx(a+bx^n)(c+dx^n)^{-1-\frac{1}{n}}}{c^2(1+n)(1+2n)}+\frac{\left(2a^2n^2\right)\int (c+dx^{\prime}}{c^2(1+n)(1+}\\ &= \frac{x(a+bx^n)^2(c+dx^n)^{-2-\frac{1}{n}}}{c(1+2n)}+\frac{2anx(a+bx^n)(c+dx^n)^{-1-\frac{1}{n}}}{c^2(1+n)(1+2n)}+\frac{2a^2n^2x(c+dx^n)^{-}}{c^3(1+n)(1+2n}\end{aligned}$$

Mathematica [A] time = 0.08, size = 113, normalized size = 0.97

$$\frac{x(c+dx^n)^{-\frac{1}{n}-2}\left(a^2\left(c^2\left(2n^2+3n+1\right)+2cdn(2n+1)x^n+2d^2n^2x^{2n}\right)+2abcx^n(2cn+c+dnx^n)+b^2c^2(n+1)x^{2n}\right)}{c^3(n+1)(2n+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^n)^2*(c + d*x^n)^(-3 - n^(-1)),x]
```

```
[Out] (x*(c + d*x^n)^(-2 - n^(-1))*(b^2*c^2*(1 + n)*x^(2*n) + 2*a*b*c*x^n*(c + 2*
c*n + d*n*x^n) + a^2*(c^2*(1 + 3*n + 2*n^2) + 2*c*d*n*(1 + 2*n)*x^n + 2*d^2
*n^2*x^(2*n))))/(c^3*(1 + n)*(1 + 2*n))
```

IntegrateAlgebraic [F] time = 0.06, size = 0, normalized size = 0.00

$$\int (a+bx^n)^2 (c+dx^n)^{-3-\frac{1}{n}}\, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x^n)^2*(c + d*x^n)^(-3 - n^(-1)),x]
```

```
[Out] Defer[IntegrateAlgebraic][(a + b*x^n)^2*(c + d*x^n)^(-3 - n^(-1)), x]
```

fricas [A] time = 0.64, size = 231, normalized size = 1.99

$$\frac{\left(2\,a^2d^3n^2+b^2c^2d+\left(b^2c^2d+2\,abcd^2\right)n\right)xx^{3\,n}+\left(6\,a^2cd^2n^2+b^2c^3+2\,abc^2d+\left(b^2c^3+6\,abc^2d+2\,a^2cd^2\right)n\right)xx^{2\,n}+\left(6\,a^2c^2dn^2+2\,abc^3+a^2c^2d+\left(4\,abc^3+5\,a^2c^2d\right)n\right)xx^n+\left(2\,a^2c^3n^2+3\,a^2c^3n+a^2c^3\right)x}{\left(2\,c^3n^2+3\,c^3n+c^3\right)\left(dx^n+c\right)^{\frac{3\,n+1}{n}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^2*(c+d*x^n)^(-3-1/n),x, algorithm="fricas")
```

```
[Out] ((2*a^2*d^3*n^2 + b^2*c^2*d + (b^2*c^2*d + 2*a*b*c*d^2)*n)*x*x^(3*n) + (6*a
^2*c*d^2*n^2 + b^2*c^3 + 2*a*b*c^2*d + (b^2*c^3 + 6*a*b*c^2*d + 2*a^2*c*d^2
)*n)*x*x^(2*n) + (6*a^2*c^2*d*n^2 + 2*a*b*c^3 + a^2*c^2*d + (4*a*b*c^3 + 5*
a^2*c^2*d)*n)*x*x^n + (2*a^2*c^3*n^2 + 3*a^2*c^3*n + a^2*c^3)*x)/((2*c^3*n^
2 + 3*c^3*n + c^3)*(d*x^n + c)^((3*n + 1)/n))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^2*(c+d*x^n)^(-3-1/n),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to divide, perhaps due to rounding er
ror%%%{8,[1,0,4,3,1,3,2,0]%%%}+%%%{12,[1,0,4,2,1,3,2,0]%%%}+%%%{6,[1,0,4,1,
1,3,2,0]%%%}+%%%{1,[1,0,4,0,1,3,2,0]%%%}+%%%{8,[0,0,4,3,2,2,0,2]%%%}+%%%{8,
[0,0,4,2,2,2,0,2]%%%}+%%%{8,[0,0,4,2,1,3,1,1]%%%}+%%%{-4,[0,0,4,2,0,4,2,0]%
%%}+%%%{2,[0,0,4,1,2,2,0,2]%%%}+%%%{8,[0,0,4,1,1,3,1,1]%%%}+%%%{-4,[0,0,4,1
,0,4,2,0]%%%}+%%%{2,[0,0,4,0,1,3,1,1]%%%}+%%%{-1,[0,0,4,0,0,4,2,0]%%%} / %%
%{8,[0,0,5,3,2,3,0,0]%%%}+%%%{12,[0,0,5,2,2,3,0,0]%%%}+%%%{6,[0,0,5,1,2,3,0
,0]%%%}+%%%{1,[0,0,5,0,2,3,0,0]%%%} Error: Bad Argument Value
```

maple [F] time = 0.80, size = 0, normalized size = 0.00

$$\int \left(b\,x^{n}+a\right)^{2} \left(d\,x^{n}+c\right)^{-\frac{1}{n}-3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^n+a)^2*(d*x^n+c)^(-1/n-3),x)
```

```
[Out] int((b*x^n+a)^2*(d*x^n+c)^(-1/n-3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (bx^n+a)^2 (dx^n+c)^{-\frac{1}{n}-3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^2*(c+d*x^n)^(-3-1/n),x, algorithm="maxima")
```

```
[Out] integrate((b*x^n + a)^2*(d*x^n + c)^(-1/n - 3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\,x^n)^2}{(c+d\,x^n)^{\frac{1}{n}+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^n)^2/(c + d*x^n)^(1/n + 3),x)
```

```
[Out] int((a + b*x^n)^2/(c + d*x^n)^(1/n + 3), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**n)**2*(c+d*x**n)**(-3-1/n),x)
```

```
[Out] Timed out
```

3.188 $\int (a+bx^n)(c+dx^n)^{-2-\frac{1}{n}}\,dx$

Optimal. Leaf size=58

$$\frac{x(a+bx^n)(c+dx^n)^{-\frac{1}{n}-1}}{c(n+1)}+\frac{anx(c+dx^n)^{-1/n}}{c^2(n+1)}$$

Rubi [A] time = 0.01, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {378, 191}

$$\frac{x(a+bx^n)(c+dx^n)^{-\frac{1}{n}-1}}{c(n+1)}+\frac{anx(c+dx^n)^{-1/n}}{c^2(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^n)*(c + d*x^n)^(-2 - n^(-1)),x]
```

```
[Out] (x*(a + b*x^n)*(c + d*x^n)^(-1 - n^(-1)))/(c*(1 + n)) + (a*n*x)/(c^2*(1 + n
)*(c + d*x^n)^n^(-1))
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 378

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> -Simp[(x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*n*(p + 1)), x] - Dist[(c
*q)/(a*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1, 0]
 && GtQ[q, 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int (a+bx^n)(c+dx^n)^{-2-\frac{1}{n}}\,dx &= \frac{x(a+bx^n)(c+dx^n)^{-1-\frac{1}{n}}}{c(1+n)}+\frac{(an)\int (c+dx^n)^{-1-\frac{1}{n}}\,dx}{c(1+n)}\\ &= \frac{x(a+bx^n)(c+dx^n)^{-1-\frac{1}{n}}}{c(1+n)}+\frac{anx(c+dx^n)^{-1/n}}{c^2(1+n)}\end{aligned}$$

Mathematica [C] time = 0.14, size = 82, normalized size = 1.41

$$\frac{x(c+dx^n)^{-\frac{n+1}{n}}\left(a(n+1)(c+dx^n)\left(\frac{dx^n}{c}+1\right)^{\frac{1}{n}}{}_2F_1\left(2+\frac{1}{n},\frac{1}{n};1+\frac{1}{n};-\frac{dx^n}{c}\right)+bcx^n\right)}{c^2(n+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^n)*(c + d*x^n)^(-2 - n^(-1)),x]
```

```
[Out] (x*(b*c*x^n + a*(1 + n)*(c + d*x^n)*(1 + (d*x^n)/c)^n^(-1)*Hypergeometric2F
1[2 + n^(-1), n^(-1), 1 + n^(-1), -((d*x^n)/c)]))/(c^2*(1 + n)*(c + d*x^n)^
((1 + n)/n))
```

IntegrateAlgebraic [F] time = 0.05, size = 0, normalized size = 0.00

$$\int (a + bx^n)(c + dx^n)^{-2-\frac{1}{n}} \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x^n)*(c + d*x^n)^(-2 - n^(-1)),x]
```

```
[Out] Defer[IntegrateAlgebraic][(a + b*x^n)*(c + d*x^n)^(-2 - n^(-1)), x]
```

fricas [A] time = 0.91, size = 85, normalized size = 1.47

$$\frac{(ad^2n + bcd)xx^{2n} + (2\,acdn + bc^2 + acd)xx^n + (ac^2n + ac^2)x}{(c^2n + c^2)(dx^n + c)^{\frac{2n+1}{n}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)*(c+d*x^n)^(-2-1/n),x, algorithm="fricas")
```

```
[Out] ((a*d^2*n + b*c*d)*x*x^(2*n) + (2*a*c*d*n + b*c^2 + a*c*d)*x*x^n + (a*c^2*n
 + a*c^2)*x)/((c^2*n + c^2)*(d*x^n + c)^((2*n + 1)/n))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)*(c+d*x^n)^(-2-1/n),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to divide, perhaps due to rounding er
ror%%%{1,[0,0,2,2,1,1,0,1]%%%}+%%%{1,[0,0,2,1,1,1,0,1]%%%}+%%%{1,[0,0,2,1,0
,2,1,0]%%%}+%%%{1,[0,0,2,0,0,2,1,0]%%%} / %%%{1,[0,0,3,2,1,2,0,0]%%%}+%%%{2
,[0,0,3,1,1,2,0,0]%%%}+%%%{1,[0,0,3,0,1,2,0,0]%%%} Error: Bad Argument Valu
e
```

maple [F] time = 0.60, size = 0, normalized size = 0.00

$$\int (b\,x^n + a)(d\,x^n + c)^{-\frac{1}{n}-2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^n+a)*(d*x^n+c)^(-1/n-2),x)
```

```
[Out] int((b*x^n+a)*(d*x^n+c)^(-1/n-2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (bx^n + a)(dx^n + c)^{-\frac{1}{n}-2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)*(c+d*x^n)^(-2-1/n),x, algorithm="maxima")
```

```
[Out] integrate((b*x^n + a)*(d*x^n + c)^(-1/n - 2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{a + b\,x^n}{(c + d\,x^n)^{\frac{1}{n}+2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^n)/(c + d*x^n)^(1/n + 2),x)
```

```
[Out] int((a + b*x^n)/(c + d*x^n)^(1/n + 2), x)
```

sympy [F(-1)]   time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**n)*(c+d*x**n)**(-2-1/n),x)
```

```
[Out] Timed out
```

3.189 $\int (c+dx^n)^{-1-\frac{1}{n}}\,dx$

Optimal. Leaf size=18

$$\frac{x(c+dx^n)^{-1/n}}{c}$$

Rubi [A] time = 0.00, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {191}

$$\frac{x(c+dx^n)^{-1/n}}{c}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^n)^(-1 - n^(-1)),x]
```

```
[Out] x/(c*(c + d*x^n)^n^(-1))
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\int (c+dx^n)^{-1-\frac{1}{n}}\,dx=\frac{x(c+dx^n)^{-1/n}}{c}$$

Mathematica [A] time = 0.04, size = 18, normalized size = 1.00

$$\frac{x(c+dx^n)^{-1/n}}{c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^n)^(-1 - n^(-1)),x]
```

```
[Out] x/(c*(c + d*x^n)^n^(-1))
```

IntegrateAlgebraic [F] time = 0.02, size = 0, normalized size = 0.00

$$\int (c+dx^n)^{-1-\frac{1}{n}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x^n)^(-1 - n^(-1)),x]
```

```
[Out] Defer[IntegrateAlgebraic][(c + d*x^n)^(-1 - n^(-1)), x]
```

fricas [A] time = 0.73, size = 31, normalized size = 1.72

$$\frac{dxx^n+cx}{(dx^n+c)^{\frac{n+1}{n}}c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*x^n)^(-1-1/n),x, algorithm="fricas")
```

```
[Out] (d*x*x^n + c*x)/((d*x^n + c)^((n + 1)/n)*c)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx^n + c)^{-\frac{1}{n}-1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*x^n)^(-1-1/n),x, algorithm="giac")
```

```
[Out] integrate((d*x^n + c)^(-1/n - 1), x)
```

maple [B] time = 0.07, size = 53, normalized size = 2.94

$$\frac{dx\,\mathrm{e}^{n\ln(x)}\mathrm{e}^{\left(-\frac{1}{n}-1\right)\ln\left(d\,\mathrm{e}^{n\ln(x)}+c\right)}}{c} + x\,\mathrm{e}^{\left(-\frac{1}{n}-1\right)\ln\left(d\,\mathrm{e}^{n\ln(x)}+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^n+c)^(-1/n-1),x)
```

```
[Out] x*exp((-1/n-1)*ln(c+d*exp(n*ln(x))))+1/c*d*x*exp(n*ln(x))*exp((-1/n-1)*ln(c
+d*exp(n*ln(x))))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx^n + c)^{-\frac{1}{n}-1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*x^n)^(-1-1/n),x, algorithm="maxima")
```

```
[Out] integrate((d*x^n + c)^(-1/n - 1), x)
```

mupad [B] time = 1.76, size = 75, normalized size = 4.17

$$\frac{d\,x^{n+1}\left(\frac{c}{d\,x^n} - \left(\frac{c}{d\,x^n} + 1\right)^{\frac{n+1}{n}} + 1\right)}{c\,n\left(\frac{n+1}{n} - 1\right)\left(c + d\,x^n\right)^{\frac{n+1}{n}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c + d*x^n)^(1/n + 1),x)
```

```
[Out] (d*x^(n + 1)*(c/(d*x^n) - (c/(d*x^n) + 1)^((n + 1)/n) + 1))/(c*n*((n + 1)/n
 - 1)*(c + d*x^n)^((n + 1)/n))
```

sympy [A] time = 33.05, size = 211, normalized size = 11.72

$$\begin{cases} -\frac{d^{-\frac{1}{n}} x x^{-n} (x^n)^{-\frac{1}{n}}}{dn} & \text{for } c = 0 \\ 0^{-1-\frac{1}{n}} x & \text{for } c = -dx^n \\ x (0^n)^{-1-\frac{1}{n}} & \text{for } c = 0^n - dx^n \\ \frac{c^2 x}{c^3 (c+dx^n)^{\frac{1}{n}} + 2c^2 dx^n (c+dx^n)^{\frac{1}{n}} + cd^2 x^{2n} (c+dx^n)^{\frac{1}{n}}} + \frac{cdxx^n}{c^3 (c+dx^n)^{\frac{1}{n}} + 2c^2 dx^n (c+dx^n)^{\frac{1}{n}} + cd^2 x^{2n} (c+dx^n)^{\frac{1}{n}}} + \frac{dxx^n}{c^2 (c+dx^n)^{\frac{1}{n}} + cdx^n (c+dx^n)^{\frac{1}{n}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*x**n)**(-1-1/n),x)
```

```
[Out] Piecewise((-d**(-1/n)*x*x**(-n)*(x**n)**(-1/n)/(d*n), Eq(c, 0)), (0**(-1 -
1/n)*x, Eq(c, -d*x**n)), (x*(0**n)**(-1 - 1/n), Eq(c, 0**n - d*x**n)), (c**
2*x/(c**3*(c + d*x**n)**(1/n) + 2*c**2*d*x**n*(c + d*x**n)**(1/n) + c*d**2*
x**(2*n)*(c + d*x**n)**(1/n)) + c*d*x*x**n/(c**3*(c + d*x**n)**(1/n) + 2*c*
*2*d*x**n*(c + d*x**n)**(1/n) + c*d**2*x**(2*n)*(c + d*x**n)**(1/n)) + d*x*
x**n/(c**2*(c + d*x**n)**(1/n) + c*d*x**n*(c + d*x**n)**(1/n)), True))
```

3.190 $\int (a+bx^n)^{\frac{adn-bc(1+n)}{(bc-ad)n}} (c+dx^n)^{\frac{ad-bcn+adn}{bcn-adn}} \, dx$

Optimal. Leaf size=57

$$\frac{x(a+bx^n)^{-\frac{bc}{n(bc-ad)}}(c+dx^n)^{\frac{ad}{n(bc-ad)}}}{ac}$$

Rubi [A] time = 0.03, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 69, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.014, Rules used = {381}

$$\frac{x(a+bx^n)^{-\frac{bc}{n(bc-ad)}}(c+dx^n)^{\frac{ad}{n(bc-ad)}}}{ac}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^n)^((a*d*n - b*c*(1 + n))/((b*c - a*d)*n))*(c + d*x^n)^((a*d -
 b*c*n + a*d*n)/(b*c*n - a*d*n)),x]
```

```
[Out] (x*(c + d*x^n)^((a*d)/((b*c - a*d)*n)))/(a*c*(a + b*x^n)^((b*c)/((b*c - a*d
)*n)))
```

Rule 381

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c), x] /; FreeQ[{a,
b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] &&
EqQ[a*d*(p + 1) + b*c*(q + 1), 0]
```

Rubi steps

$$\int (a+bx^n)^{\frac{adn-bc(1+n)}{(bc-ad)n}} (c+dx^n)^{\frac{ad-bcn+adn}{bcn-adn}} \, dx = \frac{x(a+bx^n)^{-\frac{bc}{(bc-ad)n}}(c+dx^n)^{\frac{ad}{(bc-ad)n}}}{ac}$$

Mathematica [A] time = 0.07, size = 55, normalized size = 0.96

$$\frac{x(a+bx^n)^{-\frac{bc}{bcn-adn}}(c+dx^n)^{\frac{ad}{bcn-adn}}}{ac}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^n)^((a*d*n - b*c*(1 + n))/((b*c - a*d)*n))*(c + d*x^n)^(
(a*d - b*c*n + a*d*n)/(b*c*n - a*d*n)),x]
```

```
[Out] (x*(c + d*x^n)^((a*d)/(b*c*n - a*d*n)))/(a*c*(a + b*x^n)^((b*c)/(b*c*n - a*
d*n)))
```

IntegrateAlgebraic [F] time = 0.60, size = 0, normalized size = 0.00

$$\int (a+bx^n)^{\frac{adn-bc(1+n)}{(bc-ad)n}} (c+dx^n)^{\frac{ad-bcn+adn}{bcn-adn}} \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x^n)^((a*d*n - b*c*(1 + n))/((b*c - a*d)*n))*(c +
 d*x^n)^((a*d - b*c*n + a*d*n)/(b*c*n - a*d*n)),x]
```

```
[Out] Defer[IntegrateAlgebraic][(a + b*x^n)^((a*d*n - b*c*(1 + n))/((b*c - a*d)*n
))*(c + d*x^n)^((a*d - b*c*n + a*d*n)/(b*c*n - a*d*n)), x]
```

fricas [A] time = 0.85, size = 108, normalized size = 1.89

$$\frac{\left(bdxx^{2\,n} + acx + (bc + ad)xx^{n}\right)(dx^{n} + c)^{\frac{ad-(bc-ad)n}{(bc-ad)n}}}{(bx^{n} + a)^{\frac{bc+(bc-ad)n}{(bc-ad)n}}ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^((a*d*n-b*c*(1+n))/(-a*d+b*c)/n)*(c+d*x^n)^((a*d*n-b*c*
n+a*d)/(-a*d*n+b*c*n)),x, algorithm="fricas")
```

```
[Out] (b*d*x*x^(2*n) + a*c*x + (b*c + a*d)*x*x^n)*(d*x^n + c)^((a*d - (b*c - a*d)
*n)/((b*c - a*d)*n))/((b*x^n + a)^((b*c + (b*c - a*d)*n)/((b*c - a*d)*n))*a
*c)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^{n} + a)^{\frac{bc(n+1)-adn}{(bc-ad)n}}(dx^{n} + c)^{\frac{bcn-adn-ad}{bcn-adn}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^((a*d*n-b*c*(1+n))/(-a*d+b*c)/n)*(c+d*x^n)^((a*d*n-b*c*
n+a*d)/(-a*d*n+b*c*n)),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^n + a)^((b*c*(n + 1) - a*d*n)/((b*c - a*d)*n))*(d*x^n + c
)^((b*c*n - a*d*n - a*d)/(b*c*n - a*d*n))), x)
```

maple [F] time = 0.90, size = 0, normalized size = 0.00

$$\int (b\,x^{n} + a)^{\frac{adn-(n+1)bc}{(-ad+bc)n}}(d\,x^{n} + c)^{\frac{adn-bcn+ad}{-adn+bcn}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^n+a)^((a*d*n-b*c*(n+1))/(-a*d+b*c)/n)*(d*x^n+c)^((a*d*n-b*c*n+a*d)
/(-a*d*n+b*c*n)),x)
```

```
[Out] int((b*x^n+a)^((a*d*n-b*c*(n+1))/(-a*d+b*c)/n)*(d*x^n+c)^((a*d*n-b*c*n+a*d)
/(-a*d*n+b*c*n)),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^{n} + a)^{\frac{bc(n+1)-adn}{(bc-ad)n}}(dx^{n} + c)^{\frac{bcn-adn-ad}{bcn-adn}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^((a*d*n-b*c*(1+n))/(-a*d+b*c)/n)*(c+d*x^n)^((a*d*n-b*c*
n+a*d)/(-a*d*n+b*c*n)),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^n + a)^((b*c*(n + 1) - a*d*n)/((b*c - a*d)*n))*(d*x^n + c
)^((b*c*n - a*d*n - a*d)/(b*c*n - a*d*n))), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{(a + b\,x^{n})^{\frac{a\,d\,n-b\,c\,(n+1)}{n\,(a\,d-b\,c)}}\,(c + d\,x^{n})^{\frac{a\,d+a\,d\,n-b\,c\,n}{a\,d\,n-b\,c\,n}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^n)^((a*d*n - b*c*(n + 1))/(n*(a*d - b*c)))*(c + d*x^n)^((a*
d + a*d*n - b*c*n)/(a*d*n - b*c*n))),x)
```

```
[Out] int(1/((a + b*x^n)^((a*d*n - b*c*(n + 1))/(n*(a*d - b*c)))*(c + d*x^n)^((a*
d + a*d*n - b*c*n)/(a*d*n - b*c*n))), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**n)**((a*d*n-b*c*(1+n))/(-a*d+b*c)/n)*(c+d*x**n)**((a*d*n-
b*c*n+a*d)/(-a*d*n+b*c*n)),x)
```

```
[Out] Timed out
```

3.191 $\int (a+bx^n)^2 (c+dx^n)^{-4-\frac{1}{n}}\,dx$

Optimal. Leaf size=327

$$-\frac{2a^2n^2x(c+dx^n)^{-1/n}(3adn-b(3cn+c))}{c^4(n+1)(2n+1)(3n+1)(bc-ad)}-\frac{2anx(a+bx^n)(c+dx^n)^{-\frac{1}{n}-1}(3adn-b(3cn+c))}{c^3(n+1)(2n+1)(3n+1)(bc-ad)}-\frac{x(a+bx^n)^2(c+d}{c^2\left(6n^2+\right.}$$

Rubi [A] time = 0.18, antiderivative size = 327, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {382, 378, 191}

$$-\frac{2a^2n^2x(c+dx^n)^{-1/n}(3adn-b(3cn+c))}{c^4(n+1)(2n+1)(3n+1)(bc-ad)}-\frac{x(a+bx^n)^2(c+dx^n)^{-\frac{1}{n}-2}(3adn-b(3cn+c))}{c^2\left(6n^2+5n+1\right)(bc-ad)}-\frac{2anx(a+bx^n)(c+dx^n)^{-\frac{1}{n}-1}(3adn-b(3cn+c))}{c^3(n+1)(2n+1)(3n+1)(bc-ad)}-\frac{x(a+bx^n)^3(c+dx^n)^{-\frac{1}{n}-3}(3adn-b(3cn+c))}{3acn(3n+1)(bc-ad)}-\frac{bx(a+bx^n)^3(c+dx^n)^{-\frac{1}{n}-3}}{3an(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^n)^2*(c + d*x^n)^(-4 - n^(-1)),x]
```

```
[Out] -(b*x*(a + b*x^n)^3*(c + d*x^n)^(-3 - n^(-1)))/(3*a*(b*c - a*d)*n) - ((3*a*
d*n - b*(c + 3*c*n))*x*(a + b*x^n)^3*(c + d*x^n)^(-3 - n^(-1)))/(3*a*c*(b*c
 - a*d)*n*(1 + 3*n)) - ((3*a*d*n - b*(c + 3*c*n))*x*(a + b*x^n)^2*(c + d*x^
n)^(-2 - n^(-1)))/(c^2*(b*c - a*d)*(1 + 5*n + 6*n^2)) - (2*a*n*(3*a*d*n - b
*(c + 3*c*n))*x*(a + b*x^n)*(c + d*x^n)^(-1 - n^(-1)))/(c^3*(b*c - a*d)*(1
+ n)*(1 + 2*n)*(1 + 3*n)) - (2*a^2*n^2*(3*a*d*n - b*(c + 3*c*n))*x)/(c^4*(b
*c - a*d)*(1 + n)*(1 + 2*n)*(1 + 3*n)*(c + d*x^n)^n^(-1))
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 378

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> -Simp[(x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*n*(p + 1)), x] - Dist[(c
*q)/(a*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1, 0]
 && GtQ[q, 0] && NeQ[p, -1]
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)), I
nt[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q}, x
] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !LtQ
[q, -1]) && NeQ[p, -1]
```

Rubi steps

$$\int (a+bx^n)^2 (c+dx^n)^{-4-\frac{1}{n}}\, dx = -\frac{bx(a+bx^n)^3 (c+dx^n)^{-3-\frac{1}{n}}}{3a(bc-ad)n} + \frac{\left(3+\frac{bc}{bcn-adn}\right)\int (a+bx^n)^3 (c+dx^n)^{-4-\frac{1}{n}}\, dx}{3a}$$
$$= -\frac{bx(a+bx^n)^3 (c+dx^n)^{-3-\frac{1}{n}}}{3a(bc-ad)n} + \frac{\left(3+\frac{bc}{bcn-adn}\right)x(a+bx^n)^3 (c+dx^n)^{-3-\frac{1}{n}}}{3ac(1+3n)} + \frac{(}{}$$
$$= -\frac{bx(a+bx^n)^3 (c+dx^n)^{-3-\frac{1}{n}}}{3a(bc-ad)n} + \frac{\left(3+\frac{bc}{bcn-adn}\right)x(a+bx^n)^3 (c+dx^n)^{-3-\frac{1}{n}}}{3ac(1+3n)} + \frac{n}{}$$
$$= -\frac{bx(a+bx^n)^3 (c+dx^n)^{-3-\frac{1}{n}}}{3a(bc-ad)n} + \frac{\left(3+\frac{bc}{bcn-adn}\right)x(a+bx^n)^3 (c+dx^n)^{-3-\frac{1}{n}}}{3ac(1+3n)} + \frac{n}{}$$
$$= -\frac{bx(a+bx^n)^3 (c+dx^n)^{-3-\frac{1}{n}}}{3a(bc-ad)n} + \frac{\left(3+\frac{bc}{bcn-adn}\right)x(a+bx^n)^3 (c+dx^n)^{-3-\frac{1}{n}}}{3ac(1+3n)} + \frac{n}{}$$

Mathematica [C] time = 0.46, size = 136, normalized size = 0.42

$$\frac{x(c+dx^n)^{-1/n}\left(\frac{dx^n}{c}+1\right)^{\frac{1}{n}}\left(b^2c^2\,{}_2F_1\left(2+\frac{1}{n},\frac{1}{n};1+\frac{1}{n};-\frac{dx^n}{c}\right)-(bc-ad)\left((ad-bc)\,{}_2F_1\left(4+\frac{1}{n},\frac{1}{n};1+\frac{1}{n};-\frac{dx^n}{c}\right)+2bc\,{}_2F_1\left(3+\frac{1}{n},\frac{1}{n};1+\frac{1}{n};-\frac{dx^n}{c}\right)\right)\right)}{c^4d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^n)^2*(c + d*x^n)^(-4 - n^(-1)),x]
```

```
[Out] (x*(1 + (d*x^n)/c)^n^(-1)*(b^2*c^2*Hypergeometric2F1[2 + n^(-1), n^(-1), 1
+ n^(-1), -((d*x^n)/c)] - (b*c - a*d)*(2*b*c*Hypergeometric2F1[3 + n^(-1),
n^(-1), 1 + n^(-1), -((d*x^n)/c)] + (-(b*c) + a*d)*Hypergeometric2F1[4 + n^
(-1), n^(-1), 1 + n^(-1), -((d*x^n)/c)])))/(c^4*d^2*(c + d*x^n)^n^(-1))
```

IntegrateAlgebraic [F] time = 0.07, size = 0, normalized size = 0.00

$$\int (a+bx^n)^2 (c+dx^n)^{-4-\frac{1}{n}}\, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x^n)^2*(c + d*x^n)^(-4 - n^(-1)),x]
```

```
[Out] Defer[IntegrateAlgebraic][(a + b*x^n)^2*(c + d*x^n)^(-4 - n^(-1)), x]
```

fricas [A] time = 0.87, size = 400, normalized size = 1.22

$$\frac{\left(6a^2d^4n^3+b^2c^2d^2n+\left(b^2c^2d^2+4abcd^3\right)n^2\right)xx^{4n}+\left(24a^2cd^3n^3+b^2c^3d+2\left(2b^2c^3d+8abc^2d^2+3a^2cd^3\right)n^2+\left(5b^2c^3d+4abc^2d^2\right)n\right)xx^{3n}+\left(36a^2c^2d^2n^3+b^2c^4+2abc^3d+3\left(b^2c^4+8abc^3d+7a^2c^2d^2\right)n^2+\left(4b^2c^4+14abc^3d+3a^2c^2d^2\right)n\right)xx^{2n}+\left(24a^2c^3dn^3+2abc^4+a^2c^3d+2\left(6abc^4+13a^2c^3d\right)n^2+\left(10abc^4+9a^2c^3d\right)n\right)xx^n+\left(6a^2c^4n^3+11a^2c^4n^2+6a^2c^4n+a^2c^4\right)x}{\left(6c^4n^3+11c^4n^2+6c^4n+c^4\right)(dx^n+c)^{\frac{4n+1}{n}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^2*(c+d*x^n)^(-4-1/n),x, algorithm="fricas")
```

```
[Out] ((6*a^2*d^4*n^3 + b^2*c^2*d^2*n + (b^2*c^2*d^2 + 4*a*b*c*d^3)*n^2)*x*x^(4*n
) + (24*a^2*c*d^3*n^3 + b^2*c^3*d + 2*(2*b^2*c^3*d + 8*a*b*c^2*d^2 + 3*a^2*
c*d^3)*n^2 + (5*b^2*c^3*d + 4*a*b*c^2*d^2)*n)*x*x^(3*n) + (36*a^2*c^2*d^2*n
^3 + b^2*c^4 + 2*a*b*c^3*d + 3*(b^2*c^4 + 8*a*b*c^3*d + 7*a^2*c^2*d^2)*n^2
+ (4*b^2*c^4 + 14*a*b*c^3*d + 3*a^2*c^2*d^2)*n)*x*x^(2*n) + (24*a^2*c^3*d*n
^3 + 2*a*b*c^4 + a^2*c^3*d + 2*(6*a*b*c^4 + 13*a^2*c^3*d)*n^2 + (10*a*b*c^4
 + 9*a^2*c^3*d)*n)*x*x^n + (6*a^2*c^4*n^3 + 11*a^2*c^4*n^2 + 6*a^2*c^4*n +
a^2*c^4)*x)/((6*c^4*n^3 + 11*c^4*n^2 + 6*c^4*n + c^4)*(d*x^n + c)^((4*n + 1
)/n))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^2*(c+d*x^n)^(-4-1/n),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to divide, perhaps due to rounding er
ror%%%{27,[1,0,4,3,1,3,2,0]%%%}+%%%{27,[1,0,4,2,1,3,2,0]%%%}+%%%{9,[1,0,4,1
,1,3,2,0]%%%}+%%%{1,[1,0,4,0,1,3,2,0]%%%}+%%%{27,[0,0,4,3,2,2,0,2]%%%}+%%%{
18,[0,0,4,2,2,2,0,2]%%%}+%%%{18,[0,0,4,2,1,3,1,1]%%%}+%%%{-9,[0,0,4,2,0,4,2
,0]%%%}+%%%{3,[0,0,4,1,2,2,0,2]%%%}+%%%{12,[0,0,4,1,1,3,1,1]%%%}+%%%{-6,[0,
0,4,1,0,4,2,0]%%%}+%%%{2,[0,0,4,0,1,3,1,1]%%%}+%%%{-1,[0,0,4,0,0,4,2,0]%%%}
 / %%%{27,[0,0,5,3,2,3,0,0]%%%}+%%%{27,[0,0,5,2,2,3,0,0]%%%}+%%%{9,[0,0,5,1
,2,3,0,0]%%%}+%%%{1,[0,0,5,0,2,3,0,0]%%%} Error: Bad Argument Value
```

maple [F] time = 0.77, size = 0, normalized size = 0.00

$$\int (b\,x^{n}+a)^{2} (d\,x^{n}+c)^{-\frac{1}{n}-4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^n+a)^2*(d*x^n+c)^(-1/n-4),x)
```

```
[Out] int((b*x^n+a)^2*(d*x^n+c)^(-1/n-4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (bx^{n}+a)^{2}(dx^{n}+c)^{-\frac{1}{n}-4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^2*(c+d*x^n)^(-4-1/n),x, algorithm="maxima")
```

```
[Out] integrate((b*x^n + a)^2*(d*x^n + c)^(-1/n - 4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,x^{n})^{2}}{(c+d\,x^{n})^{\frac{1}{n}+4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^n)^2/(c + d*x^n)^(1/n + 4),x)
```

```
[Out] int((a + b*x^n)^2/(c + d*x^n)^(1/n + 4), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**n)**2*(c+d*x**n)**(-4-1/n),x)
```

```
[Out] Timed out
```

3.192 $\int (a+bx^n)(c+dx^n)^{-3-\frac{1}{n}}\,dx$

Optimal. Leaf size=127

$$\frac{nx(c+dx^n)^{-1/n}(2adn+bc)}{c^3d(n+1)(2n+1)}+\frac{x(c+dx^n)^{-\frac{1}{n}-1}(2adn+bc)}{c^2d(n+1)(2n+1)}-\frac{x(bc-ad)(c+dx^n)^{-\frac{1}{n}-2}}{cd(2n+1)}$$

Rubi [A] time = 0.06, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {385, 192, 191}

$$\frac{x(c+dx^n)^{-\frac{1}{n}-1}(2adn+bc)}{c^2d(n+1)(2n+1)}+\frac{nx(c+dx^n)^{-1/n}(2adn+bc)}{c^3d(n+1)(2n+1)}-\frac{x(bc-ad)(c+dx^n)^{-\frac{1}{n}-2}}{cd(2n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^n)*(c + d*x^n)^(-3 - n^(-1)),x]
```

```
[Out] -(((b*c - a*d)*x*(c + d*x^n)^(-2 - n^(-1)))/(c*d*(1 + 2*n))) + ((b*c + 2*a*
d*n)*x*(c + d*x^n)^(-1 - n^(-1)))/(c^2*d*(1 + n)*(1 + 2*n)) + (n*(b*c + 2*a
*d*n)*x)/(c^3*d*(1 + n)*(1 + 2*n)*(c + d*x^n)^n^(-1))
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rubi steps

$$\int (a+bx^n)(c+dx^n)^{-3-\frac{1}{n}}\,dx=-\frac{(bc-ad)x(c+dx^n)^{-2-\frac{1}{n}}}{cd(1+2n)}+\frac{(bc+2adn)\int (c+dx^n)^{-2-\frac{1}{n}}\,dx}{cd(1+2n)}$$

$$=-\frac{(bc-ad)x(c+dx^n)^{-2-\frac{1}{n}}}{cd(1+2n)}+\frac{(bc+2adn)x(c+dx^n)^{-1-\frac{1}{n}}}{c^2d(1+n)(1+2n)}+\frac{(n(bc+2adn))\int (\ldots}{c^2d(1+n)\ldots}$$

$$=-\frac{(bc-ad)x(c+dx^n)^{-2-\frac{1}{n}}}{cd(1+2n)}+\frac{(bc+2adn)x(c+dx^n)^{-1-\frac{1}{n}}}{c^2d(1+n)(1+2n)}+\frac{n(bc+2adn)x(c+\ldots}{c^3d(1+n)(1-\ldots}$$

Mathematica [C] time = 0.17, size = 94, normalized size = 0.74

$$\frac{x(c+dx^n)^{-1/n}\left(\frac{dx^n}{c}+1\right)^{\frac{1}{n}}\left((ad-bc)\,{}_2F_1\left(3+\frac{1}{n},\frac{1}{n};1+\frac{1}{n};-\frac{dx^n}{c}\right)+bc\,{}_2F_1\left(2+\frac{1}{n},\frac{1}{n};1+\frac{1}{n};-\frac{dx^n}{c}\right)\right)}{c^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^n)*(c + d*x^n)^(-3 - n^(-1)),x]
```

```
[Out] (x*(1 + (d*x^n)/c)^n^(-1)*(b*c*Hypergeometric2F1[2 + n^(-1), n^(-1), 1 + n^
(-1), -((d*x^n)/c)] + (-(b*c) + a*d)*Hypergeometric2F1[3 + n^(-1), n^(-1),
1 + n^(-1), -((d*x^n)/c)]))/(c^3*d*(c + d*x^n)^n^(-1))
```

IntegrateAlgebraic [F] time = 0.05, size = 0, normalized size = 0.00

$$\int (a + bx^n)(c + dx^n)^{-3-\frac{1}{n}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x^n)*(c + d*x^n)^(-3 - n^(-1)),x]
```

```
[Out] Defer[IntegrateAlgebraic][(a + b*x^n)*(c + d*x^n)^(-3 - n^(-1)), x]
```

fricas [A] time = 0.85, size = 173, normalized size = 1.36

$$\frac{\left(2\,ad^3n^2 + bcd^2n\right)xx^{3\,n} + \left(6\,acd^2n^2 + bc^2d + \left(3\,bc^2d + 2\,acd^2\right)n\right)xx^{2\,n} + \left(6\,ac^2dn^2 + bc^3 + ac^2d + \left(2\,bc^3 + 5\,ac^2d\right)n\right)xx^n + \left(2\,ac^3n^2 + 3\,ac^3n + ac^3\right)x}{\left(2\,c^3n^2 + 3\,c^3n + c^3\right)(dx^n + c)^{\frac{3\,n+1}{n}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)*(c+d*x^n)^(-3-1/n),x, algorithm="fricas")
```

```
[Out] ((2*a*d^3*n^2 + b*c*d^2*n)*x*x^(3*n) + (6*a*c*d^2*n^2 + b*c^2*d + (3*b*c^2*
d + 2*a*c*d^2)*n)*x*x^(2*n) + (6*a*c^2*d*n^2 + b*c^3 + a*c^2*d + (2*b*c^3 +
 5*a*c^2*d)*n)*x*x^n + (2*a*c^3*n^2 + 3*a*c^3*n + a*c^3)*x)/((2*c^3*n^2 + 3
*c^3*n + c^3)*(d*x^n + c)^((3*n + 1)/n))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)*(c+d*x^n)^(-3-1/n),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to divide, perhaps due to rounding er
ror%%%{4,[0,0,2,2,1,1,0,1]%%%}+%%%{2,[0,0,2,1,1,1,0,1]%%%}+%%%{2,[0,0,2,1,0
,2,1,0]%%%}+%%%{1,[0,0,2,0,0,2,1,0]%%%} / %%%{4,[0,0,3,2,1,2,0,0]%%%}+%%%{4
,[0,0,3,1,1,2,0,0]%%%}+%%%{1,[0,0,3,0,1,2,0,0]%%%} Error: Bad Argument Valu
e
```

maple [F] time = 0.63, size = 0, normalized size = 0.00

$$\int (b\,x^n + a)(d\,x^n + c)^{-\frac{1}{n}-3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^n+a)*(d*x^n+c)^(-1/n-3),x)
```

```
[Out] int((b*x^n+a)*(d*x^n+c)^(-1/n-3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (bx^n + a)(dx^n + c)^{-\frac{1}{n}-3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)*(c+d*x^n)^(-3-1/n),x, algorithm="maxima")
```

```
[Out] integrate((b*x^n + a)*(d*x^n + c)^(-1/n - 3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a+b\,x^n}{{\left(c+d\,x^n\right)}^{\frac{1}{n}+3}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^n)/(c + d*x^n)^(1/n + 3),x)
```

```
[Out] int((a + b*x^n)/(c + d*x^n)^(1/n + 3), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**n)*(c+d*x**n)**(-3-1/n),x)
```

```
[Out] Timed out
```

3.193 $\int (c + dx^n)^{-2-\frac{1}{n}}\,dx$

Optimal. Leaf size=50

$$\frac{nx\,(c + dx^n)^{-1/n}}{c^2(n+1)} + \frac{x\,(c + dx^n)^{-\frac{1}{n}-1}}{c(n+1)}$$

Rubi [A] time = 0.01, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {192, 191}

$$\frac{nx\,(c + dx^n)^{-1/n}}{c^2(n+1)} + \frac{x\,(c + dx^n)^{-\frac{1}{n}-1}}{c(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^n)^(-2 - n^(-1)),x]
```

```
[Out] (x*(c + d*x^n)^(-1 - n^(-1)))/(c*(1 + n)) + (n*x)/(c^2*(1 + n)*(c + d*x^n)^
n^(-1))
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int (c + dx^n)^{-2-\frac{1}{n}}\,dx &= \frac{x\,(c + dx^n)^{-1-\frac{1}{n}}}{c(1+n)} + \frac{n\int (c + dx^n)^{-1-\frac{1}{n}}\,dx}{c(1+n)}\\ &= \frac{x\,(c + dx^n)^{-1-\frac{1}{n}}}{c(1+n)} + \frac{nx\,(c + dx^n)^{-1/n}}{c^2(1+n)}\end{aligned}$$

Mathematica [C] time = 0.03, size = 55, normalized size = 1.10

$$\frac{x\,(c + dx^n)^{-1/n}\left(\frac{dx^n}{c}+1\right)^{\frac{1}{n}}\,{}_2F_1\left(2+\frac{1}{n},\frac{1}{n};1+\frac{1}{n};-\frac{dx^n}{c}\right)}{c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^n)^(-2 - n^(-1)),x]
```

```
[Out] (x*(1 + (d*x^n)/c)^n^(-1)*Hypergeometric2F1[2 + n^(-1), n^(-1), 1 + n^(-1),
 -((d*x^n)/c)])/(c^2*(c + d*x^n)^n^(-1))
```

IntegrateAlgebraic [F] time = 0.02, size = 0, normalized size = 0.00

$$\int (c + dx^n)^{-2-\frac{1}{n}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x^n)^(-2 - n^(-1)),x]
```

```
[Out] Defer[IntegrateAlgebraic][(c + d*x^n)^(-2 - n^(-1)), x]
```

fricas [A] time = 0.90, size = 68, normalized size = 1.36

$$\frac{d^2nxx^{2\,n}+(2\,cdn+cd)xx^n+\left(c^2n+c^2\right)x}{\left(c^2n+c^2\right)(dx^n+c)^{\frac{2\,n+1}{n}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*x^n)^(-2-1/n),x, algorithm="fricas")
```

```
[Out] (d^2*n*x*x^(2*n) + (2*c*d*n + c*d)*x*x^n + (c^2*n + c^2)*x)/((c^2*n + c^2)*
(d*x^n + c)^((2*n + 1)/n))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int(dx^n+c)^{-\frac{1}{n}-2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*x^n)^(-2-1/n),x, algorithm="giac")
```

```
[Out] integrate((d*x^n + c)^(-1/n - 2), x)
```

maple [F] time = 0.60, size = 0, normalized size = 0.00

$$\int(d\,x^n+c)^{-\frac{1}{n}-2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^n+c)^(-1/n-2),x)
```

```
[Out] int((d*x^n+c)^(-1/n-2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int(dx^n+c)^{-\frac{1}{n}-2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*x^n)^(-2-1/n),x, algorithm="maxima")
```

```
[Out] integrate((d*x^n + c)^(-1/n - 2), x)
```

mupad [B] time = 1.76, size = 64, normalized size = 1.28

$$-\frac{x^{1-2\,n}\,{\left(\frac{c}{d\,x^n}+1\right)}^{1/n}\,{}_2\mathrm{F}_1\left(2,\frac{1}{n}+2;\ 3;\ -\frac{c}{d\,x^n}\right)}{2\,d^2\,n\,{\left(c+d\,x^n\right)}^{1/n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c + d*x^n)^(1/n + 2),x)
```

```
[Out] -(x^(1 - 2*n)*(c/(d*x^n) + 1)^(1/n)*hypergeom([2, 1/n + 2], 3, -c/(d*x^n)))
/(2*d^2*n*(c + d*x^n)^(1/n))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*x**n)**(-2-1/n),x)
```

```
[Out] Timed out
```

3.194 $\int x^5\sqrt{-c+dx}\,\sqrt{c+dx}\,\left(a+bx^2\right)\,dx$

Optimal. Leaf size=152

$$\frac{(dx-c)^{7/2}(c+dx)^{7/2}\left(ad^2+3bc^2\right)}{7d^8}+\frac{c^2(dx-c)^{5/2}(c+dx)^{5/2}\left(2ad^2+3bc^2\right)}{5d^8}+\frac{c^4(dx-c)^{3/2}(c+dx)^{3/2}\left(ad^2+bc^2\right)}{3d^8}$$

Rubi [A] time = 0.12, antiderivative size = 164, normalized size of antiderivative = 1.08, number of steps used = 6, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {460, 100, 12, 74}

$$\frac{x^4(dx-c)^{3/2}(c+dx)^{3/2}\left(3ad^2+2bc^2\right)}{21d^4}+\frac{4c^2x^2(dx-c)^{3/2}(c+dx)^{3/2}\left(3ad^2+2bc^2\right)}{105d^6}+\frac{8c^4(dx-c)^{3/2}(c+dx)^{3/2}\left(3ad^2+2bc^2\right)}{315d^8}+\frac{bx^6(dx-c)^{3/2}(c+dx)^{3/2}}{9d^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*Sqrt[-c + d*x]*Sqrt[c + d*x]*(a + b*x^2),x]
```

```
[Out] (8*c^4*(2*b*c^2 + 3*a*d^2)*(-c + d*x)^(3/2)*(c + d*x)^(3/2))/(315*d^8) + (4
*c^2*(2*b*c^2 + 3*a*d^2)*x^2*(-c + d*x)^(3/2)*(c + d*x)^(3/2))/(105*d^6) +
((2*b*c^2 + 3*a*d^2)*x^4*(-c + d*x)^(3/2)*(c + d*x)^(3/2))/(21*d^4) + (b*x^
6*(-c + d*x)^(3/2)*(c + d*x)^(3/2))/(9*d^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/(d*f*(m + n + p + 1)), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 460

```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[(d*(e*x)^
(m + 1)*(a1 + b1*x^(n/2))^(p + 1)*(a2 + b2*x^(n/2))^(p + 1))/(b1*b2*e*(m +
n*(p + 1) + 1)), x] - Dist[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/
(b1*b2*(m + n*(p + 1) + 1)), Int[(e*x)^m*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n/
2))^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, m, n, p}, x] && EqQ[non2,
n/2] && EqQ[a2*b1 + a1*b2, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rubi steps

$$\int x^5\sqrt{-c+dx}\,\sqrt{c+dx}\,\left(a+bx^2\right)dx = \frac{bx^6(-c+dx)^{3/2}(c+dx)^{3/2}}{9d^2}+\frac{1}{3}\left(3a+\frac{2bc^2}{d^2}\right)\int x^5\sqrt{-c+dx}\,\sqrt{c+dx}\,dx$$

$$= \frac{\left(2bc^2+3ad^2\right)x^4(-c+dx)^{3/2}(c+dx)^{3/2}}{21d^4}+\frac{bx^6(-c+dx)^{3/2}(c+dx)^{3/2}}{9d^2}+ \ldots$$

$$= \frac{\left(2bc^2+3ad^2\right)x^4(-c+dx)^{3/2}(c+dx)^{3/2}}{21d^4}+\frac{bx^6(-c+dx)^{3/2}(c+dx)^{3/2}}{9d^2}+ \ldots$$

$$= \frac{4c^2\left(2bc^2+3ad^2\right)x^2(-c+dx)^{3/2}(c+dx)^{3/2}}{105d^6}+\frac{\left(2bc^2+3ad^2\right)x^4(-c+dx)^{3}\ldots}{21d^4}$$

$$= \frac{4c^2\left(2bc^2+3ad^2\right)x^2(-c+dx)^{3/2}(c+dx)^{3/2}}{105d^6}+\frac{\left(2bc^2+3ad^2\right)x^4(-c+dx)^{3}\ldots}{21d^4}$$

$$= \frac{8c^4\left(2bc^2+3ad^2\right)(-c+dx)^{3/2}(c+dx)^{3/2}}{315d^8}+\frac{4c^2\left(2bc^2+3ad^2\right)x^2(-c+dx\ldots}{105d^6}$$

Mathematica [A] time = 0.10, size = 110, normalized size = 0.72

$$\frac{\sqrt{dx-c}\,\sqrt{c+dx}\,\left(d^2x^2-c^2\right)\left(3ad^2\left(8c^4+12c^2d^2x^2+15d^4x^4\right)+b\left(16c^6+24c^4d^2x^2+30c^2d^4x^4+35d^6x^6\right)\right)}{315d^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*Sqrt[-c + d*x]*Sqrt[c + d*x]*(a + b*x^2),x]
```

```
[Out] (Sqrt[-c + d*x]*Sqrt[c + d*x]*(-c^2 + d^2*x^2)*(3*a*d^2*(8*c^4 + 12*c^2*d^2
*x^2 + 15*d^4*x^4) + b*(16*c^6 + 24*c^4*d^2*x^2 + 30*c^2*d^4*x^4 + 35*d^6*x
^6)))/(315*d^8)
```

IntegrateAlgebraic [B] time = 0.22, size = 340, normalized size = 2.24

$$-\frac{8(dx-c)^{3/2}\left(-\frac{126ac^7d^2(dx-c)}{c+dx}+\frac{279ac^7d^2(dx-c)^2}{(c+dx)^2}-\frac{516ac^7d^2(dx-c)^3}{(c+dx)^3}+\frac{279ac^7d^2(dx-c)^4}{(c+dx)^4}-\frac{126ac^7d^2(dx-c)^5}{(c+dx)^5}+\frac{105ac^7d^2(dx-c)^6}{(c+dx)^6}+105ac^7d^2+\frac{126bc^9(dx-c)}{c+dx}+\frac{711bc^9(dx-c)^2}{(c+dx)^2}+\frac{356bc^9(dx-c)^3}{(c+dx)^3}+\frac{711bc^9(dx-c)^4}{(c+dx)^4}+\frac{126bc^9(dx-c)^5}{(c+dx)^5}+\frac{105bc^9(dx-c)^6}{(c+dx)^6}+105bc^9\right)}{315d^8(c+dx)^{3/2}\left(\frac{dx-c}{c+dx}-1\right)^9}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^5*Sqrt[-c + d*x]*Sqrt[c + d*x]*(a + b*x^2),x]
```

```
[Out] (-8*(-c + d*x)^(3/2)*(105*b*c^9 + 105*a*c^7*d^2 + (105*b*c^9*(-c + d*x)^6)/
(c + d*x)^6 + (105*a*c^7*d^2*(-c + d*x)^6)/(c + d*x)^6 + (126*b*c^9*(-c + d
*x)^5)/(c + d*x)^5 - (126*a*c^7*d^2*(-c + d*x)^5)/(c + d*x)^5 + (711*b*c^9*
(-c + d*x)^4)/(c + d*x)^4 + (279*a*c^7*d^2*(-c + d*x)^4)/(c + d*x)^4 + (356
*b*c^9*(-c + d*x)^3)/(c + d*x)^3 - (516*a*c^7*d^2*(-c + d*x)^3)/(c + d*x)^3
 + (711*b*c^9*(-c + d*x)^2)/(c + d*x)^2 + (279*a*c^7*d^2*(-c + d*x)^2)/(c +
 d*x)^2 + (126*b*c^9*(-c + d*x))/(c + d*x) - (126*a*c^7*d^2*(-c + d*x))/(c
+ d*x)))/(315*d^8*(c + d*x)^(3/2)*(-1 + (-c + d*x)/(c + d*x))^9)
```

fricas [A] time = 0.96, size = 114, normalized size = 0.75

$$\frac{\left(35\,bd^8x^8-16\,bc^8-24\,ac^6d^2-5\left(bc^2d^6-9\,ad^8\right)x^6-3\left(2\,bc^4d^4+3\,ac^2d^6\right)x^4-4\left(2\,bc^6d^2+3\,ac^4d^4\right)x^2\right)\sqrt{dx+c}\,\sqrt{dx-c}}{315\,d^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^2+a)*(d*x-c)^(1/2)*(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/315*(35*b*d^8*x^8 - 16*b*c^8 - 24*a*c^6*d^2 - 5*(b*c^2*d^6 - 9*a*d^8)*x^6
 - 3*(2*b*c^4*d^4 + 3*a*c^2*d^6)*x^4 - 4*(2*b*c^6*d^2 + 3*a*c^4*d^4)*x^2)*s
qrt(d*x + c)*sqrt(d*x - c)/d^8
```

giac [B] time = 0.76, size = 621, normalized size = 4.09

$$\frac{168\left(\left(\left(2\left(\left(dx+c\right)\left(4\left(dx+c\right)\left(\frac{5(dx+c)}{d^5}-\frac{31c}{d^5}\right)+\frac{321c^2}{d^5}\right)-\frac{451c^3}{d^5}\right)\left(dx+c\right)+\frac{745c^4}{d^5}\right)\left(dx+c\right)-\frac{405c^5}{d^5}\right)\sqrt{dx+c}\sqrt{dx-c}-\frac{150c^6\log\left(\left|-\sqrt{dx+c}+\sqrt{dx-c}\right|\right)}{d^5}\right)ac+3\left(\left(\left(2\left(\left(4\left(5\left(dx+c\right)\left(6\left(dx+c\right)\left(\frac{7(dx+c)}{d^7}-\frac{57c}{d^7}\right)+\frac{1219c^2}{d^7}\right)-\frac{12463c^3}{d^7}\right)\left(dx+c\right)+\frac{64233c^4}{d^7}\right)\left(dx+c\right)-\frac{53963c^5}{d^7}\right)\left(dx+c\right)+\frac{59465c^6}{d^7}\right)\left(dx+c\right)-\frac{23205c^7}{d^7}\right)\sqrt{dx+c}\sqrt{dx-c}-\frac{7350c^8\log\left(\left|-\sqrt{dx+c}+\sqrt{dx-c}\right|\right)}{d^7}\right)bc+24\left(\left(\left(2\left(\left(4\left(dx+c\right)\left(5\left(dx+c\right)\left(\frac{6(dx+c)}{d^6}-\frac{43c}{d^6}\right)+\frac{661c^2}{d^6}\right)-\frac{4551c^3}{d^6}\right)\left(dx+c\right)+\frac{4781c^4}{d^6}\right)\left(dx+c\right)-\frac{6335c^5}{d^6}\right)\left(dx+c\right)+\frac{2835c^6}{d^6}\right)\sqrt{dx+c}\sqrt{dx-c}+\frac{1050c^7\log\left(\left|-\sqrt{dx+c}+\sqrt{dx-c}\right|\right)}{d^6}\right)ad+\left(\left(\left(2\left(\left(4\left(5\left(2\left(dx+c\right)\left(7\left(dx+c\right)\left(\frac{8(dx+c)}{d^8}-\frac{73c}{d^8}\right)+\frac{2073c^2}{d^8}\right)-\frac{9833c^3}{d^8}\right)\left(dx+c\right)+\frac{75293c^4}{d^8}\right)\left(dx+c\right)-\frac{310203c^5}{d^8}\right)\left(dx+c\right)+\frac{216993c^6}{d^8}\right)\left(dx+c\right)-\frac{205275c^7}{d^8}\right)\left(dx+c\right)+\frac{69615c^8}{d^8}\right)\sqrt{dx+c}\sqrt{dx-c}+\frac{22050c^9\log\left(\left|-\sqrt{dx+c}+\sqrt{dx-c}\right|\right)}{d^8}\right)bd}{40320\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^2+a)*(d*x-c)^(1/2)*(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/40320*(168*(((2*((d*x + c)*(4*(d*x + c)*(5*(d*x + c)/d^5 - 31*c/d^5) + 32
1*c^2/d^5) - 451*c^3/d^5)*(d*x + c) + 745*c^4/d^5)*(d*x + c) - 405*c^5/d^5)
*sqrt(d*x + c)*sqrt(d*x - c) - 150*c^6*log(abs(-sqrt(d*x + c) + sqrt(d*x -
c)))/d^5)*a*c + 3*(((2*((4*(5*(d*x + c)*(6*(d*x + c)*(7*(d*x + c)/d^7 - 57*
c/d^7) + 1219*c^2/d^7) - 12463*c^3/d^7)*(d*x + c) + 64233*c^4/d^7)*(d*x + c
) - 53963*c^5/d^7)*(d*x + c) + 59465*c^6/d^7)*(d*x + c) - 23205*c^7/d^7)*sq
rt(d*x + c)*sqrt(d*x - c) - 7350*c^8*log(abs(-sqrt(d*x + c) + sqrt(d*x - c)
))/d^7)*b*c + 24*(((2*((4*(d*x + c)*(5*(d*x + c)*(6*(d*x + c)/d^6 - 43*c/d^
6) + 661*c^2/d^6) - 4551*c^3/d^6)*(d*x + c) + 4781*c^4/d^6)*(d*x + c) - 633
5*c^5/d^6)*(d*x + c) + 2835*c^6/d^6)*sqrt(d*x + c)*sqrt(d*x - c) + 1050*c^7
*log(abs(-sqrt(d*x + c) + sqrt(d*x - c)))/d^6)*a*d + (((2*((4*(5*(2*(d*x +
c)*(7*(d*x + c)*(8*(d*x + c)/d^8 - 73*c/d^8) + 2073*c^2/d^8) - 9833*c^3/d^8
)*(d*x + c) + 75293*c^4/d^8)*(d*x + c) - 310203*c^5/d^8)*(d*x + c) + 216993
*c^6/d^8)*(d*x + c) - 205275*c^7/d^8)*(d*x + c) + 69615*c^8/d^8)*sqrt(d*x +
 c)*sqrt(d*x - c) + 22050*c^9*log(abs(-sqrt(d*x + c) + sqrt(d*x - c)))/d^8)
*b*d)/d
```

maple [A] time = 0.04, size = 92, normalized size = 0.61

$$\frac{(dx+c)^{\frac{3}{2}}\left(35b\,x^6d^6+45a\,d^6x^4+30b\,c^2d^4x^4+36a\,c^2d^4x^2+24b\,c^4d^2x^2+24a\,c^4d^2+16b\,c^6\right)(dx-c)^{\frac{3}{2}}}{315d^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(b*x^2+a)*(d*x-c)^(1/2)*(d*x+c)^(1/2),x)
```

```
[Out] 1/315*(d*x+c)^(3/2)*(35*b*d^6*x^6+45*a*d^6*x^4+30*b*c^2*d^4*x^4+36*a*c^2*d^
4*x^2+24*b*c^4*d^2*x^2+24*a*c^4*d^2+16*b*c^6)*(d*x-c)^(3/2)/d^8
```

maxima [A] time = 0.50, size = 178, normalized size = 1.17

$$\frac{\left(d^2x^2-c^2\right)^{\frac{3}{2}}bx^6}{9\,d^2}+\frac{2\left(d^2x^2-c^2\right)^{\frac{3}{2}}bc^2x^4}{21\,d^4}+\frac{\left(d^2x^2-c^2\right)^{\frac{3}{2}}ax^4}{7\,d^2}+\frac{8\left(d^2x^2-c^2\right)^{\frac{3}{2}}bc^4x^2}{105\,d^6}+\frac{4\left(d^2x^2-c^2\right)^{\frac{3}{2}}ac^2x^2}{35\,d^4}+\frac{16\left(d^2x^2-c^2\right)^{\frac{3}{2}}bc^6}{315\,d^8}+\frac{8\left(d^2x^2-c^2\right)^{\frac{3}{2}}ac^4}{105\,d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^2+a)*(d*x-c)^(1/2)*(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/9*(d^2*x^2 - c^2)^(3/2)*b*x^6/d^2 + 2/21*(d^2*x^2 - c^2)^(3/2)*b*c^2*x^4/
d^4 + 1/7*(d^2*x^2 - c^2)^(3/2)*a*x^4/d^2 + 8/105*(d^2*x^2 - c^2)^(3/2)*b*c
^4*x^2/d^6 + 4/35*(d^2*x^2 - c^2)^(3/2)*a*c^2*x^2/d^4 + 16/315*(d^2*x^2 - c
^2)^(3/2)*b*c^6/d^8 + 8/105*(d^2*x^2 - c^2)^(3/2)*a*c^4/d^6
```

mupad [B] time = 1.76, size = 152, normalized size = 1.00

$$-\sqrt{d\,x-c}\,\left(\frac{\left(16\,b\,c^8+24\,a\,c^6\,d^2\right)\,\sqrt{c+d\,x}}{315\,d^8}-\frac{b\,x^8\,\sqrt{c+d\,x}}{9}+\frac{x^4\,\left(6\,b\,c^4\,d^4+9\,a\,c^2\,d^6\right)\,\sqrt{c+d\,x}}{315\,d^8}+\frac{x^2\,\left(8\,b\,c^6\,d^2+12\,a\,c^4\,d^4\right)\,\sqrt{c+d\,x}}{315\,d^8}-\frac{x^6\,\left(45\,a\,d^8-5\,b\,c^2\,d^6\right)\,\sqrt{c+d\,x}}{315\,d^8}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a + b*x^2)*(c + d*x)^(1/2)*(d*x - c)^(1/2),x)
```

```
[Out] -(d*x - c)^(1/2)*(((16*b*c^8 + 24*a*c^6*d^2)*(c + d*x)^(1/2))/(315*d^8) - (
b*x^8*(c + d*x)^(1/2))/9 + (x^4*(9*a*c^2*d^6 + 6*b*c^4*d^4)*(c + d*x)^(1/2)
```

```
)/(315*d^8) + (x^2*(12*a*c^4*d^4 + 8*b*c^6*d^2)*(c + d*x)^(1/2))/(315*d^8)
- (x^6*(45*a*d^8 - 5*b*c^2*d^6)*(c + d*x)^(1/2))/(315*d^8))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^5 \left(a + bx^2\right) \sqrt{-c + dx} \sqrt{c + dx} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(b*x**2+a)*(d*x-c)**(1/2)*(d*x+c)**(1/2),x)
```

```
[Out] Integral(x**5*(a + b*x**2)*sqrt(-c + d*x)*sqrt(c + d*x), x)
```

3.195 $\int x^3\sqrt{-c+dx}\sqrt{c+dx}\,(a+bx^2)\,dx$

Optimal. Leaf size=109

$$\frac{(dx-c)^{5/2}(c+dx)^{5/2}\left(ad^2+2bc^2\right)}{5d^6}+\frac{c^2(dx-c)^{3/2}(c+dx)^{3/2}\left(ad^2+bc^2\right)}{3d^6}+\frac{b(dx-c)^{7/2}(c+dx)^{7/2}}{7d^6}$$

Rubi [A] time = 0.09, antiderivative size = 118, normalized size of antiderivative = 1.08, number of steps used = 4, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {460, 100, 12, 74}

$$\frac{x^2(dx-c)^{3/2}(c+dx)^{3/2}\left(7ad^2+4bc^2\right)}{35d^4}+\frac{2c^2(dx-c)^{3/2}(c+dx)^{3/2}\left(7ad^2+4bc^2\right)}{105d^6}+\frac{bx^4(dx-c)^{3/2}(c+dx)^{3/2}}{7d^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Sqrt[-c + d*x]*Sqrt[c + d*x]*(a + b*x^2),x]
```

```
[Out] (2*c^2*(4*b*c^2 + 7*a*d^2)*(-c + d*x)^(3/2)*(c + d*x)^(3/2))/(105*d^6) + ((
4*b*c^2 + 7*a*d^2)*x^2*(-c + d*x)^(3/2)*(c + d*x)^(3/2))/(35*d^4) + (b*x^4*
(-c + d*x)^(3/2)*(c + d*x)^(3/2))/(7*d^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/(d*f*(m + n + p + 1)), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 460

```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[(d*(e*x)^
(m + 1)*(a1 + b1*x^(n/2))^(p + 1)*(a2 + b2*x^(n/2))^(p + 1))/(b1*b2*e*(m +
n*(p + 1) + 1)), x] - Dist[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/
(b1*b2*(m + n*(p + 1) + 1)), Int[(e*x)^m*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n/
2))^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, m, n, p}, x] && EqQ[non2,
n/2] && EqQ[a2*b1 + a1*b2, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rubi steps

$$\int x^3\sqrt{-c+dx}\,\sqrt{c+dx}\,\left(a+bx^2\right)dx = \frac{bx^4(-c+dx)^{3/2}(c+dx)^{3/2}}{7d^2} - \frac{1}{7}\left(-7a-\frac{4bc^2}{d^2}\right)\int x^3\sqrt{-c+dx}\,\sqrt{c+dx}\,dx$$
$$= \frac{\left(4bc^2+7ad^2\right)x^2(-c+dx)^{3/2}(c+dx)^{3/2}}{35d^4} + \frac{bx^4(-c+dx)^{3/2}(c+dx)^{3/2}}{7d^2} + \cdots$$
$$= \frac{\left(4bc^2+7ad^2\right)x^2(-c+dx)^{3/2}(c+dx)^{3/2}}{35d^4} + \frac{bx^4(-c+dx)^{3/2}(c+dx)^{3/2}}{7d^2} + \cdots$$
$$= \frac{2c^2\left(4bc^2+7ad^2\right)(-c+dx)^{3/2}(c+dx)^{3/2}}{105d^6} + \frac{\left(4bc^2+7ad^2\right)x^2(-c+dx)^{3/2}\cdots}{35d^4}$$

Mathematica [A] time = 0.06, size = 88, normalized size = 0.81

$$\frac{\sqrt{dx-c}\,\sqrt{c+dx}\,\left(d^2x^2-c^2\right)\left(7ad^2\left(2c^2+3d^2x^2\right)+b\left(8c^4+12c^2d^2x^2+15d^4x^4\right)\right)}{105d^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Sqrt[-c + d*x]*Sqrt[c + d*x]*(a + b*x^2),x]
```

```
[Out] (Sqrt[-c + d*x]*Sqrt[c + d*x]*(-c^2 + d^2*x^2)*(7*a*d^2*(2*c^2 + 3*d^2*x^2)
 + b*(8*c^4 + 12*c^2*d^2*x^2 + 15*d^4*x^4)))/(105*d^6)
```

IntegrateAlgebraic [B] time = 0.18, size = 246, normalized size = 2.26

$$-\frac{8(dx-c)^{3/2}\left(-\frac{56ac^5d^2(dx-c)}{c+dx}+\frac{42ac^5d^2(dx-c)^2}{(c+dx)^2}-\frac{56ac^5d^2(dx-c)^3}{(c+dx)^3}+\frac{35ac^5d^2(dx-c)^4}{(c+dx)^4}+35ac^5d^2+\frac{28bc^7(dx-c)}{c+dx}+\frac{114bc^7(dx-c)^2}{(c+dx)^2}+\frac{28bc^7(dx-c)^3}{(c+dx)^3}+\frac{35bc^7(dx-c)^4}{(c+dx)^4}+35bc^7\right)}{105d^6(c+dx)^{3/2}\left(\frac{dx-c}{c+dx}-1\right)^7}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^3*Sqrt[-c + d*x]*Sqrt[c + d*x]*(a + b*x^2),x]
```

```
[Out] (-8*(-c + d*x)^(3/2)*(35*b*c^7 + 35*a*c^5*d^2 + (35*b*c^7*(-c + d*x)^4)/(c
+ d*x)^4 + (35*a*c^5*d^2*(-c + d*x)^4)/(c + d*x)^4 + (28*b*c^7*(-c + d*x)^3
)/(c + d*x)^3 - (56*a*c^5*d^2*(-c + d*x)^3)/(c + d*x)^3 + (114*b*c^7*(-c +
d*x)^2)/(c + d*x)^2 + (42*a*c^5*d^2*(-c + d*x)^2)/(c + d*x)^2 + (28*b*c^7*(
-c + d*x))/(c + d*x) - (56*a*c^5*d^2*(-c + d*x))/(c + d*x)))/(105*d^6*(c +
d*x)^(3/2)*(-1 + (-c + d*x)/(c + d*x))^7)
```

fricas [A] time = 0.82, size = 90, normalized size = 0.83

$$\frac{\left(15\,bd^6x^6-8\,bc^6-14\,ac^4d^2-3\left(bc^2d^4-7\,ad^6\right)x^4-\left(4\,bc^4d^2+7\,ac^2d^4\right)x^2\right)\sqrt{dx+c}\,\sqrt{dx-c}}{105\,d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)*(d*x-c)^(1/2)*(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/105*(15*b*d^6*x^6 - 8*b*c^6 - 14*a*c^4*d^2 - 3*(b*c^2*d^4 - 7*a*d^6)*x^4
- (4*b*c^4*d^2 + 7*a*c^2*d^4)*x^2)*sqrt(d*x + c)*sqrt(d*x - c)/d^6
```

giac [B] time = 0.75, size = 495, normalized size = 4.54

$$\frac{70\left(\left((dx+c)\left(2(dx+c)\left(\frac{3(dx+c)}{d^3}-\frac{13c}{d^3}\right)+\frac{43c^2}{d^3}\right)-\frac{39c^3}{d^3}\right)\sqrt{dx+c}\,\sqrt{dx-c}-\frac{18c^4\log\left(\left|-\sqrt{dx+c}+\sqrt{dx-c}\right|\right)}{d^3}\right)ac+7\left(\left(\left(2\left((dx+c)\left(4(dx+c)\left(\frac{5(dx+c)}{d^5}-\frac{31c}{d^5}\right)+\frac{321c^2}{d^5}\right)-\frac{451c^3}{d^5}\right)(dx+c)+\frac{745c^4}{d^5}\right)(dx+c)-\frac{405c^5}{d^5}\right)\sqrt{dx+c}\,\sqrt{dx-c}-\frac{150c^6\log\left(\left|-\sqrt{dx+c}+\sqrt{dx-c}\right|\right)}{d^5}\right)bc+14\left(\left(\left(2(dx+c)\left(3(dx+c)\left(\frac{4(dx+c)}{d^4}-\frac{21c}{d^4}\right)+\frac{133c^2}{d^4}\right)-\frac{295c^3}{d^4}\right)(dx+c)+\frac{195c^4}{d^4}\right)\sqrt{dx+c}\,\sqrt{dx-c}+\frac{90c^5\log\left(\left|-\sqrt{dx+c}+\sqrt{dx-c}\right|\right)}{d^4}\right)ad+\left(\left(2\left(\left(\left(4(dx+c)\left(5(dx+c)\left(\frac{6(dx+c)}{d^6}-\frac{43c}{d^6}\right)+\frac{661c^2}{d^6}\right)-\frac{4551c^3}{d^6}\right)(dx+c)+\frac{4781c^4}{d^6}\right)(dx+c)-\frac{6335c^5}{d^6}\right)(dx+c)+\frac{2835c^6}{d^6}\right)\sqrt{dx+c}\,\sqrt{dx-c}+\frac{1050c^7\log\left(\left|-\sqrt{dx+c}+\sqrt{dx-c}\right|\right)}{d^6}\right)bd}{1680\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)*(d*x-c)^(1/2)*(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/1680*(70*(((d*x + c)*(2*(d*x + c)*(3*(d*x + c)/d^3 - 13*c/d^3) + 43*c^2/d
^3) - 39*c^3/d^3)*sqrt(d*x + c)*sqrt(d*x - c) - 18*c^4*log(abs(-sqrt(d*x +
c) + sqrt(d*x - c)))/d^3)*a*c + 7*(((2*((d*x + c)*(4*(d*x + c)*(5*(d*x + c)
/d^5 - 31*c/d^5) + 321*c^2/d^5) - 451*c^3/d^5)*(d*x + c) + 745*c^4/d^5)*(d*
x + c) - 405*c^5/d^5)*sqrt(d*x + c)*sqrt(d*x - c) - 150*c^6*log(abs(-sqrt(d
*x + c) + sqrt(d*x - c)))/d^5)*b*c + 14*(((2*(d*x + c)*(3*(d*x + c)*(4*(d*x
 + c)/d^4 - 21*c/d^4) + 133*c^2/d^4) - 295*c^3/d^4)*(d*x + c) + 195*c^4/d^4
)*sqrt(d*x + c)*sqrt(d*x - c) + 90*c^5*log(abs(-sqrt(d*x + c) + sqrt(d*x -
c)))/d^4)*a*d + (((2*((4*(d*x + c)*(5*(d*x + c)*(6*(d*x + c)/d^6 - 43*c/d^6
) + 661*c^2/d^6) - 4551*c^3/d^6)*(d*x + c) + 4781*c^4/d^6)*(d*x + c) - 6335
*c^5/d^6)*(d*x + c) + 2835*c^6/d^6)*sqrt(d*x + c)*sqrt(d*x - c) + 1050*c^7*
log(abs(-sqrt(d*x + c) + sqrt(d*x - c)))/d^6)*b*d)/d
```

maple [A] time = 0.05, size = 68, normalized size = 0.62

$$\frac{(dx+c)^{\frac{3}{2}}\left(15b\,d^4x^4+21a\,d^4x^2+12b\,c^2d^2x^2+14a\,c^2d^2+8b\,c^4\right)(dx-c)^{\frac{3}{2}}}{105d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x^2+a)*(d*x-c)^(1/2)*(d*x+c)^(1/2),x)
```

```
[Out] 1/105*(d*x+c)^(3/2)*(15*b*d^4*x^4+21*a*d^4*x^2+12*b*c^2*d^2*x^2+14*a*c^2*d^
2+8*b*c^4)*(d*x-c)^(3/2)/d^6
```

maxima [A] time = 0.50, size = 124, normalized size = 1.14

$$\frac{\left(d^2x^2-c^2\right)^{\frac{3}{2}}bx^4}{7\,d^2}+\frac{4\left(d^2x^2-c^2\right)^{\frac{3}{2}}bc^2x^2}{35\,d^4}+\frac{\left(d^2x^2-c^2\right)^{\frac{3}{2}}ax^2}{5\,d^2}+\frac{8\left(d^2x^2-c^2\right)^{\frac{3}{2}}bc^4}{105\,d^6}+\frac{2\left(d^2x^2-c^2\right)^{\frac{3}{2}}ac^2}{15\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)*(d*x-c)^(1/2)*(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/7*(d^2*x^2 - c^2)^(3/2)*b*x^4/d^2 + 4/35*(d^2*x^2 - c^2)^(3/2)*b*c^2*x^2/
d^4 + 1/5*(d^2*x^2 - c^2)^(3/2)*a*x^2/d^2 + 8/105*(d^2*x^2 - c^2)^(3/2)*b*c
^4/d^6 + 2/15*(d^2*x^2 - c^2)^(3/2)*a*c^2/d^4
```

mupad [B] time = 1.74, size = 118, normalized size = 1.08

$$-\sqrt{d\,x-c}\,\left(\frac{\left(8\,b\,c^6+14\,a\,c^4\,d^2\right)\,\sqrt{c+d\,x}}{105\,d^6}-\frac{b\,x^6\,\sqrt{c+d\,x}}{7}+\frac{x^2\,\left(4\,b\,c^4\,d^2+7\,a\,c^2\,d^4\right)\,\sqrt{c+d\,x}}{105\,d^6}-\frac{x^4\,\left(21\,a\,d^6-3\,b\,c^2\,d^4\right)\,\sqrt{c+d\,x}}{105\,d^6}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*x^2)*(c + d*x)^(1/2)*(d*x - c)^(1/2),x)
```

```
[Out] -(d*x - c)^(1/2)*(((8*b*c^6 + 14*a*c^4*d^2)*(c + d*x)^(1/2))/(105*d^6) - (b
*x^6*(c + d*x)^(1/2))/7 + (x^2*(7*a*c^2*d^4 + 4*b*c^4*d^2)*(c + d*x)^(1/2))
/(105*d^6) - (x^4*(21*a*d^6 - 3*b*c^2*d^4)*(c + d*x)^(1/2))/(105*d^6))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3\left(a+bx^2\right)\sqrt{-c+dx}\,\sqrt{c+dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x**2+a)*(d*x-c)**(1/2)*(d*x+c)**(1/2),x)
```

```
[Out] Integral(x**3*(a + b*x**2)*sqrt(-c + d*x)*sqrt(c + d*x), x)
```

3.196 $\int x\sqrt{-c+dx}\,\sqrt{c+dx}\,\left(a+bx^2\right)\,dx$

Optimal. Leaf size=67

$$\frac{(dx-c)^{3/2}(c+dx)^{3/2}\left(ad^2+bc^2\right)}{3d^4}+\frac{b(dx-c)^{5/2}(c+dx)^{5/2}}{5d^4}$$

Rubi [A] time = 0.04, antiderivative size = 72, normalized size of antiderivative = 1.07, number of steps used = 2, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {460, 74}

$$\frac{(dx-c)^{3/2}(c+dx)^{3/2}\left(5ad^2+2bc^2\right)}{15d^4}+\frac{bx^2(dx-c)^{3/2}(c+dx)^{3/2}}{5d^2}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sqrt[-c + d*x]*Sqrt[c + d*x]*(a + b*x^2),x]
```

```
[Out] ((2*b*c^2 + 5*a*d^2)*(-c + d*x)^(3/2)*(c + d*x)^(3/2))/(15*d^4) + (b*x^2*(-
c + d*x)^(3/2)*(c + d*x)^(3/2))/(5*d^2)
```

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 460

```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[(d*(e*x)^
(m + 1)*(a1 + b1*x^(n/2))^(p + 1)*(a2 + b2*x^(n/2))^(p + 1))/(b1*b2*e*(m +
n*(p + 1) + 1)), x] - Dist[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/
(b1*b2*(m + n*(p + 1) + 1)), Int[(e*x)^m*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n/
2))^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, m, n, p}, x] && EqQ[non2,
n/2] && EqQ[a2*b1 + a1*b2, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rubi steps

$$\begin{aligned}\int x\sqrt{-c+dx}\,\sqrt{c+dx}\,\left(a+bx^2\right)\,dx &= \frac{bx^2(-c+dx)^{3/2}(c+dx)^{3/2}}{5d^2}-\frac{1}{5}\left(-5a-\frac{2bc^2}{d^2}\right)\int x\sqrt{-c+dx}\,\sqrt{c+dx}\,dx\\ &= \frac{\left(2bc^2+5ad^2\right)(-c+dx)^{3/2}(c+dx)^{3/2}}{15d^4}+\frac{bx^2(-c+dx)^{3/2}(c+dx)^{3/2}}{5d^2}\end{aligned}$$

Mathematica [A] time = 0.05, size = 62, normalized size = 0.93

$$\frac{\sqrt{dx-c}\,\sqrt{c+dx}\,\left(d^2x^2-c^2\right)\left(5ad^2+2bc^2+3bd^2x^2\right)}{15d^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sqrt[-c + d*x]*Sqrt[c + d*x]*(a + b*x^2),x]
```

```
[Out] (Sqrt[-c + d*x]*Sqrt[c + d*x]*(-c^2 + d^2*x^2)*(2*b*c^2 + 5*a*d^2 + 3*b*d^2
*x^2))/(15*d^4)
```

IntegrateAlgebraic [B] time = 0.14, size = 152, normalized size = 2.27

$$-\frac{8(dx-c)^{3/2}\left(-\frac{10ac^3d^2(dx-c)}{c+dx}+\frac{5ac^3d^2(dx-c)^2}{(c+dx)^2}+5ac^3d^2+\frac{2bc^5(dx-c)}{c+dx}+\frac{5bc^5(dx-c)^2}{(c+dx)^2}+5bc^5\right)}{15d^4(c+dx)^{3/2}\left(\frac{dx-c}{c+dx}-1\right)^5}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x*Sqrt[-c + d*x]*Sqrt[c + d*x]*(a + b*x^2),x]
```

```
[Out] (-8*(-c + d*x)^(3/2)*(5*b*c^5 + 5*a*c^3*d^2 + (5*b*c^5*(-c + d*x)^2)/(c + d
*x)^2 + (5*a*c^3*d^2*(-c + d*x)^2)/(c + d*x)^2 + (2*b*c^5*(-c + d*x))/(c +
d*x) - (10*a*c^3*d^2*(-c + d*x))/(c + d*x)))/(15*d^4*(c + d*x)^(3/2)*(-1 +
(-c + d*x)/(c + d*x))^5)
```

fricas [A] time = 0.63, size = 66, normalized size = 0.99

$$\frac{\left(3\,bd^4x^4-2\,bc^4-5\,ac^2d^2-\left(bc^2d^2-5\,ad^4\right)x^2\right)\sqrt{dx+c}\,\sqrt{dx-c}}{15\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)*(d*x-c)^(1/2)*(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/15*(3*b*d^4*x^4 - 2*b*c^4 - 5*a*c^2*d^2 - (b*c^2*d^2 - 5*a*d^4)*x^2)*sqrt
(d*x + c)*sqrt(d*x - c)/d^4
```

giac [B] time = 0.37, size = 361, normalized size = 5.39

$$\frac{5\left(\left((dx+c)\left(2\,(dx+c)\left(\frac{3\,(dx+c)}{d^3}-\frac{13\,c}{d^3}\right)+\frac{43\,c^2}{d^3}\right)-\frac{39\,c^3}{d^3}\right)\sqrt{dx+c}\,\sqrt{dx-c}-\frac{18\,c^4\log\left(\left|-\sqrt{dx+c}+\sqrt{dx-c}\right|\right)}{d^3}\right)bc+20\left(\sqrt{dx+c}\,\sqrt{dx-c}\left((dx+c)\left(\frac{2\,(dx+c)}{d^2}-\frac{7\,c}{d^2}\right)+\frac{9\,c^2}{d^2}\right)+\frac{6\,c^3\log\left(\left|-\sqrt{dx+c}+\sqrt{dx-c}\right|\right)}{d^2}\right)ad+\left(\left(\left(2\,(dx+c)\left(3\,(dx+c)\left(\frac{4\,(dx+c)}{d^4}-\frac{21\,c}{d^4}\right)+\frac{133\,c^2}{d^4}\right)-\frac{295\,c^3}{d^4}\right)(dx+c)+\frac{195\,c^4}{d^4}\right)\sqrt{dx+c}\,\sqrt{dx-c}+\frac{90\,c^5\log\left(\left|-\sqrt{dx+c}+\sqrt{dx-c}\right|\right)}{d^4}\right)bd-\frac{60\left(2\,c^2\log\left(\left|-\sqrt{dx+c}+\sqrt{dx-c}\right|\right)-\sqrt{dx+c}\,\sqrt{dx-c}\,(dx-2\,c)\right)ac}{d}}{120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)*(d*x-c)^(1/2)*(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/120*(5*(((d*x + c)*(2*(d*x + c)*(3*(d*x + c)/d^3 - 13*c/d^3) + 43*c^2/d^3
) - 39*c^3/d^3)*sqrt(d*x + c)*sqrt(d*x - c) - 18*c^4*log(abs(-sqrt(d*x + c)
 + sqrt(d*x - c)))/d^3)*b*c + 20*(sqrt(d*x + c)*sqrt(d*x - c)*((d*x + c)*(2
*(d*x + c)/d^2 - 7*c/d^2) + 9*c^2/d^2) + 6*c^3*log(abs(-sqrt(d*x + c) + sqr
t(d*x - c)))/d^2)*a*d + (((2*(d*x + c)*(3*(d*x + c)*(4*(d*x + c)/d^4 - 21*c
/d^4) + 133*c^2/d^4) - 295*c^3/d^4)*(d*x + c) + 195*c^4/d^4)*sqrt(d*x + c)*
sqrt(d*x - c) + 90*c^5*log(abs(-sqrt(d*x + c) + sqrt(d*x - c)))/d^4)*b*d -
60*(2*c^2*log(abs(-sqrt(d*x + c) + sqrt(d*x - c))) - sqrt(d*x + c)*sqrt(d*x
 - c)*(d*x - 2*c))*a*c/d)/d
```

maple [A] time = 0.05, size = 44, normalized size = 0.66

$$\frac{(dx+c)^{\frac{3}{2}}\left(3b\,d^2x^2+5a\,d^2+2b\,c^2\right)(dx-c)^{\frac{3}{2}}}{15d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^2+a)*(d*x-c)^(1/2)*(d*x+c)^(1/2),x)
```

```
[Out] 1/15*(d*x+c)^(3/2)*(3*b*d^2*x^2+5*a*d^2+2*b*c^2)*(d*x-c)^(3/2)/d^4
```

maxima [A] time = 0.62, size = 70, normalized size = 1.04

$$\frac{\left(d^2x^2-c^2\right)^{\frac{3}{2}}bx^2}{5\,d^2}+\frac{2\left(d^2x^2-c^2\right)^{\frac{3}{2}}bc^2}{15\,d^4}+\frac{\left(d^2x^2-c^2\right)^{\frac{3}{2}}a}{3\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)*(d*x-c)^(1/2)*(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/5*(d^2*x^2 - c^2)^(3/2)*b*x^2/d^2 + 2/15*(d^2*x^2 - c^2)^(3/2)*b*c^2/d^4
+ 1/3*(d^2*x^2 - c^2)^(3/2)*a/d^2
```

mupad [**B**] time = 1.64, size = 83, normalized size = 1.24

$$\sqrt{d\,x-c}\,\left(\frac{b\,x^4\,\sqrt{c+d\,x}}{5}-\frac{\left(2\,b\,c^4+5\,a\,c^2\,d^2\right)\,\sqrt{c+d\,x}}{15\,d^4}+\frac{x^2\,\left(5\,a\,d^4-b\,c^2\,d^2\right)\,\sqrt{c+d\,x}}{15\,d^4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*x^2)*(c + d*x)^(1/2)*(d*x - c)^(1/2),x)
```

```
[Out] (d*x - c)^(1/2)*((b*x^4*(c + d*x)^(1/2))/5 - ((2*b*c^4 + 5*a*c^2*d^2)*(c +
d*x)^(1/2))/(15*d^4) + (x^2*(5*a*d^4 - b*c^2*d^2)*(c + d*x)^(1/2))/(15*d^4)
)
```

sympy [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int x\left(a+bx^2\right)\sqrt{-c+dx}\,\sqrt{c+dx}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**2+a)*(d*x-c)**(1/2)*(d*x+c)**(1/2),x)
```

```
[Out] Integral(x*(a + b*x**2)*sqrt(-c + d*x)*sqrt(c + d*x), x)
```

3.197 $\int \frac{\sqrt{-c+dx}\,\sqrt{c+dx}\,(a+bx^2)}{x}\,dx$

Optimal. Leaf size=80

$$a\sqrt{dx-c}\,\sqrt{c+dx}\;-ac\tan^{-1}\left(\frac{\sqrt{dx-c}\,\sqrt{c+dx}}{c}\right)+\frac{b(dx-c)^{3/2}(c+dx)^{3/2}}{3d^2}$$

Rubi [A] time = 0.08, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {460, 101, 12, 92, 205}

$$a\sqrt{dx-c}\,\sqrt{c+dx}\;-ac\tan^{-1}\left(\frac{\sqrt{dx-c}\,\sqrt{c+dx}}{c}\right)+\frac{b(dx-c)^{3/2}(c+dx)^{3/2}}{3d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[-c + d*x]*Sqrt[c + d*x]*(a + b*x^2))/x,x]
```

```
[Out] a*Sqrt[-c + d*x]*Sqrt[c + d*x] + (b*(-c + d*x)^(3/2)*(c + d*x)^(3/2))/(3*d^
2) - a*c*ArcTan[(Sqrt[-c + d*x]*Sqrt[c + d*x])/c]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 101

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Simp[((a + b*x)^m*(c + d*x)^n*(e + f*x)^(p + 1))/(f
*(m + n + p + 1)), x] - Dist[1/(f*(m + n + p + 1)), Int[(a + b*x)^(m - 1)*(
c + d*x)^(n - 1)*(e + f*x)^p*Simp[c*m*(b*e - a*f) + a*n*(d*e - c*f) + (d*m*
(b*e - a*f) + b*n*(d*e - c*f))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}
, x] && GtQ[m, 0] && GtQ[n, 0] && NeQ[m + n + p + 1, 0] && (IntegersQ[2*m,
2*n, 2*p] || (IntegersQ[m, n + p] || IntegersQ[p, m + n]))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 460

```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[(d*(e*x)^
(m + 1)*(a1 + b1*x^(n/2))^(p + 1)*(a2 + b2*x^(n/2))^(p + 1))/(b1*b2*e*(m +
n*(p + 1) + 1)), x] - Dist[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/
(b1*b2*(m + n*(p + 1) + 1)), Int[(e*x)^m*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n/
2))^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, m, n, p}, x] && EqQ[non2,
n/2] && EqQ[a2*b1 + a1*b2, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rubi steps

$$\int \frac{\sqrt{-c+dx}\,\sqrt{c+dx}\,\left(a+bx^2\right)}{x}\,dx = \frac{b(-c+dx)^{3/2}(c+dx)^{3/2}}{3d^2} + a\int \frac{\sqrt{-c+dx}\,\sqrt{c+dx}}{x}\,dx$$
$$= a\sqrt{-c+dx}\,\sqrt{c+dx} + \frac{b(-c+dx)^{3/2}(c+dx)^{3/2}}{3d^2} - a\int \frac{c^2}{x\sqrt{-c+dx}\,\sqrt{c+dx}}$$
$$= a\sqrt{-c+dx}\,\sqrt{c+dx} + \frac{b(-c+dx)^{3/2}(c+dx)^{3/2}}{3d^2} - \left(ac^2\right)\int \frac{1}{x\sqrt{-c+dx}\,\sqrt{c+}}$$
$$= a\sqrt{-c+dx}\,\sqrt{c+dx} + \frac{b(-c+dx)^{3/2}(c+dx)^{3/2}}{3d^2} - \left(ac^2d\right)\text{Subst}\left(\int \frac{1}{c^2d+dx}\right.$$
$$= a\sqrt{-c+dx}\,\sqrt{c+dx} + \frac{b(-c+dx)^{3/2}(c+dx)^{3/2}}{3d^2} - ac\tan^{-1}\left(\frac{\sqrt{-c+dx}\,\sqrt{c+}}{c}\right.$$

Mathematica [A] time = 0.24, size = 85, normalized size = 1.06

$$\frac{1}{3}\sqrt{dx-c}\,\sqrt{c+dx}\left(-\frac{3ac\tan^{-1}\left(\frac{\sqrt{d^2x^2-c^2}}{c}\right)}{\sqrt{d^2x^2-c^2}} + 3a + b\left(x^2-\frac{c^2}{d^2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[-c + d*x]*Sqrt[c + d*x]*(a + b*x^2))/x,x]
```

```
[Out] (Sqrt[-c + d*x]*Sqrt[c + d*x]*(3*a + b*(-(c^2/d^2) + x^2) - (3*a*c*ArcTan[S
qrt[-c^2 + d^2*x^2]/c])/Sqrt[-c^2 + d^2*x^2]))/3
```

IntegrateAlgebraic [B] time = 0.13, size = 161, normalized size = 2.01

$$-\frac{2\left(\frac{3acd^2\sqrt{dx-c}}{\sqrt{c+dx}} - \frac{6acd^2(dx-c)^{3/2}}{(c+dx)^{3/2}} + \frac{3acd^2(dx-c)^{5/2}}{(c+dx)^{5/2}} + \frac{4bc^3(dx-c)^{3/2}}{(c+dx)^{3/2}}\right)}{3d^2\left(\frac{dx-c}{c+dx}-1\right)^3} - 2ac\tan^{-1}\left(\frac{\sqrt{dx-c}}{\sqrt{c+dx}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(Sqrt[-c + d*x]*Sqrt[c + d*x]*(a + b*x^2))/x,x]
```

```
[Out] (-2*((3*a*c*d^2*(-c + d*x)^(5/2))/(c + d*x)^(5/2) + (4*b*c^3*(-c + d*x)^(3/
2))/(c + d*x)^(3/2) - (6*a*c*d^2*(-c + d*x)^(3/2))/(c + d*x)^(3/2) + (3*a*c
*d^2*Sqrt[-c + d*x])/Sqrt[c + d*x]))/(3*d^2*(-1 + (-c + d*x)/(c + d*x))^3)
- 2*a*c*ArcTan[Sqrt[-c + d*x]/Sqrt[c + d*x]]
```

fricas [A] time = 0.70, size = 80, normalized size = 1.00

$$-\frac{6\,acd^2\arctan\left(-\frac{dx-\sqrt{dx+c}\,\sqrt{dx-c}}{c}\right) - \left(bd^2x^2 - bc^2 + 3\,ad^2\right)\sqrt{dx+c}\,\sqrt{dx-c}}{3\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x-c)^(1/2)*(d*x+c)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] -1/3*(6*a*c*d^2*arctan(-(d*x - sqrt(d*x + c)*sqrt(d*x - c))/c) - (b*d^2*x^2
 - b*c^2 + 3*a*d^2)*sqrt(d*x + c)*sqrt(d*x - c))/d^2
```

giac [A] time = 0.32, size = 78, normalized size = 0.98

$$2\,ac\arctan\left(\frac{\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^{2}}{2\,c}\right)+\frac{1}{3}\,\sqrt{dx+c}\,\sqrt{dx-c}\left((dx+c)\left(\frac{(dx+c)b}{d^{2}}-\frac{2\,bc}{d^{2}}\right)+3\,a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x-c)^(1/2)*(d*x+c)^(1/2)/x,x, algorithm="giac")
```

```
[Out] 2*a*c*arctan(1/2*(sqrt(d*x + c) - sqrt(d*x - c))^2/c) + 1/3*sqrt(d*x + c)*s
qrt(d*x - c)*((d*x + c)*((d*x + c)*b/d^2 - 2*b*c/d^2) + 3*a)
```

maple [B] time = 0.10, size = 174, normalized size = 2.18

$$\frac{\sqrt{dx-c}\,\sqrt{dx+c}\,\left(3a\,c^{2}d^{2}\ln\left(-\frac{2\left(c^{2}-\sqrt{-c^{2}}\,\sqrt{d^{2}x^{2}-c^{2}}\right)}{x}\right)+\sqrt{-c^{2}}\,\sqrt{d^{2}x^{2}-c^{2}}\,b\,d^{2}x^{2}+3\sqrt{-c^{2}}\,\sqrt{d^{2}x^{2}-c^{2}}\,a\,d^{2}-\sqrt{-c^{2}}\,\sqrt{d^{2}x^{2}-c^{2}}\,b\,c^{2}\right)}{3\sqrt{d^{2}x^{2}-c^{2}}\,\sqrt{-c^{2}}\,d^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)*(d*x-c)^(1/2)*(d*x+c)^(1/2)/x,x)
```

```
[Out] 1/3*(d*x-c)^(1/2)*(d*x+c)^(1/2)*(x^2*b*d^2*(-c^2)^(1/2)*(d^2*x^2-c^2)^(1/2)
+3*ln(-2*(c^2-(-c^2)^(1/2)*(d^2*x^2-c^2)^(1/2))/x)*a*c^2*d^2+3*(-c^2)^(1/2)
*(d^2*x^2-c^2)^(1/2)*a*d^2-b*c^2*(-c^2)^(1/2)*(d^2*x^2-c^2)^(1/2))/(d^2*x^2
-c^2)^(1/2)/d^2/(-c^2)^(1/2)
```

maxima [A] time = 1.30, size = 52, normalized size = 0.65

$$ac\arcsin\left(\frac{c}{d|x|}\right)+\sqrt{d^{2}x^{2}-c^{2}}\,a+\frac{\left(d^{2}x^{2}-c^{2}\right)^{\frac{3}{2}}b}{3\,d^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x-c)^(1/2)*(d*x+c)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] a*c*arcsin(c/(d*abs(x))) + sqrt(d^2*x^2 - c^2)*a + 1/3*(d^2*x^2 - c^2)^(3/2
)*b/d^2
```

mupad [B] time = 3.60, size = 248, normalized size = 3.10

$$a\,\sqrt{-c}\,\sqrt{c}\,\ln\left(\frac{\left(\sqrt{c+d\,x}-\sqrt{c}\right)^{2}}{\left(\sqrt{-c}-\sqrt{d\,x-c}\right)^{2}}+1\right)-a\,\sqrt{-c}\,\sqrt{c}\,\ln\left(\frac{\sqrt{c+d\,x}-\sqrt{c}}{\sqrt{-c}-\sqrt{d\,x-c}}\right)-\frac{b\,\left(c^{2}-d^{2}\,x^{2}\right)\,\sqrt{c+d\,x}\,\sqrt{d\,x-c}}{3\,d^{2}}-\frac{8\,a\,\sqrt{-c}\,\sqrt{c}\,\left(\sqrt{c+d\,x}-\sqrt{c}\right)^{2}}{\left(\sqrt{-c}-\sqrt{d\,x-c}\right)^{2}\,\left(\frac{\left(\sqrt{c+d\,x}-\sqrt{c}\right)^{4}}{\left(\sqrt{-c}-\sqrt{d\,x-c}\right)^{4}}-\frac{2\,\left(\sqrt{c+d\,x}-\sqrt{c}\right)^{2}}{\left(\sqrt{-c}-\sqrt{d\,x-c}\right)^{2}}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^2)*(c + d*x)^(1/2)*(d*x - c)^(1/2))/x,x)
```

```
[Out] a*(-c)^(1/2)*c^(1/2)*log(((c + d*x)^(1/2) - c^(1/2))^2/((-c)^(1/2) - (d*x -
 c)^(1/2))^2 + 1) - a*(-c)^(1/2)*c^(1/2)*log(((c + d*x)^(1/2) - c^(1/2))/((
-c)^(1/2) - (d*x - c)^(1/2))) - (b*(c^2 - d^2*x^2)*(c + d*x)^(1/2)*(d*x - c
)^(1/2))/(3*d^2) - (8*a*(-c)^(1/2)*c^(1/2)*((c + d*x)^(1/2) - c^(1/2))^2)/(
((-c)^(1/2) - (d*x - c)^(1/2))^2*(((c + d*x)^(1/2) - c^(1/2))^4/((-c)^(1/2)
 - (d*x - c)^(1/2))^4 - (2*((c + d*x)^(1/2) - c^(1/2))^2)/((-c)^(1/2) - (d*
x - c)^(1/2))^2 + 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(a+bx^{2}\right)\sqrt{-c+dx}\,\sqrt{c+dx}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)*(d*x-c)**(1/2)*(d*x+c)**(1/2)/x,x)
```

```
[Out] Integral((a + b*x**2)*sqrt(-c + d*x)*sqrt(c + d*x)/x, x)
```

3.198 $\int \frac{\sqrt{-c+dx}\,\sqrt{c+dx}\,(a+bx^2)}{x^3}\,dx$

Optimal. Leaf size=96

$$-\frac{(2bc^2-ad^2)\tan^{-1}\left(\frac{\sqrt{dx-c}\,\sqrt{c+dx}}{c}\right)}{2c}-\frac{a\sqrt{dx-c}\,\sqrt{c+dx}}{2x^2}+b\sqrt{dx-c}\,\sqrt{c+dx}$$

Rubi [A] time = 0.08, antiderivative size = 114, normalized size of antiderivative = 1.19, number of steps used = 5, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {454, 101, 12, 92, 205}

$$\frac{1}{2}\sqrt{dx-c}\,\sqrt{c+dx}\left(2b-\frac{ad^2}{c^2}\right)-\frac{(2bc^2-ad^2)\tan^{-1}\left(\frac{\sqrt{dx-c}\,\sqrt{c+dx}}{c}\right)}{2c}+\frac{a(dx-c)^{3/2}(c+dx)^{3/2}}{2c^2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[-c + d*x]*Sqrt[c + d*x]*(a + b*x^2))/x^3,x]
```

```
[Out] ((2*b - (a*d^2)/c^2)*Sqrt[-c + d*x]*Sqrt[c + d*x])/2 + (a*(-c + d*x)^(3/2)*
(c + d*x)^(3/2))/(2*c^2*x^2) - ((2*b*c^2 - a*d^2)*ArcTan[(Sqrt[-c + d*x]*Sq
rt[c + d*x])/c])/(2*c)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 101

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Simp[((a + b*x)^m*(c + d*x)^n*(e + f*x)^(p + 1))/(f
*(m + n + p + 1)), x] - Dist[1/(f*(m + n + p + 1)), Int[(a + b*x)^(m - 1)*(
c + d*x)^(n - 1)*(e + f*x)^p*Simp[c*m*(b*e - a*f) + a*n*(d*e - c*f) + (d*m*
(b*e - a*f) + b*n*(d*e - c*f))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}
, x] && GtQ[m, 0] && GtQ[n, 0] && NeQ[m + n + p + 1, 0] && (IntegersQ[2*m,
2*n, 2*p] || (IntegersQ[m, n + p] || IntegersQ[p, m + n]))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 454

```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[(c*(e*x)^
(m + 1)*(a1 + b1*x^(n/2))^(p + 1)*(a2 + b2*x^(n/2))^(p + 1))/(a1*a2*e*(m +
1)), x] + Dist[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/(a1*a2*e^n*(
m + 1)), Int[(e*x)^(m + n)*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n/2))^p, x], x]
/; FreeQ[{a1, b1, a2, b2, c, d, e, p}, x] && EqQ[non2, n/2] && EqQ[a2*b1 +
a1*b2, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (L
```

```
tQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rubi steps

$$\int \frac{\sqrt{-c+dx}\sqrt{c+dx}\left(a+bx^2\right)}{x^3}\,dx = \frac{a(-c+dx)^{3/2}(c+dx)^{3/2}}{2c^2x^2} + \frac{1}{2}\left(2b-\frac{ad^2}{c^2}\right)\int\frac{\sqrt{-c+dx}\sqrt{c+dx}}{x}\,dx$$
$$= \frac{1}{2}\left(2b-\frac{ad^2}{c^2}\right)\sqrt{-c+dx}\sqrt{c+dx} + \frac{a(-c+dx)^{3/2}(c+dx)^{3/2}}{2c^2x^2} + \frac{1}{2}\left(-2b+\frac{ad^2}{c^2}\right.$$
$$= \frac{1}{2}\left(2b-\frac{ad^2}{c^2}\right)\sqrt{-c+dx}\sqrt{c+dx} + \frac{a(-c+dx)^{3/2}(c+dx)^{3/2}}{2c^2x^2} + \frac{1}{2}\left(-2bc^2+a\right.$$
$$= \frac{1}{2}\left(2b-\frac{ad^2}{c^2}\right)\sqrt{-c+dx}\sqrt{c+dx} + \frac{a(-c+dx)^{3/2}(c+dx)^{3/2}}{2c^2x^2} - \frac{1}{2}\left(d\left(2bc^2-\right.\right.$$
$$= \frac{1}{2}\left(2b-\frac{ad^2}{c^2}\right)\sqrt{-c+dx}\sqrt{c+dx} + \frac{a(-c+dx)^{3/2}(c+dx)^{3/2}}{2c^2x^2} - \frac{\left(2bc^2-ad^2\right)}{}$$

Mathematica [A] time = 0.06, size = 114, normalized size = 1.19

$$-\frac{\sqrt{dx-c}\sqrt{c+dx}\left(c\left(a-2bx^2\right)\sqrt{d^2x^2-c^2}+x^2\left(2bc^2-ad^2\right)\tan^{-1}\left(\frac{\sqrt{d^2x^2-c^2}}{c}\right)\right)}{2cx^2\sqrt{d^2x^2-c^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[-c + d*x]*Sqrt[c + d*x]*(a + b*x^2))/x^3,x]
```

```
[Out] -1/2*(Sqrt[-c + d*x]*Sqrt[c + d*x]*(c*(a - 2*b*x^2)*Sqrt[-c^2 + d^2*x^2] +
(2*b*c^2 - a*d^2)*x^2*ArcTan[Sqrt[-c^2 + d^2*x^2]/c]))/(c*x^2*Sqrt[-c^2 + d
^2*x^2])
```

IntegrateAlgebraic [B] time = 0.19, size = 267, normalized size = 2.78

$$\frac{\frac{ad^2(dx-c)^{5/2}}{(c+dx)^{5/2}}-\frac{2ad^2(dx-c)^{3/2}}{(c+dx)^{3/2}}+\frac{ad^2\sqrt{dx-c}}{\sqrt{c+dx}}-\frac{2bc^2(dx-c)^{5/2}}{(c+dx)^{5/2}}-\frac{4bc^2(dx-c)^{3/2}}{(c+dx)^{3/2}}-\frac{2bc^2\sqrt{dx-c}}{\sqrt{c+dx}}}{c\left(\frac{dx-c}{c+dx}+1\right)^2\left(\frac{\sqrt{dx-c}}{\sqrt{c+dx}}-1\right)\left(\frac{\sqrt{dx-c}}{\sqrt{c+dx}}+1\right)}+\frac{\left(ad^2-2bc^2\right)\tan^{-1}\left(\frac{\sqrt{dx-c}}{\sqrt{c+dx}}\right)}{c}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(Sqrt[-c + d*x]*Sqrt[c + d*x]*(a + b*x^2))/x^3,x]
```

```
[Out] ((-2*b*c^2*(-c + d*x)^(5/2))/(c + d*x)^(5/2) + (a*d^2*(-c + d*x)^(5/2))/(c
+ d*x)^(5/2) - (4*b*c^2*(-c + d*x)^(3/2))/(c + d*x)^(3/2) - (2*a*d^2*(-c +
d*x)^(3/2))/(c + d*x)^(3/2) - (2*b*c^2*Sqrt[-c + d*x])/Sqrt[c + d*x] + (a*d
^2*Sqrt[-c + d*x])/Sqrt[c + d*x])/(c*(1 + (-c + d*x)/(c + d*x))^2*(-1 + Sqr
t[-c + d*x]/Sqrt[c + d*x])*(1 + Sqrt[-c + d*x]/Sqrt[c + d*x])) + ((-2*b*c^2
 + a*d^2)*ArcTan[Sqrt[-c + d*x]/Sqrt[c + d*x]])/c
```

fricas [A] time = 0.91, size = 85, normalized size = 0.89

$$-\frac{2\left(2\,bc^2-ad^2\right)x^2\arctan\left(-\frac{dx-\sqrt{dx+c}\,\sqrt{dx-c}}{c}\right)-\left(2\,bcx^2-ac\right)\sqrt{dx+c}\,\sqrt{dx-c}}{2\,cx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x-c)^(1/2)*(d*x+c)^(1/2)/x^3,x, algorithm="fricas")
```

```
[Out] -1/2*(2*(2*b*c^2 - a*d^2)*x^2*arctan(-(d*x - sqrt(d*x + c)*sqrt(d*x - c))/c
) - (2*b*c*x^2 - a*c)*sqrt(d*x + c)*sqrt(d*x - c))/(c*x^2)
```

giac [A] time = 0.42, size = 157, normalized size = 1.64

$$\frac{\sqrt{dx+c}\,\sqrt{dx-c}\,bd + \frac{\left(2\,bc^2d-ad^3\right)\arctan\left(\frac{\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^2}{2\,c}\right)}{c} + \frac{2\left(ad^3\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^6-4\,ac^2d^3\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^2\right)}{\left(\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^4+4\,c^2\right)^2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x-c)^(1/2)*(d*x+c)^(1/2)/x^3,x, algorithm="giac")
```

```
[Out] (sqrt(d*x + c)*sqrt(d*x - c)*b*d + (2*b*c^2*d - a*d^3)*arctan(1/2*(sqrt(d*x
 + c) - sqrt(d*x - c))^2/c)/c + 2*(a*d^3*(sqrt(d*x + c) - sqrt(d*x - c))^6
- 4*a*c^2*d^3*(sqrt(d*x + c) - sqrt(d*x - c))^2)/((sqrt(d*x + c) - sqrt(d*x
 - c))^4 + 4*c^2)^2)/d
```

maple [B] time = 0.07, size = 182, normalized size = 1.90

$$-\frac{\sqrt{dx-c}\,\sqrt{dx+c}\,\left(a\,d^2x^2\ln\left(-\frac{2\left(c^2-\sqrt{-c^2}\,\sqrt{d^2x^2-c^2}\right)}{x}\right)-2b\,c^2x^2\ln\left(-\frac{2\left(c^2-\sqrt{-c^2}\,\sqrt{d^2x^2-c^2}\right)}{x}\right)-2\sqrt{-c^2}\,\sqrt{d^2x^2-c^2}\,b\,x^2+\sqrt{-c^2}\,\sqrt{d^2x^2-c^2}\,a\right)}{2\sqrt{d^2x^2-c^2}\,\sqrt{-c^2}\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)*(d*x-c)^(1/2)*(d*x+c)^(1/2)/x^3,x)
```

```
[Out] -1/2*(d*x-c)^(1/2)*(d*x+c)^(1/2)*(ln(-2*(c^2-(-c^2)^(1/2)*(d^2*x^2-c^2)^(1/
2))/x)*x^2*a*d^2-2*ln(-2*(c^2-(-c^2)^(1/2)*(d^2*x^2-c^2)^(1/2))/x)*x^2*b*c^
2-2*x^2*b*(-c^2)^(1/2)*(d^2*x^2-c^2)^(1/2)+(-c^2)^(1/2)*(d^2*x^2-c^2)^(1/2)
*a)/(d^2*x^2-c^2)^(1/2)/x^2/(-c^2)^(1/2)
```

maxima [A] time = 1.46, size = 98, normalized size = 1.02

$$bc\arcsin\left(\frac{c}{d|x|}\right)-\frac{ad^2\arcsin\left(\frac{c}{d|x|}\right)}{2\,c}+\sqrt{d^2x^2-c^2}\,b-\frac{\sqrt{d^2x^2-c^2}\,ad^2}{2\,c^2}+\frac{\left(d^2x^2-c^2\right)^{\frac{3}{2}}a}{2\,c^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x-c)^(1/2)*(d*x+c)^(1/2)/x^3,x, algorithm="maxima")
```

```
[Out] b*c*arcsin(c/(d*abs(x))) - 1/2*a*d^2*arcsin(c/(d*abs(x)))/c + sqrt(d^2*x^2
- c^2)*b - 1/2*sqrt(d^2*x^2 - c^2)*a*d^2/c^2 + 1/2*(d^2*x^2 - c^2)^(3/2)*a/
(c^2*x^2)
```

mupad [B] time = 6.89, size = 584, normalized size = 6.08

$$b\sqrt{-c}\,\sqrt{c}\,\ln\left(\frac{\left(\sqrt{c+d\,x}-\sqrt{c}\right)^2}{\left(\sqrt{-c}-\sqrt{d\,x-c}\right)^2}+1\right)-\frac{\frac{a\sqrt{-c}\,d^2}{32\,c^{3/2}}+\frac{a\sqrt{-c}\,d^2\left(\sqrt{c+d\,x}-\sqrt{c}\right)^2}{16\,c^{3/2}\left(\sqrt{-c}-\sqrt{d\,x-c}\right)^2}-\frac{15\,a\sqrt{-c}\,d^2\left(\sqrt{c+d\,x}-\sqrt{c}\right)^4}{32\,c^{3/2}\left(\sqrt{-c}-\sqrt{d\,x-c}\right)^4}}{\frac{\left(\sqrt{c+d\,x}-\sqrt{c}\right)^2}{\left(\sqrt{-c}-\sqrt{d\,x-c}\right)^2}+\frac{2\left(\sqrt{c+d\,x}-\sqrt{c}\right)^4}{\left(\sqrt{-c}-\sqrt{d\,x-c}\right)^4}+\frac{\left(\sqrt{c+d\,x}-\sqrt{c}\right)^6}{\left(\sqrt{-c}-\sqrt{d\,x-c}\right)^6}}-b\sqrt{-c}\,\sqrt{c}\,\ln\left(\frac{\sqrt{c+d\,x}-\sqrt{c}}{\sqrt{-c}-\sqrt{d\,x-c}}\right)+\frac{a\sqrt{-c}\,d^2\ln\left(\frac{\sqrt{c+d\,x}-\sqrt{c}}{\sqrt{-c}-\sqrt{d\,x-c}}\right)}{2\,c^{3/2}}-\frac{a\sqrt{-c}\,d^2\ln\left(\frac{\left(\sqrt{c+d\,x}-\sqrt{c}\right)^2}{\left(\sqrt{-c}-\sqrt{d\,x-c}\right)^2}+1\right)}{2\,c^{3/2}}-\frac{a\sqrt{-c}\,d^2\left(\sqrt{c+d\,x}-\sqrt{c}\right)^2}{32\,c^{3/2}\left(\sqrt{-c}-\sqrt{d\,x-c}\right)^2}-\frac{8\,b\sqrt{-c}\,\sqrt{c}\left(\sqrt{c+d\,x}-\sqrt{c}\right)^2}{\left(\sqrt{-c}-\sqrt{d\,x-c}\right)^2\left(\frac{\left(\sqrt{c+d\,x}-\sqrt{c}\right)^4}{\left(\sqrt{-c}-\sqrt{d\,x-c}\right)^4}-\frac{2\left(\sqrt{c+d\,x}-\sqrt{c}\right)^2}{\left(\sqrt{-c}-\sqrt{d\,x-c}\right)^2}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^2)*(c + d*x)^(1/2)*(d*x - c)^(1/2))/x^3,x)
```

```
[Out] b*(-c)^(1/2)*c^(1/2)*log(((c + d*x)^(1/2) - c^(1/2))^2/((-c)^(1/2) - (d*x -
 c)^(1/2))^2 + 1) - ((a*(-c)^(1/2)*d^2)/(32*c^(3/2)) + (a*(-c)^(1/2)*d^2*((
c + d*x)^(1/2) - c^(1/2))^2)/(16*c^(3/2)*((-c)^(1/2) - (d*x - c)^(1/2))^2)
- (15*a*(-c)^(1/2)*d^2*((c + d*x)^(1/2) - c^(1/2))^4)/(32*c^(3/2)*((-c)^(1/
2) - (d*x - c)^(1/2))^4))/(((c + d*x)^(1/2) - c^(1/2))^2/((-c)^(1/2) - (d*x
 - c)^(1/2))^2 + (2*((c + d*x)^(1/2) - c^(1/2))^4)/((-c)^(1/2) - (d*x - c)^
```

```
(1/2))^4 + ((c + d*x)^(1/2) - c^(1/2))^6/((-c)^(1/2) - (d*x - c)^(1/2))^6)
- b*(-c)^(1/2)*c^(1/2)*log(((c + d*x)^(1/2) - c^(1/2))/((-c)^(1/2) - (d*x -
 c)^(1/2))) + (a*(-c)^(1/2)*d^2*log(((c + d*x)^(1/2) - c^(1/2))/((-c)^(1/2)
 - (d*x - c)^(1/2))))/(2*c^(3/2)) - (a*(-c)^(1/2)*d^2*log(((c + d*x)^(1/2)
- c^(1/2))^2/((-c)^(1/2) - (d*x - c)^(1/2))^2 + 1))/(2*c^(3/2)) - (a*(-c)^(
1/2)*d^2*((c + d*x)^(1/2) - c^(1/2))^2)/(32*c^(3/2)*((-c)^(1/2) - (d*x - c)
^(1/2))^2) - (8*b*(-c)^(1/2)*c^(1/2)*((c + d*x)^(1/2) - c^(1/2))^2)/(((-c)^
(1/2) - (d*x - c)^(1/2))^2*(((c + d*x)^(1/2) - c^(1/2))^4/((-c)^(1/2) - (d*
x - c)^(1/2))^4 - (2*((c + d*x)^(1/2) - c^(1/2))^2)/((-c)^(1/2) - (d*x - c)
^(1/2))^2 + 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+bx^2\right)\sqrt{-c+dx}\,\sqrt{c+dx}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)*(d*x-c)**(1/2)*(d*x+c)**(1/2)/x**3,x)
```

```
[Out] Integral((a + b*x**2)*sqrt(-c + d*x)*sqrt(c + d*x)/x**3, x)
```

3.199 $\int \frac{\sqrt{-c+dx}\,\sqrt{c+dx}\,(a+bx^2)}{x^5}\,dx$

Optimal. Leaf size=121

$$-\frac{\sqrt{dx-c}\,\sqrt{c+dx}\,(ad^2+4bc^2)}{8c^2x^2}+\frac{d^2(ad^2+4bc^2)\tan^{-1}\left(\frac{\sqrt{dx-c}\,\sqrt{c+dx}}{c}\right)}{8c^3}+\frac{a(dx-c)^{3/2}(c+dx)^{3/2}}{4c^2x^4}$$

Rubi [A] time = 0.10, antiderivative size = 164, normalized size of antiderivative = 1.36, number of steps used = 5, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {454, 94, 92, 205}

$$-\frac{\sqrt{dx-c}\,(c+dx)^{3/2}(ad^2+4bc^2)}{8c^3x^2}+\frac{d\sqrt{dx-c}\,\sqrt{c+dx}\,(ad^2+4bc^2)}{8c^3x}+\frac{d^2(ad^2+4bc^2)\tan^{-1}\left(\frac{\sqrt{dx-c}\,\sqrt{c+dx}}{c}\right)}{8c^3}+\frac{a(dx-c)^{3/2}(c+dx)^{3/2}}{4c^2x^4}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[-c + d*x]*Sqrt[c + d*x]*(a + b*x^2))/x^5,x]
```

```
[Out] (d*(4*b*c^2 + a*d^2)*Sqrt[-c + d*x]*Sqrt[c + d*x])/(8*c^3*x) - ((4*b*c^2 +
a*d^2)*Sqrt[-c + d*x]*(c + d*x)^(3/2))/(8*c^3*x^2) + (a*(-c + d*x)^(3/2)*(c
 + d*x)^(3/2))/(4*c^2*x^4) + (d^2*(4*b*c^2 + a*d^2)*ArcTan[(Sqrt[-c + d*x]*
Sqrt[c + d*x])/c])/(8*c^3)
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 454

```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[(c*(e*x)^
(m + 1)*(a1 + b1*x^(n/2))^(p + 1)*(a2 + b2*x^(n/2))^(p + 1))/(a1*a2*e*(m +
1)), x] + Dist[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/(a1*a2*e^n*(
m + 1)), Int[(e*x)^(m + n)*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n/2))^p, x], x]
/; FreeQ[{a1, b1, a2, b2, c, d, e, p}, x] && EqQ[non2, n/2] && EqQ[a2*b1 +
a1*b2, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (L
tQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rubi steps

$$\int \frac{\sqrt{-c+dx}\,\sqrt{c+dx}\,(a+bx^2)}{x^5}\,dx = \frac{a(-c+dx)^{3/2}(c+dx)^{3/2}}{4c^2x^4} + \frac{1}{4}\left(4b+\frac{ad^2}{c^2}\right)\int \frac{\sqrt{-c+dx}\,\sqrt{c+dx}}{x^3}\,dx$$
$$= -\frac{(4bc^2+ad^2)\sqrt{-c+dx}\,(c+dx)^{3/2}}{8c^3x^2} + \frac{a(-c+dx)^{3/2}(c+dx)^{3/2}}{4c^2x^4} + \frac{1}{8}\left(d\left(4b+\right.\right.$$
$$= \frac{d(4bc^2+ad^2)\sqrt{-c+dx}\,\sqrt{c+dx}}{8c^3x} - \frac{(4bc^2+ad^2)\sqrt{-c+dx}\,(c+dx)^{3/2}}{8c^3x^2} + \frac{a}{}$$
$$= \frac{d(4bc^2+ad^2)\sqrt{-c+dx}\,\sqrt{c+dx}}{8c^3x} - \frac{(4bc^2+ad^2)\sqrt{-c+dx}\,(c+dx)^{3/2}}{8c^3x^2} + \frac{a}{}$$
$$= \frac{d(4bc^2+ad^2)\sqrt{-c+dx}\,\sqrt{c+dx}}{8c^3x} - \frac{(4bc^2+ad^2)\sqrt{-c+dx}\,(c+dx)^{3/2}}{8c^3x^2} + \frac{a}{}$$

Mathematica [A] time = 0.09, size = 137, normalized size = 1.13

$$\frac{\sqrt{dx-c}\,\sqrt{c+dx}\left((c^2-d^2x^2)(2ac^2-ad^2x^2+4bc^2x^2)-d^2x^4\sqrt{1-\frac{d^2x^2}{c^2}}\,(ad^2+4bc^2)\tanh^{-1}\left(\sqrt{1-\frac{d^2x^2}{c^2}}\right)\right)}{8c^2d^2x^6-8c^4x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[-c + d*x]*Sqrt[c + d*x]*(a + b*x^2))/x^5,x]
```

```
[Out] (Sqrt[-c + d*x]*Sqrt[c + d*x]*((c^2 - d^2*x^2)*(2*a*c^2 + 4*b*c^2*x^2 - a*d
^2*x^2) - d^2*(4*b*c^2 + a*d^2)*x^4*Sqrt[1 - (d^2*x^2)/c^2]*ArcTanh[Sqrt[1
- (d^2*x^2)/c^2]]))/(-8*c^4*x^4 + 8*c^2*d^2*x^6)
```

IntegrateAlgebraic [A] time = 0.18, size = 206, normalized size = 1.70

$$\frac{d^2\sqrt{dx-c}\left(\frac{dx-c}{c+dx}-1\right)\left(-\frac{6ad^2(dx-c)}{c+dx}+\frac{ad^2(dx-c)^2}{(c+dx)^2}+ad^2+\frac{8bc^2(dx-c)}{c+dx}+\frac{4bc^2(dx-c)^2}{(c+dx)^2}+4bc^2\right)}{4c^3\sqrt{c+dx}\left(\frac{dx-c}{c+dx}+1\right)^4} + \frac{(ad^4+4bc^2d^2)\tan^{-1}\left(\frac{\sqrt{dx-c}}{\sqrt{c+dx}}\right)}{4c^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(Sqrt[-c + d*x]*Sqrt[c + d*x]*(a + b*x^2))/x^5,x]
```

```
[Out] (d^2*Sqrt[-c + d*x]*(-1 + (-c + d*x)/(c + d*x))*(4*b*c^2 + a*d^2 + (4*b*c^2
*(-c + d*x)^2)/(c + d*x)^2 + (a*d^2*(-c + d*x)^2)/(c + d*x)^2 + (8*b*c^2*(-
c + d*x))/(c + d*x) - (6*a*d^2*(-c + d*x))/(c + d*x)))/(4*c^3*Sqrt[c + d*x]
*(1 + (-c + d*x)/(c + d*x))^4) + ((4*b*c^2*d^2 + a*d^4)*ArcTan[Sqrt[-c + d*
x]/Sqrt[c + d*x]])/(4*c^3)
```

fricas [A] time = 0.80, size = 100, normalized size = 0.83

$$\frac{2\left(4\,bc^2d^2+ad^4\right)x^4\arctan\left(-\frac{dx-\sqrt{dx+c}\,\sqrt{dx-c}}{c}\right)-\left(2\,ac^3+\left(4\,bc^3-acd^2\right)x^2\right)\sqrt{dx+c}\,\sqrt{dx-c}}{8\,c^3x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x-c)^(1/2)*(d*x+c)^(1/2)/x^5,x, algorithm="fricas")
```

```
[Out] 1/8*(2*(4*b*c^2*d^2 + a*d^4)*x^4*arctan(-(d*x - sqrt(d*x + c)*sqrt(d*x - c)
)/c) - (2*a*c^3 + (4*b*c^3 - a*c*d^2)*x^2)*sqrt(d*x + c)*sqrt(d*x - c))/(c^
3*x^4)
```

giac [B] time = 0.45, size = 324, normalized size = 2.68

$$-\frac{\frac{(4bc^2d^3+ad^5)\arctan\left(\frac{\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^2}{2c}\right)}{c^3}-\frac{2\left(4bc^2d^3\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^{14}-ad^5\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^{14}+16bc^4d^3\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^{10}+28ac^2d^5\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^{10}-64bc^6d^3\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^{6}-112ac^4d^5\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^{6}-256bc^8d^3\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^{2}+64ac^6d^5\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^{2}\right)}{\left(\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^4+4c^2\right)^4c^2}}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x-c)^(1/2)*(d*x+c)^(1/2)/x^5,x, algorithm="giac")
```

```
[Out] -1/4*((4*b*c^2*d^3 + a*d^5)*arctan(1/2*(sqrt(d*x + c) - sqrt(d*x - c))^2/c)
/c^3 - 2*(4*b*c^2*d^3*(sqrt(d*x + c) - sqrt(d*x - c))^14 - a*d^5*(sqrt(d*x
+ c) - sqrt(d*x - c))^14 + 16*b*c^4*d^3*(sqrt(d*x + c) - sqrt(d*x - c))^10
+ 28*a*c^2*d^5*(sqrt(d*x + c) - sqrt(d*x - c))^10 - 64*b*c^6*d^3*(sqrt(d*x
+ c) - sqrt(d*x - c))^6 - 112*a*c^4*d^5*(sqrt(d*x + c) - sqrt(d*x - c))^6 -
 256*b*c^8*d^3*(sqrt(d*x + c) - sqrt(d*x - c))^2 + 64*a*c^6*d^5*(sqrt(d*x +
 c) - sqrt(d*x - c))^2)/(((sqrt(d*x + c) - sqrt(d*x - c))^4 + 4*c^2)^4*c^2)
)/d
```

maple [B] time = 0.07, size = 226, normalized size = 1.87

$$-\frac{\sqrt{dx-c}\,\sqrt{dx+c}\left(a d^4x^4\ln\left(-\frac{2\left(c^2-\sqrt{-c^2}\,\sqrt{d^2x^2-c^2}\right)}{x}\right)+4bc^2d^2x^4\ln\left(-\frac{2\left(c^2-\sqrt{-c^2}\,\sqrt{d^2x^2-c^2}\right)}{x}\right)-\sqrt{-c^2}\,\sqrt{d^2x^2-c^2}\,a d^2x^2+4\sqrt{-c^2}\,\sqrt{d^2x^2-c^2}\,b c^2x^2+2\sqrt{-c^2}\,\sqrt{d^2x^2-c^2}\,a c^2\right)}{8\sqrt{d^2x^2-c^2}\,\sqrt{-c^2}\,c^2x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)*(d*x-c)^(1/2)*(d*x+c)^(1/2)/x^5,x)
```

```
[Out] -1/8*(d*x-c)^(1/2)*(d*x+c)^(1/2)/c^2*(ln(-2*(c^2-(-c^2)^(1/2)*(d^2*x^2-c^2)
^(1/2))/x)*x^4*a*d^4+4*ln(-2*(c^2-(-c^2)^(1/2)*(d^2*x^2-c^2)^(1/2))/x)*x^4*
b*c^2*d^2-(-c^2)^(1/2)*(d^2*x^2-c^2)^(1/2)*x^2*a*d^2+4*(-c^2)^(1/2)*(d^2*x^
2-c^2)^(1/2)*x^2*b*c^2+2*(-c^2)^(1/2)*(d^2*x^2-c^2)^(1/2)*a*c^2)/(d^2*x^2-c
^2)^(1/2)/x^4/(-c^2)^(1/2)
```

maxima [A] time = 1.35, size = 162, normalized size = 1.34

$$-\frac{bd^2\arcsin\left(\frac{c}{d|x|}\right)}{2c}-\frac{ad^4\arcsin\left(\frac{c}{d|x|}\right)}{8c^3}-\frac{\sqrt{d^2x^2-c^2}\,bd^2}{2c^2}-\frac{\sqrt{d^2x^2-c^2}\,ad^4}{8c^4}+\frac{\left(d^2x^2-c^2\right)^{\frac{3}{2}}b}{2c^2x^2}+\frac{\left(d^2x^2-c^2\right)^{\frac{3}{2}}ad^2}{8c^4x^2}+\frac{\left(d^2x^2-c^2\right)^{\frac{3}{2}}a}{4c^2x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x-c)^(1/2)*(d*x+c)^(1/2)/x^5,x, algorithm="maxima")
```

```
[Out] -1/2*b*d^2*arcsin(c/(d*abs(x)))/c - 1/8*a*d^4*arcsin(c/(d*abs(x)))/c^3 - 1/
2*sqrt(d^2*x^2 - c^2)*b*d^2/c^2 - 1/8*sqrt(d^2*x^2 - c^2)*a*d^4/c^4 + 1/2*(
d^2*x^2 - c^2)^(3/2)*b/(c^2*x^2) + 1/8*(d^2*x^2 - c^2)^(3/2)*a*d^2/(c^4*x^2
) + 1/4*(d^2*x^2 - c^2)^(3/2)*a/(c^2*x^4)
```

mupad [B] time = 15.56, size = 1004, normalized size = 8.30

$$\frac{\frac{a\sqrt{-c}\,d^4}{1024c^{7/2}}+\frac{a\sqrt{-c}\,d^4\left(\sqrt{c+dx}-\sqrt{c}\right)^2}{128c^{7/2}\left(\sqrt{-c}-\sqrt{dx-c}\right)^2}+\frac{11a\sqrt{-c}\,d^4\left(\sqrt{c+dx}-\sqrt{c}\right)^4}{512c^{7/2}\left(\sqrt{-c}-\sqrt{dx-c}\right)^4}+\frac{7a\sqrt{-c}\,d^4\left(\sqrt{c+dx}-\sqrt{c}\right)^6}{256c^{7/2}\left(\sqrt{-c}-\sqrt{dx-c}\right)^6}-\frac{239a\sqrt{-c}\,d^4\left(\sqrt{c+dx}-\sqrt{c}\right)^8}{1024c^{7/2}\left(\sqrt{-c}-\sqrt{dx-c}\right)^8}+\frac{a\sqrt{-c}\,d^4\left(\sqrt{c+dx}-\sqrt{c}\right)^{10}}{256c^{7/2}\left(\sqrt{-c}-\sqrt{dx-c}\right)^{10}}}{\frac{\left(\sqrt{c+dx}-\sqrt{c}\right)^4}{\left(\sqrt{-c}-\sqrt{dx-c}\right)^4}+\frac{4\left(\sqrt{c+dx}-\sqrt{c}\right)^6}{\left(\sqrt{-c}-\sqrt{dx-c}\right)^6}+\frac{6\left(\sqrt{c+dx}-\sqrt{c}\right)^8}{\left(\sqrt{-c}-\sqrt{dx-c}\right)^8}+\frac{4\left(\sqrt{c+dx}-\sqrt{c}\right)^{10}}{\left(\sqrt{-c}-\sqrt{dx-c}\right)^{10}}+\frac{\left(\sqrt{c+dx}-\sqrt{c}\right)^{12}}{\left(\sqrt{-c}-\sqrt{dx-c}\right)^{12}}}-\frac{\frac{b\sqrt{-c}\,d^2}{32c^{3/2}}+\frac{b\sqrt{-c}\,d^2\left(\sqrt{c+dx}-\sqrt{c}\right)^2}{16c^{3/2}\left(\sqrt{-c}-\sqrt{dx-c}\right)^2}-\frac{15b\sqrt{-c}\,d^2\left(\sqrt{c+dx}-\sqrt{c}\right)^4}{32c^{3/2}\left(\sqrt{-c}-\sqrt{dx-c}\right)^4}}{\frac{\left(\sqrt{c+dx}-\sqrt{c}\right)^2}{\left(\sqrt{-c}-\sqrt{dx-c}\right)^2}+\frac{2\left(\sqrt{c+dx}-\sqrt{c}\right)^4}{\left(\sqrt{-c}-\sqrt{dx-c}\right)^4}+\frac{\left(\sqrt{c+dx}-\sqrt{c}\right)^6}{\left(\sqrt{-c}-\sqrt{dx-c}\right)^6}}+\frac{a\sqrt{-c}\,d^4\ln\left(\frac{\sqrt{c+dx}-\sqrt{c}}{\sqrt{-c}-\sqrt{dx-c}}\right)}{8c^{7/2}}+\frac{b\sqrt{-c}\,d^2\ln\left(\frac{\sqrt{c+dx}-\sqrt{c}}{\sqrt{-c}-\sqrt{dx-c}}\right)}{2c^{3/2}}-\frac{a\sqrt{-c}\,d^4\ln\left(\frac{\left(\sqrt{c+dx}-\sqrt{c}\right)^2}{\left(\sqrt{-c}-\sqrt{dx-c}\right)^2}+1\right)}{8c^{7/2}}-\frac{b\sqrt{-c}\,d^2\ln\left(\frac{\left(\sqrt{c+dx}-\sqrt{c}\right)^2}{\left(\sqrt{-c}-\sqrt{dx-c}\right)^2}+1\right)}{2c^{3/2}}+\frac{a\sqrt{-c}\,d^4\left(\sqrt{c+dx}-\sqrt{c}\right)^2}{256c^{7/2}\left(\sqrt{-c}-\sqrt{dx-c}\right)^2}+\frac{a\sqrt{-c}\,d^4\left(\sqrt{c+dx}-\sqrt{c}\right)^4}{1024c^{7/2}\left(\sqrt{-c}-\sqrt{dx-c}\right)^4}-\frac{b\sqrt{-c}\,d^2\left(\sqrt{c+dx}-\sqrt{c}\right)^2}{32c^{3/2}\left(\sqrt{-c}-\sqrt{dx-c}\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^2)*(c + d*x)^(1/2)*(d*x - c)^(1/2))/x^5,x)
```

```
[Out] ((a*(-c)^(1/2)*d^4)/(1024*c^(7/2)) + (a*(-c)^(1/2)*d^4*((c + d*x)^(1/2) - c
^(1/2))^2)/(128*c^(7/2)*((-c)^(1/2) - (d*x - c)^(1/2))^2) + (11*a*(-c)^(1/2
)*d^4*((c + d*x)^(1/2) - c^(1/2))^4)/(512*c^(7/2)*((-c)^(1/2) - (d*x - c)^(
1/2))^4) + (7*a*(-c)^(1/2)*d^4*((c + d*x)^(1/2) - c^(1/2))^6)/(256*c^(7/2)*
((-c)^(1/2) - (d*x - c)^(1/2))^6) - (239*a*(-c)^(1/2)*d^4*((c + d*x)^(1/2)
- c^(1/2))^8)/(1024*c^(7/2)*((-c)^(1/2) - (d*x - c)^(1/2))^8) + (a*(-c)^(1/
2)*d^4*((c + d*x)^(1/2) - c^(1/2))^10)/(256*c^(7/2)*((-c)^(1/2) - (d*x - c)
```

```
^(1/2))^10))/(((c + d*x)^(1/2) - c^(1/2))^4/((-c)^(1/2) - (d*x - c)^(1/2))^
4 + (4*((c + d*x)^(1/2) - c^(1/2))^6)/((-c)^(1/2) - (d*x - c)^(1/2))^6 + (6
*((c + d*x)^(1/2) - c^(1/2))^8)/((-c)^(1/2) - (d*x - c)^(1/2))^8 + (4*((c +
 d*x)^(1/2) - c^(1/2))^10)/((-c)^(1/2) - (d*x - c)^(1/2))^10 + ((c + d*x)^(
1/2) - c^(1/2))^12/((-c)^(1/2) - (d*x - c)^(1/2))^12) - ((b*(-c)^(1/2)*d^2)
/(32*c^(3/2)) + (b*(-c)^(1/2)*d^2*((c + d*x)^(1/2) - c^(1/2))^2)/(16*c^(3/2
)*((-c)^(1/2) - (d*x - c)^(1/2))^2) - (15*b*(-c)^(1/2)*d^2*((c + d*x)^(1/2)
 - c^(1/2))^4)/(32*c^(3/2)*((-c)^(1/2) - (d*x - c)^(1/2))^4))/((((c + d*x)^(
1/2) - c^(1/2))^2/((-c)^(1/2) - (d*x - c)^(1/2))^2 + (2*((c + d*x)^(1/2) -
c^(1/2))^4)/((-c)^(1/2) - (d*x - c)^(1/2))^4 + ((c + d*x)^(1/2) - c^(1/2))^
6/((-c)^(1/2) - (d*x - c)^(1/2))^6) + (a*(-c)^(1/2)*d^4*log(((c + d*x)^(1/2
) - c^(1/2))/((-c)^(1/2) - (d*x - c)^(1/2))))/(8*c^(7/2)) + (b*(-c)^(1/2)*d
^2*log(((c + d*x)^(1/2) - c^(1/2))/((-c)^(1/2) - (d*x - c)^(1/2))))/(2*c^(3
/2)) - (a*(-c)^(1/2)*d^4*log(((c + d*x)^(1/2) - c^(1/2))^2/((-c)^(1/2) - (d
*x - c)^(1/2))^2 + 1))/(8*c^(7/2)) - (b*(-c)^(1/2)*d^2*log(((c + d*x)^(1/2)
 - c^(1/2))^2/((-c)^(1/2) - (d*x - c)^(1/2))^2 + 1))/(2*c^(3/2)) + (a*(-c)^
(1/2)*d^4*((c + d*x)^(1/2) - c^(1/2))^2)/(256*c^(7/2)*((-c)^(1/2) - (d*x -
c)^(1/2))^2) + (a*(-c)^(1/2)*d^4*((c + d*x)^(1/2) - c^(1/2))^4)/(1024*c^(7/
2)*((-c)^(1/2) - (d*x - c)^(1/2))^4) - (b*(-c)^(1/2)*d^2*((c + d*x)^(1/2) -
 c^(1/2))^2)/(32*c^(3/2)*((-c)^(1/2) - (d*x - c)^(1/2))^2)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)*(d*x-c)**(1/2)*(d*x+c)**(1/2)/x**5,x)
[Out] Timed out
```

3.200 $\int x^4\sqrt{-c+dx}\,\sqrt{c+dx}\,\left(a+bx^2\right)dx$

Optimal. Leaf size=208

$$\frac{c^2x(dx-c)^{3/2}(c+dx)^{3/2}\left(8ad^2+5bc^2\right)}{64d^6}+\frac{x^3(dx-c)^{3/2}(c+dx)^{3/2}\left(8ad^2+5bc^2\right)}{48d^4}-\frac{c^6\left(8ad^2+5bc^2\right)\tanh^{-1}\left(\frac{\sqrt{dx-}}{\sqrt{c+d}}\right.}{64d^7}$$

Rubi [A] time = 0.15, antiderivative size = 208, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.258, Rules used = {460, 100, 12, 90, 38, 63, 217, 206}

$$\frac{x^3(dx-c)^{3/2}(c+dx)^{3/2}\left(8ad^2+5bc^2\right)}{48d^4}+\frac{c^4x\sqrt{dx-c}\,\sqrt{c+dx}\left(8ad^2+5bc^2\right)}{128d^6}+\frac{c^2x(dx-c)^{3/2}(c+dx)^{3/2}\left(8ad^2+5bc^2\right)}{64d^6}-\frac{c^6\left(8ad^2+5bc^2\right)\tanh^{-1}\left(\frac{\sqrt{dx-c}}{\sqrt{c+dx}}\right)}{64d^7}+\frac{bx^5(dx-c)^{3/2}(c+dx)^{3/2}}{8d^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*Sqrt[-c + d*x]*Sqrt[c + d*x]*(a + b*x^2),x]
```

```
[Out] (c^4*(5*b*c^2 + 8*a*d^2)*x*Sqrt[-c + d*x]*Sqrt[c + d*x])/(128*d^6) + (c^2*(
5*b*c^2 + 8*a*d^2)*x*(-c + d*x)^(3/2)*(c + d*x)^(3/2))/(64*d^6) + ((5*b*c^2
 + 8*a*d^2)*x^3*(-c + d*x)^(3/2)*(c + d*x)^(3/2))/(48*d^4) + (b*x^5*(-c + d
*x)^(3/2)*(c + d*x)^(3/2))/(8*d^2) - (c^6*(5*b*c^2 + 8*a*d^2)*ArcTanh[Sqrt[
-c + d*x]/Sqrt[c + d*x]])/(64*d^7)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 38

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[(x
*(a + b*x)^m*(c + d*x)^m)/(2*m + 1), x] + Dist[(2*a*c*m)/(2*m + 1), Int[(a
+ b*x)^(m - 1)*(c + d*x)^(m - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
*c + a*d, 0] && IGtQ[m + 1/2, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*(a + b*x)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/
(d*f*(n + p + 3)), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/(d*f*(m + n + p + 1)), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
```

```
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 460

```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[(d*(e*x)^
(m + 1)*(a1 + b1*x^(n/2))^(p + 1)*(a2 + b2*x^(n/2))^(p + 1))/(b1*b2*e*(m +
n*(p + 1) + 1)), x] - Dist[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/
(b1*b2*(m + n*(p + 1) + 1)), Int[(e*x)^m*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n/
2))^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, m, n, p}, x] && EqQ[non2,
n/2] && EqQ[a2*b1 + a1*b2, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rubi steps

$$\int x^4\sqrt{-c+dx}\,\sqrt{c+dx}\,\left(a+bx^2\right)dx = \frac{bx^5(-c+dx)^{3/2}(c+dx)^{3/2}}{8d^2} - \frac{1}{8}\left(-8a-\frac{5bc^2}{d^2}\right)\int x^4\sqrt{-c+dx}\,\sqrt{c+dx}\,dx$$

$$= \frac{\left(5bc^2+8ad^2\right)x^3(-c+dx)^{3/2}(c+dx)^{3/2}}{48d^4} + \frac{bx^5(-c+dx)^{3/2}(c+dx)^{3/2}}{8d^2} +$$

$$= \frac{\left(5bc^2+8ad^2\right)x^3(-c+dx)^{3/2}(c+dx)^{3/2}}{48d^4} + \frac{bx^5(-c+dx)^{3/2}(c+dx)^{3/2}}{8d^2} +$$

$$= \frac{c^2\left(5bc^2+8ad^2\right)x(-c+dx)^{3/2}(c+dx)^{3/2}}{64d^6} + \frac{\left(5bc^2+8ad^2\right)x^3(-c+dx)^{3/2}}{48d^4}$$

$$= \frac{c^2\left(5bc^2+8ad^2\right)x(-c+dx)^{3/2}(c+dx)^{3/2}}{64d^6} + \frac{\left(5bc^2+8ad^2\right)x^3(-c+dx)^{3/2}}{48d^4}$$

$$= \frac{c^4\left(5bc^2+8ad^2\right)x\sqrt{-c+dx}\,\sqrt{c+dx}}{128d^6} + \frac{c^2\left(5bc^2+8ad^2\right)x(-c+dx)^{3/2}}{64d^6}$$

$$= \frac{c^4\left(5bc^2+8ad^2\right)x\sqrt{-c+dx}\,\sqrt{c+dx}}{128d^6} + \frac{c^2\left(5bc^2+8ad^2\right)x(-c+dx)^{3/2}}{64d^6}$$

$$= \frac{c^4\left(5bc^2+8ad^2\right)x\sqrt{-c+dx}\,\sqrt{c+dx}}{128d^6} + \frac{c^2\left(5bc^2+8ad^2\right)x(-c+dx)^{3/2}}{64d^6}$$

$$= \frac{c^4\left(5bc^2+8ad^2\right)x\sqrt{-c+dx}\,\sqrt{c+dx}}{128d^6} + \frac{c^2\left(5bc^2+8ad^2\right)x(-c+dx)^{3/2}}{64d^6}$$

Mathematica [A] time = 0.31, size = 161, normalized size = 0.77

$$\frac{\sqrt{dx-c}\,\sqrt{c+dx}\left(3\left(8ac^5d^2+5bc^7\right)\sin^{-1}\left(\frac{dx}{c}\right)-dx\sqrt{1-\frac{d^2x^2}{c^2}}\left(8ad^2\left(3c^4+2c^2d^2x^2-8d^4x^4\right)+b\left(15c^6+10c^4d^2x^2+8c^2d^4x^4-48d^6x^6\right)\right)\right)}{384d^7\sqrt{1-\frac{d^2x^2}{c^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*Sqrt[-c + d*x]*Sqrt[c + d*x]*(a + b*x^2),x]
```

```
[Out] (Sqrt[-c + d*x]*Sqrt[c + d*x]*(-(d*x*Sqrt[1 - (d^2*x^2)/c^2]*(8*a*d^2*(3*c^
4 + 2*c^2*d^2*x^2 - 8*d^4*x^4) + b*(15*c^6 + 10*c^4*d^2*x^2 + 8*c^2*d^4*x^4
 - 48*d^6*x^6))) + 3*(5*b*c^7 + 8*a*c^5*d^2)*ArcSin[(d*x)/c]))/(384*d^7*Sqr
t[1 - (d^2*x^2)/c^2])
```

IntegrateAlgebraic [A] time = 0.30, size = 385, normalized size = 1.85

$$\frac{\left(-8ac^6d^2-5bc^8\right)\tanh^{-1}\left(\frac{\sqrt{dx-c}}{\sqrt{c+dx}}\right)}{64d^7}+\frac{c^6\sqrt{dx-c}\left(\frac{dx-c}{c+dx}+1\right)\left(\frac{24ad^2(dx-c)^6}{(c+dx)^6}+\frac{304ad^2(dx-c)^5}{(c+dx)^5}-\frac{408ad^2(dx-c)^4}{(c+dx)^4}+\frac{160ad^2(dx-c)^3}{(c+dx)^3}-\frac{408ad^2(dx-c)^2}{(c+dx)^2}+\frac{304ad^2(dx-c)}{c+dx}+24ad^2+\frac{15bc^2(dx-c)^6}{(c+dx)^6}+\frac{382bc^2(dx-c)^5}{(c+dx)^5}+\frac{513bc^2(dx-c)^4}{(c+dx)^4}+\frac{1252bc^2(dx-c)^3}{(c+dx)^3}+\frac{513bc^2(dx-c)^2}{(c+dx)^2}+\frac{382bc^2(dx-c)}{c+dx}+15bc^2\right)}{192d^7\sqrt{c+dx}\left(\frac{dx-c}{c+dx}-1\right)^8}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^4*Sqrt[-c + d*x]*Sqrt[c + d*x]*(a + b*x^2),x]
```

```
[Out] (c^6*Sqrt[-c + d*x]*(1 + (-c + d*x)/(c + d*x))*(15*b*c^2 + 24*a*d^2 + (15*b
*c^2*(-c + d*x)^6)/(c + d*x)^6 + (24*a*d^2*(-c + d*x)^6)/(c + d*x)^6 + (382
*b*c^2*(-c + d*x)^5)/(c + d*x)^5 + (304*a*d^2*(-c + d*x)^5)/(c + d*x)^5 + (
513*b*c^2*(-c + d*x)^4)/(c + d*x)^4 - (408*a*d^2*(-c + d*x)^4)/(c + d*x)^4
+ (1252*b*c^2*(-c + d*x)^3)/(c + d*x)^3 + (160*a*d^2*(-c + d*x)^3)/(c + d*x
)^3 + (513*b*c^2*(-c + d*x)^2)/(c + d*x)^2 - (408*a*d^2*(-c + d*x)^2)/(c +
d*x)^2 + (382*b*c^2*(-c + d*x))/(c + d*x) + (304*a*d^2*(-c + d*x))/(c + d*x
)))/(192*d^7*Sqrt[c + d*x]*(-1 + (-c + d*x)/(c + d*x))^8) + ((-5*b*c^8 - 8*
a*c^6*d^2)*ArcTanh[Sqrt[-c + d*x]/Sqrt[c + d*x]])/(64*d^7)
```

fricas [A] time = 1.42, size = 138, normalized size = 0.66

$$\frac{\left(48\,bd^7x^7-8\left(bc^2d^5-8\,ad^7\right)x^5-2\left(5\,bc^4d^3+8\,ac^2d^5\right)x^3-3\left(5\,bc^6d+8\,ac^4d^3\right)x\right)\sqrt{dx+c}\,\sqrt{dx-c}+3\left(5\,bc^8+8\,ac^6d^2\right)\log\left(-dx+\sqrt{dx+c}\,\sqrt{dx-c}\right)}{384\,d^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)*(d*x-c)^(1/2)*(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/384*((48*b*d^7*x^7 - 8*(b*c^2*d^5 - 8*a*d^7)*x^5 - 2*(5*b*c^4*d^3 + 8*a*c
^2*d^5)*x^3 - 3*(5*b*c^6*d + 8*a*c^4*d^3)*x)*sqrt(d*x + c)*sqrt(d*x - c) +
3*(5*b*c^8 + 8*a*c^6*d^2)*log(-d*x + sqrt(d*x + c)*sqrt(d*x - c)))/d^7
```

giac [B] time = 0.63, size = 558, normalized size = 2.68

$$\frac{112\left(\left(\left(2(dx+c)\left(3(dx+c)\left(\frac{4(dx+c)}{d^4}-\frac{21c}{d^4}\right)+\frac{133c^2}{d^4}\right)-\frac{295c^3}{d^4}\right)(dx+c)+\frac{195c^4}{d^4}\right)\sqrt{dx+c}\sqrt{dx-c}+\frac{90c^5\log\left(\left|-\sqrt{dx+c}+\sqrt{dx-c}\right|\right)}{d^4}\right)ac+8\left(\left(\left(2\left(\left(4(dx+c)\left(5(dx+c)\left(\frac{6(dx+c)}{d^6}-\frac{43c}{d^6}\right)+\frac{661c^2}{d^6}\right)-\frac{4551c^3}{d^6}\right)(dx+c)+\frac{4781c^4}{d^6}\right)(dx+c)-\frac{6335c^5}{d^6}\right)(dx+c)+\frac{2835c^6}{d^6}\right)\sqrt{dx+c}\sqrt{dx-c}+\frac{1050c^7\log\left(\left|-\sqrt{dx+c}+\sqrt{dx-c}\right|\right)}{d^6}\right)bc+56\left(\left(\left(2\left((dx+c)\left(4(dx+c)\left(\frac{5(dx+c)}{d^5}-\frac{31c}{d^5}\right)+\frac{321c^2}{d^5}\right)-\frac{451c^3}{d^5}\right)(dx+c)+\frac{745c^4}{d^5}\right)(dx+c)-\frac{405c^5}{d^5}\right)\sqrt{dx+c}\sqrt{dx-c}-\frac{150c^6\log\left(\left|-\sqrt{dx+c}+\sqrt{dx-c}\right|\right)}{d^5}\right)ad+\left(\left(\left(2\left(\left(4\left(5(dx+c)\left(6(dx+c)\left(\frac{7(dx+c)}{d^7}-\frac{57c}{d^7}\right)+\frac{1219c^2}{d^7}\right)-\frac{12463c^3}{d^7}\right)(dx+c)+\frac{64233c^4}{d^7}\right)(dx+c)-\frac{53963c^5}{d^7}\right)(dx+c)+\frac{59465c^6}{d^7}\right)(dx+c)-\frac{23205c^7}{d^7}\right)\sqrt{dx+c}\sqrt{dx-c}-\frac{7350c^8\log\left(\left|-\sqrt{dx+c}+\sqrt{dx-c}\right|\right)}{d^7}\right)bd}{13440\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)*(d*x-c)^(1/2)*(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/13440*(112*(((2*(d*x + c)*(3*(d*x + c)*(4*(d*x + c)/d^4 - 21*c/d^4) + 133
*c^2/d^4) - 295*c^3/d^4)*(d*x + c) + 195*c^4/d^4)*sqrt(d*x + c)*sqrt(d*x -
c) + 90*c^5*log(abs(-sqrt(d*x + c) + sqrt(d*x - c)))/d^4)*a*c + 8*(((2*((4*
(d*x + c)*(5*(d*x + c)*(6*(d*x + c)/d^6 - 43*c/d^6) + 661*c^2/d^6) - 4551*c
^3/d^6)*(d*x + c) + 4781*c^4/d^6)*(d*x + c) - 6335*c^5/d^6)*(d*x + c) + 283
5*c^6/d^6)*sqrt(d*x + c)*sqrt(d*x - c) + 1050*c^7*log(abs(-sqrt(d*x + c) +
sqrt(d*x - c)))/d^6)*b*c + 56*(((2*((d*x + c)*(4*(d*x + c)*(5*(d*x + c)/d^5
 - 31*c/d^5) + 321*c^2/d^5) - 451*c^3/d^5)*(d*x + c) + 745*c^4/d^5)*(d*x +
c) - 405*c^5/d^5)*sqrt(d*x + c)*sqrt(d*x - c) - 150*c^6*log(abs(-sqrt(d*x +
 c) + sqrt(d*x - c)))/d^5)*a*d + (((2*((4*(5*(d*x + c)*(6*(d*x + c)*(7*(d*x
 + c)/d^7 - 57*c/d^7) + 1219*c^2/d^7) - 12463*c^3/d^7)*(d*x + c) + 64233*c^
4/d^7)*(d*x + c) - 53963*c^5/d^7)*(d*x + c) + 59465*c^6/d^7)*(d*x + c) - 23
205*c^7/d^7)*sqrt(d*x + c)*sqrt(d*x - c) - 7350*c^8*log(abs(-sqrt(d*x + c)
+ sqrt(d*x - c)))/d^7)*b*d)/d
```

maple [C] time = 0.10, size = 298, normalized size = 1.43

$$\frac{\sqrt{dx-c}\,\sqrt{dx+c}\,\left(48\sqrt{d^2x^2-c^2}\,b\,d^7x^7\mathrm{csgn}(d)+64\sqrt{d^2x^2-c^2}\,a\,d^7x^5\mathrm{csgn}(d)-8\sqrt{d^2x^2-c^2}\,b\,c^2d^5x^5\mathrm{csgn}(d)-16\sqrt{d^2x^2-c^2}\,a\,c^2d^5x^3\mathrm{csgn}(d)-10\sqrt{d^2x^2-c^2}\,b\,c^4d^3x^3\mathrm{csgn}(d)-24a\,c^6d^2\ln\left(\left(dx+\sqrt{d^2x^2-c^2}\,\mathrm{csgn}(d)\right)\mathrm{csgn}(d)\right)-24\sqrt{d^2x^2-c^2}\,a\,c^4d^3x\,\mathrm{csgn}(d)-15b\,c^8\ln\left(\left(dx+\sqrt{d^2x^2-c^2}\,\mathrm{csgn}(d)\right)\mathrm{csgn}(d)\right)-15\sqrt{d^2x^2-c^2}\,b\,c^6dx\,\mathrm{csgn}(d)\right)\mathrm{csgn}(d)}{384\sqrt{d^2x^2-c^2}\,d^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(b*x^2+a)*(d*x-c)^(1/2)*(d*x+c)^(1/2),x)
```

```
[Out] 1/384*(d*x-c)^(1/2)*(d*x+c)^(1/2)*(48*csgn(d)*x^7*b*d^7*(d^2*x^2-c^2)^(1/2)
+64*csgn(d)*x^5*a*d^7*(d^2*x^2-c^2)^(1/2)-8*csgn(d)*x^5*b*c^2*d^5*(d^2*x^2-
c^2)^(1/2)-16*csgn(d)*x^3*a*c^2*d^5*(d^2*x^2-c^2)^(1/2)-10*csgn(d)*x^3*b*c^
4*d^3*(d^2*x^2-c^2)^(1/2)-24*csgn(d)*d^3*(d^2*x^2-c^2)^(1/2)*x*a*c^4-15*csg
n(d)*d*(d^2*x^2-c^2)^(1/2)*x*b*c^6-24*ln((csgn(d)*(d^2*x^2-c^2)^(1/2)+d*x)*
csgn(d))*a*c^6*d^2-15*ln((csgn(d)*(d^2*x^2-c^2)^(1/2)+d*x)*csgn(d))*b*c^8)*
csgn(d)/(d^2*x^2-c^2)^(1/2)/d^7
```

maxima [A] time = 0.55, size = 246, normalized size = 1.18

$$\frac{\left(d^2x^2-c^2\right)^{\frac{3}{2}}bx^5}{8\,d^2}+\frac{5\left(d^2x^2-c^2\right)^{\frac{3}{2}}bc^2x^3}{48\,d^4}+\frac{\left(d^2x^2-c^2\right)^{\frac{3}{2}}ax^3}{6\,d^2}-\frac{5\,bc^8\log\left(2\,d^2x+2\,\sqrt{d^2x^2-c^2}\,d\right)}{128\,d^7}-\frac{ac^6\log\left(2\,d^2x+2\,\sqrt{d^2x^2-c^2}\,d\right)}{16\,d^5}+\frac{5\,\sqrt{d^2x^2-c^2}\,bc^6x}{128\,d^6}+\frac{\sqrt{d^2x^2-c^2}\,ac^4x}{16\,d^4}+\frac{5\left(d^2x^2-c^2\right)^{\frac{3}{2}}bc^4x}{64\,d^6}+\frac{\left(d^2x^2-c^2\right)^{\frac{3}{2}}ac^2x}{8\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)*(d*x-c)^(1/2)*(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/8*(d^2*x^2 - c^2)^(3/2)*b*x^5/d^2 + 5/48*(d^2*x^2 - c^2)^(3/2)*b*c^2*x^3/
d^4 + 1/6*(d^2*x^2 - c^2)^(3/2)*a*x^3/d^2 - 5/128*b*c^8*log(2*d^2*x + 2*sqr
t(d^2*x^2 - c^2)*d)/d^7 - 1/16*a*c^6*log(2*d^2*x + 2*sqrt(d^2*x^2 - c^2)*d)
/d^5 + 5/128*sqrt(d^2*x^2 - c^2)*b*c^6*x/d^6 + 1/16*sqrt(d^2*x^2 - c^2)*a*c
^4*x/d^4 + 5/64*(d^2*x^2 - c^2)^(3/2)*b*c^4*x/d^6 + 1/8*(d^2*x^2 - c^2)^(3/
2)*a*c^2*x/d^4
```

mupad [B] time = 39.15, size = 2314, normalized size = 11.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a + b*x^2)*(c + d*x)^(1/2)*(d*x - c)^(1/2),x)
```

```
[Out] ((35*a*c^6*((c + d*x)^(1/2) - c^(1/2))^3)/(12*((-c)^(1/2) - (d*x - c)^(1/2)
)^3) - (a*c^6*((c + d*x)^(1/2) - c^(1/2)))/(4*((-c)^(1/2) - (d*x - c)^(1/2)
)) + (757*a*c^6*((c + d*x)^(1/2) - c^(1/2))^5)/(4*((-c)^(1/2) - (d*x - c)^(
1/2))^5) + (7339*a*c^6*((c + d*x)^(1/2) - c^(1/2))^7)/(4*((-c)^(1/2) - (d*x
 - c)^(1/2))^7) + (41929*a*c^6*((c + d*x)^(1/2) - c^(1/2))^9)/(6*((-c)^(1/2
) - (d*x - c)^(1/2))^9) + (25661*a*c^6*((c + d*x)^(1/2) - c^(1/2))^11)/(2*(
(-c)^(1/2) - (d*x - c)^(1/2))^11) + (25661*a*c^6*((c + d*x)^(1/2) - c^(1/2)
)^13)/(2*((-c)^(1/2) - (d*x - c)^(1/2))^13) + (41929*a*c^6*((c + d*x)^(1/2)
 - c^(1/2))^15)/(6*((-c)^(1/2) - (d*x - c)^(1/2))^15) + (7339*a*c^6*((c + d
*x)^(1/2) - c^(1/2))^17)/(4*((-c)^(1/2) - (d*x - c)^(1/2))^17) + (757*a*c^6
*((c + d*x)^(1/2) - c^(1/2))^19)/(4*((-c)^(1/2) - (d*x - c)^(1/2))^19) + (3
5*a*c^6*((c + d*x)^(1/2) - c^(1/2))^21)/(12*((-c)^(1/2) - (d*x - c)^(1/2))^
21) - (a*c^6*((c + d*x)^(1/2) - c^(1/2))^23)/(4*((-c)^(1/2) - (d*x - c)^(1/
2))^23))/(d^5 - (12*d^5*((c + d*x)^(1/2) - c^(1/2))^2)/((-c)^(1/2) - (d*x -
 c)^(1/2))^2 + (66*d^5*((c + d*x)^(1/2) - c^(1/2))^4)/((-c)^(1/2) - (d*x -
c)^(1/2))^4 - (220*d^5*((c + d*x)^(1/2) - c^(1/2))^6)/((-c)^(1/2) - (d*x -
c)^(1/2))^6 + (495*d^5*((c + d*x)^(1/2) - c^(1/2))^8)/((-c)^(1/2) - (d*x -
c)^(1/2))^8 - (792*d^5*((c + d*x)^(1/2) - c^(1/2))^10)/((-c)^(1/2) - (d*x -
 c)^(1/2))^10 + (924*d^5*((c + d*x)^(1/2) - c^(1/2))^12)/((-c)^(1/2) - (d*x
 - c)^(1/2))^12 - (792*d^5*((c + d*x)^(1/2) - c^(1/2))^14)/((-c)^(1/2) - (d
*x - c)^(1/2))^14 + (495*d^5*((c + d*x)^(1/2) - c^(1/2))^16)/((-c)^(1/2) -
(d*x - c)^(1/2))^16 - (220*d^5*((c + d*x)^(1/2) - c^(1/2))^18)/((-c)^(1/2)
- (d*x - c)^(1/2))^18 + (66*d^5*((c + d*x)^(1/2) - c^(1/2))^20)/((-c)^(1/2)
 - (d*x - c)^(1/2))^20 - (12*d^5*((c + d*x)^(1/2) - c^(1/2))^22)/((-c)^(1/2
) - (d*x - c)^(1/2))^22 + (d^5*((c + d*x)^(1/2) - c^(1/2))^24)/((-c)^(1/2)
- (d*x - c)^(1/2))^24) - ((5*b*c^8*((c + d*x)^(1/2) - c^(1/2)))/(32*((-c)^(
1/2) - (d*x - c)^(1/2))) - (235*b*c^8*((c + d*x)^(1/2) - c^(1/2))^3)/(96*((
-c)^(1/2) - (d*x - c)^(1/2))^3) + (1723*b*c^8*((c + d*x)^(1/2) - c^(1/2))^5
```

```
)/(96*((-c)^(1/2) - (d*x - c)^(1/2))^5) + (72283*b*c^8*((c + d*x)^(1/2) - c
^(1/2))^7)/(32*((-c)^(1/2) - (d*x - c)^(1/2))^7) + (848801*b*c^8*((c + d*x)
^(1/2) - c^(1/2))^9)/(32*((-c)^(1/2) - (d*x - c)^(1/2))^9) + (4181067*b*c^8
*((c + d*x)^(1/2) - c^(1/2))^11)/(32*((-c)^(1/2) - (d*x - c)^(1/2))^11) + (
10994181*b*c^8*((c + d*x)^(1/2) - c^(1/2))^13)/(32*((-c)^(1/2) - (d*x - c)^
(1/2))^13) + (17457599*b*c^8*((c + d*x)^(1/2) - c^(1/2))^15)/(32*((-c)^(1/2
) - (d*x - c)^(1/2))^15) + (17457599*b*c^8*((c + d*x)^(1/2) - c^(1/2))^17)/
(32*((-c)^(1/2) - (d*x - c)^(1/2))^17) + (10994181*b*c^8*((c + d*x)^(1/2) -
 c^(1/2))^19)/(32*((-c)^(1/2) - (d*x - c)^(1/2))^19) + (4181067*b*c^8*((c +
 d*x)^(1/2) - c^(1/2))^21)/(32*((-c)^(1/2) - (d*x - c)^(1/2))^21) + (848801
*b*c^8*((c + d*x)^(1/2) - c^(1/2))^23)/(32*((-c)^(1/2) - (d*x - c)^(1/2))^2
3) + (72283*b*c^8*((c + d*x)^(1/2) - c^(1/2))^25)/(32*((-c)^(1/2) - (d*x -
c)^(1/2))^25) + (1723*b*c^8*((c + d*x)^(1/2) - c^(1/2))^27)/(96*((-c)^(1/2)
 - (d*x - c)^(1/2))^27) - (235*b*c^8*((c + d*x)^(1/2) - c^(1/2))^29)/(96*((
-c)^(1/2) - (d*x - c)^(1/2))^29) + (5*b*c^8*((c + d*x)^(1/2) - c^(1/2))^31)
/(32*((-c)^(1/2) - (d*x - c)^(1/2))^31))/(d^7 - (16*d^7*((c + d*x)^(1/2) -
c^(1/2))^2)/((-c)^(1/2) - (d*x - c)^(1/2))^2 + (120*d^7*((c + d*x)^(1/2) -
c^(1/2))^4)/((-c)^(1/2) - (d*x - c)^(1/2))^4 - (560*d^7*((c + d*x)^(1/2) -
c^(1/2))^6)/((-c)^(1/2) - (d*x - c)^(1/2))^6 + (1820*d^7*((c + d*x)^(1/2) -
 c^(1/2))^8)/((-c)^(1/2) - (d*x - c)^(1/2))^8 - (4368*d^7*((c + d*x)^(1/2)
- c^(1/2))^10)/((-c)^(1/2) - (d*x - c)^(1/2))^10 + (8008*d^7*((c + d*x)^(1/
2) - c^(1/2))^12)/((-c)^(1/2) - (d*x - c)^(1/2))^12 - (11440*d^7*((c + d*x)
^(1/2) - c^(1/2))^14)/((-c)^(1/2) - (d*x - c)^(1/2))^14 + (12870*d^7*((c +
d*x)^(1/2) - c^(1/2))^16)/((-c)^(1/2) - (d*x - c)^(1/2))^16 - (11440*d^7*((
c + d*x)^(1/2) - c^(1/2))^18)/((-c)^(1/2) - (d*x - c)^(1/2))^18 + (8008*d^7
*((c + d*x)^(1/2) - c^(1/2))^20)/((-c)^(1/2) - (d*x - c)^(1/2))^20 - (4368*
d^7*((c + d*x)^(1/2) - c^(1/2))^22)/((-c)^(1/2) - (d*x - c)^(1/2))^22 + (18
20*d^7*((c + d*x)^(1/2) - c^(1/2))^24)/((-c)^(1/2) - (d*x - c)^(1/2))^24 -
(560*d^7*((c + d*x)^(1/2) - c^(1/2))^26)/((-c)^(1/2) - (d*x - c)^(1/2))^26
+ (120*d^7*((c + d*x)^(1/2) - c^(1/2))^28)/((-c)^(1/2) - (d*x - c)^(1/2))^2
8 - (16*d^7*((c + d*x)^(1/2) - c^(1/2))^30)/((-c)^(1/2) - (d*x - c)^(1/2))^
30 + (d^7*((c + d*x)^(1/2) - c^(1/2))^32)/((-c)^(1/2) - (d*x - c)^(1/2))^32
) + (a*c^6*atanh(((c + d*x)^(1/2) - c^(1/2))/((-c)^(1/2) - (d*x - c)^(1/2))
))/(4*d^5) + (5*b*c^8*atanh(((c + d*x)^(1/2) - c^(1/2))/((-c)^(1/2) - (d*x
- c)^(1/2))))/(32*d^7)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^4 \left(a + bx^2\right) \sqrt{-c + dx} \sqrt{c + dx} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(b*x**2+a)*(d*x-c)**(1/2)*(d*x+c)**(1/2),x)
```

```
[Out] Integral(x**4*(a + b*x**2)*sqrt(-c + d*x)*sqrt(c + d*x), x)
```

3.201 $\int x^2\sqrt{-c+dx}\sqrt{c+dx}\left(a+bx^2\right)dx$

Optimal. Leaf size=159

$$\frac{c^2x\sqrt{dx-c}\sqrt{c+dx}\left(2ad^2+bc^2\right)}{16d^4}+\frac{x(dx-c)^{3/2}(c+dx)^{3/2}\left(2ad^2+bc^2\right)}{8d^4}-\frac{c^4\left(2ad^2+bc^2\right)\tanh^{-1}\left(\frac{\sqrt{dx-c}}{\sqrt{c+dx}}\right)}{8d^5}+\frac{bx^3(dx$$

Rubi [A] time = 0.12, antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {460, 90, 12, 38, 63, 217, 206}

$$\frac{c^2x\sqrt{dx-c}\sqrt{c+dx}\left(2ad^2+bc^2\right)}{16d^4}+\frac{x(dx-c)^{3/2}(c+dx)^{3/2}\left(2ad^2+bc^2\right)}{8d^4}-\frac{c^4\left(2ad^2+bc^2\right)\tanh^{-1}\left(\frac{\sqrt{dx-c}}{\sqrt{c+dx}}\right)}{8d^5}+\frac{bx^3(dx-c)^{3/2}(c+dx)^{3/2}}{6d^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sqrt[-c + d*x]*Sqrt[c + d*x]*(a + b*x^2),x]
```

```
[Out] (c^2*(b*c^2 + 2*a*d^2)*x*Sqrt[-c + d*x]*Sqrt[c + d*x])/(16*d^4) + ((b*c^2 +
 2*a*d^2)*x*(-c + d*x)^(3/2)*(c + d*x)^(3/2))/(8*d^4) + (b*x^3*(-c + d*x)^(
3/2)*(c + d*x)^(3/2))/(6*d^2) - (c^4*(b*c^2 + 2*a*d^2)*ArcTanh[Sqrt[-c + d*
x]/Sqrt[c + d*x]])/(8*d^5)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 38

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[(x
*(a + b*x)^m*(c + d*x)^m)/(2*m + 1), x] + Dist[(2*a*c*m)/(2*m + 1), Int[(a
+ b*x)^(m - 1)*(c + d*x)^(m - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
*c + a*d, 0] && IGtQ[m + 1/2, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*(a + b*x)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/
(d*f*(n + p + 3)), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 460

```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[(d*(e*x)^
(m + 1)*(a1 + b1*x^(n/2))^(p + 1)*(a2 + b2*x^(n/2))^(p + 1))/(b1*b2*e*(m +
n*(p + 1) + 1)), x] - Dist[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/
(b1*b2*(m + n*(p + 1) + 1)), Int[(e*x)^m*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n/
2))^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, m, n, p}, x] && EqQ[non2,
n/2] && EqQ[a2*b1 + a1*b2, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rubi steps

$$\int x^2\sqrt{-c+dx}\,\sqrt{c+dx}\,\left(a+bx^2\right)dx = \frac{bx^3(-c+dx)^{3/2}(c+dx)^{3/2}}{6d^2}+\frac{1}{2}\left(2a+\frac{bc^2}{d^2}\right)\int x^2\sqrt{-c+dx}\,\sqrt{c+dx}\,dx$$

$$=\frac{\left(bc^2+2ad^2\right)x(-c+dx)^{3/2}(c+dx)^{3/2}}{8d^4}+\frac{bx^3(-c+dx)^{3/2}(c+dx)^{3/2}}{6d^2}+\cdots$$

$$=\frac{\left(bc^2+2ad^2\right)x(-c+dx)^{3/2}(c+dx)^{3/2}}{8d^4}+\frac{bx^3(-c+dx)^{3/2}(c+dx)^{3/2}}{6d^2}+\cdots$$

$$=\frac{c^2\left(bc^2+2ad^2\right)x\sqrt{-c+dx}\,\sqrt{c+dx}}{16d^4}+\frac{\left(bc^2+2ad^2\right)x(-c+dx)^{3/2}(c+\cdots}{8d^4}$$

$$=\frac{c^2\left(bc^2+2ad^2\right)x\sqrt{-c+dx}\,\sqrt{c+dx}}{16d^4}+\frac{\left(bc^2+2ad^2\right)x(-c+dx)^{3/2}(c+\cdots}{8d^4}$$

$$=\frac{c^2\left(bc^2+2ad^2\right)x\sqrt{-c+dx}\,\sqrt{c+dx}}{16d^4}+\frac{\left(bc^2+2ad^2\right)x(-c+dx)^{3/2}(c+\cdots}{8d^4}$$

$$=\frac{c^2\left(bc^2+2ad^2\right)x\sqrt{-c+dx}\,\sqrt{c+dx}}{16d^4}+\frac{\left(bc^2+2ad^2\right)x(-c+dx)^{3/2}(c+\cdots}{8d^4}$$

Mathematica [A] time = 0.19, size = 135, normalized size = 0.85

$$\frac{\sqrt{dx-c}\,\sqrt{c+dx}\left(3\left(2ac^3d^2+bc^5\right)\sin^{-1}\left(\frac{dx}{c}\right)+dx\sqrt{1-\frac{d^2x^2}{c^2}}\left(b\left(-3c^4-2c^2d^2x^2+8d^4x^4\right)-6ad^2\left(c^2-2d^2x^2\right)\right)\right)}{48d^5\sqrt{1-\frac{d^2x^2}{c^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Sqrt[-c + d*x]*Sqrt[c + d*x]*(a + b*x^2),x]
```

```
[Out] (Sqrt[-c + d*x]*Sqrt[c + d*x]*(d*x*Sqrt[1 - (d^2*x^2)/c^2]*(-6*a*d^2*(c^2 -
 2*d^2*x^2) + b*(-3*c^4 - 2*c^2*d^2*x^2 + 8*d^4*x^4)) + 3*(b*c^5 + 2*a*c^3*
d^2)*ArcSin[(d*x)/c]))/(48*d^5*Sqrt[1 - (d^2*x^2)/c^2])
```

IntegrateAlgebraic [A] time = 0.24, size = 297, normalized size = 1.87

$$\frac{\left(-2ac^4d^2-bc^6\right)\tanh^{-1}\left(\frac{\sqrt{dx-c}}{\sqrt{c+dx}}\right)}{8d^5}+\frac{c^4\sqrt{dx-c}\left(\frac{dx-c}{c+dx}+1\right)\left(\frac{6ad^2(dx-c)^4}{(c+dx)^4}+\frac{24ad^2(dx-c)^3}{(c+dx)^3}-\frac{60ad^2(dx-c)^2}{(c+dx)^2}+\frac{24ad^2(dx-c)}{c+dx}+6ad^2+\frac{3bc^2(dx-c)^4}{(c+dx)^4}+\frac{44bc^2(dx-c)^3}{(c+dx)^3}+\frac{34bc^2(dx-c)^2}{(c+dx)^2}+\frac{44bc^2(dx-c)}{c+dx}+3bc^2\right)}{24d^5\sqrt{c+dx}\left(\frac{dx-c}{c+dx}-1\right)^6}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^2*Sqrt[-c + d*x]*Sqrt[c + d*x]*(a + b*x^2),x]
```

```
[Out] (c^4*Sqrt[-c + d*x]*(1 + (-c + d*x)/(c + d*x))*(3*b*c^2 + 6*a*d^2 + (3*b*c^
2*(-c + d*x)^4)/(c + d*x)^4 + (6*a*d^2*(-c + d*x)^4)/(c + d*x)^4 + (44*b*c^
2*(-c + d*x)^3)/(c + d*x)^3 + (24*a*d^2*(-c + d*x)^3)/(c + d*x)^3 + (34*b*c
^2*(-c + d*x)^2)/(c + d*x)^2 - (60*a*d^2*(-c + d*x)^2)/(c + d*x)^2 + (44*b*
c^2*(-c + d*x))/(c + d*x) + (24*a*d^2*(-c + d*x))/(c + d*x)))/(24*d^5*Sqrt[
c + d*x]*(-1 + (-c + d*x)/(c + d*x))^6) + ((-(b*c^6) - 2*a*c^4*d^2)*ArcTanh
[Sqrt[-c + d*x]/Sqrt[c + d*x]])/(8*d^5)
```

fricas [A] time = 0.82, size = 112, normalized size = 0.70

$$\frac{\left(8\,bd^{5}x^{5}-2\left(bc^{2}d^{3}-6\,ad^{5}\right)x^{3}-3\left(bc^{4}d+2\,ac^{2}d^{3}\right)x\right)\sqrt{dx+c}\,\sqrt{dx-c}\,+3\left(bc^{6}+2\,ac^{4}d^{2}\right)\log\left(-dx+\sqrt{dx+c}\,\sqrt{dx-c}\right)}{48\,d^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)*(d*x-c)^(1/2)*(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/48*((8*b*d^5*x^5 - 2*(b*c^2*d^3 - 6*a*d^5)*x^3 - 3*(b*c^4*d + 2*a*c^2*d^3
)*x)*sqrt(d*x + c)*sqrt(d*x - c) + 3*(b*c^6 + 2*a*c^4*d^2)*log(-d*x + sqrt(
d*x + c)*sqrt(d*x - c)))/d^5
```

giac [B] time = 0.46, size = 432, normalized size = 2.72

$$\frac{40\left(\sqrt{dx+c}\,\sqrt{dx-c}\left((dx+c)\left(\frac{2\,(dx+c)}{d^{2}}-\frac{7\,c}{d^{2}}\right)+\frac{9\,c^{2}}{d^{2}}\right)+\frac{6\,c^{3}\log\left(\left|-\sqrt{dx+c}+\sqrt{dx-c}\right|\right)}{d^{2}}\right)ac+2\left(\left(\left(2\,(dx+c)\left(3\,(dx+c)\left(\frac{4\,(dx+c)}{d^{4}}-\frac{21\,c}{d^{4}}\right)+\frac{133\,c^{2}}{d^{4}}\right)-\frac{295\,c^{3}}{d^{4}}\right)(dx+c)+\frac{195\,c^{4}}{d^{4}}\right)\sqrt{dx+c}\,\sqrt{dx-c}+\frac{90\,c^{5}\log\left(\left|-\sqrt{dx+c}+\sqrt{dx-c}\right|\right)}{d^{4}}\right)bc+10\left(\left((dx+c)\left(2\,(dx+c)\left(\frac{3\,(dx+c)}{d^{3}}-\frac{13\,c}{d^{3}}\right)+\frac{43\,c^{2}}{d^{3}}\right)-\frac{39\,c^{3}}{d^{3}}\right)\sqrt{dx+c}\,\sqrt{dx-c}-\frac{18\,c^{4}\log\left(\left|-\sqrt{dx+c}+\sqrt{dx-c}\right|\right)}{d^{3}}\right)ad+\left(\left(\left(2\left((dx+c)\left(4\,(dx+c)\left(\frac{5\,(dx+c)}{d^{5}}-\frac{31\,c}{d^{5}}\right)+\frac{321\,c^{2}}{d^{5}}\right)-\frac{451\,c^{3}}{d^{5}}\right)(dx+c)+\frac{745\,c^{4}}{d^{5}}\right)(dx+c)-\frac{405\,c^{5}}{d^{5}}\right)\sqrt{dx+c}\,\sqrt{dx-c}-\frac{150\,c^{6}\log\left(\left|-\sqrt{dx+c}+\sqrt{dx-c}\right|\right)}{d^{5}}\right)bd}{240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)*(d*x-c)^(1/2)*(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/240*(40*(sqrt(d*x + c)*sqrt(d*x - c)*((d*x + c)*(2*(d*x + c)/d^2 - 7*c/d^
2) + 9*c^2/d^2) + 6*c^3*log(abs(-sqrt(d*x + c) + sqrt(d*x - c)))/d^2)*a*c +
 2*(((2*(d*x + c)*(3*(d*x + c)*(4*(d*x + c)/d^4 - 21*c/d^4) + 133*c^2/d^4)
- 295*c^3/d^4)*(d*x + c) + 195*c^4/d^4)*sqrt(d*x + c)*sqrt(d*x - c) + 90*c^
5*log(abs(-sqrt(d*x + c) + sqrt(d*x - c)))/d^4)*b*c + 10*(((d*x + c)*(2*(d*
x + c)*(3*(d*x + c)/d^3 - 13*c/d^3) + 43*c^2/d^3) - 39*c^3/d^3)*sqrt(d*x +
c)*sqrt(d*x - c) - 18*c^4*log(abs(-sqrt(d*x + c) + sqrt(d*x - c)))/d^3)*a*d
 + (((2*((d*x + c)*(4*(d*x + c)*(5*(d*x + c)/d^5 - 31*c/d^5) + 321*c^2/d^5)
 - 451*c^3/d^5)*(d*x + c) + 745*c^4/d^5)*(d*x + c) - 405*c^5/d^5)*sqrt(d*x
+ c)*sqrt(d*x - c) - 150*c^6*log(abs(-sqrt(d*x + c) + sqrt(d*x - c)))/d^5)*
b*d)/d
```

maple [C] time = 0.07, size = 240, normalized size = 1.51

$$\frac{\sqrt{dx-c}\,\sqrt{dx+c}\,\left(8\sqrt{d^{2}x^{2}-c^{2}}\,b\,d^{5}x^{5}\mathrm{csgn}(d)+12\sqrt{d^{2}x^{2}-c^{2}}\,a\,d^{5}x^{3}\mathrm{csgn}(d)-2\sqrt{d^{2}x^{2}-c^{2}}\,b\,c^{2}d^{3}x^{3}\mathrm{csgn}(d)-6a\,c^{4}d^{2}\ln\left(\left(dx+\sqrt{d^{2}x^{2}-c^{2}}\,\mathrm{csgn}(d)\right)\mathrm{csgn}(d)\right)-6\sqrt{d^{2}x^{2}-c^{2}}\,a\,c^{2}d^{3}x\,\mathrm{csgn}(d)-3b\,c^{6}\ln\left(\left(dx+\sqrt{d^{2}x^{2}-c^{2}}\,\mathrm{csgn}(d)\right)\mathrm{csgn}(d)\right)-3\sqrt{d^{2}x^{2}-c^{2}}\,b\,c^{4}dx\,\mathrm{csgn}(d)\right)\mathrm{csgn}(d)}{48\sqrt{d^{2}x^{2}-c^{2}}\,d^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x^2+a)*(d*x-c)^(1/2)*(d*x+c)^(1/2),x)
```

```
[Out] 1/48*(d*x-c)^(1/2)*(d*x+c)^(1/2)*(8*csgn(d)*x^5*b*d^5*(d^2*x^2-c^2)^(1/2)+1
2*csgn(d)*x^3*a*d^5*(d^2*x^2-c^2)^(1/2)-2*csgn(d)*x^3*b*c^2*d^3*(d^2*x^2-c^
2)^(1/2)-6*csgn(d)*d^3*(d^2*x^2-c^2)^(1/2)*x*a*c^2-3*csgn(d)*d*(d^2*x^2-c^2
)^(1/2)*x*b*c^4-6*ln((d*x+(d^2*x^2-c^2)^(1/2)*csgn(d))*csgn(d))*a*c^4*d^2-3
*ln((d*x+(d^2*x^2-c^2)^(1/2)*csgn(d))*csgn(d))*b*c^6)*csgn(d)/(d^2*x^2-c^2)
^(1/2)/d^5
```

maxima [A] time = 0.67, size = 192, normalized size = 1.21

$$\frac{\left(d^{2}x^{2}-c^{2}\right)^{\frac{3}{2}}bx^{3}}{6\,d^{2}}-\frac{bc^{6}\log\left(2\,d^{2}x+2\,\sqrt{d^{2}x^{2}-c^{2}}\,d\right)}{16\,d^{5}}-\frac{ac^{4}\log\left(2\,d^{2}x+2\,\sqrt{d^{2}x^{2}-c^{2}}\,d\right)}{8\,d^{3}}+\frac{\sqrt{d^{2}x^{2}-c^{2}}\,bc^{4}x}{16\,d^{4}}+\frac{\sqrt{d^{2}x^{2}-c^{2}}\,ac^{2}x}{8\,d^{2}}+\frac{\left(d^{2}x^{2}-c^{2}\right)^{\frac{3}{2}}bc^{2}x}{8\,d^{4}}+\frac{\left(d^{2}x^{2}-c^{2}\right)^{\frac{3}{2}}ax}{4\,d^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)*(d*x-c)^(1/2)*(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/6*(d^2*x^2 - c^2)^(3/2)*b*x^3/d^2 - 1/16*b*c^6*log(2*d^2*x + 2*sqrt(d^2*x
^2 - c^2)*d)/d^5 - 1/8*a*c^4*log(2*d^2*x + 2*sqrt(d^2*x^2 - c^2)*d)/d^3 + 1
/16*sqrt(d^2*x^2 - c^2)*b*c^4*x/d^4 + 1/8*sqrt(d^2*x^2 - c^2)*a*c^2*x/d^2 +
 1/8*(d^2*x^2 - c^2)^(3/2)*b*c^2*x/d^4 + 1/4*(d^2*x^2 - c^2)^(3/2)*a*x/d^2
```

mupad [B] time = 42.57, size = 1681, normalized size = 10.57

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*x^2)*(c + d*x)^(1/2)*(d*x - c)^(1/2),x)
```

```
[Out] ((35*b*c^6*((c + d*x)^(1/2) - c^(1/2))^3)/(12*((-c)^(1/2) - (d*x - c)^(1/2)
)^3) - (b*c^6*((c + d*x)^(1/2) - c^(1/2)))/(4*((-c)^(1/2) - (d*x - c)^(1/2)
)) + (757*b*c^6*((c + d*x)^(1/2) - c^(1/2))^5)/(4*((-c)^(1/2) - (d*x - c)^(
1/2))^5) + (7339*b*c^6*((c + d*x)^(1/2) - c^(1/2))^7)/(4*((-c)^(1/2) - (d*x
 - c)^(1/2))^7) + (41929*b*c^6*((c + d*x)^(1/2) - c^(1/2))^9)/(6*((-c)^(1/2
) - (d*x - c)^(1/2))^9) + (25661*b*c^6*((c + d*x)^(1/2) - c^(1/2))^11)/(2*(
(-c)^(1/2) - (d*x - c)^(1/2))^11) + (25661*b*c^6*((c + d*x)^(1/2) - c^(1/2)
)^13)/(2*((-c)^(1/2) - (d*x - c)^(1/2))^13) + (41929*b*c^6*((c + d*x)^(1/2)
 - c^(1/2))^15)/(6*((-c)^(1/2) - (d*x - c)^(1/2))^15) + (7339*b*c^6*((c + d
*x)^(1/2) - c^(1/2))^17)/(4*((-c)^(1/2) - (d*x - c)^(1/2))^17) + (757*b*c^6
*((c + d*x)^(1/2) - c^(1/2))^19)/(4*((-c)^(1/2) - (d*x - c)^(1/2))^19) + (3
5*b*c^6*((c + d*x)^(1/2) - c^(1/2))^21)/(12*((-c)^(1/2) - (d*x - c)^(1/2))^
21) - (b*c^6*((c + d*x)^(1/2) - c^(1/2))^23)/(4*((-c)^(1/2) - (d*x - c)^(1/
2))^23))/(d^5 - (12*d^5*((c + d*x)^(1/2) - c^(1/2))^2)/((-c)^(1/2) - (d*x -
 c)^(1/2))^2 + (66*d^5*((c + d*x)^(1/2) - c^(1/2))^4)/((-c)^(1/2) - (d*x -
c)^(1/2))^4 - (220*d^5*((c + d*x)^(1/2) - c^(1/2))^6)/((-c)^(1/2) - (d*x -
c)^(1/2))^6 + (495*d^5*((c + d*x)^(1/2) - c^(1/2))^8)/((-c)^(1/2) - (d*x -
c)^(1/2))^8 - (792*d^5*((c + d*x)^(1/2) - c^(1/2))^10)/((-c)^(1/2) - (d*x -
 c)^(1/2))^10 + (924*d^5*((c + d*x)^(1/2) - c^(1/2))^12)/((-c)^(1/2) - (d*x
 - c)^(1/2))^12 - (792*d^5*((c + d*x)^(1/2) - c^(1/2))^14)/((-c)^(1/2) - (d
*x - c)^(1/2))^14 + (495*d^5*((c + d*x)^(1/2) - c^(1/2))^16)/((-c)^(1/2) -
(d*x - c)^(1/2))^16 - (220*d^5*((c + d*x)^(1/2) - c^(1/2))^18)/((-c)^(1/2)
- (d*x - c)^(1/2))^18 + (66*d^5*((c + d*x)^(1/2) - c^(1/2))^20)/((-c)^(1/2)
 - (d*x - c)^(1/2))^20 - (12*d^5*((c + d*x)^(1/2) - c^(1/2))^22)/((-c)^(1/2
) - (d*x - c)^(1/2))^22 + (d^5*((c + d*x)^(1/2) - c^(1/2))^24)/((-c)^(1/2)
- (d*x - c)^(1/2))^24) - ((a*c^4*((c + d*x)^(1/2) - c^(1/2)))/(2*((-c)^(1/2
) - (d*x - c)^(1/2))) + (35*a*c^4*((c + d*x)^(1/2) - c^(1/2))^3)/(2*((-c)^(
1/2) - (d*x - c)^(1/2))^3) + (273*a*c^4*((c + d*x)^(1/2) - c^(1/2))^5)/(2*(
(-c)^(1/2) - (d*x - c)^(1/2))^5) + (715*a*c^4*((c + d*x)^(1/2) - c^(1/2))^7
)/(2*((-c)^(1/2) - (d*x - c)^(1/2))^7) + (715*a*c^4*((c + d*x)^(1/2) - c^(1
/2))^9)/(2*((-c)^(1/2) - (d*x - c)^(1/2))^9) + (273*a*c^4*((c + d*x)^(1/2)
- c^(1/2))^11)/(2*((-c)^(1/2) - (d*x - c)^(1/2))^11) + (35*a*c^4*((c + d*x)
^(1/2) - c^(1/2))^13)/(2*((-c)^(1/2) - (d*x - c)^(1/2))^13) + (a*c^4*((c +
d*x)^(1/2) - c^(1/2))^15)/(2*((-c)^(1/2) - (d*x - c)^(1/2))^15))/(d^3 - (8*
d^3*((c + d*x)^(1/2) - c^(1/2))^2)/((-c)^(1/2) - (d*x - c)^(1/2))^2 + (28*d
^3*((c + d*x)^(1/2) - c^(1/2))^4)/((-c)^(1/2) - (d*x - c)^(1/2))^4 - (56*d^
3*((c + d*x)^(1/2) - c^(1/2))^6)/((-c)^(1/2) - (d*x - c)^(1/2))^6 + (70*d^3
*((c + d*x)^(1/2) - c^(1/2))^8)/((-c)^(1/2) - (d*x - c)^(1/2))^8 - (56*d^3*
((c + d*x)^(1/2) - c^(1/2))^10)/((-c)^(1/2) - (d*x - c)^(1/2))^10 + (28*d^3
*((c + d*x)^(1/2) - c^(1/2))^12)/((-c)^(1/2) - (d*x - c)^(1/2))^12 - (8*d^3
*((c + d*x)^(1/2) - c^(1/2))^14)/((-c)^(1/2) - (d*x - c)^(1/2))^14 + (d^3*(
(c + d*x)^(1/2) - c^(1/2))^16)/((-c)^(1/2) - (d*x - c)^(1/2))^16) + (a*c^4*
atanh(((c + d*x)^(1/2) - c^(1/2))/((-c)^(1/2) - (d*x - c)^(1/2))))/(2*d^3)
+ (b*c^6*atanh(((c + d*x)^(1/2) - c^(1/2))/((-c)^(1/2) - (d*x - c)^(1/2))))
/(4*d^5)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2\left(a+bx^2\right)\sqrt{-c+dx}\sqrt{c+dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x**2+a)*(d*x-c)**(1/2)*(d*x+c)**(1/2),x)
```

```
[Out] Integral(x**2*(a + b*x**2)*sqrt(-c + d*x)*sqrt(c + d*x), x)
```

3.202 $\int \sqrt{-c+dx}\sqrt{c+dx}\left(a+bx^2\right)dx$

Optimal. Leaf size=114

$$\frac{x\sqrt{dx-c}\sqrt{c+dx}\left(4ad^2+bc^2\right)}{8d^2}-\frac{c^2\left(4ad^2+bc^2\right)\tanh^{-1}\left(\frac{\sqrt{dx-c}}{\sqrt{c+dx}}\right)}{4d^3}+\frac{bx(dx-c)^{3/2}(c+dx)^{3/2}}{4d^2}$$

Rubi [A] time = 0.05, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {389, 38, 63, 217, 206}

$$\frac{x\sqrt{dx-c}\sqrt{c+dx}\left(4ad^2+bc^2\right)}{8d^2}-\frac{c^2\left(4ad^2+bc^2\right)\tanh^{-1}\left(\frac{\sqrt{dx-c}}{\sqrt{c+dx}}\right)}{4d^3}+\frac{bx(dx-c)^{3/2}(c+dx)^{3/2}}{4d^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-c + d*x]*Sqrt[c + d*x]*(a + b*x^2),x]
```

```
[Out] ((b*c^2 + 4*a*d^2)*x*Sqrt[-c + d*x]*Sqrt[c + d*x])/(8*d^2) + (b*x*(-c + d*x
)^(3/2)*(c + d*x)^(3/2))/(4*d^2) - (c^2*(b*c^2 + 4*a*d^2)*ArcTanh[Sqrt[-c +
 d*x]/Sqrt[c + d*x]])/(4*d^3)
```

Rule 38

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[(x
*(a + b*x)^m*(c + d*x)^m)/(2*m + 1), x] + Dist[(2*a*c*m)/(2*m + 1), Int[(a
+ b*x)^(m - 1)*(c + d*x)^(m - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
*c + a*d, 0] && IGtQ[m + 1/2, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 389

```
Int[((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_
.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[(d*x*(a1 + b1*x^(n/2))^(p +
1)*(a2 + b2*x^(n/2))^(p + 1))/(b1*b2*(n*(p + 1) + 1)), x] - Dist[(a1*a2*d -
 b1*b2*c*(n*(p + 1) + 1))/(b1*b2*(n*(p + 1) + 1)), Int[(a1 + b1*x^(n/2))^p*
(a2 + b2*x^(n/2))^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, n, p}, x] && Eq
Q[non2, n/2] && EqQ[a2*b1 + a1*b2, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rubi steps

$$\int \sqrt{-c+dx}\,\sqrt{c+dx}\,\left(a+bx^2\right)dx = \frac{bx(-c+dx)^{3/2}(c+dx)^{3/2}}{4d^2} - \frac{\left(-bc^2-4ad^2\right)\int \sqrt{-c+dx}\,\sqrt{c+dx}\,dx}{4d^2}$$

$$= \frac{\left(bc^2+4ad^2\right)x\sqrt{-c+dx}\,\sqrt{c+dx}}{8d^2} + \frac{bx(-c+dx)^{3/2}(c+dx)^{3/2}}{4d^2} + \frac{\left(c^2\left(-bc^2 - \right.\right.}{}$$

$$= \frac{\left(bc^2+4ad^2\right)x\sqrt{-c+dx}\,\sqrt{c+dx}}{8d^2} + \frac{bx(-c+dx)^{3/2}(c+dx)^{3/2}}{4d^2} - \frac{\left(c^2\left(bc^2 + \right.\right.}{}$$

$$= \frac{\left(bc^2+4ad^2\right)x\sqrt{-c+dx}\,\sqrt{c+dx}}{8d^2} + \frac{bx(-c+dx)^{3/2}(c+dx)^{3/2}}{4d^2} - \frac{\left(c^2\left(bc^2 + \right.\right.}{}$$

$$= \frac{\left(bc^2+4ad^2\right)x\sqrt{-c+dx}\,\sqrt{c+dx}}{8d^2} + \frac{bx(-c+dx)^{3/2}(c+dx)^{3/2}}{4d^2} - \frac{c^2\left(bc^2+4\right.}{}$$

Mathematica [A] time = 0.28, size = 129, normalized size = 1.13

$$\frac{dx\left(c^2-d^2x^2\right)\left(b\left(c^2-2d^2x^2\right)-4ad^2\right)-2c^{5/2}\sqrt{dx-c}\,\sqrt{\frac{dx}{c}+1}\,\left(4ad^2+bc^2\right)\sinh^{-1}\left(\frac{\sqrt{dx-c}}{\sqrt{2}\,\sqrt{c}}\right)}{8d^3\sqrt{dx-c}\,\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-c + d*x]*Sqrt[c + d*x]*(a + b*x^2),x]
```

```
[Out] (d*x*(c^2 - d^2*x^2)*(-4*a*d^2 + b*(c^2 - 2*d^2*x^2)) - 2*c^(5/2)*(b*c^2 +
4*a*d^2)*Sqrt[-c + d*x]*Sqrt[1 + (d*x)/c]*ArcSinh[Sqrt[-c + d*x]/(Sqrt[2]*S
qrt[c])])/(8*d^3*Sqrt[-c + d*x]*Sqrt[c + d*x])
```

IntegrateAlgebraic [A] time = 0.18, size = 207, normalized size = 1.82

$$\frac{c^2\sqrt{dx-c}\,\left(\frac{dx-c}{c+dx}+1\right)\left(-\frac{8ad^2(dx-c)}{c+dx}+\frac{4ad^2(dx-c)^2}{(c+dx)^2}+4ad^2+\frac{6bc^2(dx-c)}{c+dx}+\frac{bc^2(dx-c)^2}{(c+dx)^2}+bc^2\right)}{4d^3\sqrt{c+dx}\,\left(\frac{dx-c}{c+dx}-1\right)^4}+\frac{\left(-4ac^2d^2-bc^4\right)\tanh^{-1}\left(\frac{\sqrt{dx-c}}{\sqrt{c+dx}}\right)}{4d^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[-c + d*x]*Sqrt[c + d*x]*(a + b*x^2),x]
```

```
[Out] (c^2*Sqrt[-c + d*x]*(1 + (-c + d*x)/(c + d*x))*(b*c^2 + 4*a*d^2 + (b*c^2*(-
c + d*x)^2)/(c + d*x)^2 + (4*a*d^2*(-c + d*x)^2)/(c + d*x)^2 + (6*b*c^2*(-c
 + d*x))/(c + d*x) - (8*a*d^2*(-c + d*x))/(c + d*x)))/(4*d^3*Sqrt[c + d*x]*
(-1 + (-c + d*x)/(c + d*x))^4) + ((-(b*c^4) - 4*a*c^2*d^2)*ArcTanh[Sqrt[-c
+ d*x]/Sqrt[c + d*x]])/(4*d^3)
```

fricas [A] time = 0.87, size = 88, normalized size = 0.77

$$\frac{\left(2\,bd^3x^3-\left(bc^2d-4\,ad^3\right)x\right)\sqrt{dx+c}\,\sqrt{dx-c}+\left(bc^4+4\,ac^2d^2\right)\log\left(-dx+\sqrt{dx+c}\,\sqrt{dx-c}\right)}{8\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x-c)^(1/2)*(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/8*((2*b*d^3*x^3 - (b*c^2*d - 4*a*d^3)*x)*sqrt(d*x + c)*sqrt(d*x - c) + (b
*c^4 + 4*a*c^2*d^2)*log(-d*x + sqrt(d*x + c)*sqrt(d*x - c)))/d^3
```

giac [B] time = 0.35, size = 288, normalized size = 2.53

$$\frac{24\left(2\,c\log\left(\left|-\sqrt{dx+c}+\sqrt{dx-c}\right|\right)+\sqrt{dx+c}\,\sqrt{dx-c}\right)ac+4\left(\sqrt{dx+c}\,\sqrt{dx-c}\left((dx+c)\left(\frac{2\,(dx+c)}{d^2}-\frac{7\,c}{d^2}\right)+\frac{9\,c^2}{d^2}\right)+\frac{6\,c^3\log\left(\left|-\sqrt{dx+c}+\sqrt{dx-c}\right|\right)}{d^2}\right)bc+\left(\left((dx+c)\left(2\,(dx+c)\left(\frac{3\,(dx+c)}{d^3}-\frac{13\,c}{d^3}\right)+\frac{43\,c^2}{d^3}\right)-\frac{39\,c^3}{d^3}\right)\sqrt{dx+c}\,\sqrt{dx-c}-\frac{18\,c^4\log\left(\left|-\sqrt{dx+c}+\sqrt{dx-c}\right|\right)}{d^3}\right)bd-12\left(2\,c^2\log\left(\left|-\sqrt{dx+c}+\sqrt{dx-c}\right|\right)-\sqrt{dx+c}\,\sqrt{dx-c}\,(dx-2\,c)\right)a}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x-c)^(1/2)*(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/24*(24*(2*c*log(abs(-sqrt(d*x + c) + sqrt(d*x - c))) + sqrt(d*x + c)*sqrt
(d*x - c))*a*c + 4*(sqrt(d*x + c)*sqrt(d*x - c)*((d*x + c)*(2*(d*x + c)/d^2
 - 7*c/d^2) + 9*c^2/d^2) + 6*c^3*log(abs(-sqrt(d*x + c) + sqrt(d*x - c)))/d
^2)*b*c + (((d*x + c)*(2*(d*x + c)*(3*(d*x + c)/d^3 - 13*c/d^3) + 43*c^2/d^
3) - 39*c^3/d^3)*sqrt(d*x + c)*sqrt(d*x - c) - 18*c^4*log(abs(-sqrt(d*x + c
) + sqrt(d*x - c)))/d^3)*b*d - 12*(2*c^2*log(abs(-sqrt(d*x + c) + sqrt(d*x
- c))) - sqrt(d*x + c)*sqrt(d*x - c)*(d*x - 2*c))*a)/d
```

maple [C]   time = 0.06, size = 182, normalized size = 1.60

$$\frac{\sqrt{dx-c}\,\sqrt{dx+c}\,\left(2\sqrt{d^2x^2-c^2}\,b\,d^3x^3\mathrm{csgn}(d)-4a\,c^2d^2\ln\left(\left(dx+\sqrt{d^2x^2-c^2}\,\mathrm{csgn}(d)\right)\mathrm{csgn}(d)\right)+4\sqrt{d^2x^2-c^2}\,a\,d^3x\,\mathrm{csgn}(d)-b\,c^4\ln\left(\left(dx+\sqrt{d^2x^2-c^2}\,\mathrm{csgn}(d)\right)\mathrm{csgn}(d)\right)-\sqrt{d^2x^2-c^2}\,b\,c^2dx\,\mathrm{csgn}(d)\right)\mathrm{csgn}(d)}{8\sqrt{d^2x^2-c^2}\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)*(d*x-c)^(1/2)*(d*x+c)^(1/2),x)
```

```
[Out] 1/8*(d*x-c)^(1/2)*(d*x+c)^(1/2)*(2*csgn(d)*x^3*b*d^3*(d^2*x^2-c^2)^(1/2)+4*
csgn(d)*d^3*(d^2*x^2-c^2)^(1/2)*x*a-csgn(d)*d*(d^2*x^2-c^2)^(1/2)*x*b*c^2-4
*ln((d*x+(d^2*x^2-c^2)^(1/2)*csgn(d))*csgn(d))*a*c^2*d^2-ln((d*x+(d^2*x^2-c
^2)^(1/2)*csgn(d))*csgn(d))*b*c^4)*csgn(d)/(d^2*x^2-c^2)^(1/2)/d^3
```

maxima [A]   time = 0.54, size = 137, normalized size = 1.20

$$-\frac{bc^4\log\left(2\,d^2x+2\,\sqrt{d^2x^2-c^2}\,d\right)}{8\,d^3}-\frac{ac^2\log\left(2\,d^2x+2\,\sqrt{d^2x^2-c^2}\,d\right)}{2\,d}+\frac{1}{2}\,\sqrt{d^2x^2-c^2}\,ax+\frac{\sqrt{d^2x^2-c^2}\,bc^2x}{8\,d^2}+\frac{\left(d^2x^2-c^2\right)^{\frac{3}{2}}bx}{4\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x-c)^(1/2)*(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/8*b*c^4*log(2*d^2*x + 2*sqrt(d^2*x^2 - c^2)*d)/d^3 - 1/2*a*c^2*log(2*d^2
*x + 2*sqrt(d^2*x^2 - c^2)*d)/d + 1/2*sqrt(d^2*x^2 - c^2)*a*x + 1/8*sqrt(d^
2*x^2 - c^2)*b*c^2*x/d^2 + 1/4*(d^2*x^2 - c^2)^(3/2)*b*x/d^2
```

mupad [B]   time = 17.43, size = 734, normalized size = 6.44

$$\frac{a\,x\,\sqrt{c+d\,x}\,\sqrt{d\,x-c}}{2}-\frac{\frac{b\,c^4\left(\sqrt{c+d\,x}-\sqrt{c}\right)}{2\left(\sqrt{-c}-\sqrt{d\,x-c}\right)}+\frac{35\,b\,c^4\left(\sqrt{c+d\,x}-\sqrt{c}\right)^3}{2\left(\sqrt{-c}-\sqrt{d\,x-c}\right)^3}+\frac{273\,b\,c^4\left(\sqrt{c+d\,x}-\sqrt{c}\right)^5}{2\left(\sqrt{-c}-\sqrt{d\,x-c}\right)^5}+\frac{715\,b\,c^4\left(\sqrt{c+d\,x}-\sqrt{c}\right)^7}{2\left(\sqrt{-c}-\sqrt{d\,x-c}\right)^7}+\frac{715\,b\,c^4\left(\sqrt{c+d\,x}-\sqrt{c}\right)^9}{2\left(\sqrt{-c}-\sqrt{d\,x-c}\right)^9}+\frac{273\,b\,c^4\left(\sqrt{c+d\,x}-\sqrt{c}\right)^{11}}{2\left(\sqrt{-c}-\sqrt{d\,x-c}\right)^{11}}+\frac{35\,b\,c^4\left(\sqrt{c+d\,x}-\sqrt{c}\right)^{13}}{2\left(\sqrt{-c}-\sqrt{d\,x-c}\right)^{13}}+\frac{b\,c^4\left(\sqrt{c+d\,x}-\sqrt{c}\right)^{15}}{2\left(\sqrt{-c}-\sqrt{d\,x-c}\right)^{15}}}{d^3-\frac{8\,d^3\left(\sqrt{c+d\,x}-\sqrt{c}\right)^2}{\left(\sqrt{-c}-\sqrt{d\,x-c}\right)^2}+\frac{28\,d^3\left(\sqrt{c+d\,x}-\sqrt{c}\right)^4}{\left(\sqrt{-c}-\sqrt{d\,x-c}\right)^4}-\frac{56\,d^3\left(\sqrt{c+d\,x}-\sqrt{c}\right)^6}{\left(\sqrt{-c}-\sqrt{d\,x-c}\right)^6}+\frac{70\,d^3\left(\sqrt{c+d\,x}-\sqrt{c}\right)^8}{\left(\sqrt{-c}-\sqrt{d\,x-c}\right)^8}-\frac{56\,d^3\left(\sqrt{c+d\,x}-\sqrt{c}\right)^{10}}{\left(\sqrt{-c}-\sqrt{d\,x-c}\right)^{10}}+\frac{28\,d^3\left(\sqrt{c+d\,x}-\sqrt{c}\right)^{12}}{\left(\sqrt{-c}-\sqrt{d\,x-c}\right)^{12}}-\frac{8\,d^3\left(\sqrt{c+d\,x}-\sqrt{c}\right)^{14}}{\left(\sqrt{-c}-\sqrt{d\,x-c}\right)^{14}}+\frac{d^3\left(\sqrt{c+d\,x}-\sqrt{c}\right)^{16}}{\left(\sqrt{-c}-\sqrt{d\,x-c}\right)^{16}}}-\frac{a\,c^2\,\ln\left(d\,x+\sqrt{c+d\,x}\,\sqrt{d\,x-c}\right)}{2\,d}+\frac{b\,c^4\,\mathrm{atanh}\left(\frac{\sqrt{c+d\,x}-\sqrt{c}}{\sqrt{-c}-\sqrt{d\,x-c}}\right)}{2\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)*(c + d*x)^(1/2)*(d*x - c)^(1/2),x)
```

```
[Out] (a*x*(c + d*x)^(1/2)*(d*x - c)^(1/2))/2 - ((b*c^4*((c + d*x)^(1/2) - c^(1/2
)))/(2*((-c)^(1/2) - (d*x - c)^(1/2))) + (35*b*c^4*((c + d*x)^(1/2) - c^(1/
2))^3)/(2*((-c)^(1/2) - (d*x - c)^(1/2))^3) + (273*b*c^4*((c + d*x)^(1/2) -
 c^(1/2))^5)/(2*((-c)^(1/2) - (d*x - c)^(1/2))^5) + (715*b*c^4*((c + d*x)^(
1/2) - c^(1/2))^7)/(2*((-c)^(1/2) - (d*x - c)^(1/2))^7) + (715*b*c^4*((c +
d*x)^(1/2) - c^(1/2))^9)/(2*((-c)^(1/2) - (d*x - c)^(1/2))^9) + (273*b*c^4*
((c + d*x)^(1/2) - c^(1/2))^11)/(2*((-c)^(1/2) - (d*x - c)^(1/2))^11) + (35
*b*c^4*((c + d*x)^(1/2) - c^(1/2))^13)/(2*((-c)^(1/2) - (d*x - c)^(1/2))^13
) + (b*c^4*((c + d*x)^(1/2) - c^(1/2))^15)/(2*((-c)^(1/2) - (d*x - c)^(1/2)
)^15))/(d^3 - (8*d^3*((c + d*x)^(1/2) - c^(1/2))^2)/((-c)^(1/2) - (d*x - c)
^(1/2))^2 + (28*d^3*((c + d*x)^(1/2) - c^(1/2))^4)/((-c)^(1/2) - (d*x - c)^
(1/2))^4 - (56*d^3*((c + d*x)^(1/2) - c^(1/2))^6)/((-c)^(1/2) - (d*x - c)^(
1/2))^6 + (70*d^3*((c + d*x)^(1/2) - c^(1/2))^8)/((-c)^(1/2) - (d*x - c)^(1
/2))^8 - (56*d^3*((c + d*x)^(1/2) - c^(1/2))^10)/((-c)^(1/2) - (d*x - c)^(1
/2))^10 + (28*d^3*((c + d*x)^(1/2) - c^(1/2))^12)/((-c)^(1/2) - (d*x - c)^(
```

```
1/2))^12 - (8*d^3*((c + d*x)^(1/2) - c^(1/2))^14)/((-c)^(1/2) - (d*x - c)^(
1/2))^14 + (d^3*((c + d*x)^(1/2) - c^(1/2))^16)/((-c)^(1/2) - (d*x - c)^(1/
2))^16) - (a*c^2*log(d*x + (c + d*x)^(1/2)*(d*x - c)^(1/2)))/(2*d) + (b*c^4
*atanh(((c + d*x)^(1/2) - c^(1/2))/((-c)^(1/2) - (d*x - c)^(1/2))))/(2*d^3)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+bx^2\right)\sqrt{-c+dx}\,\sqrt{c+dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)*(d*x-c)**(1/2)*(d*x+c)**(1/2),x)
```

```
[Out] Integral((a + b*x**2)*sqrt(-c + d*x)*sqrt(c + d*x), x)
```

3.203 $\int \frac{\sqrt{-c+dx}\,\sqrt{c+dx}\,(a+bx^2)}{x^2}\,dx$

Optimal. Leaf size=104

$$\frac{1}{2}x\sqrt{dx-c}\,\sqrt{c+dx}\left(b-\frac{2ad^2}{c^2}\right)-\frac{\left(bc^2-2ad^2\right)\tanh^{-1}\left(\frac{\sqrt{dx-c}}{\sqrt{c+dx}}\right)}{d}+\frac{a(dx-c)^{3/2}(c+dx)^{3/2}}{c^2x}$$

Rubi [A] time = 0.09, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {454, 38, 63, 217, 206}

$$\frac{1}{2}x\sqrt{dx-c}\,\sqrt{c+dx}\left(b-\frac{2ad^2}{c^2}\right)-\frac{\left(bc^2-2ad^2\right)\tanh^{-1}\left(\frac{\sqrt{dx-c}}{\sqrt{c+dx}}\right)}{d}+\frac{a(dx-c)^{3/2}(c+dx)^{3/2}}{c^2x}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[-c + d*x]*Sqrt[c + d*x]*(a + b*x^2))/x^2,x]
```

```
[Out] ((b - (2*a*d^2)/c^2)*x*Sqrt[-c + d*x]*Sqrt[c + d*x])/2 + (a*(-c + d*x)^(3/2
)*(c + d*x)^(3/2))/(c^2*x) - ((b*c^2 - 2*a*d^2)*ArcTanh[Sqrt[-c + d*x]/Sqrt
[c + d*x]])/d
```

Rule 38

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[(x
*(a + b*x)^m*(c + d*x)^m)/(2*m + 1), x] + Dist[(2*a*c*m)/(2*m + 1), Int[(a
+ b*x)^(m - 1)*(c + d*x)^(m - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
*c + a*d, 0] && IGtQ[m + 1/2, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 454

```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[(c*(e*x)^
(m + 1)*(a1 + b1*x^(n/2))^(p + 1)*(a2 + b2*x^(n/2))^(p + 1))/(a1*a2*e*(m +
1)), x] + Dist[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/(a1*a2*e^n*(
m + 1)), Int[(e*x)^(m + n)*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n/2))^p, x], x]
/; FreeQ[{a1, b1, a2, b2, c, d, e, p}, x] && EqQ[non2, n/2] && EqQ[a2*b1 +
a1*b2, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (L
```

```
tQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rubi steps

$$\int \frac{\sqrt{-c+dx}\,\sqrt{c+dx}\,(a+bx^2)}{x^2}\,dx = \frac{a(-c+dx)^{3/2}(c+dx)^{3/2}}{c^2x} + \left(b-\frac{2ad^2}{c^2}\right)\int \sqrt{-c+dx}\,\sqrt{c+dx}\,dx$$
$$= \frac{1}{2}\left(b-\frac{2ad^2}{c^2}\right)x\sqrt{-c+dx}\,\sqrt{c+dx} + \frac{a(-c+dx)^{3/2}(c+dx)^{3/2}}{c^2x} + \frac{1}{2}\left(-bc^2+2\right.$$
$$= \frac{1}{2}\left(b-\frac{2ad^2}{c^2}\right)x\sqrt{-c+dx}\,\sqrt{c+dx} + \frac{a(-c+dx)^{3/2}(c+dx)^{3/2}}{c^2x} + \frac{\left(-bc^2+2ac\right.}{}$$
$$= \frac{1}{2}\left(b-\frac{2ad^2}{c^2}\right)x\sqrt{-c+dx}\,\sqrt{c+dx} + \frac{a(-c+dx)^{3/2}(c+dx)^{3/2}}{c^2x} + \frac{\left(-bc^2+2ac\right.}{}$$
$$= \frac{1}{2}\left(b-\frac{2ad^2}{c^2}\right)x\sqrt{-c+dx}\,\sqrt{c+dx} + \frac{a(-c+dx)^{3/2}(c+dx)^{3/2}}{c^2x} - \frac{\left(bc^2-2ad^2\right.}{}$$

Mathematica [A] time = 0.09, size = 101, normalized size = 0.97

$$\frac{\sqrt{dx-c}\,\sqrt{c+dx}\left(cd\left(bx^2-2a\right)\sqrt{1-\frac{d^2x^2}{c^2}}+x\left(bc^2-2ad^2\right)\sin^{-1}\left(\frac{dx}{c}\right)\right)}{2cdx\sqrt{1-\frac{d^2x^2}{c^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[-c + d*x]*Sqrt[c + d*x]*(a + b*x^2))/x^2,x]
```

```
[Out] (Sqrt[-c + d*x]*Sqrt[c + d*x]*(c*d*(-2*a + b*x^2)*Sqrt[1 - (d^2*x^2)/c^2] +
 (b*c^2 - 2*a*d^2)*x*ArcSin[(d*x)/c]))/(2*c*d*x*Sqrt[1 - (d^2*x^2)/c^2])
```

IntegrateAlgebraic [A] time = 0.20, size = 200, normalized size = 1.92

$$\frac{\left(2ad^2-bc^2\right)\tanh^{-1}\left(\frac{\sqrt{dx-c}}{\sqrt{c+dx}}\right)}{d}-\frac{\sqrt{dx-c}\left(-\frac{4ad^2(dx-c)}{c+dx}+\frac{2ad^2(dx-c)^2}{(c+dx)^2}+2ad^2-\frac{2bc^2(dx-c)}{c+dx}-\frac{bc^2(dx-c)^2}{(c+dx)^2}-bc^2\right)}{d\sqrt{c+dx}\left(\frac{dx-c}{c+dx}-1\right)^2\left(\frac{dx-c}{c+dx}+1\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(Sqrt[-c + d*x]*Sqrt[c + d*x]*(a + b*x^2))/x^2,x]
```

```
[Out] -((Sqrt[-c + d*x]*(-(b*c^2) + 2*a*d^2 - (b*c^2*(-c + d*x)^2)/(c + d*x)^2 +
(2*a*d^2*(-c + d*x)^2)/(c + d*x)^2 - (2*b*c^2*(-c + d*x))/(c + d*x) - (4*a*
d^2*(-c + d*x))/(c + d*x)))/(d*Sqrt[c + d*x]*(-1 + (-c + d*x)/(c + d*x))^2*
(1 + (-c + d*x)/(c + d*x)))) + ((-(b*c^2) + 2*a*d^2)*ArcTanh[Sqrt[-c + d*x]
/Sqrt[c + d*x]])/d
```

fricas [A] time = 0.74, size = 83, normalized size = 0.80

$$-\frac{2\,ad^2x-\left(bc^2-2\,ad^2\right)x\log\left(-dx+\sqrt{dx+c}\,\sqrt{dx-c}\right)-\left(bdx^2-2\,ad\right)\sqrt{dx+c}\,\sqrt{dx-c}}{2\,dx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x-c)^(1/2)*(d*x+c)^(1/2)/x^2,x, algorithm="fricas")
```

```
[Out] -1/2*(2*a*d^2*x - (b*c^2 - 2*a*d^2)*x*log(-d*x + sqrt(d*x + c)*sqrt(d*x - c
)) - (b*d*x^2 - 2*a*d)*sqrt(d*x + c)*sqrt(d*x - c))/(d*x)
```

giac [A] time = 0.40, size = 110, normalized size = 1.06

$$-\frac{\frac{32\,ac^2d^2}{\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^4+4\,c^2}-2\left((dx+c)b-bc\right)\sqrt{dx+c}\,\sqrt{dx-c}-\left(bc^2-2\,ad^2\right)\log\left(\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^4\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x-c)^(1/2)*(d*x+c)^(1/2)/x^2,x, algorithm="giac")
```

```
[Out] -1/4*(32*a*c^2*d^2/((sqrt(d*x + c) - sqrt(d*x - c))^4 + 4*c^2) - 2*((d*x +
c)*b - b*c)*sqrt(d*x + c)*sqrt(d*x - c) - (b*c^2 - 2*a*d^2)*log((sqrt(d*x +
 c) - sqrt(d*x - c))^4))/d
```

maple [C] time = 0.06, size = 153, normalized size = 1.47

$$\frac{\sqrt{dx-c}\,\sqrt{dx+c}\,\left(2a\,d^2x\ln\left(\left(dx+\sqrt{d^2x^2-c^2}\,\mathrm{csgn}(d)\right)\mathrm{csgn}(d)\right)-b\,c^2x\ln\left(\left(dx+\sqrt{d^2x^2-c^2}\,\mathrm{csgn}(d)\right)\mathrm{csgn}(d)\right)+\sqrt{d^2x^2-c^2}\,bd\,x^2\mathrm{csgn}(d)-2\sqrt{d^2x^2-c^2}\,ad\,\mathrm{csgn}(d)\right)\mathrm{csgn}(d)}{2\sqrt{d^2x^2-c^2}\,dx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)*(d*x-c)^(1/2)*(d*x+c)^(1/2)/x^2,x)
```

```
[Out] 1/2*(d*x-c)^(1/2)*(d*x+c)^(1/2)*(csgn(d)*x^2*b*d*(d^2*x^2-c^2)^(1/2)+2*ln((
d*x+(d^2*x^2-c^2)^(1/2)*csgn(d))*csgn(d))*x*a*d^2-ln((d*x+(d^2*x^2-c^2)^(1/
2)*csgn(d))*csgn(d))*x*b*c^2-2*csgn(d)*d*(d^2*x^2-c^2)^(1/2)*a)*csgn(d)/(d^
2*x^2-c^2)^(1/2)/x/d
```

maxima [A] time = 1.50, size = 105, normalized size = 1.01

$$-\frac{bc^2\log\left(2\,d^2x+2\,\sqrt{d^2x^2-c^2}\,d\right)}{2\,d}+ad\log\left(2\,d^2x+2\,\sqrt{d^2x^2-c^2}\,d\right)+\frac{1}{2}\,\sqrt{d^2x^2-c^2}\,bx-\frac{\sqrt{d^2x^2-c^2}\,a}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x-c)^(1/2)*(d*x+c)^(1/2)/x^2,x, algorithm="maxima")
```

```
[Out] -1/2*b*c^2*log(2*d^2*x + 2*sqrt(d^2*x^2 - c^2)*d)/d + a*d*log(2*d^2*x + 2*s
qrt(d^2*x^2 - c^2)*d) + 1/2*sqrt(d^2*x^2 - c^2)*b*x - sqrt(d^2*x^2 - c^2)*a
/x
```

mupad [B] time = 3.49, size = 243, normalized size = 2.34

$$\frac{a\,d+\frac{5\,a\,d\left(\sqrt{c+d\,x}-\sqrt{c}\right)^2}{\left(\sqrt{-c}-\sqrt{d\,x-c}\right)^2}}{\frac{4\left(\sqrt{c+d\,x}-\sqrt{c}\right)}{\sqrt{-c}-\sqrt{d\,x-c}}+\frac{4\left(\sqrt{c+d\,x}-\sqrt{c}\right)^3}{\left(\sqrt{-c}-\sqrt{d\,x-c}\right)^3}}-4\,a\,d\,\mathrm{atanh}\left(\frac{\sqrt{c+d\,x}-\sqrt{c}}{\sqrt{-c}-\sqrt{d\,x-c}}\right)+\frac{b\,x\,\sqrt{c+d\,x}\,\sqrt{d\,x-c}}{2}-\frac{b\,c^2\,\ln\left(d\,x+\sqrt{c+d\,x}\,\sqrt{d\,x-c}\right)}{2\,d}+\frac{a\,d\,\left(\sqrt{c+d\,x}-\sqrt{c}\right)}{4\,\left(\sqrt{-c}-\sqrt{d\,x-c}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^2)*(c + d*x)^(1/2)*(d*x - c)^(1/2))/x^2,x)
```

```
[Out] (a*d + (5*a*d*((c + d*x)^(1/2) - c^(1/2))^2)/((-c)^(1/2) - (d*x - c)^(1/2))
^2)/((4*((c + d*x)^(1/2) - c^(1/2)))/((-c)^(1/2) - (d*x - c)^(1/2)) + (4*((
c + d*x)^(1/2) - c^(1/2))^3)/((-c)^(1/2) - (d*x - c)^(1/2))^3) - 4*a*d*atan
h(((c + d*x)^(1/2) - c^(1/2))/((-c)^(1/2) - (d*x - c)^(1/2))) + (b*x*(c + d
*x)^(1/2)*(d*x - c)^(1/2))/2 - (b*c^2*log(d*x + (c + d*x)^(1/2)*(d*x - c)^(
1/2)))/(2*d) + (a*d*((c + d*x)^(1/2) - c^(1/2)))/(4*((-c)^(1/2) - (d*x - c)
^(1/2)))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+bx^2\right)\sqrt{-c+dx}\,\sqrt{c+dx}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)*(d*x-c)**(1/2)*(d*x+c)**(1/2)/x**2,x)
```

```
[Out] Integral((a + b*x**2)*sqrt(-c + d*x)*sqrt(c + d*x)/x**2, x)
```

3.204 $\int \frac{\sqrt{-c+dx}\sqrt{c+dx}\left(a+bx^2\right)}{x^4}\,dx$

Optimal. Leaf size=84

$$\frac{a(dx-c)^{3/2}(c+dx)^{3/2}}{3c^2x^3}-\frac{b\sqrt{dx-c}\sqrt{c+dx}}{x}+2bd\tanh^{-1}\left(\frac{\sqrt{dx-c}}{\sqrt{c+dx}}\right)$$

Rubi [A] time = 0.08, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {454, 97, 12, 63, 217, 206}

$$\frac{a(dx-c)^{3/2}(c+dx)^{3/2}}{3c^2x^3}-\frac{b\sqrt{dx-c}\sqrt{c+dx}}{x}+2bd\tanh^{-1}\left(\frac{\sqrt{dx-c}}{\sqrt{c+dx}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[-c + d*x]*Sqrt[c + d*x]*(a + b*x^2))/x^4,x]
```

```
[Out] -((b*Sqrt[-c + d*x]*Sqrt[c + d*x])/x) + (a*(-c + d*x)^(3/2)*(c + d*x)^(3/2)
)/(3*c^2*x^3) + 2*b*d*ArcTanh[Sqrt[-c + d*x]/Sqrt[c + d*x]]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 97

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p)/(b*
(m + 1)), x] - Dist[1/(b*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*
(e + f*x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ
[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 454

```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[(c*(e*x)^
(m + 1)*(a1 + b1*x^(n/2))^(p + 1)*(a2 + b2*x^(n/2))^(p + 1))/(a1*a2*e*(m +
```

```
1)), x] + Dist[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/(a1*a2*e^n*(
m + 1)), Int[(e*x)^(m + n)*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n/2))^p, x], x]
/; FreeQ[{a1, b1, a2, b2, c, d, e, p}, x] && EqQ[non2, n/2] && EqQ[a2*b1 +
a1*b2, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (L
tQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{-c+dx}\,\sqrt{c+dx}\,\left(a+bx^2\right)}{x^4}\,dx &= \frac{a(-c+dx)^{3/2}(c+dx)^{3/2}}{3c^2x^3}+b\int \frac{\sqrt{-c+dx}\,\sqrt{c+dx}}{x^2}\,dx\\
&= -\frac{b\sqrt{-c+dx}\,\sqrt{c+dx}}{x}+\frac{a(-c+dx)^{3/2}(c+dx)^{3/2}}{3c^2x^3}+b\int \frac{d^2}{\sqrt{-c+dx}\,\sqrt{c+dx}}\\
&= -\frac{b\sqrt{-c+dx}\,\sqrt{c+dx}}{x}+\frac{a(-c+dx)^{3/2}(c+dx)^{3/2}}{3c^2x^3}+\left(bd^2\right)\int \frac{1}{\sqrt{-c+dx}\,\sqrt{c+}}\\
&= -\frac{b\sqrt{-c+dx}\,\sqrt{c+dx}}{x}+\frac{a(-c+dx)^{3/2}(c+dx)^{3/2}}{3c^2x^3}+(2bd)\,\text{Subst}\left(\int \frac{1}{\sqrt{2c+}}\right.\\
&= -\frac{b\sqrt{-c+dx}\,\sqrt{c+dx}}{x}+\frac{a(-c+dx)^{3/2}(c+dx)^{3/2}}{3c^2x^3}+(2bd)\,\text{Subst}\left(\int \frac{1}{1-x^2}\right.\\
&= -\frac{b\sqrt{-c+dx}\,\sqrt{c+dx}}{x}+\frac{a(-c+dx)^{3/2}(c+dx)^{3/2}}{3c^2x^3}+2bd\tanh^{-1}\left(\frac{\sqrt{-c+dx}}{\sqrt{c+dx}}\right)
\end{aligned}
$$

Mathematica [A] time = 0.09, size = 105, normalized size = 1.25

$$
-\frac{\sqrt{dx-c}\,\sqrt{c+dx}\left(\sqrt{1-\frac{d^2x^2}{c^2}}\left(a\left(c^2-d^2x^2\right)+3bc^2x^2\right)+3bcdx^3\sin^{-1}\left(\frac{dx}{c}\right)\right)}{3c^2x^3\sqrt{1-\frac{d^2x^2}{c^2}}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[-c + d*x]*Sqrt[c + d*x]*(a + b*x^2))/x^4,x]
```

```
[Out] -1/3*(Sqrt[-c + d*x]*Sqrt[c + d*x]*(Sqrt[1 - (d^2*x^2)/c^2]*(3*b*c^2*x^2 +
a*(c^2 - d^2*x^2)) + 3*b*c*d*x^3*ArcSin[(d*x)/c]))/(c^2*x^3*Sqrt[1 - (d^2*x
^2)/c^2])
```

IntegrateAlgebraic [A] time = 0.17, size = 145, normalized size = 1.73

$$
2bd\tanh^{-1}\left(\frac{\sqrt{dx-c}}{\sqrt{c+dx}}\right)-\frac{2\sqrt{dx-c}\left(-\frac{4ad^3(dx-c)}{c+dx}+\frac{6bc^2d(dx-c)}{c+dx}+\frac{3bc^2d(dx-c)^2}{(c+dx)^2}+3bc^2d\right)}{3c^2\sqrt{c+dx}\left(\frac{dx-c}{c+dx}+1\right)^3}
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(Sqrt[-c + d*x]*Sqrt[c + d*x]*(a + b*x^2))/x^4,x]
```

```
[Out] (-2*Sqrt[-c + d*x]*(3*b*c^2*d + (3*b*c^2*d*(-c + d*x)^2)/(c + d*x)^2 + (6*b
*c^2*d*(-c + d*x))/(c + d*x) - (4*a*d^3*(-c + d*x))/(c + d*x)))/(3*c^2*Sqrt
[c + d*x]*(1 + (-c + d*x)/(c + d*x))^3) + 2*b*d*ArcTanh[Sqrt[-c + d*x]/Sqrt
[c + d*x]]
```

fricas [A] time = 0.79, size = 100, normalized size = 1.19

$$
-\frac{3\,bc^2dx^3\log\left(-dx+\sqrt{dx+c}\,\sqrt{dx-c}\right)+\left(3\,bc^2d-ad^3\right)x^3+\left(ac^2+\left(3\,bc^2-ad^2\right)x^2\right)\sqrt{dx+c}\,\sqrt{dx-c}}{3\,c^2x^3}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x-c)^(1/2)*(d*x+c)^(1/2)/x^4,x, algorithm="fricas")
```

```
[Out] -1/3*(3*b*c^2*d*x^3*log(-d*x + sqrt(d*x + c)*sqrt(d*x - c)) + (3*b*c^2*d -
a*d^3)*x^3 + (a*c^2 + (3*b*c^2 - a*d^2)*x^2)*sqrt(d*x + c)*sqrt(d*x - c))/(
c^2*x^3)
```

giac [B] time = 0.38, size = 171, normalized size = 2.04

$$-\frac{3\,bd^{2}\log\left(\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^{4}\right)+\frac{16\left(3\,bc^{2}d^{2}\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^{8}-3\,ad^{4}\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^{8}+24\,bc^{4}d^{2}\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^{4}+48\,bc^{6}d^{2}-16\,ac^{4}d^{4}\right)}{\left(\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^{4}+4\,c^{2}\right)^{3}}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x-c)^(1/2)*(d*x+c)^(1/2)/x^4,x, algorithm="giac")
```

```
[Out] -1/6*(3*b*d^2*log((sqrt(d*x + c) - sqrt(d*x - c))^4) + 16*(3*b*c^2*d^2*(sqr
t(d*x + c) - sqrt(d*x - c))^8 - 3*a*d^4*(sqrt(d*x + c) - sqrt(d*x - c))^8 +
 24*b*c^4*d^2*(sqrt(d*x + c) - sqrt(d*x - c))^4 + 48*b*c^6*d^2 - 16*a*c^4*d
^4)/((sqrt(d*x + c) - sqrt(d*x - c))^4 + 4*c^2)^3)/d
```

maple [C] time = 0.07, size = 153, normalized size = 1.82

$$\frac{\sqrt{dx-c}\,\sqrt{dx+c}\,\left(3b\,c^{2}d\,x^{3}\ln\left(\left(dx+\sqrt{d^{2}x^{2}-c^{2}}\,\mathrm{csgn}(d)\right)\mathrm{csgn}(d)\right)+\sqrt{d^{2}x^{2}-c^{2}}\,a\,d^{2}x^{2}\mathrm{csgn}(d)-3\sqrt{d^{2}x^{2}-c^{2}}\,b\,c^{2}x^{2}\mathrm{csgn}(d)-\sqrt{d^{2}x^{2}-c^{2}}\,a\,c^{2}\mathrm{csgn}(d)\right)\mathrm{csgn}(d)}{3\sqrt{d^{2}x^{2}-c^{2}}\,c^{2}x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)*(d*x-c)^(1/2)*(d*x+c)^(1/2)/x^4,x)
```

```
[Out] 1/3*(d*x-c)^(1/2)*(d*x+c)^(1/2)*(3*ln((d*x+(d^2*x^2-c^2)^(1/2)*csgn(d))*csg
n(d))*x^3*b*c^2*d+csgn(d)*x^2*a*d^2*(d^2*x^2-c^2)^(1/2)-3*csgn(d)*x^2*b*c^2
*(d^2*x^2-c^2)^(1/2)-csgn(d)*a*c^2*(d^2*x^2-c^2)^(1/2))*csgn(d)/(d^2*x^2-c^
2)^(1/2)/c^2/x^3
```

maxima [A] time = 1.47, size = 75, normalized size = 0.89

$$bd\log\left(2\,d^{2}x+2\,\sqrt{d^{2}x^{2}-c^{2}}\,d\right)-\frac{\sqrt{d^{2}x^{2}-c^{2}}\,b}{x}+\frac{\left(d^{2}x^{2}-c^{2}\right)^{\frac{3}{2}}a}{3\,c^{2}x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x-c)^(1/2)*(d*x+c)^(1/2)/x^4,x, algorithm="maxima")
```

```
[Out] b*d*log(2*d^2*x + 2*sqrt(d^2*x^2 - c^2)*d) - sqrt(d^2*x^2 - c^2)*b/x + 1/3*
(d^2*x^2 - c^2)^(3/2)*a/(c^2*x^3)
```

mupad [B] time = 3.44, size = 236, normalized size = 2.81

$$\frac{b\,d+\frac{5\,b\,d\,\left(\sqrt{c+d\,x}-\sqrt{c}\right)^{2}}{\left(\sqrt{-c}-\sqrt{d\,x-c}\right)^{2}}}{\frac{4\,\left(\sqrt{c+d\,x}-\sqrt{c}\right)}{\sqrt{-c}-\sqrt{d\,x-c}}+\frac{4\,\left(\sqrt{c+d\,x}-\sqrt{c}\right)^{3}}{\left(\sqrt{-c}-\sqrt{d\,x-c}\right)^{3}}}-4\,b\,d\,\mathrm{atanh}\left(\frac{\sqrt{c+d\,x}-\sqrt{c}}{\sqrt{-c}-\sqrt{d\,x-c}}\right)-\frac{\left(\frac{a\,\sqrt{c+d\,x}}{3}-\frac{a\,d^{2}\,x^{2}\,\sqrt{c+d\,x}}{3\,c^{2}}\right)\,\sqrt{d\,x-c}}{x^{3}}+\frac{b\,d\,\left(\sqrt{c+d\,x}-\sqrt{c}\right)}{4\,\left(\sqrt{-c}-\sqrt{d\,x-c}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^2)*(c + d*x)^(1/2)*(d*x - c)^(1/2))/x^4,x)
```

```
[Out] (b*d + (5*b*d*((c + d*x)^(1/2) - c^(1/2))^2)/((-c)^(1/2) - (d*x - c)^(1/2))
^2)/((4*((c + d*x)^(1/2) - c^(1/2)))/((-c)^(1/2) - (d*x - c)^(1/2)) + (4*((
c + d*x)^(1/2) - c^(1/2))^3)/((-c)^(1/2) - (d*x - c)^(1/2))^3) - 4*b*d*atan
```

```
h(((c + d*x)^(1/2) - c^(1/2))/((-c)^(1/2) - (d*x - c)^(1/2))) - (((a*(c + d
*x)^(1/2))/3 - (a*d^2*x^2*(c + d*x)^(1/2))/(3*c^2))*(d*x - c)^(1/2))/x^3 +
(b*d*((c + d*x)^(1/2) - c^(1/2)))/(4*((-c)^(1/2) - (d*x - c)^(1/2)))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)*(d*x-c)**(1/2)*(d*x+c)**(1/2)/x**4,x)
```

```
[Out] Timed out
```

3.205 $\int \frac{x^4\left(a+bx^2\right)}{\sqrt{-1+cx}\,\sqrt{1+cx}}\,dx$

Optimal. Leaf size=125

$$\frac{\left(6ac^2+5b\right)\cosh^{-1}(cx)}{16c^7}+\frac{x\sqrt{cx-1}\,\sqrt{cx+1}\,\left(6ac^2+5b\right)}{16c^6}+\frac{x^3\sqrt{cx-1}\,\sqrt{cx+1}\,\left(6ac^2+5b\right)}{24c^4}+\frac{bx^5\sqrt{cx-1}\,\sqrt{cx+1}}{6c^2}$$

Rubi [A] time = 0.08, antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {460, 100, 12, 90, 52}

$$\frac{x^3\sqrt{cx-1}\,\sqrt{cx+1}\,\left(6ac^2+5b\right)}{24c^4}+\frac{x\sqrt{cx-1}\,\sqrt{cx+1}\,\left(6ac^2+5b\right)}{16c^6}+\frac{\left(6ac^2+5b\right)\cosh^{-1}(cx)}{16c^7}+\frac{bx^5\sqrt{cx-1}\,\sqrt{cx+1}}{6c^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(a + b*x^2))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]),x]
```

```
[Out] ((5*b + 6*a*c^2)*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(16*c^6) + ((5*b + 6*a*c^2
)*x^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(24*c^4) + (b*x^5*Sqrt[-1 + c*x]*Sqrt[1
 + c*x])/(6*c^2) + ((5*b + 6*a*c^2)*ArcCosh[c*x])/(16*c^7)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 52

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[(b*x)/a]/b, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a + c, 0] && EqQ[b
- d, 0] && GtQ[a, 0]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*(a + b*x)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/
(d*f*(n + p + 3)), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/(d*f*(m + n + p + 1)), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 460

```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[(d*(e*x)^
(m + 1)*(a1 + b1*x^(n/2))^(p + 1)*(a2 + b2*x^(n/2))^(p + 1))/(b1*b2*e*(m +
n*(p + 1) + 1)), x] - Dist[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/
(b1*b2*(m + n*(p + 1) + 1)), Int[(e*x)^m*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n/
2))^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, m, n, p}, x] && EqQ[non2,
```

n/2] && EqQ[a2*b1 + a1*b2, 0] && NeQ[m + n*(p + 1) + 1, 0]

Rubi steps

$$\begin{aligned}\int \frac{x^4\left(a+bx^2\right)}{\sqrt{-1+cx}\,\sqrt{1+cx}}\,dx &= \frac{bx^5\sqrt{-1+cx}\,\sqrt{1+cx}}{6c^2}-\frac{1}{6}\left(-6a-\frac{5b}{c^2}\right)\int \frac{x^4}{\sqrt{-1+cx}\,\sqrt{1+cx}}\,dx\\ &= \frac{\left(5b+6ac^2\right)x^3\sqrt{-1+cx}\,\sqrt{1+cx}}{24c^4}+\frac{bx^5\sqrt{-1+cx}\,\sqrt{1+cx}}{6c^2}+\frac{\left(5b+6ac^2\right)\int \frac{3x^2}{\sqrt{-1+cx}\,\sqrt{1}}}{24c^4}\\ &= \frac{\left(5b+6ac^2\right)x^3\sqrt{-1+cx}\,\sqrt{1+cx}}{24c^4}+\frac{bx^5\sqrt{-1+cx}\,\sqrt{1+cx}}{6c^2}+\frac{\left(5b+6ac^2\right)\int \frac{x^2}{\sqrt{-1+cx}\,\sqrt{1}}}{8c^4}\\ &= \frac{\left(5b+6ac^2\right)x\sqrt{-1+cx}\,\sqrt{1+cx}}{16c^6}+\frac{\left(5b+6ac^2\right)x^3\sqrt{-1+cx}\,\sqrt{1+cx}}{24c^4}+\frac{bx^5\sqrt{-1+cx}}{6c^2}\\ &= \frac{\left(5b+6ac^2\right)x\sqrt{-1+cx}\,\sqrt{1+cx}}{16c^6}+\frac{\left(5b+6ac^2\right)x^3\sqrt{-1+cx}\,\sqrt{1+cx}}{24c^4}+\frac{bx^5\sqrt{-1+cx}}{6c^2}\end{aligned}$$

Mathematica [A] time = 0.14, size = 117, normalized size = 0.94

$$\frac{3\sqrt{c^2x^2-1}\left(6ac^2+5b\right)\tanh^{-1}\left(\frac{cx}{\sqrt{c^2x^2-1}}\right)+cx\left(c^2x^2-1\right)\left(6ac^2\left(2c^2x^2+3\right)+b\left(8c^4x^4+10c^2x^2+15\right)\right)}{48c^7\sqrt{cx-1}\,\sqrt{cx+1}}$$

Antiderivative was successfully verified.

[In] Integrate[(x^4*(a + b*x^2))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]),x]

[Out] (c*x*(-1 + c^2*x^2)*(6*a*c^2*(3 + 2*c^2*x^2) + b*(15 + 10*c^2*x^2 + 8*c^4*x^4)) + 3*(5*b + 6*a*c^2)*Sqrt[-1 + c^2*x^2]*ArcTanh[(c*x)/Sqrt[-1 + c^2*x^2]])/(48*c^7*Sqrt[-1 + c*x]*Sqrt[1 + c*x])

IntegrateAlgebraic [B] time = 0.21, size = 255, normalized size = 2.04

$$\frac{\left(\frac{(cx-1)^{3/2}}{(cx+1)^{3/2}}+\frac{\sqrt{cx-1}}{\sqrt{cx+1}}\right)\left(\frac{30ac^2(cx-1)^4}{(cx+1)^4}-\frac{72ac^2(cx-1)^3}{(cx+1)^3}+\frac{84ac^2(cx-1)^2}{(cx+1)^2}-\frac{72ac^2(cx-1)}{cx+1}+30ac^2+\frac{33b(cx-1)^4}{(cx+1)^4}-\frac{28b(cx-1)^3}{(cx+1)^3}+\frac{118b(cx-1)^2}{(cx+1)^2}-\frac{28b(cx-1)}{cx+1}+33b\right)}{24c^7\left(\frac{cx-1}{cx+1}-1\right)^6}+\frac{\left(6ac^2+5b\right)\tanh^{-1}\left(\frac{\sqrt{cx-1}}{\sqrt{cx+1}}\right)}{8c^7}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(x^4*(a + b*x^2))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]),x]

[Out] ((33*b + 30*a*c^2 + (33*b*(-1 + c*x)^4)/(1 + c*x)^4 + (30*a*c^2*(-1 + c*x)^4)/(1 + c*x)^4 - (28*b*(-1 + c*x)^3)/(1 + c*x)^3 - (72*a*c^2*(-1 + c*x)^3)/(1 + c*x)^3 + (118*b*(-1 + c*x)^2)/(1 + c*x)^2 + (84*a*c^2*(-1 + c*x)^2)/(1 + c*x)^2 - (28*b*(-1 + c*x))/(1 + c*x) - (72*a*c^2*(-1 + c*x))/(1 + c*x))*((-1 + c*x)^(3/2)/(1 + c*x)^(3/2) + Sqrt[-1 + c*x]/Sqrt[1 + c*x]))/(24*c^7*(-1 + (-1 + c*x)/(1 + c*x))^6) + ((5*b + 6*a*c^2)*ArcTanh[Sqrt[-1 + c*x]/Sqrt[1 + c*x]])/(8*c^7)

fricas [A] time = 0.88, size = 96, normalized size = 0.77

$$\frac{\left(8\,bc^5x^5+2\left(6\,ac^5+5\,bc^3\right)x^3+3\left(6\,ac^3+5\,bc\right)x\right)\sqrt{cx+1}\,\sqrt{cx-1}-3\left(6\,ac^2+5\,b\right)\log\left(-cx+\sqrt{cx+1}\,\sqrt{cx-1}\right)}{48\,c^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4*(b*x^2+a)/(c*x-1)^(1/2)/(c*x+1)^(1/2),x, algorithm="fricas")

```
[Out] 1/48*((8*b*c^5*x^5 + 2*(6*a*c^5 + 5*b*c^3)*x^3 + 3*(6*a*c^3 + 5*b*c)*x)*sqr
t(c*x + 1)*sqrt(c*x - 1) - 3*(6*a*c^2 + 5*b)*log(-c*x + sqrt(c*x + 1)*sqrt(
c*x - 1)))/c^7
```

giac [A] time = 0.23, size = 172, normalized size = 1.38

$$\frac{\left(\left(2\left((cx+1)\left(4(cx+1)\left(\frac{(cx+1)b}{c^6}-\frac{5b}{c^6}\right)+\frac{3\left(2ac^{38}+15bc^{36}\right)}{c^{42}}\right)-\frac{18ac^{38}+55bc^{36}}{c^{42}}\right)(cx+1)+\frac{54ac^{38}+85bc^{36}}{c^{42}}\right)(cx+1)-\frac{3\left(10ac^{38}+11bc^{36}\right)}{c^{42}}\right)\sqrt{cx+1}\sqrt{cx-1}-\frac{6\left(6ac^2+5b\right)\log\left(\sqrt{cx+1}-\sqrt{cx-1}\right)}{c^6}}{48c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)/(c*x-1)^(1/2)/(c*x+1)^(1/2),x, algorithm="giac")
```

```
[Out] 1/48*(((2*((c*x + 1)*(4*(c*x + 1)*((c*x + 1)*b/c^6 - 5*b/c^6) + 3*(2*a*c^38
 + 15*b*c^36)/c^42) - (18*a*c^38 + 55*b*c^36)/c^42)*(c*x + 1) + (54*a*c^38
+ 85*b*c^36)/c^42)*(c*x + 1) - 3*(10*a*c^38 + 11*b*c^36)/c^42)*sqrt(c*x + 1
)*sqrt(c*x - 1) - 6*(6*a*c^2 + 5*b)*log(sqrt(c*x + 1) - sqrt(c*x - 1))/c^6)
/c
```

maple [C] time = 0.11, size = 191, normalized size = 1.53

$$\frac{\sqrt{cx-1}\sqrt{cx+1}\left(8\sqrt{c^2x^2-1}\,bc^5x^5\mathrm{csgn}(c)+12\sqrt{c^2x^2-1}\,ac^5x^3\mathrm{csgn}(c)+10\sqrt{c^2x^2-1}\,bc^3x^3\mathrm{csgn}(c)+18\sqrt{c^2x^2-1}\,ac^3x\,\mathrm{csgn}(c)+18ac^2\ln\left(\left(cx+\sqrt{c^2x^2-1}\,\mathrm{csgn}(c)\right)\mathrm{csgn}(c)\right)+15\sqrt{c^2x^2-1}\,bcx\,\mathrm{csgn}(c)+15b\ln\left(\left(cx+\sqrt{c^2x^2-1}\,\mathrm{csgn}(c)\right)\mathrm{csgn}(c)\right)\right)\mathrm{csgn}(c)}{48\sqrt{c^2x^2-1}\,c^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(b*x^2+a)/(c*x-1)^(1/2)/(c*x+1)^(1/2),x)
```

```
[Out] 1/48*(c*x-1)^(1/2)*(c*x+1)^(1/2)*(8*csgn(c)*x^5*b*c^5*(c^2*x^2-1)^(1/2)+12*
csgn(c)*x^3*a*c^5*(c^2*x^2-1)^(1/2)+10*(c^2*x^2-1)^(1/2)*csgn(c)*c^3*x^3*b+
18*(c^2*x^2-1)^(1/2)*csgn(c)*c^3*x*a+15*(c^2*x^2-1)^(1/2)*csgn(c)*c*x*b+18*
ln(((c^2*x^2-1)^(1/2)*csgn(c)+c*x)*csgn(c))*a*c^2+15*ln(((c^2*x^2-1)^(1/2)*
csgn(c)+c*x)*csgn(c))*b)*csgn(c)/c^7/(c^2*x^2-1)^(1/2)
```

maxima [A] time = 0.55, size = 153, normalized size = 1.22

$$\frac{\sqrt{c^2x^2-1}\,bx^5}{6c^2}+\frac{\sqrt{c^2x^2-1}\,ax^3}{4c^2}+\frac{5\sqrt{c^2x^2-1}\,bx^3}{24c^4}+\frac{3\sqrt{c^2x^2-1}\,ax}{8c^4}+\frac{3a\log\left(2c^2x+2\sqrt{c^2x^2-1}\,c\right)}{8c^5}+\frac{5\sqrt{c^2x^2-1}\,bx}{16c^6}+\frac{5b\log\left(2c^2x+2\sqrt{c^2x^2-1}\,c\right)}{16c^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)/(c*x-1)^(1/2)/(c*x+1)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/6*sqrt(c^2*x^2 - 1)*b*x^5/c^2 + 1/4*sqrt(c^2*x^2 - 1)*a*x^3/c^2 + 5/24*sq
rt(c^2*x^2 - 1)*b*x^3/c^4 + 3/8*sqrt(c^2*x^2 - 1)*a*x/c^4 + 3/8*a*log(2*c^2
*x + 2*sqrt(c^2*x^2 - 1)*c)/c^5 + 5/16*sqrt(c^2*x^2 - 1)*b*x/c^6 + 5/16*b*l
og(2*c^2*x + 2*sqrt(c^2*x^2 - 1)*c)/c^7
```

mupad [B] time = 32.63, size = 1154, normalized size = 9.23

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(a + b*x^2))/((c*x - 1)^(1/2)*(c*x + 1)^(1/2)),x)
```

```
[Out] ((23*a*((c*x - 1)^(1/2) - 1i)^3)/(2*((c*x + 1)^(1/2) - 1)^3) + (333*a*((c*x
 - 1)^(1/2) - 1i)^5)/(2*((c*x + 1)^(1/2) - 1)^5) + (671*a*((c*x - 1)^(1/2)
- 1i)^7)/(2*((c*x + 1)^(1/2) - 1)^7) + (671*a*((c*x - 1)^(1/2) - 1i)^9)/(2*
((c*x + 1)^(1/2) - 1)^9) + (333*a*((c*x - 1)^(1/2) - 1i)^11)/(2*((c*x + 1)^
(1/2) - 1)^11) + (23*a*((c*x - 1)^(1/2) - 1i)^13)/(2*((c*x + 1)^(1/2) - 1)^
13) - (3*a*((c*x - 1)^(1/2) - 1i)^15)/(2*((c*x + 1)^(1/2) - 1)^15) - (3*a*(
(c*x - 1)^(1/2) - 1i))/(2*((c*x + 1)^(1/2) - 1)))/(c^5 - (8*c^5*((c*x - 1)^
(1/2) - 1i)^2)/((c*x + 1)^(1/2) - 1)^2 + (28*c^5*((c*x - 1)^(1/2) - 1i)^4)/
((c*x + 1)^(1/2) - 1)^4 - (56*c^5*((c*x - 1)^(1/2) - 1i)^6)/((c*x + 1)^(1/2
) - 1)^6 + (70*c^5*((c*x - 1)^(1/2) - 1i)^8)/((c*x + 1)^(1/2) - 1)^8 - (56*
```

```
c^5*((c*x - 1)^(1/2) - 1i)^10)/((c*x + 1)^(1/2) - 1)^10 + (28*c^5*((c*x - 1
)^(1/2) - 1i)^12)/((c*x + 1)^(1/2) - 1)^12 - (8*c^5*((c*x - 1)^(1/2) - 1i)^
14)/((c*x + 1)^(1/2) - 1)^14 + (c^5*((c*x - 1)^(1/2) - 1i)^16)/((c*x + 1)^(
1/2) - 1)^16) - ((311*b*((c*x - 1)^(1/2) - 1i)^5)/(4*((c*x + 1)^(1/2) - 1)^
5) - (175*b*((c*x - 1)^(1/2) - 1i)^3)/(12*((c*x + 1)^(1/2) - 1)^3) + (8361*
b*((c*x - 1)^(1/2) - 1i)^7)/(4*((c*x + 1)^(1/2) - 1)^7) + (42259*b*((c*x -
1)^(1/2) - 1i)^9)/(6*((c*x + 1)^(1/2) - 1)^9) + (25295*b*((c*x - 1)^(1/2) -
 1i)^11)/(2*((c*x + 1)^(1/2) - 1)^11) + (25295*b*((c*x - 1)^(1/2) - 1i)^13)
/(2*((c*x + 1)^(1/2) - 1)^13) + (42259*b*((c*x - 1)^(1/2) - 1i)^15)/(6*((c*
x + 1)^(1/2) - 1)^15) + (8361*b*((c*x - 1)^(1/2) - 1i)^17)/(4*((c*x + 1)^(1
/2) - 1)^17) + (311*b*((c*x - 1)^(1/2) - 1i)^19)/(4*((c*x + 1)^(1/2) - 1)^1
9) - (175*b*((c*x - 1)^(1/2) - 1i)^21)/(12*((c*x + 1)^(1/2) - 1)^21) + (5*b
*((c*x - 1)^(1/2) - 1i)^23)/(4*((c*x + 1)^(1/2) - 1)^23) + (5*b*((c*x - 1)^
(1/2) - 1i))/(4*((c*x + 1)^(1/2) - 1)))/(c^7 - (12*c^7*((c*x - 1)^(1/2) - 1
i)^2)/((c*x + 1)^(1/2) - 1)^2 + (66*c^7*((c*x - 1)^(1/2) - 1i)^4)/((c*x + 1
)^(1/2) - 1)^4 - (220*c^7*((c*x - 1)^(1/2) - 1i)^6)/((c*x + 1)^(1/2) - 1)^6
 + (495*c^7*((c*x - 1)^(1/2) - 1i)^8)/((c*x + 1)^(1/2) - 1)^8 - (792*c^7*((
c*x - 1)^(1/2) - 1i)^10)/((c*x + 1)^(1/2) - 1)^10 + (924*c^7*((c*x - 1)^(1/
2) - 1i)^12)/((c*x + 1)^(1/2) - 1)^12 - (792*c^7*((c*x - 1)^(1/2) - 1i)^14)
/((c*x + 1)^(1/2) - 1)^14 + (495*c^7*((c*x - 1)^(1/2) - 1i)^16)/((c*x + 1)^
(1/2) - 1)^16 - (220*c^7*((c*x - 1)^(1/2) - 1i)^18)/((c*x + 1)^(1/2) - 1)^1
8 + (66*c^7*((c*x - 1)^(1/2) - 1i)^20)/((c*x + 1)^(1/2) - 1)^20 - (12*c^7*(
(c*x - 1)^(1/2) - 1i)^22)/((c*x + 1)^(1/2) - 1)^22 + (c^7*((c*x - 1)^(1/2)
- 1i)^24)/((c*x + 1)^(1/2) - 1)^24) + (3*a*atanh(((c*x - 1)^(1/2) - 1i)/((c
*x + 1)^(1/2) - 1)))/(2*c^5) + (5*b*atanh(((c*x - 1)^(1/2) - 1i)/((c*x + 1)
^(1/2) - 1)))/(4*c^7)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(b*x**2+a)/(c*x-1)**(1/2)/(c*x+1)**(1/2),x)
```

```
[Out] Timed out
```

3.206 $\int \frac{x^3(a+bx^2)}{\sqrt{-1+cx}\sqrt{1+cx}}\,dx$

Optimal. Leaf size=103

$$\frac{2\sqrt{cx-1}\sqrt{cx+1}\,(5ac^2+4b)}{15c^6}+\frac{x^2\sqrt{cx-1}\sqrt{cx+1}\,(5ac^2+4b)}{15c^4}+\frac{bx^4\sqrt{cx-1}\sqrt{cx+1}}{5c^2}$$

Rubi [A] time = 0.07, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {460, 100, 12, 74}

$$\frac{x^2\sqrt{cx-1}\sqrt{cx+1}\,(5ac^2+4b)}{15c^4}+\frac{2\sqrt{cx-1}\sqrt{cx+1}\,(5ac^2+4b)}{15c^6}+\frac{bx^4\sqrt{cx-1}\sqrt{cx+1}}{5c^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(a + b*x^2))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]),x]
```

```
[Out] (2*(4*b + 5*a*c^2)*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(15*c^6) + ((4*b + 5*a*c^2
)*x^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(15*c^4) + (b*x^4*Sqrt[-1 + c*x]*Sqrt[1
 + c*x])/(5*c^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/(d*f*(m + n + p + 1)), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 460

```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[(d*(e*x)^
(m + 1)*(a1 + b1*x^(n/2))^(p + 1)*(a2 + b2*x^(n/2))^(p + 1))/(b1*b2*e*(m +
n*(p + 1) + 1)), x] - Dist[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/
(b1*b2*(m + n*(p + 1) + 1)), Int[(e*x)^m*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n/
2))^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, m, n, p}, x] && EqQ[non2,
n/2] && EqQ[a2*b1 + a1*b2, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rubi steps

$$\int \frac{x^3 \left(a+bx^2\right)}{\sqrt{-1+cx}\,\sqrt{1+cx}}\,dx = \frac{bx^4\sqrt{-1+cx}\,\sqrt{1+cx}}{5c^2} - \frac{1}{5}\left(-5a-\frac{4b}{c^2}\right)\int \frac{x^3}{\sqrt{-1+cx}\,\sqrt{1+cx}}\,dx$$

$$= \frac{\left(4b+5ac^2\right)x^2\sqrt{-1+cx}\,\sqrt{1+cx}}{15c^4} + \frac{bx^4\sqrt{-1+cx}\,\sqrt{1+cx}}{5c^2} + \frac{\left(4b+5ac^2\right)\int \frac{2x}{\sqrt{-1+cx}\,\sqrt{1}}}{15c^4}$$

$$= \frac{\left(4b+5ac^2\right)x^2\sqrt{-1+cx}\,\sqrt{1+cx}}{15c^4} + \frac{bx^4\sqrt{-1+cx}\,\sqrt{1+cx}}{5c^2} + \frac{\left(2\left(4b+5ac^2\right)\right)\int \frac{}{\sqrt{-1+c}}}{15c^4}$$

$$= \frac{2\left(4b+5ac^2\right)\sqrt{-1+cx}\,\sqrt{1+cx}}{15c^6} + \frac{\left(4b+5ac^2\right)x^2\sqrt{-1+cx}\,\sqrt{1+cx}}{15c^4} + \frac{bx^4\sqrt{-1+cx}}{5c^2}$$

Mathematica [A] time = 0.04, size = 70, normalized size = 0.68

$$\frac{\left(c^2x^2-1\right)\left(5ac^2\left(c^2x^2+2\right)+b\left(3c^4x^4+4c^2x^2+8\right)\right)}{15c^6\sqrt{cx-1}\,\sqrt{cx+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(a + b*x^2))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]),x]
```

```
[Out] ((-1 + c^2*x^2)*(5*a*c^2*(2 + c^2*x^2) + b*(8 + 4*c^2*x^2 + 3*c^4*x^4)))/(1
5*c^6*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

IntegrateAlgebraic [A] time = 0.13, size = 196, normalized size = 1.90

$$-\frac{2\sqrt{cx-1}\left(\frac{15ac^2(cx-1)^4}{(cx+1)^4}-\frac{40ac^2(cx-1)^3}{(cx+1)^3}+\frac{50ac^2(cx-1)^2}{(cx+1)^2}-\frac{40ac^2(cx-1)}{cx+1}+15ac^2+\frac{15b(cx-1)^4}{(cx+1)^4}-\frac{20b(cx-1)^3}{(cx+1)^3}+\frac{58b(cx-1)^2}{(cx+1)^2}-\frac{20b(cx-1)}{cx+1}+15b\right)}{15c^6\sqrt{cx+1}\left(\frac{cx-1}{cx+1}-1\right)^5}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^3*(a + b*x^2))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]),x]
```

```
[Out] (-2*Sqrt[-1 + c*x]*(15*b + 15*a*c^2 + (15*b*(-1 + c*x)^4)/(1 + c*x)^4 + (15
*a*c^2*(-1 + c*x)^4)/(1 + c*x)^4 - (20*b*(-1 + c*x)^3)/(1 + c*x)^3 - (40*a*
c^2*(-1 + c*x)^3)/(1 + c*x)^3 + (58*b*(-1 + c*x)^2)/(1 + c*x)^2 + (50*a*c^2
*(-1 + c*x)^2)/(1 + c*x)^2 - (20*b*(-1 + c*x))/(1 + c*x) - (40*a*c^2*(-1 +
c*x))/(1 + c*x)))/(15*c^6*Sqrt[1 + c*x]*(-1 + (-1 + c*x)/(1 + c*x))^5)
```

fricas [A] time = 1.05, size = 55, normalized size = 0.53

$$\frac{\left(3\,bc^4x^4+10\,ac^2+\left(5\,ac^4+4\,bc^2\right)x^2+8\,b\right)\sqrt{cx+1}\,\sqrt{cx-1}}{15\,c^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)/(c*x-1)^(1/2)/(c*x+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/15*(3*b*c^4*x^4 + 10*a*c^2 + (5*a*c^4 + 4*b*c^2)*x^2 + 8*b)*sqrt(c*x + 1)
*sqrt(c*x - 1)/c^6
```

giac [A] time = 0.22, size = 108, normalized size = 1.05

$$\frac{\left(\left((cx+1)\left(3\,(cx+1)\left(\frac{(cx+1)b}{c^5}-\frac{4\,b}{c^5}\right)+\frac{5\,ac^{27}+22\,bc^{25}}{c^{30}}\right)-\frac{10\left(ac^{27}+2\,bc^{25}\right)}{c^{30}}\right)(cx+1)+\frac{15\left(ac^{27}+bc^{25}\right)}{c^{30}}\right)\sqrt{cx+1}\,\sqrt{cx-1}}{15\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)/(c*x-1)^(1/2)/(c*x+1)^(1/2),x, algorithm="giac")
```

```
[Out] 1/15*(((c*x + 1)*(3*(c*x + 1)*((c*x + 1)*b/c^5 - 4*b/c^5) + (5*a*c^27 + 22*
b*c^25)/c^30) - 10*(a*c^27 + 2*b*c^25)/c^30)*(c*x + 1) + 15*(a*c^27 + b*c^2
5)/c^30)*sqrt(c*x + 1)*sqrt(c*x - 1)/c
```

maple [A] time = 0.05, size = 57, normalized size = 0.55

$$\frac{\sqrt{cx+1}\,\sqrt{cx-1}\,\left(3b\,x^4c^4+5a\,c^4x^2+4b\,c^2x^2+10a\,c^2+8b\right)}{15c^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x^2+a)/(c*x-1)^(1/2)/(c*x+1)^(1/2),x)
```

```
[Out] 1/15*(c*x+1)^(1/2)*(c*x-1)^(1/2)*(3*b*c^4*x^4+5*a*c^4*x^2+4*b*c^2*x^2+10*a*
c^2+8*b)/c^6
```

maxima [A] time = 0.63, size = 95, normalized size = 0.92

$$\frac{\sqrt{c^2x^2-1}\,bx^4}{5\,c^2}+\frac{\sqrt{c^2x^2-1}\,ax^2}{3\,c^2}+\frac{4\,\sqrt{c^2x^2-1}\,bx^2}{15\,c^4}+\frac{2\,\sqrt{c^2x^2-1}\,a}{3\,c^4}+\frac{8\,\sqrt{c^2x^2-1}\,b}{15\,c^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)/(c*x-1)^(1/2)/(c*x+1)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/5*sqrt(c^2*x^2 - 1)*b*x^4/c^2 + 1/3*sqrt(c^2*x^2 - 1)*a*x^2/c^2 + 4/15*sq
rt(c^2*x^2 - 1)*b*x^2/c^4 + 2/3*sqrt(c^2*x^2 - 1)*a/c^4 + 8/15*sqrt(c^2*x^2
 - 1)*b/c^6
```

mupad [B] time = 2.44, size = 108, normalized size = 1.05

$$\frac{\sqrt{c\,x-1}\,\left(\frac{10\,a\,c^2+8\,b}{15\,c^6}+\frac{b\,x^5}{5\,c}+\frac{b\,x^4}{5\,c^2}+\frac{x^2\,\left(5\,a\,c^4+4\,b\,c^2\right)}{15\,c^6}+\frac{x^3\,\left(5\,a\,c^5+4\,b\,c^3\right)}{15\,c^6}+\frac{x\,\left(10\,a\,c^3+8\,b\,c\right)}{15\,c^6}\right)}{\sqrt{c\,x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(a + b*x^2))/((c*x - 1)^(1/2)*(c*x + 1)^(1/2)),x)
```

```
[Out] ((c*x - 1)^(1/2)*((8*b + 10*a*c^2)/(15*c^6) + (b*x^5)/(5*c) + (b*x^4)/(5*c^
2) + (x^2*(5*a*c^4 + 4*b*c^2))/(15*c^6) + (x^3*(5*a*c^5 + 4*b*c^3))/(15*c^6
) + (x*(8*b*c + 10*a*c^3))/(15*c^6)))/(c*x + 1)^(1/2)
```

sympy [C] time = 62.84, size = 216, normalized size = 2.10

$$\frac{aG_{6,6}^{6,2}\left(\begin{matrix} -\frac{5}{4},-\frac{3}{4} & -1,-1,-\frac{1}{2},1 \\ -\frac{3}{2},-\frac{5}{4},-1,-\frac{3}{4},-\frac{1}{2},0 & \end{matrix} \middle| \frac{1}{c^2x^2}\right)}{4\pi^{\frac{3}{2}}c^4}+\frac{iaG_{6,6}^{2,6}\left(\begin{matrix} -2,-\frac{7}{4},-\frac{3}{2},-\frac{5}{4},-1,1 & \\ -\frac{7}{4},-\frac{5}{4} & -2,-\frac{3}{2},-\frac{3}{2},0 \end{matrix} \middle| \frac{e^{2i\pi}}{c^2x^2}\right)}{4\pi^{\frac{3}{2}}c^4}+\frac{bG_{6,6}^{6,2}\left(\begin{matrix} -\frac{9}{4},-\frac{7}{4} & -2,-2,-\frac{3}{2},1 \\ -\frac{5}{2},-\frac{9}{4},-2,-\frac{7}{4},-\frac{3}{2},0 & \end{matrix} \middle| \frac{1}{c^2x^2}\right)}{4\pi^{\frac{3}{2}}c^6}+\frac{ibG_{6,6}^{2,6}\left(\begin{matrix} -3,-\frac{11}{4},-\frac{5}{2},-\frac{9}{4},-2,1 & \\ -\frac{11}{4},-\frac{9}{4} & -3,-\frac{5}{2},-\frac{5}{2},0 \end{matrix} \middle| \frac{e^{2i\pi}}{c^2x^2}\right)}{4\pi^{\frac{3}{2}}c^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x**2+a)/(c*x-1)**(1/2)/(c*x+1)**(1/2),x)
```

```
[Out] a*meijerg(((-5/4, -3/4), (-1, -1, -1/2, 1)), ((-3/2, -5/4, -1, -3/4, -1/2,
0), ()), 1/(c**2*x**2))/(4*pi**(3/2)*c**4) + I*a*meijerg(((-2, -7/4, -3/2,
-5/4, -1, 1), ()), ((-7/4, -5/4), (-2, -3/2, -3/2, 0)), exp_polar(2*I*pi)/(
c**2*x**2))/(4*pi**(3/2)*c**4) + b*meijerg(((-9/4, -7/4), (-2, -2, -3/2, 1)
), ((-5/2, -9/4, -2, -7/4, -3/2, 0), ()), 1/(c**2*x**2))/(4*pi**(3/2)*c**6)
 + I*b*meijerg(((-3, -11/4, -5/2, -9/4, -2, 1), ()), ((-11/4, -9/4), (-3, -
5/2, -5/2, 0)), exp_polar(2*I*pi)/(c**2*x**2))/(4*pi**(3/2)*c**6)
```

3.207 $\int \frac{x^2(a+bx^2)}{\sqrt{-1+cx}\sqrt{1+cx}}\,dx$

Optimal. Leaf size=87

$$\frac{(4ac^2+3b)\cosh^{-1}(cx)}{8c^5}+\frac{x\sqrt{cx-1}\sqrt{cx+1}\,(4ac^2+3b)}{8c^4}+\frac{bx^3\sqrt{cx-1}\sqrt{cx+1}}{4c^2}$$

Rubi [A] time = 0.07, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {460, 90, 52}

$$\frac{x\sqrt{cx-1}\sqrt{cx+1}\,(4ac^2+3b)}{8c^4}+\frac{(4ac^2+3b)\cosh^{-1}(cx)}{8c^5}+\frac{bx^3\sqrt{cx-1}\sqrt{cx+1}}{4c^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a + b*x^2))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]),x]
```

```
[Out] ((3*b + 4*a*c^2)*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(8*c^4) + (b*x^3*Sqrt[-1 +
 c*x]*Sqrt[1 + c*x])/(4*c^2) + ((3*b + 4*a*c^2)*ArcCosh[c*x])/(8*c^5)
```

Rule 52

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[(b*x)/a]/b, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a + c, 0] && EqQ[b
- d, 0] && GtQ[a, 0]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*(a + b*x)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/
(d*f*(n + p + 3)), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 460

```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[(d*(e*x)^
(m + 1)*(a1 + b1*x^(n/2))^(p + 1)*(a2 + b2*x^(n/2))^(p + 1))/(b1*b2*e*(m +
n*(p + 1) + 1)), x] - Dist[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/
(b1*b2*(m + n*(p + 1) + 1)), Int[(e*x)^m*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n/
2))^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, m, n, p}, x] && EqQ[non2,
n/2] && EqQ[a2*b1 + a1*b2, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2(a+bx^2)}{\sqrt{-1+cx}\sqrt{1+cx}}\,dx &= \frac{bx^3\sqrt{-1+cx}\sqrt{1+cx}}{4c^2}-\frac{1}{4}\left(-4a-\frac{3b}{c^2}\right)\int \frac{x^2}{\sqrt{-1+cx}\sqrt{1+cx}}\,dx\\ &= \frac{(3b+4ac^2)\,x\sqrt{-1+cx}\sqrt{1+cx}}{8c^4}+\frac{bx^3\sqrt{-1+cx}\sqrt{1+cx}}{4c^2}+\frac{(3b+4ac^2)\int \frac{1}{\sqrt{-1+cx}\sqrt{1+}}}{8c^4}\\ &= \frac{(3b+4ac^2)\,x\sqrt{-1+cx}\sqrt{1+cx}}{8c^4}+\frac{bx^3\sqrt{-1+cx}\sqrt{1+cx}}{4c^2}+\frac{(3b+4ac^2)\cosh^{-1}(cx)}{8c^5}\end{aligned}$$

Mathematica [A] time = 0.11, size = 98, normalized size = 1.13

$$\frac{cx\left(c^2x^2-1\right)\left(4ac^2+b\left(2c^2x^2+3\right)\right)+\sqrt{c^2x^2-1}\left(4ac^2+3b\right)\tanh^{-1}\left(\frac{cx}{\sqrt{c^2x^2-1}}\right)}{8c^5\sqrt{cx-1}\sqrt{cx+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(a + b*x^2))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]),x]
```

```
[Out] (c*x*(-1 + c^2*x^2)*(4*a*c^2 + b*(3 + 2*c^2*x^2)) + (3*b + 4*a*c^2)*Sqrt[-1
 + c^2*x^2]*ArcTanh[(c*x)/Sqrt[-1 + c^2*x^2]])/(8*c^5*Sqrt[-1 + c*x]*Sqrt[1
 + c*x])
```

IntegrateAlgebraic [B] time = 0.16, size = 175, normalized size = 2.01

$$\frac{\sqrt{cx-1}\left(\frac{cx-1}{cx+1}+1\right)\left(-\frac{8ac^2(cx-1)}{cx+1}+\frac{4ac^2(cx-1)^2}{(cx+1)^2}+4ac^2-\frac{2b(cx-1)}{cx+1}+\frac{5b(cx-1)^2}{(cx+1)^2}+5b\right)}{4c^5\sqrt{cx+1}\left(\frac{cx-1}{cx+1}-1\right)^4}+\frac{\left(4ac^2+3b\right)\tanh^{-1}\left(\frac{\sqrt{cx-1}}{\sqrt{cx+1}}\right)}{4c^5}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^2*(a + b*x^2))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]),x]
```

```
[Out] (Sqrt[-1 + c*x]*(1 + (-1 + c*x)/(1 + c*x))*(5*b + 4*a*c^2 + (5*b*(-1 + c*x)
^2)/(1 + c*x)^2 + (4*a*c^2*(-1 + c*x)^2)/(1 + c*x)^2 - (2*b*(-1 + c*x))/(1
+ c*x) - (8*a*c^2*(-1 + c*x))/(1 + c*x)))/(4*c^5*Sqrt[1 + c*x]*(-1 + (-1 +
c*x)/(1 + c*x))^4) + ((3*b + 4*a*c^2)*ArcTanh[Sqrt[-1 + c*x]/Sqrt[1 + c*x]]
)/(4*c^5)
```

fricas [A] time = 0.83, size = 77, normalized size = 0.89

$$\frac{\left(2\,bc^3x^3+\left(4\,ac^3+3\,bc\right)x\right)\sqrt{cx+1}\,\sqrt{cx-1}-\left(4\,ac^2+3\,b\right)\log\left(-cx+\sqrt{cx+1}\,\sqrt{cx-1}\right)}{8\,c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)/(c*x-1)^(1/2)/(c*x+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/8*((2*b*c^3*x^3 + (4*a*c^3 + 3*b*c)*x)*sqrt(c*x + 1)*sqrt(c*x - 1) - (4*a
*c^2 + 3*b)*log(-c*x + sqrt(c*x + 1)*sqrt(c*x - 1)))/c^5
```

giac [A] time = 0.37, size = 121, normalized size = 1.39

$$\frac{\left((cx+1)\left(2\,(cx+1)\left(\frac{(cx+1)b}{c^4}-\frac{3\,b}{c^4}\right)+\frac{4\,ac^{18}+9\,bc^{16}}{c^{20}}\right)-\frac{4\,ac^{18}+5\,bc^{16}}{c^{20}}\right)\sqrt{cx+1}\,\sqrt{cx-1}-\frac{2\left(4\,ac^2+3\,b\right)\log\left(\sqrt{cx+1}-\sqrt{cx-1}\right)}{c^4}}{8\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)/(c*x-1)^(1/2)/(c*x+1)^(1/2),x, algorithm="giac")
```

```
[Out] 1/8*(((c*x + 1)*(2*(c*x + 1)*((c*x + 1)*b/c^4 - 3*b/c^4) + (4*a*c^18 + 9*b*
c^16)/c^20) - (4*a*c^18 + 5*b*c^16)/c^20)*sqrt(c*x + 1)*sqrt(c*x - 1) - 2*(
4*a*c^2 + 3*b)*log(sqrt(c*x + 1) - sqrt(c*x - 1))/c^4)/c
```

maple [C] time = 0.08, size = 147, normalized size = 1.69

$$\frac{\sqrt{cx-1}\,\sqrt{cx+1}\,\left(2\sqrt{c^2x^2-1}\,bc^3x^3\mathrm{csgn}(c)+4\sqrt{c^2x^2-1}\,ac^3x\,\mathrm{csgn}(c)+4ac^2\ln\left(\left(cx+\sqrt{c^2x^2-1}\,\mathrm{csgn}(c)\right)\mathrm{csgn}(c)\right)+3\sqrt{c^2x^2-1}\,bcx\,\mathrm{csgn}(c)+3b\ln\left(\left(cx+\sqrt{c^2x^2-1}\,\mathrm{csgn}(c)\right)\mathrm{csgn}(c)\right)\right)\mathrm{csgn}(c)}{8\sqrt{c^2x^2-1}\,c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x^2+a)/(c*x-1)^(1/2)/(c*x+1)^(1/2),x)
```

```
[Out] 1/8*(c*x-1)^(1/2)*(c*x+1)^(1/2)*(2*(c^2*x^2-1)^(1/2)*b*c^3*x^3*csgn(c)+4*(c
^2*x^2-1)^(1/2)*a*c^3*x*csgn(c)+3*(c^2*x^2-1)^(1/2)*b*c*x*csgn(c)+4*a*c^2*l
n((c*x+(c^2*x^2-1)^(1/2)*csgn(c))*csgn(c))+3*b*ln((c*x+(c^2*x^2-1)^(1/2)*cs
gn(c))*csgn(c)))*csgn(c)/c^5/(c^2*x^2-1)^(1/2)
```

maxima [A] time = 0.54, size = 113, normalized size = 1.30

$$\frac{\sqrt{c^2x^2-1}\,bx^3}{4\,c^2}+\frac{\sqrt{c^2x^2-1}\,ax}{2\,c^2}+\frac{a\log\left(2\,c^2x+2\,\sqrt{c^2x^2-1}\,c\right)}{2\,c^3}+\frac{3\,\sqrt{c^2x^2-1}\,bx}{8\,c^4}+\frac{3\,b\log\left(2\,c^2x+2\,\sqrt{c^2x^2-1}\,c\right)}{8\,c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)/(c*x-1)^(1/2)/(c*x+1)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/4*sqrt(c^2*x^2 - 1)*b*x^3/c^2 + 1/2*sqrt(c^2*x^2 - 1)*a*x/c^2 + 1/2*a*log
(2*c^2*x + 2*sqrt(c^2*x^2 - 1)*c)/c^3 + 3/8*sqrt(c^2*x^2 - 1)*b*x/c^4 + 3/8
*b*log(2*c^2*x + 2*sqrt(c^2*x^2 - 1)*c)/c^5
```

mupad [B] time = 22.50, size = 720, normalized size = 8.28

$$\frac{\frac{23\,b\left(\sqrt{c\,x-1}-\mathrm{i}\right)^3}{2\left(\sqrt{c\,x+1}-1\right)^3}+\frac{333\,b\left(\sqrt{c\,x-1}-\mathrm{i}\right)^5}{2\left(\sqrt{c\,x+1}-1\right)^5}+\frac{671\,b\left(\sqrt{c\,x-1}-\mathrm{i}\right)^7}{2\left(\sqrt{c\,x+1}-1\right)^7}+\frac{671\,b\left(\sqrt{c\,x-1}-\mathrm{i}\right)^9}{2\left(\sqrt{c\,x+1}-1\right)^9}+\frac{333\,b\left(\sqrt{c\,x-1}-\mathrm{i}\right)^{11}}{2\left(\sqrt{c\,x+1}-1\right)^{11}}+\frac{23\,b\left(\sqrt{c\,x-1}-\mathrm{i}\right)^{13}}{2\left(\sqrt{c\,x+1}-1\right)^{13}}-\frac{3\,b\left(\sqrt{c\,x-1}-\mathrm{i}\right)^{15}}{2\left(\sqrt{c\,x+1}-1\right)^{15}}-\frac{3\,b\left(\sqrt{c\,x-1}-\mathrm{i}\right)}{2\left(\sqrt{c\,x+1}-1\right)}}{c^5-\frac{8\,c^5\left(\sqrt{c\,x-1}-\mathrm{i}\right)^2}{\left(\sqrt{c\,x+1}-1\right)^2}+\frac{28\,c^5\left(\sqrt{c\,x-1}-\mathrm{i}\right)^4}{\left(\sqrt{c\,x+1}-1\right)^4}-\frac{56\,c^5\left(\sqrt{c\,x-1}-\mathrm{i}\right)^6}{\left(\sqrt{c\,x+1}-1\right)^6}+\frac{70\,c^5\left(\sqrt{c\,x-1}-\mathrm{i}\right)^8}{\left(\sqrt{c\,x+1}-1\right)^8}-\frac{56\,c^5\left(\sqrt{c\,x-1}-\mathrm{i}\right)^{10}}{\left(\sqrt{c\,x+1}-1\right)^{10}}+\frac{28\,c^5\left(\sqrt{c\,x-1}-\mathrm{i}\right)^{12}}{\left(\sqrt{c\,x+1}-1\right)^{12}}-\frac{8\,c^5\left(\sqrt{c\,x-1}-\mathrm{i}\right)^{14}}{\left(\sqrt{c\,x+1}-1\right)^{14}}+\frac{c^5\left(\sqrt{c\,x-1}-\mathrm{i}\right)^{16}}{\left(\sqrt{c\,x+1}-1\right)^{16}}}-\frac{\frac{14\,a\left(\sqrt{c\,x-1}-\mathrm{i}\right)^3}{\left(\sqrt{c\,x+1}-1\right)^3}+\frac{14\,a\left(\sqrt{c\,x-1}-\mathrm{i}\right)^5}{\left(\sqrt{c\,x+1}-1\right)^5}+\frac{2\,a\left(\sqrt{c\,x-1}-\mathrm{i}\right)^7}{\left(\sqrt{c\,x+1}-1\right)^7}+\frac{2\,a\left(\sqrt{c\,x-1}-\mathrm{i}\right)}{\sqrt{c\,x+1}-1}}{c^3-\frac{4\,c^3\left(\sqrt{c\,x-1}-\mathrm{i}\right)^2}{\left(\sqrt{c\,x+1}-1\right)^2}+\frac{6\,c^3\left(\sqrt{c\,x-1}-\mathrm{i}\right)^4}{\left(\sqrt{c\,x+1}-1\right)^4}-\frac{4\,c^3\left(\sqrt{c\,x-1}-\mathrm{i}\right)^6}{\left(\sqrt{c\,x+1}-1\right)^6}+\frac{c^3\left(\sqrt{c\,x-1}-\mathrm{i}\right)^8}{\left(\sqrt{c\,x+1}-1\right)^8}}+\frac{2\,a\,\mathrm{atanh}\left(\frac{\sqrt{c\,x-1}-\mathrm{i}}{\sqrt{c\,x+1}-1}\right)}{c^3}+\frac{3\,b\,\mathrm{atanh}\left(\frac{\sqrt{c\,x-1}-\mathrm{i}}{\sqrt{c\,x+1}-1}\right)}{2\,c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a + b*x^2))/((c*x - 1)^(1/2)*(c*x + 1)^(1/2)),x)
```

```
[Out] ((23*b*((c*x - 1)^(1/2) - 1i)^3)/(2*((c*x + 1)^(1/2) - 1)^3) + (333*b*((c*x
 - 1)^(1/2) - 1i)^5)/(2*((c*x + 1)^(1/2) - 1)^5) + (671*b*((c*x - 1)^(1/2)
- 1i)^7)/(2*((c*x + 1)^(1/2) - 1)^7) + (671*b*((c*x - 1)^(1/2) - 1i)^9)/(2*
((c*x + 1)^(1/2) - 1)^9) + (333*b*((c*x - 1)^(1/2) - 1i)^11)/(2*((c*x + 1)^
(1/2) - 1)^11) + (23*b*((c*x - 1)^(1/2) - 1i)^13)/(2*((c*x + 1)^(1/2) - 1)^
13) - (3*b*((c*x - 1)^(1/2) - 1i)^15)/(2*((c*x + 1)^(1/2) - 1)^15) - (3*b*(
(c*x - 1)^(1/2) - 1i))/(2*((c*x + 1)^(1/2) - 1)))/(c^5 - (8*c^5*((c*x - 1)^
(1/2) - 1i)^2)/((c*x + 1)^(1/2) - 1)^2 + (28*c^5*((c*x - 1)^(1/2) - 1i)^4)/
((c*x + 1)^(1/2) - 1)^4 - (56*c^5*((c*x - 1)^(1/2) - 1i)^6)/((c*x + 1)^(1/2
) - 1)^6 + (70*c^5*((c*x - 1)^(1/2) - 1i)^8)/((c*x + 1)^(1/2) - 1)^8 - (56*
c^5*((c*x - 1)^(1/2) - 1i)^10)/((c*x + 1)^(1/2) - 1)^10 + (28*c^5*((c*x - 1
)^(1/2) - 1i)^12)/((c*x + 1)^(1/2) - 1)^12 - (8*c^5*((c*x - 1)^(1/2) - 1i)^
14)/((c*x + 1)^(1/2) - 1)^14 + (c^5*((c*x - 1)^(1/2) - 1i)^16)/((c*x + 1)^(
1/2) - 1)^16) - ((14*a*((c*x - 1)^(1/2) - 1i)^3)/((c*x + 1)^(1/2) - 1)^3 +
(14*a*((c*x - 1)^(1/2) - 1i)^5)/((c*x + 1)^(1/2) - 1)^5 + (2*a*((c*x - 1)^(
1/2) - 1i)^7)/((c*x + 1)^(1/2) - 1)^7 + (2*a*((c*x - 1)^(1/2) - 1i))/((c*x
+ 1)^(1/2) - 1))/(c^3 - (4*c^3*((c*x - 1)^(1/2) - 1i)^2)/((c*x + 1)^(1/2) -
 1)^2 + (6*c^3*((c*x - 1)^(1/2) - 1i)^4)/((c*x + 1)^(1/2) - 1)^4 - (4*c^3*(
(c*x - 1)^(1/2) - 1i)^6)/((c*x + 1)^(1/2) - 1)^6 + (c^3*((c*x - 1)^(1/2) -
1i)^8)/((c*x + 1)^(1/2) - 1)^8) + (2*a*atanh(((c*x - 1)^(1/2) - 1i)/((c*x +
 1)^(1/2) - 1)))/c^3 + (3*b*atanh(((c*x - 1)^(1/2) - 1i)/((c*x + 1)^(1/2) -
 1)))/(2*c^5)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x**2+a)/(c*x-1)**(1/2)/(c*x+1)**(1/2),x)
```

```
[Out] Timed out
```

3.208 $\int \frac{x(a+bx^2)}{\sqrt{-1+cx}\sqrt{1+cx}}\,dx$

Optimal. Leaf size=65

$$\frac{\sqrt{cx-1}\sqrt{cx+1}\left(3ac^2+2b\right)}{3c^4}+\frac{bx^2\sqrt{cx-1}\sqrt{cx+1}}{3c^2}$$

Rubi [A] time = 0.04, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {460, 74}

$$\frac{\sqrt{cx-1}\sqrt{cx+1}\left(3ac^2+2b\right)}{3c^4}+\frac{bx^2\sqrt{cx-1}\sqrt{cx+1}}{3c^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a + b*x^2))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]),x]
```

```
[Out] ((2*b + 3*a*c^2)*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(3*c^4) + (b*x^2*Sqrt[-1 + c
*x]*Sqrt[1 + c*x])/(3*c^2)
```

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 460

```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[(d*(e*x)^
(m + 1)*(a1 + b1*x^(n/2))^(p + 1)*(a2 + b2*x^(n/2))^(p + 1))/(b1*b2*e*(m +
n*(p + 1) + 1)), x] - Dist[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/
(b1*b2*(m + n*(p + 1) + 1)), Int[(e*x)^m*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n/
2))^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, m, n, p}, x] && EqQ[non2,
n/2] && EqQ[a2*b1 + a1*b2, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x\left(a+bx^2\right)}{\sqrt{-1+cx}\sqrt{1+cx}}\,dx &= \frac{bx^2\sqrt{-1+cx}\sqrt{1+cx}}{3c^2}-\frac{1}{3}\left(-3a-\frac{2b}{c^2}\right)\int \frac{x}{\sqrt{-1+cx}\sqrt{1+cx}}\,dx\\ &= \frac{\left(2b+3ac^2\right)\sqrt{-1+cx}\sqrt{1+cx}}{3c^4}+\frac{bx^2\sqrt{-1+cx}\sqrt{1+cx}}{3c^2}\end{aligned}$$

Mathematica [A] time = 0.05, size = 52, normalized size = 0.80

$$\frac{\left(c^2x^2-1\right)\left(3ac^2+b\left(c^2x^2+2\right)\right)}{3c^4\sqrt{cx-1}\sqrt{cx+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(a + b*x^2))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]),x]
```

```
[Out] ((-1 + c^2*x^2)*(3*a*c^2 + b*(2 + c^2*x^2)))/(3*c^4*Sqrt[-1 + c*x]*Sqrt[1 +
 c*x])
```

IntegrateAlgebraic [A] time = 0.10, size = 122, normalized size = 1.88

$$-\frac{2\sqrt{cx-1}\left(-\frac{6ac^2(cx-1)}{cx+1}+\frac{3ac^2(cx-1)^2}{(cx+1)^2}+3ac^2-\frac{2b(cx-1)}{cx+1}+\frac{3b(cx-1)^2}{(cx+1)^2}+3b\right)}{3c^4\sqrt{cx+1}\left(\frac{cx-1}{cx+1}-1\right)^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x*(a + b*x^2))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]),x]
```

```
[Out] (-2*Sqrt[-1 + c*x]*(3*b + 3*a*c^2 + (3*b*(-1 + c*x)^2)/(1 + c*x)^2 + (3*a*c
^2*(-1 + c*x)^2)/(1 + c*x)^2 - (2*b*(-1 + c*x))/(1 + c*x) - (6*a*c^2*(-1 +
c*x))/(1 + c*x)))/(3*c^4*Sqrt[1 + c*x]*(-1 + (-1 + c*x)/(1 + c*x))^3)
```

fricas [A] time = 0.78, size = 37, normalized size = 0.57

$$\frac{\left(bc^2x^2+3\,ac^2+2\,b\right)\sqrt{cx+1}\,\sqrt{cx-1}}{3\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)/(c*x-1)^(1/2)/(c*x+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*(b*c^2*x^2 + 3*a*c^2 + 2*b)*sqrt(c*x + 1)*sqrt(c*x - 1)/c^4
```

giac [A] time = 0.22, size = 59, normalized size = 0.91

$$\frac{\sqrt{cx+1}\,\sqrt{cx-1}\left((cx+1)\left(\frac{(cx+1)b}{c^3}-\frac{2\,b}{c^3}\right)+\frac{3\left(ac^{11}+bc^9\right)}{c^{12}}\right)}{3\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)/(c*x-1)^(1/2)/(c*x+1)^(1/2),x, algorithm="giac")
```

```
[Out] 1/3*sqrt(c*x + 1)*sqrt(c*x - 1)*((c*x + 1)*((c*x + 1)*b/c^3 - 2*b/c^3) + 3*
(a*c^11 + b*c^9)/c^12)/c
```

maple [A] time = 0.04, size = 38, normalized size = 0.58

$$\frac{\sqrt{cx+1}\,\sqrt{cx-1}\,\left(b\,c^2x^2+3a\,c^2+2b\right)}{3c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^2+a)/(c*x-1)^(1/2)/(c*x+1)^(1/2),x)
```

```
[Out] 1/3*(c*x+1)^(1/2)*(c*x-1)^(1/2)*(b*c^2*x^2+3*a*c^2+2*b)/c^4
```

maxima [A] time = 0.47, size = 54, normalized size = 0.83

$$\frac{\sqrt{c^2x^2-1}\,bx^2}{3\,c^2}+\frac{\sqrt{c^2x^2-1}\,a}{c^2}+\frac{2\,\sqrt{c^2x^2-1}\,b}{3\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)/(c*x-1)^(1/2)/(c*x+1)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(c^2*x^2 - 1)*b*x^2/c^2 + sqrt(c^2*x^2 - 1)*a/c^2 + 2/3*sqrt(c^2*x^
2 - 1)*b/c^4
```

mupad [B] time = 2.36, size = 66, normalized size = 1.02

$$\frac{\sqrt{c\,x-1}\,\left(\frac{3\,a\,c^2+2\,b}{3\,c^4}+\frac{b\,x^3}{3\,c}+\frac{b\,x^2}{3\,c^2}+\frac{x\,\left(3\,a\,c^3+2\,b\,c\right)}{3\,c^4}\right)}{\sqrt{c\,x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a + b*x^2))/((c*x - 1)^(1/2)*(c*x + 1)^(1/2)),x)
```

```
[Out] ((c*x - 1)^(1/2)*((2*b + 3*a*c^2)/(3*c^4) + (b*x^3)/(3*c) + (b*x^2)/(3*c^2)
 + (x*(2*b*c + 3*a*c^3))/(3*c^4)))/(c*x + 1)^(1/2)
```

sympy [C] time = 41.91, size = 202, normalized size = 3.11

$$\frac{aG_{6,6}^{6,2}\left(\begin{matrix} -\frac{1}{4},\frac{1}{4} & 0,0,\frac{1}{2},1 \\ -\frac{1}{2},-\frac{1}{4},0,\frac{1}{4},\frac{1}{2},0 & \end{matrix}\middle|\frac{1}{c^2x^2}\right)}{4\pi^{\frac{3}{2}}c^2}+\frac{iaG_{6,6}^{2,6}\left(\begin{matrix} -1,-\frac{3}{4},-\frac{1}{2},-\frac{1}{4},0,1 & \\ -\frac{3}{4},-\frac{1}{4} & -1,-\frac{1}{2},-\frac{1}{2},0 \end{matrix}\middle|\frac{e^{2i\pi}}{c^2x^2}\right)}{4\pi^{\frac{3}{2}}c^2}+\frac{bG_{6,6}^{6,2}\left(\begin{matrix} -\frac{5}{4},-\frac{3}{4} & -1,-1,-\frac{1}{2},1 \\ -\frac{3}{2},-\frac{5}{4},-1,-\frac{3}{4},-\frac{1}{2},0 & \end{matrix}\middle|\frac{1}{c^2x^2}\right)}{4\pi^{\frac{3}{2}}c^4}+\frac{ibG_{6,6}^{2,6}\left(\begin{matrix} -2,-\frac{7}{4},-\frac{3}{2},-\frac{5}{4},-1,1 & \\ -\frac{7}{4},-\frac{5}{4} & -2,-\frac{3}{2},-\frac{3}{2},0 \end{matrix}\middle|\frac{e^{2i\pi}}{c^2x^2}\right)}{4\pi^{\frac{3}{2}}c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**2+a)/(c*x-1)**(1/2)/(c*x+1)**(1/2),x)
```

```
[Out] a*meijerg(((-1/4, 1/4), (0, 0, 1/2, 1)), ((-1/2, -1/4, 0, 1/4, 1/2, 0), ())
, 1/(c**2*x**2))/(4*pi**(3/2)*c**2) + I*a*meijerg(((-1, -3/4, -1/2, -1/4, 0
, 1), ()), ((-3/4, -1/4), (-1, -1/2, -1/2, 0)), exp_polar(2*I*pi)/(c**2*x**
2))/(4*pi**(3/2)*c**2) + b*meijerg(((-5/4, -3/4), (-1, -1, -1/2, 1)), ((-3/
2, -5/4, -1, -3/4, -1/2, 0), ()), 1/(c**2*x**2))/(4*pi**(3/2)*c**4) + I*b*m
eijerg(((-2, -7/4, -3/2, -5/4, -1, 1), ()), ((-7/4, -5/4), (-2, -3/2, -3/2,
 0)), exp_polar(2*I*pi)/(c**2*x**2))/(4*pi**(3/2)*c**4)
```

3.209 $\int \frac{a+bx^2}{\sqrt{-1+cx}\sqrt{1+cx}}\,dx$

Optimal. Leaf size=47

$$\frac{(2ac^2+b)\cosh^{-1}(cx)}{2c^3}+\frac{bx\sqrt{cx-1}\sqrt{cx+1}}{2c^2}$$

Rubi [A] time = 0.02, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {389, 52}

$$\frac{(2ac^2+b)\cosh^{-1}(cx)}{2c^3}+\frac{bx\sqrt{cx-1}\sqrt{cx+1}}{2c^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]),x]
```

```
[Out] (b*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(2*c^2) + ((b + 2*a*c^2)*ArcCosh[c*x])/(
2*c^3)
```

Rule 52

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[(b*x)/a]/b, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a + c, 0] && EqQ[b
- d, 0] && GtQ[a, 0]
```

Rule 389

```
Int[((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_
.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[(d*x*(a1 + b1*x^(n/2))^(p +
1)*(a2 + b2*x^(n/2))^(p + 1))/(b1*b2*(n*(p + 1) + 1)), x] - Dist[(a1*a2*d -
 b1*b2*c*(n*(p + 1) + 1))/(b1*b2*(n*(p + 1) + 1)), Int[(a1 + b1*x^(n/2))^p*
(a2 + b2*x^(n/2))^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, n, p}, x] && Eq
Q[non2, n/2] && EqQ[a2*b1 + a1*b2, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx^2}{\sqrt{-1+cx}\sqrt{1+cx}}\,dx &= \frac{bx\sqrt{-1+cx}\sqrt{1+cx}}{2c^2}-\frac{(-b-2ac^2)\int \frac{1}{\sqrt{-1+cx}\sqrt{1+cx}}\,dx}{2c^2}\\ &= \frac{bx\sqrt{-1+cx}\sqrt{1+cx}}{2c^2}+\frac{(b+2ac^2)\cosh^{-1}(cx)}{2c^3}\end{aligned}$$

Mathematica [B] time = 0.21, size = 101, normalized size = 2.15

$$\frac{4(ac^2+b)\tanh^{-1}\left(\sqrt{\frac{cx-1}{cx+1}}\right)+\frac{b\left(cx\sqrt{-(cx-1)^2}\sqrt{cx+1}-2\sqrt{cx-1}\sin^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)\right)}{\sqrt{1-cx}}}{2c^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^2)/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]),x]
```

```
[Out] ((b*(c*x*Sqrt[-(-1 + c*x)^2]*Sqrt[1 + c*x] - 2*Sqrt[-1 + c*x]*ArcSin[Sqrt[1
 - c*x]/Sqrt[2]]))/Sqrt[1 - c*x] + 4*(b + a*c^2)*ArcTanh[Sqrt[(-1 + c*x)/(1
 + c*x)]])/(2*c^3)
```

IntegrateAlgebraic [A] time = 0.13, size = 88, normalized size = 1.87

$$\frac{\left(2ac^2+b\right)\tanh^{-1}\left(\frac{\sqrt{cx-1}}{\sqrt{cx+1}}\right)}{c^3}+\frac{b\sqrt{cx-1}\left(\frac{cx-1}{cx+1}+1\right)}{c^3\sqrt{cx+1}\left(\frac{cx-1}{cx+1}-1\right)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x^2)/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]),x]
```

```
[Out] (b*Sqrt[-1 + c*x]*(1 + (-1 + c*x)/(1 + c*x)))/(c^3*Sqrt[1 + c*x]*(-1 + (-1
+ c*x)/(1 + c*x))^2) + ((b + 2*a*c^2)*ArcTanh[Sqrt[-1 + c*x]/Sqrt[1 + c*x]]
)/c^3
```

fricas [A] time = 0.88, size = 55, normalized size = 1.17

$$\frac{\sqrt{cx+1}\sqrt{cx-1}\,bcx-\left(2\,ac^2+b\right)\log\left(-cx+\sqrt{cx+1}\sqrt{cx-1}\right)}{2\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/(c*x-1)^(1/2)/(c*x+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*(sqrt(c*x + 1)*sqrt(c*x - 1)*b*c*x - (2*a*c^2 + b)*log(-c*x + sqrt(c*x
+ 1)*sqrt(c*x - 1)))/c^3
```

giac [A] time = 0.18, size = 69, normalized size = 1.47

$$\frac{\sqrt{cx+1}\sqrt{cx-1}\left(\frac{(cx+1)b}{c^2}-\frac{b}{c^2}\right)-\frac{2\left(2\,ac^2+b\right)\log\left(\sqrt{cx+1}-\sqrt{cx-1}\right)}{c^2}}{2\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/(c*x-1)^(1/2)/(c*x+1)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*(sqrt(c*x + 1)*sqrt(c*x - 1)*((c*x + 1)*b/c^2 - b/c^2) - 2*(2*a*c^2 + b
)*log(sqrt(c*x + 1) - sqrt(c*x - 1))/c^2)/c
```

maple [C] time = 0.07, size = 103, normalized size = 2.19

$$\frac{\sqrt{cx-1}\,\sqrt{cx+1}\,\left(2a\,c^2\ln\left(\left(cx+\sqrt{c^2x^2-1}\,\mathrm{csgn}(c)\right)\mathrm{csgn}(c)\right)+\sqrt{c^2x^2-1}\,bcx\,\mathrm{csgn}(c)+b\ln\left(\left(cx+\sqrt{c^2x^2-1}\,\mathrm{csgn}(c)\right)\mathrm{csgn}(c)\right)\right)\mathrm{csgn}(c)}{2\sqrt{c^2x^2-1}\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)/(c*x-1)^(1/2)/(c*x+1)^(1/2),x)
```

```
[Out] 1/2*(c*x-1)^(1/2)*(c*x+1)^(1/2)*((c^2*x^2-1)^(1/2)*b*c*x*csgn(c)+2*a*c^2*ln
((c*x+(c^2*x^2-1)^(1/2)*csgn(c))*csgn(c))+b*ln((c*x+(c^2*x^2-1)^(1/2)*csgn(
c))*csgn(c)))*csgn(c)/c^3/(c^2*x^2-1)^(1/2)
```

maxima [A] time = 0.50, size = 74, normalized size = 1.57

$$\frac{a\log\left(2\,c^2x+2\sqrt{c^2x^2-1}\,c\right)}{c}+\frac{\sqrt{c^2x^2-1}\,bx}{2\,c^2}+\frac{b\log\left(2\,c^2x+2\sqrt{c^2x^2-1}\,c\right)}{2\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/(c*x-1)^(1/2)/(c*x+1)^(1/2),x, algorithm="maxima")
```

```
[Out] a*log(2*c^2*x + 2*sqrt(c^2*x^2 - 1)*c)/c + 1/2*sqrt(c^2*x^2 - 1)*b*x/c^2 +
1/2*b*log(2*c^2*x + 2*sqrt(c^2*x^2 - 1)*c)/c^3
```

mupad [B] time = 12.69, size = 293, normalized size = 6.23

$$-\frac{\frac{14\,b\,\left(\sqrt{c\,x-1}-\mathrm{i}\right)^3}{\left(\sqrt{c\,x+1}-1\right)^3}+\frac{14\,b\,\left(\sqrt{c\,x-1}-\mathrm{i}\right)^5}{\left(\sqrt{c\,x+1}-1\right)^5}+\frac{2\,b\,\left(\sqrt{c\,x-1}-\mathrm{i}\right)^7}{\left(\sqrt{c\,x+1}-1\right)^7}+\frac{2\,b\,\left(\sqrt{c\,x-1}-\mathrm{i}\right)}{\sqrt{c\,x+1}-1}}{c^3-\frac{4\,c^3\,\left(\sqrt{c\,x-1}-\mathrm{i}\right)^2}{\left(\sqrt{c\,x+1}-1\right)^2}+\frac{6\,c^3\,\left(\sqrt{c\,x-1}-\mathrm{i}\right)^4}{\left(\sqrt{c\,x+1}-1\right)^4}-\frac{4\,c^3\,\left(\sqrt{c\,x-1}-\mathrm{i}\right)^6}{\left(\sqrt{c\,x+1}-1\right)^6}+\frac{c^3\,\left(\sqrt{c\,x-1}-\mathrm{i}\right)^8}{\left(\sqrt{c\,x+1}-1\right)^8}}+\frac{2\,b\,\mathrm{atanh}\left(\frac{\sqrt{c\,x-1}-\mathrm{i}}{\sqrt{c\,x+1}-1}\right)}{c^3}-\frac{4\,a\,\mathrm{atan}\left(\frac{c\,\left(\sqrt{c\,x-1}-\mathrm{i}\right)}{\left(\sqrt{c\,x+1}-1\right)\,\sqrt{-c^2}}\right)}{\sqrt{-c^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)/((c*x - 1)^(1/2)*(c*x + 1)^(1/2)),x)
```

```
[Out] (2*b*atanh(((c*x - 1)^(1/2) - 1i)/((c*x + 1)^(1/2) - 1)))/c^3 - ((14*b*((c*
x - 1)^(1/2) - 1i)^3)/((c*x + 1)^(1/2) - 1)^3 + (14*b*((c*x - 1)^(1/2) - 1i
)^5)/((c*x + 1)^(1/2) - 1)^5 + (2*b*((c*x - 1)^(1/2) - 1i)^7)/((c*x + 1)^(1
/2) - 1)^7 + (2*b*((c*x - 1)^(1/2) - 1i))/((c*x + 1)^(1/2) - 1))/(c^3 - (4*
c^3*((c*x - 1)^(1/2) - 1i)^2)/((c*x + 1)^(1/2) - 1)^2 + (6*c^3*((c*x - 1)^(
1/2) - 1i)^4)/((c*x + 1)^(1/2) - 1)^4 - (4*c^3*((c*x - 1)^(1/2) - 1i)^6)/((
c*x + 1)^(1/2) - 1)^6 + (c^3*((c*x - 1)^(1/2) - 1i)^8)/((c*x + 1)^(1/2) - 1
)^8) - (4*a*atan((c*((c*x - 1)^(1/2) - 1i))/(((c*x + 1)^(1/2) - 1)*(-c^2)^(
1/2))))/(-c^2)^(1/2)
```

sympy [C] time = 45.51, size = 182, normalized size = 3.87

$$\frac{aG_{6,6}^{6,2}\left(\begin{matrix}\frac{1}{4},\frac{3}{4} & \frac{1}{2},\frac{1}{2},1,1\\ 0,\frac{1}{4},\frac{1}{2},\frac{3}{4},1,0 & \end{matrix}\middle|\frac{1}{c^2x^2}\right)}{4\pi^{\frac{3}{2}}c}-\frac{iaG_{6,6}^{2,6}\left(\begin{matrix}-\frac{1}{2},-\frac{1}{4},0,\frac{1}{4},\frac{1}{2},1 & \\ -\frac{1}{4},\frac{1}{4} & -\frac{1}{2},0,0,0\end{matrix}\middle|\frac{e^{2i\pi}}{c^2x^2}\right)}{4\pi^{\frac{3}{2}}c}+\frac{bG_{6,6}^{6,2}\left(\begin{matrix}-\frac{3}{4},-\frac{1}{4} & -\frac{1}{2},-\frac{1}{2},0,1\\ -1,-\frac{3}{4},-\frac{1}{2},-\frac{1}{4},0,0 & \end{matrix}\middle|\frac{1}{c^2x^2}\right)}{4\pi^{\frac{3}{2}}c^3}-\frac{ibG_{6,6}^{2,6}\left(\begin{matrix}-\frac{3}{2},-\frac{5}{4},-1,-\frac{3}{4},-\frac{1}{2},1 & \\ -\frac{5}{4},-\frac{3}{4} & -\frac{3}{2},-1,-1,0\end{matrix}\middle|\frac{e^{2i\pi}}{c^2x^2}\right)}{4\pi^{\frac{3}{2}}c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)/(c*x-1)**(1/2)/(c*x+1)**(1/2),x)
```

```
[Out] a*meijerg(((1/4, 3/4), (1/2, 1/2, 1, 1)), ((0, 1/4, 1/2, 3/4, 1, 0), ()), 1
/(c**2*x**2))/(4*pi**(3/2)*c) - I*a*meijerg(((-1/2, -1/4, 0, 1/4, 1/2, 1),
()), ((-1/4, 1/4), (-1/2, 0, 0, 0)), exp_polar(2*I*pi)/(c**2*x**2))/(4*pi**
(3/2)*c) + b*meijerg(((-3/4, -1/4), (-1/2, -1/2, 0, 1)), ((-1, -3/4, -1/2,
-1/4, 0, 0), ()), 1/(c**2*x**2))/(4*pi**(3/2)*c**3) - I*b*meijerg(((-3/2, -
5/4, -1, -3/4, -1/2, 1), ()), ((-5/4, -3/4), (-3/2, -1, -1, 0)), exp_polar(
2*I*pi)/(c**2*x**2))/(4*pi**(3/2)*c**3)
```

3.210 $\int \frac{a+bx^2}{x\sqrt{-1+cx}\,\sqrt{1+cx}}\,dx$

Optimal. Leaf size=46

$$a\tan^{-1}\left(\sqrt{cx-1}\,\sqrt{cx+1}\right)+\frac{b\sqrt{cx-1}\,\sqrt{cx+1}}{c^2}$$

Rubi [A] time = 0.06, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {460, 92, 205}

$$a\tan^{-1}\left(\sqrt{cx-1}\,\sqrt{cx+1}\right)+\frac{b\sqrt{cx-1}\,\sqrt{cx+1}}{c^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)/(x*Sqrt[-1 + c*x]*Sqrt[1 + c*x]),x]
```

```
[Out] (b*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/c^2 + a*ArcTan[Sqrt[-1 + c*x]*Sqrt[1 + c*x
]]
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 460

```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[(d*(e*x)^
(m + 1)*(a1 + b1*x^(n/2))^(p + 1)*(a2 + b2*x^(n/2))^(p + 1))/(b1*b2*e*(m +
n*(p + 1) + 1)), x] - Dist[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/
(b1*b2*(m + n*(p + 1) + 1)), Int[(e*x)^m*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n/
2))^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, m, n, p}, x] && EqQ[non2,
n/2] && EqQ[a2*b1 + a1*b2, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{a+bx^2}{x\sqrt{-1+cx}\,\sqrt{1+cx}}\,dx &= \frac{b\sqrt{-1+cx}\,\sqrt{1+cx}}{c^2}+a\int \frac{1}{x\sqrt{-1+cx}\,\sqrt{1+cx}}\,dx\\
&= \frac{b\sqrt{-1+cx}\,\sqrt{1+cx}}{c^2}+(ac)\,\text{Subst}\left(\int \frac{1}{c+cx^2}\,dx,x,\sqrt{-1+cx}\,\sqrt{1+cx}\right)\\
&= \frac{b\sqrt{-1+cx}\,\sqrt{1+cx}}{c^2}+a\tan^{-1}\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)
\end{aligned}$$

Mathematica [A] time = 0.03, size = 66, normalized size = 1.43

$$\frac{ac^2\sqrt{c^2x^2-1}\,\tan^{-1}\left(\sqrt{c^2x^2-1}\right)+b\left(c^2x^2-1\right)}{c^2\sqrt{cx-1}\,\sqrt{cx+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)/(x*Sqrt[-1 + c*x]*Sqrt[1 + c*x]),x]
```

```
[Out] (b*(-1 + c^2*x^2) + a*c^2*Sqrt[-1 + c^2*x^2]*ArcTan[Sqrt[-1 + c^2*x^2]])/(c
^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

IntegrateAlgebraic [A] time = 0.08, size = 65, normalized size = 1.41

$$2a\tan^{-1}\left(\frac{\sqrt{cx-1}}{\sqrt{cx+1}}\right)-\frac{2b\sqrt{cx-1}}{c^2\sqrt{cx+1}\left(\frac{cx-1}{cx+1}-1\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x^2)/(x*Sqrt[-1 + c*x]*Sqrt[1 + c*x]),x]
```

```
[Out] (-2*b*Sqrt[-1 + c*x])/(c^2*Sqrt[1 + c*x]*(-1 + (-1 + c*x)/(1 + c*x))) + 2*a
*ArcTan[Sqrt[-1 + c*x]/Sqrt[1 + c*x]]
```

fricas [A] time = 0.83, size = 48, normalized size = 1.04

$$\frac{2\,ac^2\arctan\left(-cx+\sqrt{cx+1}\,\sqrt{cx-1}\right)+\sqrt{cx+1}\,\sqrt{cx-1}\,b}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/x/(c*x-1)^(1/2)/(c*x+1)^(1/2),x, algorithm="fricas")
```

```
[Out] (2*a*c^2*arctan(-c*x + sqrt(c*x + 1)*sqrt(c*x - 1)) + sqrt(c*x + 1)*sqrt(c*
x - 1)*b)/c^2
```

giac [A] time = 0.22, size = 45, normalized size = 0.98

$$-2\,a\arctan\left(\frac{1}{2}\left(\sqrt{cx+1}-\sqrt{cx-1}\right)^2\right)+\frac{\sqrt{cx+1}\,\sqrt{cx-1}\,b}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/x/(c*x-1)^(1/2)/(c*x+1)^(1/2),x, algorithm="giac")
```

```
[Out] -2*a*arctan(1/2*(sqrt(c*x + 1) - sqrt(c*x - 1))^2) + sqrt(c*x + 1)*sqrt(c*x
 - 1)*b/c^2
```

maple [A] time = 0.07, size = 62, normalized size = 1.35

$$\frac{\sqrt{cx-1}\,\sqrt{cx+1}\,\left(-a\,c^2\arctan\left(\frac{1}{\sqrt{c^2x^2-1}}\right)+\sqrt{c^2x^2-1}\,b\right)}{\sqrt{c^2x^2-1}\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)/x/(c*x-1)^(1/2)/(c*x+1)^(1/2),x)
```

```
[Out] (c*x-1)^(1/2)*(c*x+1)^(1/2)*(-arctan(1/(c^2*x^2-1)^(1/2))*a*c^2+(c^2*x^2-1)
^(1/2)*b)/(c^2*x^2-1)^(1/2)/c^2
```

maxima [A] time = 1.15, size = 29, normalized size = 0.63

$$-a\arcsin\left(\frac{1}{c|x|}\right)+\frac{\sqrt{c^2x^2-1}\,b}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/x/(c*x-1)^(1/2)/(c*x+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -a*arcsin(1/(c*abs(x))) + sqrt(c^2*x^2 - 1)*b/c^2
```

mupad [B] time = 3.86, size = 77, normalized size = 1.67

$$\frac{b\sqrt{c\,x-1}\,\sqrt{c\,x+1}}{c^2}-a\left(\ln\left(\frac{\left(\sqrt{c\,x-1}\,-\mathrm{i}\right)^2}{\left(\sqrt{c\,x+1}\,-1\right)^2}+1\right)-\ln\left(\frac{\sqrt{c\,x-1}\,-\mathrm{i}}{\sqrt{c\,x+1}\,-1}\right)\right)1\mathrm{i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)/(x*(c*x - 1)^(1/2)*(c*x + 1)^(1/2)),x)
```

```
[Out] (b*(c*x - 1)^(1/2)*(c*x + 1)^(1/2))/c^2 - a*(log(((c*x - 1)^(1/2) - 1i)^2/(
(c*x + 1)^(1/2) - 1)^2 + 1) - log(((c*x - 1)^(1/2) - 1i)/((c*x + 1)^(1/2) -
 1)))*1i
```

sympy [C] time = 40.14, size = 162, normalized size = 3.52

$$-\frac{aG_{6,6}^{5,3}\left(\begin{matrix}\frac{3}{4},\frac{5}{4},1 & 1,1,\frac{3}{2}\\ \frac{1}{2},\frac{3}{4},1,\frac{5}{4},\frac{3}{2} & 0\end{matrix}\middle|\frac{1}{c^2x^2}\right)}{4\pi^{\frac{3}{2}}}+\frac{iaG_{6,6}^{2,6}\left(\begin{matrix}0,\frac{1}{4},\frac{1}{2},\frac{3}{4},1,1 & \\ \frac{1}{4},\frac{3}{4} & 0,\frac{1}{2},\frac{1}{2},0\end{matrix}\middle|\frac{e^{2i\pi}}{c^2x^2}\right)}{4\pi^{\frac{3}{2}}}+\frac{bG_{6,6}^{6,2}\left(\begin{matrix}-\frac{1}{4},\frac{1}{4} & 0,0,\frac{1}{2},1\\ -\frac{1}{2},-\frac{1}{4},0,\frac{1}{4},\frac{1}{2},0 & \end{matrix}\middle|\frac{1}{c^2x^2}\right)}{4\pi^{\frac{3}{2}}c^2}+\frac{ibG_{6,6}^{2,6}\left(\begin{matrix}-1,-\frac{3}{4},-\frac{1}{2},-\frac{1}{4},0,1 & \\ -\frac{3}{4},-\frac{1}{4} & -1,-\frac{1}{2},-\frac{1}{2},0\end{matrix}\middle|\frac{e^{2i\pi}}{c^2x^2}\right)}{4\pi^{\frac{3}{2}}c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)/x/(c*x-1)**(1/2)/(c*x+1)**(1/2),x)
```

```
[Out] -a*meijerg(((3/4, 5/4, 1), (1, 1, 3/2)), ((1/2, 3/4, 1, 5/4, 3/2), (0,)), 1
/(c**2*x**2))/(4*pi**(3/2)) + I*a*meijerg(((0, 1/4, 1/2, 3/4, 1, 1), ()), (
(1/4, 3/4), (0, 1/2, 1/2, 0)), exp_polar(2*I*pi)/(c**2*x**2))/(4*pi**(3/2))
 + b*meijerg(((-1/4, 1/4), (0, 0, 1/2, 1)), ((-1/2, -1/4, 0, 1/4, 1/2, 0),
()), 1/(c**2*x**2))/(4*pi**(3/2)*c**2) + I*b*meijerg(((-1, -3/4, -1/2, -1/4
, 0, 1), ()), ((-3/4, -1/4), (-1, -1/2, -1/2, 0)), exp_polar(2*I*pi)/(c**2*
x**2))/(4*pi**(3/2)*c**2)
```

3.211 $\int \frac{a+bx^2}{x^2\sqrt{-1+cx}\,\sqrt{1+cx}}\,dx$

Optimal. Leaf size=33

$$\frac{a\sqrt{cx-1}\,\sqrt{cx+1}}{x}+\frac{b\cosh^{-1}(cx)}{c}$$

Rubi [A] time = 0.06, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {454, 52}

$$\frac{a\sqrt{cx-1}\,\sqrt{cx+1}}{x}+\frac{b\cosh^{-1}(cx)}{c}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)/(x^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]),x]
```

```
[Out] (a*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/x + (b*ArcCosh[c*x])/c
```

Rule 52

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[(b*x)/a]/b, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a + c, 0] && EqQ[b
- d, 0] && GtQ[a, 0]
```

Rule 454

```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[(c*(e*x)^
(m + 1)*(a1 + b1*x^(n/2))^(p + 1)*(a2 + b2*x^(n/2))^(p + 1))/(a1*a2*e*(m +
1)), x] + Dist[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/(a1*a2*e^n*(
m + 1)), Int[(e*x)^(m + n)*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n/2))^p, x], x]
/; FreeQ[{a1, b1, a2, b2, c, d, e, p}, x] && EqQ[non2, n/2] && EqQ[a2*b1 +
a1*b2, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (L
tQ[n, 0] && GtQ[m + n, -1])) && !ILtQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx^2}{x^2\sqrt{-1+cx}\,\sqrt{1+cx}}\,dx &= \frac{a\sqrt{-1+cx}\,\sqrt{1+cx}}{x}+b\int \frac{1}{\sqrt{-1+cx}\,\sqrt{1+cx}}\,dx\\ &= \frac{a\sqrt{-1+cx}\,\sqrt{1+cx}}{x}+\frac{b\cosh^{-1}(cx)}{c}\end{aligned}$$

Mathematica [B] time = 0.03, size = 73, normalized size = 2.21

$$\frac{\sqrt{c^2x^2-1}\left(\frac{a\sqrt{c^2x^2-1}}{x}+\frac{b\tanh^{-1}\left(\frac{cx}{\sqrt{c^2x^2-1}}\right)}{c}\right)}{\sqrt{cx-1}\,\sqrt{cx+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)/(x^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]),x]
```

```
[Out] (Sqrt[-1 + c^2*x^2]*((a*Sqrt[-1 + c^2*x^2])/x + (b*ArcTanh[(c*x)/Sqrt[-1 +
c^2*x^2]])/c))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

IntegrateAlgebraic [A] time = 0.09, size = 66, normalized size = 2.00

$$\frac{2ac\sqrt{cx-1}}{\sqrt{cx+1}\left(\frac{cx-1}{cx+1}+1\right)}+\frac{2b\tanh^{-1}\left(\frac{\sqrt{cx-1}}{\sqrt{cx+1}}\right)}{c}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x^2)/(x^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]),x]
```

```
[Out] (2*a*c*Sqrt[-1 + c*x])/(Sqrt[1 + c*x]*(1 + (-1 + c*x)/(1 + c*x))) + (2*b*Ar
cTanh[Sqrt[-1 + c*x]/Sqrt[1 + c*x]])/c
```

fricas [A] time = 0.87, size = 56, normalized size = 1.70

$$\frac{ac^2x+\sqrt{cx+1}\sqrt{cx-1}\,ac-bx\log\left(-cx+\sqrt{cx+1}\sqrt{cx-1}\right)}{cx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/x^2/(c*x-1)^(1/2)/(c*x+1)^(1/2),x, algorithm="fricas")
```

```
[Out] (a*c^2*x + sqrt(c*x + 1)*sqrt(c*x - 1)*a*c - b*x*log(-c*x + sqrt(c*x + 1)*s
qrt(c*x - 1)))/(c*x)
```

giac [A] time = 0.22, size = 58, normalized size = 1.76

$$\frac{\frac{16\,ac^2}{\left(\sqrt{cx+1}-\sqrt{cx-1}\right)^4+4}-b\log\left(\left(\sqrt{cx+1}-\sqrt{cx-1}\right)^4\right)}{2\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/x^2/(c*x-1)^(1/2)/(c*x+1)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*(16*a*c^2/((sqrt(c*x + 1) - sqrt(c*x - 1))^4 + 4) - b*log((sqrt(c*x + 1
) - sqrt(c*x - 1))^4))/c
```

maple [C] time = 0.07, size = 77, normalized size = 2.33

$$\frac{\sqrt{cx-1}\,\sqrt{cx+1}\,\left(\sqrt{c^2x^2-1}\,ac\,\mathrm{csgn}(c)+bx\ln\left(\left(cx+\sqrt{c^2x^2-1}\,\mathrm{csgn}(c)\right)\mathrm{csgn}(c)\right)\right)\mathrm{csgn}(c)}{\sqrt{c^2x^2-1}\,cx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)/x^2/(c*x-1)^(1/2)/(c*x+1)^(1/2),x)
```

```
[Out] (c*x-1)^(1/2)*(c*x+1)^(1/2)*(csgn(c)*c*(c^2*x^2-1)^(1/2)*a+ln((c*x+(c^2*x^2
-1)^(1/2)*csgn(c))*csgn(c))*x*b)*csgn(c)/(c^2*x^2-1)^(1/2)/c/x
```

maxima [A] time = 1.18, size = 44, normalized size = 1.33

$$\frac{b\log\left(2\,c^2x+2\,\sqrt{c^2x^2-1}\,c\right)}{c}+\frac{\sqrt{c^2x^2-1}\,a}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/x^2/(c*x-1)^(1/2)/(c*x+1)^(1/2),x, algorithm="maxima")
```

```
[Out] b*log(2*c^2*x + 2*sqrt(c^2*x^2 - 1)*c)/c + sqrt(c^2*x^2 - 1)*a/x
```

mupad [B] time = 2.59, size = 61, normalized size = 1.85

$$\frac{a\sqrt{c\,x-1}\,\sqrt{c\,x+1}}{x}-\frac{4\,b\,\mathrm{atan}\left(\frac{c\,\left(\sqrt{c\,x-1}-\mathrm{i}\right)}{\left(\sqrt{c\,x+1}-1\right)\,\sqrt{-c^2}}\right)}{\sqrt{-c^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)/(x^2*(c*x - 1)^(1/2)*(c*x + 1)^(1/2)),x)
```

```
[Out] (a*(c*x - 1)^(1/2)*(c*x + 1)^(1/2))/x - (4*b*atan((c*((c*x - 1)^(1/2) - 1i)
)/(((c*x + 1)^(1/2) - 1)*(-c^2)^(1/2))))/(-c^2)^(1/2)
```

sympy [C] time = 35.17, size = 148, normalized size = 4.48

$$-\frac{acG_{6,6}^{5,3}\left(\begin{matrix}\frac{5}{4},\frac{7}{4},1 & \frac{3}{2},\frac{3}{2},2\\ 1,\frac{5}{4},\frac{3}{2},\frac{7}{4},2 & 0\end{matrix}\middle|\frac{1}{c^2x^2}\right)}{4\pi^{\frac{3}{2}}}-\frac{iacG_{6,6}^{2,6}\left(\begin{matrix}\frac{1}{2},\frac{3}{4},1,\frac{5}{4},\frac{3}{2},1 & \\ \frac{3}{4},\frac{5}{4} & \frac{1}{2},1,1,0\end{matrix}\middle|\frac{e^{2i\pi}}{c^2x^2}\right)}{4\pi^{\frac{3}{2}}}+\frac{bG_{6,6}^{6,2}\left(\begin{matrix}\frac{1}{4},\frac{3}{4} & \frac{1}{2},\frac{1}{2},1,1\\ 0,\frac{1}{4},\frac{1}{2},\frac{3}{4},1,0 & \end{matrix}\middle|\frac{1}{c^2x^2}\right)}{4\pi^{\frac{3}{2}}c}-\frac{ibG_{6,6}^{2,6}\left(\begin{matrix}-\frac{1}{2},-\frac{1}{4},0,\frac{1}{4},\frac{1}{2},1 & \\ -\frac{1}{4},\frac{1}{4} & -\frac{1}{2},0,0,0\end{matrix}\middle|\frac{e^{2i\pi}}{c^2x^2}\right)}{4\pi^{\frac{3}{2}}c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)/x**2/(c*x-1)**(1/2)/(c*x+1)**(1/2),x)
```

```
[Out] -a*c*meijerg(((5/4, 7/4, 1), (3/2, 3/2, 2)), ((1, 5/4, 3/2, 7/4, 2), (0,)),
 1/(c**2*x**2))/(4*pi**(3/2)) - I*a*c*meijerg(((1/2, 3/4, 1, 5/4, 3/2, 1),
()), ((3/4, 5/4), (1/2, 1, 1, 0)), exp_polar(2*I*pi)/(c**2*x**2))/(4*pi**(3
/2)) + b*meijerg(((1/4, 3/4), (1/2, 1/2, 1, 1)), ((0, 1/4, 1/2, 3/4, 1, 0),
 ()), 1/(c**2*x**2))/(4*pi**(3/2)*c) - I*b*meijerg(((-1/2, -1/4, 0, 1/4, 1/
2, 1), ()), ((-1/4, 1/4), (-1/2, 0, 0, 0)), exp_polar(2*I*pi)/(c**2*x**2))/
(4*pi**(3/2)*c)
```

3.212 $\int \frac{a+bx^2}{x^3\sqrt{-1+cx}\,\sqrt{1+cx}}\,dx$

Optimal. Leaf size=60

$$\frac{1}{2}\left(ac^2+2b\right)\tan^{-1}\left(\sqrt{cx-1}\,\sqrt{cx+1}\right)+\frac{a\sqrt{cx-1}\,\sqrt{cx+1}}{2x^2}$$

Rubi [A] time = 0.06, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {454, 92, 205}

$$\frac{1}{2}\left(ac^2+2b\right)\tan^{-1}\left(\sqrt{cx-1}\,\sqrt{cx+1}\right)+\frac{a\sqrt{cx-1}\,\sqrt{cx+1}}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)/(x^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]),x]
```

```
[Out] (a*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(2*x^2) + ((2*b + a*c^2)*ArcTan[Sqrt[-1 +
c*x]*Sqrt[1 + c*x]])/2
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 454

```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[(c*(e*x)^
(m + 1)*(a1 + b1*x^(n/2))^(p + 1)*(a2 + b2*x^(n/2))^(p + 1))/(a1*a2*e*(m +
1)), x] + Dist[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/(a1*a2*e^n*(
m + 1)), Int[(e*x)^(m + n)*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n/2))^p, x], x]
/; FreeQ[{a1, b1, a2, b2, c, d, e, p}, x] && EqQ[non2, n/2] && EqQ[a2*b1 +
a1*b2, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (L
tQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{a+bx^2}{x^3\sqrt{-1+cx}\,\sqrt{1+cx}}\,dx &= \frac{a\sqrt{-1+cx}\,\sqrt{1+cx}}{2x^2}+\frac{1}{2}\left(2b+ac^2\right)\int \frac{1}{x\sqrt{-1+cx}\,\sqrt{1+cx}}\,dx\\
&= \frac{a\sqrt{-1+cx}\,\sqrt{1+cx}}{2x^2}+\frac{1}{2}\left(c\left(2b+ac^2\right)\right)\text{Subst}\left(\int \frac{1}{c+cx^2}\,dx,x,\sqrt{-1+cx}\,\sqrt{1+c
\end{aligned}$$

$$= \frac{a\sqrt{-1+cx}\,\sqrt{1+cx}}{2x^2}+\frac{1}{2}\left(2b+ac^2\right)\tan^{-1}\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)$$

Mathematica [A] time = 0.07, size = 77, normalized size = 1.28

$$\frac{x^2\sqrt{c^2x^2-1}\left(ac^2+2b\right)\tan^{-1}\left(\sqrt{c^2x^2-1}\right)+a\left(c^2x^2-1\right)}{2x^2\sqrt{cx-1}\,\sqrt{cx+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)/(x^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]),x]
```

```
[Out] (a*(-1 + c^2*x^2) + (2*b + a*c^2)*x^2*Sqrt[-1 + c^2*x^2]*ArcTan[Sqrt[-1 + c
^2*x^2]])/(2*x^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

IntegrateAlgebraic [A] time = 0.10, size = 87, normalized size = 1.45

$$(ac^2+2b)\tan^{-1}\left(\frac{\sqrt{cx-1}}{\sqrt{cx+1}}\right)-\frac{ac^2\sqrt{cx-1}\left(\frac{cx-1}{cx+1}-1\right)}{\sqrt{cx+1}\left(\frac{cx-1}{cx+1}+1\right)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x^2)/(x^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]),x]
```

```
[Out] -((a*c^2*Sqrt[-1 + c*x]*(-1 + (-1 + c*x)/(1 + c*x)))/(Sqrt[1 + c*x]*(1 + (-
1 + c*x)/(1 + c*x))^2)) + (2*b + a*c^2)*ArcTan[Sqrt[-1 + c*x]/Sqrt[1 + c*x]
]
```

fricas [A] time = 0.84, size = 57, normalized size = 0.95

$$\frac{2\left(ac^2+2b\right)x^2\arctan\left(-cx+\sqrt{cx+1}\sqrt{cx-1}\right)+\sqrt{cx+1}\sqrt{cx-1}a}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/x^3/(c*x-1)^(1/2)/(c*x+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*(2*(a*c^2 + 2*b)*x^2*arctan(-c*x + sqrt(c*x + 1)*sqrt(c*x - 1)) + sqrt(
c*x + 1)*sqrt(c*x - 1)*a)/x^2
```

giac [B] time = 0.21, size = 114, normalized size = 1.90

$$-\frac{\left(ac^3+2bc\right)\arctan\left(\frac{1}{2}\left(\sqrt{cx+1}-\sqrt{cx-1}\right)^2\right)+\frac{2\left(ac^3\left(\sqrt{cx+1}-\sqrt{cx-1}\right)^6-4ac^3\left(\sqrt{cx+1}-\sqrt{cx-1}\right)^2\right)}{\left(\left(\sqrt{cx+1}-\sqrt{cx-1}\right)^4+4\right)^2}}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/x^3/(c*x-1)^(1/2)/(c*x+1)^(1/2),x, algorithm="giac")
```

```
[Out] -((a*c^3 + 2*b*c)*arctan(1/2*(sqrt(c*x + 1) - sqrt(c*x - 1))^2) + 2*(a*c^3*
(sqrt(c*x + 1) - sqrt(c*x - 1))^6 - 4*a*c^3*(sqrt(c*x + 1) - sqrt(c*x - 1))
^2)/((sqrt(c*x + 1) - sqrt(c*x - 1))^4 + 4)^2)/c
```

maple [A] time = 0.08, size = 84, normalized size = 1.40

$$-\frac{\sqrt{cx-1}\sqrt{cx+1}\left(ac^2x^2\arctan\left(\frac{1}{\sqrt{c^2x^2-1}}\right)+2bx^2\arctan\left(\frac{1}{\sqrt{c^2x^2-1}}\right)-\sqrt{c^2x^2-1}\,a\right)}{2\sqrt{c^2x^2-1}\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)/x^3/(c*x-1)^(1/2)/(c*x+1)^(1/2),x)
```

```
[Out] -1/2*(c*x-1)^(1/2)*(c*x+1)^(1/2)*(arctan(1/(c^2*x^2-1)^(1/2))*x^2*a*c^2+2*a
rctan(1/(c^2*x^2-1)^(1/2))*x^2*b-(c^2*x^2-1)^(1/2)*a)/(c^2*x^2-1)^(1/2)/x^2
```

maxima [A] time = 1.23, size = 45, normalized size = 0.75

$$-\frac{1}{2}\,ac^2\arcsin\left(\frac{1}{c|x|}\right)-b\arcsin\left(\frac{1}{c|x|}\right)+\frac{\sqrt{c^2x^2-1}\,a}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/x^3/(c*x-1)^(1/2)/(c*x+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/2*a*c^2*arcsin(1/(c*abs(x))) - b*arcsin(1/(c*abs(x))) + 1/2*sqrt(c^2*x^2
 - 1)*a/x^2
```

mupad [B] time = 8.67, size = 297, normalized size = 4.95

$$\frac{\frac{ac^2\,1\mathrm{i}}{32}+\frac{ac^2\left(\sqrt{cx-1}-\mathrm{i}\right)^2 1\mathrm{i}}{16\left(\sqrt{cx+1}-1\right)^2}-\frac{ac^2\left(\sqrt{cx-1}-\mathrm{i}\right)^4 15\mathrm{i}}{32\left(\sqrt{cx+1}-1\right)^4}}{\frac{\left(\sqrt{cx-1}-\mathrm{i}\right)^2}{\left(\sqrt{cx+1}-1\right)^2}+\frac{2\left(\sqrt{cx-1}-\mathrm{i}\right)^4}{\left(\sqrt{cx+1}-1\right)^4}+\frac{\left(\sqrt{cx-1}-\mathrm{i}\right)^6}{\left(\sqrt{cx+1}-1\right)^6}}-b\left(\ln\left(\frac{\left(\sqrt{cx-1}-\mathrm{i}\right)^2}{\left(\sqrt{cx+1}-1\right)^2}+1\right)-\ln\left(\frac{\sqrt{cx-1}-\mathrm{i}}{\sqrt{cx+1}-1}\right)\right)1\mathrm{i}-\frac{ac^2\ln\left(\frac{\left(\sqrt{cx-1}-\mathrm{i}\right)^2}{\left(\sqrt{cx+1}-1\right)^2}+1\right)1\mathrm{i}}{2}+\frac{ac^2\ln\left(\frac{\sqrt{cx-1}-\mathrm{i}}{\sqrt{cx+1}-1}\right)1\mathrm{i}}{2}+\frac{ac^2\left(\sqrt{cx-1}-\mathrm{i}\right)^2 1\mathrm{i}}{32\left(\sqrt{cx+1}-1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)/(x^3*(c*x - 1)^(1/2)*(c*x + 1)^(1/2)),x)
```

```
[Out] ((a*c^2*1i)/32 + (a*c^2*((c*x - 1)^(1/2) - 1i)^2*1i)/(16*((c*x + 1)^(1/2) -
 1)^2) - (a*c^2*((c*x - 1)^(1/2) - 1i)^4*15i)/(32*((c*x + 1)^(1/2) - 1)^4))
/(((c*x - 1)^(1/2) - 1i)^2/((c*x + 1)^(1/2) - 1)^2 + (2*((c*x - 1)^(1/2) -
1i)^4)/((c*x + 1)^(1/2) - 1)^4 + ((c*x - 1)^(1/2) - 1i)^6/((c*x + 1)^(1/2)
- 1)^6) - b*(log(((c*x - 1)^(1/2) - 1i)^2/((c*x + 1)^(1/2) - 1)^2 + 1) - lo
g(((c*x - 1)^(1/2) - 1i)/((c*x + 1)^(1/2) - 1)))*1i - (a*c^2*log(((c*x - 1)
^(1/2) - 1i)^2/((c*x + 1)^(1/2) - 1)^2 + 1)*1i)/2 + (a*c^2*log(((c*x - 1)^(
1/2) - 1i)/((c*x + 1)^(1/2) - 1))*1i)/2 + (a*c^2*((c*x - 1)^(1/2) - 1i)^2*1
i)/(32*((c*x + 1)^(1/2) - 1)^2)
```

sympy [C] time = 63.62, size = 141, normalized size = 2.35

$$-\frac{ac^2G_{6,6}^{5,3}\left(\begin{matrix}\frac{7}{4},\frac{9}{4},1 & 2,2,\frac{5}{2}\\ \frac{3}{2},\frac{7}{4},2,\frac{9}{4},\frac{5}{2} & 0\end{matrix}\middle|\frac{1}{c^2x^2}\right)}{4\pi^{\frac{3}{2}}}+\frac{iac^2G_{6,6}^{2,6}\left(\begin{matrix}1,\frac{5}{4},\frac{3}{2},\frac{7}{4},2,1 & \\ \frac{5}{4},\frac{7}{4} & 1,\frac{3}{2},\frac{3}{2},0\end{matrix}\middle|\frac{e^{2i\pi}}{c^2x^2}\right)}{4\pi^{\frac{3}{2}}}-\frac{bG_{6,6}^{5,3}\left(\begin{matrix}\frac{3}{4},\frac{5}{4},1 & 1,1,\frac{3}{2}\\ \frac{1}{2},\frac{3}{4},1,\frac{5}{4},\frac{3}{2} & 0\end{matrix}\middle|\frac{1}{c^2x^2}\right)}{4\pi^{\frac{3}{2}}}+\frac{ibG_{6,6}^{2,6}\left(\begin{matrix}0,\frac{1}{4},\frac{1}{2},\frac{3}{4},1,1 & \\ \frac{1}{4},\frac{3}{4} & 0,\frac{1}{2},\frac{1}{2},0\end{matrix}\middle|\frac{e^{2i\pi}}{c^2x^2}\right)}{4\pi^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)/x**3/(c*x-1)**(1/2)/(c*x+1)**(1/2),x)
```

```
[Out] -a*c**2*meijerg(((7/4, 9/4, 1), (2, 2, 5/2)), ((3/2, 7/4, 2, 9/4, 5/2), (0,
)), 1/(c**2*x**2))/(4*pi**(3/2)) + I*a*c**2*meijerg(((1, 5/4, 3/2, 7/4, 2,
1), ()), ((5/4, 7/4), (1, 3/2, 3/2, 0)), exp_polar(2*I*pi)/(c**2*x**2))/(4*
pi**(3/2)) - b*meijerg(((3/4, 5/4, 1), (1, 1, 3/2)), ((1/2, 3/4, 1, 5/4, 3/
2), (0,)), 1/(c**2*x**2))/(4*pi**(3/2)) + I*b*meijerg(((0, 1/4, 1/2, 3/4, 1
, 1), ()), ((1/4, 3/4), (0, 1/2, 1/2, 0)), exp_polar(2*I*pi)/(c**2*x**2))/(
4*pi**(3/2))
```

3.213 $\int \frac{a+bx^2}{x^4\sqrt{-1+cx}\,\sqrt{1+cx}}\,dx$

Optimal. Leaf size=62

$$\frac{\sqrt{cx-1}\,\sqrt{cx+1}\,\left(2ac^2+3b\right)}{3x}+\frac{a\sqrt{cx-1}\,\sqrt{cx+1}}{3x^3}$$

Rubi [A] time = 0.06, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {454, 95}

$$\frac{\sqrt{cx-1}\,\sqrt{cx+1}\,\left(2ac^2+3b\right)}{3x}+\frac{a\sqrt{cx-1}\,\sqrt{cx+1}}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)/(x^4*Sqrt[-1 + c*x]*Sqrt[1 + c*x]),x]
```

```
[Out] (a*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(3*x^3) + ((3*b + 2*a*c^2)*Sqrt[-1 + c*x]*
Sqrt[1 + c*x])/(3*x)
```

Rule 95

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] /; FreeQ[{a, b, c, d, e, f
, m, n, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && EqQ[a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1), 0] && NeQ[m, -1]
```

Rule 454

```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[(c*(e*x)^
(m + 1)*(a1 + b1*x^(n/2))^(p + 1)*(a2 + b2*x^(n/2))^(p + 1))/(a1*a2*e*(m +
1)), x] + Dist[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/(a1*a2*e^n*(
m + 1)), Int[(e*x)^(m + n)*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n/2))^p, x], x]
/; FreeQ[{a1, b1, a2, b2, c, d, e, p}, x] && EqQ[non2, n/2] && EqQ[a2*b1 +
a1*b2, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (L
tQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx^2}{x^4\sqrt{-1+cx}\,\sqrt{1+cx}}\,dx &= \frac{a\sqrt{-1+cx}\,\sqrt{1+cx}}{3x^3}+\frac{1}{3}\left(3b+2ac^2\right)\int \frac{1}{x^2\sqrt{-1+cx}\,\sqrt{1+cx}}\,dx\\ &= \frac{a\sqrt{-1+cx}\,\sqrt{1+cx}}{3x^3}+\frac{\left(3b+2ac^2\right)\sqrt{-1+cx}\,\sqrt{1+cx}}{3x}\end{aligned}$$

Mathematica [A] time = 0.02, size = 51, normalized size = 0.82

$$\frac{\left(c^2x^2-1\right)\left(2ac^2x^2+a+3bx^2\right)}{3x^3\sqrt{cx-1}\,\sqrt{cx+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)/(x^4*Sqrt[-1 + c*x]*Sqrt[1 + c*x]),x]
```

```
[Out] ((-1 + c^2*x^2)*(a + 3*b*x^2 + 2*a*c^2*x^2))/(3*x^3*Sqrt[-1 + c*x]*Sqrt[1 +
 c*x])
```

IntegrateAlgebraic [A] time = 0.11, size = 122, normalized size = 1.97

$$\frac{2\sqrt{cx-1}\left(\frac{2ac^3(cx-1)}{cx+1}+\frac{3ac^3(cx-1)^2}{(cx+1)^2}+3ac^3+\frac{6bc(cx-1)}{cx+1}+\frac{3bc(cx-1)^2}{(cx+1)^2}+3bc\right)}{3\sqrt{cx+1}\left(\frac{cx-1}{cx+1}+1\right)^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x^2)/(x^4*Sqrt[-1 + c*x]*Sqrt[1 + c*x]),x]
```

```
[Out] (2*Sqrt[-1 + c*x]*(3*b*c + 3*a*c^3 + (3*b*c*(-1 + c*x)^2)/(1 + c*x)^2 + (3*
a*c^3*(-1 + c*x)^2)/(1 + c*x)^2 + (6*b*c*(-1 + c*x))/(1 + c*x) + (2*a*c^3*(
-1 + c*x))/(1 + c*x)))/(3*Sqrt[1 + c*x]*(1 + (-1 + c*x)/(1 + c*x))^3)
```

fricas [A] time = 0.85, size = 52, normalized size = 0.84

$$\frac{\left(2\,ac^3+3\,bc\right)x^3+\left(\left(2\,ac^2+3\,b\right)x^2+a\right)\sqrt{cx+1}\,\sqrt{cx-1}}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/x^4/(c*x-1)^(1/2)/(c*x+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*((2*a*c^3 + 3*b*c)*x^3 + ((2*a*c^2 + 3*b)*x^2 + a)*sqrt(c*x + 1)*sqrt(c
*x - 1))/x^3
```

giac [B] time = 0.21, size = 116, normalized size = 1.87

$$\frac{8\left(3\,bc^2\left(\sqrt{cx+1}-\sqrt{cx-1}\right)^8+24\,ac^4\left(\sqrt{cx+1}-\sqrt{cx-1}\right)^4+24\,bc^2\left(\sqrt{cx+1}-\sqrt{cx-1}\right)^4+32\,ac^4+48\,bc^2\right)}{3\left(\left(\sqrt{cx+1}-\sqrt{cx-1}\right)^4+4\right)^3c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/x^4/(c*x-1)^(1/2)/(c*x+1)^(1/2),x, algorithm="giac")
```

```
[Out] 8/3*(3*b*c^2*(sqrt(c*x + 1) - sqrt(c*x - 1))^8 + 24*a*c^4*(sqrt(c*x + 1) -
sqrt(c*x - 1))^4 + 24*b*c^2*(sqrt(c*x + 1) - sqrt(c*x - 1))^4 + 32*a*c^4 +
48*b*c^2)/(((sqrt(c*x + 1) - sqrt(c*x - 1))^4 + 4)^3*c)
```

maple [A] time = 0.05, size = 37, normalized size = 0.60

$$\frac{\sqrt{cx+1}\,\sqrt{cx-1}\,\left(2a\,c^2x^2+3b\,x^2+a\right)}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)/x^4/(c*x-1)^(1/2)/(c*x+1)^(1/2),x)
```

```
[Out] 1/3*(c*x+1)^(1/2)*(c*x-1)^(1/2)*(2*a*c^2*x^2+3*b*x^2+a)/x^3
```

maxima [A] time = 1.23, size = 54, normalized size = 0.87

$$\frac{2\,\sqrt{c^2x^2-1}\,ac^2}{3\,x}+\frac{\sqrt{c^2x^2-1}\,b}{x}+\frac{\sqrt{c^2x^2-1}\,a}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/x^4/(c*x-1)^(1/2)/(c*x+1)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/3*sqrt(c^2*x^2 - 1)*a*c^2/x + sqrt(c^2*x^2 - 1)*b/x + 1/3*sqrt(c^2*x^2 -
1)*a/x^3
```

mupad [B] time = 2.44, size = 53, normalized size = 0.85

$$\frac{\sqrt{c\,x-1}\,\left(\left(\frac{2\,a\,c^3}{3}+b\,c\right)\,x^3+\left(\frac{2\,a\,c^2}{3}+b\right)\,x^2+\frac{a\,c\,x}{3}+\frac{a}{3}\right)}{x^3\,\sqrt{c\,x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)/(x^4*(c*x - 1)^(1/2)*(c*x + 1)^(1/2)),x)
```

```
[Out] ((c*x - 1)^(1/2)*(a/3 + x^3*(b*c + (2*a*c^3)/3) + x^2*(b + (2*a*c^2)/3) + (
a*c*x)/3))/(x^3*(c*x + 1)^(1/2))
```

sympy [C] time = 61.44, size = 146, normalized size = 2.35

$$-\frac{ac^3G_{6,6}^{5,3}\left(\begin{matrix}\frac{9}{4},\frac{11}{4},1 & \frac{5}{2},\frac{5}{2},3\\ 2,\frac{9}{4},\frac{5}{2},\frac{11}{4},3 & 0\end{matrix}\middle|\frac{1}{c^2x^2}\right)}{4\pi^{\frac{3}{2}}}-\frac{iac^3G_{6,6}^{2,6}\left(\begin{matrix}\frac{3}{2},\frac{7}{4},2,\frac{9}{4},\frac{5}{2},1 & \\ \frac{7}{4},\frac{9}{4} & \frac{3}{2},2,2,0\end{matrix}\middle|\frac{e^{2i\pi}}{c^2x^2}\right)}{4\pi^{\frac{3}{2}}}-\frac{bcG_{6,6}^{5,3}\left(\begin{matrix}\frac{5}{4},\frac{7}{4},1 & \frac{3}{2},\frac{3}{2},2\\ 1,\frac{5}{4},\frac{3}{2},\frac{7}{4},2 & 0\end{matrix}\middle|\frac{1}{c^2x^2}\right)}{4\pi^{\frac{3}{2}}}-\frac{ibcG_{6,6}^{2,6}\left(\begin{matrix}\frac{1}{2},\frac{3}{4},1,\frac{5}{4},\frac{3}{2},1 & \\ \frac{3}{4},\frac{5}{4} & \frac{1}{2},1,1,0\end{matrix}\middle|\frac{e^{2i\pi}}{c^2x^2}\right)}{4\pi^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)/x**4/(c*x-1)**(1/2)/(c*x+1)**(1/2),x)
```

```
[Out] -a*c**3*meijerg(((9/4, 11/4, 1), (5/2, 5/2, 3)), ((2, 9/4, 5/2, 11/4, 3), (
0,)), 1/(c**2*x**2))/(4*pi**(3/2)) - I*a*c**3*meijerg(((3/2, 7/4, 2, 9/4, 5
/2, 1), ()), ((7/4, 9/4), (3/2, 2, 2, 0)), exp_polar(2*I*pi)/(c**2*x**2))/(
4*pi**(3/2)) - b*c*meijerg(((5/4, 7/4, 1), (3/2, 3/2, 2)), ((1, 5/4, 3/2, 7
/4, 2), (0,)), 1/(c**2*x**2))/(4*pi**(3/2)) - I*b*c*meijerg(((1/2, 3/4, 1,
5/4, 3/2, 1), ()), ((3/4, 5/4), (1/2, 1, 1, 0)), exp_polar(2*I*pi)/(c**2*x*
*2))/(4*pi**(3/2))
```

3.214 $\int \frac{a+bx^2}{x^5\sqrt{-1+cx}\sqrt{1+cx}}\,dx$

Optimal. Leaf size=99

$$\frac{\sqrt{cx-1}\sqrt{cx+1}\left(3ac^2+4b\right)}{8x^2}+\frac{1}{8}c^2\left(3ac^2+4b\right)\tan^{-1}\left(\sqrt{cx-1}\sqrt{cx+1}\right)+\frac{a\sqrt{cx-1}\sqrt{cx+1}}{4x^4}$$

Rubi [A] time = 0.08, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {454, 103, 12, 92, 205}

$$\frac{\sqrt{cx-1}\sqrt{cx+1}\left(3ac^2+4b\right)}{8x^2}+\frac{1}{8}c^2\left(3ac^2+4b\right)\tan^{-1}\left(\sqrt{cx-1}\sqrt{cx+1}\right)+\frac{a\sqrt{cx-1}\sqrt{cx+1}}{4x^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)/(x^5*Sqrt[-1 + c*x]*Sqrt[1 + c*x]),x]
```

```
[Out] (a*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(4*x^4) + ((4*b + 3*a*c^2)*Sqrt[-1 + c*x]*
Sqrt[1 + c*x])/(8*x^2) + (c^2*(4*b + 3*a*c^2)*ArcTan[Sqrt[-1 + c*x]*Sqrt[1
+ c*x]])/8
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 454

```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[(c*(e*x)^
(m + 1)*(a1 + b1*x^(n/2))^(p + 1)*(a2 + b2*x^(n/2))^(p + 1))/(a1*a2*e*(m +
1)), x] + Dist[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/(a1*a2*e^n*(
m + 1)), Int[(e*x)^(m + n)*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n/2))^p, x], x]
/; FreeQ[{a1, b1, a2, b2, c, d, e, p}, x] && EqQ[non2, n/2] && EqQ[a2*b1 +
a1*b2, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (L
tQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{a+bx^2}{x^5\sqrt{-1+cx}\,\sqrt{1+cx}}\,dx &= \frac{a\sqrt{-1+cx}\,\sqrt{1+cx}}{4x^4} + \frac{1}{4}\left(4b+3ac^2\right)\int \frac{1}{x^3\sqrt{-1+cx}\,\sqrt{1+cx}}\,dx \\
&= \frac{a\sqrt{-1+cx}\,\sqrt{1+cx}}{4x^4} + \frac{\left(4b+3ac^2\right)\sqrt{-1+cx}\,\sqrt{1+cx}}{8x^2} + \frac{1}{8}\left(4b+3ac^2\right)\int \frac{\cdots}{x\sqrt{-1+\cdots}} \\
&= \frac{a\sqrt{-1+cx}\,\sqrt{1+cx}}{4x^4} + \frac{\left(4b+3ac^2\right)\sqrt{-1+cx}\,\sqrt{1+cx}}{8x^2} + \frac{1}{8}\left(c^2\left(4b+3ac^2\right)\right)\int \frac{\cdots}{x\sqrt{\cdots}} \\
&= \frac{a\sqrt{-1+cx}\,\sqrt{1+cx}}{4x^4} + \frac{\left(4b+3ac^2\right)\sqrt{-1+cx}\,\sqrt{1+cx}}{8x^2} + \frac{1}{8}\left(c^3\left(4b+3ac^2\right)\right)\text{Subst}\left(\cdots\right) \\
&= \frac{a\sqrt{-1+cx}\,\sqrt{1+cx}}{4x^4} + \frac{\left(4b+3ac^2\right)\sqrt{-1+cx}\,\sqrt{1+cx}}{8x^2} + \frac{1}{8}c^2\left(4b+3ac^2\right)\tan^{-1}\left(\sqrt{\cdots}\right)
\end{aligned}
$$

Mathematica [A] time = 0.10, size = 102, normalized size = 1.03

$$
\frac{\left(c^2x^2-1\right)\left(a\left(3c^2x^2+2\right)+4bx^2\right)-c^2x^4\sqrt{1-c^2x^2}\left(3ac^2+4b\right)\tanh^{-1}\left(\sqrt{1-c^2x^2}\right)}{8x^4\sqrt{cx-1}\,\sqrt{cx+1}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)/(x^5*Sqrt[-1 + c*x]*Sqrt[1 + c*x]),x]
```

```
[Out] ((-1 + c^2*x^2)*(4*b*x^2 + a*(2 + 3*c^2*x^2)) - c^2*(4*b + 3*a*c^2)*x^4*Sqr
t[1 - c^2*x^2]*ArcTanh[Sqrt[1 - c^2*x^2]])/(8*x^4*Sqrt[-1 + c*x]*Sqrt[1 + c
*x])
```

IntegrateAlgebraic [A] time = 0.16, size = 175, normalized size = 1.77

$$
\frac{1}{4}\left(3ac^4+4bc^2\right)\tan^{-1}\left(\frac{\sqrt{cx-1}}{\sqrt{cx+1}}\right)-\frac{c^2\sqrt{cx-1}\left(\frac{cx-1}{cx+1}-1\right)\left(\frac{2ac^2(cx-1)}{cx+1}+\frac{5ac^2(cx-1)^2}{(cx+1)^2}+5ac^2+\frac{8b(cx-1)}{cx+1}+\frac{4b(cx-1)^2}{(cx+1)^2}+4b\right)}{4\sqrt{cx+1}\left(\frac{cx-1}{cx+1}+1\right)^4}
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x^2)/(x^5*Sqrt[-1 + c*x]*Sqrt[1 + c*x]),x]
```

```
[Out] -1/4*(c^2*Sqrt[-1 + c*x]*(-1 + (-1 + c*x)/(1 + c*x))*(4*b + 5*a*c^2 + (4*b*
(-1 + c*x)^2)/(1 + c*x)^2 + (5*a*c^2*(-1 + c*x)^2)/(1 + c*x)^2 + (8*b*(-1 +
 c*x))/(1 + c*x) + (2*a*c^2*(-1 + c*x))/(1 + c*x)))/(Sqrt[1 + c*x]*(1 + (-1
 + c*x)/(1 + c*x))^4) + ((4*b*c^2 + 3*a*c^4)*ArcTan[Sqrt[-1 + c*x]/Sqrt[1 +
 c*x]])/4
```

fricas [A] time = 0.59, size = 78, normalized size = 0.79

$$
\frac{2\left(3\,ac^4+4\,bc^2\right)x^4\arctan\left(-cx+\sqrt{cx+1}\,\sqrt{cx-1}\right)+\left(\left(3\,ac^2+4\,b\right)x^2+2\,a\right)\sqrt{cx+1}\,\sqrt{cx-1}}{8\,x^4}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/x^5/(c*x-1)^(1/2)/(c*x+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/8*(2*(3*a*c^4 + 4*b*c^2)*x^4*arctan(-c*x + sqrt(c*x + 1)*sqrt(c*x - 1)) +
 ((3*a*c^2 + 4*b)*x^2 + 2*a)*sqrt(c*x + 1)*sqrt(c*x - 1))/x^4
```

giac [B] time = 0.22, size = 268, normalized size = 2.71

$$-\frac{\left(3\,ac^{5}+4\,bc^{3}\right)\arctan\left(\frac{1}{2}\left(\sqrt{cx+1}-\sqrt{cx-1}\right)^{2}\right)+\frac{2\left(3\,ac^{5}\left(\sqrt{cx+1}-\sqrt{cx-1}\right)^{14}+4\,bc^{3}\left(\sqrt{cx+1}-\sqrt{cx-1}\right)^{14}+44\,ac^{5}\left(\sqrt{cx+1}-\sqrt{cx-1}\right)^{10}+16\,bc^{3}\left(\sqrt{cx+1}-\sqrt{cx-1}\right)^{10}-176\,ac^{5}\left(\sqrt{cx+1}-\sqrt{cx-1}\right)^{6}-64\,bc^{3}\left(\sqrt{cx+1}-\sqrt{cx-1}\right)^{6}-192\,ac^{5}\left(\sqrt{cx+1}-\sqrt{cx-1}\right)^{2}-256\,bc^{3}\left(\sqrt{cx+1}-\sqrt{cx-1}\right)^{2}\right)}{\left(\left(\sqrt{cx+1}-\sqrt{cx-1}\right)^{4}+4\right)^{4}}}{4\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/x^5/(c*x-1)^(1/2)/(c*x+1)^(1/2),x, algorithm="giac")
```

```
[Out] -1/4*((3*a*c^5 + 4*b*c^3)*arctan(1/2*(sqrt(c*x + 1) - sqrt(c*x - 1))^2) + 2
*(3*a*c^5*(sqrt(c*x + 1) - sqrt(c*x - 1))^14 + 4*b*c^3*(sqrt(c*x + 1) - sqr
t(c*x - 1))^14 + 44*a*c^5*(sqrt(c*x + 1) - sqrt(c*x - 1))^10 + 16*b*c^3*(sq
rt(c*x + 1) - sqrt(c*x - 1))^10 - 176*a*c^5*(sqrt(c*x + 1) - sqrt(c*x - 1))
^6 - 64*b*c^3*(sqrt(c*x + 1) - sqrt(c*x - 1))^6 - 192*a*c^5*(sqrt(c*x + 1)
- sqrt(c*x - 1))^2 - 256*b*c^3*(sqrt(c*x + 1) - sqrt(c*x - 1))^2)/((sqrt(c*
x + 1) - sqrt(c*x - 1))^4 + 4)^4)/c
```

maple [A] time = 0.07, size = 125, normalized size = 1.26

$$-\frac{\sqrt{cx-1}\,\sqrt{cx+1}\,\left(3a\,c^{4}x^{4}\arctan\left(\frac{1}{\sqrt{c^{2}x^{2}-1}}\right)+4b\,c^{2}x^{4}\arctan\left(\frac{1}{\sqrt{c^{2}x^{2}-1}}\right)-3\sqrt{c^{2}x^{2}-1}\,a\,c^{2}x^{2}-4\sqrt{c^{2}x^{2}-1}\,b\,x^{2}-2\sqrt{c^{2}x^{2}-1}\,a\right)}{8\sqrt{c^{2}x^{2}-1}\,x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)/x^5/(c*x-1)^(1/2)/(c*x+1)^(1/2),x)
```

```
[Out] -1/8*(c*x-1)^(1/2)*(c*x+1)^(1/2)*(3*arctan(1/(c^2*x^2-1)^(1/2))*x^4*a*c^4+4
*arctan(1/(c^2*x^2-1)^(1/2))*x^4*b*c^2-3*(c^2*x^2-1)^(1/2)*x^2*a*c^2-4*(c^2
*x^2-1)^(1/2)*x^2*b-2*(c^2*x^2-1)^(1/2)*a)/(c^2*x^2-1)^(1/2)/x^4
```

maxima [A] time = 1.11, size = 85, normalized size = 0.86

$$-\frac{3}{8}\,ac^{4}\arcsin\left(\frac{1}{c|x|}\right)-\frac{1}{2}\,bc^{2}\arcsin\left(\frac{1}{c|x|}\right)+\frac{3\,\sqrt{c^{2}x^{2}-1}\,ac^{2}}{8\,x^{2}}+\frac{\sqrt{c^{2}x^{2}-1}\,b}{2\,x^{2}}+\frac{\sqrt{c^{2}x^{2}-1}\,a}{4\,x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/x^5/(c*x-1)^(1/2)/(c*x+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -3/8*a*c^4*arcsin(1/(c*abs(x))) - 1/2*b*c^2*arcsin(1/(c*abs(x))) + 3/8*sqrt
(c^2*x^2 - 1)*a*c^2/x^2 + 1/2*sqrt(c^2*x^2 - 1)*b/x^2 + 1/4*sqrt(c^2*x^2 -
1)*a/x^4
```

mupad [B] time = 21.45, size = 650, normalized size = 6.57

$$\frac{\frac{b\,c^{2}\,1\mathrm{i}}{32}+\frac{b\,c^{2}\,{\left(\sqrt{c\,x-1}-\mathrm{i}\right)}^{2}\,1\mathrm{i}}{16\,{\left(\sqrt{c\,x+1}-1\right)}^{2}}-\frac{b\,c^{2}\,{\left(\sqrt{c\,x-1}-\mathrm{i}\right)}^{4}\,15\mathrm{i}}{32\,{\left(\sqrt{c\,x+1}-1\right)}^{4}}}{\frac{{\left(\sqrt{c\,x-1}-\mathrm{i}\right)}^{2}}{{\left(\sqrt{c\,x+1}-1\right)}^{2}}+\frac{2\,{\left(\sqrt{c\,x-1}-\mathrm{i}\right)}^{4}}{{\left(\sqrt{c\,x+1}-1\right)}^{4}}+\frac{{\left(\sqrt{c\,x-1}-\mathrm{i}\right)}^{6}}{{\left(\sqrt{c\,x+1}-1\right)}^{6}}}-\frac{\frac{a\,c^{4}\,1\mathrm{i}}{1024}-\frac{a\,c^{4}\,{\left(\sqrt{c\,x-1}-\mathrm{i}\right)}^{2}\,3\mathrm{i}}{128\,{\left(\sqrt{c\,x+1}-1\right)}^{2}}-\frac{a\,c^{4}\,{\left(\sqrt{c\,x-1}-\mathrm{i}\right)}^{4}\,53\mathrm{i}}{512\,{\left(\sqrt{c\,x+1}-1\right)}^{4}}+\frac{a\,c^{4}\,{\left(\sqrt{c\,x-1}-\mathrm{i}\right)}^{6}\,87\mathrm{i}}{256\,{\left(\sqrt{c\,x+1}-1\right)}^{6}}+\frac{a\,c^{4}\,{\left(\sqrt{c\,x-1}-\mathrm{i}\right)}^{8}\,657\mathrm{i}}{1024\,{\left(\sqrt{c\,x+1}-1\right)}^{8}}+\frac{a\,c^{4}\,{\left(\sqrt{c\,x-1}-\mathrm{i}\right)}^{10}\,121\mathrm{i}}{256\,{\left(\sqrt{c\,x+1}-1\right)}^{10}}}{\frac{{\left(\sqrt{c\,x-1}-\mathrm{i}\right)}^{4}}{{\left(\sqrt{c\,x+1}-1\right)}^{4}}+\frac{4\,{\left(\sqrt{c\,x-1}-\mathrm{i}\right)}^{6}}{{\left(\sqrt{c\,x+1}-1\right)}^{6}}+\frac{6\,{\left(\sqrt{c\,x-1}-\mathrm{i}\right)}^{8}}{{\left(\sqrt{c\,x+1}-1\right)}^{8}}+\frac{4\,{\left(\sqrt{c\,x-1}-\mathrm{i}\right)}^{10}}{{\left(\sqrt{c\,x+1}-1\right)}^{10}}+\frac{{\left(\sqrt{c\,x-1}-\mathrm{i}\right)}^{12}}{{\left(\sqrt{c\,x+1}-1\right)}^{12}}}-\frac{a\,c^{4}\,\ln\left(\frac{{\left(\sqrt{c\,x-1}-\mathrm{i}\right)}^{2}}{{\left(\sqrt{c\,x+1}-1\right)}^{2}}+1\right)\,3\mathrm{i}}{8}-\frac{b\,c^{2}\,\ln\left(\frac{{\left(\sqrt{c\,x-1}-\mathrm{i}\right)}^{2}}{{\left(\sqrt{c\,x+1}-1\right)}^{2}}+1\right)\,1\mathrm{i}}{2}+\frac{a\,c^{4}\,\ln\left(\frac{\sqrt{c\,x-1}-\mathrm{i}}{\sqrt{c\,x+1}-1}\right)\,3\mathrm{i}}{8}+\frac{b\,c^{2}\,\ln\left(\frac{\sqrt{c\,x-1}-\mathrm{i}}{\sqrt{c\,x+1}-1}\right)\,1\mathrm{i}}{2}+\frac{a\,c^{4}\,{\left(\sqrt{c\,x-1}-\mathrm{i}\right)}^{2}\,7\mathrm{i}}{256\,{\left(\sqrt{c\,x+1}-1\right)}^{2}}-\frac{a\,c^{4}\,{\left(\sqrt{c\,x-1}-\mathrm{i}\right)}^{4}\,1\mathrm{i}}{1024\,{\left(\sqrt{c\,x+1}-1\right)}^{4}}+\frac{b\,c^{2}\,{\left(\sqrt{c\,x-1}-\mathrm{i}\right)}^{2}\,1\mathrm{i}}{32\,{\left(\sqrt{c\,x+1}-1\right)}^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)/(x^5*(c*x - 1)^(1/2)*(c*x + 1)^(1/2)),x)
```

```
[Out] ((b*c^2*1i)/32 + (b*c^2*((c*x - 1)^(1/2) - 1i)^2*1i)/(16*((c*x + 1)^(1/2) -
 1)^2) - (b*c^2*((c*x - 1)^(1/2) - 1i)^4*15i)/(32*((c*x + 1)^(1/2) - 1)^4))
/((((c*x - 1)^(1/2) - 1i)^2/((c*x + 1)^(1/2) - 1)^2 + (2*((c*x - 1)^(1/2) -
1i)^4)/((c*x + 1)^(1/2) - 1)^4 + ((c*x - 1)^(1/2) - 1i)^6/((c*x + 1)^(1/2)
- 1)^6) - ((a*c^4*1i)/1024 - (a*c^4*((c*x - 1)^(1/2) - 1i)^2*3i)/(128*((c*x
 + 1)^(1/2) - 1)^2) - (a*c^4*((c*x - 1)^(1/2) - 1i)^4*53i)/(512*((c*x + 1)^
(1/2) - 1)^4) + (a*c^4*((c*x - 1)^(1/2) - 1i)^6*87i)/(256*((c*x + 1)^(1/2)
- 1)^6) + (a*c^4*((c*x - 1)^(1/2) - 1i)^8*657i)/(1024*((c*x + 1)^(1/2) - 1)
^8) + (a*c^4*((c*x - 1)^(1/2) - 1i)^10*121i)/(256*((c*x + 1)^(1/2) - 1)^10)
)/((((c*x - 1)^(1/2) - 1i)^4/((c*x + 1)^(1/2) - 1)^4 + (4*((c*x - 1)^(1/2) -
 1i)^6)/((c*x + 1)^(1/2) - 1)^6 + (6*((c*x - 1)^(1/2) - 1i)^8)/((c*x + 1)^(
```

```
1/2) - 1)^8 + (4*((c*x - 1)^(1/2) - 1i)^10)/((c*x + 1)^(1/2) - 1)^10 + ((c*
x - 1)^(1/2) - 1i)^12/((c*x + 1)^(1/2) - 1)^12) - (a*c^4*log(((c*x - 1)^(1/
2) - 1i)^2/((c*x + 1)^(1/2) - 1)^2 + 1)*3i)/8 - (b*c^2*log(((c*x - 1)^(1/2)
 - 1i)^2/((c*x + 1)^(1/2) - 1)^2 + 1)*1i)/2 + (a*c^4*log(((c*x - 1)^(1/2) -
 1i)/((c*x + 1)^(1/2) - 1))*3i)/8 + (b*c^2*log(((c*x - 1)^(1/2) - 1i)/((c*x
 + 1)^(1/2) - 1))*1i)/2 + (a*c^4*((c*x - 1)^(1/2) - 1i)^2*7i)/(256*((c*x +
1)^(1/2) - 1)^2) - (a*c^4*((c*x - 1)^(1/2) - 1i)^4*1i)/(1024*((c*x + 1)^(1/
2) - 1)^4) + (b*c^2*((c*x - 1)^(1/2) - 1i)^2*1i)/(32*((c*x + 1)^(1/2) - 1)^
2)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)/x**5/(c*x-1)**(1/2)/(c*x+1)**(1/2),x)
[Out] Timed out
```

3.215 $\int \frac{x^4(a+bx^2)}{\sqrt{-c+dx}\sqrt{c+dx}}\,dx$

Optimal. Leaf size=164

$$\frac{c^2x\sqrt{dx-c}\sqrt{c+dx}\left(6ad^2+5bc^2\right)}{16d^6}+\frac{x^3\sqrt{dx-c}\sqrt{c+dx}\left(6ad^2+5bc^2\right)}{24d^4}+\frac{c^4\left(6ad^2+5bc^2\right)\tanh^{-1}\left(\frac{\sqrt{dx-c}}{\sqrt{c+dx}}\right)}{8d^7}+\frac{bx}{}$$

Rubi [A] time = 0.12, antiderivative size = 164, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {460, 100, 12, 90, 63, 217, 206}

$$\frac{x^3\sqrt{dx-c}\sqrt{c+dx}\left(6ad^2+5bc^2\right)}{24d^4}+\frac{c^2x\sqrt{dx-c}\sqrt{c+dx}\left(6ad^2+5bc^2\right)}{16d^6}+\frac{c^4\left(6ad^2+5bc^2\right)\tanh^{-1}\left(\frac{\sqrt{dx-c}}{\sqrt{c+dx}}\right)}{8d^7}+\frac{bx^5\sqrt{dx-c}\sqrt{c+dx}}{6d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(a + b*x^2))/(Sqrt[-c + d*x]*Sqrt[c + d*x]),x]
```

```
[Out] (c^2*(5*b*c^2 + 6*a*d^2)*x*Sqrt[-c + d*x]*Sqrt[c + d*x])/(16*d^6) + ((5*b*c
^2 + 6*a*d^2)*x^3*Sqrt[-c + d*x]*Sqrt[c + d*x])/(24*d^4) + (b*x^5*Sqrt[-c +
 d*x]*Sqrt[c + d*x])/(6*d^2) + (c^4*(5*b*c^2 + 6*a*d^2)*ArcTanh[Sqrt[-c + d
*x]/Sqrt[c + d*x]])/(8*d^7)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*(a + b*x)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/
(d*f*(n + p + 3)), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/(d*f*(m + n + p + 1)), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 460

```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[(d*(e*x)^
(m + 1)*(a1 + b1*x^(n/2))^(p + 1)*(a2 + b2*x^(n/2))^(p + 1))/(b1*b2*e*(m +
n*(p + 1) + 1)), x] - Dist[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/
(b1*b2*(m + n*(p + 1) + 1)), Int[(e*x)^m*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n/
2))^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, m, n, p}, x] && EqQ[non2,
n/2] && EqQ[a2*b1 + a1*b2, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rubi steps

$$\int \frac{x^4\left(a+bx^2\right)}{\sqrt{-c+dx}\sqrt{c+dx}}\,dx = \frac{bx^5\sqrt{-c+dx}\sqrt{c+dx}}{6d^2} - \frac{1}{6}\left(-6a-\frac{5bc^2}{d^2}\right)\int \frac{x^4}{\sqrt{-c+dx}\sqrt{c+dx}}\,dx$$

$$= \frac{\left(5bc^2+6ad^2\right)x^3\sqrt{-c+dx}\sqrt{c+dx}}{24d^4} + \frac{bx^5\sqrt{-c+dx}\sqrt{c+dx}}{6d^2} + \frac{\left(5bc^2+6ad^2\right)\int \frac{\cdots}{\sqrt{-c\cdots}}}{24d^4}$$

$$= \frac{\left(5bc^2+6ad^2\right)x^3\sqrt{-c+dx}\sqrt{c+dx}}{24d^4} + \frac{bx^5\sqrt{-c+dx}\sqrt{c+dx}}{6d^2} + \frac{\left(c^2\left(5bc^2+6ad^2\right)\right)\int \cdots}{8d^4}$$

$$= \frac{c^2\left(5bc^2+6ad^2\right)x\sqrt{-c+dx}\sqrt{c+dx}}{16d^6} + \frac{\left(5bc^2+6ad^2\right)x^3\sqrt{-c+dx}\sqrt{c+dx}}{24d^4} + \frac{bx^5\sqrt{\cdots}}{\cdots}$$

$$= \frac{c^2\left(5bc^2+6ad^2\right)x\sqrt{-c+dx}\sqrt{c+dx}}{16d^6} + \frac{\left(5bc^2+6ad^2\right)x^3\sqrt{-c+dx}\sqrt{c+dx}}{24d^4} + \frac{bx^5\sqrt{\cdots}}{\cdots}$$

$$= \frac{c^2\left(5bc^2+6ad^2\right)x\sqrt{-c+dx}\sqrt{c+dx}}{16d^6} + \frac{\left(5bc^2+6ad^2\right)x^3\sqrt{-c+dx}\sqrt{c+dx}}{24d^4} + \frac{bx^5\sqrt{\cdots}}{\cdots}$$

$$= \frac{c^2\left(5bc^2+6ad^2\right)x\sqrt{-c+dx}\sqrt{c+dx}}{16d^6} + \frac{\left(5bc^2+6ad^2\right)x^3\sqrt{-c+dx}\sqrt{c+dx}}{24d^4} + \frac{bx^5\sqrt{\cdots}}{\cdots}$$

$$= \frac{c^2\left(5bc^2+6ad^2\right)x\sqrt{-c+dx}\sqrt{c+dx}}{16d^6} + \frac{\left(5bc^2+6ad^2\right)x^3\sqrt{-c+dx}\sqrt{c+dx}}{24d^4} + \frac{bx^5\sqrt{\cdots}}{\cdots}$$

Mathematica [A] time = 0.12, size = 148, normalized size = 0.90

$$\frac{3c^4\sqrt{d^2x^2-c^2}\left(6ad^2+5bc^2\right)\tanh^{-1}\left(\frac{dx}{\sqrt{d^2x^2-c^2}}\right)+dx\left(d^2x^2-c^2\right)\left(6ad^2\left(3c^2+2d^2x^2\right)+b\left(15c^4+10c^2d^2x^2+8d^4x^4\right)\right)}{48d^7\sqrt{dx-c}\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(a + b*x^2))/(Sqrt[-c + d*x]*Sqrt[c + d*x]),x]
```

```
[Out] (d*x*(-c^2 + d^2*x^2)*(6*a*d^2*(3*c^2 + 2*d^2*x^2) + b*(15*c^4 + 10*c^2*d^2
*x^2 + 8*d^4*x^4)) + 3*c^4*(5*b*c^2 + 6*a*d^2)*Sqrt[-c^2 + d^2*x^2]*ArcTanh
[(d*x)/Sqrt[-c^2 + d^2*x^2]])/(48*d^7*Sqrt[-c + d*x]*Sqrt[c + d*x])
```

IntegrateAlgebraic [A] time = 0.24, size = 297, normalized size = 1.81

$$\frac{\left(6ac^4d^2+5bc^6\right)\tanh^{-1}\left(\frac{\sqrt{dx-c}}{\sqrt{c+dx}}\right)}{8d^7}+\frac{c^4\sqrt{dx-c}\left(\frac{dx-c}{c+dx}+1\right)\left(\frac{30ad^2(dx-c)^4}{(c+dx)^4}-\frac{72ad^2(dx-c)^3}{(c+dx)^3}+\frac{84ad^2(dx-c)^2}{(c+dx)^2}-\frac{72ad^2(dx-c)}{c+dx}+30ad^2+\frac{33bc^2(dx-c)^4}{(c+dx)^4}-\frac{28bc^2(dx-c)^3}{(c+dx)^3}+\frac{118bc^2(dx-c)^2}{(c+dx)^2}-\frac{28bc^2(dx-c)}{c+dx}+33bc^2\right)}{24d^7\sqrt{c+dx}\left(\frac{dx-c}{c+dx}-1\right)^6}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^4*(a + b*x^2))/(Sqrt[-c + d*x]*Sqrt[c + d*x]),x]
```

```
[Out] (c^4*Sqrt[-c + d*x]*(1 + (-c + d*x)/(c + d*x))*(33*b*c^2 + 30*a*d^2 + (33*b
*c^2*(-c + d*x)^4)/(c + d*x)^4 + (30*a*d^2*(-c + d*x)^4)/(c + d*x)^4 - (28*
b*c^2*(-c + d*x)^3)/(c + d*x)^3 - (72*a*d^2*(-c + d*x)^3)/(c + d*x)^3 + (11
8*b*c^2*(-c + d*x)^2)/(c + d*x)^2 + (84*a*d^2*(-c + d*x)^2)/(c + d*x)^2 - (
28*b*c^2*(-c + d*x))/(c + d*x) - (72*a*d^2*(-c + d*x))/(c + d*x)))/(24*d^7*
Sqrt[c + d*x]*(-1 + (-c + d*x)/(c + d*x))^6) + ((5*b*c^6 + 6*a*c^4*d^2)*Arc
Tanh[Sqrt[-c + d*x]/Sqrt[c + d*x]])/(8*d^7)
```

fricas [A] time = 0.78, size = 115, normalized size = 0.70

$$\frac{\left(8\,bd^5x^5+2\left(5\,bc^2d^3+6\,ad^5\right)x^3+3\left(5\,bc^4d+6\,ac^2d^3\right)x\right)\sqrt{dx+c}\,\sqrt{dx-c}-3\left(5\,bc^6+6\,ac^4d^2\right)\log\left(-dx+\sqrt{dx+c}\,\sqrt{dx-c}\right)}{48\,d^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)/(d*x-c)^(1/2)/(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/48*((8*b*d^5*x^5 + 2*(5*b*c^2*d^3 + 6*a*d^5)*x^3 + 3*(5*b*c^4*d + 6*a*c^2
*d^3)*x)*sqrt(d*x + c)*sqrt(d*x - c) - 3*(5*b*c^6 + 6*a*c^4*d^2)*log(-d*x +
 sqrt(d*x + c)*sqrt(d*x - c)))/d^7
```

giac [A] time = 0.28, size = 203, normalized size = 1.24

$$\frac{\left(\left(2\left((dx+c)\left(4\,(dx+c)\left(\frac{(dx+c)b}{d^6}-\frac{5\,bc}{d^6}\right)+\frac{3\left(15\,bc^2d^{36}+2\,ad^{38}\right)}{d^{42}}\right)-\frac{55\,bc^3d^{36}+18\,acd^{38}}{d^{42}}\right)(dx+c)+\frac{85\,bc^4d^{36}+54\,ac^2d^{38}}{d^{42}}\right)(dx+c)-\frac{3\left(11\,bc^5d^{36}+10\,ac^3d^{38}\right)}{d^{42}}\right)\sqrt{dx+c}\,\sqrt{dx-c}-\frac{6\left(5\,bc^6+6\,ac^4d^2\right)\log\left(\left|-\sqrt{dx+c}+\sqrt{dx-c}\right|\right)}{d^6}}{48\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)/(d*x-c)^(1/2)/(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/48*(((2*((d*x + c)*(4*(d*x + c)*((d*x + c)*b/d^6 - 5*b*c/d^6) + 3*(15*b*c
^2*d^36 + 2*a*d^38)/d^42) - (55*b*c^3*d^36 + 18*a*c*d^38)/d^42)*(d*x + c) +
 (85*b*c^4*d^36 + 54*a*c^2*d^38)/d^42)*(d*x + c) - 3*(11*b*c^5*d^36 + 10*a*
c^3*d^38)/d^42)*sqrt(d*x + c)*sqrt(d*x - c) - 6*(5*b*c^6 + 6*a*c^4*d^2)*log
(abs(-sqrt(d*x + c) + sqrt(d*x - c)))/d^6)/d
```

maple [C] time = 0.10, size = 240, normalized size = 1.46

$$\frac{\sqrt{dx-c}\,\sqrt{dx+c}\,\left(8\sqrt{d^2x^2-c^2}\,b\,d^5x^5\mathrm{csgn}(d)+12\sqrt{d^2x^2-c^2}\,a\,d^5x^3\mathrm{csgn}(d)+10\sqrt{d^2x^2-c^2}\,b\,c^2d^3x^3\mathrm{csgn}(d)+18a\,c^4d^2\ln\left(\left(dx+\sqrt{d^2x^2-c^2}\,\mathrm{csgn}(d)\right)\mathrm{csgn}(d)\right)+18\sqrt{d^2x^2-c^2}\,a\,c^2d^3x\,\mathrm{csgn}(d)+15b\,c^6\ln\left(\left(dx+\sqrt{d^2x^2-c^2}\,\mathrm{csgn}(d)\right)\mathrm{csgn}(d)\right)+15\sqrt{d^2x^2-c^2}\,b\,c^4dx\,\mathrm{csgn}(d)\right)\mathrm{csgn}(d)}{48\sqrt{d^2x^2-c^2}\,d^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(b*x^2+a)/(d*x-c)^(1/2)/(d*x+c)^(1/2),x)
```

```
[Out] 1/48*(d*x-c)^(1/2)*(d*x+c)^(1/2)*(8*(d^2*x^2-c^2)^(1/2)*b*d^5*x^5*csgn(d)+1
2*(d^2*x^2-c^2)^(1/2)*a*d^5*x^3*csgn(d)+10*(d^2*x^2-c^2)^(1/2)*b*c^2*d^3*x^
3*csgn(d)+18*(d^2*x^2-c^2)^(1/2)*a*c^2*d^3*x*csgn(d)+15*(d^2*x^2-c^2)^(1/2)
*b*c^4*d*x*csgn(d)+18*a*c^4*d^2*ln((d*x+(d^2*x^2-c^2)^(1/2)*csgn(d))*csgn(d
))+15*b*c^6*ln((d*x+(d^2*x^2-c^2)^(1/2)*csgn(d))*csgn(d)))*csgn(d)/d^7/(d^2
*x^2-c^2)^(1/2)
```

maxima [A] time = 0.57, size = 196, normalized size = 1.20

$$\frac{\sqrt{d^2x^2-c^2}\,bx^5}{6\,d^2}+\frac{5\,\sqrt{d^2x^2-c^2}\,bc^2x^3}{24\,d^4}+\frac{\sqrt{d^2x^2-c^2}\,ax^3}{4\,d^2}+\frac{5\,bc^6\log\left(2\,d^2x+2\,\sqrt{d^2x^2-c^2}\,d\right)}{16\,d^7}+\frac{3\,ac^4\log\left(2\,d^2x+2\,\sqrt{d^2x^2-c^2}\,d\right)}{8\,d^5}+\frac{5\,\sqrt{d^2x^2-c^2}\,bc^4x}{16\,d^6}+\frac{3\,\sqrt{d^2x^2-c^2}\,ac^2x}{8\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)/(d*x-c)^(1/2)/(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/6*sqrt(d^2*x^2 - c^2)*b*x^5/d^2 + 5/24*sqrt(d^2*x^2 - c^2)*b*c^2*x^3/d^4
+ 1/4*sqrt(d^2*x^2 - c^2)*a*x^3/d^2 + 5/16*b*c^6*log(2*d^2*x + 2*sqrt(d^2*x
^2 - c^2)*d)/d^7 + 3/8*a*c^4*log(2*d^2*x + 2*sqrt(d^2*x^2 - c^2)*d)/d^5 + 5
/16*sqrt(d^2*x^2 - c^2)*b*c^4*x/d^6 + 3/8*sqrt(d^2*x^2 - c^2)*a*c^2*x/d^4
```

mupad [B] time = 42.66, size = 1682, normalized size = 10.26

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(a + b*x^2))/((c + d*x)^(1/2)*(d*x - c)^(1/2)),x)
```

```
[Out] ((5*b*c^6*((c + d*x)^(1/2) - c^(1/2)))/(4*((-c)^(1/2) - (d*x - c)^(1/2))) -
 (175*b*c^6*((c + d*x)^(1/2) - c^(1/2))^3)/(12*((-c)^(1/2) - (d*x - c)^(1/2
))^3) + (311*b*c^6*((c + d*x)^(1/2) - c^(1/2))^5)/(4*((-c)^(1/2) - (d*x - c
)^(1/2))^5) + (8361*b*c^6*((c + d*x)^(1/2) - c^(1/2))^7)/(4*((-c)^(1/2) - (
d*x - c)^(1/2))^7) + (42259*b*c^6*((c + d*x)^(1/2) - c^(1/2))^9)/(6*((-c)^(
1/2) - (d*x - c)^(1/2))^9) + (25295*b*c^6*((c + d*x)^(1/2) - c^(1/2))^11)/(
2*((-c)^(1/2) - (d*x - c)^(1/2))^11) + (25295*b*c^6*((c + d*x)^(1/2) - c^(1
/2))^13)/(2*((-c)^(1/2) - (d*x - c)^(1/2))^13) + (42259*b*c^6*((c + d*x)^(1
/2) - c^(1/2))^15)/(6*((-c)^(1/2) - (d*x - c)^(1/2))^15) + (8361*b*c^6*((c
+ d*x)^(1/2) - c^(1/2))^17)/(4*((-c)^(1/2) - (d*x - c)^(1/2))^17) + (311*b*
c^6*((c + d*x)^(1/2) - c^(1/2))^19)/(4*((-c)^(1/2) - (d*x - c)^(1/2))^19) -
 (175*b*c^6*((c + d*x)^(1/2) - c^(1/2))^21)/(12*((-c)^(1/2) - (d*x - c)^(1/
2))^21) + (5*b*c^6*((c + d*x)^(1/2) - c^(1/2))^23)/(4*((-c)^(1/2) - (d*x -
c)^(1/2))^23))/(d^7 - (12*d^7*((c + d*x)^(1/2) - c^(1/2))^2)/((-c)^(1/2) -
(d*x - c)^(1/2))^2 + (66*d^7*((c + d*x)^(1/2) - c^(1/2))^4)/((-c)^(1/2) - (
d*x - c)^(1/2))^4 - (220*d^7*((c + d*x)^(1/2) - c^(1/2))^6)/((-c)^(1/2) - (
d*x - c)^(1/2))^6 + (495*d^7*((c + d*x)^(1/2) - c^(1/2))^8)/((-c)^(1/2) - (
d*x - c)^(1/2))^8 - (792*d^7*((c + d*x)^(1/2) - c^(1/2))^10)/((-c)^(1/2) -
(d*x - c)^(1/2))^10 + (924*d^7*((c + d*x)^(1/2) - c^(1/2))^12)/((-c)^(1/2)
- (d*x - c)^(1/2))^12 - (792*d^7*((c + d*x)^(1/2) - c^(1/2))^14)/((-c)^(1/2
) - (d*x - c)^(1/2))^14 + (495*d^7*((c + d*x)^(1/2) - c^(1/2))^16)/((-c)^(1
/2) - (d*x - c)^(1/2))^16 - (220*d^7*((c + d*x)^(1/2) - c^(1/2))^18)/((-c)^
(1/2) - (d*x - c)^(1/2))^18 + (66*d^7*((c + d*x)^(1/2) - c^(1/2))^20)/((-c)
^(1/2) - (d*x - c)^(1/2))^20 - (12*d^7*((c + d*x)^(1/2) - c^(1/2))^22)/((-c
)^(1/2) - (d*x - c)^(1/2))^22 + (d^7*((c + d*x)^(1/2) - c^(1/2))^24)/((-c)^
(1/2) - (d*x - c)^(1/2))^24) - ((23*a*c^4*((c + d*x)^(1/2) - c^(1/2))^3)/(2
*((-c)^(1/2) - (d*x - c)^(1/2))^3) - (3*a*c^4*((c + d*x)^(1/2) - c^(1/2)))/
(2*((-c)^(1/2) - (d*x - c)^(1/2))) + (333*a*c^4*((c + d*x)^(1/2) - c^(1/2))
^5)/(2*((-c)^(1/2) - (d*x - c)^(1/2))^5) + (671*a*c^4*((c + d*x)^(1/2) - c^
(1/2))^7)/(2*((-c)^(1/2) - (d*x - c)^(1/2))^7) + (671*a*c^4*((c + d*x)^(1/2
) - c^(1/2))^9)/(2*((-c)^(1/2) - (d*x - c)^(1/2))^9) + (333*a*c^4*((c + d*x
)^(1/2) - c^(1/2))^11)/(2*((-c)^(1/2) - (d*x - c)^(1/2))^11) + (23*a*c^4*((
c + d*x)^(1/2) - c^(1/2))^13)/(2*((-c)^(1/2) - (d*x - c)^(1/2))^13) - (3*a*
c^4*((c + d*x)^(1/2) - c^(1/2))^15)/(2*((-c)^(1/2) - (d*x - c)^(1/2))^15))/
(d^5 - (8*d^5*((c + d*x)^(1/2) - c^(1/2))^2)/((-c)^(1/2) - (d*x - c)^(1/2))
^2 + (28*d^5*((c + d*x)^(1/2) - c^(1/2))^4)/((-c)^(1/2) - (d*x - c)^(1/2))^
4 - (56*d^5*((c + d*x)^(1/2) - c^(1/2))^6)/((-c)^(1/2) - (d*x - c)^(1/2))^6
 + (70*d^5*((c + d*x)^(1/2) - c^(1/2))^8)/((-c)^(1/2) - (d*x - c)^(1/2))^8
- (56*d^5*((c + d*x)^(1/2) - c^(1/2))^10)/((-c)^(1/2) - (d*x - c)^(1/2))^10
 + (28*d^5*((c + d*x)^(1/2) - c^(1/2))^12)/((-c)^(1/2) - (d*x - c)^(1/2))^1
2 - (8*d^5*((c + d*x)^(1/2) - c^(1/2))^14)/((-c)^(1/2) - (d*x - c)^(1/2))^1
4 + (d^5*((c + d*x)^(1/2) - c^(1/2))^16)/((-c)^(1/2) - (d*x - c)^(1/2))^16)
 - (3*a*c^4*atanh(((c + d*x)^(1/2) - c^(1/2))/((-c)^(1/2) - (d*x - c)^(1/2)
)))/(2*d^5) - (5*b*c^6*atanh(((c + d*x)^(1/2) - c^(1/2))/((-c)^(1/2) - (d*x
 - c)^(1/2))))/(4*d^7)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(b*x**2+a)/(d*x-c)**(1/2)/(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

3.216 $\int \frac{x^3(a+bx^2)}{\sqrt{-c+dx}\sqrt{c+dx}}\,dx$

Optimal. Leaf size=118

$$\frac{2c^2\sqrt{dx-c}\sqrt{c+dx}\left(5ad^2+4bc^2\right)}{15d^6}+\frac{x^2\sqrt{dx-c}\sqrt{c+dx}\left(5ad^2+4bc^2\right)}{15d^4}+\frac{bx^4\sqrt{dx-c}\sqrt{c+dx}}{5d^2}$$

Rubi [A] time = 0.09, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {460, 100, 12, 74}

$$\frac{x^2\sqrt{dx-c}\sqrt{c+dx}\left(5ad^2+4bc^2\right)}{15d^4}+\frac{2c^2\sqrt{dx-c}\sqrt{c+dx}\left(5ad^2+4bc^2\right)}{15d^6}+\frac{bx^4\sqrt{dx-c}\sqrt{c+dx}}{5d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(a + b*x^2))/(Sqrt[-c + d*x]*Sqrt[c + d*x]),x]
```

```
[Out] (2*c^2*(4*b*c^2 + 5*a*d^2)*Sqrt[-c + d*x]*Sqrt[c + d*x])/(15*d^6) + ((4*b*c
^2 + 5*a*d^2)*x^2*Sqrt[-c + d*x]*Sqrt[c + d*x])/(15*d^4) + (b*x^4*Sqrt[-c +
 d*x]*Sqrt[c + d*x])/(5*d^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/(d*f*(m + n + p + 1)), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 460

```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[(d*(e*x)^
(m + 1)*(a1 + b1*x^(n/2))^(p + 1)*(a2 + b2*x^(n/2))^(p + 1))/(b1*b2*e*(m +
n*(p + 1) + 1)), x] - Dist[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/
(b1*b2*(m + n*(p + 1) + 1)), Int[(e*x)^m*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n/
2))^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, m, n, p}, x] && EqQ[non2,
n/2] && EqQ[a2*b1 + a1*b2, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rubi steps

$$\int \frac{x^3\left(a+bx^2\right)}{\sqrt{-c+dx}\,\sqrt{c+dx}}\,dx = \frac{bx^4\sqrt{-c+dx}\,\sqrt{c+dx}}{5d^2} - \frac{1}{5}\left(-5a-\frac{4bc^2}{d^2}\right)\int \frac{x^3}{\sqrt{-c+dx}\,\sqrt{c+dx}}\,dx$$

$$= \frac{\left(4bc^2+5ad^2\right)x^2\sqrt{-c+dx}\,\sqrt{c+dx}}{15d^4} + \frac{bx^4\sqrt{-c+dx}\,\sqrt{c+dx}}{5d^2} + \frac{\left(4bc^2+5ad^2\right)\int}{15d}$$

$$= \frac{\left(4bc^2+5ad^2\right)x^2\sqrt{-c+dx}\,\sqrt{c+dx}}{15d^4} + \frac{bx^4\sqrt{-c+dx}\,\sqrt{c+dx}}{5d^2} + \frac{\left(2c^2\left(4bc^2+5ad^2\right.\right.}{1}$$

$$= \frac{2c^2\left(4bc^2+5ad^2\right)\sqrt{-c+dx}\,\sqrt{c+dx}}{15d^6} + \frac{\left(4bc^2+5ad^2\right)x^2\sqrt{-c+dx}\,\sqrt{c+dx}}{15d^4} + bx^4$$

Mathematica [A] time = 0.06, size = 87, normalized size = 0.74

$$\frac{\left(d^2x^2-c^2\right)\left(5ad^2\left(2c^2+d^2x^2\right)+b\left(8c^4+4c^2d^2x^2+3d^4x^4\right)\right)}{15d^6\sqrt{dx-c}\,\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(a + b*x^2))/(Sqrt[-c + d*x]*Sqrt[c + d*x]),x]
```

```
[Out] ((-c^2 + d^2*x^2)*(5*a*d^2*(2*c^2 + d^2*x^2) + b*(8*c^4 + 4*c^2*d^2*x^2 + 3
*d^4*x^4)))/(15*d^6*Sqrt[-c + d*x]*Sqrt[c + d*x])
```

IntegrateAlgebraic [B] time = 0.15, size = 246, normalized size = 2.08

$$-\frac{2\sqrt{dx-c}\left(-\frac{40ac^3d^2(dx-c)}{c+dx}+\frac{50ac^3d^2(dx-c)^2}{(c+dx)^2}-\frac{40ac^3d^2(dx-c)^3}{(c+dx)^3}+\frac{15ac^3d^2(dx-c)^4}{(c+dx)^4}+15ac^3d^2-\frac{20bc^5(dx-c)}{c+dx}+\frac{58bc^5(dx-c)^2}{(c+dx)^2}-\frac{20bc^5(dx-c)^3}{(c+dx)^3}+\frac{15bc^5(dx-c)^4}{(c+dx)^4}+15bc^5\right)}{15d^6\sqrt{c+dx}\left(\frac{dx-c}{c+dx}-1\right)^5}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^3*(a + b*x^2))/(Sqrt[-c + d*x]*Sqrt[c + d*x]),x]
```

```
[Out] (-2*Sqrt[-c + d*x]*(15*b*c^5 + 15*a*c^3*d^2 + (15*b*c^5*(-c + d*x)^4)/(c +
d*x)^4 + (15*a*c^3*d^2*(-c + d*x)^4)/(c + d*x)^4 - (20*b*c^5*(-c + d*x)^3)/
(c + d*x)^3 - (40*a*c^3*d^2*(-c + d*x)^3)/(c + d*x)^3 + (58*b*c^5*(-c + d*x
)^2)/(c + d*x)^2 + (50*a*c^3*d^2*(-c + d*x)^2)/(c + d*x)^2 - (20*b*c^5*(-c
+ d*x))/(c + d*x) - (40*a*c^3*d^2*(-c + d*x))/(c + d*x)))/(15*d^6*Sqrt[c +
d*x]*(-1 + (-c + d*x)/(c + d*x))^5)
```

fricas [A] time = 0.78, size = 66, normalized size = 0.56

$$\frac{\left(3\,bd^4x^4+8\,bc^4+10\,ac^2d^2+\left(4\,bc^2d^2+5\,ad^4\right)x^2\right)\sqrt{dx+c}\,\sqrt{dx-c}}{15\,d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)/(d*x-c)^(1/2)/(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/15*(3*b*d^4*x^4 + 8*b*c^4 + 10*a*c^2*d^2 + (4*b*c^2*d^2 + 5*a*d^4)*x^2)*s
qrt(d*x + c)*sqrt(d*x - c)/d^6
```

giac [A] time = 0.25, size = 124, normalized size = 1.05

$$\frac{\left(\left((dx+c)\left(3\,(dx+c)\left(\frac{(dx+c)b}{d^5}-\frac{4\,bc}{d^5}\right)+\frac{22\,bc^2d^{25}+5\,ad^{27}}{d^{30}}\right)-\frac{10\left(2\,bc^3d^{25}+acd^{27}\right)}{d^{30}}\right)(dx+c)+\frac{15\left(bc^4d^{25}+ac^2d^{27}\right)}{d^{30}}\right)\sqrt{dx+c}\,\sqrt{dx-c}}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)/(d*x-c)^(1/2)/(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/15*(((d*x + c)*(3*(d*x + c)*((d*x + c)*b/d^5 - 4*b*c/d^5) + (22*b*c^2*d^2
5 + 5*a*d^27)/d^30) - 10*(2*b*c^3*d^25 + a*c*d^27)/d^30)*(d*x + c) + 15*(b*
c^4*d^25 + a*c^2*d^27)/d^30)*sqrt(d*x + c)*sqrt(d*x - c)/d
```

maple [A] time = 0.05, size = 68, normalized size = 0.58

$$\frac{\sqrt{dx+c}\,\left(3b\,d^4x^4+5a\,d^4x^2+4b\,c^2d^2x^2+10a\,c^2d^2+8b\,c^4\right)\sqrt{dx-c}}{15d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x^2+a)/(d*x-c)^(1/2)/(d*x+c)^(1/2),x)
```

```
[Out] 1/15*(d*x+c)^(1/2)*(3*b*d^4*x^4+5*a*d^4*x^2+4*b*c^2*d^2*x^2+10*a*c^2*d^2+8*
b*c^4)/d^6*(d*x-c)^(1/2)
```

maxima [A] time = 0.55, size = 124, normalized size = 1.05

$$\frac{\sqrt{d^2x^2-c^2}\,bx^4}{5\,d^2}+\frac{4\,\sqrt{d^2x^2-c^2}\,bc^2x^2}{15\,d^4}+\frac{\sqrt{d^2x^2-c^2}\,ax^2}{3\,d^2}+\frac{8\,\sqrt{d^2x^2-c^2}\,bc^4}{15\,d^6}+\frac{2\,\sqrt{d^2x^2-c^2}\,ac^2}{3\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)/(d*x-c)^(1/2)/(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/5*sqrt(d^2*x^2 - c^2)*b*x^4/d^2 + 4/15*sqrt(d^2*x^2 - c^2)*b*c^2*x^2/d^4
+ 1/3*sqrt(d^2*x^2 - c^2)*a*x^2/d^2 + 8/15*sqrt(d^2*x^2 - c^2)*b*c^4/d^6 +
2/3*sqrt(d^2*x^2 - c^2)*a*c^2/d^4
```

mupad [B] time = 2.70, size = 130, normalized size = 1.10

$$\frac{\sqrt{d\,x-c}\,\left(\frac{8\,b\,c^5+10\,a\,c^3\,d^2}{15\,d^6}+\frac{x^3\,\left(4\,b\,c^2\,d^3+5\,a\,d^5\right)}{15\,d^6}+\frac{x\,\left(8\,b\,c^4\,d+10\,a\,c^2\,d^3\right)}{15\,d^6}+\frac{b\,x^5}{5\,d}+\frac{x^2\,\left(4\,b\,c^3\,d^2+5\,a\,c\,d^4\right)}{15\,d^6}+\frac{b\,c\,x^4}{5\,d^2}\right)}{\sqrt{c+d\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(a + b*x^2))/((c + d*x)^(1/2)*(d*x - c)^(1/2)),x)
```

```
[Out] ((d*x - c)^(1/2)*((8*b*c^5 + 10*a*c^3*d^2)/(15*d^6) + (x^3*(5*a*d^5 + 4*b*c
^2*d^3))/(15*d^6) + (x*(10*a*c^2*d^3 + 8*b*c^4*d))/(15*d^6) + (b*x^5)/(5*d)
 + (x^2*(4*b*c^3*d^2 + 5*a*c*d^4))/(15*d^6) + (b*c*x^4)/(5*d^2)))/(c + d*x)
^(1/2)
```

sympy [C] time = 70.84, size = 240, normalized size = 2.03

$$\frac{ac^3G_{6,6}^{6,2}\left(\begin{matrix}-\frac{5}{4},-\frac{3}{4} & -1,-1,-\frac{1}{2},1\\ -\frac{3}{2},-\frac{5}{4},-1,-\frac{3}{4},-\frac{1}{2},0 & \end{matrix}\middle|\frac{c^2}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}d^4}+\frac{iac^3G_{6,6}^{2,6}\left(\begin{matrix}-2,-\frac{7}{4},-\frac{3}{2},-\frac{5}{4},-1,1 & \\ -\frac{7}{4},-\frac{5}{4} & -2,-\frac{3}{2},-\frac{3}{2},0\end{matrix}\middle|\frac{c^2e^{2i\pi}}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}d^4}+\frac{bc^5G_{6,6}^{6,2}\left(\begin{matrix}-\frac{9}{4},-\frac{7}{4} & -2,-2,-\frac{3}{2},1\\ -\frac{5}{2},-\frac{9}{4},-2,-\frac{7}{4},-\frac{3}{2},0 & \end{matrix}\middle|\frac{c^2}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}d^6}+\frac{ibc^5G_{6,6}^{2,6}\left(\begin{matrix}-3,-\frac{11}{4},-\frac{5}{2},-\frac{9}{4},-2,1 & \\ -\frac{11}{4},-\frac{9}{4} & -3,-\frac{5}{2},-\frac{5}{2},0\end{matrix}\middle|\frac{c^2e^{2i\pi}}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x**2+a)/(d*x-c)**(1/2)/(d*x+c)**(1/2),x)
```

```
[Out] a*c**3*meijerg(((-5/4, -3/4), (-1, -1, -1/2, 1)), ((-3/2, -5/4, -1, -3/4, -
1/2, 0), ()), c**2/(d**2*x**2))/(4*pi**(3/2)*d**4) + I*a*c**3*meijerg(((-2,
 -7/4, -3/2, -5/4, -1, 1), ()), ((-7/4, -5/4), (-2, -3/2, -3/2, 0)), c**2*e
xp_polar(2*I*pi)/(d**2*x**2))/(4*pi**(3/2)*d**4) + b*c**5*meijerg(((-9/4, -
7/4), (-2, -2, -3/2, 1)), ((-5/2, -9/4, -2, -7/4, -3/2, 0), ()), c**2/(d**2
*x**2))/(4*pi**(3/2)*d**6) + I*b*c**5*meijerg(((-3, -11/4, -5/2, -9/4, -2,
1), ()), ((-11/4, -9/4), (-3, -5/2, -5/2, 0)), c**2*exp_polar(2*I*pi)/(d**2
*x**2))/(4*pi**(3/2)*d**6)
```

3.217 $\int \frac{x^2(a+bx^2)}{\sqrt{-c+dx}\,\sqrt{c+dx}}\,dx$

Optimal. Leaf size=118

$$\frac{c^2\left(4ad^2+3bc^2\right)\tanh^{-1}\left(\frac{\sqrt{dx-c}}{\sqrt{c+dx}}\right)}{4d^5}+\frac{x\sqrt{dx-c}\,\sqrt{c+dx}\,\left(4ad^2+3bc^2\right)}{8d^4}+\frac{bx^3\sqrt{dx-c}\,\sqrt{c+dx}}{4d^2}$$

Rubi [A] time = 0.10, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {460, 90, 12, 63, 217, 206}

$$\frac{x\sqrt{dx-c}\,\sqrt{c+dx}\,\left(4ad^2+3bc^2\right)}{8d^4}+\frac{c^2\left(4ad^2+3bc^2\right)\tanh^{-1}\left(\frac{\sqrt{dx-c}}{\sqrt{c+dx}}\right)}{4d^5}+\frac{bx^3\sqrt{dx-c}\,\sqrt{c+dx}}{4d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a + b*x^2))/(Sqrt[-c + d*x]*Sqrt[c + d*x]),x]
```

```
[Out] ((3*b*c^2 + 4*a*d^2)*x*Sqrt[-c + d*x]*Sqrt[c + d*x])/(8*d^4) + (b*x^3*Sqrt[
-c + d*x]*Sqrt[c + d*x])/(4*d^2) + (c^2*(3*b*c^2 + 4*a*d^2)*ArcTanh[Sqrt[-c
 + d*x]/Sqrt[c + d*x]])/(4*d^5)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*(a + b*x)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/
(d*f*(n + p + 3)), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 460

```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[(d*(e*x)^
```

```
(m + 1)*(a1 + b1*x^(n/2))^(p + 1)*(a2 + b2*x^(n/2))^(p + 1))/(b1*b2*e*(m +
n*(p + 1) + 1)), x] - Dist[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/
(b1*b2*(m + n*(p + 1) + 1)), Int[(e*x)^m*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n/
2))^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, m, n, p}, x] && EqQ[non2,
n/2] && EqQ[a2*b1 + a1*b2, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rubi steps

$$\int \frac{x^2\left(a+bx^2\right)}{\sqrt{-c+dx}\,\sqrt{c+dx}}\,dx = \frac{bx^3\sqrt{-c+dx}\,\sqrt{c+dx}}{4d^2} - \frac{1}{4}\left(-4a-\frac{3bc^2}{d^2}\right)\int \frac{x^2}{\sqrt{-c+dx}\,\sqrt{c+dx}}\,dx$$

$$= \frac{\left(3bc^2+4ad^2\right)x\sqrt{-c+dx}\,\sqrt{c+dx}}{8d^4} + \frac{bx^3\sqrt{-c+dx}\,\sqrt{c+dx}}{4d^2} + \frac{\left(3bc^2+4ad^2\right)\int \frac{}{\sqrt{-c+}}}{8d^4}$$

$$= \frac{\left(3bc^2+4ad^2\right)x\sqrt{-c+dx}\,\sqrt{c+dx}}{8d^4} + \frac{bx^3\sqrt{-c+dx}\,\sqrt{c+dx}}{4d^2} + \frac{\left(c^2\left(3bc^2+4ad^2\right)\right)\int}{8d^4}$$

$$= \frac{\left(3bc^2+4ad^2\right)x\sqrt{-c+dx}\,\sqrt{c+dx}}{8d^4} + \frac{bx^3\sqrt{-c+dx}\,\sqrt{c+dx}}{4d^2} + \frac{\left(c^2\left(3bc^2+4ad^2\right)\right)\mathrm{Su}}{}$$

$$= \frac{\left(3bc^2+4ad^2\right)x\sqrt{-c+dx}\,\sqrt{c+dx}}{8d^4} + \frac{bx^3\sqrt{-c+dx}\,\sqrt{c+dx}}{4d^2} + \frac{\left(c^2\left(3bc^2+4ad^2\right)\right)\mathrm{Su}}{}$$

$$= \frac{\left(3bc^2+4ad^2\right)x\sqrt{-c+dx}\,\sqrt{c+dx}}{8d^4} + \frac{bx^3\sqrt{-c+dx}\,\sqrt{c+dx}}{4d^2} + \frac{c^2\left(3bc^2+4ad^2\right)\tanh}{4d^5}$$

Mathematica [A] time = 0.10, size = 121, normalized size = 1.03

$$\frac{dx\left(d^2x^2-c^2\right)\left(4ad^2+3bc^2+2bd^2x^2\right)+c^2\sqrt{d^2x^2-c^2}\left(4ad^2+3bc^2\right)\tanh^{-1}\left(\frac{dx}{\sqrt{d^2x^2-c^2}}\right)}{8d^5\sqrt{dx-c}\,\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(a + b*x^2))/(Sqrt[-c + d*x]*Sqrt[c + d*x]),x]
```

```
[Out] (d*x*(-c^2 + d^2*x^2)*(3*b*c^2 + 4*a*d^2 + 2*b*d^2*x^2) + c^2*(3*b*c^2 + 4*
a*d^2)*Sqrt[-c^2 + d^2*x^2]*ArcTanh[(d*x)/Sqrt[-c^2 + d^2*x^2]])/(8*d^5*Sqr
t[-c + d*x]*Sqrt[c + d*x])
```

IntegrateAlgebraic [A] time = 0.19, size = 209, normalized size = 1.77

$$\frac{c^2\sqrt{dx-c}\left(\frac{dx-c}{c+dx}+1\right)\left(-\frac{8ad^2(dx-c)}{c+dx}+\frac{4ad^2(dx-c)^2}{(c+dx)^2}+4ad^2-\frac{2bc^2(dx-c)}{c+dx}+\frac{5bc^2(dx-c)^2}{(c+dx)^2}+5bc^2\right)}{4d^5\sqrt{c+dx}\left(\frac{dx-c}{c+dx}-1\right)^4}+\frac{\left(4ac^2d^2+3bc^4\right)\tanh^{-1}\left(\frac{\sqrt{dx-c}}{\sqrt{c+dx}}\right)}{4d^5}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^2*(a + b*x^2))/(Sqrt[-c + d*x]*Sqrt[c + d*x]),x]
```

```
[Out] (c^2*Sqrt[-c + d*x]*(1 + (-c + d*x)/(c + d*x))*(5*b*c^2 + 4*a*d^2 + (5*b*c^
2*(-c + d*x)^2)/(c + d*x)^2 + (4*a*d^2*(-c + d*x)^2)/(c + d*x)^2 - (2*b*c^2
*(-c + d*x))/(c + d*x) - (8*a*d^2*(-c + d*x))/(c + d*x)))/(4*d^5*Sqrt[c + d
*x]*(-1 + (-c + d*x)/(c + d*x))^4) + ((3*b*c^4 + 4*a*c^2*d^2)*ArcTanh[Sqrt[
-c + d*x]/Sqrt[c + d*x]])/(4*d^5)
```

fricas [A] time = 1.20, size = 90, normalized size = 0.76

$$\frac{\left(2\,bd^3x^3+\left(3\,bc^2d+4\,ad^3\right)x\right)\sqrt{dx+c}\,\sqrt{dx-c}-\left(3\,bc^4+4\,ac^2d^2\right)\log\left(-dx+\sqrt{dx+c}\,\sqrt{dx-c}\right)}{8\,d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)/(d*x-c)^(1/2)/(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/8*((2*b*d^3*x^3 + (3*b*c^2*d + 4*a*d^3)*x)*sqrt(d*x + c)*sqrt(d*x - c) -
(3*b*c^4 + 4*a*c^2*d^2)*log(-d*x + sqrt(d*x + c)*sqrt(d*x - c)))/d^5
```

giac [A] time = 0.23, size = 140, normalized size = 1.19

$$\frac{\left((dx+c)\left(2\,(dx+c)\left(\frac{(dx+c)b}{d^4}-\frac{3\,bc}{d^4}\right)+\frac{9\,bc^2d^{16}+4\,ad^{18}}{d^{20}}\right)-\frac{5\,bc^3d^{16}+4\,acd^{18}}{d^{20}}\right)\sqrt{dx+c}\,\sqrt{dx-c}-\frac{2\left(3\,bc^4+4\,ac^2d^2\right)\log\left(\left|-\sqrt{dx+c}+\sqrt{dx-c}\right|\right)}{d^4}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)/(d*x-c)^(1/2)/(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/8*(((d*x + c)*(2*(d*x + c)*((d*x + c)*b/d^4 - 3*b*c/d^4) + (9*b*c^2*d^16
+ 4*a*d^18)/d^20) - (5*b*c^3*d^16 + 4*a*c*d^18)/d^20)*sqrt(d*x + c)*sqrt(d*
x - c) - 2*(3*b*c^4 + 4*a*c^2*d^2)*log(abs(-sqrt(d*x + c) + sqrt(d*x - c)))
/d^4)/d
```

maple [C] time = 0.08, size = 182, normalized size = 1.54

$$\frac{\sqrt{dx-c}\,\sqrt{dx+c}\,\left(2\sqrt{d^2x^2-c^2}\,b\,d^3x^3\mathrm{csgn}(d)+4a\,c^2d^2\ln\left(\left(dx+\sqrt{d^2x^2-c^2}\,\mathrm{csgn}(d)\right)\mathrm{csgn}(d)\right)+4\sqrt{d^2x^2-c^2}\,a\,d^3x\,\mathrm{csgn}(d)+3b\,c^4\ln\left(\left(dx+\sqrt{d^2x^2-c^2}\,\mathrm{csgn}(d)\right)\mathrm{csgn}(d)\right)+3\sqrt{d^2x^2-c^2}\,b\,c^2dx\,\mathrm{csgn}(d)\right)\mathrm{csgn}(d)}{8\sqrt{d^2x^2-c^2}\,d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x^2+a)/(d*x-c)^(1/2)/(d*x+c)^(1/2),x)
```

```
[Out] 1/8*(d*x-c)^(1/2)*(d*x+c)^(1/2)*(2*(d^2*x^2-c^2)^(1/2)*b*d^3*x^3*csgn(d)+4*
(d^2*x^2-c^2)^(1/2)*a*d^3*x*csgn(d)+3*(d^2*x^2-c^2)^(1/2)*b*c^2*d*x*csgn(d)
+4*a*c^2*d^2*ln((d*x+(d^2*x^2-c^2)^(1/2)*csgn(d))*csgn(d))+3*b*c^4*ln((d*x+
(d^2*x^2-c^2)^(1/2)*csgn(d))*csgn(d)))*csgn(d)/d^5/(d^2*x^2-c^2)^(1/2)
```

maxima [A] time = 0.64, size = 142, normalized size = 1.20

$$\frac{\sqrt{d^2x^2-c^2}\,bx^3}{4\,d^2}+\frac{3\,bc^4\log\left(2\,d^2x+2\,\sqrt{d^2x^2-c^2}\,d\right)}{8\,d^5}+\frac{ac^2\log\left(2\,d^2x+2\,\sqrt{d^2x^2-c^2}\,d\right)}{2\,d^3}+\frac{3\,\sqrt{d^2x^2-c^2}\,bc^2x}{8\,d^4}+\frac{\sqrt{d^2x^2-c^2}\,ax}{2\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)/(d*x-c)^(1/2)/(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/4*sqrt(d^2*x^2 - c^2)*b*x^3/d^2 + 3/8*b*c^4*log(2*d^2*x + 2*sqrt(d^2*x^2
- c^2)*d)/d^5 + 1/2*a*c^2*log(2*d^2*x + 2*sqrt(d^2*x^2 - c^2)*d)/d^3 + 3/8*
sqrt(d^2*x^2 - c^2)*b*c^2*x/d^4 + 1/2*sqrt(d^2*x^2 - c^2)*a*x/d^2
```

mupad [B] time = 25.51, size = 1048, normalized size = 8.88

$$\frac{\frac{2\,a\,c^2\left(\sqrt{c+d\,x}-\sqrt{c}\right)}{\sqrt{-c}-\sqrt{d\,x-c}}+\frac{14\,a\,c^2\left(\sqrt{c+d\,x}-\sqrt{c}\right)^3}{\left(\sqrt{-c}-\sqrt{d\,x-c}\right)^3}+\frac{14\,a\,c^2\left(\sqrt{c+d\,x}-\sqrt{c}\right)^5}{\left(\sqrt{-c}-\sqrt{d\,x-c}\right)^5}+\frac{2\,a\,c^2\left(\sqrt{c+d\,x}-\sqrt{c}\right)^7}{\left(\sqrt{-c}-\sqrt{d\,x-c}\right)^7}}{d^3-\frac{4\,d^3\left(\sqrt{c+d\,x}-\sqrt{c}\right)^2}{\left(\sqrt{-c}-\sqrt{d\,x-c}\right)^2}+\frac{6\,d^3\left(\sqrt{c+d\,x}-\sqrt{c}\right)^4}{\left(\sqrt{-c}-\sqrt{d\,x-c}\right)^4}-\frac{4\,d^3\left(\sqrt{c+d\,x}-\sqrt{c}\right)^6}{\left(\sqrt{-c}-\sqrt{d\,x-c}\right)^6}+\frac{d^3\left(\sqrt{c+d\,x}-\sqrt{c}\right)^8}{\left(\sqrt{-c}-\sqrt{d\,x-c}\right)^8}}-\frac{\frac{23\,b\,c^4\left(\sqrt{c+d\,x}-\sqrt{c}\right)^3}{2\left(\sqrt{-c}-\sqrt{d\,x-c}\right)^3}-\frac{3\,b\,c^4\left(\sqrt{c+d\,x}-\sqrt{c}\right)}{2\left(\sqrt{-c}-\sqrt{d\,x-c}\right)}+\frac{333\,b\,c^4\left(\sqrt{c+d\,x}-\sqrt{c}\right)^5}{2\left(\sqrt{-c}-\sqrt{d\,x-c}\right)^5}+\frac{671\,b\,c^4\left(\sqrt{c+d\,x}-\sqrt{c}\right)^7}{2\left(\sqrt{-c}-\sqrt{d\,x-c}\right)^7}+\frac{671\,b\,c^4\left(\sqrt{c+d\,x}-\sqrt{c}\right)^9}{2\left(\sqrt{-c}-\sqrt{d\,x-c}\right)^9}+\frac{333\,b\,c^4\left(\sqrt{c+d\,x}-\sqrt{c}\right)^{11}}{2\left(\sqrt{-c}-\sqrt{d\,x-c}\right)^{11}}+\frac{23\,b\,c^4\left(\sqrt{c+d\,x}-\sqrt{c}\right)^{13}}{2\left(\sqrt{-c}-\sqrt{d\,x-c}\right)^{13}}-\frac{3\,b\,c^4\left(\sqrt{c+d\,x}-\sqrt{c}\right)^{15}}{2\left(\sqrt{-c}-\sqrt{d\,x-c}\right)^{15}}}{d^5-\frac{8\,d^5\left(\sqrt{c+d\,x}-\sqrt{c}\right)^2}{\left(\sqrt{-c}-\sqrt{d\,x-c}\right)^2}+\frac{28\,d^5\left(\sqrt{c+d\,x}-\sqrt{c}\right)^4}{\left(\sqrt{-c}-\sqrt{d\,x-c}\right)^4}-\frac{56\,d^5\left(\sqrt{c+d\,x}-\sqrt{c}\right)^6}{\left(\sqrt{-c}-\sqrt{d\,x-c}\right)^6}+\frac{70\,d^5\left(\sqrt{c+d\,x}-\sqrt{c}\right)^8}{\left(\sqrt{-c}-\sqrt{d\,x-c}\right)^8}-\frac{56\,d^5\left(\sqrt{c+d\,x}-\sqrt{c}\right)^{10}}{\left(\sqrt{-c}-\sqrt{d\,x-c}\right)^{10}}+\frac{28\,d^5\left(\sqrt{c+d\,x}-\sqrt{c}\right)^{12}}{\left(\sqrt{-c}-\sqrt{d\,x-c}\right)^{12}}-\frac{8\,d^5\left(\sqrt{c+d\,x}-\sqrt{c}\right)^{14}}{\left(\sqrt{-c}-\sqrt{d\,x-c}\right)^{14}}+\frac{d^5\left(\sqrt{c+d\,x}-\sqrt{c}\right)^{16}}{\left(\sqrt{-c}-\sqrt{d\,x-c}\right)^{16}}}-\frac{2\,a\,c^2\,\mathrm{atanh}\left(\frac{\sqrt{c+d\,x}-\sqrt{c}}{\sqrt{-c}-\sqrt{d\,x-c}}\right)}{d^3}-\frac{3\,b\,c^4\,\mathrm{atanh}\left(\frac{\sqrt{c+d\,x}-\sqrt{c}}{\sqrt{-c}-\sqrt{d\,x-c}}\right)}{2\,d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a + b*x^2))/((c + d*x)^(1/2)*(d*x - c)^(1/2)),x)
```

```
[Out] ((2*a*c^2*((c + d*x)^(1/2) - c^(1/2)))/((-c)^(1/2) - (d*x - c)^(1/2)) + (14
*a*c^2*((c + d*x)^(1/2) - c^(1/2))^3)/((-c)^(1/2) - (d*x - c)^(1/2))^3 + (1
4*a*c^2*((c + d*x)^(1/2) - c^(1/2))^5)/((-c)^(1/2) - (d*x - c)^(1/2))^5 + (
2*a*c^2*((c + d*x)^(1/2) - c^(1/2))^7)/((-c)^(1/2) - (d*x - c)^(1/2))^7)/(d
^3 - (4*d^3*((c + d*x)^(1/2) - c^(1/2))^2)/((-c)^(1/2) - (d*x - c)^(1/2))^2
 + (6*d^3*((c + d*x)^(1/2) - c^(1/2))^4)/((-c)^(1/2) - (d*x - c)^(1/2))^4 -
 (4*d^3*((c + d*x)^(1/2) - c^(1/2))^6)/((-c)^(1/2) - (d*x - c)^(1/2))^6 + (
```

```
d^3*((c + d*x)^(1/2) - c^(1/2))^8)/((-c)^(1/2) - (d*x - c)^(1/2))^8) - ((23
*b*c^4*((c + d*x)^(1/2) - c^(1/2))^3)/(2*((-c)^(1/2) - (d*x - c)^(1/2))^3)
- (3*b*c^4*((c + d*x)^(1/2) - c^(1/2)))/(2*((-c)^(1/2) - (d*x - c)^(1/2)))
+ (333*b*c^4*((c + d*x)^(1/2) - c^(1/2))^5)/(2*((-c)^(1/2) - (d*x - c)^(1/2
))^5) + (671*b*c^4*((c + d*x)^(1/2) - c^(1/2))^7)/(2*((-c)^(1/2) - (d*x - c
)^(1/2))^7) + (671*b*c^4*((c + d*x)^(1/2) - c^(1/2))^9)/(2*((-c)^(1/2) - (d
*x - c)^(1/2))^9) + (333*b*c^4*((c + d*x)^(1/2) - c^(1/2))^11)/(2*((-c)^(1/
2) - (d*x - c)^(1/2))^11) + (23*b*c^4*((c + d*x)^(1/2) - c^(1/2))^13)/(2*((
-c)^(1/2) - (d*x - c)^(1/2))^13) - (3*b*c^4*((c + d*x)^(1/2) - c^(1/2))^15)
/(2*((-c)^(1/2) - (d*x - c)^(1/2))^15))/(d^5 - (8*d^5*((c + d*x)^(1/2) - c^
(1/2))^2)/((-c)^(1/2) - (d*x - c)^(1/2))^2 + (28*d^5*((c + d*x)^(1/2) - c^(
1/2))^4)/((-c)^(1/2) - (d*x - c)^(1/2))^4 - (56*d^5*((c + d*x)^(1/2) - c^(1
/2))^6)/((-c)^(1/2) - (d*x - c)^(1/2))^6 + (70*d^5*((c + d*x)^(1/2) - c^(1/
2))^8)/((-c)^(1/2) - (d*x - c)^(1/2))^8 - (56*d^5*((c + d*x)^(1/2) - c^(1/2
))^10)/((-c)^(1/2) - (d*x - c)^(1/2))^10 + (28*d^5*((c + d*x)^(1/2) - c^(1/
2))^12)/((-c)^(1/2) - (d*x - c)^(1/2))^12 - (8*d^5*((c + d*x)^(1/2) - c^(1/
2))^14)/((-c)^(1/2) - (d*x - c)^(1/2))^14 + (d^5*((c + d*x)^(1/2) - c^(1/2)
)^16)/((-c)^(1/2) - (d*x - c)^(1/2))^16) - (2*a*c^2*atanh(((c + d*x)^(1/2)
- c^(1/2))/((-c)^(1/2) - (d*x - c)^(1/2))))/d^3 - (3*b*c^4*atanh(((c + d*x)
^(1/2) - c^(1/2))/((-c)^(1/2) - (d*x - c)^(1/2))))/(2*d^5)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x**2+a)/(d*x-c)**(1/2)/(d*x+c)**(1/2),x)
[Out] Timed out
```

3.218 $\int \frac{x(a+bx^2)}{\sqrt{-c+dx}\,\sqrt{c+dx}}\,dx$

Optimal. Leaf size=72

$$\frac{\sqrt{dx-c}\,\sqrt{c+dx}\,\left(3ad^2+2bc^2\right)}{3d^4}+\frac{bx^2\sqrt{dx-c}\,\sqrt{c+dx}}{3d^2}$$

Rubi [A] time = 0.05, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {460, 74}

$$\frac{\sqrt{dx-c}\,\sqrt{c+dx}\,\left(3ad^2+2bc^2\right)}{3d^4}+\frac{bx^2\sqrt{dx-c}\,\sqrt{c+dx}}{3d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a + b*x^2))/(Sqrt[-c + d*x]*Sqrt[c + d*x]),x]
```

```
[Out] ((2*b*c^2 + 3*a*d^2)*Sqrt[-c + d*x]*Sqrt[c + d*x])/(3*d^4) + (b*x^2*Sqrt[-c
 + d*x]*Sqrt[c + d*x])/(3*d^2)
```

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 460

```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[(d*(e*x)^
(m + 1)*(a1 + b1*x^(n/2))^(p + 1)*(a2 + b2*x^(n/2))^(p + 1))/(b1*b2*e*(m +
n*(p + 1) + 1)), x] - Dist[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/
(b1*b2*(m + n*(p + 1) + 1)), Int[(e*x)^m*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n/
2))^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, m, n, p}, x] && EqQ[non2,
n/2] && EqQ[a2*b1 + a1*b2, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x\left(a+bx^2\right)}{\sqrt{-c+dx}\,\sqrt{c+dx}}\,dx &= \frac{bx^2\sqrt{-c+dx}\,\sqrt{c+dx}}{3d^2}-\frac{1}{3}\left(-3a-\frac{2bc^2}{d^2}\right)\int \frac{x}{\sqrt{-c+dx}\,\sqrt{c+dx}}\,dx\\ &= \frac{\left(2bc^2+3ad^2\right)\sqrt{-c+dx}\,\sqrt{c+dx}}{3d^4}+\frac{bx^2\sqrt{-c+dx}\,\sqrt{c+dx}}{3d^2}\end{aligned}$$

Mathematica [A] time = 0.04, size = 61, normalized size = 0.85

$$\frac{\left(d^2x^2-c^2\right)\left(3ad^2+2bc^2+bd^2x^2\right)}{3d^4\sqrt{dx-c}\,\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(a + b*x^2))/(Sqrt[-c + d*x]*Sqrt[c + d*x]),x]
```

```
[Out] ((-c^2 + d^2*x^2)*(2*b*c^2 + 3*a*d^2 + b*d^2*x^2))/(3*d^4*Sqrt[-c + d*x]*Sq
rt[c + d*x])
```

IntegrateAlgebraic [B] time = 0.13, size = 146, normalized size = 2.03

$$-\frac{2\sqrt{dx-c}\left(-\frac{6acd^2(dx-c)}{c+dx}+\frac{3acd^2(dx-c)^2}{(c+dx)^2}+3acd^2-\frac{2bc^3(dx-c)}{c+dx}+\frac{3bc^3(dx-c)^2}{(c+dx)^2}+3bc^3\right)}{3d^4\sqrt{c+dx}\left(\frac{dx-c}{c+dx}-1\right)^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x*(a + b*x^2))/(Sqrt[-c + d*x]*Sqrt[c + d*x]),x]
```

```
[Out] (-2*Sqrt[-c + d*x]*(3*b*c^3 + 3*a*c*d^2 + (3*b*c^3*(-c + d*x)^2)/(c + d*x)^
2 + (3*a*c*d^2*(-c + d*x)^2)/(c + d*x)^2 - (2*b*c^3*(-c + d*x))/(c + d*x) -
 (6*a*c*d^2*(-c + d*x))/(c + d*x)))/(3*d^4*Sqrt[c + d*x]*(-1 + (-c + d*x)/(
c + d*x))^3)
```

fricas [A] time = 0.76, size = 42, normalized size = 0.58

$$\frac{\left(bd^2x^2+2\,bc^2+3\,ad^2\right)\sqrt{dx+c}\,\sqrt{dx-c}}{3\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)/(d*x-c)^(1/2)/(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*(b*d^2*x^2 + 2*b*c^2 + 3*a*d^2)*sqrt(d*x + c)*sqrt(d*x - c)/d^4
```

giac [A] time = 0.20, size = 65, normalized size = 0.90

$$\frac{\sqrt{dx+c}\,\sqrt{dx-c}\left((dx+c)\left(\frac{(dx+c)b}{d^3}-\frac{2\,bc}{d^3}\right)+\frac{3\left(bc^2d^9+ad^{11}\right)}{d^{12}}\right)}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)/(d*x-c)^(1/2)/(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/3*sqrt(d*x + c)*sqrt(d*x - c)*((d*x + c)*((d*x + c)*b/d^3 - 2*b*c/d^3) +
3*(b*c^2*d^9 + a*d^11)/d^12)/d
```

maple [A] time = 0.04, size = 43, normalized size = 0.60

$$\frac{\sqrt{dx+c}\,\left(b\,d^2x^2+3a\,d^2+2b\,c^2\right)\sqrt{dx-c}}{3d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^2+a)/(d*x-c)^(1/2)/(d*x+c)^(1/2),x)
```

```
[Out] 1/3*(d*x+c)^(1/2)*(b*d^2*x^2+3*a*d^2+2*b*c^2)/d^4*(d*x-c)^(1/2)
```

maxima [A] time = 0.64, size = 69, normalized size = 0.96

$$\frac{\sqrt{d^2x^2-c^2}\,bx^2}{3\,d^2}+\frac{2\,\sqrt{d^2x^2-c^2}\,bc^2}{3\,d^4}+\frac{\sqrt{d^2x^2-c^2}\,a}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)/(d*x-c)^(1/2)/(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(d^2*x^2 - c^2)*b*x^2/d^2 + 2/3*sqrt(d^2*x^2 - c^2)*b*c^2/d^4 + sqr
t(d^2*x^2 - c^2)*a/d^2
```

mupad [B] time = 2.66, size = 76, normalized size = 1.06

$$\frac{\sqrt{d\,x-c}\,\left(\frac{2\,b\,c^3+3\,a\,c\,d^2}{3\,d^4}+\frac{b\,x^3}{3\,d}+\frac{x\,\left(2\,b\,c^2\,d+3\,a\,d^3\right)}{3\,d^4}+\frac{b\,c\,x^2}{3\,d^2}\right)}{\sqrt{c+d\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a + b*x^2))/((c + d*x)^(1/2)*(d*x - c)^(1/2)),x)
```

```
[Out] ((d*x - c)^(1/2)*((2*b*c^3 + 3*a*c*d^2)/(3*d^4) + (b*x^3)/(3*d) + (x*(3*a*d
^3 + 2*b*c^2*d))/(3*d^4) + (b*c*x^2)/(3*d^2)))/(c + d*x)^(1/2)
```

sympy [C] time = 44.70, size = 223, normalized size = 3.10

$$\frac{acG_{6,6}^{6,2}\left(\begin{matrix} -\frac{1}{4},\frac{1}{4} & 0,0,\frac{1}{2},1 \\ -\frac{1}{2},-\frac{1}{4},0,\frac{1}{4},\frac{1}{2},0 & \end{matrix}\middle|\frac{c^2}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}d^2}+\frac{iacG_{6,6}^{2,6}\left(\begin{matrix} -1,-\frac{3}{4},-\frac{1}{2},-\frac{1}{4},0,1 & \\ -\frac{3}{4},-\frac{1}{4} & -1,-\frac{1}{2},-\frac{1}{2},0\end{matrix}\middle|\frac{c^2e^{2i\pi}}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}d^2}+\frac{bc^3G_{6,6}^{6,2}\left(\begin{matrix} -\frac{5}{4},-\frac{3}{4} & -1,-1,-\frac{1}{2},1 \\ -\frac{3}{2},-\frac{5}{4},-1,-\frac{3}{4},-\frac{1}{2},0 & \end{matrix}\middle|\frac{c^2}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}d^4}+\frac{ibc^3G_{6,6}^{2,6}\left(\begin{matrix} -2,-\frac{7}{4},-\frac{3}{2},-\frac{5}{4},-1,1 & \\ -\frac{7}{4},-\frac{5}{4} & -2,-\frac{3}{2},-\frac{3}{2},0\end{matrix}\middle|\frac{c^2e^{2i\pi}}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**2+a)/(d*x-c)**(1/2)/(d*x+c)**(1/2),x)
```

```
[Out] a*c*meijerg(((-1/4, 1/4), (0, 0, 1/2, 1)), ((-1/2, -1/4, 0, 1/4, 1/2, 0), (
)), c**2/(d**2*x**2))/(4*pi**(3/2)*d**2) + I*a*c*meijerg(((-1, -3/4, -1/2,
-1/4, 0, 1), ()), ((-3/4, -1/4), (-1, -1/2, -1/2, 0)), c**2*exp_polar(2*I*p
i)/(d**2*x**2))/(4*pi**(3/2)*d**2) + b*c**3*meijerg(((-5/4, -3/4), (-1, -1,
 -1/2, 1)), ((-3/2, -5/4, -1, -3/4, -1/2, 0), ()), c**2/(d**2*x**2))/(4*pi*
*(3/2)*d**4) + I*b*c**3*meijerg(((-2, -7/4, -3/2, -5/4, -1, 1), ()), ((-7/4
, -5/4), (-2, -3/2, -3/2, 0)), c**2*exp_polar(2*I*pi)/(d**2*x**2))/(4*pi**(
3/2)*d**4)
```

3.219 $\int \frac{a+bx^2}{\sqrt{-c+dx}\,\sqrt{c+dx}}\,dx$

Optimal. Leaf size=68

$$\frac{(2ad^2+bc^2)\tanh^{-1}\left(\frac{\sqrt{dx-c}}{\sqrt{c+dx}}\right)}{d^3}+\frac{bx\sqrt{dx-c}\,\sqrt{c+dx}}{2d^2}$$

Rubi [A] time = 0.03, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {389, 63, 217, 206}

$$\frac{(2ad^2+bc^2)\tanh^{-1}\left(\frac{\sqrt{dx-c}}{\sqrt{c+dx}}\right)}{d^3}+\frac{bx\sqrt{dx-c}\,\sqrt{c+dx}}{2d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)/(Sqrt[-c + d*x]*Sqrt[c + d*x]),x]
```

```
[Out] (b*x*Sqrt[-c + d*x]*Sqrt[c + d*x])/(2*d^2) + ((b*c^2 + 2*a*d^2)*ArcTanh[Sqr
t[-c + d*x]/Sqrt[c + d*x]])/d^3
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 389

```
Int[((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_
.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[(d*x*(a1 + b1*x^(n/2))^(p +
1)*(a2 + b2*x^(n/2))^(p + 1))/(b1*b2*(n*(p + 1) + 1)), x] - Dist[(a1*a2*d -
 b1*b2*c*(n*(p + 1) + 1))/(b1*b2*(n*(p + 1) + 1)), Int[(a1 + b1*x^(n/2))^p*
(a2 + b2*x^(n/2))^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, n, p}, x] && Eq
Q[non2, n/2] && EqQ[a2*b1 + a1*b2, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rubi steps

$$\int \frac{a+bx^2}{\sqrt{-c+dx}\sqrt{c+dx}}\,dx = \frac{bx\sqrt{-c+dx}\sqrt{c+dx}}{2d^2} - \frac{\left(-bc^2-2ad^2\right)\int \frac{1}{\sqrt{-c+dx}\sqrt{c+dx}}\,dx}{2d^2}$$
$$= \frac{bx\sqrt{-c+dx}\sqrt{c+dx}}{2d^2} + \frac{\left(bc^2+2ad^2\right)\text{Subst}\left(\int \frac{1}{\sqrt{2c+x^2}}\,dx, x, \sqrt{-c+dx}\right)}{d^3}$$
$$= \frac{bx\sqrt{-c+dx}\sqrt{c+dx}}{2d^2} + \frac{\left(bc^2+2ad^2\right)\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{\sqrt{-c+dx}}{\sqrt{c+dx}}\right)}{d^3}$$
$$= \frac{bx\sqrt{-c+dx}\sqrt{c+dx}}{2d^2} + \frac{\left(bc^2+2ad^2\right)\tanh^{-1}\left(\frac{\sqrt{-c+dx}}{\sqrt{c+dx}}\right)}{d^3}$$

Mathematica [A] time = 0.35, size = 119, normalized size = 1.75

$$\frac{4\left(ad^2+bc^2\right)\tanh^{-1}\left(\frac{\sqrt{dx-c}}{\sqrt{c+dx}}\right) - \frac{2bc^{5/2}\sqrt{\frac{dx}{c}+1}\,\sinh^{-1}\left(\frac{\sqrt{dx-c}}{\sqrt{2}\sqrt{c}}\right)}{\sqrt{c+dx}} + bdx\sqrt{dx-c}\sqrt{c+dx}}{2d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)/(Sqrt[-c + d*x]*Sqrt[c + d*x]),x]
```

```
[Out] (b*d*x*Sqrt[-c + d*x]*Sqrt[c + d*x] - (2*b*c^(5/2)*Sqrt[1 + (d*x)/c]*ArcSin
h[Sqrt[-c + d*x]/(Sqrt[2]*Sqrt[c])])/Sqrt[c + d*x] + 4*(b*c^2 + a*d^2)*ArcT
anh[Sqrt[-c + d*x]/Sqrt[c + d*x]])/(2*d^3)
```

IntegrateAlgebraic [A] time = 0.14, size = 103, normalized size = 1.51

$$\frac{\left(2ad^2+bc^2\right)\tanh^{-1}\left(\frac{\sqrt{dx-c}}{\sqrt{c+dx}}\right)}{d^3} + \frac{bc^2\sqrt{dx-c}\left(\frac{dx-c}{c+dx}+1\right)}{d^3\sqrt{c+dx}\left(\frac{dx-c}{c+dx}-1\right)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x^2)/(Sqrt[-c + d*x]*Sqrt[c + d*x]),x]
```

```
[Out] (b*c^2*Sqrt[-c + d*x]*(1 + (-c + d*x)/(c + d*x)))/(d^3*Sqrt[c + d*x]*(-1 +
(-c + d*x)/(c + d*x))^2) + ((b*c^2 + 2*a*d^2)*ArcTanh[Sqrt[-c + d*x]/Sqrt[c
 + d*x]])/d^3
```

fricas [A] time = 0.86, size = 63, normalized size = 0.93

$$\frac{\sqrt{dx+c}\,\sqrt{dx-c}\,bdx - \left(bc^2+2\,ad^2\right)\log\left(-dx+\sqrt{dx+c}\,\sqrt{dx-c}\right)}{2\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/(d*x-c)^(1/2)/(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*(sqrt(d*x + c)*sqrt(d*x - c)*b*d*x - (b*c^2 + 2*a*d^2)*log(-d*x + sqrt(
d*x + c)*sqrt(d*x - c)))/d^3
```

giac [A] time = 0.26, size = 79, normalized size = 1.16

$$\frac{\sqrt{dx+c}\,\sqrt{dx-c}\left(\frac{(dx+c)b}{d^2}-\frac{bc}{d^2}\right) - \frac{2\left(bc^2+2\,ad^2\right)\log\left(\left|-\sqrt{dx+c}+\sqrt{dx-c}\right|\right)}{d^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/(d*x-c)^(1/2)/(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*(sqrt(d*x + c)*sqrt(d*x - c)*((d*x + c)*b/d^2 - b*c/d^2) - 2*(b*c^2 + 2
*a*d^2)*log(abs(-sqrt(d*x + c) + sqrt(d*x - c)))/d^2)/d
```

maple [C] time = 0.07, size = 124, normalized size = 1.82

$$\frac{\sqrt{dx-c}\,\sqrt{dx+c}\,\left(2a\,d^2\ln\left(\left(dx+\sqrt{d^2x^2-c^2}\,\mathrm{csgn}(d)\right)\mathrm{csgn}(d)\right)+b\,c^2\ln\left(\left(dx+\sqrt{d^2x^2-c^2}\,\mathrm{csgn}(d)\right)\mathrm{csgn}(d)\right)+\sqrt{d^2x^2-c^2}\,bdx\,\mathrm{csgn}(d)\right)\mathrm{csgn}(d)}{2\sqrt{d^2x^2-c^2}\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)/(d*x-c)^(1/2)/(d*x+c)^(1/2),x)
```

```
[Out] 1/2*(d*x-c)^(1/2)*(d*x+c)^(1/2)/d^3*((d^2*x^2-c^2)^(1/2)*csgn(d)*d*x*b+ln((
d*x+(d^2*x^2-c^2)^(1/2)*csgn(d))*csgn(d))*b*c^2+2*ln((d*x+(d^2*x^2-c^2)^(1/
2)*csgn(d))*csgn(d))*a*d^2)/(d^2*x^2-c^2)^(1/2)*csgn(d)
```

maxima [A] time = 0.46, size = 89, normalized size = 1.31

$$\frac{bc^2\log\left(2\,d^2x+2\,\sqrt{d^2x^2-c^2}\,d\right)}{2\,d^3}+\frac{a\log\left(2\,d^2x+2\,\sqrt{d^2x^2-c^2}\,d\right)}{d}+\frac{\sqrt{d^2x^2-c^2}\,bx}{2\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/(d*x-c)^(1/2)/(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*b*c^2*log(2*d^2*x + 2*sqrt(d^2*x^2 - c^2)*d)/d^3 + a*log(2*d^2*x + 2*sq
rt(d^2*x^2 - c^2)*d)/d + 1/2*sqrt(d^2*x^2 - c^2)*b*x/d^2
```

mupad [B] time = 10.80, size = 417, normalized size = 6.13

$$\frac{\frac{2\,b\,c^2\left(\sqrt{c+d\,x}-\sqrt{c}\right)}{\sqrt{-c}-\sqrt{d\,x-c}}+\frac{14\,b\,c^2\left(\sqrt{c+d\,x}-\sqrt{c}\right)^3}{\left(\sqrt{-c}-\sqrt{d\,x-c}\right)^3}+\frac{14\,b\,c^2\left(\sqrt{c+d\,x}-\sqrt{c}\right)^5}{\left(\sqrt{-c}-\sqrt{d\,x-c}\right)^5}+\frac{2\,b\,c^2\left(\sqrt{c+d\,x}-\sqrt{c}\right)^7}{\left(\sqrt{-c}-\sqrt{d\,x-c}\right)^7}}{d^3-\frac{4\,d^3\left(\sqrt{c+d\,x}-\sqrt{c}\right)^2}{\left(\sqrt{-c}-\sqrt{d\,x-c}\right)^2}+\frac{6\,d^3\left(\sqrt{c+d\,x}-\sqrt{c}\right)^4}{\left(\sqrt{-c}-\sqrt{d\,x-c}\right)^4}-\frac{4\,d^3\left(\sqrt{c+d\,x}-\sqrt{c}\right)^6}{\left(\sqrt{-c}-\sqrt{d\,x-c}\right)^6}+\frac{d^3\left(\sqrt{c+d\,x}-\sqrt{c}\right)^8}{\left(\sqrt{-c}-\sqrt{d\,x-c}\right)^8}}+\frac{4\,a\,\mathrm{atan}\left(\frac{d\left(\sqrt{-c}-\sqrt{d\,x-c}\right)}{\sqrt{-d^2}\,\left(\sqrt{c+d\,x}-\sqrt{c}\right)}\right)}{\sqrt{-d^2}}-\frac{2\,b\,c^2\,\mathrm{atanh}\left(\frac{\sqrt{c+d\,x}-\sqrt{c}}{\sqrt{-c}-\sqrt{d\,x-c}}\right)}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)/((c + d*x)^(1/2)*(d*x - c)^(1/2)),x)
```

```
[Out] ((2*b*c^2*((c + d*x)^(1/2) - c^(1/2)))/((-c)^(1/2) - (d*x - c)^(1/2)) + (14
*b*c^2*((c + d*x)^(1/2) - c^(1/2))^3)/((-c)^(1/2) - (d*x - c)^(1/2))^3 + (1
4*b*c^2*((c + d*x)^(1/2) - c^(1/2))^5)/((-c)^(1/2) - (d*x - c)^(1/2))^5 + (
2*b*c^2*((c + d*x)^(1/2) - c^(1/2))^7)/((-c)^(1/2) - (d*x - c)^(1/2))^7)/(d
^3 - (4*d^3*((c + d*x)^(1/2) - c^(1/2))^2)/((-c)^(1/2) - (d*x - c)^(1/2))^2
 + (6*d^3*((c + d*x)^(1/2) - c^(1/2))^4)/((-c)^(1/2) - (d*x - c)^(1/2))^4 -
 (4*d^3*((c + d*x)^(1/2) - c^(1/2))^6)/((-c)^(1/2) - (d*x - c)^(1/2))^6 + (
d^3*((c + d*x)^(1/2) - c^(1/2))^8)/((-c)^(1/2) - (d*x - c)^(1/2))^8) + (4*a
*atan((d*((-c)^(1/2) - (d*x - c)^(1/2)))/((-d^2)^(1/2)*((c + d*x)^(1/2) - c
^(1/2)))))/(-d^2)^(1/2) - (2*b*c^2*atanh(((c + d*x)^(1/2) - c^(1/2))/((-c)^
(1/2) - (d*x - c)^(1/2))))/d^3
```

sympy [C] time = 41.78, size = 199, normalized size = 2.93

$$\frac{aG_{6,6}^{6,2}\left(\begin{matrix}\frac{1}{4},\frac{3}{4} & \frac{1}{2},\frac{1}{2},1,1\\0,\frac{1}{4},\frac{1}{2},\frac{3}{4},1,0 & \end{matrix}\middle|\frac{c^2}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}d}-\frac{iaG_{6,6}^{2,6}\left(\begin{matrix}-\frac{1}{2},-\frac{1}{4},0,\frac{1}{4},\frac{1}{2},1 & \\-\frac{1}{4},\frac{1}{4} & -\frac{1}{2},0,0,0\end{matrix}\middle|\frac{c^2e^{2i\pi}}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}d}+\frac{bc^2G_{6,6}^{6,2}\left(\begin{matrix}-\frac{3}{4},-\frac{1}{4} & -\frac{1}{2},-\frac{1}{2},0,1\\-1,-\frac{3}{4},-\frac{1}{2},-\frac{1}{4},0,0 & \end{matrix}\middle|\frac{c^2}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}d^3}-\frac{ibc^2G_{6,6}^{2,6}\left(\begin{matrix}-\frac{3}{2},-\frac{5}{4},-1,-\frac{3}{4},-\frac{1}{2},1 & \\-\frac{5}{4},-\frac{3}{4} & -\frac{3}{2},-1,-1,0\end{matrix}\middle|\frac{c^2e^{2i\pi}}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)/(d*x-c)**(1/2)/(d*x+c)**(1/2),x)
```

```
[Out] a*meijerg(((1/4, 3/4), (1/2, 1/2, 1, 1)), ((0, 1/4, 1/2, 3/4, 1, 0), ()), c
**2/(d**2*x**2))/(4*pi**(3/2)*d) - I*a*meijerg(((-1/2, -1/4, 0, 1/4, 1/2, 1
), ()), ((-1/4, 1/4), (-1/2, 0, 0, 0)), c**2*exp_polar(2*I*pi)/(d**2*x**2))
/(4*pi**(3/2)*d) + b*c**2*meijerg(((-3/4, -1/4), (-1/2, -1/2, 0, 1)), ((-1,
 -3/4, -1/2, -1/4, 0, 0), ()), c**2/(d**2*x**2))/(4*pi**(3/2)*d**3) - I*b*c
**2*meijerg(((-3/2, -5/4, -1, -3/4, -1/2, 1), ()), ((-5/4, -3/4), (-3/2, -1
, -1, 0)), c**2*exp_polar(2*I*pi)/(d**2*x**2))/(4*pi**(3/2)*d**3)
```

3.220 $\int \frac{a+bx^2}{x\sqrt{-c+dx}\,\sqrt{c+dx}}\,dx$

Optimal. Leaf size=56

$$\frac{a\tan^{-1}\left(\frac{\sqrt{dx-c}\,\sqrt{c+dx}}{c}\right)}{c}+\frac{b\sqrt{dx-c}\,\sqrt{c+dx}}{d^2}$$

Rubi [A] time = 0.07, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {460, 92, 205}

$$\frac{a\tan^{-1}\left(\frac{\sqrt{dx-c}\,\sqrt{c+dx}}{c}\right)}{c}+\frac{b\sqrt{dx-c}\,\sqrt{c+dx}}{d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)/(x*Sqrt[-c + d*x]*Sqrt[c + d*x]),x]
```

```
[Out] (b*Sqrt[-c + d*x]*Sqrt[c + d*x])/d^2 + (a*ArcTan[(Sqrt[-c + d*x]*Sqrt[c + d
*x])/c])/c
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 460

```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[(d*(e*x)^
(m + 1)*(a1 + b1*x^(n/2))^(p + 1)*(a2 + b2*x^(n/2))^(p + 1))/(b1*b2*e*(m +
n*(p + 1) + 1)), x] - Dist[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/
(b1*b2*(m + n*(p + 1) + 1)), Int[(e*x)^m*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n/
2))^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, m, n, p}, x] && EqQ[non2,
n/2] && EqQ[a2*b1 + a1*b2, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{a+bx^2}{x\sqrt{-c+dx}\,\sqrt{c+dx}}\,dx &= \frac{b\sqrt{-c+dx}\,\sqrt{c+dx}}{d^2}+a\int \frac{1}{x\sqrt{-c+dx}\,\sqrt{c+dx}}\,dx\\
&= \frac{b\sqrt{-c+dx}\,\sqrt{c+dx}}{d^2}+(ad)\,\text{Subst}\left(\int \frac{1}{c^2d+dx^2}\,dx,x,\sqrt{-c+dx}\,\sqrt{c+dx}\right)\\
&= \frac{b\sqrt{-c+dx}\,\sqrt{c+dx}}{d^2}+\frac{a\tan^{-1}\left(\frac{\sqrt{-c+dx}\,\sqrt{c+dx}}{c}\right)}{c}
\end{aligned}$$

Mathematica [A] time = 0.04, size = 87, normalized size = 1.55

$$\frac{ad^2\sqrt{d^2x^2-c^2}\,\tan^{-1}\left(\frac{\sqrt{d^2x^2-c^2}}{c}\right)-bc^3+bcd^2x^2}{cd^2\sqrt{dx-c}\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)/(x*Sqrt[-c + d*x]*Sqrt[c + d*x]),x]
```

```
[Out] (-(b*c^3) + b*c*d^2*x^2 + a*d^2*Sqrt[-c^2 + d^2*x^2]*ArcTan[Sqrt[-c^2 + d^2
*x^2]/c])/(c*d^2*Sqrt[-c + d*x]*Sqrt[c + d*x])
```

IntegrateAlgebraic [A] time = 0.11, size = 75, normalized size = 1.34

$$\frac{2a\tan^{-1}\left(\frac{\sqrt{dx-c}}{\sqrt{c+dx}}\right)}{c}-\frac{2bc\sqrt{dx-c}}{d^2\sqrt{c+dx}\left(\frac{dx-c}{c+dx}-1\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x^2)/(x*Sqrt[-c + d*x]*Sqrt[c + d*x]),x]
```

```
[Out] (-2*b*c*Sqrt[-c + d*x])/(d^2*Sqrt[c + d*x]*(-1 + (-c + d*x)/(c + d*x))) + (
2*a*ArcTan[Sqrt[-c + d*x]/Sqrt[c + d*x]])/c
```

fricas [A] time = 0.85, size = 61, normalized size = 1.09

$$\frac{2\,ad^2\arctan\left(-\frac{dx-\sqrt{dx+c}\sqrt{dx-c}}{c}\right)+\sqrt{dx+c}\sqrt{dx-c}\,bc}{cd^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/x/(d*x-c)^(1/2)/(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] (2*a*d^2*arctan(-(d*x - sqrt(d*x + c)*sqrt(d*x - c))/c) + sqrt(d*x + c)*sqr
t(d*x - c)*b*c)/(c*d^2)
```

giac [A] time = 0.21, size = 55, normalized size = 0.98

$$-\frac{2\,a\arctan\left(\frac{\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^2}{2\,c}\right)}{c}+\frac{\sqrt{dx+c}\sqrt{dx-c}\,b}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/x/(d*x-c)^(1/2)/(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] -2*a*arctan(1/2*(sqrt(d*x + c) - sqrt(d*x - c))^2/c)/c + sqrt(d*x + c)*sqrt
(d*x - c)*b/d^2
```

maple [B] time = 0.08, size = 108, normalized size = 1.93

$$\frac{\left(-a\,d^2\ln\left(-\frac{2\left(c^2-\sqrt{-c^2}\sqrt{d^2x^2-c^2}\right)}{x}\right)+\sqrt{-c^2}\sqrt{d^2x^2-c^2}\,b\right)\sqrt{dx-c}\sqrt{dx+c}}{\sqrt{d^2x^2-c^2}\sqrt{-c^2}\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)/x/(d*x-c)^(1/2)/(d*x+c)^(1/2),x)
```

```
[Out] (-ln(-2*(c^2-(-c^2)^(1/2)*(d^2*x^2-c^2)^(1/2))/x)*a*d^2+b*(-c^2)^(1/2)*(d^2
*x^2-c^2)^(1/2))*(d*x-c)^(1/2)*(d*x+c)^(1/2)/(d^2*x^2-c^2)^(1/2)/d^2/(-c^2)
^(1/2)
```

maxima [A] time = 1.34, size = 37, normalized size = 0.66

$$-\frac{a \arcsin\left(\frac{c}{d|x|}\right)}{c} + \frac{\sqrt{d^2x^2 - c^2}\, b}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/x/(d*x-c)^(1/2)/(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] -a*arcsin(c/(d*abs(x)))/c + sqrt(d^2*x^2 - c^2)*b/d^2
```

mupad [B] time = 3.97, size = 108, normalized size = 1.93

$$\frac{b\sqrt{c+d\,x}\,\sqrt{d\,x-c}}{d^2} - \frac{a\sqrt{-c}\left(\ln\left(\frac{\left(\sqrt{c+d\,x}-\sqrt{c}\right)^2}{\left(\sqrt{-c}-\sqrt{d\,x-c}\right)^2}+1\right)-\ln\left(\frac{\sqrt{c+d\,x}-\sqrt{c}}{\sqrt{-c}-\sqrt{d\,x-c}}\right)\right)}{c^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)/(x*(c + d*x)^(1/2)*(d*x - c)^(1/2)),x)
```

```
[Out] (b*(c + d*x)^(1/2)*(d*x - c)^(1/2))/d^2 - (a*(-c)^(1/2)*(log(((c + d*x)^(1/
2) - c^(1/2))^2/((-c)^(1/2) - (d*x - c)^(1/2))^2 + 1) - log(((c + d*x)^(1/2
) - c^(1/2))/((-c)^(1/2) - (d*x - c)^(1/2)))))/c^(3/2)
```

sympy [C] time = 39.16, size = 178, normalized size = 3.18

$$-\frac{aG_{6,6}^{5,3}\left(\begin{matrix}\frac{3}{4},\frac{5}{4},1 & 1,1,\frac{3}{2}\\ \frac{1}{2},\frac{3}{4},1,\frac{5}{4},\frac{3}{2} & 0\end{matrix}\middle|\frac{c^2}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}c} + \frac{iaG_{6,6}^{2,6}\left(\begin{matrix}0,\frac{1}{4},\frac{1}{2},\frac{3}{4},1,1 & \\ \frac{1}{4},\frac{3}{4} & 0,\frac{1}{2},\frac{1}{2},0\end{matrix}\middle|\frac{c^2e^{2i\pi}}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}c} + \frac{bcG_{6,6}^{6,2}\left(\begin{matrix}-\frac{1}{4},\frac{1}{4} & 0,0,\frac{1}{2},1\\ -\frac{1}{2},-\frac{1}{4},0,\frac{1}{4},\frac{1}{2},0 & \end{matrix}\middle|\frac{c^2}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}d^2} + \frac{ibcG_{6,6}^{2,6}\left(\begin{matrix}-1,-\frac{3}{4},-\frac{1}{2},-\frac{1}{4},0,1 & \\ -\frac{3}{4},-\frac{1}{4} & -1,-\frac{1}{2},-\frac{1}{2},0\end{matrix}\middle|\frac{c^2e^{2i\pi}}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)/x/(d*x-c)**(1/2)/(d*x+c)**(1/2),x)
```

```
[Out] -a*meijerg(((3/4, 5/4, 1), (1, 1, 3/2)), ((1/2, 3/4, 1, 5/4, 3/2), (0,)), c
**2/(d**2*x**2))/(4*pi**(3/2)*c) + I*a*meijerg(((0, 1/4, 1/2, 3/4, 1, 1), (
)), ((1/4, 3/4), (0, 1/2, 1/2, 0)), c**2*exp_polar(2*I*pi)/(d**2*x**2))/(4*
pi**(3/2)*c) + b*c*meijerg(((-1/4, 1/4), (0, 0, 1/2, 1)), ((-1/2, -1/4, 0,
1/4, 1/2, 0), ()), c**2/(d**2*x**2))/(4*pi**(3/2)*d**2) + I*b*c*meijerg(((-
1, -3/4, -1/2, -1/4, 0, 1), ()), ((-3/4, -1/4), (-1, -1/2, -1/2, 0)), c**2*
exp_polar(2*I*pi)/(d**2*x**2))/(4*pi**(3/2)*d**2)
```

3.221 $\int \frac{a+bx^2}{x^2\sqrt{-c+dx}\,\sqrt{c+dx}}\,dx$

Optimal. Leaf size=57

$$\frac{a\sqrt{dx-c}\,\sqrt{c+dx}}{c^2x}+\frac{2b\tanh^{-1}\left(\frac{\sqrt{dx-c}}{\sqrt{c+dx}}\right)}{d}$$

Rubi [A] time = 0.07, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {454, 63, 217, 206}

$$\frac{a\sqrt{dx-c}\,\sqrt{c+dx}}{c^2x}+\frac{2b\tanh^{-1}\left(\frac{\sqrt{dx-c}}{\sqrt{c+dx}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)/(x^2*Sqrt[-c + d*x]*Sqrt[c + d*x]),x]
```

```
[Out] (a*Sqrt[-c + d*x]*Sqrt[c + d*x])/(c^2*x) + (2*b*ArcTanh[Sqrt[-c + d*x]/Sqrt
[c + d*x]])/d
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 454

```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[(c*(e*x)^
(m + 1)*(a1 + b1*x^(n/2))^(p + 1)*(a2 + b2*x^(n/2))^(p + 1))/(a1*a2*e*(m +
1)), x] + Dist[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/(a1*a2*e^n*(
m + 1)), Int[(e*x)^(m + n)*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n/2))^p, x], x]
/; FreeQ[{a1, b1, a2, b2, c, d, e, p}, x] && EqQ[non2, n/2] && EqQ[a2*b1 +
a1*b2, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (L
tQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rubi steps

$$\int \frac{a+bx^2}{x^2\sqrt{-c+dx}\,\sqrt{c+dx}}\,dx = \frac{a\sqrt{-c+dx}\,\sqrt{c+dx}}{c^2x} + b\int \frac{1}{\sqrt{-c+dx}\,\sqrt{c+dx}}\,dx$$
$$= \frac{a\sqrt{-c+dx}\,\sqrt{c+dx}}{c^2x} + \frac{(2b)\,\text{Subst}\left(\int \frac{1}{\sqrt{2c+x^2}}\,dx, x, \sqrt{-c+dx}\right)}{d}$$
$$= \frac{a\sqrt{-c+dx}\,\sqrt{c+dx}}{c^2x} + \frac{(2b)\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{\sqrt{-c+dx}}{\sqrt{c+dx}}\right)}{d}$$
$$= \frac{a\sqrt{-c+dx}\,\sqrt{c+dx}}{c^2x} + \frac{2b\tanh^{-1}\left(\frac{\sqrt{-c+dx}}{\sqrt{c+dx}}\right)}{d}$$

Mathematica [A] time = 0.04, size = 90, normalized size = 1.58

$$\frac{\sqrt{d^2x^2-c^2}\left(\frac{a\sqrt{d^2x^2-c^2}}{c^2x} + \frac{b\tanh^{-1}\left(\frac{dx}{\sqrt{d^2x^2-c^2}}\right)}{d}\right)}{\sqrt{dx-c}\,\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)/(x^2*Sqrt[-c + d*x]*Sqrt[c + d*x]),x]
```

```
[Out] (Sqrt[-c^2 + d^2*x^2]*((a*Sqrt[-c^2 + d^2*x^2])/(c^2*x) + (b*ArcTanh[(d*x)/
Sqrt[-c^2 + d^2*x^2]])/d))/(Sqrt[-c + d*x]*Sqrt[c + d*x])
```

IntegrateAlgebraic [A] time = 0.12, size = 75, normalized size = 1.32

$$\frac{2ad\sqrt{dx-c}}{c^2\sqrt{c+dx}\left(\frac{dx-c}{c+dx}+1\right)} + \frac{2b\tanh^{-1}\left(\frac{\sqrt{dx-c}}{\sqrt{c+dx}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x^2)/(x^2*Sqrt[-c + d*x]*Sqrt[c + d*x]),x]
```

```
[Out] (2*a*d*Sqrt[-c + d*x])/(c^2*Sqrt[c + d*x]*(1 + (-c + d*x)/(c + d*x))) + (2*
b*ArcTanh[Sqrt[-c + d*x]/Sqrt[c + d*x]])/d
```

fricas [A] time = 1.09, size = 68, normalized size = 1.19

$$-\frac{bc^2x\log\left(-dx+\sqrt{dx+c}\,\sqrt{dx-c}\right) - ad^2x - \sqrt{dx+c}\,\sqrt{dx-c}\,ad}{c^2dx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/x^2/(d*x-c)^(1/2)/(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] -(b*c^2*x*log(-d*x + sqrt(d*x + c)*sqrt(d*x - c)) - a*d^2*x - sqrt(d*x + c)
*sqrt(d*x - c)*a*d)/(c^2*d*x)
```

giac [A] time = 0.22, size = 66, normalized size = 1.16

$$\frac{\frac{16\,ad^2}{\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^4+4\,c^2} - b\log\left(\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^4\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/x^2/(d*x-c)^(1/2)/(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*(16*a*d^2/((sqrt(d*x + c) - sqrt(d*x - c))^4 + 4*c^2) - b*log((sqrt(d*x
 + c) - sqrt(d*x - c))^4))/d
```

maple [C] time = 0.07, size = 97, normalized size = 1.70

$$\frac{\sqrt{dx-c}\,\sqrt{dx+c}\,\left(b\,c^{2}x\ln\left(\left(dx+\sqrt{d^{2}x^{2}-c^{2}}\,\operatorname{csgn}(d)\right)\operatorname{csgn}(d)\right)+\sqrt{d^{2}x^{2}-c^{2}}\,ad\operatorname{csgn}(d)\right)\operatorname{csgn}(d)}{\sqrt{d^{2}x^{2}-c^{2}}\,c^{2}dx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)/x^2/(d*x-c)^(1/2)/(d*x+c)^(1/2),x)
```

```
[Out] (d*x-c)^(1/2)*(d*x+c)^(1/2)/c^2*(b*c^2*x*ln((d*x+(d^2*x^2-c^2)^(1/2)*csgn(d
))*csgn(d))+(d^2*x^2-c^2)^(1/2)*a*d*csgn(d))*csgn(d)/(d^2*x^2-c^2)^(1/2)/d/
x
```

maxima [A] time = 1.26, size = 55, normalized size = 0.96

$$\frac{b\log\left(2\,d^{2}x+2\,\sqrt{d^{2}x^{2}-c^{2}}\,d\right)}{d}+\frac{\sqrt{d^{2}x^{2}-c^{2}}\,a}{c^{2}x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/x^2/(d*x-c)^(1/2)/(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] b*log(2*d^2*x + 2*sqrt(d^2*x^2 - c^2)*d)/d + sqrt(d^2*x^2 - c^2)*a/(c^2*x)
```

mupad [B] time = 2.94, size = 77, normalized size = 1.35

$$\frac{4\,b\,\mathrm{atan}\left(\frac{d\,\left(\sqrt{-c}-\sqrt{d\,x-c}\right)}{\sqrt{-d^{2}}\,\left(\sqrt{c+d\,x}-\sqrt{c}\right)}\right)}{\sqrt{-d^{2}}}+\frac{a\,\sqrt{c+d\,x}\,\sqrt{d\,x-c}}{c^{2}\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)/(x^2*(c + d*x)^(1/2)*(d*x - c)^(1/2)),x)
```

```
[Out] (4*b*atan((d*((-c)^(1/2) - (d*x - c)^(1/2)))/((-d^2)^(1/2)*((c + d*x)^(1/2)
 - c^(1/2)))))/(-d^2)^(1/2) + (a*(c + d*x)^(1/2)*(d*x - c)^(1/2))/(c^2*x)
```

sympy [C] time = 36.82, size = 165, normalized size = 2.89

$$-\frac{adG_{6,6}^{5,3}\left(\begin{matrix}\frac{5}{4},\frac{7}{4},1 & \frac{3}{2},\frac{3}{2},2\\ 1,\frac{5}{4},\frac{3}{2},\frac{7}{4},2 & 0\end{matrix}\middle|\frac{c^{2}}{d^{2}x^{2}}\right)}{4\pi^{\frac{3}{2}}c^{2}}-\frac{iadG_{6,6}^{2,6}\left(\begin{matrix}\frac{1}{2},\frac{3}{4},1,\frac{5}{4},\frac{3}{2},1 & \\ \frac{3}{4},\frac{5}{4} & \frac{1}{2},1,1,0\end{matrix}\middle|\frac{c^{2}e^{2i\pi}}{d^{2}x^{2}}\right)}{4\pi^{\frac{3}{2}}c^{2}}+\frac{bG_{6,6}^{6,2}\left(\begin{matrix}\frac{1}{4},\frac{3}{4} & \frac{1}{2},\frac{1}{2},1,1\\ 0,\frac{1}{4},\frac{1}{2},\frac{3}{4},1,0 & \end{matrix}\middle|\frac{c^{2}}{d^{2}x^{2}}\right)}{4\pi^{\frac{3}{2}}d}-\frac{ibG_{6,6}^{2,6}\left(\begin{matrix}-\frac{1}{2},-\frac{1}{4},0,\frac{1}{4},\frac{1}{2},1 & \\ -\frac{1}{4},\frac{1}{4} & -\frac{1}{2},0,0,0\end{matrix}\middle|\frac{c^{2}e^{2i\pi}}{d^{2}x^{2}}\right)}{4\pi^{\frac{3}{2}}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)/x**2/(d*x-c)**(1/2)/(d*x+c)**(1/2),x)
```

```
[Out] -a*d*meijerg(((5/4, 7/4, 1), (3/2, 3/2, 2)), ((1, 5/4, 3/2, 7/4, 2), (0,)),
 c**2/(d**2*x**2))/(4*pi**(3/2)*c**2) - I*a*d*meijerg(((1/2, 3/4, 1, 5/4, 3
/2, 1), ()), ((3/4, 5/4), (1/2, 1, 1, 0)), c**2*exp_polar(2*I*pi)/(d**2*x**
2))/(4*pi**(3/2)*c**2) + b*meijerg(((1/4, 3/4), (1/2, 1/2, 1, 1)), ((0, 1/4
, 1/2, 3/4, 1, 0), ()), c**2/(d**2*x**2))/(4*pi**(3/2)*d) - I*b*meijerg(((-
1/2, -1/4, 0, 1/4, 1/2, 1), ()), ((-1/4, 1/4), (-1/2, 0, 0, 0)), c**2*exp_p
olar(2*I*pi)/(d**2*x**2))/(4*pi**(3/2)*d)
```

3.222 $\int \frac{a+bx^2}{x^3\sqrt{-c+dx}\,\sqrt{c+dx}}\,dx$

Optimal. Leaf size=76

$$\frac{(ad^2+2bc^2)\tan^{-1}\left(\frac{\sqrt{dx-c}\,\sqrt{c+dx}}{c}\right)}{2c^3}+\frac{a\sqrt{dx-c}\,\sqrt{c+dx}}{2c^2x^2}$$

Rubi [A] time = 0.07, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {454, 92, 205}

$$\frac{(ad^2+2bc^2)\tan^{-1}\left(\frac{\sqrt{dx-c}\,\sqrt{c+dx}}{c}\right)}{2c^3}+\frac{a\sqrt{dx-c}\,\sqrt{c+dx}}{2c^2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)/(x^3*Sqrt[-c + d*x]*Sqrt[c + d*x]),x]
```

```
[Out] (a*Sqrt[-c + d*x]*Sqrt[c + d*x])/(2*c^2*x^2) + ((2*b*c^2 + a*d^2)*ArcTan[(S
qrt[-c + d*x]*Sqrt[c + d*x])/c])/(2*c^3)
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 454

```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[(c*(e*x)^
(m + 1)*(a1 + b1*x^(n/2))^(p + 1)*(a2 + b2*x^(n/2))^(p + 1))/(a1*a2*e*(m +
1)), x] + Dist[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/(a1*a2*e^n*(
m + 1)), Int[(e*x)^(m + n)*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n/2))^p, x], x]
/; FreeQ[{a1, b1, a2, b2, c, d, e, p}, x] && EqQ[non2, n/2] && EqQ[a2*b1 +
a1*b2, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (L
tQ[n, 0] && GtQ[m + n, -1])) && !ILtQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx^2}{x^3\sqrt{-c+dx}\,\sqrt{c+dx}}\,dx &= \frac{a\sqrt{-c+dx}\,\sqrt{c+dx}}{2c^2x^2}+\frac{1}{2}\left(2b+\frac{ad^2}{c^2}\right)\int \frac{1}{x\sqrt{-c+dx}\,\sqrt{c+dx}}\,dx\\ &= \frac{a\sqrt{-c+dx}\,\sqrt{c+dx}}{2c^2x^2}+\frac{1}{2}\left(d\left(2b+\frac{ad^2}{c^2}\right)\right)\text{Subst}\left(\int \frac{1}{c^2d+dx^2}\,dx,x,\sqrt{-c+dx}\,\sqrt{c+\right.\\ &= \frac{a\sqrt{-c+dx}\,\sqrt{c+dx}}{2c^2x^2}+\frac{(2bc^2+ad^2)\tan^{-1}\left(\frac{\sqrt{-c+dx}\,\sqrt{c+dx}}{c}\right)}{2c^3}\end{aligned}$$

Mathematica [A] time = 0.10, size = 102, normalized size = 1.34

$$\frac{x^2\sqrt{d^2x^2-c^2}\left(ad^2+2bc^2\right)\tan^{-1}\left(\frac{\sqrt{d^2x^2-c^2}}{c}\right)+a\left(cd^2x^2-c^3\right)}{2c^3x^2\sqrt{dx-c}\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)/(x^3*Sqrt[-c + d*x]*Sqrt[c + d*x]),x]
```

```
[Out] (a*(-c^3 + c*d^2*x^2) + (2*b*c^2 + a*d^2)*x^2*Sqrt[-c^2 + d^2*x^2]*ArcTan[S
qrt[-c^2 + d^2*x^2]/c])/(2*c^3*x^2*Sqrt[-c + d*x]*Sqrt[c + d*x])
```

IntegrateAlgebraic [A] time = 0.12, size = 104, normalized size = 1.37

$$\frac{\left(ad^2+2bc^2\right)\tan^{-1}\left(\frac{\sqrt{dx-c}}{\sqrt{c+dx}}\right)}{c^3}-\frac{ad^2\sqrt{dx-c}\left(\frac{dx-c}{c+dx}-1\right)}{c^3\sqrt{c+dx}\left(\frac{dx-c}{c+dx}+1\right)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x^2)/(x^3*Sqrt[-c + d*x]*Sqrt[c + d*x]),x]
```

```
[Out] -((a*d^2*Sqrt[-c + d*x]*(-1 + (-c + d*x)/(c + d*x)))/(c^3*Sqrt[c + d*x]*(1
+ (-c + d*x)/(c + d*x))^2)) + ((2*b*c^2 + a*d^2)*ArcTan[Sqrt[-c + d*x]/Sqrt
[c + d*x]])/c^3
```

fricas [A] time = 0.81, size = 73, normalized size = 0.96

$$\frac{2\left(2\,bc^2+ad^2\right)x^2\arctan\left(-\frac{dx-\sqrt{dx+c}\,\sqrt{dx-c}}{c}\right)+\sqrt{dx+c}\,\sqrt{dx-c}\,ac}{2\,c^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/x^3/(d*x-c)^(1/2)/(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*(2*(2*b*c^2 + a*d^2)*x^2*arctan(-(d*x - sqrt(d*x + c)*sqrt(d*x - c))/c)
 + sqrt(d*x + c)*sqrt(d*x - c)*a*c)/(c^3*x^2)
```

giac [B] time = 0.49, size = 141, normalized size = 1.86

$$-\frac{\frac{\left(2\,bc^2d+ad^3\right)\arctan\left(\frac{\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^2}{2\,c}\right)}{c^3}+\frac{2\left(ad^3\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^6-4\,ac^2d^3\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^2\right)}{\left(\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^4+4\,c^2\right)^2c^2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/x^3/(d*x-c)^(1/2)/(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] -((2*b*c^2*d + a*d^3)*arctan(1/2*(sqrt(d*x + c) - sqrt(d*x - c))^2/c)/c^3 +
 2*(a*d^3*(sqrt(d*x + c) - sqrt(d*x - c))^6 - 4*a*c^2*d^3*(sqrt(d*x + c) -
sqrt(d*x - c))^2)/(((sqrt(d*x + c) - sqrt(d*x - c))^4 + 4*c^2)^2*c^2))/d
```

maple [B] time = 0.07, size = 158, normalized size = 2.08

$$-\frac{\sqrt{dx-c}\,\sqrt{dx+c}\left(a\,d^2x^2\ln\left(-\frac{2\left(c^2-\sqrt{-c^2}\,\sqrt{d^2x^2-c^2}\right)}{x}\right)+2b\,c^2x^2\ln\left(-\frac{2\left(c^2-\sqrt{-c^2}\,\sqrt{d^2x^2-c^2}\right)}{x}\right)-\sqrt{-c^2}\,\sqrt{d^2x^2-c^2}\,a\right)}{2\sqrt{d^2x^2-c^2}\,\sqrt{-c^2}\,c^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)/x^3/(d*x-c)^(1/2)/(d*x+c)^(1/2),x)
```

```
[Out] -1/2*(d*x-c)^(1/2)*(d*x+c)^(1/2)/c^2*(a*d^2*x^2*ln(-2*(c^2-(-c^2)^(1/2)*(d^
2*x^2-c^2)^(1/2))/x)+2*b*c^2*x^2*ln(-2*(c^2-(-c^2)^(1/2)*(d^2*x^2-c^2)^(1/2
))/x)-(-c^2)^(1/2)*(d^2*x^2-c^2)^(1/2)*a)/(d^2*x^2-c^2)^(1/2)/x^2/(-c^2)^(1
/2)
```

maxima [A] time = 1.29, size = 60, normalized size = 0.79

$$-\frac{b\arcsin\left(\frac{c}{d|x|}\right)}{c}-\frac{ad^2\arcsin\left(\frac{c}{d|x|}\right)}{2\,c^3}+\frac{\sqrt{d^2x^2-c^2}\,a}{2\,c^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/x^3/(d*x-c)^(1/2)/(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] -b*arcsin(c/(d*abs(x)))/c - 1/2*a*d^2*arcsin(c/(d*abs(x)))/c^3 + 1/2*sqrt(d
^2*x^2 - c^2)*a/(c^2*x^2)
```

mupad [B] time = 7.50, size = 457, normalized size = 6.01

$$\frac{a\,(-c)^{3/2}\,d^2\,\ln\left(\frac{\left(\sqrt{c+d\,x}-\sqrt{c}\right)^2}{\left(\sqrt{-c}-\sqrt{d\,x-c}\right)^2}+1\right)}{2\,c^{9/2}}-\frac{b\,\sqrt{-c}\,\left(\ln\left(\frac{\left(\sqrt{c+d\,x}-\sqrt{c}\right)^2}{\left(\sqrt{-c}-\sqrt{d\,x-c}\right)^2}+1\right)-\ln\left(\frac{\sqrt{c+d\,x}-\sqrt{c}}{\sqrt{-c}-\sqrt{d\,x-c}}\right)\right)}{c^{3/2}}-\frac{a\,(-c)^{3/2}\,d^2\,\ln\left(\frac{\sqrt{c+d\,x}-\sqrt{c}}{\sqrt{-c}-\sqrt{d\,x-c}}\right)}{2\,c^{9/2}}-\frac{\frac{a\,(-c)^{3/2}\,d^2}{32\,c^{9/2}}+\frac{a\,(-c)^{3/2}\,d^2\,\left(\sqrt{c+d\,x}-\sqrt{c}\right)^2}{16\,c^{9/2}\,\left(\sqrt{-c}-\sqrt{d\,x-c}\right)^2}-\frac{15\,a\,(-c)^{3/2}\,d^2\,\left(\sqrt{c+d\,x}-\sqrt{c}\right)^4}{32\,c^{9/2}\,\left(\sqrt{-c}-\sqrt{d\,x-c}\right)^4}}{\frac{\left(\sqrt{c+d\,x}-\sqrt{c}\right)^2}{\left(\sqrt{-c}-\sqrt{d\,x-c}\right)^2}+\frac{2\,\left(\sqrt{c+d\,x}-\sqrt{c}\right)^4}{\left(\sqrt{-c}-\sqrt{d\,x-c}\right)^4}+\frac{\left(\sqrt{c+d\,x}-\sqrt{c}\right)^6}{\left(\sqrt{-c}-\sqrt{d\,x-c}\right)^6}}+\frac{a\,d^2\,\left(\sqrt{c+d\,x}-\sqrt{c}\right)^2}{32\,(-c)^{3/2}\,c^{3/2}\,\left(\sqrt{-c}-\sqrt{d\,x-c}\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)/(x^3*(c + d*x)^(1/2)*(d*x - c)^(1/2)),x)
```

```
[Out] (a*(-c)^(3/2)*d^2*log(((c + d*x)^(1/2) - c^(1/2))^2/((-c)^(1/2) - (d*x - c)
^(1/2))^2 + 1))/(2*c^(9/2)) - (b*(-c)^(1/2)*(log(((c + d*x)^(1/2) - c^(1/2)
)^2/((-c)^(1/2) - (d*x - c)^(1/2))^2 + 1) - log(((c + d*x)^(1/2) - c^(1/2))
/((-c)^(1/2) - (d*x - c)^(1/2)))))/c^(3/2) - (a*(-c)^(3/2)*d^2*log(((c + d*
x)^(1/2) - c^(1/2))/((-c)^(1/2) - (d*x - c)^(1/2))))/(2*c^(9/2)) - ((a*(-c)
^(3/2)*d^2)/(32*c^(9/2)) + (a*(-c)^(3/2)*d^2*((c + d*x)^(1/2) - c^(1/2))^2)
/(16*c^(9/2)*((-c)^(1/2) - (d*x - c)^(1/2))^2) - (15*a*(-c)^(3/2)*d^2*((c +
 d*x)^(1/2) - c^(1/2))^4)/(32*c^(9/2)*((-c)^(1/2) - (d*x - c)^(1/2))^4))/((
(c + d*x)^(1/2) - c^(1/2))^2/((-c)^(1/2) - (d*x - c)^(1/2))^2 + (2*((c + d*
x)^(1/2) - c^(1/2))^4)/((-c)^(1/2) - (d*x - c)^(1/2))^4 + ((c + d*x)^(1/2)
- c^(1/2))^6/((-c)^(1/2) - (d*x - c)^(1/2))^6) + (a*d^2*((c + d*x)^(1/2) -
c^(1/2))^2)/(32*(-c)^(3/2)*c^(3/2)*((-c)^(1/2) - (d*x - c)^(1/2))^2)
```

sympy [C] time = 69.67, size = 162, normalized size = 2.13

$$-\frac{ad^2G_{6,6}^{5,3}\left(\begin{matrix}\frac{7}{4},\frac{9}{4},1 & 2,2,\frac{5}{2}\\ \frac{3}{2},\frac{7}{4},2,\frac{9}{4},\frac{5}{2} & 0\end{matrix}\middle|\frac{c^2}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}c^3}+\frac{iad^2G_{6,6}^{2,6}\left(\begin{matrix}1,\frac{5}{4},\frac{3}{2},\frac{7}{4},2,1 & \\ \frac{5}{4},\frac{7}{4} & 1,\frac{3}{2},\frac{3}{2},0\end{matrix}\middle|\frac{c^2e^{2i\pi}}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}c^3}-\frac{bG_{6,6}^{5,3}\left(\begin{matrix}\frac{3}{4},\frac{5}{4},1 & 1,1,\frac{3}{2}\\ \frac{1}{2},\frac{3}{4},1,\frac{5}{4},\frac{3}{2} & 0\end{matrix}\middle|\frac{c^2}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}c}+\frac{ibG_{6,6}^{2,6}\left(\begin{matrix}0,\frac{1}{4},\frac{1}{2},\frac{3}{4},1,1 & \\ \frac{1}{4},\frac{3}{4} & 0,\frac{1}{2},\frac{1}{2},0\end{matrix}\middle|\frac{c^2e^{2i\pi}}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)/x**3/(d*x-c)**(1/2)/(d*x+c)**(1/2),x)
```

```
[Out] -a*d**2*meijerg(((7/4, 9/4, 1), (2, 2, 5/2)), ((3/2, 7/4, 2, 9/4, 5/2), (0,
)), c**2/(d**2*x**2))/(4*pi**(3/2)*c**3) + I*a*d**2*meijerg(((1, 5/4, 3/2,
7/4, 2, 1), ()), ((5/4, 7/4), (1, 3/2, 3/2, 0)), c**2*exp_polar(2*I*pi)/(d*
*2*x**2))/(4*pi**(3/2)*c**3) - b*meijerg(((3/4, 5/4, 1), (1, 1, 3/2)), ((1/
2, 3/4, 1, 5/4, 3/2), (0,)), c**2/(d**2*x**2))/(4*pi**(3/2)*c) + I*b*meijer
g(((0, 1/4, 1/2, 3/4, 1, 1), ()), ((1/4, 3/4), (0, 1/2, 1/2, 0)), c**2*exp_
polar(2*I*pi)/(d**2*x**2))/(4*pi**(3/2)*c)
```

3.223 $\int \frac{a+bx^2}{x^4\sqrt{-c+dx}\sqrt{c+dx}}\,dx$

Optimal. Leaf size=75

$$\frac{\sqrt{dx-c}\sqrt{c+dx}\left(2ad^2+3bc^2\right)}{3c^4x}+\frac{a\sqrt{dx-c}\sqrt{c+dx}}{3c^2x^3}$$

Rubi [A] time = 0.07, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {454, 95}

$$\frac{\sqrt{dx-c}\sqrt{c+dx}\left(2ad^2+3bc^2\right)}{3c^4x}+\frac{a\sqrt{dx-c}\sqrt{c+dx}}{3c^2x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)/(x^4*Sqrt[-c + d*x]*Sqrt[c + d*x]),x]
```

```
[Out] (a*Sqrt[-c + d*x]*Sqrt[c + d*x])/(3*c^2*x^3) + ((3*b*c^2 + 2*a*d^2)*Sqrt[-c
 + d*x]*Sqrt[c + d*x])/(3*c^4*x)
```

Rule 95

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] /; FreeQ[{a, b, c, d, e, f
, m, n, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && EqQ[a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1), 0] && NeQ[m, -1]
```

Rule 454

```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[(c*(e*x)^
(m + 1)*(a1 + b1*x^(n/2))^(p + 1)*(a2 + b2*x^(n/2))^(p + 1))/(a1*a2*e*(m +
1)), x] + Dist[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/(a1*a2*e^n*(
m + 1)), Int[(e*x)^(m + n)*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n/2))^p, x], x]
/; FreeQ[{a1, b1, a2, b2, c, d, e, p}, x] && EqQ[non2, n/2] && EqQ[a2*b1 +
a1*b2, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (L
tQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx^2}{x^4\sqrt{-c+dx}\sqrt{c+dx}}\,dx &= \frac{a\sqrt{-c+dx}\sqrt{c+dx}}{3c^2x^3}+\frac{1}{3}\left(3b+\frac{2ad^2}{c^2}\right)\int \frac{1}{x^2\sqrt{-c+dx}\sqrt{c+dx}}\,dx\\ &= \frac{a\sqrt{-c+dx}\sqrt{c+dx}}{3c^2x^3}+\frac{\left(3bc^2+2ad^2\right)\sqrt{-c+dx}\sqrt{c+dx}}{3c^4x}\end{aligned}$$

Mathematica [A] time = 0.03, size = 66, normalized size = 0.88

$$-\frac{\left(c^2-d^2x^2\right)\left(a\left(c^2+2d^2x^2\right)+3bc^2x^2\right)}{3c^4x^3\sqrt{dx-c}\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)/(x^4*Sqrt[-c + d*x]*Sqrt[c + d*x]),x]
```

```
[Out] -1/3*((c^2 - d^2*x^2)*(3*b*c^2*x^2 + a*(c^2 + 2*d^2*x^2)))/(c^4*x^3*Sqrt[-c
 + d*x]*Sqrt[c + d*x])
```

IntegrateAlgebraic [A] time = 0.14, size = 146, normalized size = 1.95

$$\frac{2\sqrt{dx-c}\left(\frac{2ad^3(dx-c)}{c+dx}+\frac{3ad^3(dx-c)^2}{(c+dx)^2}+3ad^3+\frac{6bc^2d(dx-c)}{c+dx}+\frac{3bc^2d(dx-c)^2}{(c+dx)^2}+3bc^2d\right)}{3c^4\sqrt{c+dx}\left(\frac{dx-c}{c+dx}+1\right)^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x^2)/(x^4*Sqrt[-c + d*x]*Sqrt[c + d*x]),x]
```

```
[Out] (2*Sqrt[-c + d*x]*(3*b*c^2*d + 3*a*d^3 + (3*b*c^2*d*(-c + d*x)^2)/(c + d*x)
^2 + (3*a*d^3*(-c + d*x)^2)/(c + d*x)^2 + (6*b*c^2*d*(-c + d*x))/(c + d*x)
+ (2*a*d^3*(-c + d*x))/(c + d*x)))/(3*c^4*Sqrt[c + d*x]*(1 + (-c + d*x)/(c
+ d*x))^3)
```

fricas [A] time = 0.74, size = 67, normalized size = 0.89

$$\frac{\left(3\,bc^2d+2\,ad^3\right)x^3+\left(ac^2+\left(3\,bc^2+2\,ad^2\right)x^2\right)\sqrt{dx+c}\,\sqrt{dx-c}}{3\,c^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/x^4/(d*x-c)^(1/2)/(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*((3*b*c^2*d + 2*a*d^3)*x^3 + (a*c^2 + (3*b*c^2 + 2*a*d^2)*x^2)*sqrt(d*x
 + c)*sqrt(d*x - c))/(c^4*x^3)
```

giac [B] time = 0.24, size = 137, normalized size = 1.83

$$\frac{8\left(3\,bd^2\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^8+24\,bc^2d^2\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^4+24\,ad^4\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^4+48\,bc^4d^2+32\,ac^2d^4\right)}{3\left(\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^4+4\,c^2\right)^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/x^4/(d*x-c)^(1/2)/(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] 8/3*(3*b*d^2*(sqrt(d*x + c) - sqrt(d*x - c))^8 + 24*b*c^2*d^2*(sqrt(d*x + c
) - sqrt(d*x - c))^4 + 24*a*d^4*(sqrt(d*x + c) - sqrt(d*x - c))^4 + 48*b*c^
4*d^2 + 32*a*c^2*d^4)/(((sqrt(d*x + c) - sqrt(d*x - c))^4 + 4*c^2)^3*d)
```

maple [A] time = 0.04, size = 49, normalized size = 0.65

$$\frac{\sqrt{dx+c}\,\left(2a\,d^2x^2+3b\,c^2x^2+a\,c^2\right)\sqrt{dx-c}}{3c^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)/x^4/(d*x-c)^(1/2)/(d*x+c)^(1/2),x)
```

```
[Out] 1/3*(d*x+c)^(1/2)*(2*a*d^2*x^2+3*b*c^2*x^2+a*c^2)/x^3/c^4*(d*x-c)^(1/2)
```

maxima [A] time = 1.32, size = 75, normalized size = 1.00

$$\frac{\sqrt{d^2x^2-c^2}\,b}{c^2x}+\frac{2\,\sqrt{d^2x^2-c^2}\,ad^2}{3\,c^4x}+\frac{\sqrt{d^2x^2-c^2}\,a}{3\,c^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/x^4/(d*x-c)^(1/2)/(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] sqrt(d^2*x^2 - c^2)*b/(c^2*x) + 2/3*sqrt(d^2*x^2 - c^2)*a*d^2/(c^4*x) + 1/3
*sqrt(d^2*x^2 - c^2)*a/(c^2*x^3)
```

mupad [B] time = 2.77, size = 79, normalized size = 1.05

$$\frac{\sqrt{d\,x-c}\,\left(\frac{a}{3\,c}+\frac{x^2\,\left(3\,b\,c^3+2\,a\,c\,d^2\right)}{3\,c^4}+\frac{x^3\,\left(3\,b\,c^2\,d+2\,a\,d^3\right)}{3\,c^4}+\frac{a\,d\,x}{3\,c^2}\right)}{x^3\,\sqrt{c+d\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)/(x^4*(c + d*x)^(1/2)*(d*x - c)^(1/2)),x)
```

```
[Out] ((d*x - c)^(1/2)*(a/(3*c) + (x^2*(3*b*c^3 + 2*a*c*d^2))/(3*c^4) + (x^3*(2*a
*d^3 + 3*b*c^2*d))/(3*c^4) + (a*d*x)/(3*c^2)))/(x^3*(c + d*x)^(1/2))
```

sympy [C] time = 70.80, size = 170, normalized size = 2.27

$$-\frac{ad^3G_{6,6}^{5,3}\left(\begin{matrix}\frac{9}{4},\frac{11}{4},1 & \frac{5}{2},\frac{5}{2},3\\ 2,\frac{9}{4},\frac{5}{2},\frac{11}{4},3 & 0\end{matrix}\middle|\frac{c^2}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}c^4}-\frac{iad^3G_{6,6}^{2,6}\left(\begin{matrix}\frac{3}{2},\frac{7}{4},2,\frac{9}{4},\frac{5}{2},1 & \\ \frac{7}{4},\frac{9}{4} & \frac{3}{2},2,2,0\end{matrix}\middle|\frac{c^2e^{2i\pi}}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}c^4}-\frac{bdG_{6,6}^{5,3}\left(\begin{matrix}\frac{5}{4},\frac{7}{4},1 & \frac{3}{2},\frac{3}{2},2\\ 1,\frac{5}{4},\frac{3}{2},\frac{7}{4},2 & 0\end{matrix}\middle|\frac{c^2}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}c^2}-\frac{ibdG_{6,6}^{2,6}\left(\begin{matrix}\frac{1}{2},\frac{3}{4},1,\frac{5}{4},\frac{3}{2},1 & \\ \frac{3}{4},\frac{5}{4} & \frac{1}{2},1,1,0\end{matrix}\middle|\frac{c^2e^{2i\pi}}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)/x**4/(d*x-c)**(1/2)/(d*x+c)**(1/2),x)
```

```
[Out] -a*d**3*meijerg(((9/4, 11/4, 1), (5/2, 5/2, 3)), ((2, 9/4, 5/2, 11/4, 3), (
0,)), c**2/(d**2*x**2))/(4*pi**(3/2)*c**4) - I*a*d**3*meijerg(((3/2, 7/4, 2
, 9/4, 5/2, 1), ()), ((7/4, 9/4), (3/2, 2, 2, 0)), c**2*exp_polar(2*I*pi)/(
d**2*x**2))/(4*pi**(3/2)*c**4) - b*d*meijerg(((5/4, 7/4, 1), (3/2, 3/2, 2))
, ((1, 5/4, 3/2, 7/4, 2), (0,)), c**2/(d**2*x**2))/(4*pi**(3/2)*c**2) - I*b
*d*meijerg(((1/2, 3/4, 1, 5/4, 3/2, 1), ()), ((3/4, 5/4), (1/2, 1, 1, 0)),
c**2*exp_polar(2*I*pi)/(d**2*x**2))/(4*pi**(3/2)*c**2)
```

3.224 $\int \frac{a+bx^2}{x^5\sqrt{-c+dx}\sqrt{c+dx}}\,dx$

Optimal. Leaf size=123

$$\frac{d^2\left(3ad^2+4bc^2\right)\tan^{-1}\left(\frac{\sqrt{dx-c}\sqrt{c+dx}}{c}\right)}{8c^5}+\frac{\sqrt{dx-c}\sqrt{c+dx}\left(3ad^2+4bc^2\right)}{8c^4x^2}+\frac{a\sqrt{dx-c}\sqrt{c+dx}}{4c^2x^4}$$

Rubi [A] time = 0.10, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {454, 103, 12, 92, 205}

$$\frac{\sqrt{dx-c}\sqrt{c+dx}\left(3ad^2+4bc^2\right)}{8c^4x^2}+\frac{d^2\left(3ad^2+4bc^2\right)\tan^{-1}\left(\frac{\sqrt{dx-c}\sqrt{c+dx}}{c}\right)}{8c^5}+\frac{a\sqrt{dx-c}\sqrt{c+dx}}{4c^2x^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)/(x^5*Sqrt[-c + d*x]*Sqrt[c + d*x]),x]
```

```
[Out] (a*Sqrt[-c + d*x]*Sqrt[c + d*x])/(4*c^2*x^4) + ((4*b*c^2 + 3*a*d^2)*Sqrt[-c
 + d*x]*Sqrt[c + d*x])/(8*c^4*x^2) + (d^2*(4*b*c^2 + 3*a*d^2)*ArcTan[(Sqrt[
-c + d*x]*Sqrt[c + d*x])/c])/(8*c^5)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 454

```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[(c*(e*x)^
(m + 1)*(a1 + b1*x^(n/2))^(p + 1)*(a2 + b2*x^(n/2))^(p + 1))/(a1*a2*e*(m +
1)), x] + Dist[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/(a1*a2*e^n*(
m + 1)), Int[(e*x)^(m + n)*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n/2))^p, x], x]
/; FreeQ[{a1, b1, a2, b2, c, d, e, p}, x] && EqQ[non2, n/2] && EqQ[a2*b1 +
a1*b2, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (L
```

```
tQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rubi steps

$$\int \frac{a+bx^2}{x^5\sqrt{-c+dx}\sqrt{c+dx}}\,dx = \frac{a\sqrt{-c+dx}\sqrt{c+dx}}{4c^2x^4} + \frac{1}{4}\left(4b+\frac{3ad^2}{c^2}\right)\int \frac{1}{x^3\sqrt{-c+dx}\sqrt{c+dx}}\,dx$$
$$= \frac{a\sqrt{-c+dx}\sqrt{c+dx}}{4c^2x^4} + \frac{\left(4bc^2+3ad^2\right)\sqrt{-c+dx}\sqrt{c+dx}}{8c^4x^2} + \frac{\left(4bc^2+3ad^2\right)\int \frac{}{x\sqrt{-}}}{8c^4}$$
$$= \frac{a\sqrt{-c+dx}\sqrt{c+dx}}{4c^2x^4} + \frac{\left(4bc^2+3ad^2\right)\sqrt{-c+dx}\sqrt{c+dx}}{8c^4x^2} + \frac{\left(d^2\left(4bc^2+3ad^2\right)\right)}{8c}$$
$$= \frac{a\sqrt{-c+dx}\sqrt{c+dx}}{4c^2x^4} + \frac{\left(4bc^2+3ad^2\right)\sqrt{-c+dx}\sqrt{c+dx}}{8c^4x^2} + \left(d^3\left(4bc^2+3ad^2\right)\right)$$
$$= \frac{a\sqrt{-c+dx}\sqrt{c+dx}}{4c^2x^4} + \frac{\left(4bc^2+3ad^2\right)\sqrt{-c+dx}\sqrt{c+dx}}{8c^4x^2} + \frac{d^2\left(4bc^2+3ad^2\right)\tan}{8c}$$

Mathematica [A] time = 0.11, size = 144, normalized size = 1.17

$$-\frac{\left(c^2-d^2x^2\right)\left(c^2\sqrt{1-\frac{d^2x^2}{c^2}}\left(2ac^2+3ad^2x^2+4bc^2x^2\right)+d^2x^4\left(3ad^2+4bc^2\right)\tanh^{-1}\left(\sqrt{1-\frac{d^2x^2}{c^2}}\right)\right)}{8c^6x^4\sqrt{dx-c}\sqrt{c+dx}\sqrt{1-\frac{d^2x^2}{c^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)/(x^5*Sqrt[-c + d*x]*Sqrt[c + d*x]),x]
```

```
[Out] -1/8*((c^2 - d^2*x^2)*(c^2*(2*a*c^2 + 4*b*c^2*x^2 + 3*a*d^2*x^2)*Sqrt[1 - (
d^2*x^2)/c^2] + d^2*(4*b*c^2 + 3*a*d^2)*x^4*ArcTanh[Sqrt[1 - (d^2*x^2)/c^2]
]))/(c^6*x^4*Sqrt[-c + d*x]*Sqrt[c + d*x]*Sqrt[1 - (d^2*x^2)/c^2])
```

IntegrateAlgebraic [A] time = 0.19, size = 209, normalized size = 1.70

$$\frac{\left(3ad^4+4bc^2d^2\right)\tan^{-1}\left(\frac{\sqrt{dx-c}}{\sqrt{c+dx}}\right)}{4c^5} - \frac{d^2\sqrt{dx-c}\left(\frac{dx-c}{c+dx}-1\right)\left(\frac{2ad^2(dx-c)}{c+dx}+\frac{5ad^2(dx-c)^2}{(c+dx)^2}+5ad^2+\frac{8bc^2(dx-c)}{c+dx}+\frac{4bc^2(dx-c)^2}{(c+dx)^2}+4bc^2\right)}{4c^5\sqrt{c+dx}\left(\frac{dx-c}{c+dx}+1\right)^4}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x^2)/(x^5*Sqrt[-c + d*x]*Sqrt[c + d*x]),x]
```

```
[Out] -1/4*(d^2*Sqrt[-c + d*x]*(-1 + (-c + d*x)/(c + d*x))*(4*b*c^2 + 5*a*d^2 + (
4*b*c^2*(-c + d*x)^2)/(c + d*x)^2 + (5*a*d^2*(-c + d*x)^2)/(c + d*x)^2 + (8
*b*c^2*(-c + d*x))/(c + d*x) + (2*a*d^2*(-c + d*x))/(c + d*x)))/(c^5*Sqrt[c
 + d*x]*(1 + (-c + d*x)/(c + d*x))^4) + ((4*b*c^2*d^2 + 3*a*d^4)*ArcTan[Sqr
t[-c + d*x]/Sqrt[c + d*x]])/(4*c^5)
```

fricas [A] time = 0.86, size = 100, normalized size = 0.81

$$\frac{2\left(4\,bc^2d^2+3\,ad^4\right)x^4\arctan\left(-\frac{dx-\sqrt{dx+c}\sqrt{dx-c}}{c}\right)+\left(2\,ac^3+\left(4\,bc^3+3\,acd^2\right)x^2\right)\sqrt{dx+c}\sqrt{dx-c}}{8\,c^5x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/x^5/(d*x-c)^(1/2)/(d*x+c)^(1/2),x, algorithm="fricas")
```

[Out] 1/8*(2*(4*b*c^2*d^2 + 3*a*d^4)*x^4*arctan(-(d*x - sqrt(d*x + c)*sqrt(d*x - c))/c) + (2*a*c^3 + (4*b*c^3 + 3*a*c*d^2)*x^2)*sqrt(d*x + c)*sqrt(d*x - c))/(c^5*x^4)

giac [B] time = 0.27, size = 325, normalized size = 2.64

$$-\frac{\frac{(4bc^2d^3+3ad^5)\arctan\left(\frac{\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^2}{2c}\right)}{c^5}+\frac{2\left(4bc^2d^3\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^{14}+3ad^5\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^{14}+16bc^4d^3\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^{10}+44ac^2d^5\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^{10}-64bc^6d^3\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^{6}-176ac^4d^5\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^{6}-256bc^8d^3\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^{2}-192ac^6d^5\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^{2}\right)}{\left(\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^4+4c^2\right)^4c^4}}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^2+a)/x^5/(d*x-c)^(1/2)/(d*x+c)^(1/2),x, algorithm="giac")

[Out] -1/4*((4*b*c^2*d^3 + 3*a*d^5)*arctan(1/2*(sqrt(d*x + c) - sqrt(d*x - c))^2/c)/c^5 + 2*(4*b*c^2*d^3*(sqrt(d*x + c) - sqrt(d*x - c))^14 + 3*a*d^5*(sqrt(d*x + c) - sqrt(d*x - c))^14 + 16*b*c^4*d^3*(sqrt(d*x + c) - sqrt(d*x - c))^10 + 44*a*c^2*d^5*(sqrt(d*x + c) - sqrt(d*x - c))^10 - 64*b*c^6*d^3*(sqrt(d*x + c) - sqrt(d*x - c))^6 - 176*a*c^4*d^5*(sqrt(d*x + c) - sqrt(d*x - c))^6 - 256*b*c^8*d^3*(sqrt(d*x + c) - sqrt(d*x - c))^2 - 192*a*c^6*d^5*(sqrt(d*x + c) - sqrt(d*x - c))^2)/(((sqrt(d*x + c) - sqrt(d*x - c))^4 + 4*c^2)^4*c^4))/d

maple [B] time = 0.07, size = 227, normalized size = 1.85

$$-\frac{\sqrt{dx-c}\,\sqrt{dx+c}\,\left(3a\,d^4x^4\ln\left(-\frac{2\left(c^2-\sqrt{-c^2}\,\sqrt{d^2x^2-c^2}\right)}{x}\right)+4b\,c^2d^2x^4\ln\left(-\frac{2\left(c^2-\sqrt{-c^2}\,\sqrt{d^2x^2-c^2}\right)}{x}\right)-3\sqrt{-c^2}\,\sqrt{d^2x^2-c^2}\,a\,d^2x^2-4\sqrt{-c^2}\,\sqrt{d^2x^2-c^2}\,b\,c^2x^2-2\sqrt{-c^2}\,\sqrt{d^2x^2-c^2}\,a\,c^2\right)}{8\sqrt{d^2x^2-c^2}\,\sqrt{-c^2}\,c^4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x^2+a)/x^5/(d*x-c)^(1/2)/(d*x+c)^(1/2),x)

[Out] -1/8*(d*x-c)^(1/2)*(d*x+c)^(1/2)/c^4*(3*ln(-2*(c^2-(-c^2)^(1/2)*(d^2*x^2-c^2)^(1/2))/x)*x^4*a*d^4+4*b*c^2*d^2*x^4*ln(-2*(c^2-(-c^2)^(1/2)*(d^2*x^2-c^2)^(1/2))/x)-3*(-c^2)^(1/2)*(d^2*x^2-c^2)^(1/2)*x^2*a*d^2-4*(-c^2)^(1/2)*(d^2*x^2-c^2)^(1/2)*x^2*b*c^2-2*(-c^2)^(1/2)*(d^2*x^2-c^2)^(1/2)*a*c^2)/(d^2*x^2-c^2)^(1/2)/x^4/(-c^2)^(1/2)

maxima [A] time = 1.21, size = 114, normalized size = 0.93

$$-\frac{bd^2\arcsin\left(\frac{c}{d|x|}\right)}{2c^3}-\frac{3ad^4\arcsin\left(\frac{c}{d|x|}\right)}{8c^5}+\frac{\sqrt{d^2x^2-c^2}\,b}{2c^2x^2}+\frac{3\sqrt{d^2x^2-c^2}\,ad^2}{8c^4x^2}+\frac{\sqrt{d^2x^2-c^2}\,a}{4c^2x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^2+a)/x^5/(d*x-c)^(1/2)/(d*x+c)^(1/2),x, algorithm="maxima")

[Out] -1/2*b*d^2*arcsin(c/(d*abs(x)))/c^3 - 3/8*a*d^4*arcsin(c/(d*abs(x)))/c^5 + 1/2*sqrt(d^2*x^2 - c^2)*b/(c^2*x^2) + 3/8*sqrt(d^2*x^2 - c^2)*a*d^2/(c^4*x^2) + 1/4*sqrt(d^2*x^2 - c^2)*a/(c^2*x^4)

mupad [B] time = 19.13, size = 1005, normalized size = 8.17

$$\frac{3a\sqrt{-c}\,d^4\ln\left(\frac{\sqrt{c+dx}-\sqrt{c}}{\sqrt{-c}-\sqrt{dx-c}}\right)}{8c^{11/2}}-\frac{\frac{b(-c)^{3/2}d^2}{32c^{9/2}}+\frac{b(-c)^{3/2}d^2\left(\sqrt{c+dx}-\sqrt{c}\right)^2}{16c^{9/2}\left(\sqrt{-c}-\sqrt{dx-c}\right)^2}-\frac{15b(-c)^{3/2}d^2\left(\sqrt{c+dx}-\sqrt{c}\right)^4}{32c^{9/2}\left(\sqrt{-c}-\sqrt{dx-c}\right)^4}}{\frac{\left(\sqrt{c+dx}-\sqrt{c}\right)^2}{\left(\sqrt{-c}-\sqrt{dx-c}\right)^2}+\frac{2\left(\sqrt{c+dx}-\sqrt{c}\right)^4}{\left(\sqrt{-c}-\sqrt{dx-c}\right)^4}+\frac{\left(\sqrt{c+dx}-\sqrt{c}\right)^6}{\left(\sqrt{-c}-\sqrt{dx-c}\right)^6}}-\frac{\frac{a\sqrt{-c}\,d^4}{1024c^{11/2}}-\frac{3a\sqrt{-c}\,d^4\left(\sqrt{c+dx}-\sqrt{c}\right)^2}{128c^{11/2}\left(\sqrt{-c}-\sqrt{dx-c}\right)^2}-\frac{53a\sqrt{-c}\,d^4\left(\sqrt{c+dx}-\sqrt{c}\right)^4}{512c^{11/2}\left(\sqrt{-c}-\sqrt{dx-c}\right)^4}+\frac{87a\sqrt{-c}\,d^4\left(\sqrt{c+dx}-\sqrt{c}\right)^6}{256c^{11/2}\left(\sqrt{-c}-\sqrt{dx-c}\right)^6}+\frac{657a\sqrt{-c}\,d^4\left(\sqrt{c+dx}-\sqrt{c}\right)^8}{1024c^{11/2}\left(\sqrt{-c}-\sqrt{dx-c}\right)^8}+\frac{121a\sqrt{-c}\,d^4\left(\sqrt{c+dx}-\sqrt{c}\right)^{10}}{256c^{11/2}\left(\sqrt{-c}-\sqrt{dx-c}\right)^{10}}}{\frac{\left(\sqrt{c+dx}-\sqrt{c}\right)^4}{\left(\sqrt{-c}-\sqrt{dx-c}\right)^4}+\frac{4\left(\sqrt{c+dx}-\sqrt{c}\right)^6}{\left(\sqrt{-c}-\sqrt{dx-c}\right)^6}+\frac{6\left(\sqrt{c+dx}-\sqrt{c}\right)^8}{\left(\sqrt{-c}-\sqrt{dx-c}\right)^8}+\frac{4\left(\sqrt{c+dx}-\sqrt{c}\right)^{10}}{\left(\sqrt{-c}-\sqrt{dx-c}\right)^{10}}+\frac{\left(\sqrt{c+dx}-\sqrt{c}\right)^{12}}{\left(\sqrt{-c}-\sqrt{dx-c}\right)^{12}}}-\frac{b(-c)^{3/2}d^2\ln\left(\frac{\sqrt{c+dx}-\sqrt{c}}{\sqrt{-c}-\sqrt{dx-c}}\right)}{2c^{9/2}}-\frac{3a\sqrt{-c}\,d^4\ln\left(\frac{\left(\sqrt{c+dx}-\sqrt{c}\right)^2}{\left(\sqrt{-c}-\sqrt{dx-c}\right)^2}+1\right)}{8c^{11/2}}+\frac{b(-c)^{3/2}d^2\ln\left(\frac{\left(\sqrt{c+dx}-\sqrt{c}\right)^2}{\left(\sqrt{-c}-\sqrt{dx-c}\right)^2}+1\right)}{2c^{9/2}}-\frac{7ad^4\left(\sqrt{c+dx}-\sqrt{c}\right)^2}{256\sqrt{-c}\,c^{9/2}\left(\sqrt{-c}-\sqrt{dx-c}\right)^2}+\frac{ad^4\left(\sqrt{c+dx}-\sqrt{c}\right)^4}{1024\sqrt{-c}\,c^{9/2}\left(\sqrt{-c}-\sqrt{dx-c}\right)^4}+\frac{bd^2\left(\sqrt{c+dx}-\sqrt{c}\right)^2}{32(-c)^{3/2}c^{3/2}\left(\sqrt{-c}-\sqrt{dx-c}\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*x^2)/(x^5*(c + d*x)^(1/2)*(d*x - c)^(1/2)),x)

[Out] (3*a*(-c)^(1/2)*d^4*log(((c + d*x)^(1/2) - c^(1/2))/((-c)^(1/2) - (d*x - c)^(1/2))))/(8*c^(11/2)) - ((b*(-c)^(3/2)*d^2)/(32*c^(9/2)) + (b*(-c)^(3/2)*d^2*((c + d*x)^(1/2) - c^(1/2))^2)/(16*c^(9/2)*((-c)^(1/2) - (d*x - c)^(1/2))^2) - (15*b*(-c)^(3/2)*d^2*((c + d*x)^(1/2) - c^(1/2))^4)/(32*c^(9/2)*((-c

```
)^(1/2) - (d*x - c)^(1/2))^4))/(((c + d*x)^(1/2) - c^(1/2))^2/((-c)^(1/2) -
 (d*x - c)^(1/2))^2 + (2*((c + d*x)^(1/2) - c^(1/2))^4)/((-c)^(1/2) - (d*x
- c)^(1/2))^4 + ((c + d*x)^(1/2) - c^(1/2))^6/((-c)^(1/2) - (d*x - c)^(1/2)
)^6) - ((a*(-c)^(1/2)*d^4)/(1024*c^(11/2)) - (3*a*(-c)^(1/2)*d^4*((c + d*x)
^(1/2) - c^(1/2))^2)/(128*c^(11/2)*((-c)^(1/2) - (d*x - c)^(1/2))^2) - (53*
a*(-c)^(1/2)*d^4*((c + d*x)^(1/2) - c^(1/2))^4)/(512*c^(11/2)*((-c)^(1/2) -
 (d*x - c)^(1/2))^4) + (87*a*(-c)^(1/2)*d^4*((c + d*x)^(1/2) - c^(1/2))^6)/
(256*c^(11/2)*((-c)^(1/2) - (d*x - c)^(1/2))^6) + (657*a*(-c)^(1/2)*d^4*((c
 + d*x)^(1/2) - c^(1/2))^8)/(1024*c^(11/2)*((-c)^(1/2) - (d*x - c)^(1/2))^8
) + (121*a*(-c)^(1/2)*d^4*((c + d*x)^(1/2) - c^(1/2))^10)/(256*c^(11/2)*((-
c)^(1/2) - (d*x - c)^(1/2))^10))/(((c + d*x)^(1/2) - c^(1/2))^4/((-c)^(1/2)
 - (d*x - c)^(1/2))^4 + (4*((c + d*x)^(1/2) - c^(1/2))^6)/((-c)^(1/2) - (d*
x - c)^(1/2))^6 + (6*((c + d*x)^(1/2) - c^(1/2))^8)/((-c)^(1/2) - (d*x - c)
^(1/2))^8 + (4*((c + d*x)^(1/2) - c^(1/2))^10)/((-c)^(1/2) - (d*x - c)^(1/2
))^10 + ((c + d*x)^(1/2) - c^(1/2))^12/((-c)^(1/2) - (d*x - c)^(1/2))^12) -
 (b*(-c)^(3/2)*d^2*log(((c + d*x)^(1/2) - c^(1/2))/((-c)^(1/2) - (d*x - c)^
(1/2))))/(2*c^(9/2)) - (3*a*(-c)^(1/2)*d^4*log(((c + d*x)^(1/2) - c^(1/2))^
2/((-c)^(1/2) - (d*x - c)^(1/2))^2 + 1))/(8*c^(11/2)) + (b*(-c)^(3/2)*d^2*l
og(((c + d*x)^(1/2) - c^(1/2))^2/((-c)^(1/2) - (d*x - c)^(1/2))^2 + 1))/(2*
c^(9/2)) - (7*a*d^4*((c + d*x)^(1/2) - c^(1/2))^2)/(256*(-c)^(1/2)*c^(9/2)*
((-c)^(1/2) - (d*x - c)^(1/2))^2) + (a*d^4*((c + d*x)^(1/2) - c^(1/2))^4)/(
1024*(-c)^(1/2)*c^(9/2)*((-c)^(1/2) - (d*x - c)^(1/2))^4) + (b*d^2*((c + d*
x)^(1/2) - c^(1/2))^2)/(32*(-c)^(3/2)*c^(3/2)*((-c)^(1/2) - (d*x - c)^(1/2)
)^2)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)/x**5/(d*x-c)**(1/2)/(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

3.225 $\int \frac{x^4(a+bx^2)}{(-c+dx)^{3/2}(c+dx)^{3/2}}\,dx$

Optimal. Leaf size=161

$$\frac{3c^2(4ad^2+5bc^2)\tanh^{-1}\left(\frac{\sqrt{dx-c}}{\sqrt{c+dx}}\right)}{4d^7}+\frac{3x\sqrt{dx-c}\sqrt{c+dx}\,(4ad^2+5bc^2)}{8d^6}-\frac{x^3(4ad^2+5bc^2)}{4d^4\sqrt{dx-c}\sqrt{c+dx}}+\frac{bx^5}{4d^2\sqrt{dx-c}\sqrt{c+dx}}$$

Rubi [A] time = 0.12, antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.258, Rules used = {460, 98, 21, 90, 12, 63, 217, 206}

$$-\frac{x^3(4ad^2+5bc^2)}{4d^4\sqrt{dx-c}\sqrt{c+dx}}+\frac{3x\sqrt{dx-c}\sqrt{c+dx}\,(4ad^2+5bc^2)}{8d^6}+\frac{3c^2(4ad^2+5bc^2)\tanh^{-1}\left(\frac{\sqrt{dx-c}}{\sqrt{c+dx}}\right)}{4d^7}+\frac{bx^5}{4d^2\sqrt{dx-c}\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(a + b*x^2))/((-c + d*x)^(3/2)*(c + d*x)^(3/2)),x]
```

```
[Out] -((5*b*c^2 + 4*a*d^2)*x^3)/(4*d^4*Sqrt[-c + d*x]*Sqrt[c + d*x]) + (b*x^5)/(
4*d^2*Sqrt[-c + d*x]*Sqrt[c + d*x]) + (3*(5*b*c^2 + 4*a*d^2)*x*Sqrt[-c + d*
x]*Sqrt[c + d*x])/(8*d^6) + (3*c^2*(5*b*c^2 + 4*a*d^2)*ArcTanh[Sqrt[-c + d*
x]/Sqrt[c + d*x]])/(4*d^7)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*(a + b*x)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/
(d*f*(n + p + 3)), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
```

```
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 460

```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[(d*(e*x)^
(m + 1)*(a1 + b1*x^(n/2))^(p + 1)*(a2 + b2*x^(n/2))^(p + 1))/(b1*b2*e*(m +
n*(p + 1) + 1)), x] - Dist[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/
(b1*b2*(m + n*(p + 1) + 1)), Int[(e*x)^m*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n/
2))^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, m, n, p}, x] && EqQ[non2,
n/2] && EqQ[a2*b1 + a1*b2, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rubi steps

$$\int \frac{x^4\left(a+bx^2\right)}{(-c+dx)^{3/2}(c+dx)^{3/2}}\,dx = \frac{bx^5}{4d^2\sqrt{-c+dx}\,\sqrt{c+dx}} - \frac{1}{4}\left(-4a-\frac{5bc^2}{d^2}\right)\int \frac{x^4}{(-c+dx)^{3/2}(c+dx)^{3/2}}\,dx$$

$$= -\frac{\left(5bc^2+4ad^2\right)x^3}{4d^4\sqrt{-c+dx}\,\sqrt{c+dx}} + \frac{bx^5}{4d^2\sqrt{-c+dx}\,\sqrt{c+dx}} - \frac{\left(4a+\frac{5bc^2}{d^2}\right)\int \frac{x^2\left(-3c^2-3cdx\cdots\right.}{\sqrt{-c+dx}\,(c+dx)\cdots}}{4cd^2}$$

$$= -\frac{\left(5bc^2+4ad^2\right)x^3}{4d^4\sqrt{-c+dx}\,\sqrt{c+dx}} + \frac{bx^5}{4d^2\sqrt{-c+dx}\,\sqrt{c+dx}} + \frac{\left(3\left(5bc^2+4ad^2\right)\right)\int \frac{\cdots}{\sqrt{-c+\cdots}}}{4d^4}$$

$$= -\frac{\left(5bc^2+4ad^2\right)x^3}{4d^4\sqrt{-c+dx}\,\sqrt{c+dx}} + \frac{bx^5}{4d^2\sqrt{-c+dx}\,\sqrt{c+dx}} + \frac{3\left(5bc^2+4ad^2\right)x\sqrt{-c+d\cdots}}{8d^6}$$

$$= -\frac{\left(5bc^2+4ad^2\right)x^3}{4d^4\sqrt{-c+dx}\,\sqrt{c+dx}} + \frac{bx^5}{4d^2\sqrt{-c+dx}\,\sqrt{c+dx}} + \frac{3\left(5bc^2+4ad^2\right)x\sqrt{-c+d\cdots}}{8d^6}$$

$$= -\frac{\left(5bc^2+4ad^2\right)x^3}{4d^4\sqrt{-c+dx}\,\sqrt{c+dx}} + \frac{bx^5}{4d^2\sqrt{-c+dx}\,\sqrt{c+dx}} + \frac{3\left(5bc^2+4ad^2\right)x\sqrt{-c+d\cdots}}{8d^6}$$

$$= -\frac{\left(5bc^2+4ad^2\right)x^3}{4d^4\sqrt{-c+dx}\,\sqrt{c+dx}} + \frac{bx^5}{4d^2\sqrt{-c+dx}\,\sqrt{c+dx}} + \frac{3\left(5bc^2+4ad^2\right)x\sqrt{-c+d\cdots}}{8d^6}$$

$$= -\frac{\left(5bc^2+4ad^2\right)x^3}{4d^4\sqrt{-c+dx}\,\sqrt{c+dx}} + \frac{bx^5}{4d^2\sqrt{-c+dx}\,\sqrt{c+dx}} + \frac{3\left(5bc^2+4ad^2\right)x\sqrt{-c+d\cdots}}{8d^6}$$

Mathematica [A] time = 0.16, size = 119, normalized size = 0.74

$$\frac{3c^3\sqrt{1-\frac{d^2x^2}{c^2}}\left(4ad^2+5bc^2\right)\sin^{-1}\left(\frac{dx}{c}\right)+4ad^3x\left(d^2x^2-3c^2\right)+bdx\left(-15c^4+5c^2d^2x^2+2d^4x^4\right)}{8d^7\sqrt{dx-c}\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(a + b*x^2))/((-c + d*x)^(3/2)*(c + d*x)^(3/2)),x]
```

```
[Out] (4*a*d^3*x*(-3*c^2 + d^2*x^2) + b*d*x*(-15*c^4 + 5*c^2*d^2*x^2 + 2*d^4*x^4)
 + 3*c^3*(5*b*c^2 + 4*a*d^2)*Sqrt[1 - (d^2*x^2)/c^2]*ArcSin[(d*x)/c])/(8*d^
7*Sqrt[-c + d*x]*Sqrt[c + d*x])
```

IntegrateAlgebraic [A] time = 0.26, size = 297, normalized size = 1.84

$$\frac{3\left(4ac^2d^2+5bc^4\right)\tanh^{-1}\left(\frac{\sqrt{dx-c}}{\sqrt{c+dx}}\right)}{4d^7}-\frac{c^2\sqrt{c+dx}\left(\frac{dx-c}{c+dx}+1\right)\left(\frac{2ad^2(dx-c)^4}{(c+dx)^4}-\frac{12ad^2(dx-c)^3}{(c+dx)^3}+\frac{20ad^2(dx-c)^2}{(c+dx)^2}-\frac{12ad^2(dx-c)}{c+dx}+2ad^2+\frac{2bc^2(dx-c)^4}{(c+dx)^4}-\frac{17bc^2(dx-c)^3}{(c+dx)^3}+\frac{22bc^2(dx-c)^2}{(c+dx)^2}-\frac{17bc^2(dx-c)}{c+dx}+2bc^2\right)}{4d^7\sqrt{dx-c}\left(\frac{dx-c}{c+dx}-1\right)^4}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^4*(a + b*x^2))/((-c + d*x)^(3/2)*(c + d*x)^(3/2)),x]
```

```
[Out] -1/4*(c^2*Sqrt[c + d*x]*(1 + (-c + d*x)/(c + d*x))*(2*b*c^2 + 2*a*d^2 + (2*
b*c^2*(-c + d*x)^4)/(c + d*x)^4 + (2*a*d^2*(-c + d*x)^4)/(c + d*x)^4 - (17*
b*c^2*(-c + d*x)^3)/(c + d*x)^3 - (12*a*d^2*(-c + d*x)^3)/(c + d*x)^3 + (22
*b*c^2*(-c + d*x)^2)/(c + d*x)^2 + (20*a*d^2*(-c + d*x)^2)/(c + d*x)^2 - (1
7*b*c^2*(-c + d*x))/(c + d*x) - (12*a*d^2*(-c + d*x))/(c + d*x)))/(d^7*Sqrt
[-c + d*x]*(-1 + (-c + d*x)/(c + d*x))^4) + (3*(5*b*c^4 + 4*a*c^2*d^2)*ArcT
anh[Sqrt[-c + d*x]/Sqrt[c + d*x]])/(4*d^7)
```

fricas [A] time = 0.86, size = 190, normalized size = 1.18

$$\frac{8\,bc^6+8\,ac^4d^2-8\left(bc^4d^2+ac^2d^4\right)x^2+\left(2\,bd^5x^5+\left(5\,bc^2d^3+4\,ad^5\right)x^3-3\left(5\,bc^4d+4\,ac^2d^3\right)x\right)\sqrt{dx+c}\,\sqrt{dx-c}+3\left(5\,bc^6+4\,ac^4d^2-\left(5\,bc^4d^2+4\,ac^2d^4\right)x^2\right)\log\left(-dx+\sqrt{dx+c}\,\sqrt{dx-c}\right)}{8\left(d^9x^2-c^2d^7\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)/(d*x-c)^(3/2)/(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/8*(8*b*c^6 + 8*a*c^4*d^2 - 8*(b*c^4*d^2 + a*c^2*d^4)*x^2 + (2*b*d^5*x^5 +
 (5*b*c^2*d^3 + 4*a*d^5)*x^3 - 3*(5*b*c^4*d + 4*a*c^2*d^3)*x)*sqrt(d*x + c)
*sqrt(d*x - c) + 3*(5*b*c^6 + 4*a*c^4*d^2 - (5*b*c^4*d^2 + 4*a*c^2*d^4)*x^2
)*log(-d*x + sqrt(d*x + c)*sqrt(d*x - c)))/(d^9*x^2 - c^2*d^7)
```

giac [A] time = 0.40, size = 214, normalized size = 1.33

$$\frac{\left(\left((dx+c)\left(2\,(dx+c)\left(\frac{(dx+c)b}{d^7}-\frac{5\,bc}{d^7}\right)+\frac{25\,bc^2d^{35}+4\,ad^{37}}{d^{42}}\right)-\frac{35\,bc^3d^{35}+12\,acd^{37}}{d^{42}}\right)(dx+c)+\frac{2\left(7\,bc^4d^{35}+2\,ac^2d^{37}\right)}{d^{42}}\right)\sqrt{dx+c}}{8\,\sqrt{dx-c}}-\frac{3\left(5\,bc^4+4\,ac^2d^2\right)\log\left(\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^2\right)}{8\,d^7}-\frac{2\left(bc^5+ac^3d^2\right)}{\left(\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^2+2\,c\right)d^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)/(d*x-c)^(3/2)/(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] 1/8*(((d*x + c)*(2*(d*x + c)*((d*x + c)*b/d^7 - 5*b*c/d^7) + (25*b*c^2*d^35
 + 4*a*d^37)/d^42) - (35*b*c^3*d^35 + 12*a*c*d^37)/d^42)*(d*x + c) + 2*(7*b
*c^4*d^35 + 2*a*c^2*d^37)/d^42)*sqrt(d*x + c)/sqrt(d*x - c) - 3/8*(5*b*c^4
+ 4*a*c^2*d^2)*log((sqrt(d*x + c) - sqrt(d*x - c))^2)/d^7 - 2*(b*c^5 + a*c^
3*d^2)/(((sqrt(d*x + c) - sqrt(d*x - c))^2 + 2*c)*d^7)
```

maple [C] time = 0.09, size = 316, normalized size = 1.96

$$\frac{\left(2\sqrt{d^2x^2-c^2}\,b\,d^5x^5\mathrm{csgn}(d)+12a\,c^2d^4x^2\ln\left(\left(dx+\sqrt{d^2x^2-c^2}\,\mathrm{csgn}(d)\right)\mathrm{csgn}(d)\right)+4\sqrt{d^2x^2-c^2}\,a\,d^5x^3\mathrm{csgn}(d)+15b\,c^4d^2x^2\ln\left(\left(dx+\sqrt{d^2x^2-c^2}\,\mathrm{csgn}(d)\right)\mathrm{csgn}(d)\right)+5\sqrt{d^2x^2-c^2}\,b\,c^2d^3x^3\mathrm{csgn}(d)-12a\,c^4d^2\ln\left(\left(dx+\sqrt{d^2x^2-c^2}\,\mathrm{csgn}(d)\right)\mathrm{csgn}(d)\right)-12\sqrt{d^2x^2-c^2}\,a\,c^2d^3x\,\mathrm{csgn}(d)-15b\,c^6\ln\left(\left(dx+\sqrt{d^2x^2-c^2}\,\mathrm{csgn}(d)\right)\mathrm{csgn}(d)\right)-15\sqrt{d^2x^2-c^2}\,b\,c^4dx\,\mathrm{csgn}(d)\right)\mathrm{csgn}(d)}{8\sqrt{d^2x^2-c^2}\,\sqrt{dx+c}\,\sqrt{dx-c}\,d^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(b*x^2+a)/(d*x-c)^(3/2)/(d*x+c)^(3/2),x)
```

```
[Out] 1/8*(2*(d^2*x^2-c^2)^(1/2)*b*d^5*x^5*csgn(d)+4*(d^2*x^2-c^2)^(1/2)*a*d^5*x^
3*csgn(d)+5*(d^2*x^2-c^2)^(1/2)*b*c^2*d^3*x^3*csgn(d)+12*ln((d*x+(d^2*x^2-c
^2)^(1/2)*csgn(d))*csgn(d))*x^2*a*c^2*d^4+15*ln((d*x+(d^2*x^2-c^2)^(1/2)*cs
gn(d))*csgn(d))*x^2*b*c^4*d^2-12*(d^2*x^2-c^2)^(1/2)*a*c^2*d^3*x*csgn(d)-15
*(d^2*x^2-c^2)^(1/2)*b*c^4*d*x*csgn(d)-12*a*c^4*d^2*ln((d*x+(d^2*x^2-c^2)^(
1/2)*csgn(d))*csgn(d))-15*b*c^6*ln((d*x+(d^2*x^2-c^2)^(1/2)*csgn(d))*csgn(d
)))*csgn(d)/(d^2*x^2-c^2)^(1/2)/d^7/(d*x+c)^(1/2)/(d*x-c)^(1/2)
```

maxima [A] time = 0.49, size = 196, normalized size = 1.22

$$\frac{bx^5}{4\sqrt{d^2x^2-c^2}\,d^2}+\frac{5\,bc^2x^3}{8\sqrt{d^2x^2-c^2}\,d^4}+\frac{ax^3}{2\sqrt{d^2x^2-c^2}\,d^2}-\frac{15\,bc^4x}{8\sqrt{d^2x^2-c^2}\,d^6}-\frac{3\,ac^2x}{2\sqrt{d^2x^2-c^2}\,d^4}+\frac{15\,bc^4\log\left(2\,d^2x+2\sqrt{d^2x^2-c^2}\,d\right)}{8\,d^7}+\frac{3\,ac^2\log\left(2\,d^2x+2\sqrt{d^2x^2-c^2}\,d\right)}{2\,d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)/(d*x-c)^(3/2)/(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/4*b*x^5/(sqrt(d^2*x^2 - c^2)*d^2) + 5/8*b*c^2*x^3/(sqrt(d^2*x^2 - c^2)*d^
4) + 1/2*a*x^3/(sqrt(d^2*x^2 - c^2)*d^2) - 15/8*b*c^4*x/(sqrt(d^2*x^2 - c^2
)*d^6) - 3/2*a*c^2*x/(sqrt(d^2*x^2 - c^2)*d^4) + 15/8*b*c^4*log(2*d^2*x + 2
*sqrt(d^2*x^2 - c^2)*d)/d^7 + 3/2*a*c^2*log(2*d^2*x + 2*sqrt(d^2*x^2 - c^2)
*d)/d^5
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^4\,\left(b\,x^2+a\right)}{{\left(c+d\,x\right)}^{3/2}\,{\left(d\,x-c\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(a + b*x^2))/((c + d*x)^(3/2)*(d*x - c)^(3/2)),x)
```

```
[Out] int((x^4*(a + b*x^2))/((c + d*x)^(3/2)*(d*x - c)^(3/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(b*x**2+a)/(d*x-c)**(3/2)/(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

3.226 $\int \frac{x^3(a+bx^2)}{(-c+dx)^{3/2}(c+dx)^{3/2}} dx$

Optimal. Leaf size=115

$$\frac{2\sqrt{dx-c}\sqrt{c+dx}\,(3ad^2+4bc^2)}{3d^6}-\frac{x^2(3ad^2+4bc^2)}{3d^4\sqrt{dx-c}\sqrt{c+dx}}+\frac{bx^4}{3d^2\sqrt{dx-c}\sqrt{c+dx}}$$

Rubi [A] time = 0.09, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {460, 98, 21, 74}

$$-\frac{x^2(3ad^2+4bc^2)}{3d^4\sqrt{dx-c}\sqrt{c+dx}}+\frac{2\sqrt{dx-c}\sqrt{c+dx}\,(3ad^2+4bc^2)}{3d^6}+\frac{bx^4}{3d^2\sqrt{dx-c}\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(a + b*x^2))/((-c + d*x)^(3/2)*(c + d*x)^(3/2)),x]
```

```
[Out] -((4*b*c^2 + 3*a*d^2)*x^2)/(3*d^4*Sqrt[-c + d*x]*Sqrt[c + d*x]) + (b*x^4)/(
3*d^2*Sqrt[-c + d*x]*Sqrt[c + d*x]) + (2*(4*b*c^2 + 3*a*d^2)*Sqrt[-c + d*x]
*Sqrt[c + d*x])/(3*d^6)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 460

```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[(d*(e*x)^
(m + 1)*(a1 + b1*x^(n/2))^(p + 1)*(a2 + b2*x^(n/2))^(p + 1))/(b1*b2*e*(m +
n*(p + 1) + 1)), x] - Dist[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/
(b1*b2*(m + n*(p + 1) + 1)), Int[(e*x)^m*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n/
2))^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, m, n, p}, x] && EqQ[non2,
n/2] && EqQ[a2*b1 + a1*b2, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rubi steps

$$\int \frac{x^3\left(a+bx^2\right)}{(-c+dx)^{3/2}(c+dx)^{3/2}}\,dx = \frac{bx^4}{3d^2\sqrt{-c+dx}\,\sqrt{c+dx}} - \frac{1}{3}\left(-3a-\frac{4bc^2}{d^2}\right)\int \frac{x^3}{(-c+dx)^{3/2}(c+dx)^{3/2}}\,dx$$
$$= -\frac{\left(4bc^2+3ad^2\right)x^2}{3d^4\sqrt{-c+dx}\,\sqrt{c+dx}} + \frac{bx^4}{3d^2\sqrt{-c+dx}\,\sqrt{c+dx}} - \frac{\left(3a+\frac{4bc^2}{d^2}\right)\int \frac{x\left(-2c^2-2cdx\right)}{\sqrt{-c+dx}\,(c+dx)}}{3cd^2}$$
$$= -\frac{\left(4bc^2+3ad^2\right)x^2}{3d^4\sqrt{-c+dx}\,\sqrt{c+dx}} + \frac{bx^4}{3d^2\sqrt{-c+dx}\,\sqrt{c+dx}} + \frac{\left(2\left(3a+\frac{4bc^2}{d^2}\right)\right)\int \frac{x}{\sqrt{-c+dx}\,\sqrt{}}}{3d^2}$$
$$= -\frac{\left(4bc^2+3ad^2\right)x^2}{3d^4\sqrt{-c+dx}\,\sqrt{c+dx}} + \frac{bx^4}{3d^2\sqrt{-c+dx}\,\sqrt{c+dx}} + \frac{2\left(4bc^2+3ad^2\right)\sqrt{-c+dx}}{3d^6}$$

Mathematica [A] time = 0.05, size = 72, normalized size = 0.63

$$\frac{-6ac^2d^2+3ad^4x^2-8bc^4+4bc^2d^2x^2+bd^4x^4}{3d^6\sqrt{dx-c}\,\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(a + b*x^2))/((-c + d*x)^(3/2)*(c + d*x)^(3/2)),x]
```

```
[Out] (-8*b*c^4 - 6*a*c^2*d^2 + 4*b*c^2*d^2*x^2 + 3*a*d^4*x^2 + b*d^4*x^4)/(3*d^6
*Sqrt[-c + d*x]*Sqrt[c + d*x])
```

IntegrateAlgebraic [B] time = 0.16, size = 236, normalized size = 2.05

$$\frac{\sqrt{c+dx}\left(\frac{3acd^2(dx-c)^4}{(c+dx)^4}-\frac{24acd^2(dx-c)^3}{(c+dx)^3}+\frac{42acd^2(dx-c)^2}{(c+dx)^2}-\frac{24acd^2(dx-c)}{c+dx}+3acd^2+\frac{3bc^3(dx-c)^4}{(c+dx)^4}-\frac{36bc^3(dx-c)^3}{(c+dx)^3}+\frac{50bc^3(dx-c)^2}{(c+dx)^2}-\frac{36bc^3(dx-c)}{c+dx}+3bc^3\right)}{6d^6\sqrt{dx-c}\left(\frac{dx-c}{c+dx}-1\right)^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^3*(a + b*x^2))/((-c + d*x)^(3/2)*(c + d*x)^(3/2)),x]
```

```
[Out] (Sqrt[c + d*x]*(3*b*c^3 + 3*a*c*d^2 + (3*b*c^3*(-c + d*x)^4)/(c + d*x)^4 +
(3*a*c*d^2*(-c + d*x)^4)/(c + d*x)^4 - (36*b*c^3*(-c + d*x)^3)/(c + d*x)^3
- (24*a*c*d^2*(-c + d*x)^3)/(c + d*x)^3 + (50*b*c^3*(-c + d*x)^2)/(c + d*x)
^2 + (42*a*c*d^2*(-c + d*x)^2)/(c + d*x)^2 - (36*b*c^3*(-c + d*x))/(c + d*x
) - (24*a*c*d^2*(-c + d*x))/(c + d*x)))/(6*d^6*Sqrt[-c + d*x]*(-1 + (-c + d
*x)/(c + d*x))^3)
```

fricas [A] time = 0.66, size = 80, normalized size = 0.70

$$\frac{\left(bd^4x^4-8\,bc^4-6\,ac^2d^2+\left(4\,bc^2d^2+3\,ad^4\right)x^2\right)\sqrt{dx+c}\,\sqrt{dx-c}}{3\left(d^8x^2-c^2d^6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)/(d*x-c)^(3/2)/(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/3*(b*d^4*x^4 - 8*b*c^4 - 6*a*c^2*d^2 + (4*b*c^2*d^2 + 3*a*d^4)*x^2)*sqrt(
d*x + c)*sqrt(d*x - c)/(d^8*x^2 - c^2*d^6)
```

giac [B] time = 0.43, size = 200, normalized size = 1.74

$$\frac{\left(2\,(dx+c)\left((dx+c)\left(\frac{(dx+c)b}{d^6}-\frac{4\,bc}{d^6}\right)+\frac{10\,bc^2d^{24}+3\,ad^{26}}{d^{30}}\right)-\frac{3\left(9\,bc^3d^{24}+5\,acd^{26}\right)}{d^{30}}\right)\sqrt{dx+c}}{6\,\sqrt{dx-c}}+\frac{2\left(b^2c^8+2\,abc^6d^2+a^2c^4d^4\right)}{\left(bc^4\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^2+ac^2d^2\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^2+2\,bc^5+2\,ac^3d^2\right)d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)/(d*x-c)^(3/2)/(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] 1/6*(2*(d*x + c)*((d*x + c)*((d*x + c)*b/d^6 - 4*b*c/d^6) + (10*b*c^2*d^24
+ 3*a*d^26)/d^30) - 3*(9*b*c^3*d^24 + 5*a*c*d^26)/d^30)*sqrt(d*x + c)/sqrt(
d*x - c) + 2*(b^2*c^8 + 2*a*b*c^6*d^2 + a^2*c^4*d^4)/((b*c^4*(sqrt(d*x + c)
 - sqrt(d*x - c))^2 + a*c^2*d^2*(sqrt(d*x + c) - sqrt(d*x - c))^2 + 2*b*c^5
 + 2*a*c^3*d^2)*d^6)
```

maple [A] time = 0.05, size = 68, normalized size = 0.59

$$-\frac{-b\,d^4x^4-3a\,d^4x^2-4b\,c^2d^2x^2+6a\,c^2d^2+8b\,c^4}{3\sqrt{dx+c}\,\sqrt{dx-c}\,d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x^2+a)/(d*x-c)^(3/2)/(d*x+c)^(3/2),x)
```

```
[Out] -1/3*(-b*d^4*x^4-3*a*d^4*x^2-4*b*c^2*d^2*x^2+6*a*c^2*d^2+8*b*c^4)/(d*x+c)^(
1/2)/d^6/(d*x-c)^(1/2)
```

maxima [A] time = 0.46, size = 123, normalized size = 1.07

$$\frac{bx^4}{3\sqrt{d^2x^2-c^2}\,d^2}+\frac{4\,bc^2x^2}{3\sqrt{d^2x^2-c^2}\,d^4}+\frac{ax^2}{\sqrt{d^2x^2-c^2}\,d^2}-\frac{8\,bc^4}{3\sqrt{d^2x^2-c^2}\,d^6}-\frac{2\,ac^2}{\sqrt{d^2x^2-c^2}\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)/(d*x-c)^(3/2)/(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/3*b*x^4/(sqrt(d^2*x^2 - c^2)*d^2) + 4/3*b*c^2*x^2/(sqrt(d^2*x^2 - c^2)*d^
4) + a*x^2/(sqrt(d^2*x^2 - c^2)*d^2) - 8/3*b*c^4/(sqrt(d^2*x^2 - c^2)*d^6)
- 2*a*c^2/(sqrt(d^2*x^2 - c^2)*d^4)
```

mupad [B] time = 2.80, size = 90, normalized size = 0.78

$$\frac{\sqrt{d\,x-c}\,\left(\frac{x^2\,\left(4\,b\,c^2\,d^2+3\,a\,d^4\right)}{3\,d^7}-\frac{8\,b\,c^4+6\,a\,c^2\,d^2}{3\,d^7}+\frac{b\,x^4}{3\,d^3}\right)}{x\,\sqrt{c+d\,x}-\frac{c\,\sqrt{c+d\,x}}{d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(a + b*x^2))/((c + d*x)^(3/2)*(d*x - c)^(3/2)),x)
```

```
[Out] ((d*x - c)^(1/2)*((x^2*(3*a*d^4 + 4*b*c^2*d^2))/(3*d^7) - (8*b*c^4 + 6*a*c^
2*d^2)/(3*d^7) + (b*x^4)/(3*d^3)))/(x*(c + d*x)^(1/2) - (c*(c + d*x)^(1/2))
/d)
```

sympy [C] time = 177.26, size = 226, normalized size = 1.97

$$a\left(\frac{cG_{6,6}^{6,2}\left(\begin{matrix} -\frac{3}{4}, -\frac{1}{4} & -1, 0, \frac{1}{2}, 1 \\ -\frac{3}{4}, -\frac{1}{2}, -\frac{1}{4}, 0, \frac{1}{2}, 0 & \end{matrix} \middle| {\frac{c^2}{d^2x^2}} \right)}{2\pi^{\frac{3}{2}}d^4}-\frac{icG_{6,6}^{2,6}\left(\begin{matrix} -2, -\frac{3}{2}, -\frac{5}{4}, -1, -\frac{3}{4}, 1 & \\ -\frac{5}{4}, -\frac{3}{4} & -2, -\frac{3}{2}, -\frac{1}{2}, 0 \end{matrix} \middle| {\frac{c^2e^{2i\pi}}{d^2x^2}} \right)}{2\pi^{\frac{3}{2}}d^4}\right)+b\left(\frac{c^3G_{6,6}^{6,2}\left(\begin{matrix} -\frac{7}{4}, -\frac{5}{4} & -2, -1, -\frac{1}{2}, 1 \\ -\frac{7}{4}, -\frac{3}{2}, -\frac{5}{4}, -1, -\frac{1}{2}, 0 & \end{matrix} \middle| {\frac{c^2}{d^2x^2}} \right)}{2\pi^{\frac{3}{2}}d^6}-\frac{ic^3G_{6,6}^{2,6}\left(\begin{matrix} -3, -\frac{5}{2}, -\frac{9}{4}, -2, -\frac{7}{4}, 1 & \\ -\frac{9}{4}, -\frac{7}{4} & -3, -\frac{5}{2}, -\frac{3}{2}, 0 \end{matrix} \middle| {\frac{c^2e^{2i\pi}}{d^2x^2}} \right)}{2\pi^{\frac{3}{2}}d^6}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x**2+a)/(d*x-c)**(3/2)/(d*x+c)**(3/2),x)
```

```
[Out] a*(c*meijerg(((-3/4, -1/4), (-1, 0, 1/2, 1)), ((-3/4, -1/2, -1/4, 0, 1/2, 0
), ()), c**2/(d**2*x**2))/(2*pi**(3/2)*d**4) - I*c*meijerg(((-2, -3/2, -5/4
, -1, -3/4, 1), ()), ((-5/4, -3/4), (-2, -3/2, -1/2, 0)), c**2*exp_polar(2*
```

```
I*pi)/(d**2*x**2))/(2*pi**(3/2)*d**4)) + b*(c**3*meijerg(((-7/4, -5/4), (-2
, -1, -1/2, 1)), ((-7/4, -3/2, -5/4, -1, -1/2, 0), ()), c**2/(d**2*x**2))/(
2*pi**(3/2)*d**6) - I*c**3*meijerg(((-3, -5/2, -9/4, -2, -7/4, 1), ()), ((-
9/4, -7/4), (-3, -5/2, -3/2, 0)), c**2*exp_polar(2*I*pi)/(d**2*x**2))/(2*pi
**(3/2)*d**6))
```

3.227 $\int \frac{x^2(a+bx^2)}{(-c+dx)^{3/2}(c+dx)^{3/2}}\,dx$

Optimal. Leaf size=152

$$-\frac{\sqrt{dx-c}\,(2ad^2+3bc^2)}{2d^5\sqrt{c+dx}}-\frac{c\,(2ad^2+3bc^2)}{2d^5\sqrt{dx-c}\,\sqrt{c+dx}}+\frac{(2ad^2+3bc^2)\tanh^{-1}\left(\frac{\sqrt{dx-c}}{\sqrt{c+dx}}\right)}{d^5}+\frac{bx^3}{2d^2\sqrt{dx-c}\,\sqrt{c+dx}}$$

Rubi [A] time = 0.11, antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {460, 89, 12, 78, 63, 217, 206}

$$-\frac{\sqrt{dx-c}\,(2ad^2+3bc^2)}{2d^5\sqrt{c+dx}}-\frac{c\,(2ad^2+3bc^2)}{2d^5\sqrt{dx-c}\,\sqrt{c+dx}}+\frac{(2ad^2+3bc^2)\tanh^{-1}\left(\frac{\sqrt{dx-c}}{\sqrt{c+dx}}\right)}{d^5}+\frac{bx^3}{2d^2\sqrt{dx-c}\,\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a + b*x^2))/((-c + d*x)^(3/2)*(c + d*x)^(3/2)),x]
```

```
[Out] -(c*(3*b*c^2 + 2*a*d^2))/(2*d^5*Sqrt[-c + d*x]*Sqrt[c + d*x]) + (b*x^3)/(2*
d^2*Sqrt[-c + d*x]*Sqrt[c + d*x]) - ((3*b*c^2 + 2*a*d^2)*Sqrt[-c + d*x])/(2
*d^5*Sqrt[c + d*x]) + ((3*b*c^2 + 2*a*d^2)*ArcTanh[Sqrt[-c + d*x]/Sqrt[c +
d*x]])/d^5
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 460

```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[(d*(e*x)^
(m + 1)*(a1 + b1*x^(n/2))^(p + 1)*(a2 + b2*x^(n/2))^(p + 1))/(b1*b2*e*(m +
n*(p + 1) + 1)), x] - Dist[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/
(b1*b2*(m + n*(p + 1) + 1)), Int[(e*x)^m*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n/
2))^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, m, n, p}, x] && EqQ[non2,
n/2] && EqQ[a2*b1 + a1*b2, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rubi steps

$$\int \frac{x^2\left(a+bx^2\right)}{(-c+dx)^{3/2}(c+dx)^{3/2}}\,dx = \frac{bx^3}{2d^2\sqrt{-c+dx}\,\sqrt{c+dx}} - \frac{1}{2}\left(-2a-\frac{3bc^2}{d^2}\right)\int \frac{x^2}{(-c+dx)^{3/2}(c+dx)^{3/2}}\,dx$$

$$= -\frac{c\left(3bc^2+2ad^2\right)}{2d^5\sqrt{-c+dx}\,\sqrt{c+dx}} + \frac{bx^3}{2d^2\sqrt{-c+dx}\,\sqrt{c+dx}} - \frac{\left(-2a-\frac{3bc^2}{d^2}\right)\int \frac{cd^2x}{\sqrt{-c+dx}\,(c+d}}{2cd^3}$$

$$= -\frac{c\left(3bc^2+2ad^2\right)}{2d^5\sqrt{-c+dx}\,\sqrt{c+dx}} + \frac{bx^3}{2d^2\sqrt{-c+dx}\,\sqrt{c+dx}} + \frac{\left(3bc^2+2ad^2\right)\int \frac{x}{\sqrt{-c+dx}\,(c+}}{2d^3}$$

$$= -\frac{c\left(3bc^2+2ad^2\right)}{2d^5\sqrt{-c+dx}\,\sqrt{c+dx}} + \frac{bx^3}{2d^2\sqrt{-c+dx}\,\sqrt{c+dx}} - \frac{\left(3bc^2+2ad^2\right)\sqrt{-c+dx}}{2d^5\sqrt{c+dx}}$$

$$= -\frac{c\left(3bc^2+2ad^2\right)}{2d^5\sqrt{-c+dx}\,\sqrt{c+dx}} + \frac{bx^3}{2d^2\sqrt{-c+dx}\,\sqrt{c+dx}} - \frac{\left(3bc^2+2ad^2\right)\sqrt{-c+dx}}{2d^5\sqrt{c+dx}}$$

$$= -\frac{c\left(3bc^2+2ad^2\right)}{2d^5\sqrt{-c+dx}\,\sqrt{c+dx}} + \frac{bx^3}{2d^2\sqrt{-c+dx}\,\sqrt{c+dx}} - \frac{\left(3bc^2+2ad^2\right)\sqrt{-c+dx}}{2d^5\sqrt{c+dx}}$$

$$= -\frac{c\left(3bc^2+2ad^2\right)}{2d^5\sqrt{-c+dx}\,\sqrt{c+dx}} + \frac{bx^3}{2d^2\sqrt{-c+dx}\,\sqrt{c+dx}} - \frac{\left(3bc^2+2ad^2\right)\sqrt{-c+dx}}{2d^5\sqrt{c+dx}}$$

Mathematica [A] time = 0.12, size = 90, normalized size = 0.59

$$\frac{c\sqrt{1-\frac{d^2x^2}{c^2}}\left(2ad^2+3bc^2\right)\sin^{-1}\left(\frac{dx}{c}\right)-2ad^3x-3bc^2dx+bd^3x^3}{2d^5\sqrt{dx-c}\,\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(a + b*x^2))/((-c + d*x)^(3/2)*(c + d*x)^(3/2)),x]
```

```
[Out] (-3*b*c^2*d*x - 2*a*d^3*x + b*d^3*x^3 + c*(3*b*c^2 + 2*a*d^2)*Sqrt[1 - (d^2
*x^2)/c^2]*ArcSin[(d*x)/c])/(2*d^5*Sqrt[-c + d*x]*Sqrt[c + d*x])
```

IntegrateAlgebraic [A] time = 0.19, size = 196, normalized size = 1.29

$$\frac{\left(2ad^2+3bc^2\right)\tanh^{-1}\left(\frac{\sqrt{dx-c}}{\sqrt{c+dx}}\right)}{d^5}-\frac{\sqrt{c+dx}\left(\frac{dx-c}{c+dx}+1\right)\left(-\frac{2ad^2(dx-c)}{c+dx}+\frac{ad^2(dx-c)^2}{(c+dx)^2}+ad^2-\frac{4bc^2(dx-c)}{c+dx}+\frac{bc^2(dx-c)^2}{(c+dx)^2}+bc^2\right)}{2d^5\sqrt{dx-c}\left(\frac{dx-c}{c+dx}-1\right)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^2*(a + b*x^2))/((-c + d*x)^(3/2)*(c + d*x)^(3/2)),x]
```

```
[Out] -1/2*(Sqrt[c + d*x]*(1 + (-c + d*x)/(c + d*x))*(b*c^2 + a*d^2 + (b*c^2*(-c
+ d*x)^2)/(c + d*x)^2 + (a*d^2*(-c + d*x)^2)/(c + d*x)^2 - (4*b*c^2*(-c + d
*x))/(c + d*x) - (2*a*d^2*(-c + d*x))/(c + d*x)))/(d^5*Sqrt[-c + d*x]*(-1 +
 (-c + d*x)/(c + d*x))^2) + ((3*b*c^2 + 2*a*d^2)*ArcTanh[Sqrt[-c + d*x]/Sqr
t[c + d*x]])/d^5
```

fricas [A] time = 0.67, size = 159, normalized size = 1.05

$$\frac{2\,bc^4+2\,ac^2d^2-2\left(bc^2d^2+ad^4\right)x^2+\left(bd^3x^3-\left(3\,bc^2d+2\,ad^3\right)x\right)\sqrt{dx+c}\,\sqrt{dx-c}+\left(3\,bc^4+2\,ac^2d^2-\left(3\,bc^2d^2+2\,ad^4\right)x^2\right)\log\left(-dx+\sqrt{dx+c}\,\sqrt{dx-c}\right)}{2\left(d^7x^2-c^2d^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)/(d*x-c)^(3/2)/(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/2*(2*b*c^4 + 2*a*c^2*d^2 - 2*(b*c^2*d^2 + a*d^4)*x^2 + (b*d^3*x^3 - (3*b*
c^2*d + 2*a*d^3)*x)*sqrt(d*x + c)*sqrt(d*x - c) + (3*b*c^4 + 2*a*c^2*d^2 -
(3*b*c^2*d^2 + 2*a*d^4)*x^2)*log(-d*x + sqrt(d*x + c)*sqrt(d*x - c)))/(d^7*
x^2 - c^2*d^5)
```

giac [A] time = 0.32, size = 147, normalized size = 0.97

$$\frac{\sqrt{dx+c}\left((dx+c)\left(\frac{(dx+c)b}{d^5}-\frac{3\,bc}{d^5}\right)+\frac{bc^2d^{15}-ad^{17}}{d^{20}}\right)}{2\,\sqrt{dx-c}}-\frac{\left(3\,bc^2+2\,ad^2\right)\log\left(\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^2\right)}{2\,d^5}-\frac{2\left(bc^3+acd^2\right)}{\left(\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^2+2\,c\right)d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)/(d*x-c)^(3/2)/(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(d*x + c)*((d*x + c)*((d*x + c)*b/d^5 - 3*b*c/d^5) + (b*c^2*d^15 -
a*d^17)/d^20)/sqrt(d*x - c) - 1/2*(3*b*c^2 + 2*a*d^2)*log((sqrt(d*x + c) -
sqrt(d*x - c))^2)/d^5 - 2*(b*c^3 + a*c*d^2)/(((sqrt(d*x + c) - sqrt(d*x - c
))^2 + 2*c)*d^5)
```

maple [C] time = 0.08, size = 254, normalized size = 1.67

$$\frac{\left(2a\,d^4x^2\ln\left(\left(dx+\sqrt{d^2x^2-c^2}\,\mathrm{csgn}(d)\right)\mathrm{csgn}(d)\right)+3b\,c^2d^2x^2\ln\left(\left(dx+\sqrt{d^2x^2-c^2}\,\mathrm{csgn}(d)\right)\mathrm{csgn}(d)\right)+\sqrt{d^2x^2-c^2}\,b\,d^3x^3\mathrm{csgn}(d)-2a\,c^2d^2\ln\left(\left(dx+\sqrt{d^2x^2-c^2}\,\mathrm{csgn}(d)\right)\mathrm{csgn}(d)\right)-2\sqrt{d^2x^2-c^2}\,a\,d^3x\,\mathrm{csgn}(d)-3b\,c^4\ln\left(\left(dx+\sqrt{d^2x^2-c^2}\,\mathrm{csgn}(d)\right)\mathrm{csgn}(d)\right)-3\sqrt{d^2x^2-c^2}\,b\,c^2dx\,\mathrm{csgn}(d)\right)\mathrm{csgn}(d)}{2\sqrt{d^2x^2-c^2}\,\sqrt{dx+c}\,\sqrt{dx-c}\,d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x^2+a)/(d*x-c)^(3/2)/(d*x+c)^(3/2),x)
```

```
[Out] 1/2*((d^2*x^2-c^2)^(1/2)*b*d^3*x^3*csgn(d)+2*ln((d*x+(d^2*x^2-c^2)^(1/2)*cs
gn(d))*csgn(d))*x^2*a*d^4+3*ln((d*x+(d^2*x^2-c^2)^(1/2)*csgn(d))*csgn(d))*x
^2*b*c^2*d^2-2*(d^2*x^2-c^2)^(1/2)*a*d^3*x*csgn(d)-3*(d^2*x^2-c^2)^(1/2)*b*
c^2*d*x*csgn(d)-2*a*c^2*d^2*ln((d*x+(d^2*x^2-c^2)^(1/2)*csgn(d))*csgn(d))-3
*b*c^4*ln((d*x+(d^2*x^2-c^2)^(1/2)*csgn(d))*csgn(d)))*csgn(d)/(d^2*x^2-c^2)
^(1/2)/d^5/(d*x+c)^(1/2)/(d*x-c)^(1/2)
```

maxima [A] time = 0.50, size = 138, normalized size = 0.91

$$\frac{bx^3}{2\,\sqrt{d^2x^2-c^2}\,d^2}-\frac{3\,bc^2x}{2\,\sqrt{d^2x^2-c^2}\,d^4}-\frac{ax}{\sqrt{d^2x^2-c^2}\,d^2}+\frac{3\,bc^2\log\left(2\,d^2x+2\,\sqrt{d^2x^2-c^2}\,d\right)}{2\,d^5}+\frac{a\log\left(2\,d^2x+2\,\sqrt{d^2x^2-c^2}\,d\right)}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)/(d*x-c)^(3/2)/(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/2*b*x^3/(sqrt(d^2*x^2 - c^2)*d^2) - 3/2*b*c^2*x/(sqrt(d^2*x^2 - c^2)*d^4)
 - a*x/(sqrt(d^2*x^2 - c^2)*d^2) + 3/2*b*c^2*log(2*d^2*x + 2*sqrt(d^2*x^2 -
 c^2)*d)/d^5 + a*log(2*d^2*x + 2*sqrt(d^2*x^2 - c^2)*d)/d^3
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2\,\left(b\,x^2+a\right)}{{\left(c+d\,x\right)}^{3/2}\,{\left(d\,x-c\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a + b*x^2))/((c + d*x)^(3/2)*(d*x - c)^(3/2)),x)
```

```
[Out] int((x^2*(a + b*x^2))/((c + d*x)^(3/2)*(d*x - c)^(3/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x**2+a)/(d*x-c)**(3/2)/(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

3.228 $\int \frac{x(a+bx^2)}{(-c+dx)^{3/2}(c+dx)^{3/2}}\,dx$

Optimal. Leaf size=76

$$\frac{\sqrt{dx-c}\,\sqrt{c+dx}\,\left(ad^2+2bc^2\right)}{c^2d^4}-\frac{x^2\left(\frac{a}{c^2}+\frac{b}{d^2}\right)}{\sqrt{dx-c}\,\sqrt{c+dx}}$$

Rubi [A] time = 0.05, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {458, 74}

$$\frac{\sqrt{dx-c}\,\sqrt{c+dx}\,\left(ad^2+2bc^2\right)}{c^2d^4}-\frac{x^2\left(\frac{a}{c^2}+\frac{b}{d^2}\right)}{\sqrt{dx-c}\,\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a + b*x^2))/((-c + d*x)^(3/2)*(c + d*x)^(3/2)),x]
```

```
[Out] -(((a/c^2 + b/d^2)*x^2)/(Sqrt[-c + d*x]*Sqrt[c + d*x])) + ((2*b*c^2 + a*d^2
)*Sqrt[-c + d*x]*Sqrt[c + d*x])/(c^2*d^4)
```

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 458

```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b1*b2*
c - a1*a2*d)*(e*x)^(m + 1)*(a1 + b1*x^(n/2))^(p + 1)*(a2 + b2*x^(n/2))^(p +
 1))/(a1*a2*b1*b2*e*n*(p + 1)), x] - Dist[(a1*a2*d*(m + 1) - b1*b2*c*(m + n
*(p + 1) + 1))/(a1*a2*b1*b2*n*(p + 1)), Int[(e*x)^m*(a1 + b1*x^(n/2))^(p +
1)*(a2 + b2*x^(n/2))^(p + 1), x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, m,
n}, x] && EqQ[non2, n/2] && EqQ[a2*b1 + a1*b2, 0] && LtQ[p, -1] && (( !Inte
gerQ[p + 1/2] && NeQ[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p +
 1/2, 0] && LeQ[-1, m, -(n*(p + 1))]))
```

Rubi steps

$$\begin{aligned}\int \frac{x\left(a+bx^2\right)}{(-c+dx)^{3/2}(c+dx)^{3/2}}\,dx &= -\frac{\left(\frac{a}{c^2}+\frac{b}{d^2}\right)x^2}{\sqrt{-c+dx}\,\sqrt{c+dx}}-\left(-\frac{a}{c^2}-\frac{2b}{d^2}\right)\int \frac{x}{\sqrt{-c+dx}\,\sqrt{c+dx}}\,dx\\ &= -\frac{\left(\frac{a}{c^2}+\frac{b}{d^2}\right)x^2}{\sqrt{-c+dx}\,\sqrt{c+dx}}+\frac{\left(\frac{a}{c^2}+\frac{2b}{d^2}\right)\sqrt{-c+dx}\,\sqrt{c+dx}}{d^2}\end{aligned}$$

Mathematica [A] time = 0.07, size = 45, normalized size = 0.59

$$\frac{-ad^2-2bc^2+bd^2x^2}{d^4\sqrt{dx-c}\,\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(a + b*x^2))/((-c + d*x)^(3/2)*(c + d*x)^(3/2)),x]
[Out] (-2*b*c^2 - a*d^2 + b*d^2*x^2)/(d^4*Sqrt[-c + d*x]*Sqrt[c + d*x])
```

IntegrateAlgebraic [B] time = 0.14, size = 173, normalized size = 2.28

$$\frac{\sqrt{c+dx}\left(-\frac{2ad^2(dx-c)}{c+dx}+\frac{ad^2(dx-c)^2}{(c+dx)^2}+ad^2-\frac{6bc^2(dx-c)}{c+dx}+\frac{bc^2(dx-c)^2}{(c+dx)^2}+bc^2\right)}{2cd^4\sqrt{dx-c}\left(\frac{\sqrt{dx-c}}{\sqrt{c+dx}}-1\right)\left(\frac{\sqrt{dx-c}}{\sqrt{c+dx}}+1\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x*(a + b*x^2))/((-c + d*x)^(3/2)*(c + d*x)^(3/2)),x]
[Out] (Sqrt[c + d*x]*(b*c^2 + a*d^2 + (b*c^2*(-c + d*x)^2)/(c + d*x)^2 + (a*d^2*(
-c + d*x)^2)/(c + d*x)^2 - (6*b*c^2*(-c + d*x))/(c + d*x) - (2*a*d^2*(-c +
d*x))/(c + d*x)))/(2*c*d^4*Sqrt[-c + d*x]*(-1 + Sqrt[-c + d*x]/Sqrt[c + d*x
])*(1 + Sqrt[-c + d*x]/Sqrt[c + d*x]))
```

fricas [A] time = 0.77, size = 56, normalized size = 0.74

$$\frac{\left(bd^2x^2-2\,bc^2-ad^2\right)\sqrt{dx+c}\,\sqrt{dx-c}}{d^6x^2-c^2d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)/(d*x-c)^(3/2)/(d*x+c)^(3/2),x, algorithm="fricas")
[Out] (b*d^2*x^2 - 2*b*c^2 - a*d^2)*sqrt(d*x + c)*sqrt(d*x - c)/(d^6*x^2 - c^2*d^
4)
```

giac [B] time = 0.28, size = 152, normalized size = 2.00

$$\frac{\sqrt{dx+c}\left(\frac{2\,(dx+c)b}{d^4}-\frac{5\,bc^2d^8+ad^{10}}{cd^{12}}\right)}{2\,\sqrt{dx-c}}+\frac{2\left(b^2c^4+2\,abc^2d^2+a^2d^4\right)}{\left(bc^2\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^2+ad^2\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^2+2\,bc^3+2\,acd^2\right)d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)/(d*x-c)^(3/2)/(d*x+c)^(3/2),x, algorithm="giac")
[Out] 1/2*sqrt(d*x + c)*(2*(d*x + c)*b/d^4 - (5*b*c^2*d^8 + a*d^10)/(c*d^12))/sqr
t(d*x - c) + 2*(b^2*c^4 + 2*a*b*c^2*d^2 + a^2*d^4)/((b*c^2*(sqrt(d*x + c) -
 sqrt(d*x - c))^2 + a*d^2*(sqrt(d*x + c) - sqrt(d*x - c))^2 + 2*b*c^3 + 2*a
*c*d^2)*d^4)
```

maple [A] time = 0.04, size = 43, normalized size = 0.57

$$-\frac{-b\,d^2x^2+a\,d^2+2b\,c^2}{\sqrt{dx+c}\,\sqrt{dx-c}\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^2+a)/(d*x-c)^(3/2)/(d*x+c)^(3/2),x)
[Out] -(-b*d^2*x^2+a*d^2+2*b*c^2)/(d*x+c)^(1/2)/d^4/(d*x-c)^(1/2)
```

maxima [A] time = 0.58, size = 69, normalized size = 0.91

$$\frac{bx^2}{\sqrt{d^2x^2-c^2}\,d^2}-\frac{2\,bc^2}{\sqrt{d^2x^2-c^2}\,d^4}-\frac{a}{\sqrt{d^2x^2-c^2}\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)/(d*x-c)^(3/2)/(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] b*x^2/(sqrt(d^2*x^2 - c^2)*d^2) - 2*b*c^2/(sqrt(d^2*x^2 - c^2)*d^4) - a/(sq
rt(d^2*x^2 - c^2)*d^2)
```

mupad [B] time = 2.75, size = 67, normalized size = 0.88

$$\frac{a\,d^2\,\sqrt{d\,x-c}\,+2\,b\,c^2\,\sqrt{d\,x-c}\,-b\,d^2\,x^2\,\sqrt{d\,x-c}}{d^4\,\sqrt{c+d\,x}\,\left(c-d\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a + b*x^2))/((c + d*x)^(3/2)*(d*x - c)^(3/2)),x)
```

```
[Out] (a*d^2*(d*x - c)^(1/2) + 2*b*c^2*(d*x - c)^(1/2) - b*d^2*x^2*(d*x - c)^(1/2
))/(d^4*(c + d*x)^(1/2)*(c - d*x))
```

sympy [C] time = 136.30, size = 201, normalized size = 2.64

$$a\left(-\frac{G_{6,6}^{5,3}\left(\begin{matrix}\frac{1}{4},\frac{3}{4},1 & 0,1,\frac{3}{2}\\ \frac{1}{4},\frac{1}{2},\frac{3}{4},1,\frac{3}{2} & 0\end{matrix}\middle|\frac{c^2}{d^2x^2}\right)}{2\pi^{\frac{3}{2}}cd^2}-\frac{iG_{6,6}^{2,6}\left(\begin{matrix}-1,-\frac{1}{2},-\frac{1}{4},0,\frac{1}{4},1 & \\ -\frac{1}{4},\frac{1}{4} & -1,-\frac{1}{2},\frac{1}{2},0\end{matrix}\middle|\frac{c^2e^{2i\pi}}{d^2x^2}\right)}{2\pi^{\frac{3}{2}}cd^2}\right)+b\left(\frac{cG_{6,6}^{6,2}\left(\begin{matrix}-\frac{3}{4},-\frac{1}{4} & -1,0,\frac{1}{2},1\\ -\frac{3}{4},-\frac{1}{2},-\frac{1}{4},0,\frac{1}{2},0 & \end{matrix}\middle|\frac{c^2}{d^2x^2}\right)}{2\pi^{\frac{3}{2}}d^4}-\frac{icG_{6,6}^{2,6}\left(\begin{matrix}-2,-\frac{3}{2},-\frac{5}{4},-1,-\frac{3}{4},1 & \\ -\frac{5}{4},-\frac{3}{4} & -2,-\frac{3}{2},-\frac{1}{2},0\end{matrix}\middle|\frac{c^2e^{2i\pi}}{d^2x^2}\right)}{2\pi^{\frac{3}{2}}d^4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**2+a)/(d*x-c)**(3/2)/(d*x+c)**(3/2),x)
```

```
[Out] a*(-meijerg(((1/4, 3/4, 1), (0, 1, 3/2)), ((1/4, 1/2, 3/4, 1, 3/2), (0,)),
c**2/(d**2*x**2))/(2*pi**(3/2)*c*d**2) - I*meijerg(((-1, -1/2, -1/4, 0, 1/4
, 1), ()), ((-1/4, 1/4), (-1, -1/2, 1/2, 0)), c**2*exp_polar(2*I*pi)/(d**2*
x**2))/(2*pi**(3/2)*c*d**2)) + b*(c*meijerg(((-3/4, -1/4), (-1, 0, 1/2, 1))
, ((-3/4, -1/2, -1/4, 0, 1/2, 0), ()), c**2/(d**2*x**2))/(2*pi**(3/2)*d**4)
 - I*c*meijerg(((-2, -3/2, -5/4, -1, -3/4, 1), ()), ((-5/4, -3/4), (-2, -3/
2, -1/2, 0)), c**2*exp_polar(2*I*pi)/(d**2*x**2))/(2*pi**(3/2)*d**4))
```

3.229 $\int \frac{a+bx^2}{(-c+dx)^{3/2}(c+dx)^{3/2}}\,dx$

Optimal. Leaf size=63

$$\frac{2b\tanh^{-1}\left(\frac{\sqrt{dx-c}}{\sqrt{c+dx}}\right)}{d^3}-\frac{x\left(\frac{a}{c^2}+\frac{b}{d^2}\right)}{\sqrt{dx-c}\,\sqrt{c+dx}}$$

Rubi [A] time = 0.03, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {386, 63, 217, 206}

$$\frac{2b\tanh^{-1}\left(\frac{\sqrt{dx-c}}{\sqrt{c+dx}}\right)}{d^3}-\frac{x\left(\frac{a}{c^2}+\frac{b}{d^2}\right)}{\sqrt{dx-c}\,\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)/((-c + d*x)^(3/2)*(c + d*x)^(3/2)),x]
```

```
[Out] -(((a/c^2 + b/d^2)*x)/(Sqrt[-c + d*x]*Sqrt[c + d*x])) + (2*b*ArcTanh[Sqrt[-
c + d*x]/Sqrt[c + d*x]])/d^3
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 386

```
Int[((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_
.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b1*b2*c - a1*a2*d)*x*(a1
+ b1*x^(n/2))^(p + 1)*(a2 + b2*x^(n/2))^(p + 1))/(a1*a2*b1*b2*n*(p + 1)), x
] - Dist[(a1*a2*d - b1*b2*c*(n*(p + 1) + 1))/(a1*a2*b1*b2*n*(p + 1)), Int[(
a1 + b1*x^(n/2))^(p + 1)*(a2 + b2*x^(n/2))^(p + 1), x], x] /; FreeQ[{a1, b1
, a2, b2, c, d, n}, x] && EqQ[non2, n/2] && EqQ[a2*b1 + a1*b2, 0] && (LtQ[p
, -1] || ILtQ[1/n + p, 0])
```

Rubi steps

$$\int \frac{a+bx^2}{(-c+dx)^{3/2}(c+dx)^{3/2}}\,dx = -\frac{\left(\frac{a}{c^2}+\frac{b}{d^2}\right)x}{\sqrt{-c+dx}\,\sqrt{c+dx}} + \frac{b\int \frac{1}{\sqrt{-c+dx}\,\sqrt{c+dx}}\,dx}{d^2}$$
$$= -\frac{\left(\frac{a}{c^2}+\frac{b}{d^2}\right)x}{\sqrt{-c+dx}\,\sqrt{c+dx}} + \frac{(2b)\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{2c+x^2}}\,dx, x, \sqrt{-c+dx}\right)}{d^3}$$
$$= -\frac{\left(\frac{a}{c^2}+\frac{b}{d^2}\right)x}{\sqrt{-c+dx}\,\sqrt{c+dx}} + \frac{(2b)\,\mathrm{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{\sqrt{-c+dx}}{\sqrt{c+dx}}\right)}{d^3}$$
$$= -\frac{\left(\frac{a}{c^2}+\frac{b}{d^2}\right)x}{\sqrt{-c+dx}\,\sqrt{c+dx}} + \frac{2b\tanh^{-1}\left(\frac{\sqrt{-c+dx}}{\sqrt{c+dx}}\right)}{d^3}$$

Mathematica [A] time = 0.24, size = 86, normalized size = 1.37

$$\frac{2bc^{5/2}\sqrt{\frac{dx}{c}+1}\,\sinh^{-1}\left(\frac{\sqrt{dx-c}}{\sqrt{2}\,\sqrt{c}}\right) - \frac{dx\left(ad^2+bc^2\right)}{\sqrt{dx-c}}}{c^2d^3\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)/((-c + d*x)^(3/2)*(c + d*x)^(3/2)),x]
```

```
[Out] (-((d*(b*c^2 + a*d^2)*x)/Sqrt[-c + d*x]) + 2*b*c^(5/2)*Sqrt[1 + (d*x)/c]*Ar
cSinh[Sqrt[-c + d*x]/(Sqrt[2]*Sqrt[c])])/(c^2*d^3*Sqrt[c + d*x])
```

IntegrateAlgebraic [A] time = 0.15, size = 87, normalized size = 1.38

$$\frac{2b\tanh^{-1}\left(\frac{\sqrt{dx-c}}{\sqrt{c+dx}}\right)}{d^3} - \frac{\sqrt{c+dx}\left(\frac{dx-c}{c+dx}+1\right)\left(ad^2+bc^2\right)}{2c^2d^3\sqrt{dx-c}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x^2)/((-c + d*x)^(3/2)*(c + d*x)^(3/2)),x]
```

```
[Out] -1/2*((b*c^2 + a*d^2)*Sqrt[c + d*x]*(1 + (-c + d*x)/(c + d*x)))/(c^2*d^3*Sq
rt[-c + d*x]) + (2*b*ArcTanh[Sqrt[-c + d*x]/Sqrt[c + d*x]])/d^3
```

fricas [B] time = 0.84, size = 129, normalized size = 2.05

$$\frac{bc^4+ac^2d^2-\left(bc^2d+ad^3\right)\sqrt{dx+c}\,\sqrt{dx-c}\,x-\left(bc^2d^2+ad^4\right)x^2-\left(bc^2d^2x^2-bc^4\right)\log\left(-dx+\sqrt{dx+c}\,\sqrt{dx-c}\right)}{c^2d^5x^2-c^4d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/(d*x-c)^(3/2)/(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] (b*c^4 + a*c^2*d^2 - (b*c^2*d + a*d^3)*sqrt(d*x + c)*sqrt(d*x - c)*x - (b*c
^2*d^2 + a*d^4)*x^2 - (b*c^2*d^2*x^2 - b*c^4)*log(-d*x + sqrt(d*x + c)*sqrt
(d*x - c)))/(c^2*d^5*x^2 - c^4*d^3)
```

giac [B] time = 0.28, size = 113, normalized size = 1.79

$$-\frac{b\log\left(\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^2\right)}{d^3} - \frac{2\left(bc^2+ad^2\right)}{\left(\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^2+2c\right)cd^3} - \frac{\left(bc^2d^3+ad^5\right)\sqrt{dx+c}}{2\sqrt{dx-c}\,c^2d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/(d*x-c)^(3/2)/(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] -b*log((sqrt(d*x + c) - sqrt(d*x - c))^2)/d^3 - 2*(b*c^2 + a*d^2)/(((sqrt(d
*x + c) - sqrt(d*x - c))^2 + 2*c)*c*d^3) - 1/2*(b*c^2*d^3 + a*d^5)*sqrt(d*x
 + c)/(sqrt(d*x - c)*c^2*d^6)
```

maple [C] time = 0.07, size = 160, normalized size = 2.54

$$\frac{\left(b\,c^{2}d^{2}x^{2}\ln\left(\left(dx+\sqrt{\left(dx-c\right)\left(dx+c\right)}\,\mathrm{csgn}\left(d\right)\right)\mathrm{csgn}\left(d\right)\right)-\sqrt{d^{2}x^{2}-c^{2}}\,a\,d^{3}x\,\mathrm{csgn}\left(d\right)-b\,c^{4}\ln\left(\left(dx+\sqrt{\left(dx-c\right)\left(dx+c\right)}\,\mathrm{csgn}\left(d\right)\right)\mathrm{csgn}\left(d\right)\right)-\sqrt{d^{2}x^{2}-c^{2}}\,b\,c^{2}dx\,\mathrm{csgn}\left(d\right)\right)\mathrm{csgn}\left(d\right)}{\sqrt{d^{2}x^{2}-c^{2}}\,\sqrt{dx+c}\,\sqrt{dx-c}\,c^{2}d^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)/(d*x-c)^(3/2)/(d*x+c)^(3/2),x)
```

```
[Out] (ln((csgn(d)*((d*x-c)*(d*x+c))^(1/2)+d*x)*csgn(d))*x^2*b*c^2*d^2-(d^2*x^2-c
^2)^(1/2)*a*d^3*x*csgn(d)-(d^2*x^2-c^2)^(1/2)*b*c^2*d*x*csgn(d)-ln((csgn(d)
*((d*x-c)*(d*x+c))^(1/2)+d*x)*csgn(d))*b*c^4)*csgn(d)/(d^2*x^2-c^2)^(1/2)/c
^2/d^3/(d*x+c)^(1/2)/(d*x-c)^(1/2)
```

maxima [A] time = 0.55, size = 76, normalized size = 1.21

$$-\frac{ax}{\sqrt{d^{2}x^{2}-c^{2}}\,c^{2}}-\frac{bx}{\sqrt{d^{2}x^{2}-c^{2}}\,d^{2}}+\frac{b\log\left(2\,d^{2}x+2\,\sqrt{d^{2}x^{2}-c^{2}}\,d\right)}{d^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/(d*x-c)^(3/2)/(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] -a*x/(sqrt(d^2*x^2 - c^2)*c^2) - b*x/(sqrt(d^2*x^2 - c^2)*d^2) + b*log(2*d^
2*x + 2*sqrt(d^2*x^2 - c^2)*d)/d^3
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{b\,x^{2}+a}{{\left(c+d\,x\right)}^{3/2}\,{\left(d\,x-c\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)/((c + d*x)^(3/2)*(d*x - c)^(3/2)),x)
```

```
[Out] int((a + b*x^2)/((c + d*x)^(3/2)*(d*x - c)^(3/2)), x)
```

sympy [C] time = 112.36, size = 182, normalized size = 2.89

$$a\left(-\frac{G_{6,6}^{5,3}\left(\begin{matrix}\frac{3}{4},\frac{5}{4},1 & \frac{1}{2},\frac{3}{2},2\\ \frac{3}{4},1,\frac{5}{4},\frac{3}{2},2 & 0\end{matrix}\middle|\frac{c^{2}}{d^{2}x^{2}}\right)}{2\pi^{\frac{3}{2}}c^{2}d}+\frac{i G_{6,6}^{2,6}\left(\begin{matrix}-\frac{1}{2},0,\frac{1}{4},\frac{1}{2},\frac{3}{4},1 & \\ \frac{1}{4},\frac{3}{4} & -\frac{1}{2},0,1,0\end{matrix}\middle|\frac{c^{2}e^{2i\pi}}{d^{2}x^{2}}\right)}{2\pi^{\frac{3}{2}}c^{2}d}\right)+b\left(\frac{G_{6,6}^{6,2}\left(\begin{matrix}-\frac{1}{4},\frac{1}{4} & -\frac{1}{2},\frac{1}{2},1,1\\ -\frac{1}{4},0,\frac{1}{4},\frac{1}{2},1,0 & \end{matrix}\middle|\frac{c^{2}}{d^{2}x^{2}}\right)}{2\pi^{\frac{3}{2}}d^{3}}+\frac{i G_{6,6}^{2,6}\left(\begin{matrix}-\frac{3}{2},-1,-\frac{3}{4},-\frac{1}{2},-\frac{1}{4},1 & \\ -\frac{3}{4},-\frac{1}{4} & -\frac{3}{2},-1,0,0\end{matrix}\middle|\frac{c^{2}e^{2i\pi}}{d^{2}x^{2}}\right)}{2\pi^{\frac{3}{2}}d^{3}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)/(d*x-c)**(3/2)/(d*x+c)**(3/2),x)
```

```
[Out] a*(-meijerg(((3/4, 5/4, 1), (1/2, 3/2, 2)), ((3/4, 1, 5/4, 3/2, 2), (0,)),
c**2/(d**2*x**2))/(2*pi**(3/2)*c**2*d) + I*meijerg(((-1/2, 0, 1/4, 1/2, 3/4
, 1), ()), ((1/4, 3/4), (-1/2, 0, 1, 0)), c**2*exp_polar(2*I*pi)/(d**2*x**2
))/(2*pi**(3/2)*c**2*d)) + b*(meijerg(((-1/4, 1/4), (-1/2, 1/2, 1, 1)), ((-
1/4, 0, 1/4, 1/2, 1, 0), ()), c**2/(d**2*x**2))/(2*pi**(3/2)*d**3) + I*meij
erg(((-3/2, -1, -3/4, -1/2, -1/4, 1), ()), ((-3/4, -1/4), (-3/2, -1, 0, 0))
, c**2*exp_polar(2*I*pi)/(d**2*x**2))/(2*pi**(3/2)*d**3))
```

3.230 $\int \frac{a+bx^2}{x(-c+dx)^{3/2}(c+dx)^{3/2}}\,dx$

Optimal. Leaf size=65

$$-\frac{\frac{a}{c^2}+\frac{b}{d^2}}{\sqrt{dx-c}\,\sqrt{c+dx}}-\frac{a\tan^{-1}\left(\frac{\sqrt{dx-c}\,\sqrt{c+dx}}{c}\right)}{c^3}$$

Rubi [A] time = 0.08, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {458, 92, 205}

$$-\frac{\frac{a}{c^2}+\frac{b}{d^2}}{\sqrt{dx-c}\,\sqrt{c+dx}}-\frac{a\tan^{-1}\left(\frac{\sqrt{dx-c}\,\sqrt{c+dx}}{c}\right)}{c^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)/(x*(-c + d*x)^(3/2)*(c + d*x)^(3/2)),x]
```

```
[Out] -((a/c^2 + b/d^2)/(Sqrt[-c + d*x]*Sqrt[c + d*x])) - (a*ArcTan[(Sqrt[-c + d*
x]*Sqrt[c + d*x])/c])/c^3
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 458

```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b1*b2*
c - a1*a2*d)*(e*x)^(m + 1)*(a1 + b1*x^(n/2))^(p + 1)*(a2 + b2*x^(n/2))^(p +
 1))/(a1*a2*b1*b2*e*n*(p + 1)), x] - Dist[(a1*a2*d*(m + 1) - b1*b2*c*(m + n
*(p + 1) + 1))/(a1*a2*b1*b2*n*(p + 1)), Int[(e*x)^m*(a1 + b1*x^(n/2))^(p +
1)*(a2 + b2*x^(n/2))^(p + 1), x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, m,
n}, x] && EqQ[non2, n/2] && EqQ[a2*b1 + a1*b2, 0] && LtQ[p, -1] && (( !Inte
gerQ[p + 1/2] && NeQ[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p +
 1/2, 0] && LeQ[-1, m, -(n*(p + 1))]))
```

Rubi steps

$$\begin{aligned}
\int \frac{a+bx^2}{x(-c+dx)^{3/2}(c+dx)^{3/2}}\,dx &= -\frac{\frac{a}{c^2}+\frac{b}{d^2}}{\sqrt{-c+dx}\,\sqrt{c+dx}}-\frac{a\int \frac{1}{x\sqrt{-c+dx}\,\sqrt{c+dx}}\,dx}{c^2}\\
&= -\frac{\frac{a}{c^2}+\frac{b}{d^2}}{\sqrt{-c+dx}\,\sqrt{c+dx}}-\frac{(ad)\,\text{Subst}\left(\int \frac{1}{c^2d+dx^2}\,dx, x, \sqrt{-c+dx}\,\sqrt{c+dx}\right)}{c^2}\\
&= -\frac{\frac{a}{c^2}+\frac{b}{d^2}}{\sqrt{-c+dx}\,\sqrt{c+dx}}-\frac{a\tan^{-1}\left(\frac{\sqrt{-c+dx}\,\sqrt{c+dx}}{c}\right)}{c^3}
\end{aligned}$$

Mathematica [A] time = 0.04, size = 84, normalized size = 1.29

$$-\frac{ad^2\sqrt{d^2x^2-c^2}\tan^{-1}\left(\frac{\sqrt{d^2x^2-c^2}}{c}\right)+acd^2+bc^3}{c^3d^2\sqrt{dx-c}\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)/(x*(-c + d*x)^(3/2)*(c + d*x)^(3/2)),x]
```

```
[Out] -((b*c^3 + a*c*d^2 + a*d^2*Sqrt[-c^2 + d^2*x^2]*ArcTan[Sqrt[-c^2 + d^2*x^2]
/c])/(c^3*d^2*Sqrt[-c + d*x]*Sqrt[c + d*x]))
```

IntegrateAlgebraic [A] time = 0.11, size = 87, normalized size = 1.34

$$\frac{\sqrt{c+dx}\left(\frac{dx-c}{c+dx}-1\right)(ad^2+bc^2)}{2c^3d^2\sqrt{dx-c}}-\frac{2a\tan^{-1}\left(\frac{\sqrt{dx-c}}{\sqrt{c+dx}}\right)}{c^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x^2)/(x*(-c + d*x)^(3/2)*(c + d*x)^(3/2)),x]
```

```
[Out] ((b*c^2 + a*d^2)*Sqrt[c + d*x]*(-1 + (-c + d*x)/(c + d*x)))/(2*c^3*d^2*Sqrt
[-c + d*x]) - (2*a*ArcTan[Sqrt[-c + d*x]/Sqrt[c + d*x]])/c^3
```

fricas [A] time = 0.81, size = 101, normalized size = 1.55

$$-\frac{(bc^3+acd^2)\sqrt{dx+c}\sqrt{dx-c}+2(ad^4x^2-ac^2d^2)\arctan\left(-\frac{dx-\sqrt{dx+c}\sqrt{dx-c}}{c}\right)}{c^3d^4x^2-c^5d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/x/(d*x-c)^(3/2)/(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] -((b*c^3 + a*c*d^2)*sqrt(d*x + c)*sqrt(d*x - c) + 2*(a*d^4*x^2 - a*c^2*d^2)
*arctan(-(d*x - sqrt(d*x + c)*sqrt(d*x - c))/c))/(c^3*d^4*x^2 - c^5*d^2)
```

giac [B] time = 0.36, size = 115, normalized size = 1.77

$$\frac{2\,a\arctan\left(\frac{\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^2}{2\,c}\right)}{c^3}-\frac{(bc^2+ad^2)\sqrt{dx+c}}{2\sqrt{dx-c}\,c^3d^2}+\frac{2\,(bc^2+ad^2)}{\left(\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^2+2\,c\right)c^2d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/x/(d*x-c)^(3/2)/(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] 2*a*arctan(1/2*(sqrt(d*x + c) - sqrt(d*x - c))^2/c)/c^3 - 1/2*(b*c^2 + a*d^
2)*sqrt(d*x + c)/(sqrt(d*x - c)*c^3*d^2) + 2*(b*c^2 + a*d^2)/(((sqrt(d*x +
c) - sqrt(d*x - c))^2 + 2*c)*c^2*d^2)
```

maple [B] time = 0.08, size = 188, normalized size = 2.89

$$\frac{a\,d^4x^2\ln\left(-\frac{2\left(c^2-\sqrt{-c^2}\,\sqrt{d^2x^2-c^2}\right)}{x}\right)-a\,c^2d^2\ln\left(-\frac{2\left(c^2-\sqrt{-c^2}\,\sqrt{d^2x^2-c^2}\right)}{x}\right)-\sqrt{-c^2}\,\sqrt{d^2x^2-c^2}\,a\,d^2-\sqrt{-c^2}\,\sqrt{d^2x^2-c^2}\,b\,c^2}{\sqrt{-c^2}\,\sqrt{d^2x^2-c^2}\,\sqrt{dx+c}\,\sqrt{dx-c}\,c^2d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)/x/(d*x-c)^(3/2)/(d*x+c)^(3/2),x)
```

[Out] 1/c^2*(ln(-2*(c^2-(-c^2)^(1/2)*(d^2*x^2-c^2)^(1/2))/x)*x^2*a*d^4-ln(-2*(c^2 -(-c^2)^(1/2)*(d^2*x^2-c^2)^(1/2))/x)*a*c^2*d^2-(-c^2)^(1/2)*(d^2*x^2-c^2)^ (1/2)*a*d^2-b*c^2*(-c^2)^(1/2)*(d^2*x^2-c^2)^(1/2))/(-c^2)^(1/2)/(d^2*x^2-c ^2)^(1/2)/d^2/(d*x+c)^(1/2)/(d*x-c)^(1/2)

maxima [A] time = 1.46, size = 58, normalized size = 0.89

$$\frac{a \arcsin\left(\frac{c}{d|x|}\right)}{c^3} - \frac{a}{\sqrt{d^2x^2 - c^2}\, c^2} - \frac{b}{\sqrt{d^2x^2 - c^2}\, d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^2+a)/x/(d*x-c)^(3/2)/(d*x+c)^(3/2),x, algorithm="maxima")

[Out] a*arcsin(c/(d*abs(x)))/c^3 - a/(sqrt(d^2*x^2 - c^2)*c^2) - b/(sqrt(d^2*x^2 - c^2)*d^2)

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{b\,x^2 + a}{x\,(c + d\,x)^{3/2}\,(d\,x - c)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*x^2)/(x*(c + d*x)^(3/2)*(d*x - c)^(3/2)),x)

[Out] int((a + b*x^2)/(x*(c + d*x)^(3/2)*(d*x - c)^(3/2)), x)

sympy [C] time = 136.44, size = 172, normalized size = 2.65

$$a \left(- \frac{G_{6,6}^{5,3}\left(\begin{matrix} \frac{5}{4}, \frac{7}{4}, 1 & 1, 2, \frac{5}{2} \\ \frac{5}{4}, \frac{3}{2}, \frac{7}{4}, 2, \frac{5}{2} & 0 \end{matrix} \middle| \frac{c^2}{d^2x^2} \right)}{2\pi^{\frac{3}{2}} c^3} - \frac{i G_{6,6}^{2,6}\left(\begin{matrix} 0, \frac{1}{2}, \frac{3}{4}, 1, \frac{5}{4}, 1 & \\ \frac{3}{4}, \frac{5}{4} & 0, \frac{1}{2}, \frac{3}{2}, 0 \end{matrix} \middle| \frac{c^2 e^{2i\pi}}{d^2x^2} \right)}{2\pi^{\frac{3}{2}} c^3} \right) + b \left(- \frac{G_{6,6}^{5,3}\left(\begin{matrix} \frac{1}{4}, \frac{3}{4}, 1 & 0, 1, \frac{3}{2} \\ \frac{1}{4}, \frac{1}{2}, \frac{3}{4}, 1, \frac{3}{2} & 0 \end{matrix} \middle| \frac{c^2}{d^2x^2} \right)}{2\pi^{\frac{3}{2}} c d^2} - \frac{i G_{6,6}^{2,6}\left(\begin{matrix} -1, -\frac{1}{2}, -\frac{1}{4}, 0, \frac{1}{4}, 1 & \\ -\frac{1}{4}, \frac{1}{4} & -1, -\frac{1}{2}, \frac{1}{2}, 0 \end{matrix} \middle| \frac{c^2 e^{2i\pi}}{d^2x^2} \right)}{2\pi^{\frac{3}{2}} c d^2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x**2+a)/x/(d*x-c)**(3/2)/(d*x+c)**(3/2),x)

[Out] a*(-meijerg(((5/4, 7/4, 1), (1, 2, 5/2)), ((5/4, 3/2, 7/4, 2, 5/2), (0,)), c**2/(d**2*x**2))/(2*pi**(3/2)*c**3) - I*meijerg(((0, 1/2, 3/4, 1, 5/4, 1), ()), ((3/4, 5/4), (0, 1/2, 3/2, 0)), c**2*exp_polar(2*I*pi)/(d**2*x**2))/(2*pi**(3/2)*c**3)) + b*(-meijerg(((1/4, 3/4, 1), (0, 1, 3/2)), ((1/4, 1/2, 3/4, 1, 3/2), (0,)), c**2/(d**2*x**2))/(2*pi**(3/2)*c*d**2) - I*meijerg(((- 1, -1/2, -1/4, 0, 1/4, 1), ()), ((-1/4, 1/4), (-1, -1/2, 1/2, 0)), c**2*exp _polar(2*I*pi)/(d**2*x**2))/(2*pi**(3/2)*c*d**2))

3.231 $\int \frac{a+bx^2}{x^2(-c+dx)^{3/2}(c+dx)^{3/2}}\,dx$

Optimal. Leaf size=67

$$\frac{a}{c^2x\sqrt{dx-c}\sqrt{c+dx}}-\frac{x\left(2ad^2+bc^2\right)}{c^4\sqrt{dx-c}\sqrt{c+dx}}$$

Rubi [A] time = 0.08, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {454, 39}

$$\frac{a}{c^2x\sqrt{dx-c}\sqrt{c+dx}}-\frac{x\left(2ad^2+bc^2\right)}{c^4\sqrt{dx-c}\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)/(x^2*(-c + d*x)^(3/2)*(c + d*x)^(3/2)),x]
```

```
[Out] a/(c^2*x*Sqrt[-c + d*x]*Sqrt[c + d*x]) - ((b*c^2 + 2*a*d^2)*x)/(c^4*Sqrt[-c
 + d*x]*Sqrt[c + d*x])
```

Rule 39

```
Int[1/(((a_) + (b_.)*(x_))^(3/2)*((c_) + (d_.)*(x_))^(3/2)), x_Symbol] :> S
imp[x/(a*c*Sqrt[a + b*x]*Sqrt[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[b*c + a*d, 0]
```

Rule 454

```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[(c*(e*x)^
(m + 1)*(a1 + b1*x^(n/2))^(p + 1)*(a2 + b2*x^(n/2))^(p + 1))/(a1*a2*e*(m +
1)), x] + Dist[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/(a1*a2*e^n*(
m + 1)), Int[(e*x)^(m + n)*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n/2))^p, x], x]
/; FreeQ[{a1, b1, a2, b2, c, d, e, p}, x] && EqQ[non2, n/2] && EqQ[a2*b1 +
a1*b2, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (L
tQ[n, 0] && GtQ[m + n, -1])) && !ILtQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx^2}{x^2(-c+dx)^{3/2}(c+dx)^{3/2}}\,dx &= \frac{a}{c^2x\sqrt{-c+dx}\sqrt{c+dx}}+\left(b+\frac{2ad^2}{c^2}\right)\int \frac{1}{(-c+dx)^{3/2}(c+dx)^{3/2}}\,dx\\ &= \frac{a}{c^2x\sqrt{-c+dx}\sqrt{c+dx}}-\frac{\left(bc^2+2ad^2\right)x}{c^4\sqrt{-c+dx}\sqrt{c+dx}}\end{aligned}$$

Mathematica [A] time = 0.03, size = 51, normalized size = 0.76

$$\frac{a\left(c^2-2d^2x^2\right)-bc^2x^2}{c^4x\sqrt{dx-c}\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)/(x^2*(-c + d*x)^(3/2)*(c + d*x)^(3/2)),x]
```

```
[Out] (-(b*c^2*x^2) + a*(c^2 - 2*d^2*x^2))/(c^4*x*Sqrt[-c + d*x]*Sqrt[c + d*x])
```

IntegrateAlgebraic [B] time = 0.15, size = 146, normalized size = 2.18

$$\frac{\sqrt{c+dx}\left(-\frac{6ad^2(dx-c)}{c+dx}-\frac{ad^2(dx-c)^2}{(c+dx)^2}-ad^2-\frac{2bc^2(dx-c)}{c+dx}-\frac{bc^2(dx-c)^2}{(c+dx)^2}-bc^2\right)}{2c^4d\sqrt{dx-c}\left(\frac{dx-c}{c+dx}+1\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x^2)/(x^2*(-c + d*x)^(3/2)*(c + d*x)^(3/2)),x]
```

```
[Out] (Sqrt[c + d*x]*(-(b*c^2) - a*d^2 - (b*c^2*(-c + d*x)^2)/(c + d*x)^2 - (a*d^
2*(-c + d*x)^2)/(c + d*x)^2 - (2*b*c^2*(-c + d*x))/(c + d*x) - (6*a*d^2*(-c
 + d*x))/(c + d*x)))/(2*c^4*d*Sqrt[-c + d*x]*(1 + (-c + d*x)/(c + d*x)))
```

fricas [A] time = 0.78, size = 103, normalized size = 1.54

$$-\frac{\left(bc^2d^2+2\,ad^4\right)x^3-\left(ac^2d-\left(bc^2d+2\,ad^3\right)x^2\right)\sqrt{dx+c}\,\sqrt{dx-c}-\left(bc^4+2\,ac^2d^2\right)x}{c^4d^3x^3-c^6dx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/x^2/(d*x-c)^(3/2)/(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] -((b*c^2*d^2 + 2*a*d^4)*x^3 - (a*c^2*d - (b*c^2*d + 2*a*d^3)*x^2)*sqrt(d*x
+ c)*sqrt(d*x - c) - (b*c^4 + 2*a*c^2*d^2)*x)/(c^4*d^3*x^3 - c^6*d*x)
```

giac [B] time = 0.46, size = 219, normalized size = 3.27

$$-\frac{\left(bc^2+ad^2\right)\sqrt{dx+c}}{2\,\sqrt{dx-c}\,c^4d}-\frac{2\left(bc^2\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^4+ad^2\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^4+4\,acd^2\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^2+4\,bc^4+12\,ac^2d^2\right)}{\left(\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^6+2\,c\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^4+4\,c^2\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^2+8\,c^3\right)c^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/x^2/(d*x-c)^(3/2)/(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] -1/2*(b*c^2 + a*d^2)*sqrt(d*x + c)/(sqrt(d*x - c)*c^4*d) - 2*(b*c^2*(sqrt(d
*x + c) - sqrt(d*x - c))^4 + a*d^2*(sqrt(d*x + c) - sqrt(d*x - c))^4 + 4*a*
c*d^2*(sqrt(d*x + c) - sqrt(d*x - c))^2 + 4*b*c^4 + 12*a*c^2*d^2)/(((sqrt(d
*x + c) - sqrt(d*x - c))^6 + 2*c*(sqrt(d*x + c) - sqrt(d*x - c))^4 + 4*c^2*
(sqrt(d*x + c) - sqrt(d*x - c))^2 + 8*c^3)*c^3*d)
```

maple [A] time = 0.05, size = 48, normalized size = 0.72

$$\frac{-2a\,d^2x^2-b\,c^2x^2+a\,c^2}{\sqrt{dx+c}\,\sqrt{dx-c}\,c^4x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)/x^2/(d*x-c)^(3/2)/(d*x+c)^(3/2),x)
```

```
[Out] (-2*a*d^2*x^2-b*c^2*x^2+a*c^2)/(d*x+c)^(1/2)/x/c^4/(d*x-c)^(1/2)
```

maxima [A] time = 1.36, size = 71, normalized size = 1.06

$$-\frac{bx}{\sqrt{d^2x^2-c^2}\,c^2}-\frac{2\,ad^2x}{\sqrt{d^2x^2-c^2}\,c^4}+\frac{a}{\sqrt{d^2x^2-c^2}\,c^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/x^2/(d*x-c)^(3/2)/(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] -b*x/(sqrt(d^2*x^2 - c^2)*c^2) - 2*a*d^2*x/(sqrt(d^2*x^2 - c^2)*c^4) + a/(s
qrt(d^2*x^2 - c^2)*c^2*x)
```

mupad [B] time = 2.87, size = 73, normalized size = 1.09

$$\frac{2\,a\,d^2\,x^2\,\sqrt{d\,x-c}-a\,c^2\,\sqrt{d\,x-c}+b\,c^2\,x^2\,\sqrt{d\,x-c}}{c^4\,x\,\sqrt{c+d\,x}\,\left(c-d\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)/(x^2*(c + d*x)^(3/2)*(d*x - c)^(3/2)),x)
```

```
[Out] (2*a*d^2*x^2*(d*x - c)^(1/2) - a*c^2*(d*x - c)^(1/2) + b*c^2*x^2*(d*x - c)^
(1/2))/(c^4*x*(c + d*x)^(1/2)*(c - d*x))
```

sympy [C] time = 136.13, size = 165, normalized size = 2.46

$$a\left(-\frac{dG_{6,6}^{5,3}\left(\begin{matrix}\frac{7}{4},\frac{9}{4},1 & \frac{3}{2},\frac{5}{2},3\\ \frac{7}{4},2,\frac{9}{4},\frac{5}{2},3 & 0\end{matrix}\middle|\frac{c^2}{d^2x^2}\right)}{2\pi^{\frac{3}{2}}c^4}+\frac{idG_{6,6}^{2,6}\left(\begin{matrix}\frac{1}{2},1,\frac{5}{4},\frac{3}{2},\frac{7}{4},1 & \\ \frac{5}{4},\frac{7}{4} & \frac{1}{2},1,2,0\end{matrix}\middle|\frac{c^2e^{2i\pi}}{d^2x^2}\right)}{2\pi^{\frac{3}{2}}c^4}\right)+b\left(-\frac{G_{6,6}^{5,3}\left(\begin{matrix}\frac{3}{4},\frac{5}{4},1 & \frac{1}{2},\frac{3}{2},2\\ \frac{3}{4},1,\frac{5}{4},\frac{3}{2},2 & 0\end{matrix}\middle|\frac{c^2}{d^2x^2}\right)}{2\pi^{\frac{3}{2}}c^2d}+\frac{iG_{6,6}^{2,6}\left(\begin{matrix}-\frac{1}{2},0,\frac{1}{4},\frac{1}{2},\frac{3}{4},1 & \\ \frac{1}{4},\frac{3}{4} & -\frac{1}{2},0,1,0\end{matrix}\middle|\frac{c^2e^{2i\pi}}{d^2x^2}\right)}{2\pi^{\frac{3}{2}}c^2d}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)/x**2/(d*x-c)**(3/2)/(d*x+c)**(3/2),x)
```

```
[Out] a*(-d*meijerg(((7/4, 9/4, 1), (3/2, 5/2, 3)), ((7/4, 2, 9/4, 5/2, 3), (0,))
, c**2/(d**2*x**2))/(2*pi**(3/2)*c**4) + I*d*meijerg(((1/2, 1, 5/4, 3/2, 7/
4, 1), ()), ((5/4, 7/4), (1/2, 1, 2, 0)), c**2*exp_polar(2*I*pi)/(d**2*x**2
))/(2*pi**(3/2)*c**4)) + b*(-meijerg(((3/4, 5/4, 1), (1/2, 3/2, 2)), ((3/4,
 1, 5/4, 3/2, 2), (0,)), c**2/(d**2*x**2))/(2*pi**(3/2)*c**2*d) + I*meijerg
(((-1/2, 0, 1/4, 1/2, 3/4, 1), ()), ((1/4, 3/4), (-1/2, 0, 1, 0)), c**2*exp
_polar(2*I*pi)/(d**2*x**2))/(2*pi**(3/2)*c**2*d))
```

3.232 $\int \frac{a+bx^2}{x^3(-c+dx)^{3/2}(c+dx)^{3/2}}\,dx$

Optimal. Leaf size=117

$$-\frac{\left(3ad^2+2bc^2\right)\tan^{-1}\left(\frac{\sqrt{dx-c}\sqrt{c+dx}}{c}\right)}{2c^5}-\frac{3ad^2+2bc^2}{2c^4\sqrt{dx-c}\sqrt{c+dx}}+\frac{a}{2c^2x^2\sqrt{dx-c}\sqrt{c+dx}}$$

Rubi [A] time = 0.10, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {454, 104, 21, 92, 205}

$$-\frac{3ad^2+2bc^2}{2c^4\sqrt{dx-c}\sqrt{c+dx}}-\frac{\left(3ad^2+2bc^2\right)\tan^{-1}\left(\frac{\sqrt{dx-c}\sqrt{c+dx}}{c}\right)}{2c^5}+\frac{a}{2c^2x^2\sqrt{dx-c}\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)/(x^3*(-c + d*x)^(3/2)*(c + d*x)^(3/2)),x]
```

```
[Out] -(2*b*c^2 + 3*a*d^2)/(2*c^4*Sqrt[-c + d*x]*Sqrt[c + d*x]) + a/(2*c^2*x^2*Sq
rt[-c + d*x]*Sqrt[c + d*x]) - ((2*b*c^2 + 3*a*d^2)*ArcTan[(Sqrt[-c + d*x]*S
qrt[c + d*x])/c])/(2*c^5)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 104

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegersQ
[2*m, 2*n, 2*p]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 454

```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[(c*(e*x)^
(m + 1)*(a1 + b1*x^(n/2))^(p + 1)*(a2 + b2*x^(n/2))^(p + 1))/(a1*a2*e*(m +
1)), x] + Dist[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/(a1*a2*e^n*(
m + 1)), Int[(e*x)^(m + n)*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n/2))^p, x], x]
```

```
/; FreeQ[{a1, b1, a2, b2, c, d, e, p}, x] && EqQ[non2, n/2] && EqQ[a2*b1 +
a1*b2, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (L
tQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rubi steps

$$\int \frac{a+bx^2}{x^3(-c+dx)^{3/2}(c+dx)^{3/2}}\,dx = \frac{a}{2c^2x^2\sqrt{-c+dx}\,\sqrt{c+dx}} + \frac{1}{2}\left(2b+\frac{3ad^2}{c^2}\right)\int \frac{1}{x(-c+dx)^{3/2}(c+dx)^{3/2}}\,dx$$

$$= -\frac{2bc^2+3ad^2}{2c^4\sqrt{-c+dx}\,\sqrt{c+dx}} + \frac{a}{2c^2x^2\sqrt{-c+dx}\,\sqrt{c+dx}} + \frac{\left(-2b-\frac{3ad^2}{c^2}\right)\int \frac{c\ldots}{x\sqrt{-c+\ldots}}}{2c^2d}$$

$$= -\frac{2bc^2+3ad^2}{2c^4\sqrt{-c+dx}\,\sqrt{c+dx}} + \frac{a}{2c^2x^2\sqrt{-c+dx}\,\sqrt{c+dx}} - \frac{\left(2bc^2+3ad^2\right)\int \frac{\ldots}{x\sqrt{-c\ldots}}}{2c^4}$$

$$= -\frac{2bc^2+3ad^2}{2c^4\sqrt{-c+dx}\,\sqrt{c+dx}} + \frac{a}{2c^2x^2\sqrt{-c+dx}\,\sqrt{c+dx}} - \frac{\left(d\left(2bc^2+3ad^2\right)\right)\text{Sub}\ldots}{\ldots}$$

$$= -\frac{2bc^2+3ad^2}{2c^4\sqrt{-c+dx}\,\sqrt{c+dx}} + \frac{a}{2c^2x^2\sqrt{-c+dx}\,\sqrt{c+dx}} - \frac{\left(2bc^2+3ad^2\right)\tan^{-1}\left(\ldots\right.}{2c^5}$$

Mathematica [C] time = 0.03, size = 75, normalized size = 0.64

$$\frac{ac^2-x^2\left(3ad^2+2bc^2\right)\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};1-\frac{d^2x^2}{c^2}\right)}{2c^4x^2\sqrt{dx-c}\,\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)/(x^3*(-c + d*x)^(3/2)*(c + d*x)^(3/2)),x]
```

```
[Out] (a*c^2 - (2*b*c^2 + 3*a*d^2)*x^2*Hypergeometric2F1[-1/2, 1, 1/2, 1 - (d^2*x
^2)/c^2])/(2*c^4*x^2*Sqrt[-c + d*x]*Sqrt[c + d*x])
```

IntegrateAlgebraic [A] time = 0.18, size = 196, normalized size = 1.68

$$\frac{\sqrt{c+dx}\left(\frac{dx-c}{c+dx}-1\right)\left(\frac{4ad^2(dx-c)}{c+dx}+\frac{ad^2(dx-c)^2}{(c+dx)^2}+ad^2+\frac{2bc^2(dx-c)}{c+dx}+\frac{bc^2(dx-c)^2}{(c+dx)^2}+bc^2\right)}{2c^5\sqrt{dx-c}\left(\frac{dx-c}{c+dx}+1\right)^2}+\frac{\left(-3ad^2-2bc^2\right)\tan^{-1}\left(\frac{\sqrt{dx-c}}{\sqrt{c+dx}}\right)}{c^5}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x^2)/(x^3*(-c + d*x)^(3/2)*(c + d*x)^(3/2)),x]
```

```
[Out] (Sqrt[c + d*x]*(-1 + (-c + d*x)/(c + d*x))*(b*c^2 + a*d^2 + (b*c^2*(-c + d*
x)^2)/(c + d*x)^2 + (a*d^2*(-c + d*x)^2)/(c + d*x)^2 + (2*b*c^2*(-c + d*x))
/(c + d*x) + (4*a*d^2*(-c + d*x))/(c + d*x)))/(2*c^5*Sqrt[-c + d*x]*(1 + (-
c + d*x)/(c + d*x))^2) + ((-2*b*c^2 - 3*a*d^2)*ArcTan[Sqrt[-c + d*x]/Sqrt[c
 + d*x]])/c^5
```

fricas [A] time = 0.72, size = 138, normalized size = 1.18

$$\frac{\left(ac^3-\left(2\,bc^3+3\,acd^2\right)x^2\right)\sqrt{dx+c}\,\sqrt{dx-c}-2\left(\left(2\,bc^2d^2+3\,ad^4\right)x^4-\left(2\,bc^4+3\,ac^2d^2\right)x^2\right)\arctan\left(-\frac{dx-\sqrt{dx+c}\,\sqrt{dx-c}}{c}\right)}{2\left(c^5d^2x^4-c^7x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/x^3/(d*x-c)^(3/2)/(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/2*((a*c^3 - (2*b*c^3 + 3*a*c*d^2)*x^2)*sqrt(d*x + c)*sqrt(d*x - c) - 2*((
2*b*c^2*d^2 + 3*a*d^4)*x^4 - (2*b*c^4 + 3*a*c^2*d^2)*x^2)*arctan(-(d*x - sq
rt(d*x + c)*sqrt(d*x - c))/c))/(c^5*d^2*x^4 - c^7*x^2)
```

giac [B] time = 0.54, size = 211, normalized size = 1.80

$$\frac{\left(2\,bc^{2}+3\,ad^{2}\right)\arctan\left(\frac{\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^{2}}{2\,c}\right)}{c^{5}}-\frac{\left(bc^{2}+ad^{2}\right)\sqrt{dx+c}}{2\,\sqrt{dx-c}\,c^{5}}+\frac{2\left(bc^{2}+ad^{2}\right)}{\left(\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^{2}+2\,c\right)c^{4}}+\frac{2\left(ad^{2}\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^{6}-4\,ac^{2}d^{2}\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^{2}\right)}{\left(\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^{4}+4\,c^{2}\right)^{2}c^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/x^3/(d*x-c)^(3/2)/(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] (2*b*c^2 + 3*a*d^2)*arctan(1/2*(sqrt(d*x + c) - sqrt(d*x - c))^2/c)/c^5 - 1
/2*(b*c^2 + a*d^2)*sqrt(d*x + c)/(sqrt(d*x - c)*c^5) + 2*(b*c^2 + a*d^2)/((
(sqrt(d*x + c) - sqrt(d*x - c))^2 + 2*c)*c^4) + 2*(a*d^2*(sqrt(d*x + c) - s
qrt(d*x - c))^6 - 4*a*c^2*d^2*(sqrt(d*x + c) - sqrt(d*x - c))^2)/(((sqrt(d*
x + c) - sqrt(d*x - c))^4 + 4*c^2)^2*c^4)
```

maple [B] time = 0.09, size = 315, normalized size = 2.69

$$\frac{3a\,d^{4}x^{4}\ln\left(-\frac{2\left(c^{2}-\sqrt{-c^{2}}\,\sqrt{d^{2}x^{2}-c^{2}}\right)}{x}\right)+2b\,c^{2}d^{2}x^{4}\ln\left(-\frac{2\left(c^{2}-\sqrt{-c^{2}}\,\sqrt{d^{2}x^{2}-c^{2}}\right)}{x}\right)-3a\,c^{2}d^{2}x^{2}\ln\left(-\frac{2\left(c^{2}-\sqrt{-c^{2}}\,\sqrt{d^{2}x^{2}-c^{2}}\right)}{x}\right)-2b\,c^{4}x^{2}\ln\left(-\frac{2\left(c^{2}-\sqrt{-c^{2}}\,\sqrt{d^{2}x^{2}-c^{2}}\right)}{x}\right)-3\sqrt{-c^{2}}\,\sqrt{d^{2}x^{2}-c^{2}}\,a\,d^{2}x^{2}-2\sqrt{-c^{2}}\,\sqrt{d^{2}x^{2}-c^{2}}\,b\,c^{2}x^{2}+\sqrt{-c^{2}}\,\sqrt{d^{2}x^{2}-c^{2}}\,a\,c^{2}}{2\sqrt{-c^{2}}\,\sqrt{d^{2}x^{2}-c^{2}}\,\sqrt{dx+c}\,\sqrt{dx-c}\,c^{4}x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)/x^3/(d*x-c)^(3/2)/(d*x+c)^(3/2),x)
```

```
[Out] 1/2/c^4*(3*a*d^4*x^4*ln(-2*(c^2-(-c^2)^(1/2)*(d^2*x^2-c^2)^(1/2))/x)+2*b*c^
2*d^2*x^4*ln(-2*(c^2-(-c^2)^(1/2)*(d^2*x^2-c^2)^(1/2))/x)-3*ln(-2*(c^2-(-c^
2)^(1/2)*(d^2*x^2-c^2)^(1/2))/x)*x^2*a*c^2*d^2-2*ln(-2*(c^2-(-c^2)^(1/2)*(d
^2*x^2-c^2)^(1/2))/x)*x^2*b*c^4-3*(-c^2)^(1/2)*(d^2*x^2-c^2)^(1/2)*a*d^2*x^
2-2*(-c^2)^(1/2)*(d^2*x^2-c^2)^(1/2)*b*c^2*x^2+(-c^2)^(1/2)*(d^2*x^2-c^2)^(
1/2)*a*c^2)/(-c^2)^(1/2)/x^2/(d^2*x^2-c^2)^(1/2)/(d*x+c)^(1/2)/(d*x-c)^(1/2
)
```

maxima [A] time = 1.26, size = 104, normalized size = 0.89

$$\frac{b\arcsin\left(\frac{c}{d|x|}\right)}{c^{3}}+\frac{3\,ad^{2}\arcsin\left(\frac{c}{d|x|}\right)}{2\,c^{5}}-\frac{b}{\sqrt{d^{2}x^{2}-c^{2}}\,c^{2}}-\frac{3\,ad^{2}}{2\,\sqrt{d^{2}x^{2}-c^{2}}\,c^{4}}+\frac{a}{2\,\sqrt{d^{2}x^{2}-c^{2}}\,c^{2}x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/x^3/(d*x-c)^(3/2)/(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] b*arcsin(c/(d*abs(x)))/c^3 + 3/2*a*d^2*arcsin(c/(d*abs(x)))/c^5 - b/(sqrt(d
^2*x^2 - c^2)*c^2) - 3/2*a*d^2/(sqrt(d^2*x^2 - c^2)*c^4) + 1/2*a/(sqrt(d^2*
x^2 - c^2)*c^2*x^2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{b\,x^{2}+a}{x^{3}\,{\left(c+d\,x\right)}^{3/2}\,{\left(d\,x-c\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)/(x^3*(c + d*x)^(3/2)*(d*x - c)^(3/2)),x)
```

```
[Out] int((a + b*x^2)/(x^3*(c + d*x)^(3/2)*(d*x - c)^(3/2)), x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)/x**3/(d*x-c)**(3/2)/(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

3.233 $\int \frac{a+bx^2}{x^4(-c+dx)^{3/2}(c+dx)^{3/2}}\,dx$

Optimal. Leaf size=119

$$-\frac{2d^2x\left(4ad^2+3bc^2\right)}{3c^6\sqrt{dx-c}\,\sqrt{c+dx}}+\frac{4ad^2+3bc^2}{3c^4x\sqrt{dx-c}\,\sqrt{c+dx}}+\frac{a}{3c^2x^3\sqrt{dx-c}\,\sqrt{c+dx}}$$

Rubi [A] time = 0.10, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {454, 103, 12, 39}

$$-\frac{2d^2x\left(4ad^2+3bc^2\right)}{3c^6\sqrt{dx-c}\,\sqrt{c+dx}}+\frac{4ad^2+3bc^2}{3c^4x\sqrt{dx-c}\,\sqrt{c+dx}}+\frac{a}{3c^2x^3\sqrt{dx-c}\,\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)/(x^4*(-c + d*x)^(3/2)*(c + d*x)^(3/2)),x]
```

```
[Out] a/(3*c^2*x^3*Sqrt[-c + d*x]*Sqrt[c + d*x]) + (3*b*c^2 + 4*a*d^2)/(3*c^4*x*S
qrt[-c + d*x]*Sqrt[c + d*x]) - (2*d^2*(3*b*c^2 + 4*a*d^2)*x)/(3*c^6*Sqrt[-c
 + d*x]*Sqrt[c + d*x])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 39

```
Int[1/(((a_) + (b_.)*(x_))^(3/2)*((c_) + (d_.)*(x_))^(3/2)), x_Symbol] :> S
imp[x/(a*c*Sqrt[a + b*x]*Sqrt[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[b*c + a*d, 0]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 454

```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[(c*(e*x)^
(m + 1)*(a1 + b1*x^(n/2))^(p + 1)*(a2 + b2*x^(n/2))^(p + 1))/(a1*a2*e*(m +
1)), x] + Dist[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/(a1*a2*e^n*(
m + 1)), Int[(e*x)^(m + n)*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n/2))^p, x], x]
/; FreeQ[{a1, b1, a2, b2, c, d, e, p}, x] && EqQ[non2, n/2] && EqQ[a2*b1 +
a1*b2, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (L
tQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{a+bx^2}{x^4(-c+dx)^{3/2}(c+dx)^{3/2}}\,dx &= \frac{a}{3c^2x^3\sqrt{-c+dx}\,\sqrt{c+dx}} + \frac{1}{3}\left(3b+\frac{4ad^2}{c^2}\right)\int \frac{1}{x^2(-c+dx)^{3/2}(c+dx)^{3/2}}\,dx \\
&= \frac{a}{3c^2x^3\sqrt{-c+dx}\,\sqrt{c+dx}} + \frac{3bc^2+4ad^2}{3c^4x\sqrt{-c+dx}\,\sqrt{c+dx}} + \frac{\left(3b+\frac{4ad^2}{c^2}\right)\int \frac{2\cdots}{(-c+dx)^{3/2}\cdots}}{3c^2} \\
&= \frac{a}{3c^2x^3\sqrt{-c+dx}\,\sqrt{c+dx}} + \frac{3bc^2+4ad^2}{3c^4x\sqrt{-c+dx}\,\sqrt{c+dx}} + \frac{\left(2d^2\left(3b+\frac{4ad^2}{c^2}\right)\right)\int \frac{\cdots}{(-\cdots}}{3c^2} \\
&= \frac{a}{3c^2x^3\sqrt{-c+dx}\,\sqrt{c+dx}} + \frac{3bc^2+4ad^2}{3c^4x\sqrt{-c+dx}\,\sqrt{c+dx}} - \frac{2d^2\left(3bc^2+4ad^2\right)x}{3c^6\sqrt{-c+dx}\,\sqrt{c+dx}}
\end{aligned}
$$

Mathematica [A] time = 0.03, size = 77, normalized size = 0.65

$$
\frac{a\left(c^4+4c^2d^2x^2-8d^4x^4\right)+3bc^2x^2\left(c^2-2d^2x^2\right)}{3c^6x^3\sqrt{dx-c}\,\sqrt{c+dx}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)/(x^4*(-c + d*x)^(3/2)*(c + d*x)^(3/2)),x]
```

```
[Out] (3*b*c^2*x^2*(c^2 - 2*d^2*x^2) + a*(c^4 + 4*c^2*d^2*x^2 - 8*d^4*x^4))/(3*c^
6*x^3*Sqrt[-c + d*x]*Sqrt[c + d*x])
```

IntegrateAlgebraic [A] time = 0.18, size = 236, normalized size = 1.98

$$
\frac{\sqrt{c+dx}\left(-\frac{3ad^3(dx-c)^4}{(c+dx)^4}-\frac{36ad^3(dx-c)^3}{(c+dx)^3}-\frac{50ad^3(dx-c)^2}{(c+dx)^2}-\frac{36ad^3(dx-c)}{c+dx}-3ad^3-\frac{3bc^2d(dx-c)^4}{(c+dx)^4}-\frac{24bc^2d(dx-c)^3}{(c+dx)^3}-\frac{42bc^2d(dx-c)^2}{(c+dx)^2}-\frac{24bc^2d(dx-c)}{c+dx}-3bc^2d\right)}{6c^6\sqrt{dx-c}\left(\frac{dx-c}{c+dx}+1\right)^3}
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x^2)/(x^4*(-c + d*x)^(3/2)*(c + d*x)^(3/2)),x]
```

```
[Out] (Sqrt[c + d*x]*(-3*b*c^2*d - 3*a*d^3 - (3*b*c^2*d*(-c + d*x)^4)/(c + d*x)^4
 - (3*a*d^3*(-c + d*x)^4)/(c + d*x)^4 - (24*b*c^2*d*(-c + d*x)^3)/(c + d*x)
^3 - (36*a*d^3*(-c + d*x)^3)/(c + d*x)^3 - (42*b*c^2*d*(-c + d*x)^2)/(c + d
*x)^2 - (50*a*d^3*(-c + d*x)^2)/(c + d*x)^2 - (24*b*c^2*d*(-c + d*x))/(c +
d*x) - (36*a*d^3*(-c + d*x))/(c + d*x)))/(6*c^6*Sqrt[-c + d*x]*(1 + (-c + d
*x)/(c + d*x))^3)
```

fricas [A] time = 0.80, size = 132, normalized size = 1.11

$$
-\frac{2\left(3\,bc^2d^3+4\,ad^5\right)x^5-2\left(3\,bc^4d+4\,ac^2d^3\right)x^3-\left(ac^4-2\left(3\,bc^2d^2+4\,ad^4\right)x^4+\left(3\,bc^4+4\,ac^2d^2\right)x^2\right)\sqrt{dx+c}\,\sqrt{dx-c}}{3\left(c^6d^2x^5-c^8x^3\right)}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/x^4/(d*x-c)^(3/2)/(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/3*(2*(3*b*c^2*d^3 + 4*a*d^5)*x^5 - 2*(3*b*c^4*d + 4*a*c^2*d^3)*x^3 - (a*
c^4 - 2*(3*b*c^2*d^2 + 4*a*d^4)*x^4 + (3*b*c^4 + 4*a*c^2*d^2)*x^2)*sqrt(d*x
 + c)*sqrt(d*x - c))/(c^6*d^2*x^5 - c^8*x^3)
```

giac [B] time = 0.73, size = 242, normalized size = 2.03

$$
-\frac{\left(bc^2d+ad^3\right)\sqrt{dx+c}}{2\sqrt{dx-c}\,c^6}-\frac{2\left(bc^2d+ad^3\right)}{\left(\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^2+2\,c\right)c^5}-\frac{8\left(3\,bc^2d\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^8+3\,ad^3\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^8+24\,bc^4d\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^4+48\,ac^2d^3\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^4+48\,bc^6d+80\,ac^4d^3\right)}{3\left(\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^4+4\,c^2\right)^3c^4}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/x^4/(d*x-c)^(3/2)/(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] -1/2*(b*c^2*d + a*d^3)*sqrt(d*x + c)/(sqrt(d*x - c)*c^6) - 2*(b*c^2*d + a*d
^3)/(((sqrt(d*x + c) - sqrt(d*x - c))^2 + 2*c)*c^5) - 8/3*(3*b*c^2*d*(sqrt(
d*x + c) - sqrt(d*x - c))^8 + 3*a*d^3*(sqrt(d*x + c) - sqrt(d*x - c))^8 + 2
4*b*c^4*d*(sqrt(d*x + c) - sqrt(d*x - c))^4 + 48*a*c^2*d^3*(sqrt(d*x + c) -
 sqrt(d*x - c))^4 + 48*b*c^6*d + 80*a*c^4*d^3)/(((sqrt(d*x + c) - sqrt(d*x
- c))^4 + 4*c^2)^3*c^4)
```

maple [A] time = 0.05, size = 73, normalized size = 0.61

$$\frac{-8a\,d^4x^4-6b\,c^2d^2x^4+4a\,c^2d^2x^2+3b\,c^4x^2+a\,c^4}{3\sqrt{dx+c}\,\sqrt{dx-c}\,c^6x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)/x^4/(d*x-c)^(3/2)/(d*x+c)^(3/2),x)
```

```
[Out] 1/3*(-8*a*d^4*x^4-6*b*c^2*d^2*x^4+4*a*c^2*d^2*x^2+3*b*c^4*x^2+a*c^4)/(d*x+c
)^(1/2)/x^3/c^6/(d*x-c)^(1/2)
```

maxima [A] time = 1.34, size = 125, normalized size = 1.05

$$-\frac{2\,bd^2x}{\sqrt{d^2x^2-c^2}\,c^4}-\frac{8\,ad^4x}{3\,\sqrt{d^2x^2-c^2}\,c^6}+\frac{b}{\sqrt{d^2x^2-c^2}\,c^2x}+\frac{4\,ad^2}{3\,\sqrt{d^2x^2-c^2}\,c^4x}+\frac{a}{3\,\sqrt{d^2x^2-c^2}\,c^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/x^4/(d*x-c)^(3/2)/(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] -2*b*d^2*x/(sqrt(d^2*x^2 - c^2)*c^4) - 8/3*a*d^4*x/(sqrt(d^2*x^2 - c^2)*c^6
) + b/(sqrt(d^2*x^2 - c^2)*c^2*x) + 4/3*a*d^2/(sqrt(d^2*x^2 - c^2)*c^4*x) +
 1/3*a/(sqrt(d^2*x^2 - c^2)*c^2*x^3)
```

mupad [B] time = 2.90, size = 104, normalized size = 0.87

$$\frac{\sqrt{d\,x-c}\,\left(\frac{a}{3\,c^2\,d}+\frac{x^2\,\left(3\,b\,c^4+4\,a\,c^2\,d^2\right)}{3\,c^6\,d}-\frac{x^4\,\left(6\,b\,c^2\,d^2+8\,a\,d^4\right)}{3\,c^6\,d}\right)}{x^4\,\sqrt{c+d\,x}-\frac{c\,x^3\,\sqrt{c+d\,x}}{d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)/(x^4*(c + d*x)^(3/2)*(d*x - c)^(3/2)),x)
```

```
[Out] ((d*x - c)^(1/2)*(a/(3*c^2*d) + (x^2*(3*b*c^4 + 4*a*c^2*d^2))/(3*c^6*d) - (
x^4*(8*a*d^4 + 6*b*c^2*d^2))/(3*c^6*d)))/(x^4*(c + d*x)^(1/2) - (c*x^3*(c +
 d*x)^(1/2))/d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)/x**4/(d*x-c)**(3/2)/(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

3.234 $\int \frac{a+bx^2}{x^5(-c+dx)^{3/2}(c+dx)^{3/2}}\,dx$

Optimal. Leaf size=166

$$-\frac{3d^2\left(5ad^2+4bc^2\right)\tan^{-1}\left(\frac{\sqrt{dx-c}\,\sqrt{c+dx}}{c}\right)}{8c^7}-\frac{3d^2\left(5ad^2+4bc^2\right)}{8c^6\sqrt{dx-c}\,\sqrt{c+dx}}+\frac{5ad^2+4bc^2}{8c^4x^2\sqrt{dx-c}\,\sqrt{c+dx}}+\frac{a}{4c^2x^4\sqrt{dx-c}\,\sqrt{c+dx}}$$

Rubi [A] time = 0.12, antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {454, 103, 12, 104, 21, 92, 205}

$$\frac{5ad^2+4bc^2}{8c^4x^2\sqrt{dx-c}\,\sqrt{c+dx}}-\frac{3d^2\left(5ad^2+4bc^2\right)}{8c^6\sqrt{dx-c}\,\sqrt{c+dx}}-\frac{3d^2\left(5ad^2+4bc^2\right)\tan^{-1}\left(\frac{\sqrt{dx-c}\,\sqrt{c+dx}}{c}\right)}{8c^7}+\frac{a}{4c^2x^4\sqrt{dx-c}\,\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)/(x^5*(-c + d*x)^(3/2)*(c + d*x)^(3/2)),x]
```

```
[Out] (-3*d^2*(4*b*c^2 + 5*a*d^2))/(8*c^6*Sqrt[-c + d*x]*Sqrt[c + d*x]) + a/(4*c^
2*x^4*Sqrt[-c + d*x]*Sqrt[c + d*x]) + (4*b*c^2 + 5*a*d^2)/(8*c^4*x^2*Sqrt[-
c + d*x]*Sqrt[c + d*x]) - (3*d^2*(4*b*c^2 + 5*a*d^2)*ArcTan[(Sqrt[-c + d*x]
*Sqrt[c + d*x])/c])/(8*c^7)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 104

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
```

```
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegersQ
[2*m, 2*n, 2*p]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 454

```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[(c*(e*x)^
(m + 1)*(a1 + b1*x^(n/2))^(p + 1)*(a2 + b2*x^(n/2))^(p + 1))/(a1*a2*e*(m +
1)), x] + Dist[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/(a1*a2*e^n*(
m + 1)), Int[(e*x)^(m + n)*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n/2))^p, x], x]
/; FreeQ[{a1, b1, a2, b2, c, d, e, p}, x] && EqQ[non2, n/2] && EqQ[a2*b1 +
a1*b2, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (L
tQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{a+bx^2}{x^5(-c+dx)^{3/2}(c+dx)^{3/2}}\,dx &= \frac{a}{4c^2x^4\sqrt{-c+dx}\,\sqrt{c+dx}}+\frac{1}{4}\left(4b+\frac{5ad^2}{c^2}\right)\int \frac{1}{x^3(-c+dx)^{3/2}(c+dx)^{3/2}}\,dx \\
&= \frac{a}{4c^2x^4\sqrt{-c+dx}\,\sqrt{c+dx}}+\frac{4bc^2+5ad^2}{8c^4x^2\sqrt{-c+dx}\,\sqrt{c+dx}}+\frac{\left(4bc^2+5ad^2\right)\int \frac{3}{x(-c+dx)}}{8c^4} \\
&= \frac{a}{4c^2x^4\sqrt{-c+dx}\,\sqrt{c+dx}}+\frac{4bc^2+5ad^2}{8c^4x^2\sqrt{-c+dx}\,\sqrt{c+dx}}+\frac{\left(3d^2\left(4bc^2+5ad^2\right)\right)\int \frac{}{x}}{8c^4} \\
&= -\frac{3d^2\left(4bc^2+5ad^2\right)}{8c^6\sqrt{-c+dx}\,\sqrt{c+dx}}+\frac{a}{4c^2x^4\sqrt{-c+dx}\,\sqrt{c+dx}}+\frac{4bc^2+5ad^2}{8c^4x^2\sqrt{-c+dx}\,\sqrt{c+dx}} \\
&= -\frac{3d^2\left(4bc^2+5ad^2\right)}{8c^6\sqrt{-c+dx}\,\sqrt{c+dx}}+\frac{a}{4c^2x^4\sqrt{-c+dx}\,\sqrt{c+dx}}+\frac{4bc^2+5ad^2}{8c^4x^2\sqrt{-c+dx}\,\sqrt{c+dx}} \\
&= -\frac{3d^2\left(4bc^2+5ad^2\right)}{8c^6\sqrt{-c+dx}\,\sqrt{c+dx}}+\frac{a}{4c^2x^4\sqrt{-c+dx}\,\sqrt{c+dx}}+\frac{4bc^2+5ad^2}{8c^4x^2\sqrt{-c+dx}\,\sqrt{c+dx}} \\
&= -\frac{3d^2\left(4bc^2+5ad^2\right)}{8c^6\sqrt{-c+dx}\,\sqrt{c+dx}}+\frac{a}{4c^2x^4\sqrt{-c+dx}\,\sqrt{c+dx}}+\frac{4bc^2+5ad^2}{8c^4x^2\sqrt{-c+dx}\,\sqrt{c+dx}}
\end{aligned}$$

Mathematica [C] time = 0.03, size = 78, normalized size = 0.47

$$\frac{ac^4-d^2x^4\left(5ad^2+4bc^2\right)\,{}_2F_1\left(-\frac{1}{2},2;\frac{1}{2};1-\frac{d^2x^2}{c^2}\right)}{4c^6x^4\sqrt{dx-c}\,\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)/(x^5*(-c + d*x)^(3/2)*(c + d*x)^(3/2)),x]
```

```
[Out] (a*c^4 - d^2*(4*b*c^2 + 5*a*d^2)*x^4*Hypergeometric2F1[-1/2, 2, 1/2, 1 - (d
^2*x^2)/c^2])/(4*c^6*x^4*Sqrt[-c + d*x]*Sqrt[c + d*x])
```

IntegrateAlgebraic [A] time = 0.25, size = 297, normalized size = 1.79

$$\frac{d^2\sqrt{c+dx}\left(\frac{dx-c}{c+dx}-1\right)\left(\frac{2ad^2(dx-c)^4}{(c+dx)^4}+\frac{17ad^2(dx-c)^3}{(c+dx)^3}+\frac{22ad^2(dx-c)^2}{(c+dx)^2}+\frac{17ad^2(dx-c)}{c+dx}+2ad^2+\frac{2bc^2(dx-c)^4}{(c+dx)^4}+\frac{12bc^2(dx-c)^3}{(c+dx)^3}+\frac{20bc^2(dx-c)^2}{(c+dx)^2}+\frac{12bc^2(dx-c)}{c+dx}+2bc^2\right)}{4c^7\sqrt{dx-c}\left(\frac{dx-c}{c+dx}+1\right)^4}-\frac{3\left(5ad^4+4bc^2d^2\right)\tan^{-1}\left(\frac{\sqrt{dx-c}}{\sqrt{c+dx}}\right)}{4c^7}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x^2)/(x^5*(-c + d*x)^(3/2)*(c + d*x)^(3/2)),x]
```

```
[Out] (d^2*Sqrt[c + d*x]*(-1 + (-c + d*x)/(c + d*x))*(2*b*c^2 + 2*a*d^2 + (2*b*c^
2*(-c + d*x)^4)/(c + d*x)^4 + (2*a*d^2*(-c + d*x)^4)/(c + d*x)^4 + (12*b*c^
2*(-c + d*x)^3)/(c + d*x)^3 + (17*a*d^2*(-c + d*x)^3)/(c + d*x)^3 + (20*b*c
^2*(-c + d*x)^2)/(c + d*x)^2 + (22*a*d^2*(-c + d*x)^2)/(c + d*x)^2 + (12*b*
c^2*(-c + d*x))/(c + d*x) + (17*a*d^2*(-c + d*x))/(c + d*x)))/(4*c^7*Sqrt[-
c + d*x]*(1 + (-c + d*x)/(c + d*x))^4) - (3*(4*b*c^2*d^2 + 5*a*d^4)*ArcTan[
Sqrt[-c + d*x]/Sqrt[c + d*x]])/(4*c^7)
```

fricas [A] time = 0.88, size = 165, normalized size = 0.99

$$\frac{\left(2\,ac^5-3\left(4\,bc^3d^2+5\,acd^4\right)x^4+\left(4\,bc^5+5\,ac^3d^2\right)x^2\right)\sqrt{dx+c}\,\sqrt{dx-c}-6\left(\left(4\,bc^2d^4+5\,ad^6\right)x^6-\left(4\,bc^4d^2+5\,ac^2d^4\right)x^4\right)\arctan\left(-\frac{dx-\sqrt{dx+c}\,\sqrt{dx-c}}{c}\right)}{8\left(c^7d^2x^6-c^9x^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/x^5/(d*x-c)^(3/2)/(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/8*((2*a*c^5 - 3*(4*b*c^3*d^2 + 5*a*c*d^4)*x^4 + (4*b*c^5 + 5*a*c^3*d^2)*x
^2)*sqrt(d*x + c)*sqrt(d*x - c) - 6*((4*b*c^2*d^4 + 5*a*d^6)*x^6 - (4*b*c^4
*d^2 + 5*a*c^2*d^4)*x^4)*arctan(-(d*x - sqrt(d*x + c)*sqrt(d*x - c))/c))/(c
^7*d^2*x^6 - c^9*x^4)
```

giac [B] time = 0.79, size = 402, normalized size = 2.42

$$\frac{3\left(4\,bc^2d^2+5\,ad^4\right)\arctan\left(\frac{\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^2}{2\,c}\right)}{4\,c^7}-\frac{\left(bc^2d^2+ad^4\right)\sqrt{dx+c}}{2\,\sqrt{dx-c}\,c^7}+\frac{2\left(bc^2d^2+ad^4\right)}{\left(\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^2+2\,c\right)c^6}+\frac{4\,bc^2d^2\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^{14}+7\,ad^4\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^{14}+16\,bc^4d^2\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^{10}+60\,ac^2d^4\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^{10}-64\,bc^6d^2\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^6-240\,ac^4d^4\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^6-256\,bc^8d^2\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^2-448\,ac^6d^4\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^2}{2\left(\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^4+4\,c^2\right)^4c^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/x^5/(d*x-c)^(3/2)/(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] 3/4*(4*b*c^2*d^2 + 5*a*d^4)*arctan(1/2*(sqrt(d*x + c) - sqrt(d*x - c))^2/c)
/c^7 - 1/2*(b*c^2*d^2 + a*d^4)*sqrt(d*x + c)/(sqrt(d*x - c)*c^7) + 2*(b*c^2
*d^2 + a*d^4)/(((sqrt(d*x + c) - sqrt(d*x - c))^2 + 2*c)*c^6) + 1/2*(4*b*c^
2*d^2*(sqrt(d*x + c) - sqrt(d*x - c))^14 + 7*a*d^4*(sqrt(d*x + c) - sqrt(d*
x - c))^14 + 16*b*c^4*d^2*(sqrt(d*x + c) - sqrt(d*x - c))^10 + 60*a*c^2*d^4
*(sqrt(d*x + c) - sqrt(d*x - c))^10 - 64*b*c^6*d^2*(sqrt(d*x + c) - sqrt(d*
x - c))^6 - 240*a*c^4*d^4*(sqrt(d*x + c) - sqrt(d*x - c))^6 - 256*b*c^8*d^2
*(sqrt(d*x + c) - sqrt(d*x - c))^2 - 448*a*c^6*d^4*(sqrt(d*x + c) - sqrt(d*
x - c))^2)/(((sqrt(d*x + c) - sqrt(d*x - c))^4 + 4*c^2)^4*c^6)
```

maple [B] time = 0.08, size = 387, normalized size = 2.33

$$\frac{15a\,d^6x^6\ln\left(-\frac{2\left(c^2-\sqrt{-c^2}\,\sqrt{d^2x^2-c^2}\right)}{x}\right)+12b\,c^2d^4x^6\ln\left(-\frac{2\left(c^2-\sqrt{-c^2}\,\sqrt{d^2x^2-c^2}\right)}{x}\right)-15a\,c^2d^4x^4\ln\left(-\frac{2\left(c^2-\sqrt{-c^2}\,\sqrt{d^2x^2-c^2}\right)}{x}\right)-12b\,c^4d^2x^4\ln\left(-\frac{2\left(c^2-\sqrt{-c^2}\,\sqrt{d^2x^2-c^2}\right)}{x}\right)-15\sqrt{-c^2}\,\sqrt{d^2x^2-c^2}\,a\,d^4x^4-12\sqrt{-c^2}\,\sqrt{d^2x^2-c^2}\,b\,c^2d^2x^4+5\sqrt{d^2x^2-c^2}\,\sqrt{-c^2}\,a\,c^2d^2x^2+4\sqrt{d^2x^2-c^2}\,\sqrt{-c^2}\,b\,c^4x^2+2\sqrt{d^2x^2-c^2}\,\sqrt{-c^2}\,a\,c^4}{8\sqrt{-c^2}\,\sqrt{d^2x^2-c^2}\,\sqrt{dx+c}\,\sqrt{dx-c}\,c^6x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)/x^5/(d*x-c)^(3/2)/(d*x+c)^(3/2),x)
```

```
[Out] 1/8/c^6*(15*ln(-2*(c^2-(-c^2)^(1/2)*(d^2*x^2-c^2)^(1/2))/x)*x^6*a*d^6+12*ln
(-2*(c^2-(-c^2)^(1/2)*(d^2*x^2-c^2)^(1/2))/x)*x^6*b*c^2*d^4-15*ln(-2*(c^2-(
-c^2)^(1/2)*(d^2*x^2-c^2)^(1/2))/x)*x^4*a*c^2*d^4-12*ln(-2*(c^2-(-c^2)^(1/2
)*(d^2*x^2-c^2)^(1/2))/x)*x^4*b*c^4*d^2-15*(-c^2)^(1/2)*(d^2*x^2-c^2)^(1/2)
*x^4*a*d^4-12*(-c^2)^(1/2)*(d^2*x^2-c^2)^(1/2)*x^4*b*c^2*d^2+5*x^2*a*c^2*d^
```

```
2*(d^2*x^2-c^2)^(1/2)*(-c^2)^(1/2)+4*x^2*b*c^4*(d^2*x^2-c^2)^(1/2)*(-c^2)^(
1/2)+2*a*c^4*(d^2*x^2-c^2)^(1/2)*(-c^2)^(1/2))/(-c^2)^(1/2)/x^4/(d^2*x^2-c^
2)^(1/2)/(d*x+c)^(1/2)/(d*x-c)^(1/2)
```

maxima [A] time = 1.48, size = 162, normalized size = 0.98

$$\frac{3\,bd^2\arcsin\left(\frac{c}{d|x|}\right)}{2\,c^5}+\frac{15\,ad^4\arcsin\left(\frac{c}{d|x|}\right)}{8\,c^7}-\frac{3\,bd^2}{2\,\sqrt{d^2x^2-c^2}\,c^4}-\frac{15\,ad^4}{8\,\sqrt{d^2x^2-c^2}\,c^6}+\frac{b}{2\,\sqrt{d^2x^2-c^2}\,c^2x^2}+\frac{5\,ad^2}{8\,\sqrt{d^2x^2-c^2}\,c^4x^2}+\frac{a}{4\,\sqrt{d^2x^2-c^2}\,c^2x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/x^5/(d*x-c)^(3/2)/(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] 3/2*b*d^2*arcsin(c/(d*abs(x)))/c^5 + 15/8*a*d^4*arcsin(c/(d*abs(x)))/c^7 -
3/2*b*d^2/(sqrt(d^2*x^2 - c^2)*c^4) - 15/8*a*d^4/(sqrt(d^2*x^2 - c^2)*c^6)
+ 1/2*b/(sqrt(d^2*x^2 - c^2)*c^2*x^2) + 5/8*a*d^2/(sqrt(d^2*x^2 - c^2)*c^4*
x^2) + 1/4*a/(sqrt(d^2*x^2 - c^2)*c^2*x^4)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{b\,x^2+a}{x^5\,{\left(c+d\,x\right)}^{3/2}\,{\left(d\,x-c\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)/(x^5*(c + d*x)^(3/2)*(d*x - c)^(3/2)),x)
```

```
[Out] int((a + b*x^2)/(x^5*(c + d*x)^(3/2)*(d*x - c)^(3/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)/x**5/(d*x-c)**(3/2)/(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

3.235 $\int \frac{1+c^2x^2}{x\sqrt{-1+cx}\sqrt{1+cx}}\,dx$

Optimal. Leaf size=40

$$\sqrt{cx-1}\sqrt{cx+1}+\tan^{-1}\left(\sqrt{cx-1}\sqrt{cx+1}\right)$$

Rubi [A] time = 0.05, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {460, 92, 205}

$$\sqrt{cx-1}\sqrt{cx+1}+\tan^{-1}\left(\sqrt{cx-1}\sqrt{cx+1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + c^2*x^2)/(x*Sqrt[-1 + c*x]*Sqrt[1 + c*x]),x]
```

```
[Out] Sqrt[-1 + c*x]*Sqrt[1 + c*x] + ArcTan[Sqrt[-1 + c*x]*Sqrt[1 + c*x]]
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 460

```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[(d*(e*x)^
(m + 1)*(a1 + b1*x^(n/2))^(p + 1)*(a2 + b2*x^(n/2))^(p + 1))/(b1*b2*e*(m +
n*(p + 1) + 1)), x] - Dist[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/
(b1*b2*(m + n*(p + 1) + 1)), Int[(e*x)^m*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n/
2))^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, m, n, p}, x] && EqQ[non2,
n/2] && EqQ[a2*b1 + a1*b2, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1+c^2x^2}{x\sqrt{-1+cx}\sqrt{1+cx}}\,dx &= \sqrt{-1+cx}\sqrt{1+cx}+\int \frac{1}{x\sqrt{-1+cx}\sqrt{1+cx}}\,dx\\
&= \sqrt{-1+cx}\sqrt{1+cx}+c\,\text{Subst}\left(\int \frac{1}{c+cx^2}\,dx,x,\sqrt{-1+cx}\sqrt{1+cx}\right)\\
&= \sqrt{-1+cx}\sqrt{1+cx}+\tan^{-1}\left(\sqrt{-1+cx}\sqrt{1+cx}\right)
\end{aligned}$$

Mathematica [A] time = 0.03, size = 56, normalized size = 1.40

$$\frac{c^2x^2+\sqrt{c^2x^2-1}\,\tan^{-1}\left(\sqrt{c^2x^2-1}\right)-1}{\sqrt{cx-1}\sqrt{cx+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + c^2*x^2)/(x*Sqrt[-1 + c*x]*Sqrt[1 + c*x]),x]
```

```
[Out] (-1 + c^2*x^2 + Sqrt[-1 + c^2*x^2]*ArcTan[Sqrt[-1 + c^2*x^2]])/(Sqrt[-1 + c
*x]*Sqrt[1 + c*x])
```

IntegrateAlgebraic [A] time = 0.07, size = 60, normalized size = 1.50

$$2\tan^{-1}\left(\frac{\sqrt{cx-1}}{\sqrt{cx+1}}\right)-\frac{2\sqrt{cx-1}}{\sqrt{cx+1}\left(\frac{cx-1}{cx+1}-1\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 + c^2*x^2)/(x*Sqrt[-1 + c*x]*Sqrt[1 + c*x]),x]
```

```
[Out] (-2*Sqrt[-1 + c*x])/(Sqrt[1 + c*x]*(-1 + (-1 + c*x)/(1 + c*x))) + 2*ArcTan[
Sqrt[-1 + c*x]/Sqrt[1 + c*x]]
```

fricas [A] time = 0.89, size = 39, normalized size = 0.98

$$\sqrt{cx+1}\sqrt{cx-1}+2\arctan\left(-cx+\sqrt{cx+1}\sqrt{cx-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^2+1)/x/(c*x-1)^(1/2)/(c*x+1)^(1/2),x, algorithm="fricas")
```

```
[Out] sqrt(c*x + 1)*sqrt(c*x - 1) + 2*arctan(-c*x + sqrt(c*x + 1)*sqrt(c*x - 1))
```

giac [A] time = 0.17, size = 40, normalized size = 1.00

$$\sqrt{cx+1}\sqrt{cx-1}-2\arctan\left(\frac{1}{2}\left(\sqrt{cx+1}-\sqrt{cx-1}\right)^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^2+1)/x/(c*x-1)^(1/2)/(c*x+1)^(1/2),x, algorithm="giac")
```

```
[Out] sqrt(c*x + 1)*sqrt(c*x - 1) - 2*arctan(1/2*(sqrt(c*x + 1) - sqrt(c*x - 1))^
2)
```

maple [A] time = 0.07, size = 53, normalized size = 1.32

$$\frac{\sqrt{cx-1}\,\sqrt{cx+1}\,\left(-\arctan\left(\frac{1}{\sqrt{c^2x^2-1}}\right)+\sqrt{c^2x^2-1}\right)}{\sqrt{c^2x^2-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*x^2+1)/x/(c*x-1)^(1/2)/(c*x+1)^(1/2),x)
```

```
[Out] (c*x-1)^(1/2)*(c*x+1)^(1/2)/(c^2*x^2-1)^(1/2)*((c^2*x^2-1)^(1/2)-arctan(1/(
c^2*x^2-1)^(1/2)))
```

maxima [A] time = 1.35, size = 23, normalized size = 0.58

$$\sqrt{c^2x^2-1}-\arcsin\left(\frac{1}{c|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^2+1)/x/(c*x-1)^(1/2)/(c*x+1)^(1/2),x, algorithm="maxima")
```

```
[Out] sqrt(c^2*x^2 - 1) - arcsin(1/(c*abs(x)))
```

mupad [B] time = 3.65, size = 72, normalized size = 1.80

$$\sqrt{c\,x-1}\,\sqrt{c\,x+1}-\ln\left(\frac{\left(\sqrt{c\,x-1}-\mathrm{i}\right)^2}{\left(\sqrt{c\,x+1}-1\right)^2}+1\right)\,1\mathrm{i}+\ln\left(\frac{\sqrt{c\,x-1}-\mathrm{i}}{\sqrt{c\,x+1}-1}\right)\,1\mathrm{i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*x^2 + 1)/(x*(c*x - 1)^(1/2)*(c*x + 1)^(1/2)),x)
```

```
[Out] log(((c*x - 1)^(1/2) - 1i)/((c*x + 1)^(1/2) - 1))*1i - log(((c*x - 1)^(1/2)
 - 1i)^2/((c*x + 1)^(1/2) - 1)^2 + 1)*1i + (c*x - 1)^(1/2)*(c*x + 1)^(1/2)
```

sympy [C] time = 30.11, size = 148, normalized size = 3.70

$$\frac{G_{6,6}^{6,2}\left(\begin{matrix} -\frac{1}{4},\frac{1}{4} & 0,0,\frac{1}{2},1 \\ -\frac{1}{2},-\frac{1}{4},0,\frac{1}{4},\frac{1}{2},0 & \end{matrix}\middle|\frac{1}{c^2x^2}\right)}{4\pi^{\frac{3}{2}}}-\frac{G_{6,6}^{5,3}\left(\begin{matrix} \frac{3}{4},\frac{5}{4},1 & 1,1,\frac{3}{2} \\ \frac{1}{2},\frac{3}{4},1,\frac{5}{4},\frac{3}{2} & 0 \end{matrix}\middle|\frac{1}{c^2x^2}\right)}{4\pi^{\frac{3}{2}}}+\frac{iG_{6,6}^{2,6}\left(\begin{matrix} -1,-\frac{3}{4},-\frac{1}{2},-\frac{1}{4},0,1 & \\ -\frac{3}{4},-\frac{1}{4} & -1,-\frac{1}{2},-\frac{1}{2},0 \end{matrix}\middle|\frac{e^{2i\pi}}{c^2x^2}\right)}{4\pi^{\frac{3}{2}}}+\frac{iG_{6,6}^{2,6}\left(\begin{matrix} 0,\frac{1}{4},\frac{1}{2},\frac{3}{4},1,1 & \\ \frac{1}{4},\frac{3}{4} & 0,\frac{1}{2},\frac{1}{2},0 \end{matrix}\middle|\frac{e^{2i\pi}}{c^2x^2}\right)}{4\pi^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c**2*x**2+1)/x/(c*x-1)**(1/2)/(c*x+1)**(1/2),x)
```

```
[Out] meijerg(((-1/4, 1/4), (0, 0, 1/2, 1)), ((-1/2, -1/4, 0, 1/4, 1/2, 0), ()),
1/(c**2*x**2))/(4*pi**(3/2)) - meijerg(((3/4, 5/4, 1), (1, 1, 3/2)), ((1/2,
 3/4, 1, 5/4, 3/2), (0,)), 1/(c**2*x**2))/(4*pi**(3/2)) + I*meijerg(((-1, -
3/4, -1/2, -1/4, 0, 1), ()), ((-3/4, -1/4), (-1, -1/2, -1/2, 0)), exp_polar
(2*I*pi)/(c**2*x**2))/(4*pi**(3/2)) + I*meijerg(((0, 1/4, 1/2, 3/4, 1, 1),
()), ((1/4, 3/4), (0, 1/2, 1/2, 0)), exp_polar(2*I*pi)/(c**2*x**2))/(4*pi**
(3/2))
```

3.236 $\int \frac{x^{-\frac{2b^2c+a^2d}{b^2c+a^2d}}\left(c+dx^2\right)}{\sqrt{-a+bx}\sqrt{a+bx}}\,dx$

Optimal. Leaf size=53

$$\sqrt{bx-a}\sqrt{a+bx}\left(\frac{c}{a^2}+\frac{d}{b^2}\right)x^{-\frac{b^2c}{a^2d+b^2c}}$$

Rubi [A] time = 0.09, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 57, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.018, Rules used = {450}

$$\sqrt{bx-a}\sqrt{a+bx}\left(\frac{c}{a^2}+\frac{d}{b^2}\right)x^{-\frac{b^2c}{a^2d+b^2c}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)/(x^((2*b^2*c + a^2*d)/(b^2*c + a^2*d))*Sqrt[-a + b*x]*Sqrt[
a + b*x]),x]
```

```
[Out] ((c/a^2 + d/b^2)*Sqrt[-a + b*x]*Sqrt[a + b*x])/x^((b^2*c)/(b^2*c + a^2*d))
```

Rule 450

```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[(c*(e*x)^
(m + 1)*(a1 + b1*x^(n/2))^(p + 1)*(a2 + b2*x^(n/2))^(p + 1))/(a1*a2*e*(m +
1)), x] /; FreeQ[{a1, b1, a2, b2, c, d, e, m, n, p}, x] && EqQ[non2, n/2] &
& EqQ[a2*b1 + a1*b2, 0] && EqQ[a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1
), 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{x^{-\frac{2b^2c+a^2d}{b^2c+a^2d}}\left(c+dx^2\right)}{\sqrt{-a+bx}\sqrt{a+bx}}\,dx=\left(\frac{c}{a^2}+\frac{d}{b^2}\right)x^{-\frac{b^2c}{b^2c+a^2d}}\sqrt{-a+bx}\sqrt{a+bx}$$

Mathematica [C] time = 0.31, size = 244, normalized size = 4.60

$$\frac{\sqrt{1-\frac{b^2x^2}{a^2}}\left(a^2d+b^2c\right)x^{-\frac{b^2c}{a^2d+b^2c}}\left(b^2dx^2\,{}_2F_1\left(\frac{1}{2},\frac{2da^2+b^2c}{2da^2+2b^2c};\frac{4da^2+3b^2c}{2da^2+2b^2c};\frac{b^2x^2}{a^2}\right)-\left(2a^2d+b^2c\right){}_2F_1\left(\frac{1}{2},-\frac{b^2c}{2\left(da^2+b^2c\right)};\frac{2da^2+b^2c}{2da^2+2b^2c};\frac{b^2x^2}{a^2}\right)\right)}{b^2\sqrt{bx-a}\sqrt{a+bx}\left(2a^2d+b^2c\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)/(x^((2*b^2*c + a^2*d)/(b^2*c + a^2*d))*Sqrt[-a + b*x]
*Sqrt[a + b*x]),x]
```

```
[Out] ((b^2*c + a^2*d)*Sqrt[1 - (b^2*x^2)/a^2]*(-((b^2*c + 2*a^2*d)*Hypergeometri
c2F1[1/2, -1/2*(b^2*c)/(b^2*c + a^2*d), (b^2*c + 2*a^2*d)/(2*b^2*c + 2*a^2*
d), (b^2*x^2)/a^2]) + b^2*d*x^2*Hypergeometric2F1[1/2, (b^2*c + 2*a^2*d)/(2
*b^2*c + 2*a^2*d), (3*b^2*c + 4*a^2*d)/(2*b^2*c + 2*a^2*d), (b^2*x^2)/a^2])
)/(b^2*(b^2*c + 2*a^2*d)*x^((b^2*c)/(b^2*c + a^2*d))*Sqrt[-a + b*x]*Sqrt[a
+ b*x])
```

IntegrateAlgebraic [F] time = 24.09, size = 0, normalized size = 0.00

$$\int \frac{x^{-\frac{2b^2c+a^2d}{b^2c+a^2d}}\left(c+dx^2\right)}{\sqrt{-a+bx}\sqrt{a+bx}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x^2)/(x^((2*b^2*c + a^2*d)/(b^2*c + a^2*d))*Sqrt[
-a + b*x]*Sqrt[a + b*x]),x]
```

```
[Out] Defer[IntegrateAlgebraic][(c + d*x^2)/(x^((2*b^2*c + a^2*d)/(b^2*c + a^2*d)
)*Sqrt[-a + b*x]*Sqrt[a + b*x]), x]
```

fricas [A] time = 0.86, size = 65, normalized size = 1.23

$$\frac{\left(b^2c+a^2d\right)\sqrt{bx+a}\sqrt{bx-a}\,x}{a^2b^2x^{\frac{2\,b^2c+a^2d}{b^2c+a^2d}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(x^((a^2*d+2*b^2*c)/(a^2*d+b^2*c)))/(b*x-a)^(1/2)/(b*x+
a)^(1/2),x, algorithm="fricas")
```

```
[Out] (b^2*c + a^2*d)*sqrt(b*x + a)*sqrt(b*x - a)*x/(a^2*b^2*x^((2*b^2*c + a^2*d)
/(b^2*c + a^2*d)))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{dx^2+c}{\sqrt{bx+a}\sqrt{bx-a}\,x^{\frac{2\,b^2c+a^2d}{b^2c+a^2d}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(x^((a^2*d+2*b^2*c)/(a^2*d+b^2*c)))/(b*x-a)^(1/2)/(b*x+
a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((d*x^2 + c)/(sqrt(b*x + a)*sqrt(b*x - a)*x^((2*b^2*c + a^2*d)/(b^
2*c + a^2*d))), x)
```

maple [A] time = 0.05, size = 66, normalized size = 1.25

$$\frac{\left(a^2d+b^2c\right)\sqrt{bx+a}\,\sqrt{bx-a}\,x\,x^{-\frac{a^2d+2b^2c}{a^2d+b^2c}}}{a^2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)/(x^((a^2*d+2*b^2*c)/(a^2*d+b^2*c)))/(b*x-a)^(1/2)/(b*x+a)^(1/
2),x)
```

```
[Out] x*(a^2*d+b^2*c)*(b*x+a)^(1/2)/b^2/a^2/(x^((a^2*d+2*b^2*c)/(a^2*d+b^2*c)))*(
b*x-a)^(1/2)
```

maxima [A] time = 1.24, size = 79, normalized size = 1.49

$$\frac{\left(b^2c+a^2d\right)\sqrt{bx+a}\,\sqrt{bx-a}\,xe^{\left(-\frac{2\,b^2c\log(x)}{b^2c+a^2d}-\frac{a^2d\log(x)}{b^2c+a^2d}\right)}}{a^2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(x^((a^2*d+2*b^2*c)/(a^2*d+b^2*c)))/(b*x-a)^(1/2)/(b*x+
a)^(1/2),x, algorithm="maxima")
```

```
[Out] (b^2*c + a^2*d)*sqrt(b*x + a)*sqrt(b*x - a)*x*e^(-2*b^2*c*log(x)/(b^2*c + a
^2*d) - a^2*d*log(x)/(b^2*c + a^2*d))/(a^2*b^2)
```

mupad [B] time = 3.27, size = 96, normalized size = 1.81

$$-\frac{\frac{x\left(d\,a^4+c\,a^2\,b^2\right)}{a^2\,b^2}-\frac{x^3\left(d\,a^2\,b^2+c\,b^4\right)}{a^2\,b^2}}{x^{\frac{d\,a^2+2\,c\,b^2}{d\,a^2+c\,b^2}}\,\sqrt{a+b\,x}\,\sqrt{b\,x-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2)/(x^((a^2*d + 2*b^2*c)/(a^2*d + b^2*c))*(a + b*x)^(1/2)*(b*x
 - a)^(1/2)),x)
```

```
[Out] -((x*(a^4*d + a^2*b^2*c))/(a^2*b^2) - (x^3*(b^4*c + a^2*b^2*d))/(a^2*b^2))/
(x^((a^2*d + 2*b^2*c)/(a^2*d + b^2*c))*(a + b*x)^(1/2)*(b*x - a)^(1/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)/(x**((a**2*d+2*b**2*c)/(a**2*d+b**2*c)))/(b*x-a)**(1/2
)/(b*x+a)**(1/2),x)
```

```
[Out] Timed out
```

3.237 $\int \frac{1}{\sqrt{-1-\sqrt{x}}\sqrt{-1+\sqrt{x}}\sqrt{1+x}}\,dx$

Optimal. Leaf size=36

$$\frac{\sqrt{1-x}\sin^{-1}(x)}{\sqrt{-\sqrt{x}-1}\sqrt{\sqrt{x}-1}}$$

Rubi [A] time = 0.02, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.094$, Rules used = {519, 41, 216}

$$\frac{\sqrt{1-x}\sin^{-1}(x)}{\sqrt{-\sqrt{x}-1}\sqrt{\sqrt{x}-1}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[-1 - Sqrt[x]]*Sqrt[-1 + Sqrt[x]]*Sqrt[1 + x]),x]
```

```
[Out] (Sqrt[1 - x]*ArcSin[x])/(Sqrt[-1 - Sqrt[x]]*Sqrt[-1 + Sqrt[x]])
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 519

```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p
_)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_), x_Symbol] :> Dist[((a1 + b1*x^(n/2)
)^FracPart[p]*(a2 + b2*x^(n/2))^FracPart[p])/(a1*a2 + b1*b2*x^n)^FracPart[p
], Int[u*(a1*a2 + b1*b2*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a1, b1, a2,
b2, c, d, n, p, q}, x] && EqQ[non2, n/2] && EqQ[a2*b1 + a1*b2, 0] &&  !(EqQ
[n, 2] && IGtQ[q, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt{-1-\sqrt{x}}\sqrt{-1+\sqrt{x}}\sqrt{1+x}}\,dx &= \frac{\sqrt{1-x}\int \frac{1}{\sqrt{1-x}\sqrt{1+x}}\,dx}{\sqrt{-1-\sqrt{x}}\sqrt{-1+\sqrt{x}}}\\
&= \frac{\sqrt{1-x}\int \frac{1}{\sqrt{1-x^2}}\,dx}{\sqrt{-1-\sqrt{x}}\sqrt{-1+\sqrt{x}}}\\
&= \frac{\sqrt{1-x}\sin^{-1}(x)}{\sqrt{-1-\sqrt{x}}\sqrt{-1+\sqrt{x}}}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 36, normalized size = 1.00

$$\frac{\sqrt{1-x}\sin^{-1}(x)}{\sqrt{-\sqrt{x}-1}\sqrt{\sqrt{x}-1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[-1 - Sqrt[x]]*Sqrt[-1 + Sqrt[x]]*Sqrt[1 + x]),x]
[Out] (Sqrt[1 - x]*ArcSin[x])/(Sqrt[-1 - Sqrt[x]]*Sqrt[-1 + Sqrt[x]])
```

IntegrateAlgebraic [C] time = 7.62, size = 44, normalized size = 1.22

$$-i\log\left(-x+i\sqrt{-\sqrt{x}-1}\sqrt{\sqrt{x}-1}\sqrt{x+1}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(Sqrt[-1 - Sqrt[x]]*Sqrt[-1 + Sqrt[x]]*Sqrt[1 + x]),x]
[Out] (-I)*Log[-x + I*Sqrt[-1 - Sqrt[x]]*Sqrt[-1 + Sqrt[x]]*Sqrt[1 + x]]
```

fricas [C] time = 0.87, size = 69, normalized size = 1.92

$$-i\log\left(\frac{\sqrt{x+1}\sqrt{\sqrt{x}-1}\sqrt{-\sqrt{x}-1}+ix-1}{x}\right)+i\log\left(\frac{\sqrt{x+1}\sqrt{\sqrt{x}-1}\sqrt{-\sqrt{x}-1}-ix-1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)^(1/2)/(-1-x^(1/2))^(1/2)/(-1+x^(1/2))^(1/2),x, algorithm=
"fricas")
[Out] -I*log((sqrt(x + 1)*sqrt(sqrt(x) - 1)*sqrt(-sqrt(x) - 1) + I*x - 1)/x) + I*
log((sqrt(x + 1)*sqrt(sqrt(x) - 1)*sqrt(-sqrt(x) - 1) - I*x - 1)/x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\sqrt{x+1}\sqrt{\sqrt{x}-1}\sqrt{-\sqrt{x}-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)^(1/2)/(-1-x^(1/2))^(1/2)/(-1+x^(1/2))^(1/2),x, algorithm=
"giac")
[Out] integrate(1/(sqrt(x + 1)*sqrt(sqrt(x) - 1)*sqrt(-sqrt(x) - 1)), x)
```

maple [F] time = 0.86, size = 0, normalized size = 0.00

$$\int\frac{1}{\sqrt{x+1}\,\sqrt{-\sqrt{x}-1}\,\sqrt{\sqrt{x}-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x+1)^(1/2)/(-1-x^(1/2))^(1/2)/(-1+x^(1/2))^(1/2),x)
[Out] int(1/(x+1)^(1/2)/(-1-x^(1/2))^(1/2)/(-1+x^(1/2))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x+1}\sqrt{\sqrt{x}-1}\sqrt{-\sqrt{x}-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)^(1/2)/(-1-x^(1/2))^(1/2)/(-1+x^(1/2))^(1/2),x, algorithm=
"maxima")
```

```
[Out] integrate(1/(sqrt(x + 1)*sqrt(sqrt(x) - 1)*sqrt(-sqrt(x) - 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{\sqrt{\sqrt{x}-1}\,\sqrt{-\sqrt{x}-1}\,\sqrt{x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((x^(1/2) - 1)^(1/2)*(- x^(1/2) - 1)^(1/2)*(x + 1)^(1/2)),x)
```

```
[Out] int(1/((x^(1/2) - 1)^(1/2)*(- x^(1/2) - 1)^(1/2)*(x + 1)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-\sqrt{x}-1}\sqrt{\sqrt{x}-1}\sqrt{x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)**(1/2)/(-1-x**(1/2))**(1/2)/(-1+x**(1/2))**(1/2),x)
```

```
[Out] Integral(1/(sqrt(-sqrt(x) - 1)*sqrt(sqrt(x) - 1)*sqrt(x + 1)), x)
```

3.238 $\int \frac{1}{\sqrt{a-b\sqrt{x}}\,\sqrt{a+b\sqrt{x}}\,\sqrt{a^2+b^2x}}\,dx$

Optimal. Leaf size=75

$$-\frac{2\sqrt{a^2-b^2x}\,\tan^{-1}\left(\frac{\sqrt{a^2-b^2x}}{\sqrt{a^2+b^2x}}\right)}{b^2\sqrt{a-b\sqrt{x}}\,\sqrt{a+b\sqrt{x}}}$$

Rubi [A] time = 0.05, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.098, Rules used = {519, 63, 217, 203}

$$-\frac{2\sqrt{a^2-b^2x}\,\tan^{-1}\left(\frac{\sqrt{a^2-b^2x}}{\sqrt{a^2+b^2x}}\right)}{b^2\sqrt{a-b\sqrt{x}}\,\sqrt{a+b\sqrt{x}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a - b*Sqrt[x]]*Sqrt[a + b*Sqrt[x]]*Sqrt[a^2 + b^2*x]),x]
```

```
[Out] (-2*Sqrt[a^2 - b^2*x]*ArcTan[Sqrt[a^2 - b^2*x]/Sqrt[a^2 + b^2*x]])/(b^2*Sqr
t[a - b*Sqrt[x]]*Sqrt[a + b*Sqrt[x]])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 519

```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p
_)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_), x_Symbol] :> Dist[((a1 + b1*x^(n/2)
)^FracPart[p]*(a2 + b2*x^(n/2))^FracPart[p])/(a1*a2 + b1*b2*x^n)^FracPart[p
], Int[u*(a1*a2 + b1*b2*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a1, b1, a2,
b2, c, d, n, p, q}, x] && EqQ[non2, n/2] && EqQ[a2*b1 + a1*b2, 0] &&  !(EqQ
[n, 2] && IGtQ[q, 0])
```

Rubi steps

$$\int \frac{1}{\sqrt{a-b\sqrt{x}}\sqrt{a+b\sqrt{x}}\sqrt{a^2+b^2x}}\,dx = \frac{\sqrt{a^2-b^2x}\int \frac{1}{\sqrt{a^2-b^2x}\sqrt{a^2+b^2x}}\,dx}{\sqrt{a-b\sqrt{x}}\sqrt{a+b\sqrt{x}}}$$

$$= -\frac{\left(2\sqrt{a^2-b^2x}\right)\text{Subst}\left(\int \frac{1}{\sqrt{2a^2-x^2}}\,dx, x, \sqrt{a^2-b^2x}\right)}{b^2\sqrt{a-b\sqrt{x}}\sqrt{a+b\sqrt{x}}}$$

$$= -\frac{\left(2\sqrt{a^2-b^2x}\right)\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \frac{\sqrt{a^2-b^2x}}{\sqrt{a^2+b^2x}}\right)}{b^2\sqrt{a-b\sqrt{x}}\sqrt{a+b\sqrt{x}}}$$

$$= -\frac{2\sqrt{a^2-b^2x}\tan^{-1}\left(\frac{\sqrt{a^2-b^2x}}{\sqrt{a^2+b^2x}}\right)}{b^2\sqrt{a-b\sqrt{x}}\sqrt{a+b\sqrt{x}}}$$

Mathematica [A] time = 0.06, size = 75, normalized size = 1.00

$$-\frac{2\sqrt{a^2-b^2x}\tan^{-1}\left(\frac{\sqrt{a^2-b^2x}}{\sqrt{a^2+b^2x}}\right)}{b^2\sqrt{a-b\sqrt{x}}\sqrt{a+b\sqrt{x}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[a - b*Sqrt[x]]*Sqrt[a + b*Sqrt[x]]*Sqrt[a^2 + b^2*x]),x]
```

```
[Out] (-2*Sqrt[a^2 - b^2*x]*ArcTan[Sqrt[a^2 - b^2*x]/Sqrt[a^2 + b^2*x]])/(b^2*Sqr
t[a - b*Sqrt[x]]*Sqrt[a + b*Sqrt[x]])
```

IntegrateAlgebraic [C] time = 10.95, size = 58, normalized size = 0.77

$$-\frac{i\log\left(\sqrt{a-b\sqrt{x}}\sqrt{a+b\sqrt{x}}\sqrt{a^2+b^2x}+ib^2x\right)}{b^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(Sqrt[a - b*Sqrt[x]]*Sqrt[a + b*Sqrt[x]]*Sqrt[a^2 + b^
2*x]),x]
```

```
[Out] ((-I)*Log[I*b^2*x + Sqrt[a - b*Sqrt[x]]*Sqrt[a + b*Sqrt[x]]*Sqrt[a^2 + b^2*
x]])/b^2
```

fricas [A] time = 0.72, size = 50, normalized size = 0.67

$$-\frac{2\arctan\left(-\frac{a^2-\sqrt{b^2x+a^2}\sqrt{b\sqrt{x}+a}\sqrt{-b\sqrt{x}+a}}{b^2x}\right)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*x+a^2)^(1/2)/(a-b*x^(1/2))^(1/2)/(a+b*x^(1/2))^(1/2),x, al
gorithm="fricas")
```

```
[Out] -2*arctan(-(a^2 - sqrt(b^2*x + a^2)*sqrt(b*sqrt(x) + a)*sqrt(-b*sqrt(x) + a
))/(b^2*x))/b^2
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{b^2x+a^2}\sqrt{b\sqrt{x}+a}\sqrt{-b\sqrt{x}+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*x+a^2)^(1/2)/(a-b*x^(1/2))^(1/2)/(a+b*x^(1/2))^(1/2),x, al
gorithm="giac")
```

```
[Out] integrate(1/(sqrt(b^2*x + a^2)*sqrt(b*sqrt(x) + a)*sqrt(-b*sqrt(x) + a)), x
)
```

maple [F] time = 0.88, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{b^2x+a^2}\,\sqrt{-b\sqrt{x}+a}\,\sqrt{b\sqrt{x}+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b^2*x+a^2)^(1/2)/(a-b*x^(1/2))^(1/2)/(b*x^(1/2)+a)^(1/2),x)
```

```
[Out] int(1/(b^2*x+a^2)^(1/2)/(a-b*x^(1/2))^(1/2)/(b*x^(1/2)+a)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{b^2x+a^2}\sqrt{b\sqrt{x}+a}\sqrt{-b\sqrt{x}+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*x+a^2)^(1/2)/(a-b*x^(1/2))^(1/2)/(a+b*x^(1/2))^(1/2),x, al
gorithm="maxima")
```

```
[Out] integrate(1/(sqrt(b^2*x + a^2)*sqrt(b*sqrt(x) + a)*sqrt(-b*sqrt(x) + a)), x
)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{a+b\,\sqrt{x}}\,\sqrt{a-b\,\sqrt{x}}\,\sqrt{a^2+x\,b^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^(1/2))^(1/2)*(a - b*x^(1/2))^(1/2)*(b^2*x + a^2)^(1/2)),x)
```

```
[Out] int(1/((a + b*x^(1/2))^(1/2)*(a - b*x^(1/2))^(1/2)*(b^2*x + a^2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a-b\sqrt{x}}\,\sqrt{a+b\sqrt{x}}\,\sqrt{a^2+b^2x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b**2*x+a**2)**(1/2)/(a-b*x**(1/2))**(1/2)/(a+b*x**(1/2))**(1/2
),x)
```

```
[Out] Integral(1/(sqrt(a - b*sqrt(x))*sqrt(a + b*sqrt(x))*sqrt(a**2 + b**2*x)), x
)
```

3.239

$$\int \left(a-bx^{n/2}\right)^p \left(a+bx^{n/2}\right)^p \left(\frac{a^2d(1+p)}{b^2\left(1+\frac{-1-2n-np}{n}\right)}+dx^n\right)^{\frac{-1-2n-np}{n}} dx$$

Optimal. Leaf size=96

$$-\frac{b^2x(np+n+1)\left(a-bx^{n/2}\right)^{p+1}\left(a+bx^{n/2}\right)^{p+1}\left(dx^n-\frac{a^2dn(p+1)}{b^2(np+n+1)}\right)^{-\frac{np+n+1}{n}}}{a^4dn(p+1)}$$

Rubi [A] time = 0.12, antiderivative size = 104, normalized size of antiderivative = 1.08, number of steps used = 2, number of rules used = 2, integrand size = 76, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.026, Rules used = {519, 381}

$$-\frac{b^2x(np+n+1)\left(a^2-b^2x^n\right)\left(a-bx^{n/2}\right)^p\left(a+bx^{n/2}\right)^p\left(dx^n-\frac{a^2dn(p+1)}{b^2(np+n+1)}\right)^{-\frac{np+n+1}{n}}}{a^4dn(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*x^(n/2))^p*(a + b*x^(n/2))^p*((a^2*d*(1 + p))/(b^2*(1 + (-1 - 2*
n - n*p)/n)) + d*x^n)^((-1 - 2*n - n*p)/n),x]
```

```
[Out] -((b^2*(1 + n + n*p)*x*(a - b*x^(n/2))^p*(a + b*x^(n/2))^p*(a^2 - b^2*x^n))
/(a^4*d*n*(1 + p)*(-((a^2*d*n*(1 + p))/(b^2*(1 + n + n*p))) + d*x^n)^((1 +
n + n*p)/n)))
```

Rule 381

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c), x] /; FreeQ[{a,
b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] &&
EqQ[a*d*(p + 1) + b*c*(q + 1), 0]
```

Rule 519

```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p
_)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_), x_Symbol] :> Dist[((a1 + b1*x^(n/2)
)^FracPart[p]*(a2 + b2*x^(n/2))^FracPart[p])/(a1*a2 + b1*b2*x^n)^FracPart[p
], Int[u*(a1*a2 + b1*b2*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a1, b1, a2,
b2, c, d, n, p, q}, x] && EqQ[non2, n/2] && EqQ[a2*b1 + a1*b2, 0] && !(EqQ
[n, 2] && IGtQ[q, 0])
```

Rubi steps

$$\int \left(a-bx^{n/2}\right)^p \left(a+bx^{n/2}\right)^p \left(\frac{a^2d(1+p)}{b^2\left(1+\frac{-1-2n-np}{n}\right)}+dx^n\right)^{\frac{-1-2n-np}{n}} dx = \left(\left(a-bx^{n/2}\right)^p\left(a+bx^{n/2}\right)^p\left(a^2-b^2x^n\right)^{-p}\right)\int$$

$$= -\frac{b^2(1+n+np)x\left(a-bx^{n/2}\right)^p\left(a+bx^{n/2}\right)^p}{a^4dn(1}$$

Mathematica [A] time = 0.39, size = 103, normalized size = 1.07

$$-\frac{b^2x(np+n+1)\left(a^2-b^2x^n\right)\left(a-bx^{n/2}\right)^p\left(a+bx^{n/2}\right)^p\left(d\left(x^n-\frac{a^2n(p+1)}{b^2(np+n+1)}\right)\right)^{-\frac{np+n+1}{n}}}{a^4dn(p+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b*x^(n/2))^p*(a + b*x^(n/2))^p*((a^2*d*(1 + p))/(b^2*(1 + (-
1 - 2*n - n*p)/n)) + d*x^n)^((-1 - 2*n - n*p)/n),x]
```

```
[Out] -((b^2*(1 + n + n*p)*x*(a - b*x^(n/2))^p*(a + b*x^(n/2))^p*(a^2 - b^2*x^n))
/(a^4*d*n*(1 + p)*(d*(-((a^2*n*(1 + p))/(b^2*(1 + n + n*p))) + x^n))^((1 +
n + n*p)/n)))
```

IntegrateAlgebraic [F] time = 1.41, size = 0, normalized size = 0.00

$$\int \left(a-bx^{n/2}\right)^p\left(a+bx^{n/2}\right)^p\left(\frac{a^2d(1+p)}{b^2\left(1+\frac{-1-2n-np}{n}\right)}+dx^n\right)^{\frac{-1-2n-np}{n}}dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a - b*x^(n/2))^p*(a + b*x^(n/2))^p*((a^2*d*(1 + p))/(b^
2*(1 + (-1 - 2*n - n*p)/n)) + d*x^n)^((-1 - 2*n - n*p)/n),x]
```

```
[Out] Defer[IntegrateAlgebraic][(a - b*x^(n/2))^p*(a + b*x^(n/2))^p*((a^2*d*(1 +
p))/(b^2*(1 + (-1 - 2*n - n*p)/n)) + d*x^n)^((-1 - 2*n - n*p)/n), x]
```

fricas [A] time = 0.85, size = 180, normalized size = 1.88

$$\frac{\left(\left(b^4np+b^4n+b^4\right)xx^{2n}-\left(2a^2b^2np+2a^2b^2n+a^2b^2\right)xx^n+\left(a^4np+a^4n\right)x\right)\left(bx^{\frac{1}{2}n}+a\right)^p\left(-bx^{\frac{1}{2}n}+a\right)^p}{\left(a^4np+a^4n\right)\left(-\frac{a^2dnp+a^2dn-\left(b^2dnp+b^2dn+b^2d\right)x^n}{b^2np+b^2n+b^2}\right)^{\frac{np+2n+1}{n}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-b*x^(1/2*n))^p*(a+b*x^(1/2*n))^p*(a^2*d*(1+p)/b^2/(1+(-n*p-2*n
-1)/n)+d*x^n)^((-n*p-2*n-1)/n),x, algorithm="fricas")
```

```
[Out] ((b^4*n*p + b^4*n + b^4)*x*x^(2*n) - (2*a^2*b^2*n*p + 2*a^2*b^2*n + a^2*b^2
)*x*x^n + (a^4*n*p + a^4*n)*x)*(b*x^(1/2*n) + a)^p*(-b*x^(1/2*n) + a)^p/((a
^4*n*p + a^4*n)*(-(a^2*d*n*p + a^2*d*n - (b^2*d*n*p + b^2*d*n + b^2*d)*x^n)
/(b^2*n*p + b^2*n + b^2))^((n*p + 2*n + 1)/n))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^{\frac{1}{2}n}+a\right)^p\left(-bx^{\frac{1}{2}n}+a\right)^p}{\left(dx^n-\frac{a^2d(p+1)}{b^2\left(\frac{np+2n+1}{n}-1\right)}\right)^{\frac{np+2n+1}{n}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-b*x^(1/2*n))^p*(a+b*x^(1/2*n))^p*(a^2*d*(1+p)/b^2/(1+(-n*p-2*n
-1)/n)+d*x^n)^((-n*p-2*n-1)/n),x, algorithm="giac")
```

```
[Out] integrate((b*x^(1/2*n) + a)^p*(-b*x^(1/2*n) + a)^p/(d*x^n - a^2*d*(p + 1)/(
b^2*((n*p + 2*n + 1)/n - 1)))^((n*p + 2*n + 1)/n), x)
```

maple [F] time = 2.17, size = 0, normalized size = 0.00

$$\int \left(-b\,x^{\frac{n}{2}}+a\right)^{p} \left(b\,x^{\frac{n}{2}}+a\right)^{p} \left(d\,x^{n}+\frac{\left(p+1\right) a^{2} d}{\left(\frac{-np-2n-1}{n}+1\right) b^{2}}\right)^{\frac{-np-2n-1}{n}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a-b*x^(1/2*n))^p*(a+b*x^(1/2*n))^p*(a^2*d*(p+1)/b^2/(1+(-n*p-2*n-1)/n)
+d*x^n)^((-n*p-2*n-1)/n),x)
```

```
[Out] int((a-b*x^(1/2*n))^p*(a+b*x^(1/2*n))^p*(a^2*d*(p+1)/b^2/(1+(-n*p-2*n-1)/n)
+d*x^n)^((-n*p-2*n-1)/n),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^{\frac{1}{2}n}+a\right)^{p} \left(-bx^{\frac{1}{2}n}+a\right)^{p}}{\left(dx^{n}-\frac{a^{2}d\left(p+1\right)}{b^{2}\left(\frac{np+2n+1}{n}-1\right)}\right)^{\frac{np+2n+1}{n}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-b*x^(1/2*n))^p*(a+b*x^(1/2*n))^p*(a^2*d*(1+p)/b^2/(1+(-n*p-2*n
-1)/n)+d*x^n)^((-n*p-2*n-1)/n),x, algorithm="maxima")
```

```
[Out] integrate((b*x^(1/2*n) + a)^p*(-b*x^(1/2*n) + a)^p/(d*x^n - a^2*d*(p + 1)/(
b^2*((n*p + 2*n + 1)/n - 1)))^((n*p + 2*n + 1)/n), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+b\,x^{n/2}\right)^{p}\,\left(a-b\,x^{n/2}\right)^{p}}{\left(d\,x^{n}-\frac{a^{2}\,d\,\left(p+1\right)}{b^{2}\,\left(\frac{2\,n+n\,p+1}{n}-1\right)}\right)^{\frac{2\,n+n\,p+1}{n}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^(n/2))^p*(a - b*x^(n/2))^p)/(d*x^n - (a^2*d*(p + 1))/(b^2*((2
*n + n*p + 1)/n - 1)))^((2*n + n*p + 1)/n),x)
```

```
[Out] int(((a + b*x^(n/2))^p*(a - b*x^(n/2))^p)/(d*x^n - (a^2*d*(p + 1))/(b^2*((2
*n + n*p + 1)/n - 1)))^((2*n + n*p + 1)/n), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-b*x**(1/2*n))**p*(a+b*x**(1/2*n))**p*(a**2*d*(1+p)/b**2/(1+(-n
*p-2*n-1)/n)+d*x**n)**((-n*p-2*n-1)/n),x)
```

```
[Out] Timed out
```

Chapter 4

Appendix

Local contents

4.1 Download section

The following zip files contain the raw integrals used in this test.

This is a subset of Rubi test suite thanks to Albert Rich, which includes only the algebraic integrals with elementray optimal antiderivatives. It also includes a subset of a test file provided thanks to Sam Blake.

Mathematica format Mathematica_syntax_CAS_integration_elementary_version.zip

Maple and Mupad format Maple_syntax_CAS_integration_elementary_version.zip

Sympy format SYMPY_syntax_CAS_integration_elementary_version.zip

Sage math format SAGE_syntax_CAS_integration_elementary_version.zip

4.2 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.2.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] :=
  If[ExpnType[result]<=ExpnType[optimal],
    If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
      If[LeafCount[result]<=2*LeafCount[optimal],
        "A",
      "B"],
    "C"],
  If[FreeQ[result,Integrate] && FreeQ[result,Int],
    "C",
  "F"]]


(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
```

```
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
        Exp,Log,
        Sin,Cos,Tan,Cot,Sec,Csc,
        ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
        Sinh,Cosh,Tanh,Coth,Sech,Csch,
        ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
        Erf, Erfc, Erfi,
        FresnelS, FresnelC,
        ExpIntegralE, ExpIntegralEi, LogIntegral,
        SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
        Gamma, LogGamma, PolyGamma,
        Zeta, PolyLog, ProductLog,
        EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
```

```
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.2.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems

GradeAntiderivative := proc(result,optimal)
local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal,
   debug:=false;

      leaf_count_result:=leafcount(result);
      #do NOT call ExpnType() if leaf size is too large. Recursion problem
      if leaf_count_result > 500000 then
          return "B";
      fi;

      leaf_count_optimal:=leafcount(optimal);

      ExpnType_result:=ExpnType(result);
      ExpnType_optimal:=ExpnType(optimal);

      if debug then
         print("ExpnType_result",ExpnType_result," ExpnType_optimal=",
   ExpnType_optimal);
      fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

  #This check below actually is not needed, since I only
  #call this grading only for passed integrals. i.e. I check
  #for "F" before calling this. But no harm of keeping it here.
  #just in case.


  if not type(result,freeof('int')) then
     return "F";
  end if;


  if ExpnType_result<=ExpnType_optimal then
      if debug then
           print("ExpnType_result<=ExpnType_optimal");
      fi;
```

```
      if is_contains_complex(result) then
         if is_contains_complex(optimal) then
            if debug then
               print("both result and optimal complex");
            fi;
            #both result and optimal complex
            if leaf_count_result<=2*leaf_count_optimal then
               return "A";
            else
               return "B";
            end if
         else #result contains complex but optimal is not
            if debug then
               print("result contains complex but optimal is not");
            fi;
            return "C";
         end if
      else # result do not contain complex
           # this assumes optimal do not as well
           if debug then
               print("result do not contain complex, this assumes optimal do
   not as well");
           fi;
           if leaf_count_result<=2*leaf_count_optimal then
              if debug then
                 print("leaf_count_result<=2*leaf_count_optimal");
              fi;
              return "A";
           else
              if debug then
                 print("leaf_count_result>2*leaf_count_optimal");
              fi;
              return "B";
           end if
      end if
  else #ExpnType(result) > ExpnType(optimal)
      if debug then
         print("ExpnType(result) > ExpnType(optimal)");
      fi;
      return "C";
  end if

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
```

```
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:
```

```
AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.2.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
```

```
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
           return True
        else:
           return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn
   )))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,
   ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or
   type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,
   expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType
   ,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
```

```
def grade_antiderivative(result,optimal):

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        return "F"

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
   well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```

4.2.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
```

```
    else:
        return False

def is_elementary_function(func):
    debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    debug=False
    if debug: print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','
    sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e',
           'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','
    hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in
    sagemath

def is_atom(expn):

    debug=False
    if debug: print ("Enter is_atom")


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-
    sagemath-equivalent-to-atomic-type-in-maple/
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
```

```
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens")
    :
            return expn in expn.parent().base_ring() or expn in expn.parent().
    gens()
        return False

    except AttributeError as error:
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],
    Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.
    args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer
    )
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],
    Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],
    Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(
    expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()
    [1])) #max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #
    isinstance(expn,Add) or isinstance(expn,Mul)
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn
    .func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(
    expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #
    is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
```

```
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    if debug: print ("Enter grade_antiderivative for sagemath")

    leaf_count_result  = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    if debug: print ("leaf_count_result=", leaf_count_result, "
    leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",
    expnType_optimal)

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
    well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```